TPWA for 2-meson photoproduction (preliminary!!) - I

- **5-dimensional** phase-space:

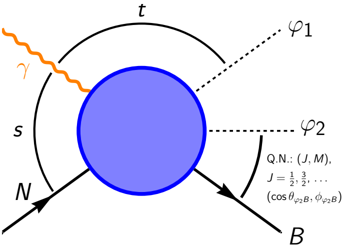
$$\left\{s, t, m^2_{\varphi_2 B}, \Omega_{\varphi_2 B} = (\theta_{\varphi_2 B}, \phi_{\varphi_2 B})\right\}$$

- 8 helicity-configurations:

$$\lambda_\gamma = \pm 1,\ \lambda_N = \pm\tfrac{1}{2},\ \lambda_B = \pm\tfrac{1}{2}.$$

- $\varphi_2 B$ angular-momentum Q.N.'s:

$$J = \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2}, \ldots, \infty \text{ and } M = -J, \ldots, +J.$$



[cf. Talk by V. Mathieu (Monday)]

TPWA for 2-meson photoproduction (preliminary!!) - I

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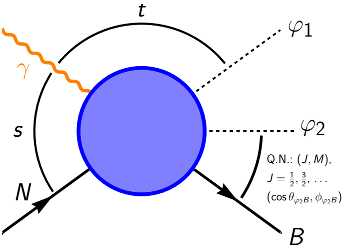

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⇒ TPWA:

$$\mathcal{A}_{\lambda_\gamma;\lambda_N \lambda_B} \left(s, t, m_{\varphi_2 B}^2, \Omega_{\varphi_2 B} \right) = \sum_{J=\frac{1}{2},\frac{3}{2},\ldots}^{J_{\mathrm{max}}} \sum_{M=-J}^{+J} \mathcal{T}_{\lambda_\gamma, M;\lambda_N \lambda_B}^{(J)} \left(s, t, m_{\varphi_2 B}^2 \right) D_{M,-\lambda_B}^{*J} \left(\Omega_{\varphi_2 B} \right).$$

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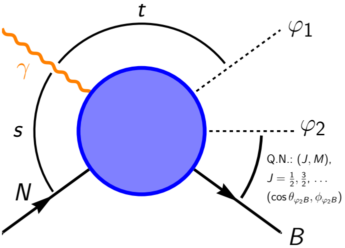

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$*$) Assume: 8 functions $\left| \mathcal{A}_{\lambda_\gamma; \lambda_N \lambda_B} \right|^2$ uniquely fixed from **8 pol.-measurements**:

$$\left\{ \left| \mathcal{A}_{+;++} \right|^2, \left| \mathcal{A}_{+;+-} \right|^2, \ldots, \left| \mathcal{A}_{-;--} \right|^2 \right\} \Leftrightarrow \left\{ I_0, P_z, P_{z'}, \mathcal{O}_{zz'}, I^{\odot}, P_z^{\odot}, P_{z'}^{\odot}, \mathcal{O}_{zz'}^{\odot} \right\}.$$

$\rightarrow$ Forget dependence on $(\lambda_\gamma; \lambda_N \lambda_B)$ from now on (fix $\lambda_B = 1/2$ in D-fct.)

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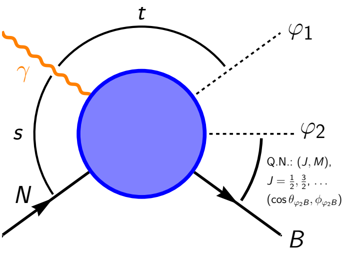

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$\Rightarrow$ Consider discrete partial-wave ambiguities for the squared-modulus function

$$\mathcal{A}\left(\Omega_{\varphi_2 B}\right) \mathcal{A}^*\left(\Omega_{\varphi_2 B}\right).$$

TPWA for 2-meson photoproduction (preliminary!!) - II

*) Introduce **new angular variables** $u := e^{i\theta_{\varphi_2 B}}$ and $v := e^{i\phi_{\varphi_2 B}}$

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I: $[u - u_n(v)]$ (i.e.: $u_n(v) \rightarrow u_n(v)$) or II: $\left[\frac{1}{u} - u_n^*(v)\right]$ $\left(\text{i.e.: } u_n(v) \rightarrow \frac{1}{u_n^*(v)}\right)$.

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⇒ 2-meson TPWA is **much better constrained, most likely unique!**

cf.: [I. S. Stefanescu, J. Math. Phys. **26** (9), 2141-2160 (1985)]
& [W. A. Smith *et al.* [JPAC], Phys. Rev. D **108**, no.7, 076001 (2023)]