Discussion Session
Exploring Excited Nucleons

E45

PHY-2012826
PHY-2310034

Philip Cole Lamar University

June 20, 2024

The Panel

Izabela Ciepal – Institute of Nuclear Physics PAS, Kraków (Timelike Regime) [HADES]

Phil Cole – Lamar University, Chair of Session [JLab + E45 + BGOOD]

Ralf Gothe – University of South Carolina (electroproduction of baryon resonances) [JLab]

Shinhyung Kim – Kyungpook National University (πN -> ππN) [J-PARC-E45]

Yannick Wunderlich – University of Bonn (Amplitude Analysis)

There will be a short presentation on the exploring the production of N*s
with pion and electron beams, followed by a panel discussion and open forum.

Why this Session (see Abstract)

- **The focus of the session will be on the spectrum and structure of nucleon resonances (N*), as revealed through N* electro-excitation amplitudes.** Such fundamental information on the mechanisms of strong-coupling QCD is crucial to validating any proposed solution to the theory and explaining the emergence of mass.

- **Precise data from both electron and pion beams are necessary for developing robust approaches for amplitude analyses, capable of delivering sound results for N* quantum numbers and excitation amplitudes over a broad range of q^2 to be interpreted starting from the QCD Lagrangian.**

- **Our "workshop" will foster the synergies necessary to move this effort forward in a timely way.**

Why this Session (see Abstract)

- **We will focus on how to coordinate the physics of performing dynamical coupled-channel analyses on the reaction data of πN and $\gamma^{(*)}N \to \pi\pi N$ and $K\Lambda/K\Sigma$.**
- **This will require a full amplitude and partial-wave analysis to constrain the pion- and electron-induced N* production.**
- **We will also discuss spanning the gap of the space-like ($q^2 < 0$) and time-like ($q^2 > 0$) divide anchored by the photoproduction data at $q^2 = 0$.**

(Context for this proposal is provided in Exploring the production of N*s with pion and electron beams, published December 2022 in the quarterly online journal *The Innovation Platform*.)

Getting the word out for why N*s to Policy Makers

NUCLEAR, PARTICLE, & ASTRO PHYSICS | PROFILE

How CLAS12 and BGOOD are exploring nucleon excitations

Professors Philip L Cole (Lamar University), Kyungseon Joo (University of Connecticut), and Hartmut Schmieden (University of Bonn) are exploring the properties of subatomic matter through the excited

quarks, in order of mass: up, down, strange, charm, bottom, and top (*u*,*d*,*s*,*c*,*b*,*t*).

right. The Hall D complex is located at the opposite end of the facility in a cluster of rectangular buildings on the upper left. Image courtesy of Jefferson Lab

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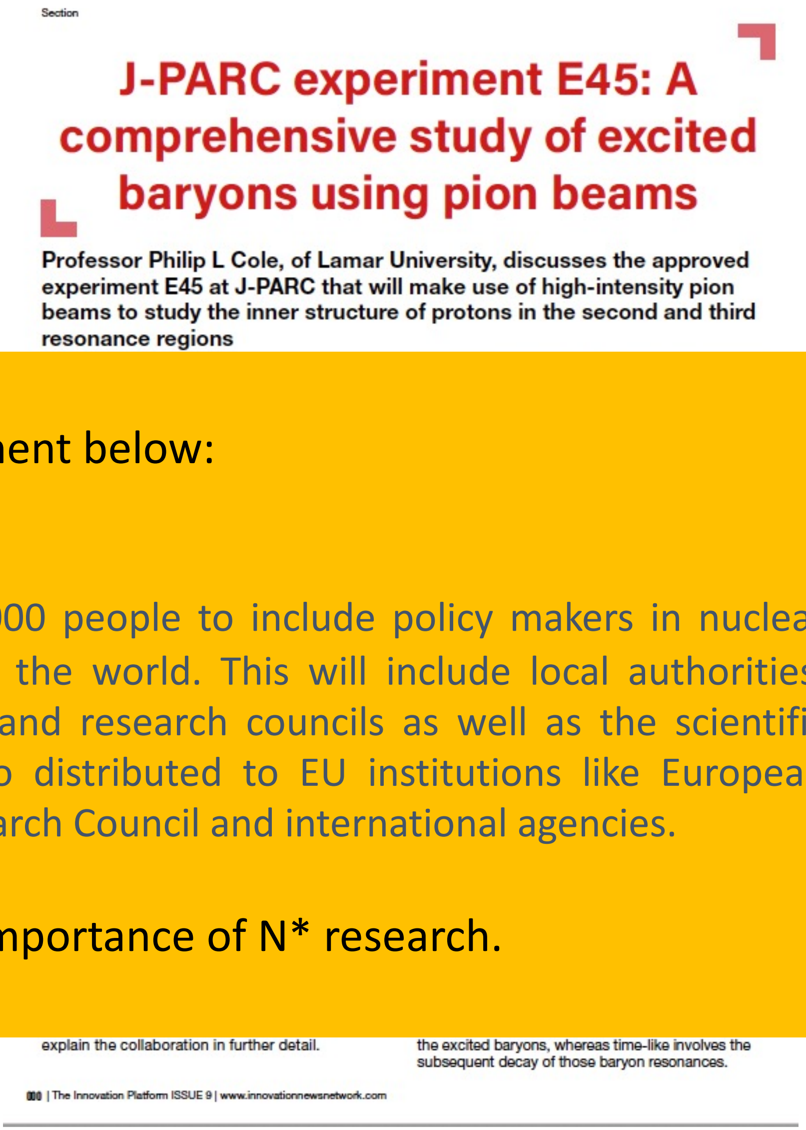

Section

J-PARC experiment E45: A comprehensive study of excited baryons using pion beams

Professor Philip L Cole, of Lamar University, discusses the approved experiment E45 at J-PARC that will make use of high-intensity pion beams to study the inner structure of protons in the second and third resonance regions

explain the collaboration in further detail.

the excited baryons, whereas time-like involves the subsequent decay of those baryon resonances.

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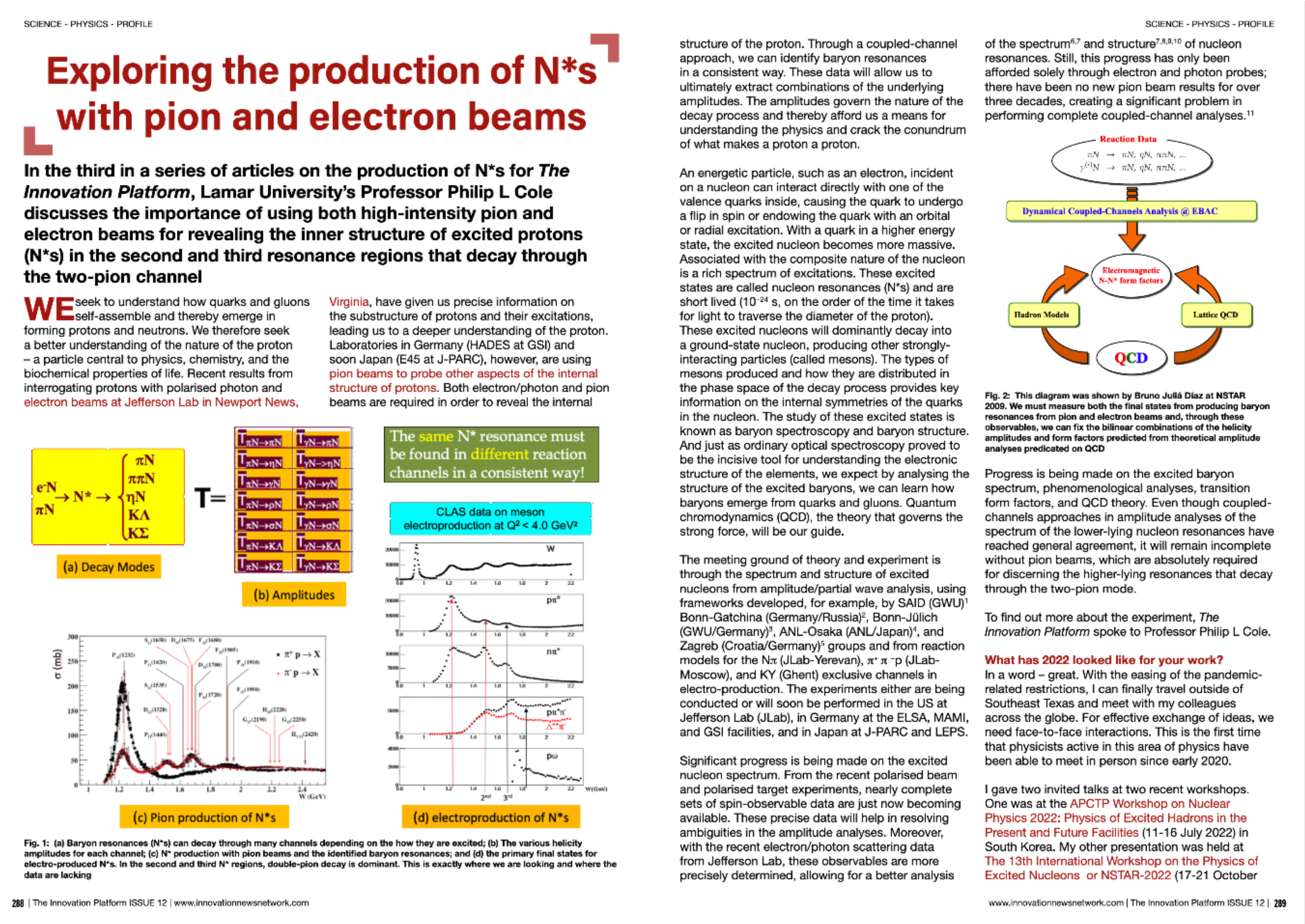
The purpose of the articles was guided by the statement below:

The publication will be distributed digitally to over 200 000 people to include policy makers in nuclear physics in Europe, the US, Canada, Asia and the rest of the world. This will include local authorities, regional governments, government agencies, ministries and research councils as well as the scientific community, industry and academic institutions. It is also distributed to EU institutions like European Parliament, European Commission and the European Research Council and international agencies.

We seek to get the word out on the timeliness and importance of N* research.

Exploring the production of N*s with pion and electron beams
This article stresses the need for **Coupled–Channel Analysis**



Exploring the production of N*s with pion and electron beams

In the third in a series of articles on the production of N*s for *The Innovation Platform*, Lamar University's Professor Philip L Cole discusses the importance of using both high-intensity pion and electron beams for revealing the inner structure of excited protons (N*s) in the second and third resonance regions that decay through the two-pion channel

WE seek to understand how quarks and gluons self-assemble and thereby emerge in forming protons and neutrons. We therefore seek a better understanding of the nature of the proton – a particle central to physics, chemistry, and the biochemical properties of life. Recent results from interrogating protons with polarised photon and electron beams at Jefferson Lab in Newport News, Virginia, have given us precise information on the substructure of protons and their excitations, leading us to a deeper understanding of the proton. Laboratories in Germany (HADES at GSI) and soon Japan (E45 at J-PARC), however, are using pion beams to probe other aspects of the internal structure of protons. Both electron/photon and pion beams are required in order to reveal the internal structure of the proton. Through a coupled-channel approach, we can identify baryon resonances in a consistent way. These data will allow us to ultimately extract combinations of the underlying amplitudes. The amplitudes govern the nature of the decay process and thereby afford us a means for understanding the physics and crack the conundrum of what makes a proton a proton.

The same N* resonance must be found in different reaction channels in a consistent way!

Fig. 1: (a) Baryon resonances (N*s) can decay through many channels depending on the how they are excited; (b) The various helicity amplitudes for each channel; (c) N* production with pion beams and the identified baryon resonances; and (d) the primary final states for electro-produced N*s. In the second and third N* regions, double-pion decay is dominant. This is exactly where we are looking and where the data are lacking

An energetic particle, such as an electron, incident on a nucleon can interact directly with one of the valence quarks inside, causing the quark to undergo a flip in spin or endowing the quark with an orbital or radial excitation. With a quark in a higher energy state, the excited nucleon becomes more massive. Associated with the composite nature of the nucleon is a rich spectrum of excitations. These excited states are called nucleon resonances (N*s) and are short lived (10^{-24} s, on the order of the time it takes for light to traverse the diameter of the proton). These excited nucleons will dominantly decay into a ground-state nucleon, producing other strongly-interacting particles (called mesons). The types of mesons produced and how they are distributed in the phase space of the decay process provides key information on the internal symmetries of the quarks in the nucleon. The study of these excited states is known as baryon spectroscopy and baryon structure. And just as ordinary optical spectroscopy proved to be the incisive tool for understanding the electronic structure of the elements, we expect by analysing the structure of the excited baryons, we can learn how baryons emerge from quarks and gluons. Quantum chromodynamics (QCD), the theory that governs the strong force, will be our guide.

The meeting ground of theory and experiment is through the spectrum and structure of excited nucleons from amplitude/partial wave analysis, using frameworks developed, for example, by SAID (GWU)[1] Bonn-Gatchina (Germany/Russia)[2], Bonn-Jülich (GWU/Germany)[3], ANL-Osaka (ANL/Japan)[4], and Zagreb (Croatia/Germany)[5] groups and from reaction models for the $N\pi$ (JLab-Yerevan), $\pi^+ \pi^- p$ (JLab-Moscow), and KY (Ghent) exclusive channels in electro-production. The experiments either are being conducted or will soon be performed in the US at Jefferson Lab (JLab), in Germany at the ELSA, MAMI, and GSI facilities, and in Japan at J-PARC and LEPS.

Significant progress is being made on the excited nucleon spectrum. From the recent polarised beam and polarised target experiments, nearly complete sets of spin-observable data are just now becoming available. These precise data will help in resolving ambiguities in the amplitude analyses. Moreover, with the recent electron/photon scattering data from Jefferson Lab, these observables are more precisely determined, allowing for a better analysis of the spectrum[6,7] and structure[7,8,9,10] of nucleon resonances. Still, this progress has only been afforded solely through electron and photon probes; there have been no new pion beam results for over three decades, creating a significant problem in performing complete coupled-channel analyses.[11]

Fig. 2: This diagram was shown by Bruno Juliá Díaz at NSTAR 2009. We must measure both the final states from producing baryon resonances from pion and electron beams and, through these observables, we can fix the bilinear combinations of the helicity amplitudes and form factors predicted from theoretical amplitude analyses predicated on QCD

Progress is being made on the excited baryon spectrum, phenomenological analyses, transition form factors, and QCD theory. Even though coupled-channels approaches in amplitude analyses of the spectrum of the lower-lying nucleon resonances have reached general agreement, it will remain incomplete without pion beams, which are absolutely required for discerning the higher-lying resonances that decay through the two-pion mode.

To find out more about the experiment, *The Innovation Platform* spoke to Professor Philip L Cole.

What has 2022 looked like for your work?

In a word – great. With the easing of the pandemic-related restrictions, I can finally travel outside of Southeast Texas and meet with my colleagues across the globe. For effective exchange of ideas, we need face-to-face interactions. This is the first time that physicists active in this area of physics have been able to meet in person since early 2020.

I gave two invited talks at two recent workshops. One was at the APCTP Workshop on Nuclear Physics 2022: Physics of Excited Hadrons in the Present and Future Facilities (11-16 July 2022) in South Korea. My other presentation was held at The 13th International Workshop on the Physics of Excited Nucleons or NSTAR-2022 (17-21 October

Issue 12 (December 2022) pages 288 to 292

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Exploring the p… electron beams
This article str… nnel Analysis

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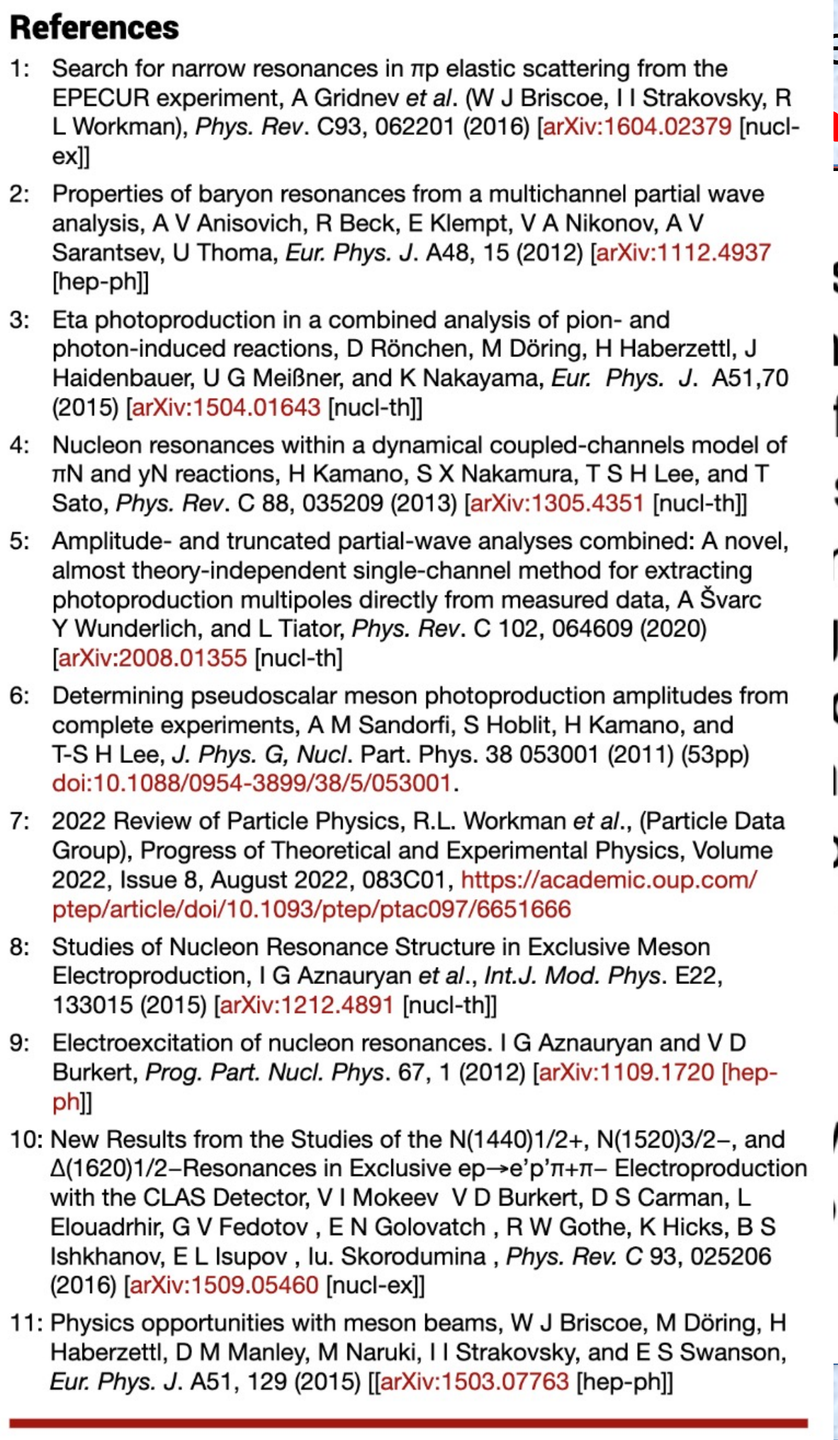

References

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2: Properties of baryon resonances from a multichannel partial wave analysis, A V Anisovich, R Beck, E Klempt, V A Nikonov, A V Sarantsev, U Thoma, *Eur. Phys. J*. A48, 15 (2012) [arXiv:1112.4937 [hep-ph]]

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7: 2022 Review of Particle Physics, R.L. Workman *et al*., (Particle Data Group), Progress of Theoretical and Experimental Physics, Volume 2022, Issue 8, August 2022, 083C01, https://academic.oup.com/ptep/article/doi/10.1093/ptep/ptac097/6651666

8: Studies of Nucleon Resonance Structure in Exclusive Meson Electroproduction, I G Aznauryan *et al*., *Int.J. Mod. Phys*. E22, 133015 (2015) [arXiv:1212.4891 [nucl-th]]

9: Electroexcitation of nucleon resonances. I G Aznauryan and V D Burkert, *Prog. Part. Nucl. Phys*. 67, 1 (2012) [arXiv:1109.1720 [hep-ph]]

10: New Results from the Studies of the N(1440)1/2+, N(1520)3/2−, and Δ(1620)1/2−Resonances in Exclusive ep→e′p′π+π− Electroproduction with the CLAS Detector, V I Mokeev V D Burkert, D S Carman, L Elouadrhir, G V Fedotov , E N Golovatch , R W Gothe, K Hicks, B S Ishkhanov, E L Isupov , Iu. Skorodumina , *Phys. Rev. C* 93, 025206 (2016) [arXiv:1509.05460 [nucl-ex]]

11: Physics opportunities with meson beams, W J Briscoe, M Döring, H Haberzettl, D M Manley, M Naruki, I I Strakovsky, and E S Swanson, *Eur. Phys. J*. A51, 129 (2015) [[arXiv:1503.07763 [hep-ph]]

8 to 292

Over 40 people attended on a Sunday morning for our workshop in Hawaii (first day and the sun was shining and the water was nice)

Exploring Excited Nucleons with Meson, Electron, and Photon Beams

The focus of the **workshop** will be on the spectrum and structure of nucleon resonances (N*), as revealed through N* electroexcitation amplitudes. Such fundamental information on the mechanisms of strong-coupling QCD is crucial to validating any proposed solution to the theory and explaining the emergence of mass. Precise data from both electron and pion beams are necessary for developing robust approaches for amplitude analyses, capable of delivering sound results for N* quantum numbers and excitation amplitudes over a broad range of q^2 to be interpreted starting from the QCD Lagrangian.

Co-Chairs: Philip Cole (Lamar University)
Hiroyuki Sako (Japan Atomic Energy Agency)

Workshop 1WEA/1WEB (9 am to 12:30 pm)
Sunday, November 26, 2023
Hilton Waikoloa Village – Queens 4

The 2023 Fall Meeting of the Division of Nuclear Physics of the American Physical Society and the Physical Society of Japan

Craig Roberts (Institute for Nonperturbative Physics, Nanjing University)
Insights into Strong QCD from Baryon Structure

Tetsuo Hyodo (Tokyo Metropolitan University)
Compositeness of Hadrons and its Application to Baryon Resonances

Patrick Achenbach (Jefferson Lab)
Studies of Hadron Form Factors

Toshikazu Hashimoto (Research Center for Nuclear Physics, Osaka Univ.)
Studies of Baryon Resonances with Meson Photoproduction for the LEPS2/BGOegg Experiment at SPring-8

Gilberto Ramalho (Soongsil University and OMEG Institute)
Exposing Nucleon Resonance Structure using Electron and Pion Beams

Shinhyung Kim (Korea University)
N* Spectroscopy in 3-Body Hadronic Reactions with HypTPC at J-PARC

Pion and electron beams are crucial for Coupled-Channels Analysis

This was
presented at
NSTAR-2009
Bruno Julia Diaz
Beijing, PRC

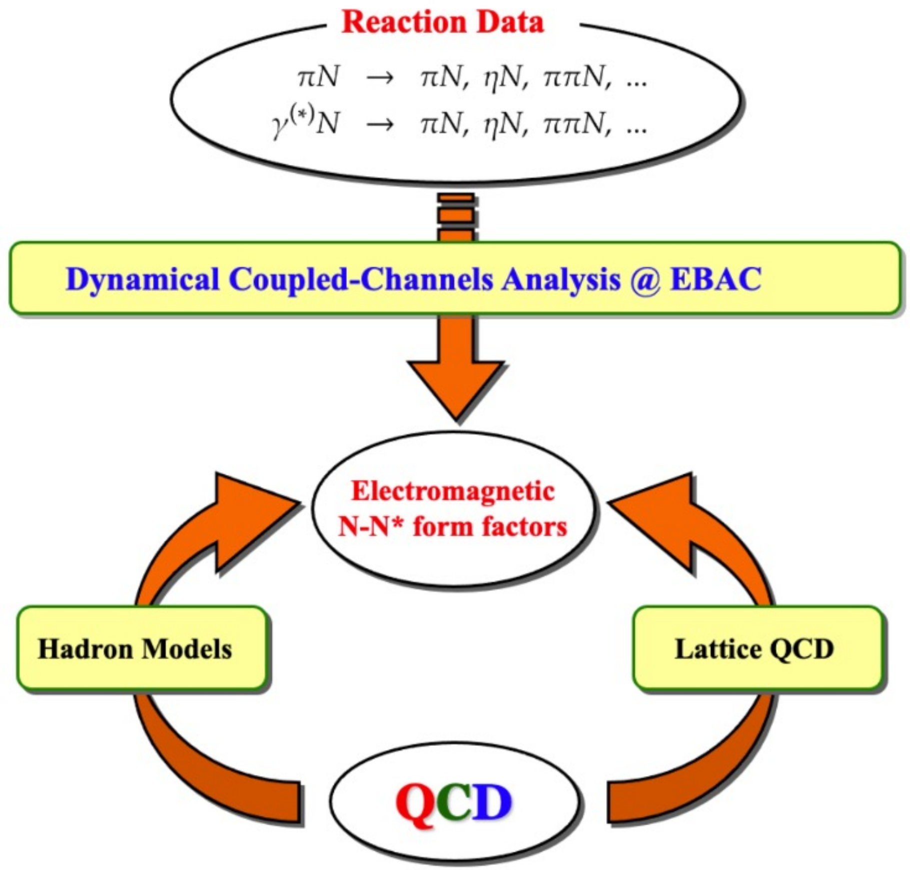

Fig. 2: This diagram was shown by Bruno Juliá Díaz at NSTAR 2009. We must measure both the final states from producing baryon resonances from pion and electron beams and, through these observables, we can fix the bilinear combinations of the helicity amplitudes and form factors predicted from theoretical amplitude analyses predicated on QCD

Establishing N* parameters within a global coupled channel analyses of the data of experiments with electromagnetic &hadronic probes

$$\begin{matrix} e^{-}N \\ \pi N \end{matrix} \rightarrow N^{*} \rightarrow \left\{ \begin{matrix} \pi N \\ \pi\pi N \\ \eta N \\ K\Lambda \\ K\Sigma \end{matrix} \right.$$

Accounting for the FSI within coupled channel approaches is vital to gain insight into the N* spectrum/structure in a way consistent with the restrictions imposed by a general unitarity condition.

$$T = \begin{pmatrix}
T_{\pi N\to\pi N} & T_{\eta N\to\pi N} & T_{\gamma N\to\pi N} & T_{\rho N\to\pi N} & T_{\sigma N\to\pi N} & T_{K\Lambda\to\pi N} & T_{K\Sigma\to\pi N} \\
T_{\pi N\to\eta N} & T_{\eta N\to\eta N} & T_{\gamma N\text{->}\eta N} & T_{\rho N\to\eta N} & T_{\sigma N\to\eta N} & T_{K\Lambda\to\eta N} & T_{K\Sigma\to\eta N} \\
T_{\pi N\to\gamma N} & T_{\eta N\to\gamma N} & T_{\gamma N\to\gamma N} & T_{\rho N\to\gamma N} & T_{\sigma N\to\gamma N} & T_{K\Lambda\to\gamma N} & T_{K\Sigma\text{->}\gamma N} \\
T_{\pi N\to\rho N} & T_{\eta N\to\rho N} & T_{\gamma N\to\rho N} & T_{\rho N\to\rho N} & T_{\sigma N\to\rho N} & T_{K\Lambda\to\rho N} & T_{K\Sigma\to\rho N} \\
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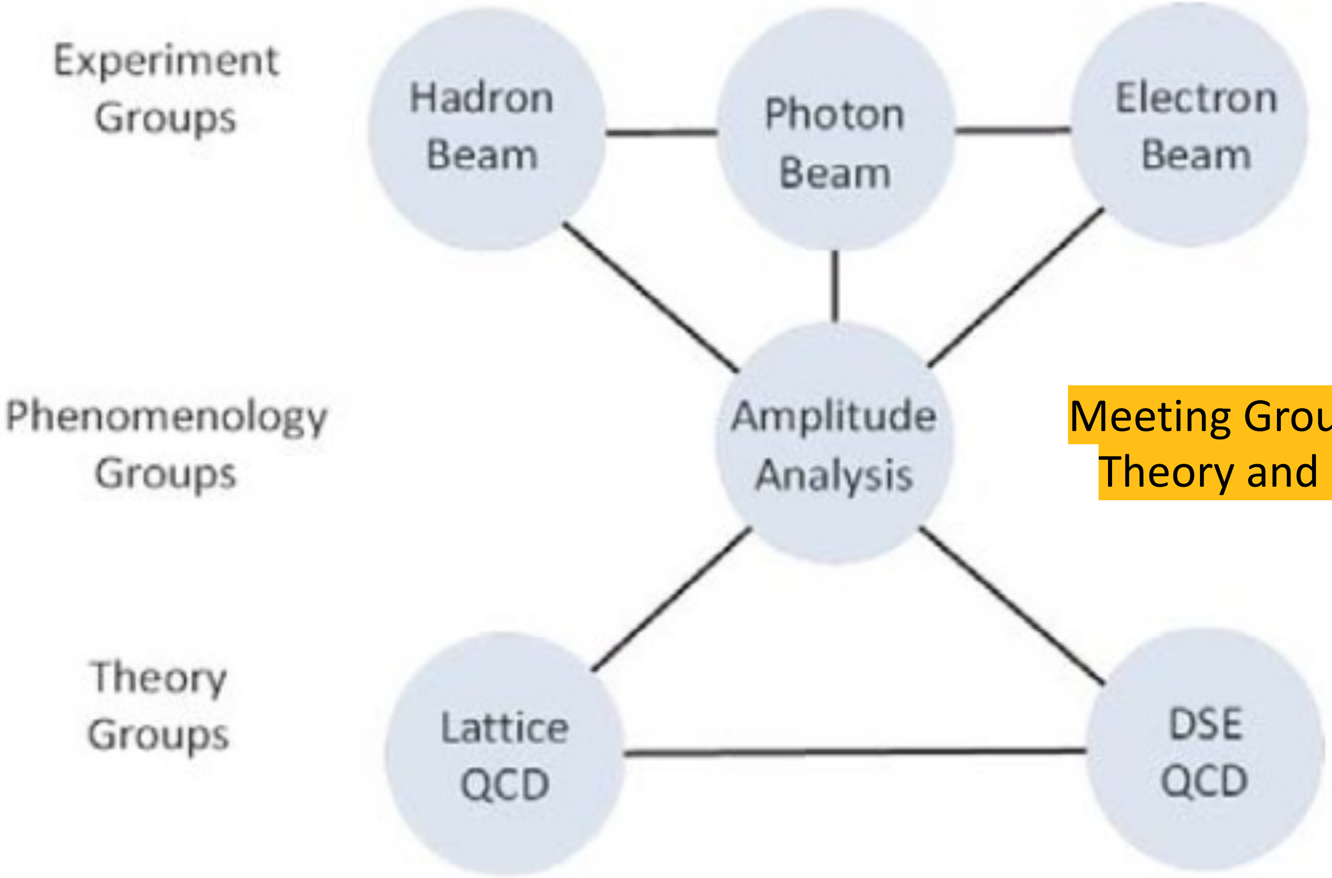
\end{pmatrix}$$

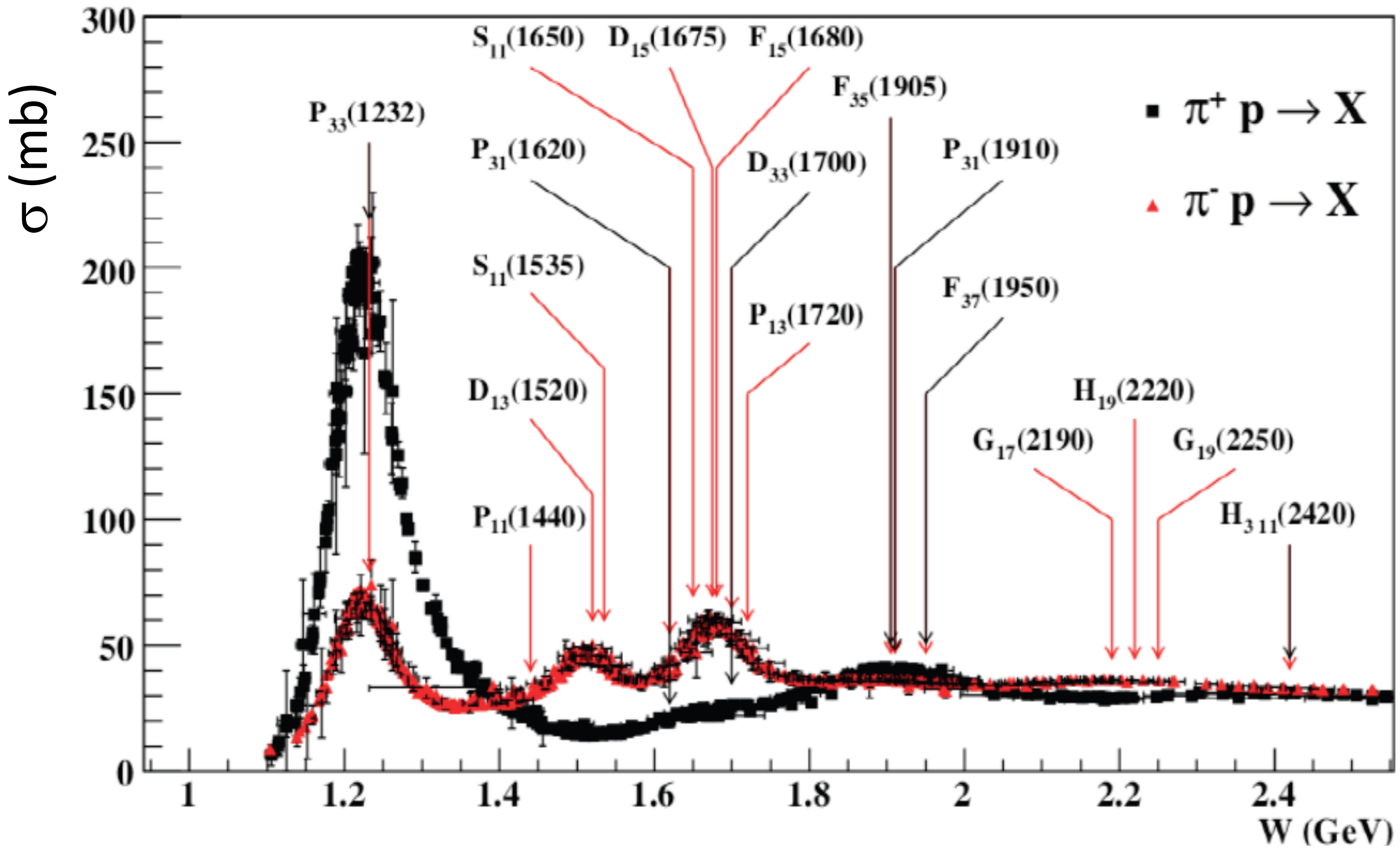
A Campaign – across continents and NP subfields



Meeting Ground between Theory and Experiment

From our PIRE proposal of 2016 – continuation of the 2009 US/PRC one

Baryon resonances (N*s and Δ*s)

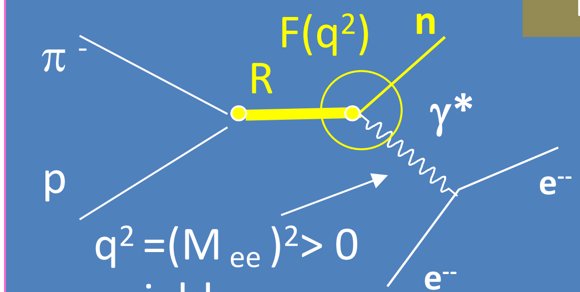


- N*s are broadly overlapping

Electromagnetic baryonic transitions in time-like and space-like regions: towards a global picture?

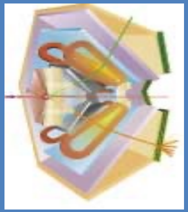
Time-Like electromagnetic form factors

preliminary studies with HADES/GSI



$q^2 = (M_{ee})^2 > 0$
variable

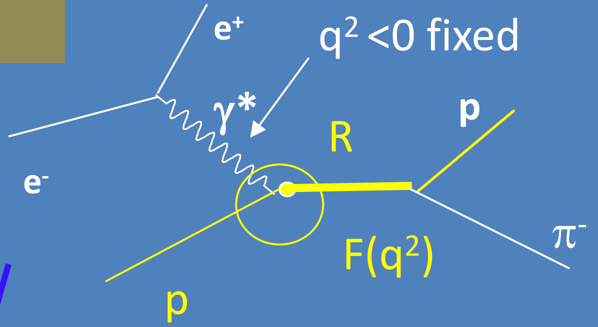
Inverse pion electroproduction

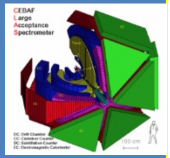


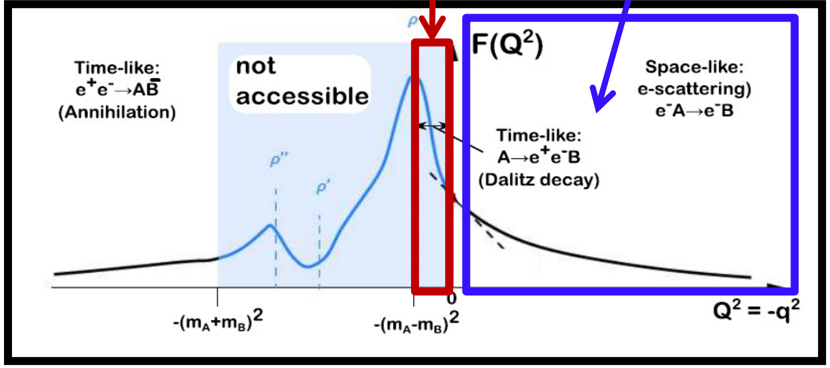
Space-Like electromagnetic form factors

Precise data from JLab/CLAS up to $-q^2=4$ GeV^2

$q^2 < 0$ fixed







Béatrice Ramstein ECT* 2017

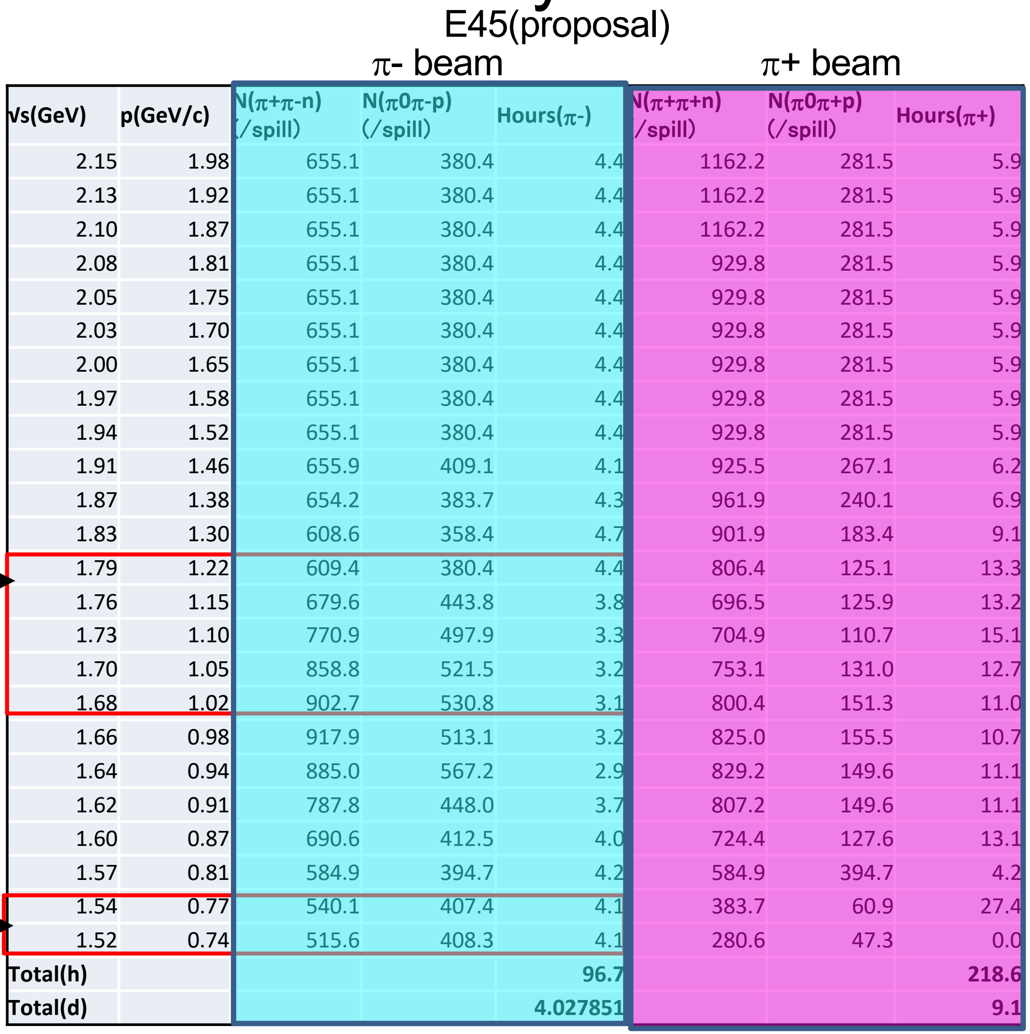
- ***Theoretical tools: Dispersion Relations, Dyson-Schwinger, Vector Dominance, Constituent Quarks ?***

E45 expected data summary

HADES
- only π^- beam
- $\sqrt{s}$~1.5 GeV run done
- Plan for $\sqrt{s}$~1.7 GeV
- No continuous energy scan

HADES Proposed

HADES Completed



E45(proposal)

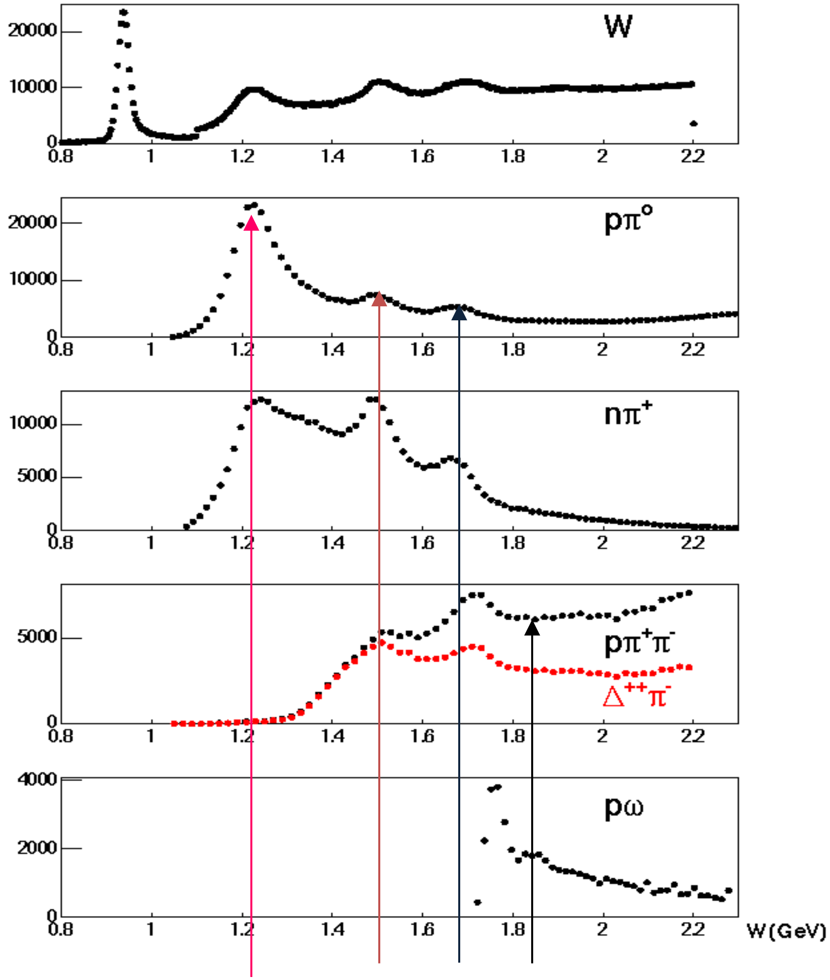
		π- beam			π+ beam		
√s(GeV)	p(GeV/c)	N(π+π-n) (/spill)	N(π0π-p) (/spill)	Hours(π-)	N(π+π+n) (/spill)	N(π0π+p) (/spill)	Hours(π+)
2.15	1.98	655.1	380.4	4.4	1162.2	281.5	5.9
2.13	1.92	655.1	380.4	4.4	1162.2	281.5	5.9
2.10	1.87	655.1	380.4	4.4	1162.2	281.5	5.9
2.08	1.81	655.1	380.4	4.4	929.8	281.5	5.9
2.05	1.75	655.1	380.4	4.4	929.8	281.5	5.9
2.03	1.70	655.1	380.4	4.4	929.8	281.5	5.9
2.00	1.65	655.1	380.4	4.4	929.8	281.5	5.9
1.97	1.58	655.1	380.4	4.4	929.8	281.5	5.9
1.94	1.52	655.1	380.4	4.4	929.8	281.5	5.9
1.91	1.46	655.9	409.1	4.1	925.5	267.1	6.2
1.87	1.38	654.2	383.7	4.3	961.9	240.1	6.9
1.83	1.30	608.6	358.4	4.7	901.9	183.4	9.1
1.79	1.22	609.4	380.4	4.4	806.4	125.1	13.3
1.76	1.15	679.6	443.8	3.8	696.5	125.9	13.2
1.73	1.10	770.9	497.9	3.3	704.9	110.7	15.1
1.70	1.05	858.8	521.5	3.2	753.1	131.0	12.7
1.68	1.02	902.7	530.8	3.1	800.4	151.3	11.0
1.66	0.98	917.9	513.1	3.2	825.0	155.5	10.7
1.64	0.94	885.0	567.2	2.9	829.2	149.6	11.1
1.62	0.91	787.8	448.0	3.7	807.2	149.6	11.1
1.60	0.87	690.6	412.5	4.0	724.4	127.6	13.1
1.57	0.81	584.9	394.7	4.2	584.9	394.7	4.2
1.54	0.77	540.1	407.4	4.1	383.7	60.9	27.4
1.52	0.74	515.6	408.3	4.1	280.6	47.3	0.0
Total(h)				**96.7**			**218.6**
Total(d)				**4.027851**			**9.1**

Why Nπ/Nππ electroproduction channels are important

- **Nπ/Nππ channels are the two major contributors in N* excitation region;**
- **these two channels combined are sensitive to almost all excited proton states;**
- **they are strongly coupled by πN→ππN final state interaction;**
- **may substantially affect exclusive channels having smaller cross sections, such as ηp, KΛ, and KΣ.**

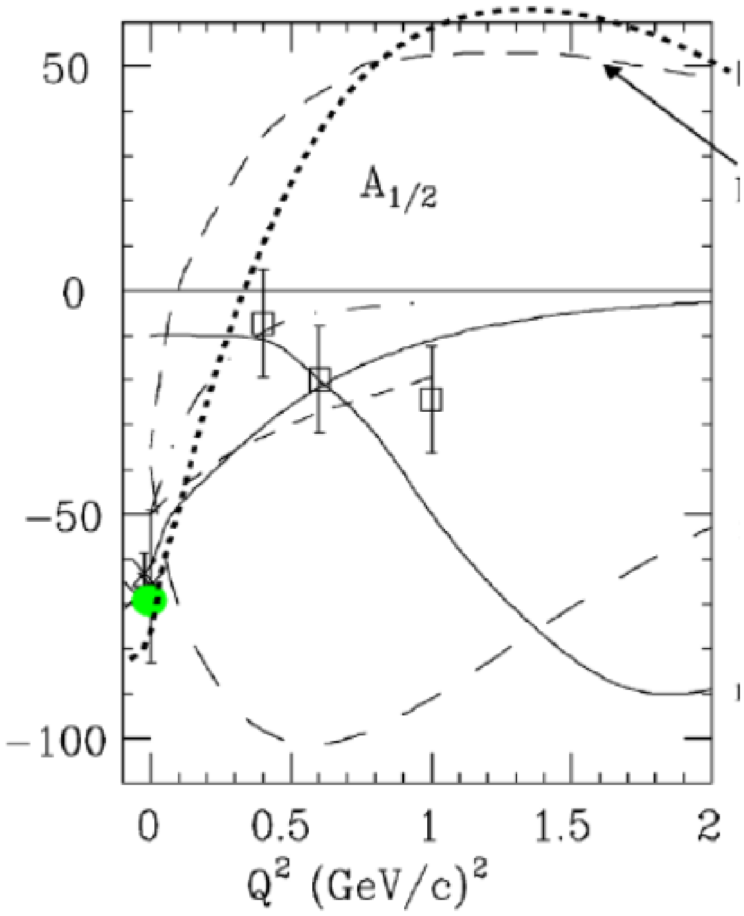
Therefore, knowledge on Nπ/Nππ electroproduction and πN → ππN mechanisms are key for the entire N* Program (Spectrum and Structure)

CLAS data on meson electroproduction at $Q^2 < 4.0$ GeV^2

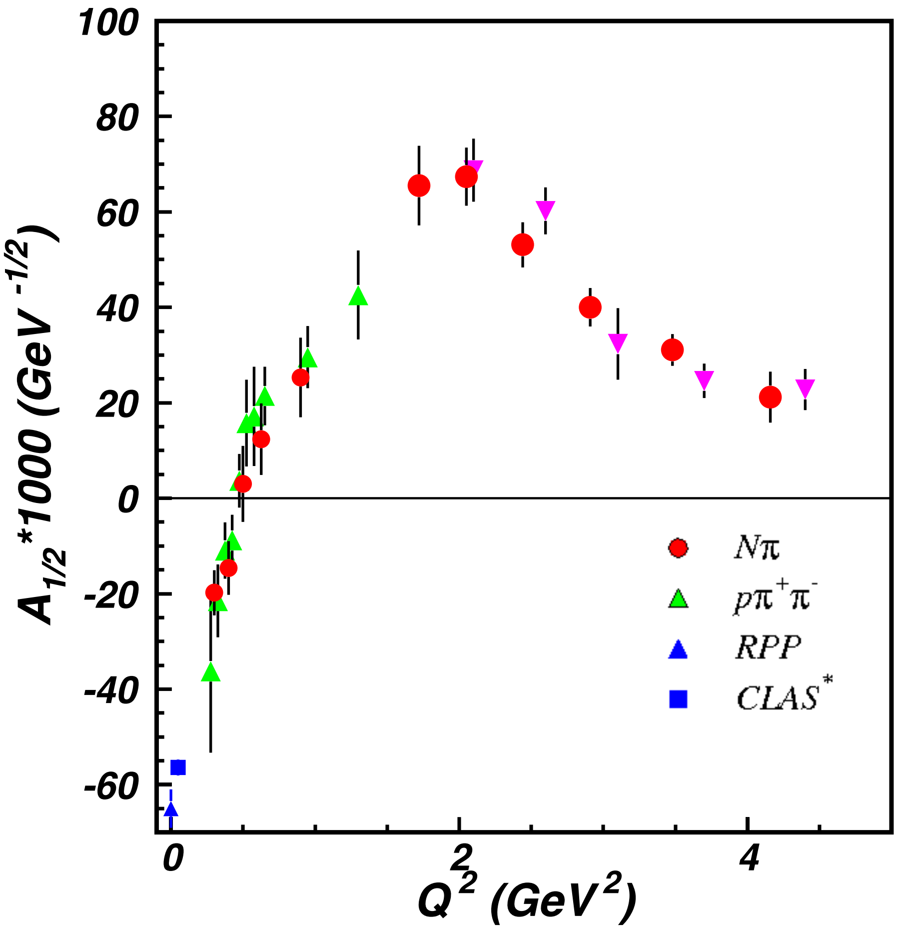


Roper Resonance in 2002 and 2023

2002



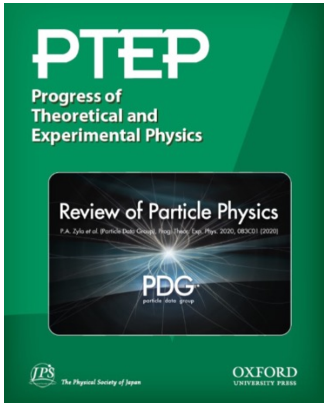
V. Burkert, Baryons 2002

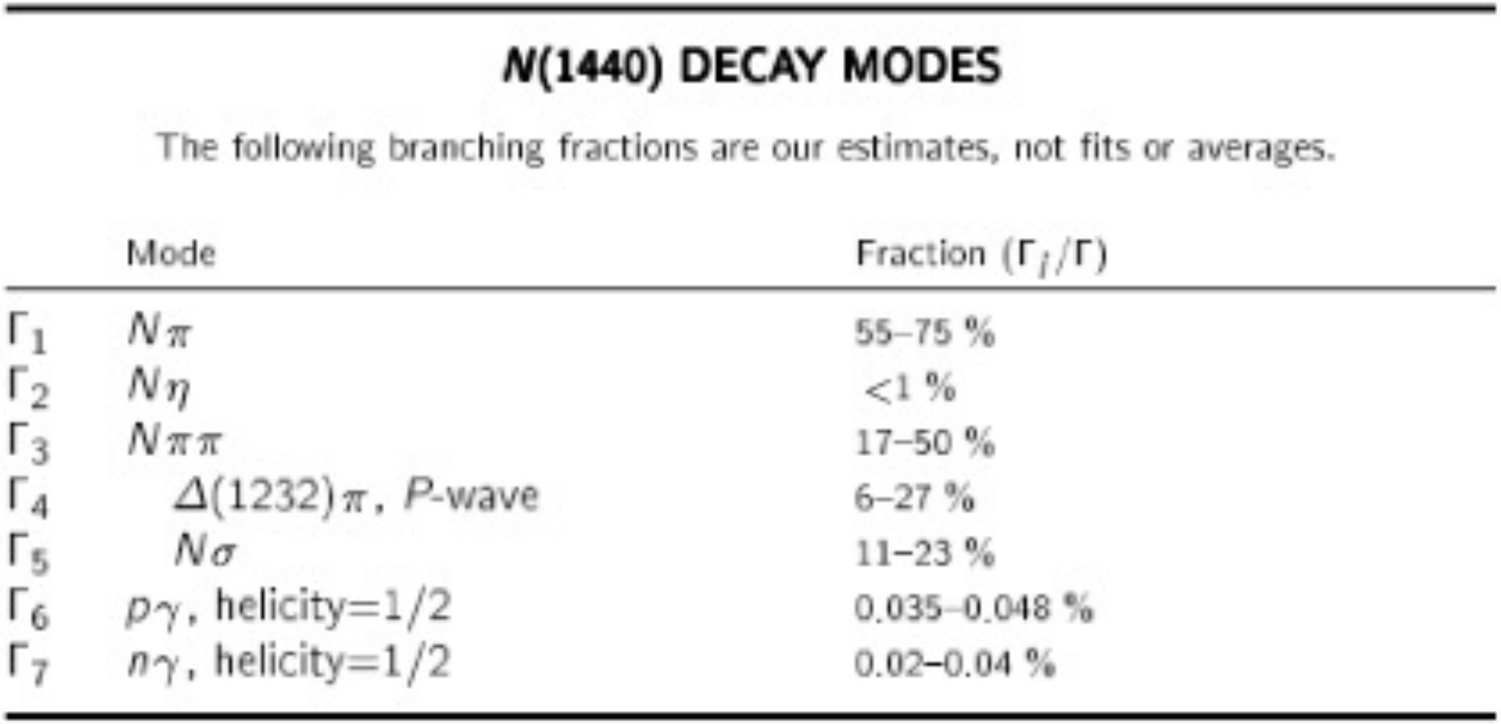


V.D. Burkert, eprint::2212.08980 [hep-ph], D.S. Carman, R.W. Gothe, V.I. Mokeev, and C.D. Roberts, Particles 6, 416 (2023)

$N(1440)\ 1/2^+$

Decay Modes





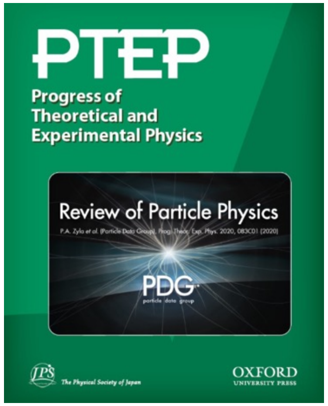
N(1440) DECAY MODES

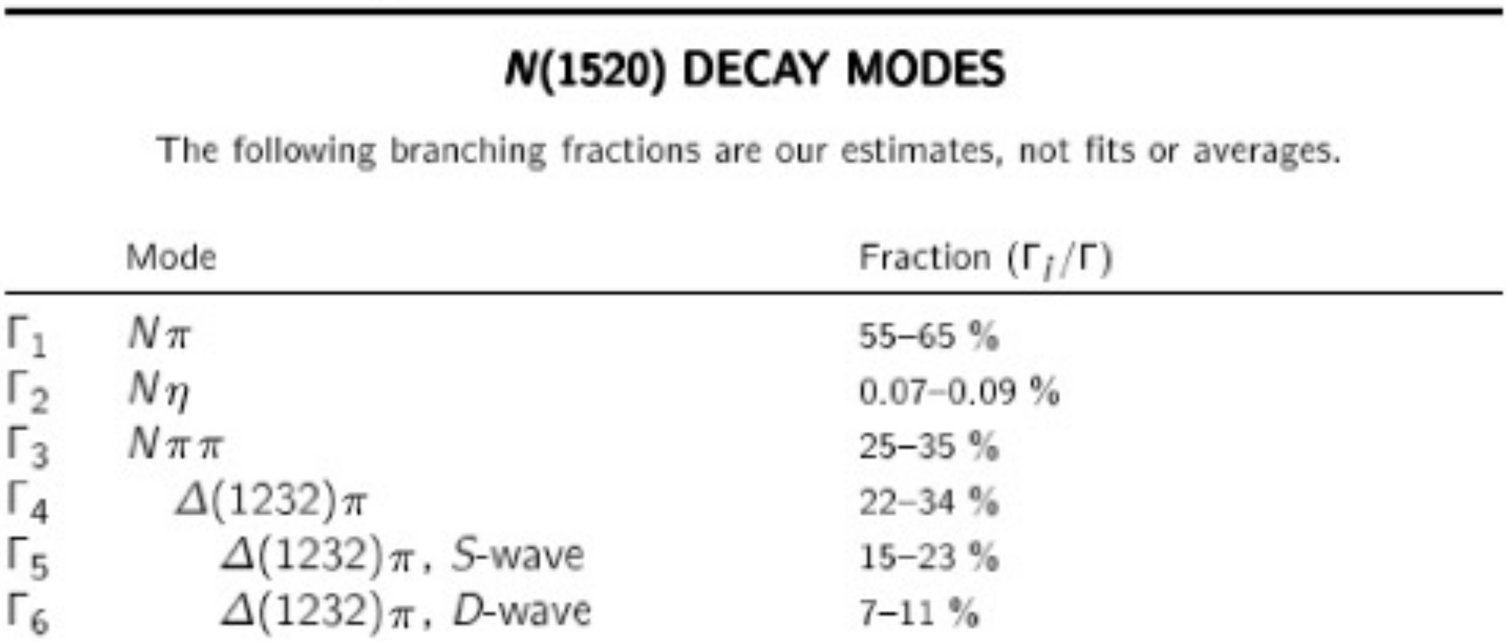
The following branching fractions are our estimates, not fits or averages.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	55–75 %
Γ_2	$N\eta$	<1 %
Γ_3	$N\pi\pi$	17–50 %
Γ_4	$\Delta(1232)\pi$, P-wave	6–27 %
Γ_5	$N\sigma$	11–23 %
Γ_6	$p\gamma$, helicity=1/2	0.035–0.048 %
Γ_7	$n\gamma$, helicity=1/2	0.02–0.04 %

$N(1520)\ 3/2^-$

Decay Modes



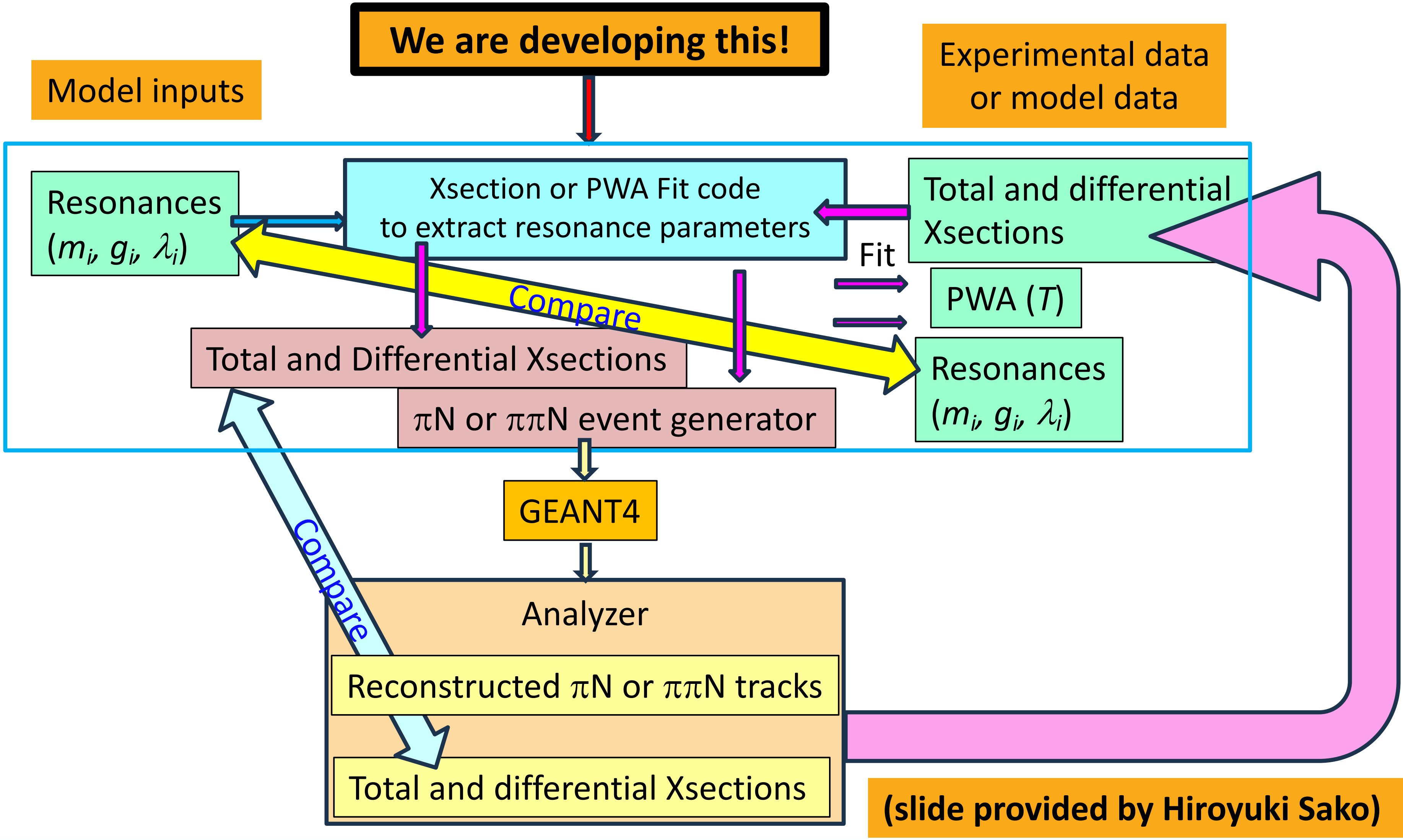


N(1520) DECAY MODES

The following branching fractions are our estimates, not fits or averages.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	55–65 %
Γ_2	$N\eta$	0.07–0.09 %
Γ_3	$N\pi\pi$	25–35 %
Γ_4	$\Delta(1232)\pi$	22–34 %
Γ_5	$\Delta(1232)\pi$, S-wave	15–23 %
Γ_6	$\Delta(1232)\pi$, D-wave	7–11 %

E45 data and the PWA EBAC Model



We are developing this!

Model inputs

Experimental data
or model data

Resonances
(m_i, g_i, λ_i)

Xsection or PWA Fit code
to extract resonance parameters

Total and differential
Xsections

Fit

PWA (T)

Compare

Total and Differential Xsections

Resonances
(m_i, g_i, λ_i)

πN or ππN event generator

GEANT4

Compare

Analyzer

Reconstructed πN or ππN tracks

Total and differential Xsections

(slide provided by Hiroyuki Sako)

Wherefore & Whither EBAC?
We need theoretical HELP to resuscitate and resurrect

- This is not a job for Experimentalists
- We need Theoretical Guidance
- Who will pick up the mantle for Coupled Channel Analysis?
- Who can lead the way in Reaction Modelling?
- Who is available in Asia/North America/Europe?

Employing pion and electron beams
for insight into the
spectrum/structure of N*s
through their two-pion decays

Some Questions to Ponder

1. How to compare Helicity Amplitudes between SL and TL?
2. What are the relevant d.o.f. as a function of q^2 for the SL and TL regimes?
3. **Amplitude Analysis**
4. **Coupled Channel Approach**
5. **WHITE PAPER for joining forces?**

NSTAR2024
June 17th-21st

NSTAR2024
June 17th-21st

NSTAR2024
June 17th-21st

- **Baryon spectrum through meson photoproduction**
- **Baryon resonances in experiments with hadron beams and in the e^+e^- collisions**
- **Baryon resonances in ion collisions and their role in cosmology**
- **Baryon structure through meson electroproduction, transition form factors, and time-like form factors**
- **Amplitude analyses and baryon parameter extraction**
- **Effective field theory, Phenomenological models, and Functional methods**
- **Baryon spectrum and structure from first principles of QCD**
- **Exotic Hadrons**
- **Advances in modeling of baryon spectrum & structure**
- **Facilities and future projects**

PWA for E45 (Today)

Strategy proposed by **T.-S. Harry-Lee**

[See: Note on the determination of Partial-Wave Amplitdes (PWA) from $\pi N \to \pi\pi N$ data]

T.-S. Harry Lee – December 8, 2022

- Approximate PWA to average intermediate resonance momentum k to reduce variable (k, W) → (W)
  - $\pi(\mathbf{k_0})N(-\mathbf{k_0}) \to \Delta(\mathbf{k})\pi(-\mathbf{k}) \to \pi(\mathbf{k_1})N(\mathbf{k_2})\pi(-\mathbf{k})$
  - (J: total angular momentum, $L\pi N$, $L\pi\Delta$: spin-angular momenta)

$$T^J_{L_{\pi\Delta},L_{\pi N}}(k,W) \sim \bar{T}^J_{L_{\pi\Delta},L_{\pi N}}(W)$$

  - Validity confirmed with ANL-Osaka model
- Parameterize averaged ANL-Osaka PWA with polynomials as a function of W

(slides provided by Hiroyuki Sako)

PWA for E45 (cont.)

- Fit E45 differential cross sections with the averaged parameterized PWA, using the initial parameters from ANL-Osaka PWA
  - Development of a code to calculate differential cross sections from PWA was started in **Kyungson Joo's group**
- Use separable model based on CMB approach to fit E45 averaged PWA and extract new PWA and resonance poles
  - Development of separable model in JAEA and KNU started. So far only a simple isobar model with 2-body reaction (pN→pN or pΔ)
- Use a sophisticated coupled-channel model to fit to the E45 data to obtain the resonance information

Only theorists can do this, and we need your support!

Strategy for PWA code development proposed by **T.S. Harry-Lee**

1. Study validity of averaged PWA with comparison of cross sections of ANL-Osaka model

   →Already confirmed by **T. S. Harry-Lee**

2. (1) Parameterize the averaged PWA of ANL-Osaka with the polynomials as functions of *W*

$$\bar{T}_{\alpha,\beta}(W) = \sum_i a^J_{\alpha\beta}(i) W^i \qquad (14)$$

(2) Calculate the differential cross sections from the averaged PWA

→Being developed by **Kyungseon Joo's group** but only 2-body reactions so far)

(3) Fit the differential cross sections of E45 with the initial parameters of (14) and obtain new averaged PWA

Go over if time

(slides provided by Hiroyuki Sako)

Procedures of PWA code development (Cont.)

3. Develop CMB model which is on-shell approximation and separable model (each pole can be resolved) of ANL-Osaka model to repeat Procedure 2. **Then extract resonance poles from CMB model.**
   - We started this development with an isobar model.
4. Extract resonance poles from ANL-Osaka with the fit results of 2. **(only by ANL-Osaka experts).** Solving Coupled-Channel equations required.

Go over if time

(slides provided by Hiroyuki Sako)

Averaged partial wave amplitudes

- Approximation to reduce the number of kinematic variables for 3-body reactions
- Consider reaction

$\pi(\mathbf{k_0})N(-\mathbf{k_0}) \rightarrow \Delta(\mathbf{k})\pi(-\mathbf{k}) \rightarrow \pi(\mathbf{k_1})N(\mathbf{k_2})\pi(-\mathbf{k})$

  - $\mathbf{k_0}$: Initial π momentum
  - $\mathbf{k}$: Intermediate Δ momentum
  - $\mathbf{k_1}$ $\mathbf{k_2}$: π and N momenta decayed from Δ
  - $\mathbf{k} = \mathbf{k_1} + \mathbf{k_2}$

Go over if time

(slides provided by Hiroyuki Sako)

Cross sections

- Total cross section

$$\sigma^{tot}(W) = \sum_{J} \sum_{L_{\pi\Delta},L_{\pi N}} d\sigma^{J}_{L_{\pi\Delta},L_{\pi N}}(W)$$

J: Total angular momentum

LπΔ: angular momentum of πΔ

LπN: angular momentum of πN

Partial wave cross section of (J, LπΔ , LπN) with the Partial Wave Amplitude T

$$d\sigma^{J}_{L_{\pi\Delta},L_{\pi N}}(W) = \frac{4\pi}{k_0^2}(J+\frac{1}{2}) \int_{m_\pi+m_N}^{W-m_\pi} d\,M |\Gamma^{1/2}_{\pi\Delta}(M,W) T^{J}_{L_{\pi\Delta},L_{\pi N}}(k,k_0;W) \rho^{1/2}_{\pi N}(W)|^2 \tag{3}$$

- M: invariant mass of final πN
- W: invariant mass of initial πN

Go over if time

(slides provided by Hiroyuki Sako)

$$\Gamma^{1/2}_{\pi\Delta}(M,W) = \pi[\frac{kE_\pi(k)E_M(k)}{W}][\frac{qE_\pi(q)E_N(q)}{M}]\times|\frac{h_{\pi N,\Delta}(q)}{W - E_\pi(k) - E_\Delta(k) - \Sigma(k,W)}|^2 \quad (4)$$

$$\rho_{\pi N}(W) = \pi\frac{k_0E_\pi(k_0)E_N(k_0)}{W} \quad (5)$$

$$W = E_\pi(k_0) + E_N(k_0)$$

$$W = E_\pi(k) + E_M(k)$$

$$M = E_\pi(q) + E_N(q)$$

Go over if time

(slides provided by Hiroyuki Sako)

Averaged PWA

$$T^J_{L_{\pi\Delta},L_{\pi N}}(k,k_0;W) \rightarrow \bar{T}^J_{L_{\pi\Delta},L_{\pi N}}(W) = T^J_{L_{\pi\Delta},L_{\pi N}}(\boxed{\bar{k}},k_0;W) \tag{10}$$

Averaged over intermediate Δ momentum k

$$\bar{k} = \frac{\int \Gamma_{\pi\Delta}(M,W)\,k\,dM}{\int \Gamma_{\pi\Delta}(M,W)\,dM}, \tag{8}$$

$$W = E_\pi(\bar{k}) + [\bar{M}^2 + \bar{k}^2]^{1/2} \tag{9}$$

$$d\bar{\sigma}^J_{L_{\pi\Delta},L_{\pi N}}(W) = \frac{4\pi}{k_0^2}(J+\frac{1}{2})|\rho^{1/2}_{\pi\Delta}(W)\bar{T}^J_{L_{\pi\Delta},L_{\pi N}}(W)\rho^{1/2}_{\pi N}(W)|^2 \tag{11}$$

Go over if time

(slides provided by Hiroyuki Sako)

PWA code Status

Goal: Develop an analysis code to fit to measured cross sections and extract resonance information in $\pi N \to \pi\pi N$ reactions

First step: Fit to $\pi N \to MB$ 2-body reactions with **a simple isobar model**

The code was created in C++ and ROOT by Shinhyung Kim based on T.S.H. Lee's Fortran code.

- 1 resonance (Δ) and 1 channel (πN): done by Shinhyung
- 2 resonances and 1 channel (πN): done
- 2 resonances and 2 channels (πN and $\pi\Delta$):done
- M resonances and N channels→ Implemented (2/12)

  with C++ complex matrix library Eigen

  to be checked with M≧3, N≧3

Next step

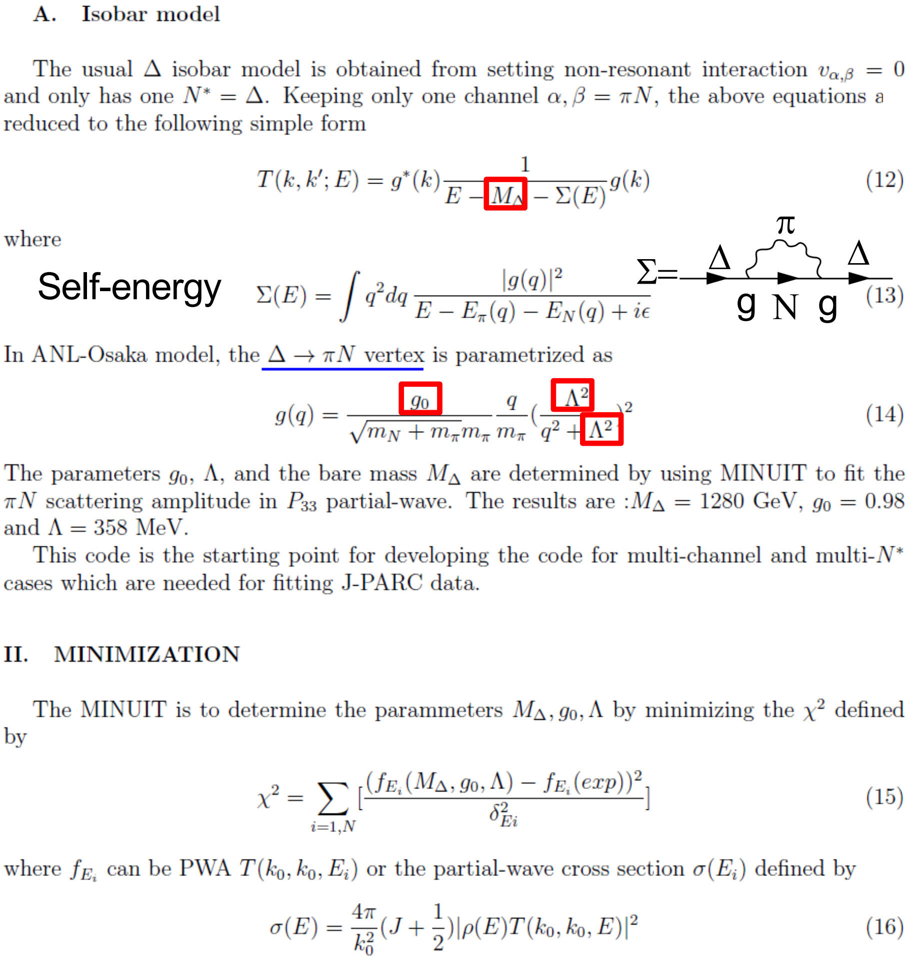
- Optimization of fitting algorithms

  Fit is very difficult with even for 2 resonances and 1 channel
- Implement Generator for GEANT4
  - 2-body decays
  - 3-body decays

Go over if time

(slides provided by Hiroyuki Sako)

1 resonance + 1 channel



A. Isobar model

The usual Δ isobar model is obtained from setting non-resonant interaction $v_{\alpha,\beta} = 0$ and only has one $N^* = \Delta$. Keeping only one channel $\alpha, \beta = \pi N$, the above equations a reduced to the following simple form

$$T(k,k';E) = g^*(k)\frac{1}{E - \boxed{M_\Delta} - \Sigma(E)}g(k) \tag{12}$$

where

Self-energy

$$\Sigma(E) = \int q^2 dq \frac{|g(q)|^2}{E - E_\pi(q) - E_N(q) + i\epsilon} \tag{13}$$

In ANL-Osaka model, the $\Delta \to \pi N$ vertex is parametrized as

$$g(q) = \frac{\boxed{g_0}}{\sqrt{m_N + m_\pi m_\pi}}\frac{q}{m_\pi}\left(\frac{\boxed{\Lambda^2}}{q^2 + \boxed{\Lambda^2}}\right)^2 \tag{14}$$

The parameters g_0, Λ, and the bare mass M_Δ are determined by using MINUIT to fit the πN scattering amplitude in P_{33} partial-wave. The results are :$M_\Delta = 1280$ GeV, $g_0 = 0.98$ and $\Lambda = 358$ MeV.

This code is the starting point for developing the code for multi-channel and multi-N^* cases which are needed for fitting J-PARC data.

II. MINIMIZATION

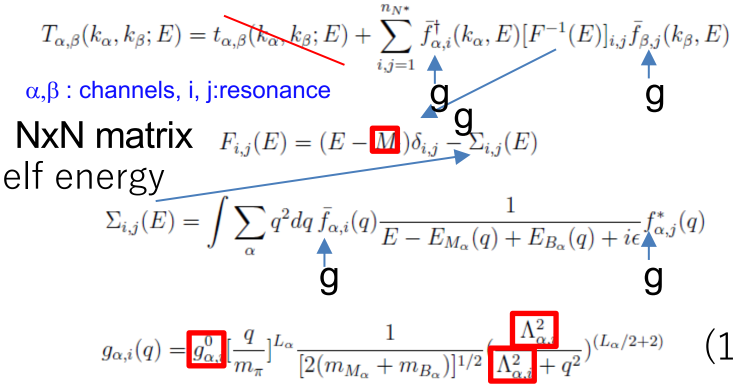
The MINUIT is to determine the parammeters M_Δ, g_0, Λ by minimizing the χ^2 defined by

$$\chi^2 = \sum_{i=1,N}\left[\frac{(f_{E_i}(M_\Delta, g_0, \Lambda) - f_{E_i}(exp))^2}{\delta^2_{Ei}}\right] \tag{15}$$

where f_{E_i} can be PWA $T(k_0, k_0, E_i)$ or the partial-wave cross section $\sigma(E_i)$ defined by

$$\sigma(E) = \frac{4\pi}{k_0^2}(J + \frac{1}{2})|\rho(E)T(k_0,k_0,E)|^2 \tag{16}$$

2 resonances + 1 channel



continuum

resonance

$$T_{\alpha,\beta}(k_\alpha,k_\beta;E) = \cancel{t_{\alpha,\beta}(k_\alpha,k_\beta;E)} + \sum_{i,j=1}^{n_{N^*}} \bar{f}^\dagger_{\alpha,i}(k_\alpha,E)[F^{-1}(E)]_{i,j}\bar{f}_{\beta,j}(k_\beta,E) \tag{17}$$

g

g

g

α,β : channels, i, j:resonance

NxN matrix

$$F_{i,j}(E) = (E - \boxed{M})\delta_{i,j} - \Sigma_{i,j}(E)$$

Self energy

$$\Sigma_{i,j}(E) = \int \sum_\alpha q^2 dq\, \bar{f}_{\alpha,i}(q)\frac{1}{E - E_{M_\alpha}(q) + E_{B_\alpha}(q) + i\epsilon}\bar{f}^*_{\alpha,j}(q) \tag{18}$$

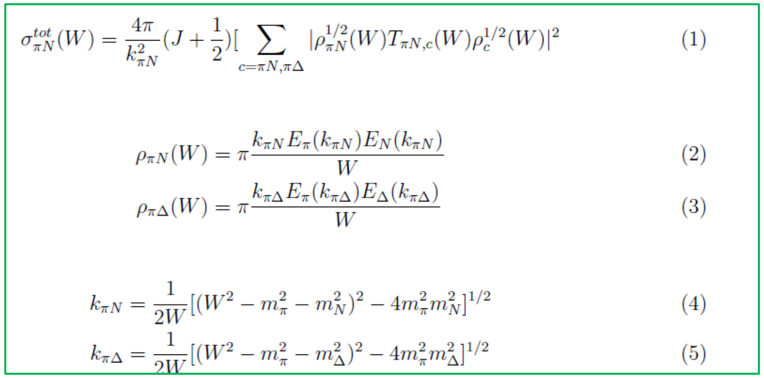
g

g

$$g_{\alpha,i}(q) = \boxed{g^0_{\alpha,i}}\left[\frac{q}{m_\pi}\right]^{L_\alpha}\frac{1}{[2(m_{M_\alpha} + m_{B_\alpha})]^{1/2}}\left(\frac{\boxed{\Lambda^2_{\alpha,i}}}{\boxed{\Lambda^2_{\alpha,i}} + q^2}\right)^{(L_\alpha/2+2)} \tag{19}$$

- Fit with total cross section

$$\chi^2 = \sum_{j=1}^{N}\sum_{c=1}^{N_c}\left[\sigma^{tot}_{\pi N,c}(E_j) - \sigma^{tot}_{\pi N,c(\mathrm{exp}\ )}(E_j)\right]^2 \tag{20}$$



$$\sigma^{tot}_{\pi N}(W) = \frac{4\pi}{k^2_{\pi N}}(J + \frac{1}{2})[\sum_{c=\pi N,\pi\Delta}|\rho^{1/2}_{\pi N}(W)T_{\pi N,c}(W)\rho^{1/2}_c(W)|^2 \tag{1}$$

$$\rho_{\pi N}(W) = \pi\frac{k_{\pi N}E_\pi(k_{\pi N})E_N(k_{\pi N})}{W} \tag{2}$$

$$\rho_{\pi\Delta}(W) = \pi\frac{k_{\pi\Delta}E_\pi(k_{\pi\Delta})E_\Delta(k_{\pi\Delta})}{W} \tag{3}$$

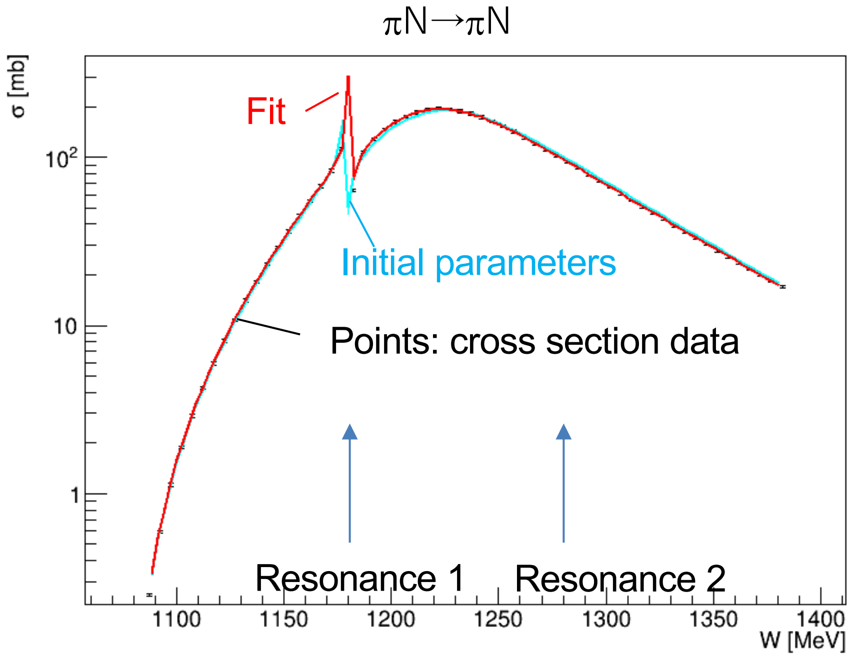
$$k_{\pi N} = \frac{1}{2W}[(W^2 - m_\pi^2 - m_N^2)^2 - 4m_\pi^2 m_N^2]^{1/2} \tag{4}$$

$$k_{\pi\Delta} = \frac{1}{2W}[(W^2 - m_\pi^2 - m_\Delta^2)^2 - 4m_\pi^2 m_\Delta^2]^{1/2} \tag{5}$$

Go over if time

(slides provided by Hiroyuki Sako)

Fit result example (2 channels, w/ 2 hypothetical resonances)


$\pi N \to \pi N$

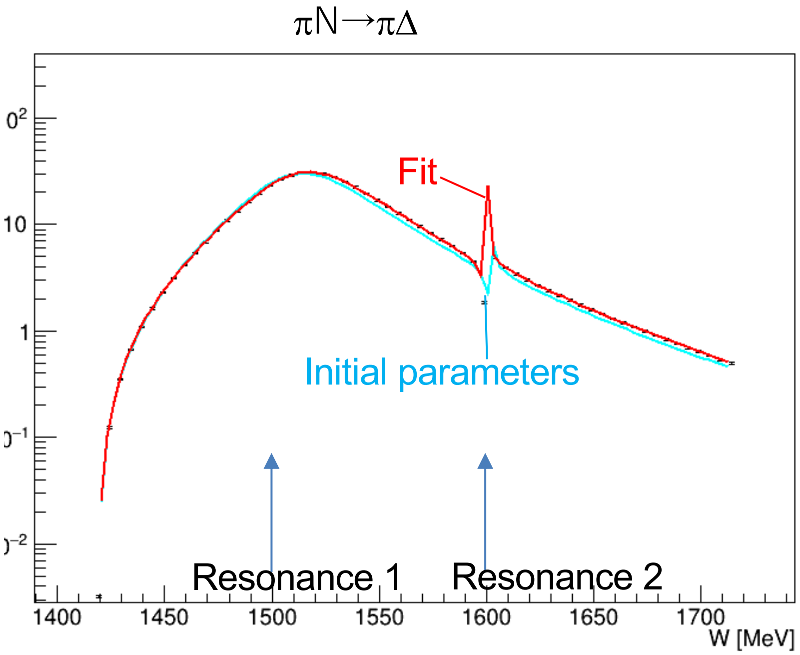
Fit

Initial parameters

Points: cross section data

Resonance 1

Resonance 2


$\pi N \to \pi \Delta$

Fit

Initial parameters

Resonance 1

Resonance 2

Go over if time

(slides provided by Hiroyuki Sako)

Development of Newton-Raphson method

$$\chi^2 = \sum_{j=1}^{N}\sum_{c=1}^{N_c}\left[\sigma^{tot}_{\pi N,c}(E_j) - \sigma^{tot}_{\pi N,c(\exp\;\;)}(E_j)\right]^2$$

Usually χ^2 (1-dimension) is minimized in many algorithms. χ^2 of the total cross section residuals of all data points

- Simplex, gradient descent, … (implemented in Minuit library)

Instead of using χ^2, use residual vector of data-model at each data point!

- $S_i=f(x_1,x_2,\ldots x_n, E_i)-y(E_i)$: $x_1,\ldots x_n$, model parameters, E_i: energy of i-th data point
  - f: modeled cross section, y : measured cross section data
  - $\chi^2 = \Sigma S_i^2$ (i=1,…n, data points)

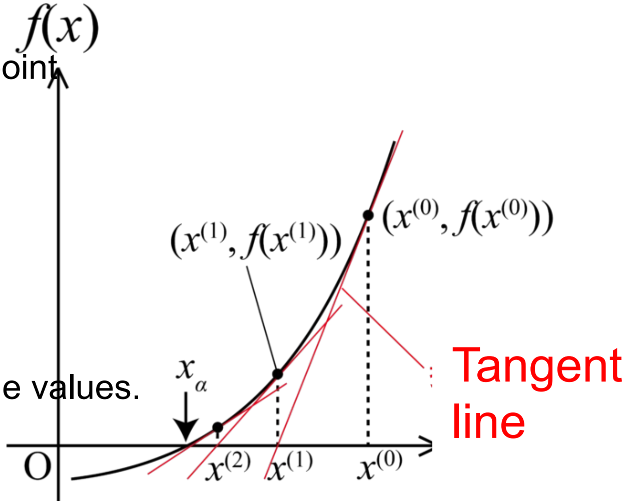
Require **S=0,** instead of minimizing χ^2

And solve it with Newton-Raphson method (iterative)

- $x_{n+1} = x_n - (\partial S(x_n)/\partial x)^{-1} S(x_n)$
- $\partial S(x_n)/\partial x$: Jacobian: $J_{ij}= \partial S_j/\partial x_i$

Nonlinear equation is approximated to a linear solver problem!

Newton-Raphson method is very fast if the initial parameters are close to the true values.



Tangent line

Test with 2 resonances and 1 channel

χ^2 minimization (Minuit): 340 iterations

Newton-Raphson: 20 iterations

But, Newton-Raphson method can converge only if the initial parameters are close enough to the true values.

Go over if time

(slides provided by Hiroyuki Sako)


Thank
You
Merci
Gracias
Spacibo
Danke
TerimaKasih
Hvala
Arigatô
KamSahHamnida
Mèsi
Xièxie
BarakAllahuFiik
Takk
aDank
TapadhLeat
Köszönöm
Murakoze
Grazzi
Grazie
Nouari
Shukriya
Buznyg
WaadMahadsantahay
ThintKo
FaafetaiLava
Ngiyabonga
Enkosi
M-Sapo
Bedankt
Zikomo
TapadhLeibh
AsanteSana
Waila
Blagodaram
Rahmat
Dhanyavaad
Trugarez
Aabhar
Chokrane
Kiitos
Dziakuju
Mauruuru
GoRaibhMaithAgat
KyayTzuTinPaTe
Mochchakkeram
Matondo
Welalin
TangioTumas
Dhanyavad
Tanan
Sukriya
TeşekkürEderim
Sobodi
AabhariAahe
Barkal
KurreSumanga
GratiasAgimus
Mercé
Mamnun
TesekkurEdirem
Yekeniele
Dhanyabaad
Taiku
KeYaLeboha
Tanemirt
Vinaka
Dankie
Tenki
KhobChaiDeu
Chnorakaloutioun
Milesker
TänanVäga
Ačiū
KopKhunKrap
Tau
Nanni
Barka
Dankon
Dankje
Dankie
Motashakkeram
Toda
Dakujem
aNiKiè
Paldies
DjereDieuf
Gratzias
ANiKiè
Trugéré
Akpé
Misaotra
Tak
Sagolun
Mersi
Nizzik ajr
Aguyjé
Diolch
EskerrikAsko Dankewol
Rahmet
Dhanyavadagalu
Grandmercé
Dhanyavadalu
Sulpáy
GratiasAgo
Tanmirt
Akiba
Gràcies
Blagodaria
NajisTuke
Nandri
Spas
MaloTeOfa
Gràçie
Gracie
KopKhunKha
Faleminderit
Merkzi
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Wado
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CámOn
Efharisto
DankJuWel
Marahaba
Saha
Multumesc
Meharbani
ÇoxSagOlun
Nirringrazzjak
Bayarlalaa
Obrigado
Dîky
TakkFyri
Salamat
Miigwetch