Approaches to complete experiments

Complete-experiment analysis (CEA)

*) Generic meson-prod. reaction $\mathcal{P}N \to \{\varphi_i\}\, B$,
described by $N_{\mathcal{A}}$ spin-amplitudes $b_1, \ldots, b_{N_{\mathcal{A}}}$,
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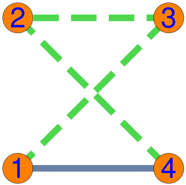

is complete ✓

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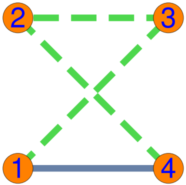

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Truncated partial-wave analysis (TPWA)

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$$\mathcal{T}_{\mu_1\mu_2,\lambda_1\lambda_2}(s,t) = e^{i(\lambda-\mu)\phi} \times \sum_{j=\max(|\lambda|,|\mu|)}^{\infty} (2j+1)\mathcal{T}_{\mu,\lambda}^{j}(s)\, d_{\mu,\lambda}^{j}(\theta),$$

(where $\lambda := \lambda_1 - \lambda_2$, $\mu := \mu_1 - \mu_2$ and $\{b_i\} \Leftrightarrow \{H_i\} = \{\mathcal{T}_{\pm\pm,\pm\pm}\}$.), at some maximal angular momentum j_{max} (ℓ_{max}) and search for complete experiments for determination of the $\left\{\mathcal{T}_{\mu,\lambda}^{j}\right\}$.

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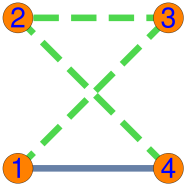

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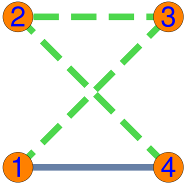

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Surprise: *all* ambiguites can be resolved using less than $2N_{\mathcal{A}} = 8$ observables!

[YW, R. Beck and L. Tiator PRC **89**, 055203 (2014)]

[YW, arXiv:2008.00514 [nucl-th]]

Testing completeness of photoproduction TPWA

$*)$ I-O studies using model-data (MAID2007, $\gamma p \rightarrow \pi^0 p$), set: $\left\{ \sigma_0, \check{\Sigma}, \check{T}, \check{P}, \check{F} \right\}$

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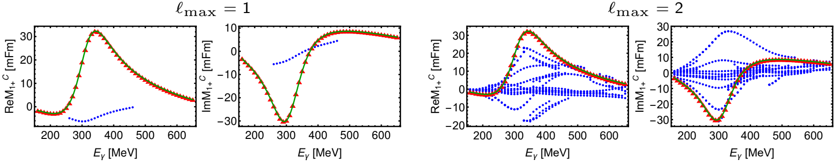

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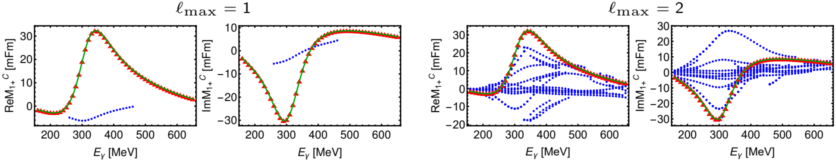
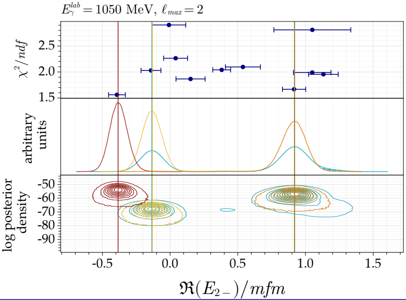
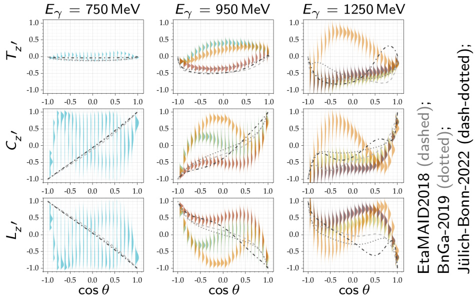


$\ast$) Bayesian inference for (real) $\gamma p \rightarrow \eta p$ data, set $\left\{ \sigma_0, \check{\Sigma}, \check{T}, \check{E}, \check{F}, \check{G} \right\}$

cf.: [P. Kroenert, YW, F. Afzal and A. Thiel, Phys. Rev. C **109**, no.4, 045206 (2024)]





Dreams of a 'coupled-channels complete experiment'

Consider *channel-space* $\{|\pi N\rangle, |\gamma N\rangle, |\pi\pi N\rangle\}$, i.e.:

$$(\mathcal{T}_{fi}) = \left[\begin{array}{ccc} \mathcal{T}_{\pi N, \pi N} & \mathcal{T}_{\pi N, \gamma N} & \mathcal{T}_{\pi N, \pi\pi N} \\ \mathcal{T}_{\gamma N, \pi N} & \mathcal{T}_{\gamma N, \gamma N} \simeq 0 & \mathcal{T}_{\gamma N, \pi\pi N} \\ \mathcal{T}_{\pi\pi N, \pi N} & \mathcal{T}_{\pi\pi N, \gamma N} & \mathcal{T}_{\pi\pi N, \pi\pi N} \end{array} \right].$$

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$\hookrightarrow$ Measure individual complete experiments with perfect *phase-space coverage and overlap* among individual reactions (complete exp.'s determinable using *graphs*):

Reaction	Example complete experiment (yields $\lvert b_i \rvert$ & ϕ_{ij})
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$\Rightarrow$ Missing phase-information $e^{i\phi_{fi}}$ fixed and resonance-spectrum (hopefully) unique!

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Issues: - Can we assume perfect time-reversal inv., to relate $3 \rightarrow 2$ to $2 \rightarrow 3$ processes?

- $3 \rightarrow 3$-process $\pi\pi N \rightarrow \pi\pi N$ unmeasurable. Does this hurt the proposal?