TPWA for 2-meson final state (preliminary!!) - I

- **5-dimensional** phase-space:

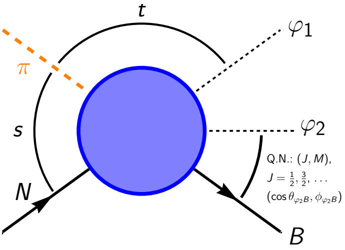
$$\left\{ s, t, m^2_{\varphi_2 B}, \Omega_{\varphi_2 B} = \left(\theta_{\varphi_2 B}, \phi_{\varphi_2 B} \right) \right\}$$

- 4 helicity-configurations:

$$\lambda_N = \pm \frac{1}{2},\ \lambda_B = \pm \frac{1}{2}.$$

- $\varphi_2 B$ angular-momentum Q.N.'s:

$$J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots, \infty \text{ and } M = -J, \ldots, +J.$$



[cf. Talk by V. Mathieu (Monday)]

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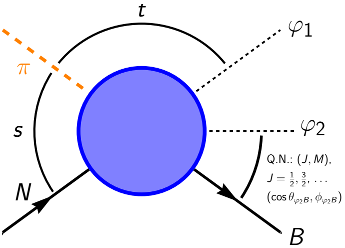

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$\Rightarrow$ TPWA:

$$\mathcal{A}_{\lambda_N \lambda_B}\left(s, t, m^2_{\varphi_2 B}, \Omega_{\varphi_2 B} \right) = \sum_{J=\frac{1}{2},\frac{3}{2},\ldots}^{J_{\mathrm{max}}} \sum_{M=-J}^{+J} \mathcal{T}^{(J)}_{M;\lambda_N \lambda_B}\left(s, t, m^2_{\varphi_2 B} \right) D^{*J}_{M,-\lambda_B}\left(\Omega_{\varphi_2 B} \right).$$

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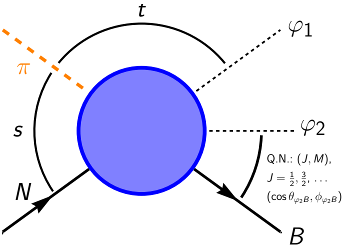

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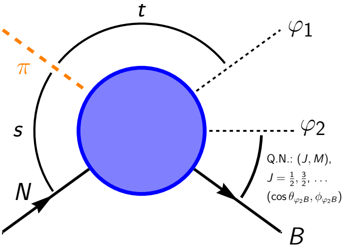

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$\Rightarrow$ Consider discrete partial-wave ambiguities for the squared-modulus function

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$\Rightarrow$ 2-meson TPWA is **much better constrained, most likely unique!**

cf.: [I. S. Stefanescu, J. Math. Phys. **26** (9), 2141-2160 (1985)]

& [W. A. Smith *et al.* [JPAC], Phys. Rev. D **108**, no.7, 076001 (2023)]