Properties of X(3872) from hadronic potentials coupled to quarks

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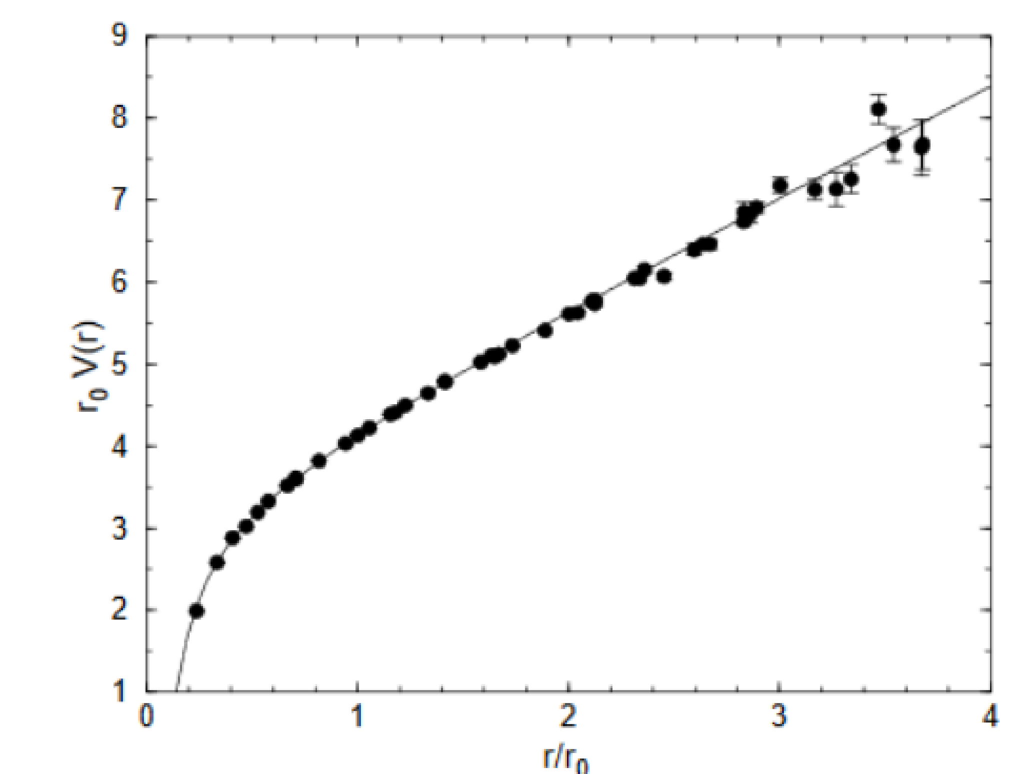
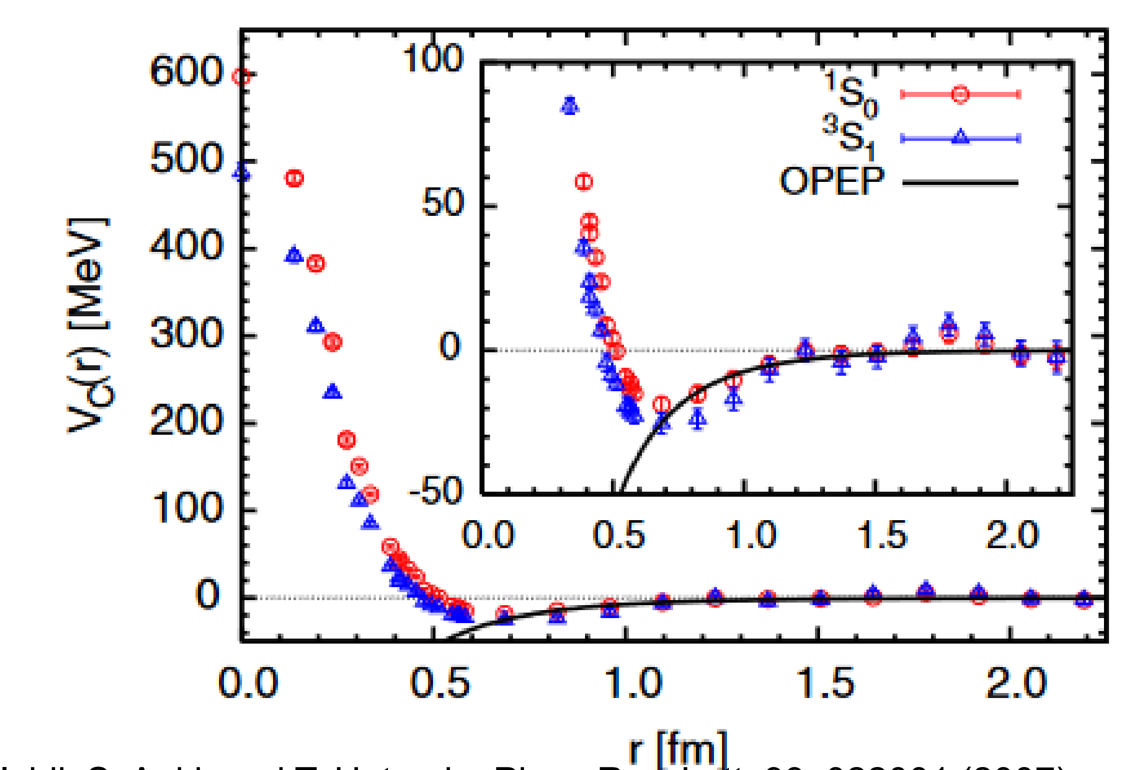
The formulation of this talk is based on
[I. Terashima and T. Hyodo, PhysRevC.108.035204 (2023)]

Numerical calculation by LQCD



CP-PACS, A. Ali Khan, et al., Phys. Rev. D **65**, 054505 (2002)



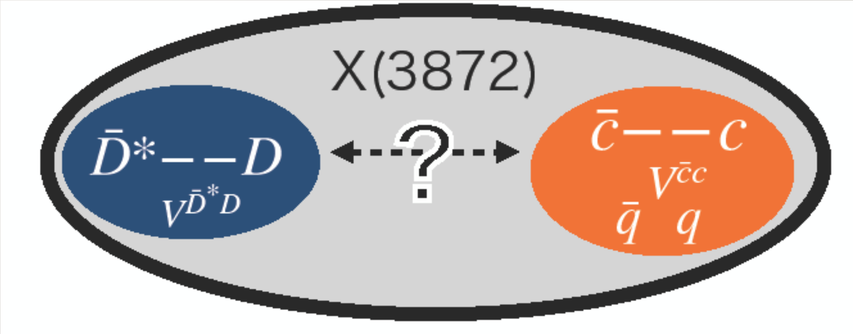
N. Ishii, S. Aoki, and T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007)

- ➢Inter-quark (static) potential
- ➢Inter-hadron (NN) potential

■Quark-antiquark potentials and hadron-hadron potentials have been studied independently

Exotic hadron $X(3872)$

- There is no restriction by QCD which prohibits the mixing with each d.o.f
  - States with same quantum numbers mix by definition
- Structure of $X(3872)$ [A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, PTEP **2016** (2016)]
  - **Mixing with quark and hadron** degrees of freedom
  - Not enough experimental data and lattice QCD results
  - How about a channel coupling between **quark** and **hadron** degrees of freedom like $\mathbf{X(3872)}$?



Channel coupling

✓ Formulation according to Feshbach method [H. Feshbach, Ann. Phys. 5, 357 (1958); ibid., 19, 287 (1962)]

- Hamiltonian H with channel between quark potential V^q and hadron V^h

$$H = \begin{pmatrix} T^q & 0 \\ 0 & T^h + \Delta \end{pmatrix} + \begin{pmatrix} V^q & V^t \\ V^t & V^h \end{pmatrix}$$

T^q, T^h:Kinetic energy

Δ:Threshold energy

V^t:Transition potential

- Schrödinger equation with wave functions of quark and hadron channels $|q\rangle, |h\rangle$

$$H \begin{pmatrix} |q\rangle \\ |h\rangle \end{pmatrix} = E \begin{pmatrix} |q\rangle \\ |h\rangle \end{pmatrix}$$

➢ Two set of equations with quark and hadron channels are obtained

Effective potential

- Eliminate quark channel to obtain an effective Hamiltonian of hadron channel $H^h_{\text{eff}}(E)$

with, $H^h_{\text{eff}}(E)\,|h\rangle = E\,|h\rangle\,,$ $V^h_{\text{eff}}(E)$

$$H^h_{\text{eff}}(E) = T^h + \Delta^h + V^h + V^t G^q(E) V^t$$

✓ No approximation

✓ G_q is the Green function of quark channel

$$G_q(E) = (E - (T^q + V^q))^{-1}$$

➢ Quark channel contribution by coupled channels

- Coordinate representation with initial relative coordinate $\boldsymbol{r}$ and final $\boldsymbol{r}'$

$$\langle \boldsymbol{r}'_h \mid V^h_{\text{eff}}(E) \mid \boldsymbol{r}_h\rangle = \langle \boldsymbol{r}'_h \mid V^h \mid \boldsymbol{r}_h\rangle + \sum_n \frac{\langle \boldsymbol{r}'_h \mid V^t \mid \phi_n\rangle \langle \phi_n \mid V^t \mid \boldsymbol{r}_h\rangle}{E - E_n}$$

➢ Quark channel contribution. Sum of discrete eigenstates E_n

◆ Energy dependent potential (denominator depends on E)

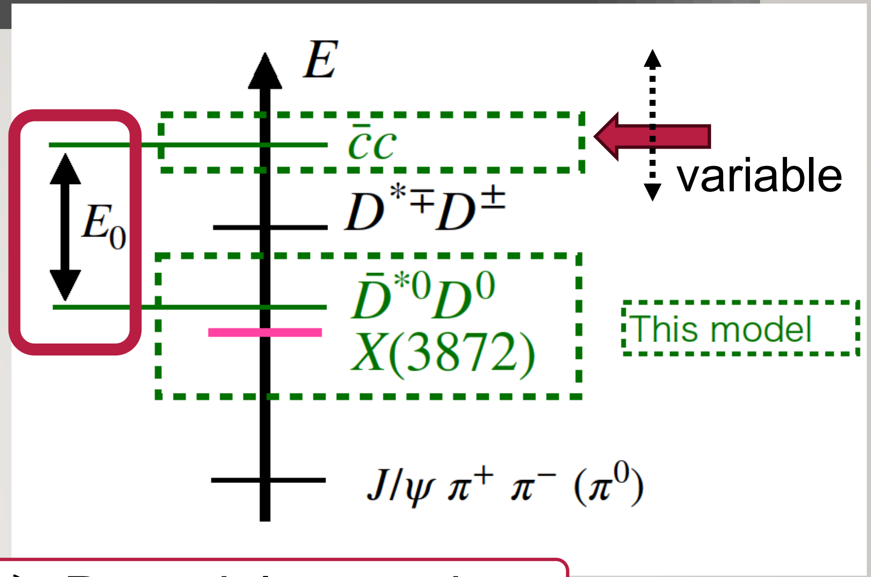
◆ Non-local potential (numerator depends on $\boldsymbol{r}, \boldsymbol{r}'$ independently)

Formulation of $X(3872)$

◆ Quark channel : $\bar{c}c$

$$H = \begin{pmatrix} T^q & 0 \\ 0 & T^h + \Delta \end{pmatrix} + \begin{pmatrix} V^q & V^t \\ V^t & V^h \end{pmatrix}$$

◆ Hadron channel : $D^0\,\bar{D}^{*0}$

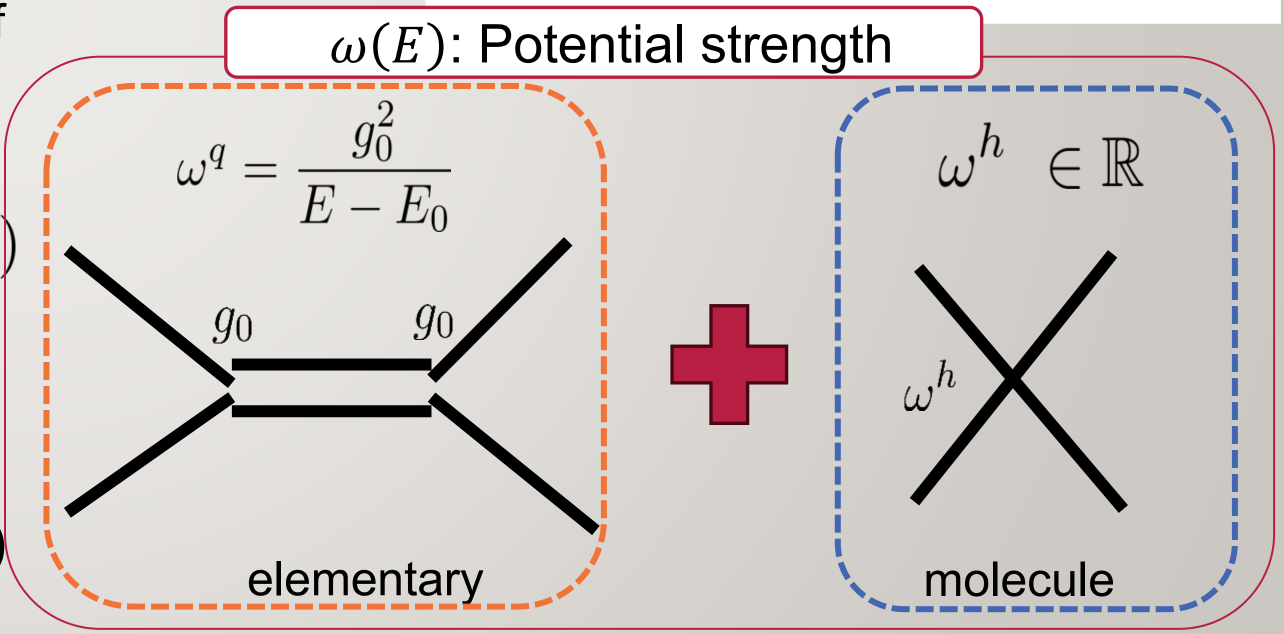


$$\langle \boldsymbol{r}'_h \mid V^t \mid \boldsymbol{r}_h \rangle = \omega(E)\frac{e^{-\mu r}}{r}\frac{e^{-\mu r'}}{r'}$$

✓ Separable

μ: cut-off

$$V_{\text{eff}}^{\bar{D}^* D}(\boldsymbol{r}, \boldsymbol{r}', E) = \left[\omega^q(E) + \omega^h(E)\right] V(\boldsymbol{r}) V(\boldsymbol{r}')$$
$$= \omega(E) V(\boldsymbol{r}) V(\boldsymbol{r}')$$



g_0: coupling constant

➢ Determine to reproduce mass of $X(3872)$

Wave function ψ and phase shift δ

- The wave function $\psi_k(r)$ and the phase shift $\delta(k)$ can be obtained analytically in our formulation

$$\langle \boldsymbol{r}_h' \mid V^t \mid \boldsymbol{r}_h \rangle = \omega(E) \frac{e^{-\mu r}}{r} \frac{e^{-\mu r'}}{r'}$$

- Scattering wave function ψ

$$\psi_k(r) = \frac{\sin[kr + \delta(k)] - \sin\delta(k) e^{-\mu r}}{kr}$$

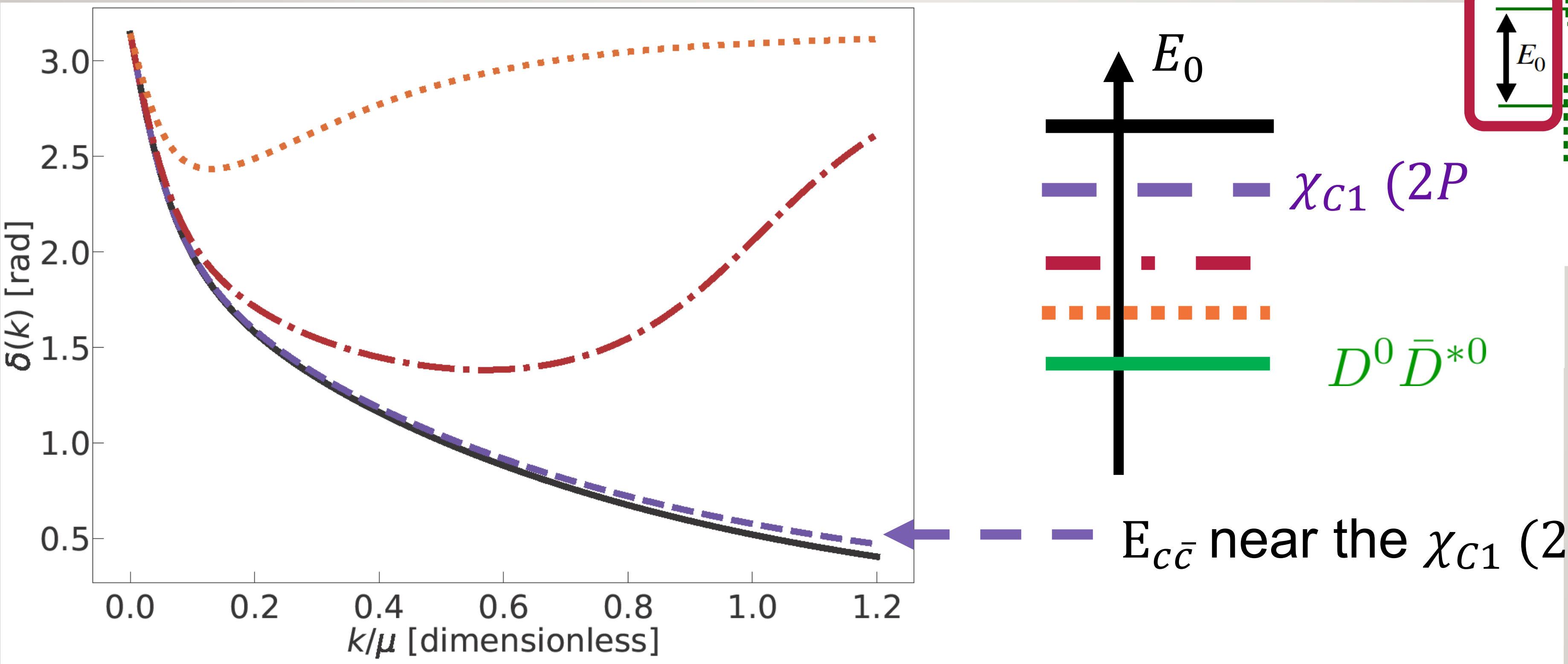
- Phase shift δ

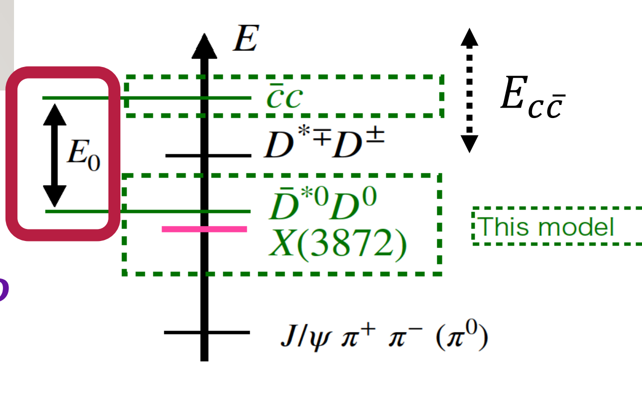
$$k \cot\delta(k) = -\frac{\mu[4\pi m\omega(E) + \mu^3]}{8\pi m\omega(E)} + \frac{1}{2\mu}\left[1 - \frac{2\mu^3}{4\pi m\omega(E)}\right] k^2 - \frac{1}{8\pi m\omega(E)} k^4$$

Scattering length a_0

Result: E_0 dependence of $V^h_{\rm eff}(r, r', E)$

■Compare exact phase shift $\delta(k)$ with different E_0





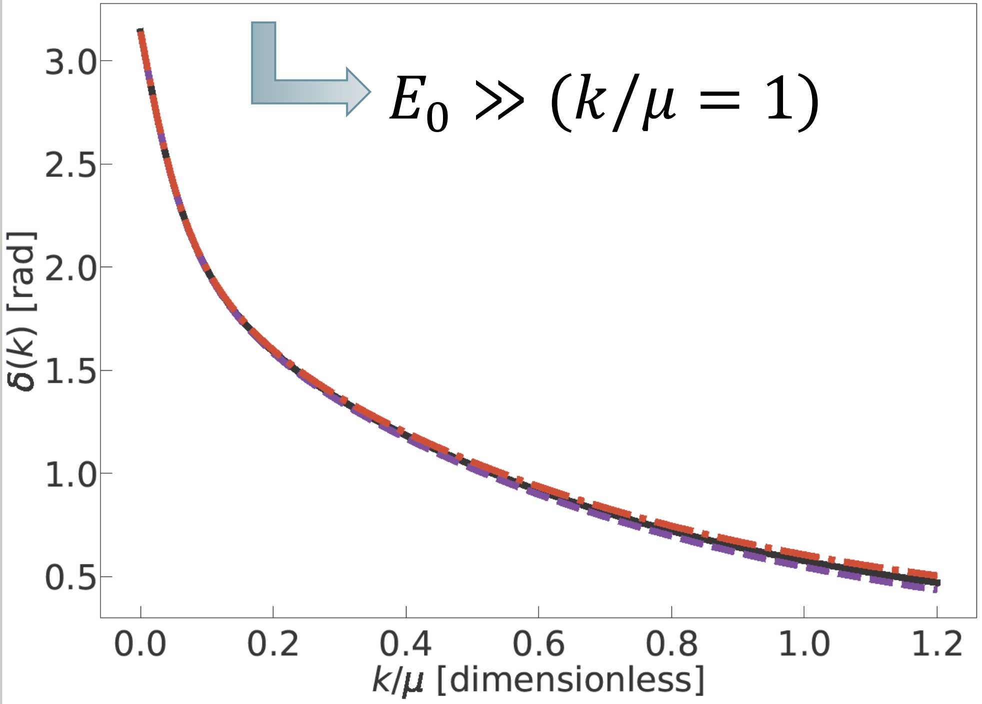
➢ E_0 dependence of exact $\delta(k)$ is large for small E_0

➢Binding energy is fixed so that $\delta(k)$ does not change in small k reagion

Result: V^h dependence of exact $\delta(k)$

◆Large E_0 ($\simeq E_{\chi_{C1}(2P)}$)

$E_0 \gg (k/\mu = 1)$

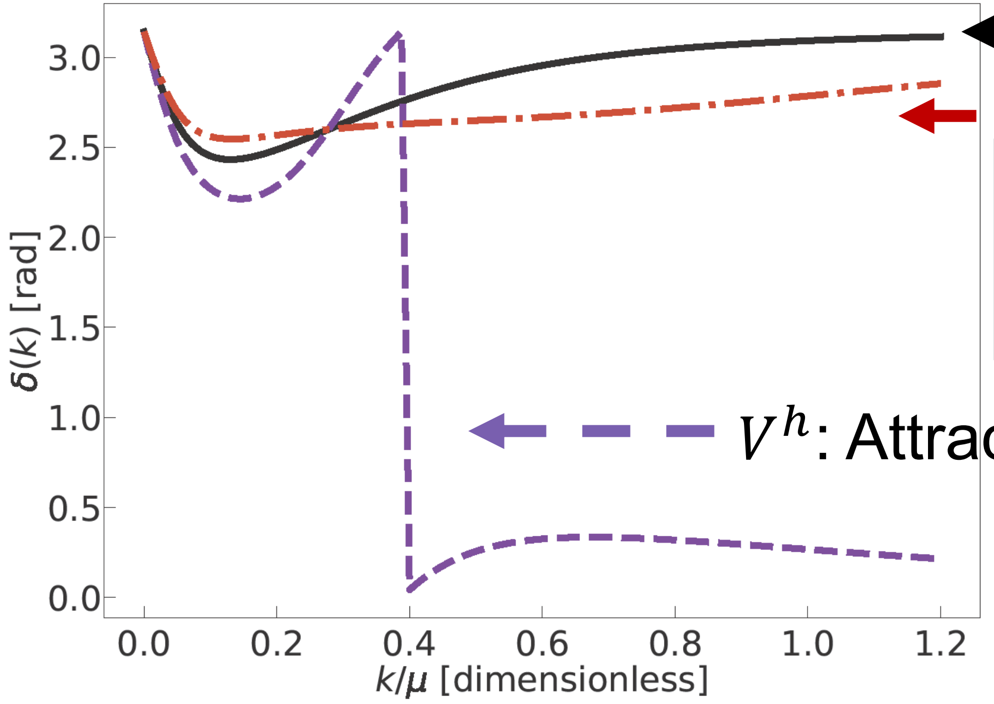


◆ Small E_0 ($\simeq \frac{1}{100} E_{\chi_{C1}(2P)}$)

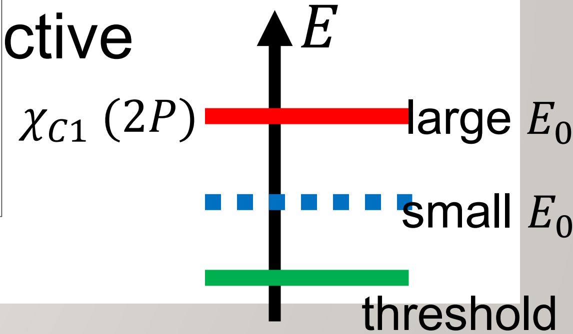
$E_0 \simeq (k/\mu = 1)$

$V^h = 0$

V^h: Repulsive



V^h: Attractive



- ➢ V^h dependence of exact $\delta(k)$ is large for small E_0
  - ✓ Quark potential strength $\omega^q = \frac{g_0^2}{E - E_0} \approx -\frac{g_0^2}{E_0} - \frac{E g_0^2}{E_0^2}$ is suppressed when E_0 is large

Result : Compositeness

- **Compositeness** is also calculatable analytically by considering Lippmann–Schwinger equation or the bound state wave function
  - Compositeness corresponds to elementary for 0, molecule for 1
- When quark-ch. energy is close to the threshold energy of meson creation, effect of the hadron-ch. is great

Quark channel energy	Binding energy [KeV]	Hadron channel potential	Compositeness [dimensionless]	Scattering length [fm]
$\chi_{C1}(2P)$	40	None	**0.991**	24.5
$\chi_{C1}(2P)$ / 100	40	Attractive	**0.719**	20.78
$\chi_{C1}(2P)$ / 100	40	None	**0.549**	17.87
$\chi_{C1}(2P)$ / 100	40	Repulsive	**0.444**	15.77

Let us see the $V_{\text{eff}}^{h}(\boldsymbol{r},\boldsymbol{r}',E)$

- To visualize the effective potential, we need to,

$$V_{\text{eff}}^{h}(\boldsymbol{r},\boldsymbol{r}',E) \quad \rangle \quad V_{\text{eff}}^{h}(\boldsymbol{r},E)$$

- Choose 2 ways to approximate and compare each in the next steps
  - Fixed parameters
    - E_0: $\chi_{c_1}(2P)$
    - μ : mass of π (lightest exchanging meson)
    - g_o : reproduce mass of $X(3872)$
    - $\omega_h = 0$: focus only the effect of the channel coupling

Local approximations

✓Approximation of non-local potential to local one by two different methods

[S.Aoki and K.Yazaki, PTEP 2022, no.3, 033B04 (2022)]

① Formal derivative expansion

- Express non-local potential in terms of derivatives of delta function by **Taylor expansion** at $\boldsymbol{r} = \boldsymbol{r}'$ directly

② Derivative expansion by HAL QCD method

- Construct the potential from wave function $\psi_{k_0}(r)$ obtained from Schrödinger equation with non-local potentials at momentum k_0

➤**Solve for potentials inversely** to construct the local potentials

HAL QCD method in detail

Energy dependent

order of derivative

- Schrödinger equation with **non-local potential** at $n+1$ points of k_i $(i = 0, 1, \cdots, n)$

$$-\frac{1}{2m}\nabla^2\psi_{k_i}(\boldsymbol{r}) + \int d^3\boldsymbol{r}'\, V_n(\boldsymbol{r}, \boldsymbol{r}', E)\psi_{k_i}(\boldsymbol{r}') = E_{k_i}\psi_{k_i}(\boldsymbol{r})$$

Unknown: $\psi_{k_i}(\boldsymbol{r})$

Obtain wavefunctions $\psi_{k_i}(\boldsymbol{r})$

Assume

- Wave functions $\psi_{k_i}(\boldsymbol{r})$ satisfy the Schrödinger equation with local potentials

$$\left(-\frac{1}{2m}\boldsymbol{\nabla}^2 + V_n(\boldsymbol{r}, \boldsymbol{\nabla})\right)\psi_{k_i}(\boldsymbol{r}) = E_{k_i}\psi_{k_i}(\boldsymbol{r}),$$

Unknown: $V_n(\boldsymbol{r}, \boldsymbol{\nabla})$

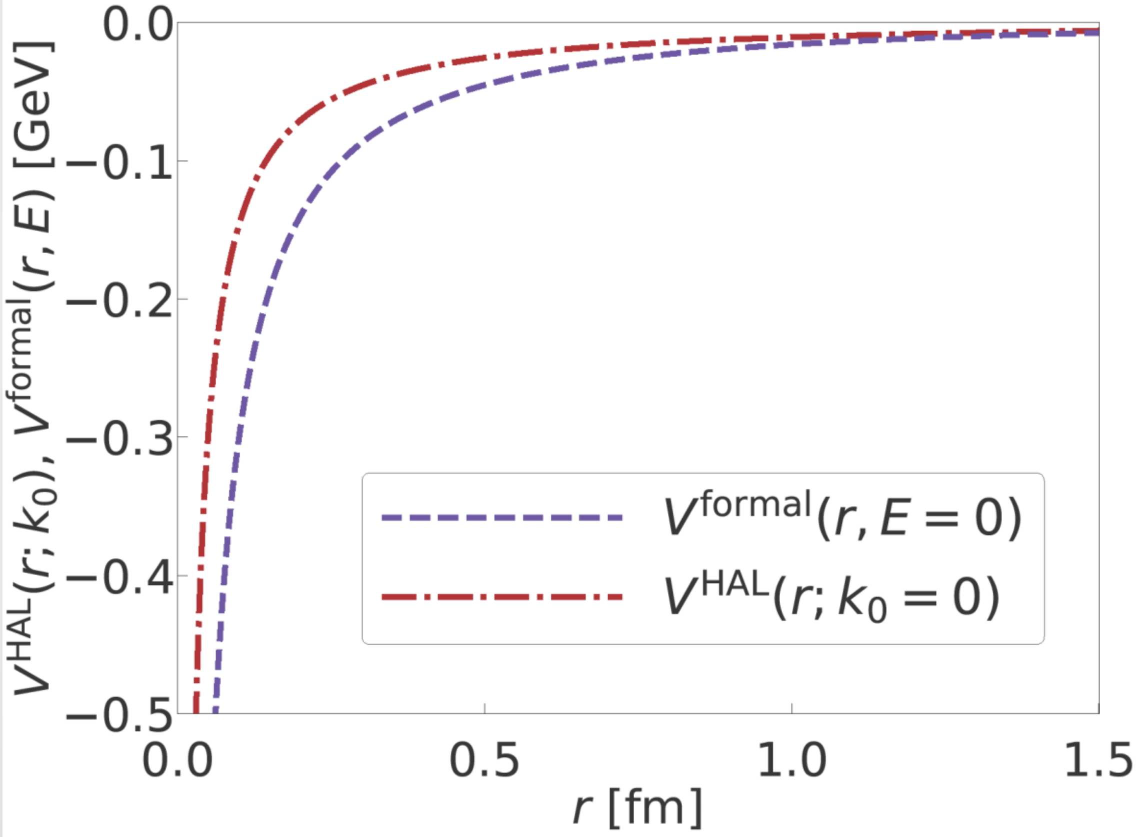
- Obtain local potential $V_n(\boldsymbol{r}, \boldsymbol{\nabla})$ by solving above equation for the potential inversely

Obtain $\psi = \psi_{k_i}$ exactly by solving local Schrödinger equation at $E = E_{k_i}$,
so that the $V_n(\boldsymbol{r}, \boldsymbol{\nabla})$ reproduces exact phase shift which is derived from $V_n(\boldsymbol{r}, \boldsymbol{r}', E)$

Result : comparison of V^{HAL} and V^{formal}

- Compare approximated potentials for $X(3872)$
  - V^{HAL} and V^{formal} from the same non-local potential

> - Both **potentials** are attractive in short-range
> - **Strengths** of potential are quantitatively different



✓How about physical observables from these potentials?

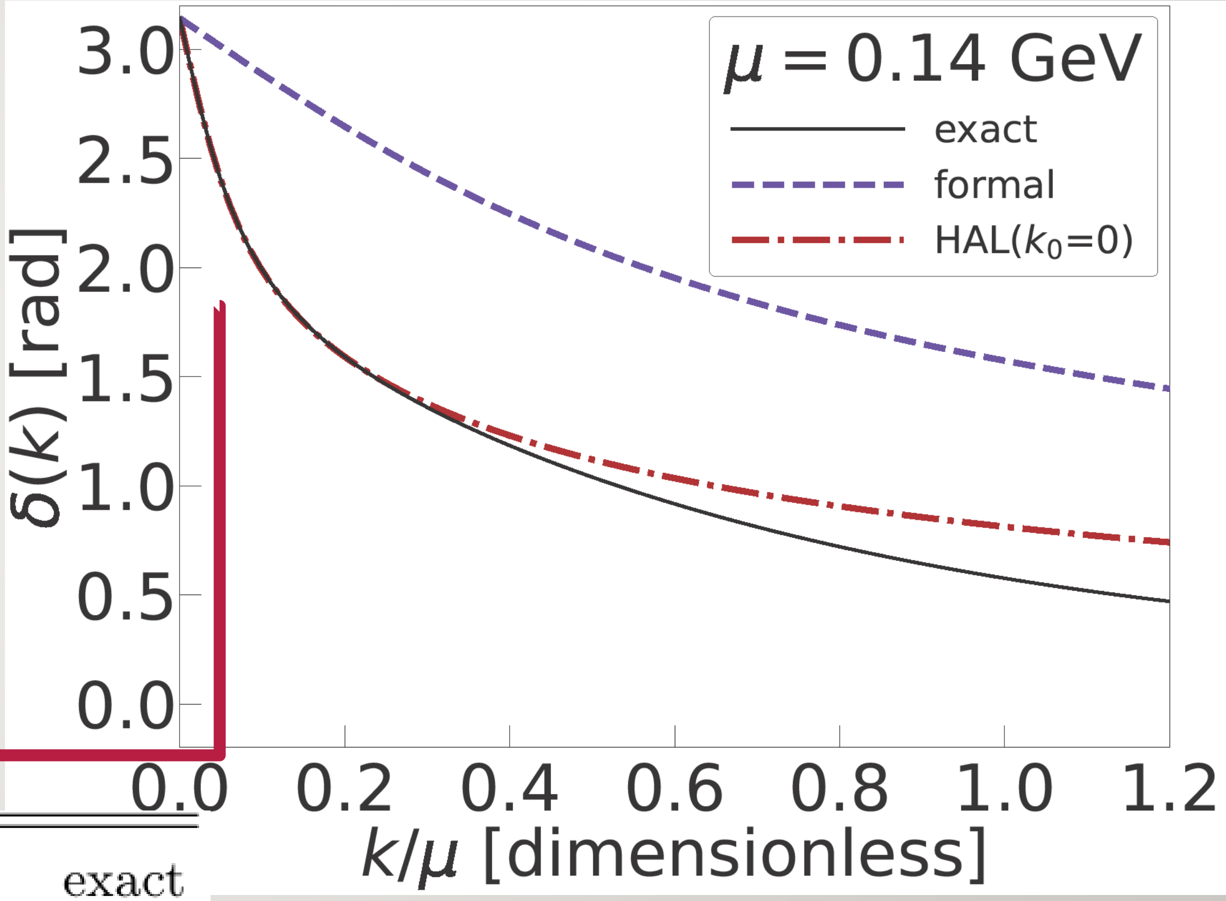
Result : Phase sift $\delta(k)$

- Compare phase shifts $\delta(k)$ from $V^{\text{formal}}(r, E)$ and $V^{\text{HAL}}(r; k_0 = 0)$ with exact $\delta(k)$ from non-local potential

- $\delta(k)$ from HAL QCD method reproduces exact $\delta(k)$, especially for small k

Scattering lengths are obtained from the gradient at $k = 0$

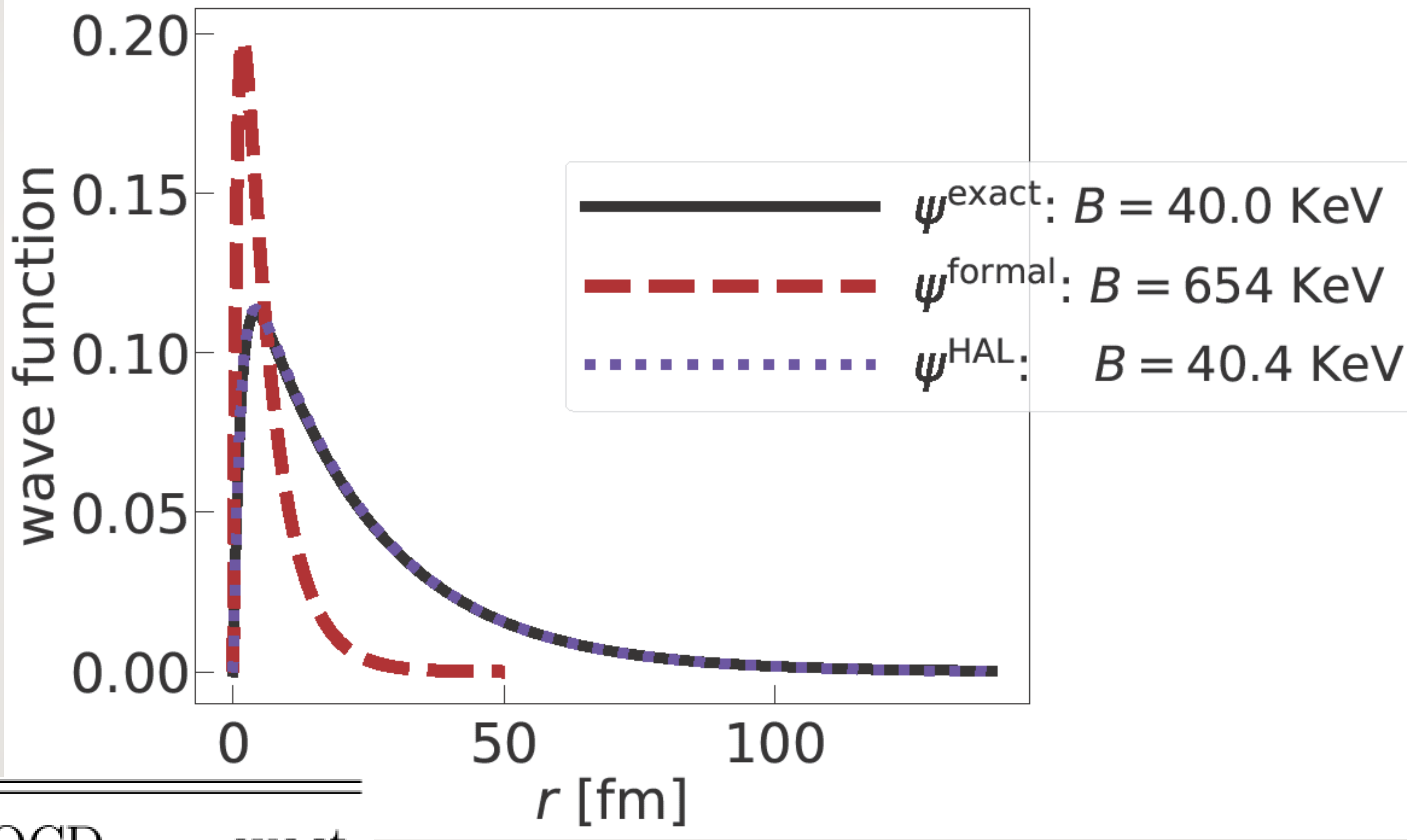
	formal	HAL QCD	exact
scattering length [fm]	6.55	24.48	24.48



$\delta(k)$ as a function of dimensionless k/μ

Result : wave function

- Binding energies change by localization, so the wave function also change
- Results corresponds to scattering length



	formal	HAL QCD	exact
scattering length [fm]	6.55	24.48	24.48

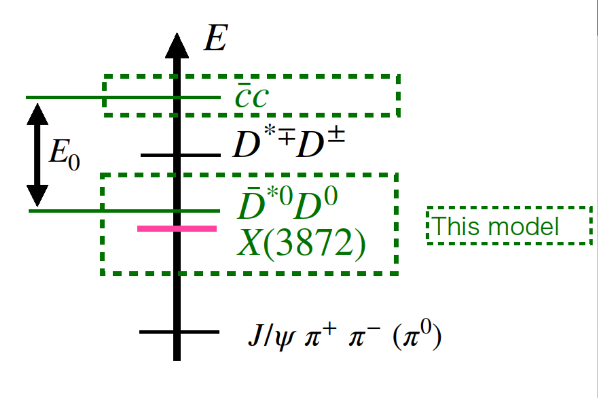
Summary

[I. Terashima and T. Hyodo, PhysRevC.108.035204 (2023)]

◆Channel coupling between quark and hadron d.o.f

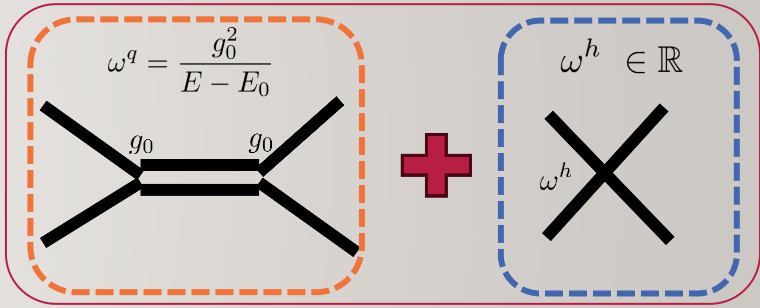
$$H = \begin{pmatrix} T^q & 0 \\ 0 & T^h + \Delta \end{pmatrix} + \begin{pmatrix} V^q & V^t \\ V^t & V^h \end{pmatrix}$$

◆Channel coupling between $c\bar{c}$ and $D\bar{D}^*$ in $X(3872)$



✓Formulate with explicit hadron d.o.f

$$V_{\rm eff}^{\bar{D}^* D}(\boldsymbol{r}, \boldsymbol{r}', E) = [\omega^q(E) + \omega^h(E)]V(\boldsymbol{r})V(\boldsymbol{r}')$$



✓Phase shift $\delta(k)$ depends on E_0 when E_0 is small enough

➢Phase shift $\delta(k)$ depends on V^h when E_0 is small enough

✓Convert non-local E-dependent potential to local by
(I) Formal derivative expansion, (II) HAL QCD method

✓ $V^{\rm formal}$ and $V^{\rm HAL}$ are quantitatively different

➢ $V^{\rm HAL}$ reproduces the exact $\delta(k)$ better than $V^{\rm formal}$