Low-energy constants in the chiral Lagrangian with baryon fields from Lattice QCD data

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- ✔ **Results from two flavor ensembles**
- ✔ **The chiral SU(3) Lagrangian with baryon fields**
- ✔ **Chiral extrapolation for baryon masses on Lattice QCD ensembles**
- ✔ **Summary and outlook**

arXiv:1801.06417, arXiv:1907.00714, arXiv:2301.06387, arXiv:2404.02769, arXiv:2406.07442

Chiral extrapolation for QCD with light quarks

✔ Sustainable approach

- use Lattice QCD data where they are 'cheap'
- exploit the nonlinearity of chiral symmetry
  to predict experimental data where taking Lattice QCD data is expensive

✔ the low-energy constants (LEC) of the chiral Lagrangian
that determine the quark-mass dependence of a hadron mass
impact the scattering processes involving the Goldstone bosons and that hadron

- critical challenge: what is the convergence radius of a chiral expansion
- conventional expansion schemes appear often very slow (if at all) convergent

✔ novel expansion scheme in terms of on-shell masses

- pioneered for various hadrons on flavour SU(3) ensembles
- chiral expansion is not necessarily smooth - first order transitions are possible
- revisted for flavour SU(2) chiral expansions

Chiral extrapolation for QCD with up and down quarks

✔ consider $m_{u,d} \simeq 2 - 5$ MeV to be small in QCD

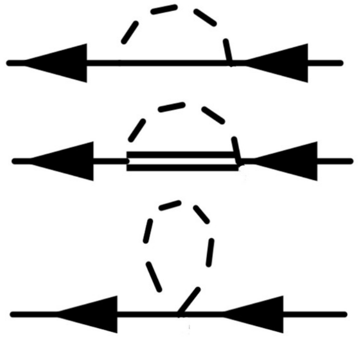
- approximate $\mathrm{SU}(2)_{L} \otimes \mathrm{SU}(2)_{R}$ chiral symmetry
- apply χPT in terms of the chiral Lagrangian

✔ how-to power count in the presence of heavy fields?

- controversial how to deal with the $\Delta(1232)$ baryon
- conventional expansion schemes appear very slow (if at all) convergent

✔ baryon masses at the one-loop level

- pioneered computation with on-shell masses in loops
- converging chiral expansion of loop contributions
- possibility of parametric phase transitions



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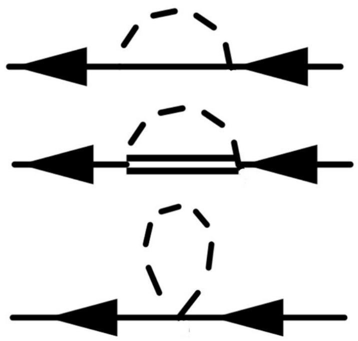

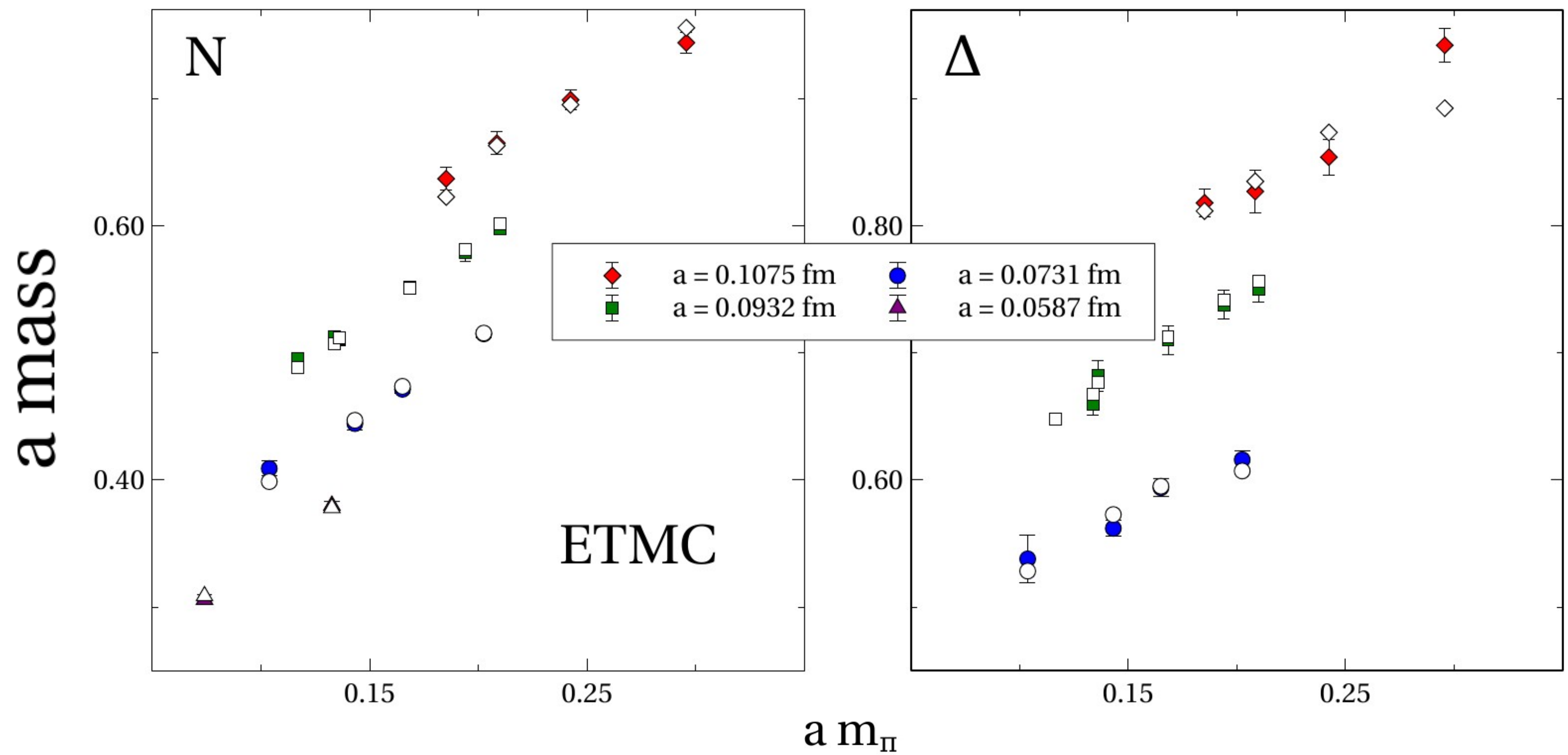

MFML, U. Sauerwein, RGE Timmermans, Eur.Phys.J.C 80 (2020) 9, 844

Global fit to lattice data from ETMC, CLS and RQCD

✔ Global fit to results on flavour SU(2) ensembles

- consider nucleon and isobar masses and axial-vector form factor
- use novel form of one-loop expressions according to [1]

  for the first time also induced pseudo-scalar form factor
- in total 124 lattice data points with $\chi^2/N_{dof} \sim 1.077$
- consider all data points with $m_\pi < 525$ MeV

✔ How much strangeness is needed?

- huge discrepancies with empirical values
- $G_A(0) = 1.2732(23)$
- $\sigma_{\pi N} = 58(5)$ MeV
- $f_\pi = (92.21 \pm 0.14)$ MeV

	Fit results
$G_A(0)$	$1.2284(^{+0.0021}_{-0.0059})$
$\langle r_A^2 \rangle$ [fm^2]	$0.2014(^{+0.0032}_{-0.0035})$
g_P	$8.2521(^{+0.039}_{-0.039})$
$\sigma_{\pi N}$ [MeV]	$42.22(^{+0.02}_{-0.05})$
$\sigma_{\pi \Delta}$ [MeV]	$35.27(^{+0.01}_{-0.06})$
f_π [MeV]	$84.96(^{+0.29}_{-0.82})$

[1] F. Hermsen, T. Isken, MFML and D. Thoma, arXiv:2402.04905

Lattice QCD for baryon octet and decuplet masses

✔ Baryon masses on CLS ensembles from Regensburg

- large set of ensembles at different β values, quark masses and volumes
- ensembles at fixed m_s or $m_u = m_d = m_s$ or $m_u + m_d + m_s$ :: crucial for chiral SU(3)
- there are about 400 data points with $m_\pi, m_K < 550$ MeV
- a significant continuum limit extrapolation appears feasible

arXiv:2211.03744

✔ The challenge of a global fit

- finite volume effects from chiral one-loop contributions are considered
- leading and subleading LEC have a quadratice lattice scale dependence
- global scale-setting with baryon octet and decplet masses

  physical baryon octet and decuplet masses are always reproduced
- acurracy level : self-consistent one-loop at $\mathrm{N}^3\mathrm{LO}$

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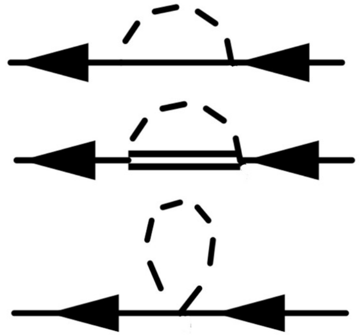
✔ Ensembles with physical pion masses: challenges

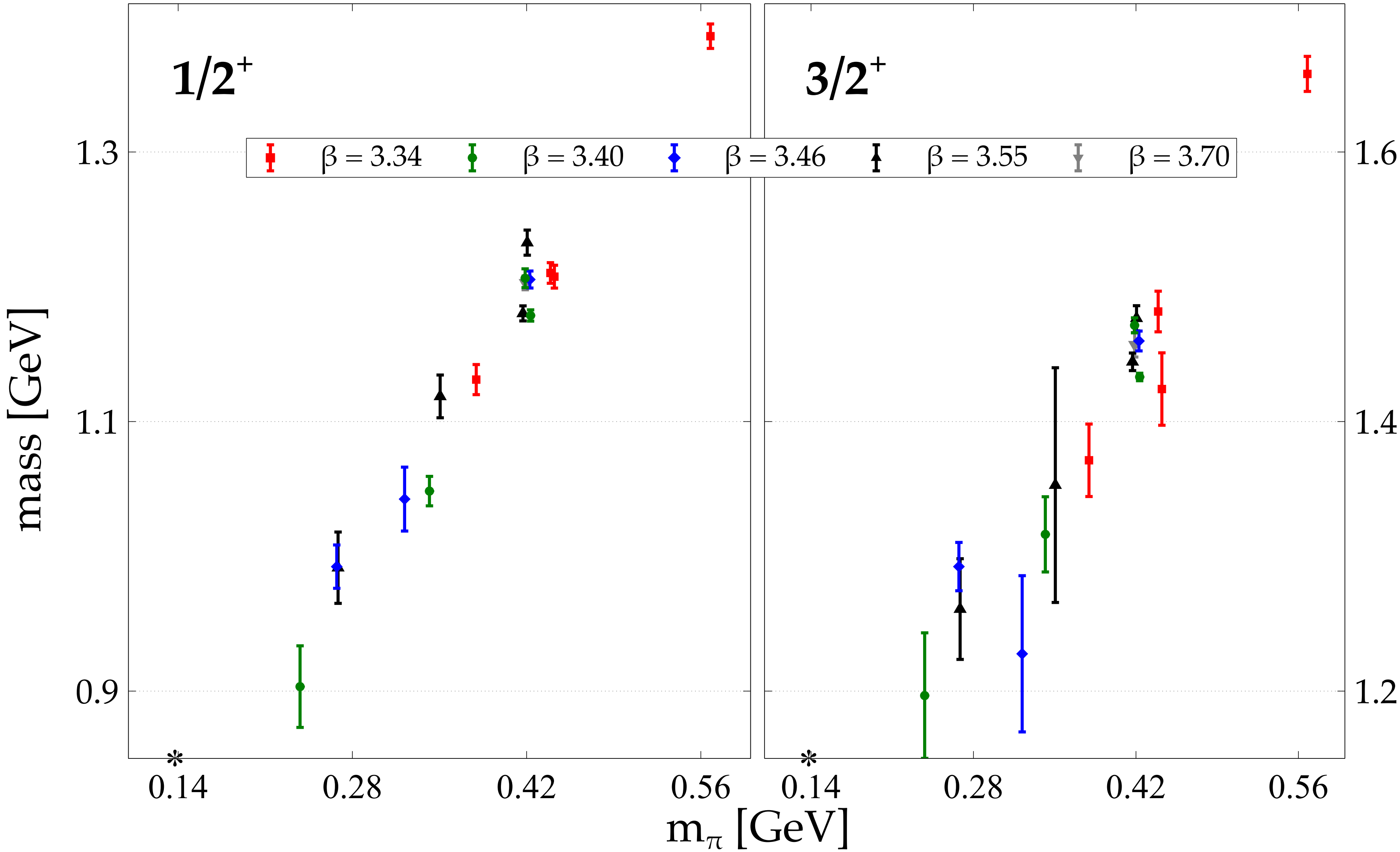
- excited state contamination of exponential signals?
- infinite volume extrapolation of isobar states?
- from an EFT point of view unphysical quark-masses are more interesting!



Chiral extrapolation for QCD with strange quarks

✔ baryon masses on CLS ensembles from Regensburg arXiv:2211.03744

- at flavor symmetric points $m_u = m_d = m_s$ (improve decuplet data !)

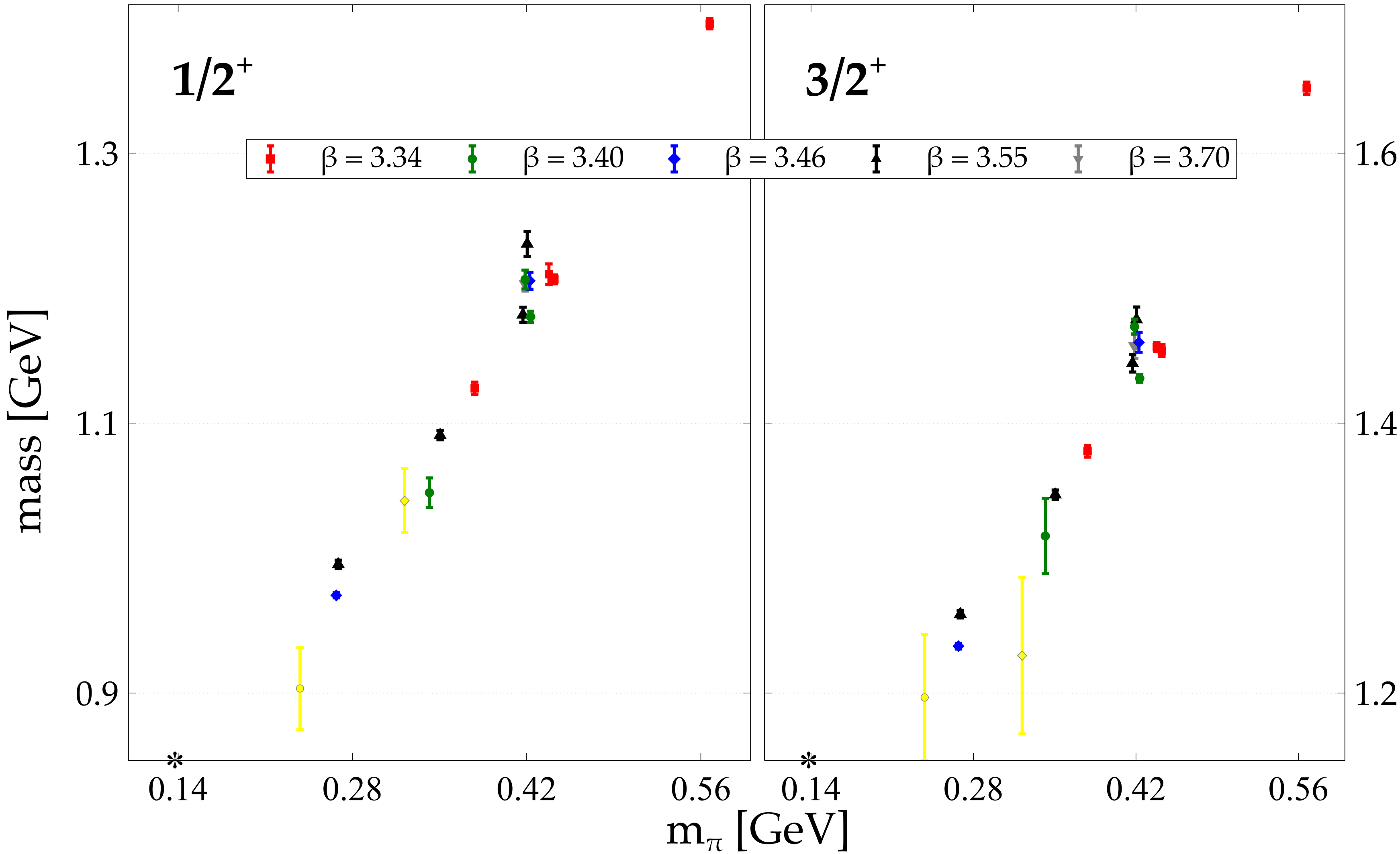


MFML, Y. Heo, X.-Y. Guo, Eur. Phys.J.C 83 (2023) 5, 440

Chiral extrapolation for QCD with strange quarks

✔ baryon masses on CLS ensembles improved at GSI by Renwick J. Hudspith

- at flavor symmetric points $m_u = m_d = m_s$ (more accurate decuplet masses !)



MFML, Y. Heo, RJ Hudspith, arXiv:2406.07442

The chiral SU(3) Lagrangian with baryon fields

$$\Phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\,\eta \end{pmatrix}$$

Goldstone boson octet $(J^P = 0^-)$

baryon octet $(J^P = \frac{1}{2}^+)$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\,\Lambda \end{pmatrix}$$

✔ Leading order terms

covariant derivative $\partial_\mu = \partial_\mu + \cdots$

$$\begin{aligned}
\mathcal{L} \;&=\; \mathrm{tr}\left\{\bar{B}(i\,\partial\cdot\gamma - M_{[8]})\,B\right\} + F\,\mathrm{tr}\left\{\bar{B}\,\gamma^\mu\gamma_5\,[i\,U_\mu, B]\right\} + D\,\mathrm{tr}\left\{\bar{B}\,\gamma^\mu\gamma_5\,\{i\,U_\mu,\,B\}\right\} \\
&-\; \mathrm{tr}\left\{\bar{B}_\mu\cdot\left((i\,\partial\cdot\gamma - M_{[10]})\,g^{\mu\nu} - i\,(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu) + \gamma^\mu(i\,\partial\cdot\gamma + M_{[10]})\gamma^\nu\right) B_\nu\right\} \\
&+\; C\left(\mathrm{tr}\left\{(\bar{B}_\mu\cdot i\,U^\mu)\,B\right\} + \mathrm{h.c.}\right) + H\,\mathrm{tr}\left\{(\bar{B}^\mu\cdot\gamma_\nu\gamma_5\,B_\mu)\,i\,U^\nu\right\}
\end{aligned}$$

- $U_\mu = \frac{1}{2}\,u^\dagger\left(\partial_\mu\, e^{i\,\frac{\Phi}{f}}\right)u^\dagger - \frac{i}{2}\,u^\dagger\,(v_\mu + a_\mu)\,u + \frac{i}{2}\,u\,(v_\mu - a_\mu)\,u^\dagger$ with $u = e^{i\,\frac{\Phi}{2\,f}}$
- from $B \to B' + e + \bar{\nu}_e$: $F \simeq 0.45$ and $D \simeq 0.80$
- from large-N_c: $H = 9\,F - 3\,D$ and $C = 2\,D$

Chiral symmetry breaking terms

$$\mathcal{L}_\chi^{(2)} = 2\, b_0 \,\mathrm{tr}\,\left(\bar{B}\, B\right) \mathrm{tr}\,(\chi_+) + 2\, b_D \,\mathrm{tr}\,\left(\bar{B}\,\{\chi_+,\, B\}\right) + 2\, b_F \,\mathrm{tr}\,\left(\bar{B}\,[\chi_+,\, B]\right)$$
$$-\; 2\, d_0 \,\mathrm{tr}\,\left(\bar{B}_\mu \cdot B^\mu\right) \mathrm{tr}\,(\chi_+) - 2\, d_D \,\mathrm{tr}\,\left(\left(\bar{B}_\mu \cdot B^\mu\right) \chi_+\right)$$

$$\chi_+ = \chi_0 - \frac{1}{8\, f^2}\left\{\Phi, \left\{\Phi,\, \chi_0\right\}\right\} + \mathcal{O}\left(\Phi^4\right)$$

$$\text{quark} - \text{mass matrix} \qquad \chi_0 \sim \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

✔ Relevance of low-energy parameters

- quark-mass dependence of the baryon masses $\leftrightarrow$ lattice QCD
- meson-baryon scattering $\leftrightarrow$ resonances in QCD
- nucleon sigma terms, $\langle N|\bar{u}\, u|N\rangle$, $\langle N|\bar{d}\, d|N\rangle$ and $\langle N|\bar{s}\, s|N\rangle$ relevant in WIMP scenarios – ATLAS

see e.g. arXiv:1805.09795

Quark-mass dependence of the baryon masses

✔ A challenge

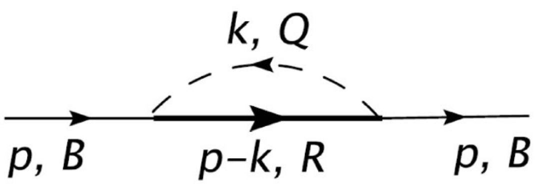
- 'poor' convergence in the heavy-baryon formulation of χPT

$$\text{e.g.} \qquad M_\Xi = (1018 + 1311 - 1007)\,\text{MeV} = 1322\,\text{MeV}$$

- conventional χPT inconsistent with three-flavor QCD lattice simulations?

✔ Multi-scale problem: how to powercount?

$$\frac{m_\pi}{M_N} \sim \frac{m_\pi}{M_\Delta} \sim \frac{m_\pi}{4\pi\, f} \sim Q$$



- insist on flavour SU(3) symmetric counting

$$\frac{m_Q}{M_B} \sim Q \quad \text{with} \quad Q \in [8] \qquad \text{and} \qquad B \in [8], [10]$$

- how to count mass differences?

$$M_R - M_B \sim Q \quad \text{with} \quad B \in [8] \quad \text{and} \quad R \in [10] \qquad \text{else} \qquad \sim Q^2$$

Quark-mass dependence of the baryon masses

✔ A challenge

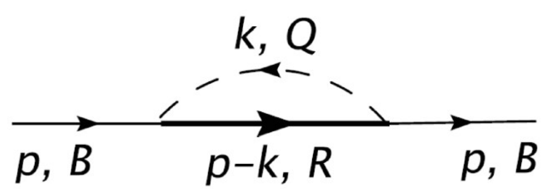
- ’poor’ convergence in the heavy-baryon formulation of χPT

$$\text{e.g.} \qquad M_\Xi = (1018 + 1311 - 1007)\,\text{MeV} = 1322\,\text{MeV}$$

- conventional χPT inconsistent with three-flavor QCD lattice simulations?

✔ Bubble-loops depend critically on internal masses

$$\Sigma_B(p) = \sum_{Q\in[8]} \sum_{R\in[8],[10]}$$



$$+ \cdots$$

- chiral expansion in terms of physical meson and baryon masses
- reorganize conventional χPT keeping its model independence
- renormalization scale and reparametrization invariance

Quark-mass dependence of the baryon masses

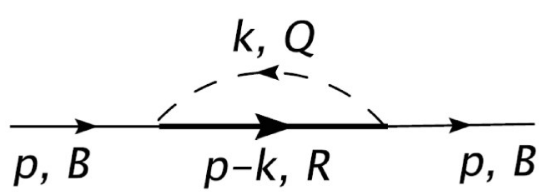
✔ Good convergence of reordered chiral expansion

- use physical meson and baryon masses
- the full one-loop contributions can be decomposed into chiral moments
- taking empirical masses the N^4LO effects are less than 8 MeV

✔ Baryon masses determined by a non-linear system

$$M_B - \Sigma_B(M_B) = \begin{cases} M_{[8]} & \text{for} \quad B \in [8] \\ M_{[10]} & \text{for} \quad B \in [10] \end{cases}$$

$$\Sigma_B(p) = \sum_{Q \in [8]} \sum_{R \in [8],[10]}$$



$$+ \cdots$$

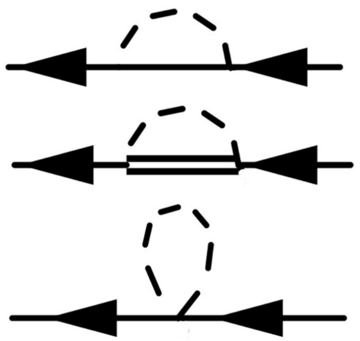
- numerical challenge

Low-energy parameters from lattice QCD simulations

✔ A global fit to baryon masses on CLS ensembles

- consider all ensembles with $m_\pi < 550$ MeV and $m_K < 550$ MeV
- finite volume effects from chiral one-loop contributions are considered
- leading and subleading LEC have a quadratice lattice scale dependence
- global scale-setting with baryon octet and decplet masses

  physical baryon octet and decuplet masses are always reproduced
- acurracy level : self-consistent one-loop at N^{3}LO
- determine systematic error such that $\chi^2/d.o.f. \simeq 1$



✔ Sum rules from QCD in the limit of large N_c

- significant parameter reduction
- use such relations only for LEC relevant at N^{3}LO
- we adjust 44= $30_{\mathrm{LEC}} + 14_{\mathrm{Lattice}}$ parameters to the lattice data set
- use GENEVA on heterogenous MPI grids with 2000 ranks ::
  Comput. Softw. Big Sci. 7 (2023) 1, 4

Predictions for LEC in the chiral limit

✔ The impact of axial couplings

- from large-N_c: $H = 9\,F - 3\,D$ and $C = 2\,D$
- may use empirical tree-level estimates for F, D but D, H from large-N_c?
- fit all to the Lattice Data set - do not use large-N_c relations here

✔ A fit to baryon masses

	Fit arXiv:2301.06387	Fit arXiv:2406.07442
f [MeV]	92.4^*	82.39(77)
F	0.51^*	0.4887(63)
D	0.72^*	0.4800(39)
C	1.44^*	0.9537(80)
H	2.43^*	1.8230(143)
M [MeV]	804.3(1)	865.1(9)
$M + \Delta$ [MeV]	1115.2(1)	1117.8(11.5)

- direct fit to the data set with $\chi^2/d.o.f. = 0.96$

Quark-masses from Lattice QCD ensembles

$$m_\pi^2 = 2\,B_0\,m - \frac{1}{18\,f^2}\left\{-10\,m_\pi^2 + 4\,m_K^2 - 3\,m_\eta^2\right\}\bar I_\pi - \frac{1}{6\,f^2}\,m_\pi^2\,\bar I_\eta$$
$$+ \frac{8}{f^2}\,m_\pi^2\,(m_\pi^2 + 2\,m_K^2)\,(2\,L_6 - L_4) + \frac{8}{f^2}\,m_\pi^4\,(2\,L_8 - L_5)\,,$$
$$m_K^2 = B_0\,(m + m_s) - \frac{1}{6\,f^2}\left\{m_\pi^2 - 4\,m_K^2 + 3\,m_\eta^2\right\}\bar I_K + \frac{1}{3\,f^2}\,m_K^2\,\bar I_\eta$$
$$+ \frac{12}{f^2}\,m_K^2\,(m_\pi^2 + m_\eta^2)\,(2\,L_6 - L_4) + \frac{8}{f^2}\,m_K^4\,(2\,L_8 - L_5)\,,$$
$$m_\eta^2 = \frac{2}{3}\,B_0\,(m + 2\,m_s) - \frac{1}{2\,f^2}\,m_\pi^2\,\bar I_\pi - \frac{1}{6\,f^2}\left\{7\,m_\eta^2 - 4\,m_K^2\right\}\bar I_\eta + \frac{4}{3\,f^2}\,m_K^2\,\bar I_K$$
$$+ \frac{24}{f^2}\,m_\eta^2\,(2\,m_K^2 - m_\eta^2)\,(2\,L_6 - L_4) + \frac{8}{f^2}\,m_\eta^4\,(2\,L_8 - L_5)$$
$$+ \frac{16}{5\,f^2}\,(3\,m_\pi^4 - 8\,m_K^4 - 8\,m_\eta^2\,m_K^2 + 13\,m_\eta^4)\,(3\,L_7 + L_8)\,,$$

$$\bar I_Q = \frac{m_Q^2}{(4\,\pi)^2}\,\log\left(\frac{m_Q^2}{\mu^2}\right) + \text{finite} - \text{box corrections}$$

✔ Use on-shell meson masses

- for given pion and kaon mass determine $m = m_u = m_d$ and m_s
- such an analysis determines Gasser and Leutwyler LEC

Predictions for quark-mass ratios on lattice ensembles

✔ How to fit the lattice data?

- take pion and kaon mass of the ensemble → compute quark masses
- this requires the low-energy constants $L_4 - 2\,L_6, L_5 - 2\,L_8, L_8 + 3\,L_7$
- we do not fit to the quark-mass ratios (not given) by the lattice groups!

✔ A fit to baryon masses

- renormalization scale $\mu = 0.77$ GeV

	Fit arXiv:2301.06387	Fit arXiv:2406.07442
$10^3\,(2\,L_6 - L_4)$	0.0411(3)	0.0235(47)
$10^3\,(2\,L_8 - L_5)$	0.0826(12)	-0.0631(117)
$10^3\,(L_8 + 3\,L_7)$	-0.4768(4)	-0.3155(46)
f [MeV]	92.4*	82.4(8)
m_s/m	26.2(1)	27.5(1)

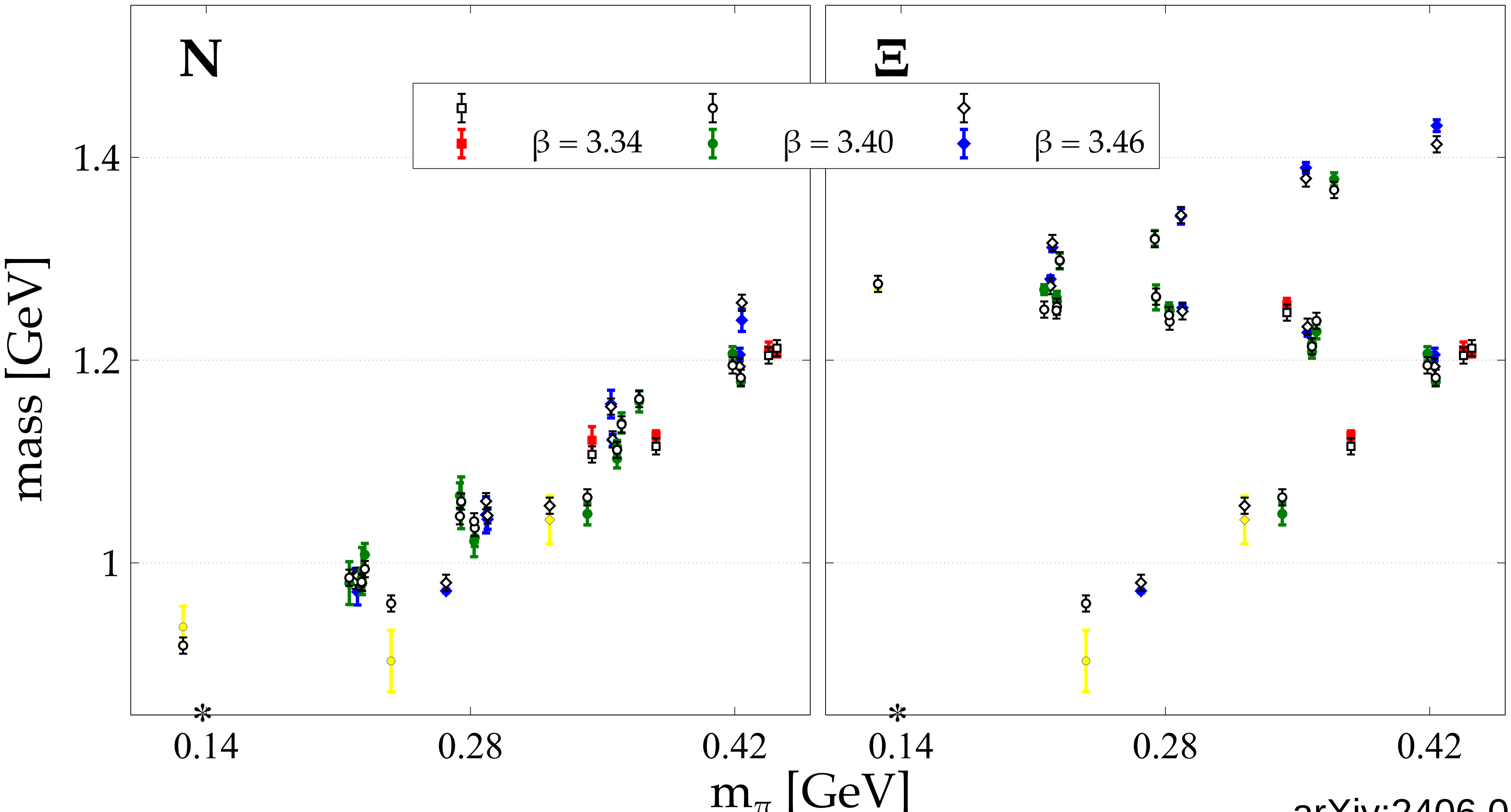
- small statistical error from global fits
- with improved data values for f and m_s/m compatible with FLAG

Pion-mass dependence of the baryon octet masses

- consider all ensembles with $m_\pi < 550$ MeV and $m_K < 550$ MeV
- leading and subleading LEC have a quadratice lattice scale dependence

$$M \to M + a^2\,\gamma_{M_8}\,,\; b_0 \to b_0 + a^2\,\gamma_{b_0}\,,\quad b_D \to b_F + a^2\,\gamma_{b_D}\,,\quad b_F \to b_F + a^2\,\gamma_{b_F}$$

- with systematic error of 8 MeV :: $\chi^2/d.o.f. = 0.96$



arXiv:2406.07442

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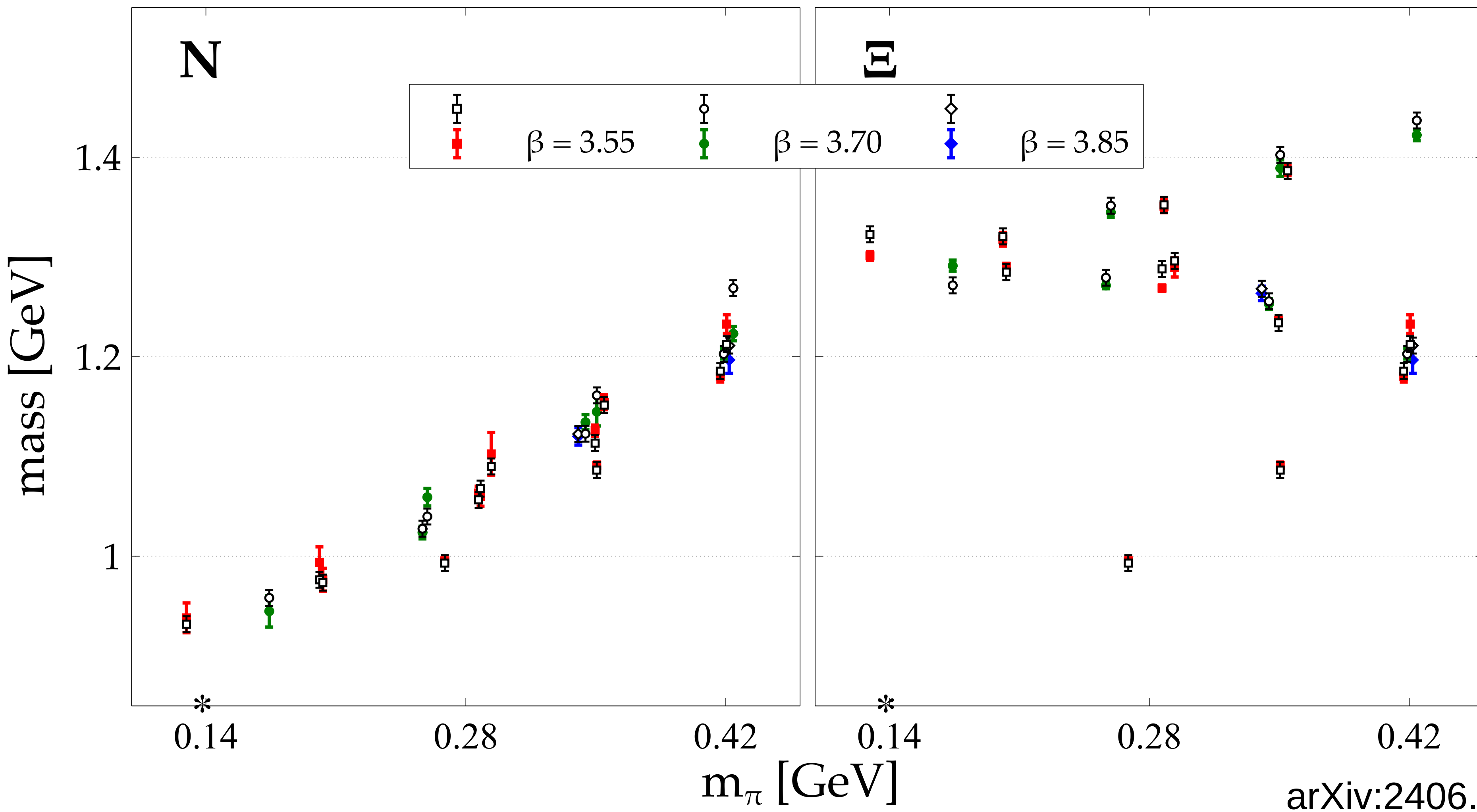

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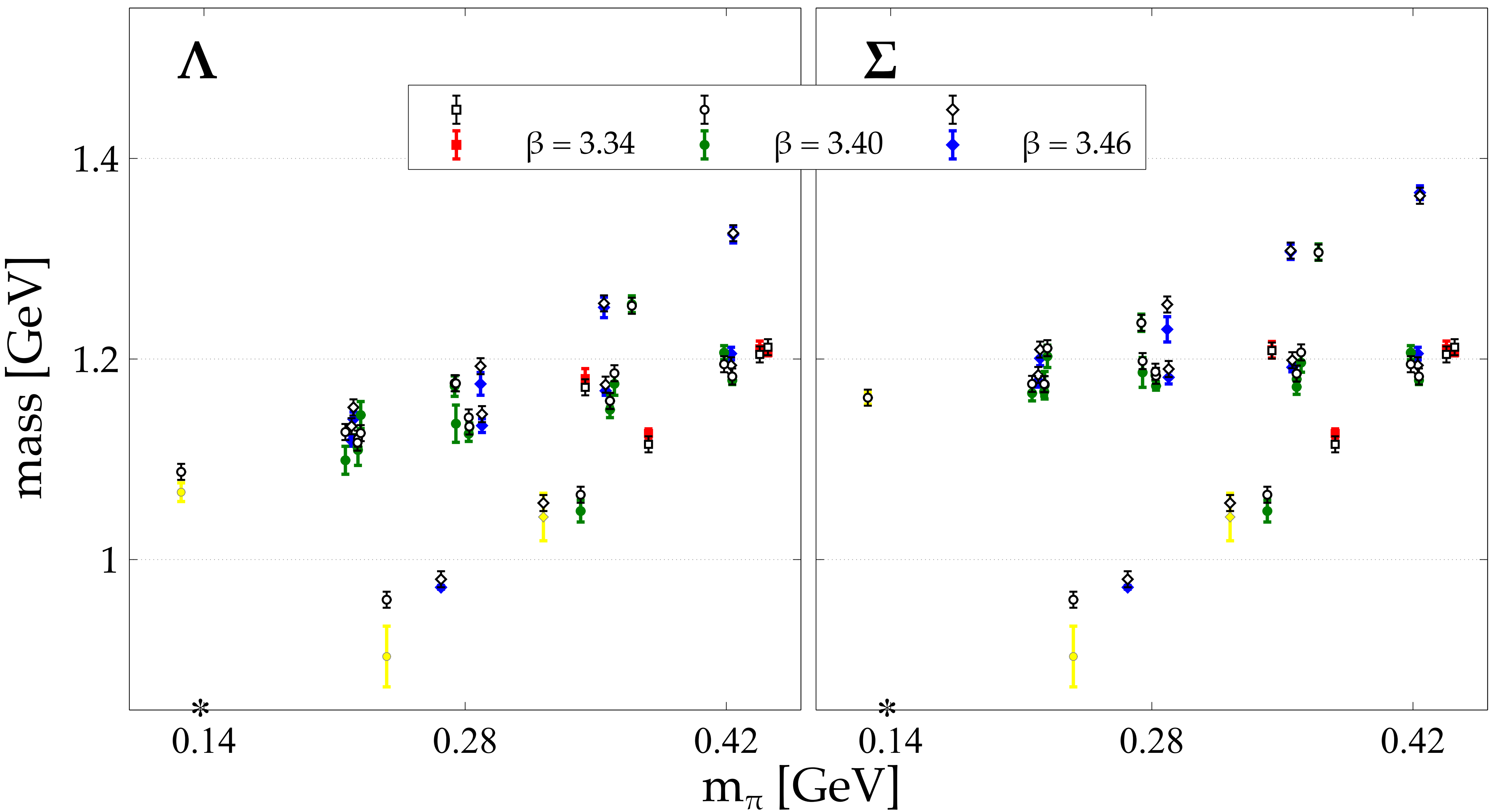

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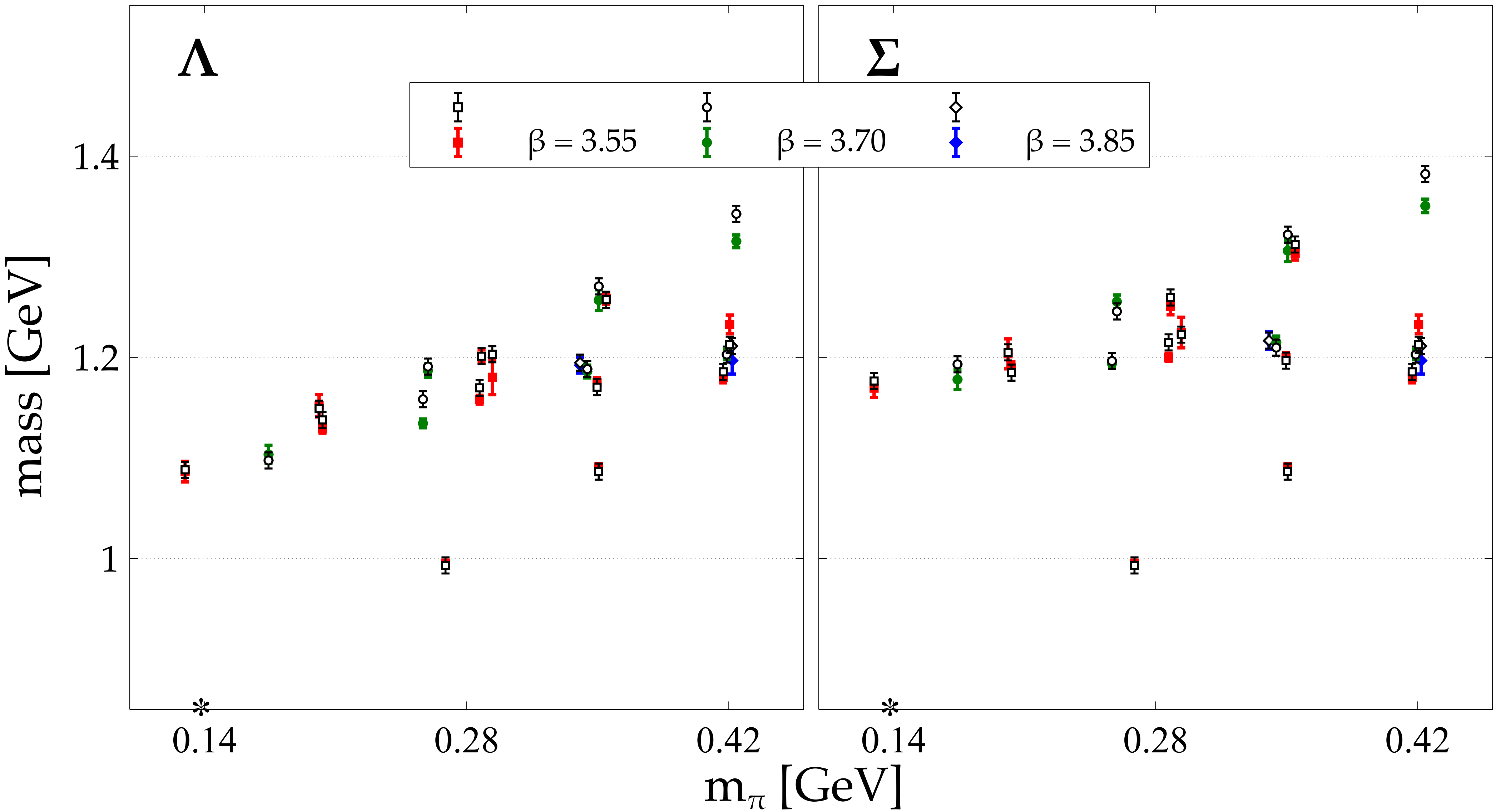
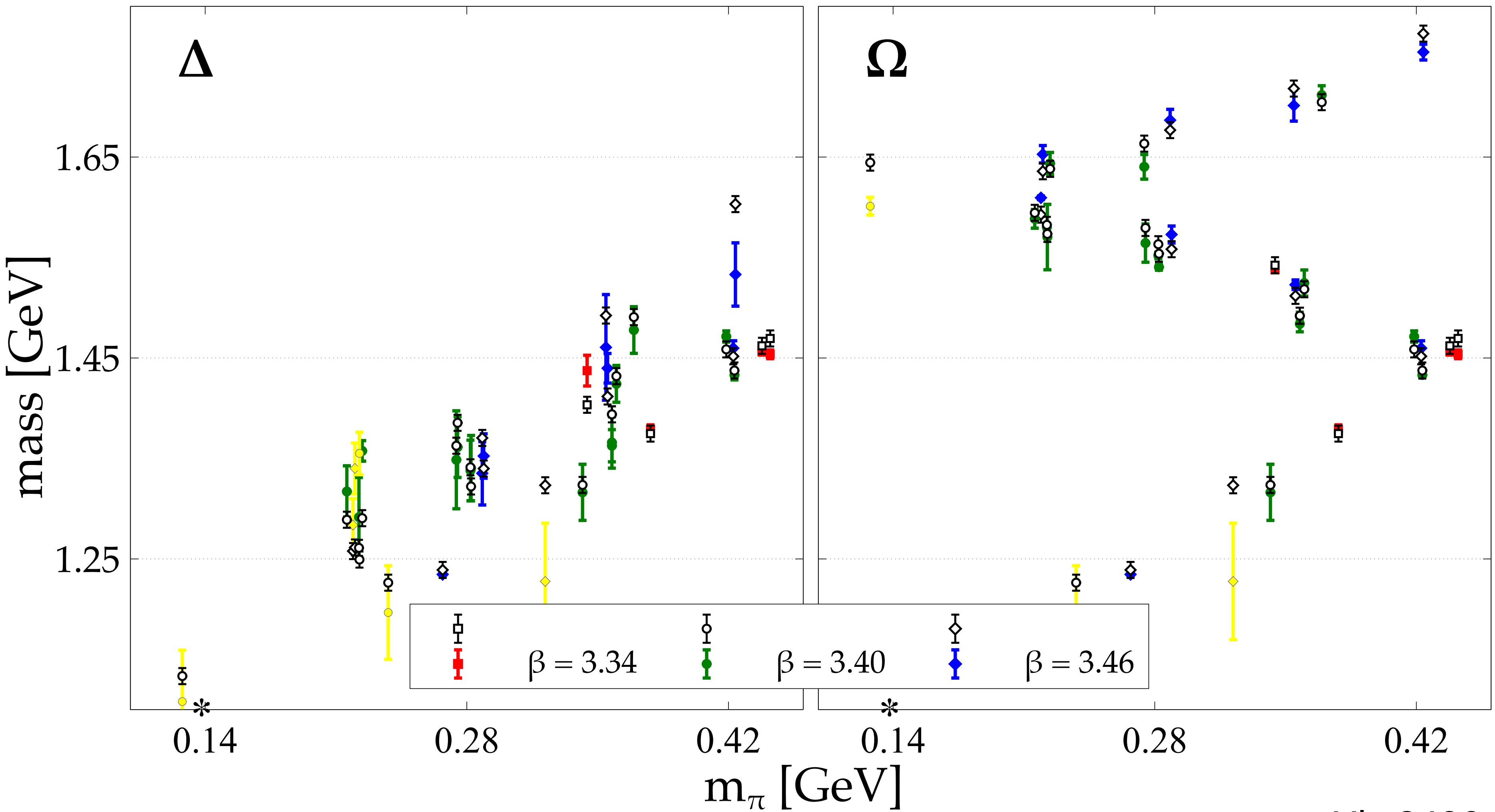


arXiv:2406.07442

Pion-mass dependence of the baryon decuplet masses

- consider all ensembles with $m_\pi < 550$ MeV and $m_K < 550$ MeV
- leading and subleading LEC have a quadratice lattice scale dependence
- with systematic error of 8 MeV :: $\chi^2/d.o.f. = 0.96$



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Pion-mass dependence of the baryon decuplet masses

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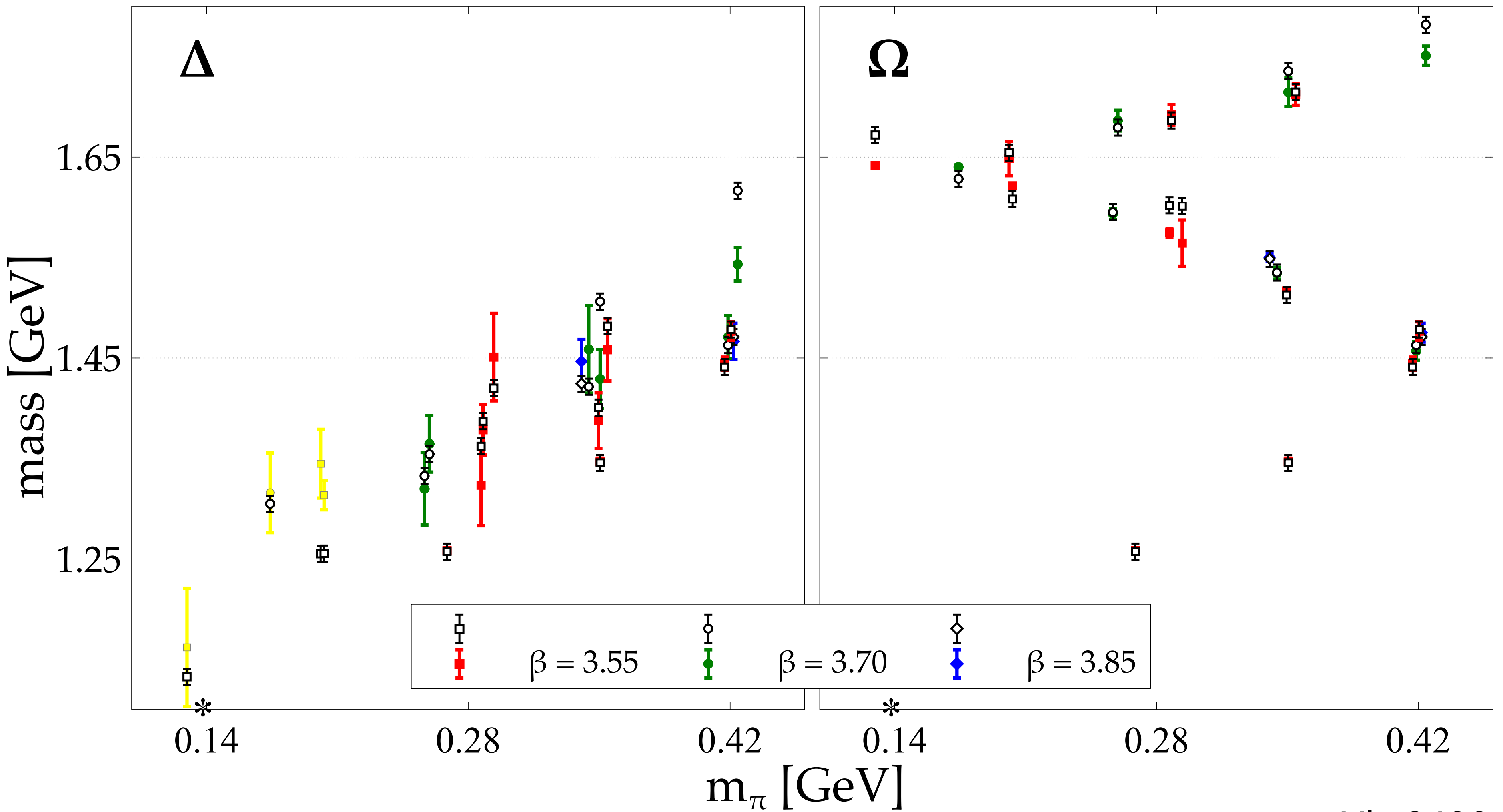

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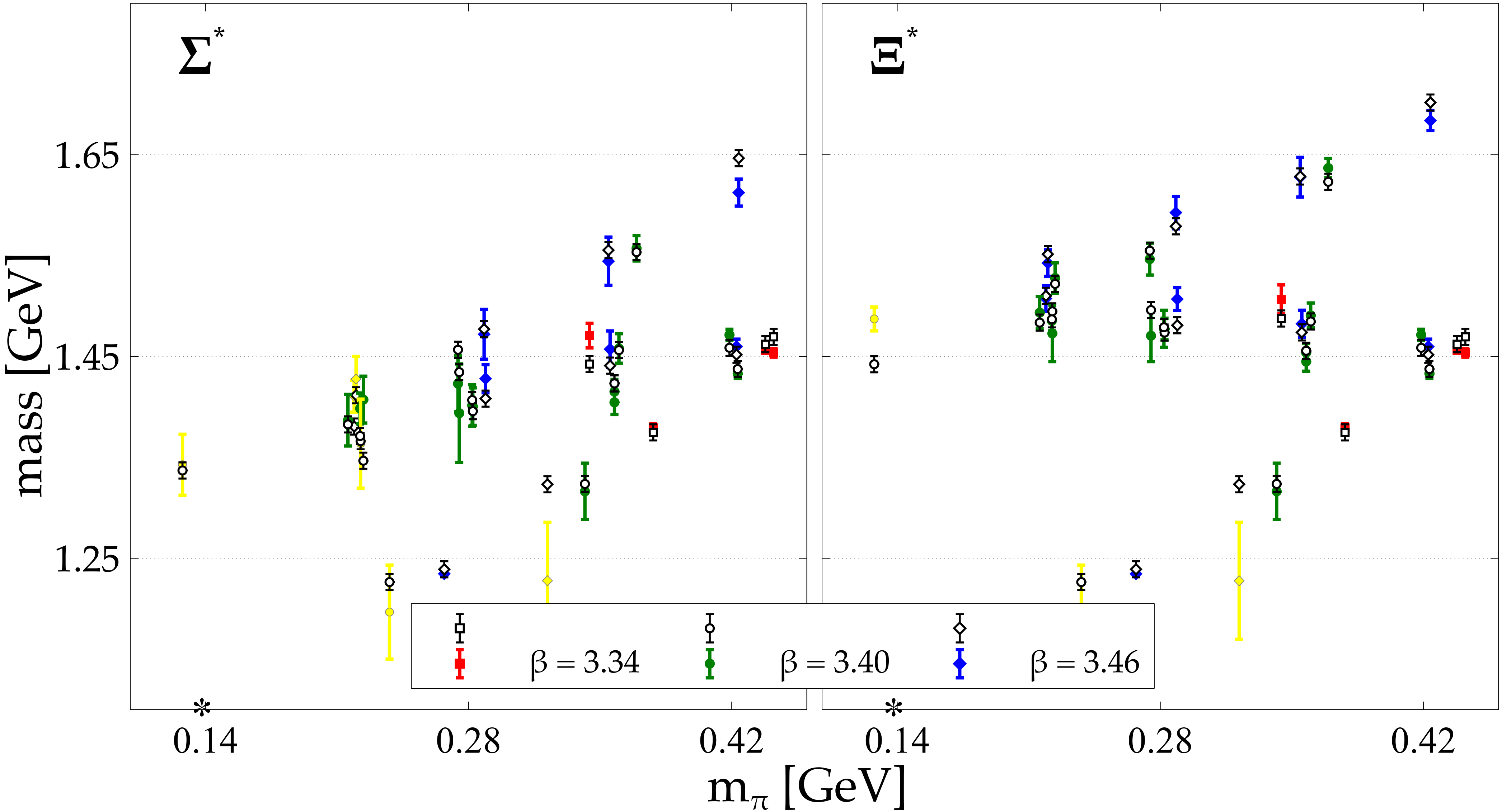

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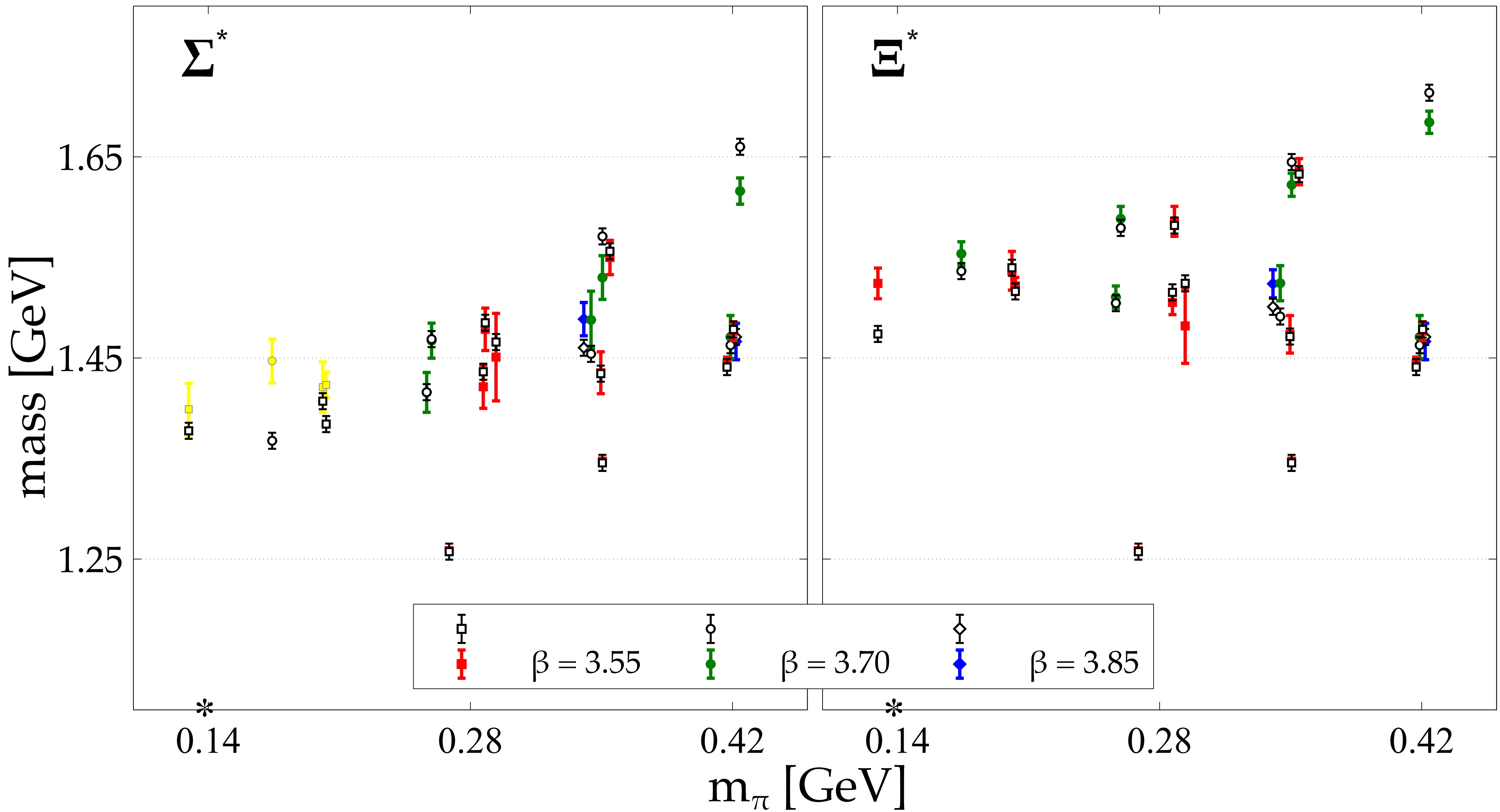

arXiv:2406.07442

Predictions for sigma terms

$$\sigma_{\pi N} = m \frac{\partial}{\partial m} m_N$$

✔ $\sigma_{\pi N}$ from pion-nucleon scattering and pionic atom data

- empirical value $\sigma_{\pi N} = 59.0(3.5)$ MeV (M. Hoferichter et al., arXiv:2305.07045)
- significant tension with QCD lattice results (only some typical cases shown)

$\sigma_{\pi N} = 45.8(7.4)(2.8)$ MeV	Y. B. Yan et al., Phys. Rev. D 94 (2016) 054503
$\sigma_{\pi N} = 35(6)$ MeV	G. S. Bali et al., Phys. Rev. D 93 (2016) 094504
$\sigma_{\pi N} = 59.6(7.4)$ MeV	R. Gupta et al., arXiv:2105.12095

✔ From CLS ensembles

- $\sigma_{\pi N} = 43.9(4.7)$ MeV — G. S. Bali et al., arXiv:2211.03744
- $\sigma_{\pi N} = 43.6(3.8)$ MeV — A. Agadjanov et al., arXiv::2303.08741
- $\sigma_{\pi N} = 58.7(1.2)$ MeV — MFML et al., arXiv:2301.06387

  isobar masses from arXiv:2211.03744 (Regensburg)
- $\sigma_{\pi N} = 42.4(4)$ MeV — MFML et al., arXiv:2406.07442

  improved isobar masses (in part from Renwick J. Hudspith at GSI)

Summary & Outlook

✔ Chiral extrapolation of hadron masses

- resummed χPT : use physical masses in the loops
  - chiral expansion with up, down and strange quarks is useful
- so far we considered baryon masses at N^3LO
  - fits to masses of ground states with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$
  - quantitative reproduction of the available lattice data set
- predict a large number of low-energy constants for the chiral Lagrangian of QCD
  - obtain a pion-nucleon sigma term as expected form the Lattice community
  - the decuplet baryons play an instrumental role

✔ QCD spectroscopy with coupled-channel dynamcis

- curent QCD lattice data provide many LEC relevant for scattering processes
- use as input in systematic coupled-channel computations
- analyze and predict the quark-mass dependence of hadron resonances in QCD

thanks to: Yonggoo Heo and Jamie Hudspith