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Hybrid baryons in a constituent model

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- Bound state of three quarks within an **excited** gluonic field: **hybrid baryon**
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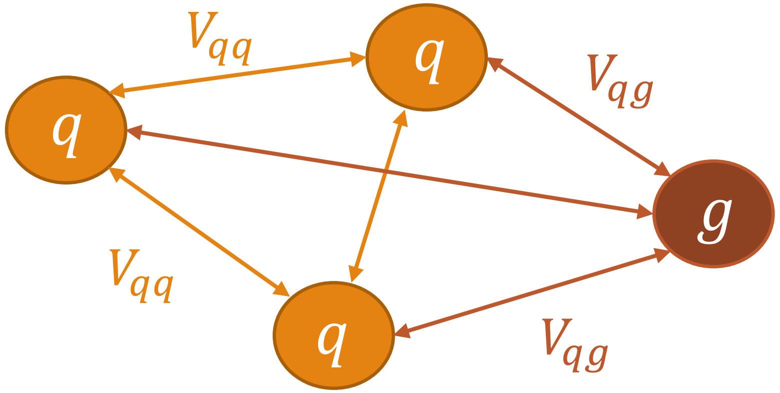
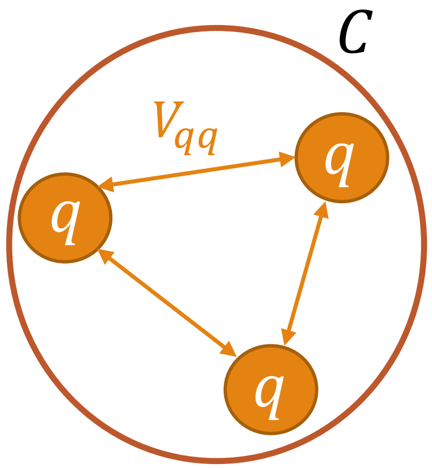


Quark core model

- Analogous to the quark-diquark model in baryons
- Interaction between quarks → quark core C
  - ➢ 3-body system



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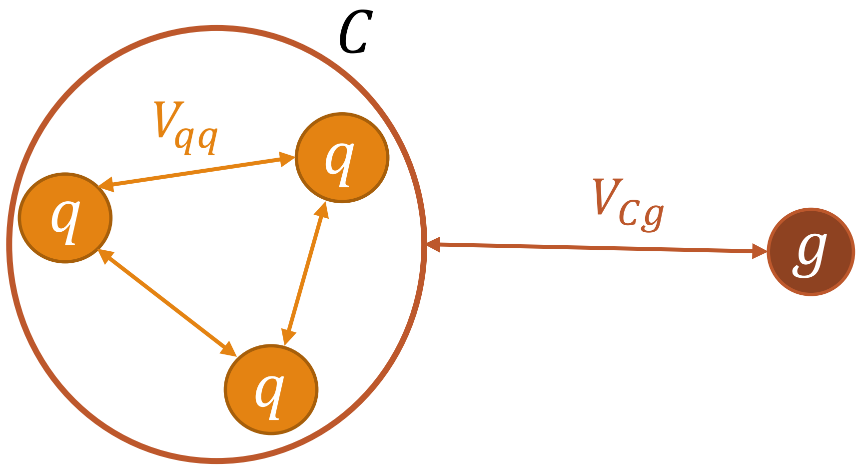
- Interaction between the core and the gluon
  - ➢ 2-body system



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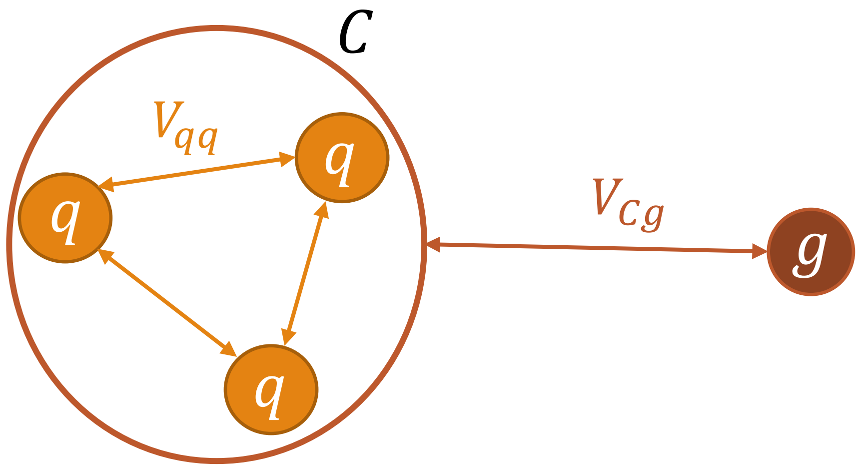
- Helicity of the gluon easier to consider



Quark core Hamiltonian

- Similar structure to the baryon (3 quarks)
  - Fülcher-inspired potential [1]

$$V_{qqq}(r) = \sum_{i<j} A\, F^2(i)|\boldsymbol{r}_i - \boldsymbol{r}_j| + B\, F(i) \cdot F(j) \left(|\boldsymbol{r}_i - \boldsymbol{r}_j|\right)^{-\mathbf{1}}$$

[1] L.P. Fulcher, Phys. Rev. D **50**, 447 (1994)

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Short-range interaction (OGE)

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Casimir of the pair $q_i q_j$

[1] L.P. Fulcher, Phys. Rev. D **50**, 447 (1994)

Quark core Hamiltonian

- Computation of $F(i) \cdot F(j)$ via the colour w.f.
- Hadrons are colour singlets $\mathbf{1}$ (confinement)

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$SU(3)$ coupling rule:

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1}^{A} \oplus \mathbf{8}^{MA} \oplus \mathbf{8}^{MS} \oplus \mathbf{10}^{S}$$

Quark core Hamiltonian

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- Hadrons are colour singlets $\mathbf{1}$ (confinement)
  - Quarks in ordinary baryon form colour singlet $\mathbf{1}$
  - Quarks in hybrid baryon form colour octet since $\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \cdots$
  - Colour w.f. ϕ has a mixed symmetry

Constituent gluon

$SU(3)$ coupling rule:
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1}^{A} \oplus \mathbf{8}^{MA} \oplus \mathbf{8}^{MS} \oplus \mathbf{10}^{S}$

Quark core Hamiltonian

- Computation of $F(i) \cdot F(j)$ via the colour w.f.
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- Three identical quarks qqq
  - Total w.f. is anti-symmetric

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  - Spatial w.f. ψ is symmetric $(L = 0)$

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$$|qqq\rangle = \psi^S \xi^S \underbrace{(\chi^{MS}\phi^{MA} - \chi^{MA}\phi^{MS})}_{AS}$$

→ Spin-colour w.f.

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→ Spin-colour w.f.

  - Spin-1/2 only
  - $F(i) \cdot F(j) = -1/6$ for every pairs

Quark core Hamiltonian

- Quark core Hamiltonian

$$H_C = \sum_i \sqrt{\boldsymbol{p}_i^2 + m^2} + \sum_{i<j} \frac{4A}{3} F^2(i)\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right| - \frac{B}{6}\left(\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|\right)^{-\mathbf{1}}$$

- Resolution of the Schrödinger equation by the expansion in oscillator bases [2]
  - ➢ Acces to the mass m_C of the core, and the « size » $1/\lambda$ of the system

[2] B. Silvestre-Brac, R. Bonnaz, C. Semay, and F. Brau, arXiv:2003.11028

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State	m_B	m_C	Δ	λ
ccc	4.822	5.119	0.297	0.825
bbb	14.401	14.894	0.493	1.261

Mass of ordinary baryon m_B, quark core m_C, difference Δ and size $1/\lambda$ [3]

[3] L. Cimino, C.T. Willemyns, C. Semay, arXiv:2406.07912

Core – gluon Hamiltonian

- Core – gluon interaction like gluon – gluon [4]

$$V_{gg}(r) = A'r - B'\frac{1}{r}$$

[4] V. Mathieu, F. Buisseret, C. Semay, Phys. Rev. D **77**, 114022 (2008)

Core – gluon Hamiltonian

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$$V_{gg}(r) = A'r - B'\frac{1}{r}$$

- Spatial extension of the core considered

$$V_{Cg}(\boldsymbol{r}) = \int d\boldsymbol{r}\, \rho(\boldsymbol{r}) V_{gg}(|\boldsymbol{R} + \boldsymbol{r}|)$$

  - $\rho(\boldsymbol{r})$ is the quark density

$$\rho(\boldsymbol{r}) = \frac{\lambda^3}{\pi^{3/2}} e^{-\lambda^2 r^2}$$

[4] V. Mathieu, F. Buisseret, C. Semay, Phys. Rev. D **77**, 114022 (2008)

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$$H_{Cg} = \sqrt{p_C^2 + m_C^2} + \sqrt{p_g^2} + A' \left[\frac{e^{-\lambda^2 r^2}}{\sqrt{\pi}\lambda} + \left(r + \frac{1}{2\lambda^2 r} \right) \mathrm{erf}(r) \right] - B' \frac{\mathrm{erf}(r)}{r}$$

- Resolution of the Schrödinger equation by the Lagrange-mesh method [5]

[5] D. Baye, Phys. Rep. **565**, 1 (2015)

Helicity formalism

- Coupling of the spin J_C of the quark core and the helicity $\lambda_g = \pm 1$ of the gluon
  - Helicity formalism of Jacob and Wick [6]
  - Well-known for 2-body systems but less for 3- or 4-body systems: **quark core model**

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- Basis of states $\left|H_\pm; J^P; \lambda_1\lambda_2\right\rangle$ with well-defined J^P quantum numbers

$$\left|H_\pm; J^P; \lambda_1\lambda_2\right\rangle = \frac{1}{\sqrt{2}}\left[|JM;\lambda_1\lambda_2\rangle \pm |JM;-\lambda_1-\lambda_2\rangle\right]$$

$$|JM;\lambda_1\lambda_2\rangle = \left(\frac{2J+1}{4\pi}\right)^{\frac{1}{2}} \int \mathrm{d}\Omega\, D^{J*}_{M,\lambda_1-\lambda_2}(\phi,\theta,-\phi)|\theta\phi;\lambda_1\lambda_2\rangle \otimes |(\pi-\theta)(\pi+\phi);\lambda_1\lambda_2\rangle$$

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  - Expansion in canonical states $\left|{}^{2S+1}L_J\right\rangle$

$$|JM;\lambda_1\lambda_2\rangle = \sum_{L,S} \left(\frac{2L+1}{2J+1}\right)^{1/2} (L\ 0\ S\ \lambda_1 - \lambda_2\ |\ J\lambda_1 - \lambda_2)(s_1\ \lambda_1\ s_2\ - \lambda_2\ |\ S\lambda_1 - \lambda_2)\left|{}^{2S+1}L_J\right\rangle$$

  - States of non-relativistic quantum mechanics

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$$\left|H_+; \left(k+\frac{1}{2}\right)^P; \frac{1}{2} 1\right\rangle \text{ with } P = (-1)^k \Rightarrow \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \ldots$$

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$1/2^+$	1.842	1.784
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$1/2^-$	2.131	2.013
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Mass gap between hybrid $qqqg$ and ordinary qqq baryon [3]

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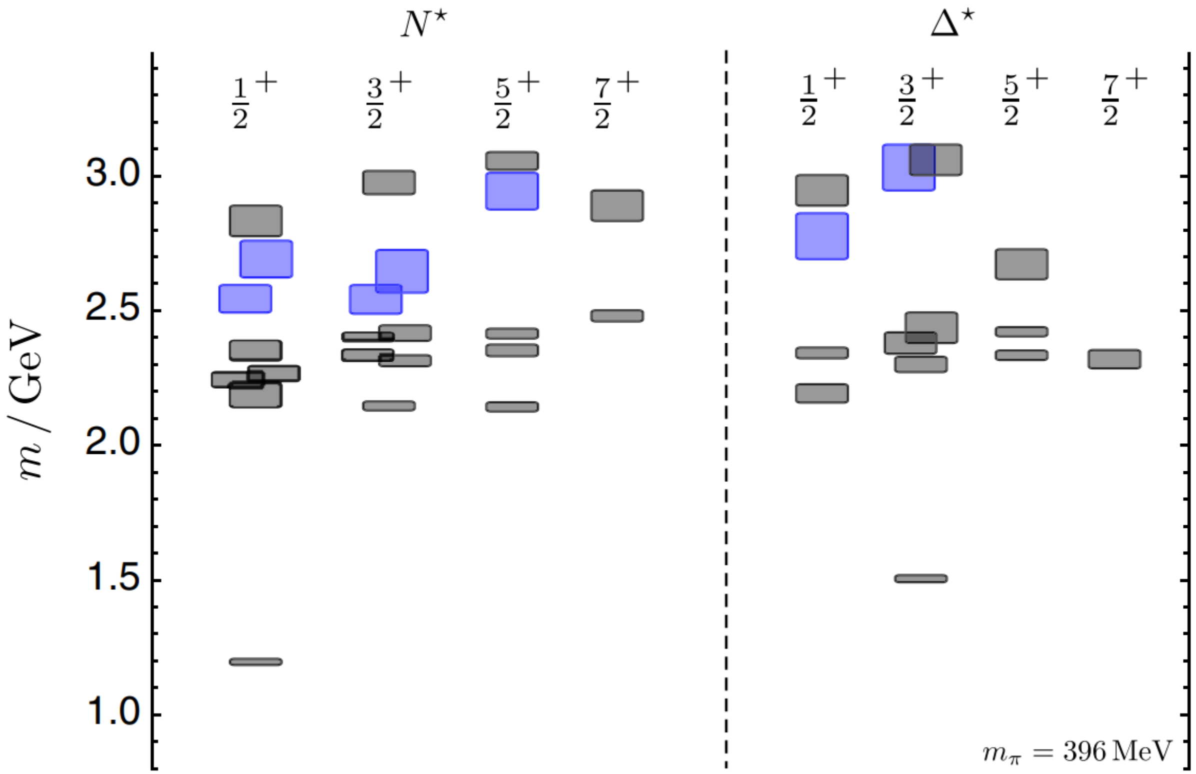
Thank you for your attention

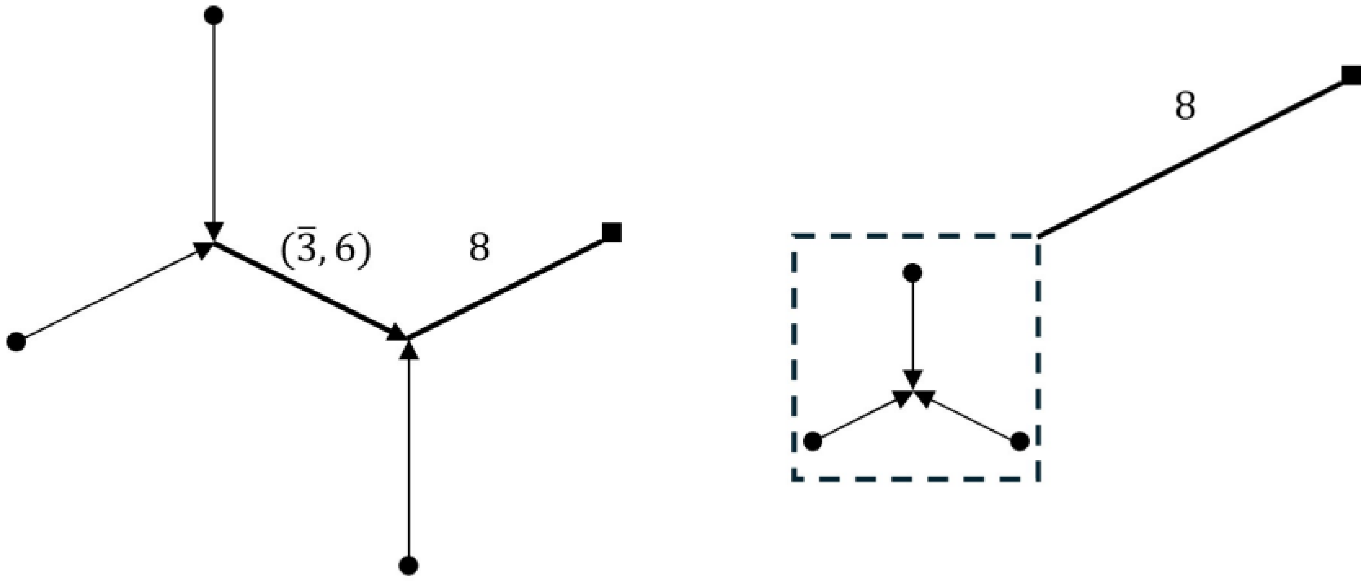
Lattice QCD results

- Lattice QCD results for light hybrid baryons [1]



[1] J.J. Dudek, R.G. Edwards, Phys. Rev. D **85**, 054016 (2012)

Flux tubes of hybrid baryons



(a) (b)