Baryon form factors from dispersion theory and functional method

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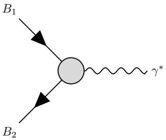
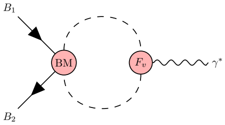
4. Summary and outlook:
   microscopic match between dispersion theory and functional methods

Baryon electromagnetic structure

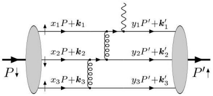
We try to understand the structure of the baryons.



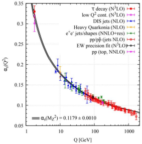
Baryon form factors



At low energies



At very high energies



QCD running coupling

How large is $\langle 0 | \, qqq \, | B \rangle$ and $\langle 0 | \, \text{Meson Baryon} \, | B \rangle$, quantitatively?

Baryon electromagnetic structure

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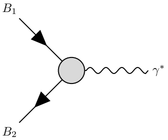
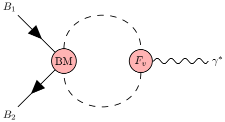


Baryon form factors



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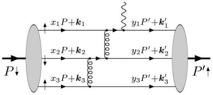

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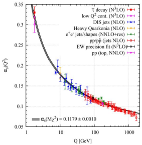
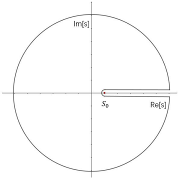


QCD running coupling

How large is $\langle 0|\, qqq \,|B\rangle$ and $\langle 0|$ Meson Baryon $|B\rangle$, quantitatively?

Need model-independent tool $\rightarrow$ Dispersion theory



Unitarity cut $[4m_\pi^2, \infty)$

Axiomatic QFT

→ Form factors are analytic functions in the complex plane.

Unitarity+analyticity

→ the location of cut, branch point, singularities...

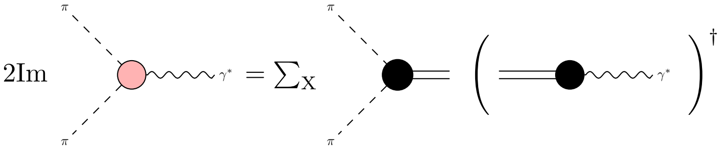
Cauchy integral Formula:

$$F(q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\text{Im}\, F(s)}{s - q^2 - i\epsilon} ds$$

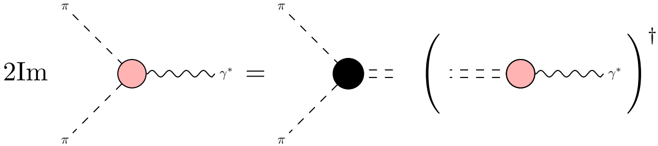
(Dispersion relation)

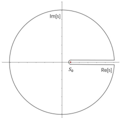
Pion-vector form factor

Optical theorem for pion vector form factor F_v:



$X = 2\pi, 4\pi, K\bar{K}, N\bar{N}, \ldots$
Keep only $X = 2\pi$

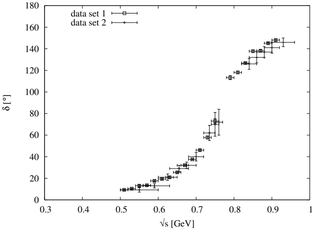




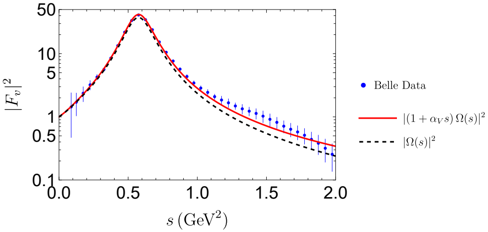
Only need pion p-wave scattering amp.
parameterised by phase-shift δ_1
$\delta_1 \Rightarrow$ well measured by experiments!

δ_1 contains ρ meson information

$$\xrightarrow[\textit{relation}]{\text{Dispersion}} F_v(s) \approx \Omega(s) = exp[\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\delta_1(s')}{s'(s'-s)}]$$

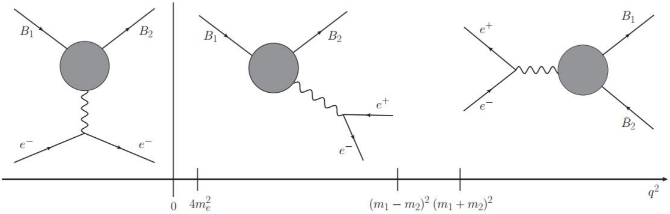


Pion p-wave phase shift [2]



Pion vector FF from dispersion theory

Experiments' status

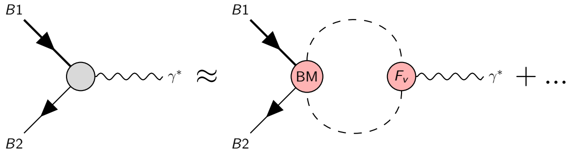


Space-like and time-like form factors [3].

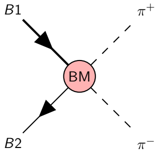
1. Space-like form factors accessible from Jlab and MAMI $e^- B_1 \to e^- B_2$.
2. Time-like form factors will be accessible in the future in the process $B_1 \to B_2 e^- e^+$ from PANDA+HADES.
3. BES, Belle for scattering region.

Dipersion theory for baryon form factors

Optical theorem for baryon form factors:



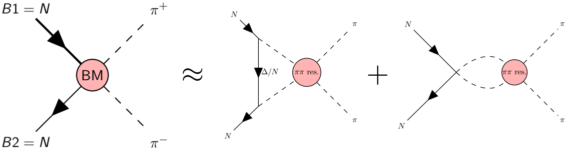
BM blob: non-perturbative **B**aryon-**M**eson interaction
Impossible to measure
$\Sigma^0\bar{\Lambda} \to 2\pi$,$\Sigma^*(1385)\bar{\Lambda} \to 2\pi$,
$N^*(1520)\bar{N} \to 2\pi$,$N\bar{N} \to 2\pi$.



Calculate BM blob using Muskhelishvili-Omnès formalism

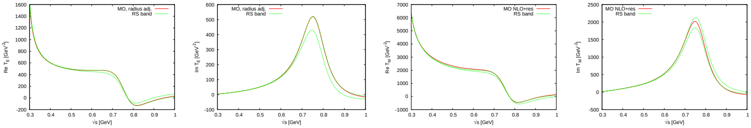
Nucleon FFs as a test case

Motivation: is Muskhelishvili-Omnès formalism reliable?
One can compare to non-perturbative analysis $N\bar{N} \to 2\pi$



$$T_{E/M}(s) = K_{E/M}(s) + \Omega(s)\, s \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{K_{E/M}(s')\, \sin\delta(s')}{|\Omega(s')|\,(s' - s - i\epsilon)\, s'} + P_{E/M}\Omega(s)$$

$N\bar{N} \to 2\pi$ p-wave amplitudes ($2m_\pi \le E \le 1$ GeV)
Green: fully dispersive analysis, Red: Muskhelishvili-Omnès formalism

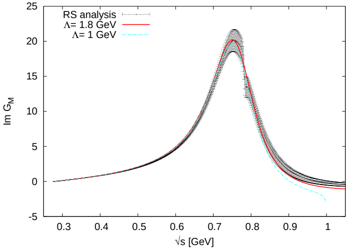


(a) T_E (Re) (b) T_E(Im) (c) T_M (Re) (d) T_M (Im)

Once-subtracted dispersion relation:

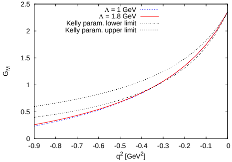
$$G_{M/E}(q^2) = G_{M/E}(0) + \frac{q^2}{12\pi}\int_{4m_\pi^2}^{\infty} \frac{ds}{\pi}\, \frac{T_{M/E}(s)\, p_{\rm c.m.}^3(s)\, F_\pi^{V*}(s)}{s^{3/2}\,(s-q^2-i\epsilon)}\,.$$

$$G_E(0) = \frac{1}{2},\; G_M(0) = \frac{1}{2}(1+\kappa_p-\kappa_n).$$

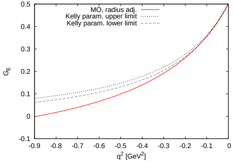


Im $G_M(q^2)$ for $4m_\pi^2 < q^2 < 1\mathrm{GeV}^2$

Space-like FFs:



$G_M(q^2)$ $-0.9\,\mathrm{GeV}^2 < q^2 < 0$



$G_E(q^2)$ $-0.9\,\mathrm{GeV}^2 < q^2 < 0$

Quark mass dependence of nucleon isovector FFs

Lattice QCD needs chiral extrapolation $\rightarrow$ ChPT

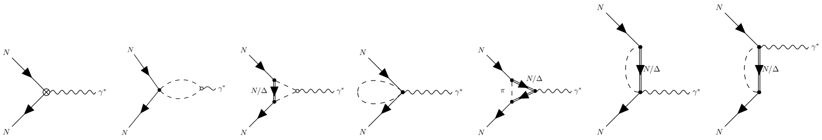
Low-lying vector mesons e.g. ρ, ω not included in ChPT

$\rightarrow$ ChPT doesn't describe nucleon FFs well.

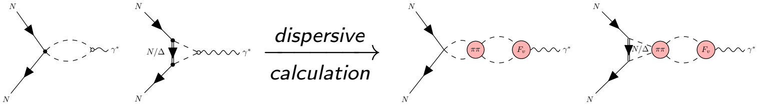
Alternative: Dispersively Modified ChPT

Phys. Rev. D 108, 114021 F. Alvarado, DA, L. Alvarez-Ruso, S. Leupold

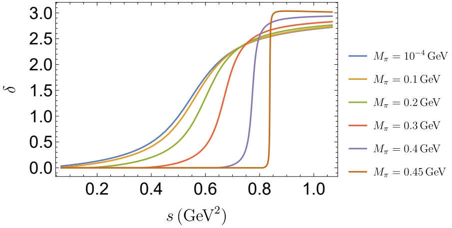
NNLO ChPT:



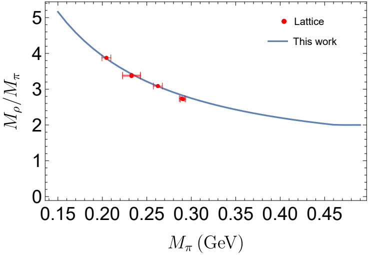
In Dispersively Modified ChPT, diagrams with a 2-pion cut are re-summed:



Prediction from the inverse amplitude method:
The pion mass dependence of the phase shift:



$\delta(s)$ for different values of M_π



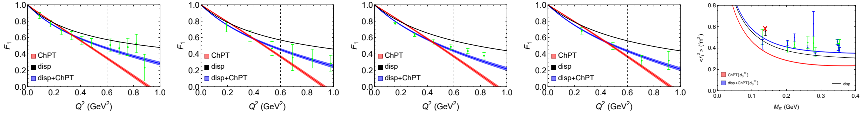
m_ρ/M_π as a function of M_π

Model-independent, consistent with chiral power counting, systematically improvable.

$\rightarrow$ **application to lattice (radii and magnetic moment extraction)**

Apply dispersively modified ChPT to Lattice QCD Lattice data from PhysRevD.103.094522

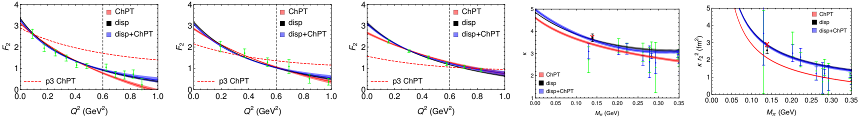
Dirac form factor:



$M_\pi = 0.203$ GeV $M_\pi = 0.278$ GeV $M_\pi = 0.347$ GeV $M_\pi = 0.350$ GeV Dirac radius (M_π)

Blue: Dispersively modified ChPT, Red: ChPT $\mathcal{O}(p^3)$

Pauli form factor:



$M_\pi = 0.203$ GeV $M_\pi = 0.278$ GeV $M_\pi = 0.347$ GeV $\kappa(M_\pi)$ Pauli radius (M_π)

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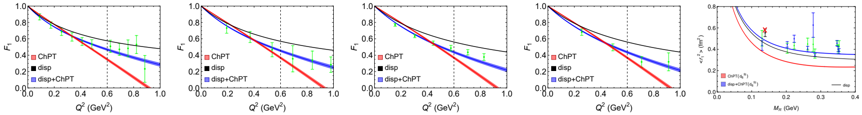
Nucleon FFs at unphysical quark masses

Apply dispersively modified ChPT to Lattice QCD Lattice data from PhysRevD.103.094522

Dirac form factor:



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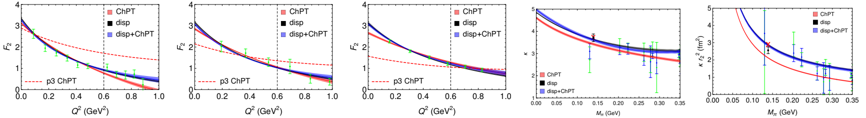

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Possible to generalize to hyperons and resonances. → More on Wednesday (S.Leupold) at 9 AM

$N^*(1520)$ TFFs

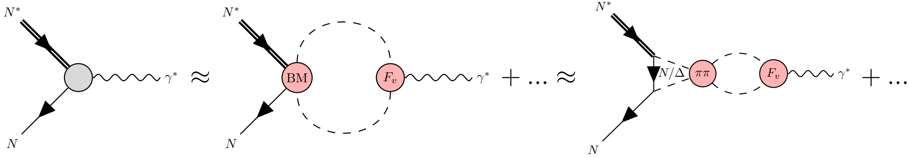
$N^*(1520)$ $I = 1/2$ and $J^P = 3/2^-$.

$$\langle N | j_\mu | N^* \rangle = e\, \bar{u}_N(p_N)\, \Gamma_{\mu\nu}(q)\, u^\nu_{N^*}(p_{N^*})$$

with

$$\Gamma^{\mu\nu}(q) := i\left(\gamma^\mu q^\nu - \not{q} g^{\mu\nu}\right) m_N F_1(q^2) + \sigma^{\mu\alpha} q_\alpha q^\nu F_2(q^2) + +i\left(q^\mu q^\nu - q^2 g^{\mu\nu}\right) F_3(q^2)\,,$$

where $q^\mu := p^\mu_{N^*} - p^\mu_N$.
We focus on isovector TFFs:



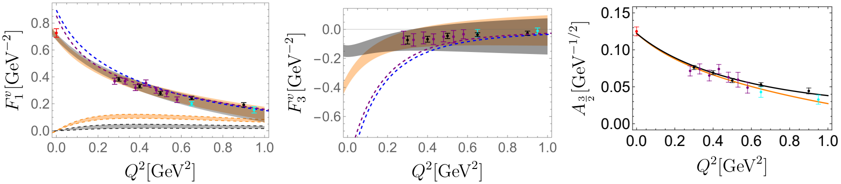
Dispersive machinery for the TFFs at low energy.

Theory meets experiments: space-like TFFs of $N^*(1520)$

Isovector TFFs $:= \frac{1}{2}(F_i^{\text{proton}} - F_i^{\text{neutron}}) \quad i = 1, 2, 3.$

Once-subtracted dispersion relation $\rightarrow$ fix the photon point to the PDG

1 (physical) parameter P_i for each $F_i \Rightarrow \;\; N\gamma^* \rightarrow N^*(1520)$



F1 F3 $A_{\frac{3}{2}}$

Orange and gray: this work[1]. Data: Jlab.

Dashed blue : MAID isovector estimate

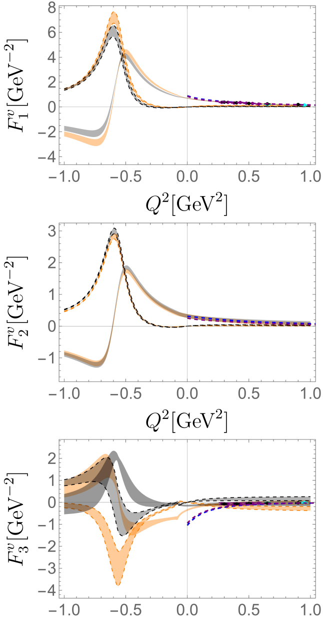
Full lines: Real part, Dashed lines: Imaginary part.

Fixing P_i predicts $\rightarrow \Gamma(\rho N, L = 0) \approx 1.5 \times 10^{-2}$ GeV

HADES measurement: $\Gamma(\rho N, L = 0) = (1.4 \pm 0.3) \times 10^{-2}$ GeV

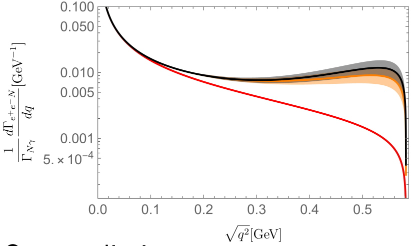
[1] 2 possible sign choices of $N^* - \Delta - \pi$ coupling

Theory predictions: time-like TFFs of $N^*(1520)$



$N^* \to N e^+ e^-$

(Assuming isovector dominance)



Our prediction:

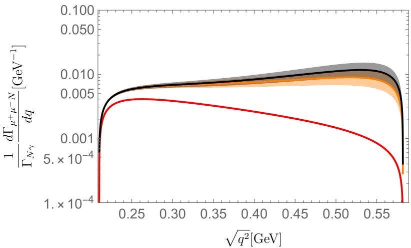
$\Gamma_{N^* \to Nee} / \Gamma_{N^* \to N\gamma} = 1.1 \times 10^{-3}$

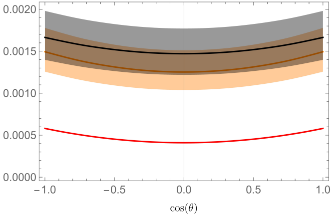
$\Gamma^{\text{QED}}_{N^* \to Nee} / \Gamma_{N^* \to N\gamma} = 0.9 \times 10^{-3}$

Results can be tested by HADES.

Theory predictions: Time-like TFFs

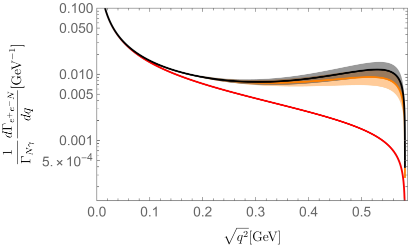
$N^* \to N\mu^+\mu^-$

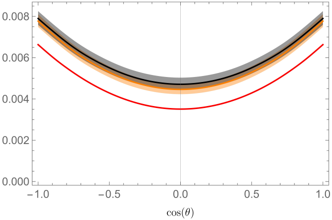




$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos(\theta)}$$

$N^* \to Ne^+e^-$





$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos(\theta)}$$

Summary and outlook

Summary:

1. Dispersion theory is a **reliable** and **model-independent tool**.
2. We make many predictions and are eager to be tested by experimentalists!

Outlook:

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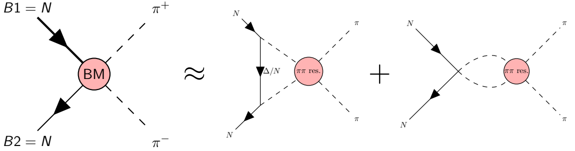
Can one calculate the contact terms from functional methods?



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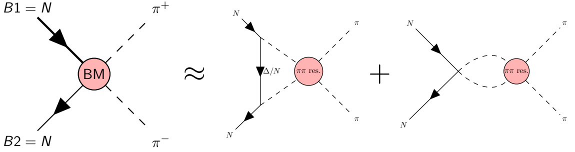

Yes

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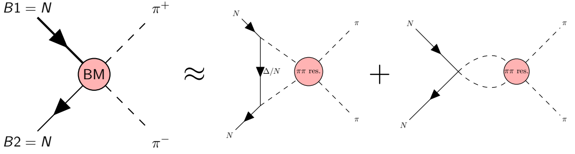
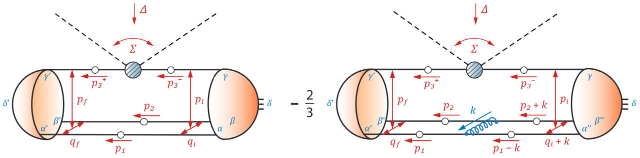


Yes

Ongoing project with Prof. G. Eichmann, Prof. C. Fischer and Prof. S. Leupold.

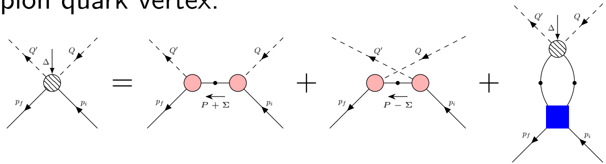
Outlook: match the hadrons with quarks and gluons

Calculate the contact term from Dyson-Schwinger equations (DSEs):



Pion nucleon handbag diagrams

DSE for the pion quark vertex:



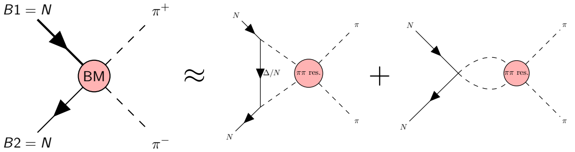
Preliminary results for the $N\pi$ scattering length:

$$a_{0+}^{-} = 4.7 \pm 1.7\,\mathrm{MeV}^{-1}$$

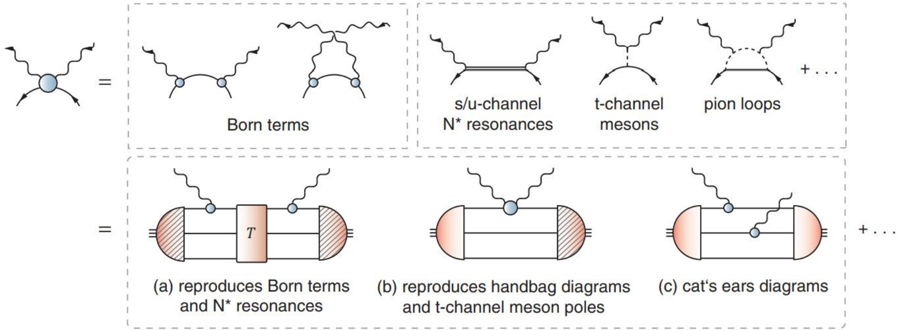
tree level ChPT prediction: $a_{0+}^{-} = 5.71\,\mathrm{MeV}^{-1}$

Thank you for listening!

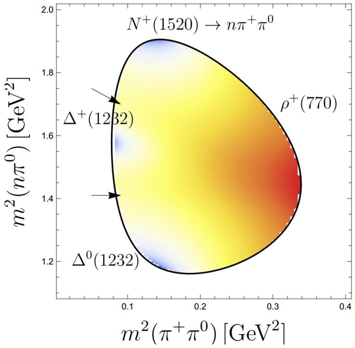
Match the hadrons with quarks and gluons



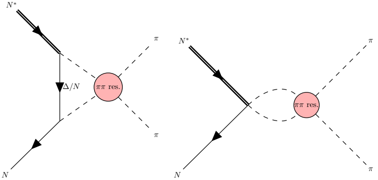
Compton scattering amplitudes (Gernot Eichmann et al):



Our dispersive prediction $N^+(1520) \to n\pi^+\pi^0$ as an example



$\frac{d\Gamma}{dm^2_{N\pi}\, dm^2_{\pi\pi}}$

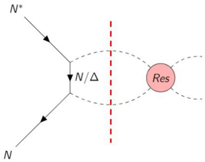


→ can be used to test the quality of isobar model predictions.

$$T_i(s) = K_i(s) + \Omega(s)\, P_i + T_i^{\text{anom}}(s) + \Omega(s)\, s \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{K_i(s')\, \sin\delta(s')}{|\Omega(s')|\,(s'-s-i\epsilon)\, s'} .$$

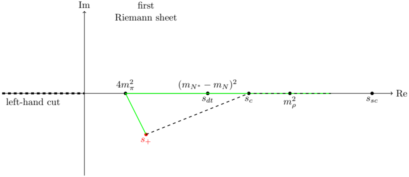
Anomalous threshold condition

$m_{exc}^2 < \frac{1}{2}(m_{N^*}^2 + m_N^2 - 2m_\pi^2)$
$m_{exc} = m_N$ (see back up slides for rigorous derivation)



Cutkosky cutting rules

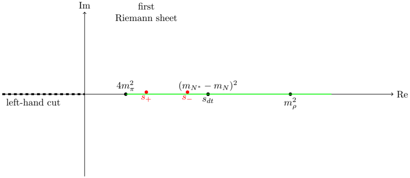
N exchange



$$T(s) = \frac{1}{2\pi i}\int_{4m_\pi^2}^{\infty} \frac{disc_{UNI}\,T(z)}{z-c}\,dz + \frac{1}{2\pi i}\int_\gamma \frac{d\gamma}{dt}\frac{disc_{ANOM}\,T((\gamma(t))}{\gamma(t)-s}\,dt$$

Δ exchange

Two singularities on the second Riemann Sheet



Unitarity cut $[4m_\pi^2, \infty)$

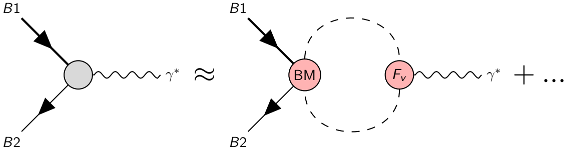
Previous studies by Uppsala Group

Nucleon sector:

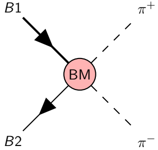
Nucleon isovector FFs (Leupold) [4]
$N^*(1520)$ TFFs (An, Leupold) (in preparation)
Quark mass dependence of nucleon FFs (An, Alvarado, Leupold, Alvarez-Ruso)
$\Delta(J^P = \frac{3}{2}^+) \to N(J^P = \frac{1}{2}^+)$ (Aung, Leupold, Perotti) (in preparation)

Hyperon sector:

$\Sigma(J^P = \frac{1}{2}^+) \to \Lambda(J^P = \frac{1}{2}^+)$ (Granados, Leupold, Perotti) [5]
$\Sigma^*(J^P = \frac{3}{2}^+) \to \Lambda(J^P = \frac{1}{2}^+)$ (Junker, Leupold, Perotti, Vitos) [6]



BM blob: non-perturbative **B**aryon-**M**eson interaction
Impossible to measure $\Sigma^0\bar{\Lambda} \to 2\pi$,$\Sigma^*(1385)\bar{\Lambda} \to 2\pi$, $N^*(1520)\bar{N} \to 2\pi$,$N\bar{N} \to 2\pi$.



Calculate BM blob using Muskhelishvili-Omnès formalism

$N^*(1520)$ TFFs: form factor dispersion relation

Subtracted dispersion relations for TFFs:

$$F_i(q^2) = F_i(0) + \frac{q^2}{12\pi} \int_{4m_\pi^2}^{\Lambda^2} \frac{\mathrm{d}s}{\pi} \frac{T_i(s)\, p_{\mathrm{c.m.}}^3(s)\, F_\pi^{V*}(s)}{s^{3/2}\,(s - q^2 - i\epsilon)} + F_i^{\mathrm{anom}}(q^2) \text{ for } i = 1, 2, 3.$$

$T_i \sim N^* N \to 2\pi$ amplitudes calculated from Muskhelishvili-Omnès formalism:

$$T_i(s) = K_i(s) + \Omega(s)\, P_i + T_i^{\mathrm{anom}}(s) + \Omega(s)\, s \int_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s'}{\pi} \frac{K_i(s')\, \sin\delta(s')}{|\Omega(s')|\,(s' - s - i\epsilon)\, s'}\,.$$

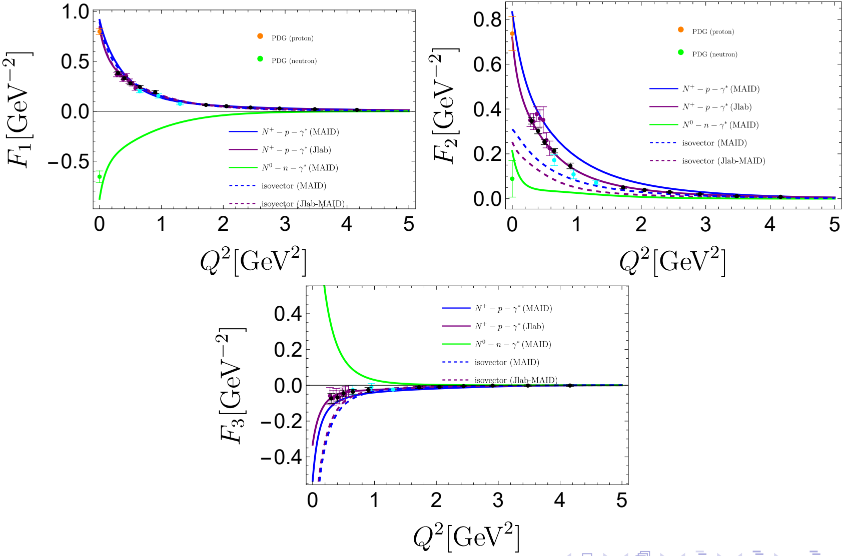
$P_{i=1,2,3}$ are fit parameters (contact term interactions).

Branching ratios of N^* by PDG:

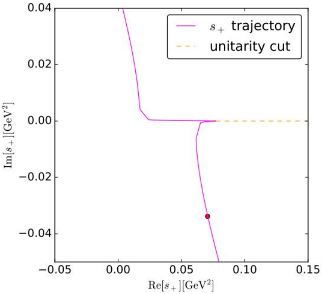
1. $N\pi$ — $55 - 65\%$
2. $\Delta(1232)\pi$, (S-wave) — $15 - 23\%$
3. $\Delta(1232)\pi$, (D-wave) — $7 - 11\%$
4. $N\rho$, $S = \frac{3}{2}$, (S-wave) — $10 - 16\%$
5. $N\rho$, $S = \frac{1}{2}$, (D-wave) — $0.2 - 0.4\%$
6. $N\rho$, $S = \frac{3}{2}$, (D-wave) — ≈ 0
7. $N\eta$ — $0.07 - 0.08\%$

Experimental data on space-like TFFs

We can only calculate isovector TFFs:$=\frac{1}{2}(F_i^{proton} - F_i^{neutron}), i = 1, 2, 3$



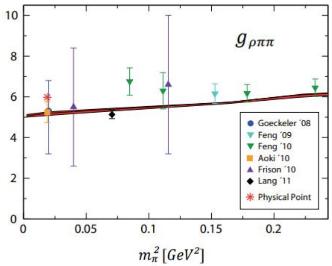
Anomalous singularity



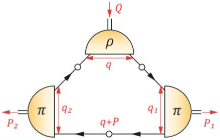
Trajectory of a singularity in the complex plane[8]

Calculating the couplings in quark di-quark approach in Dyson-Schwinger equations and Bethe-Salpeter equations

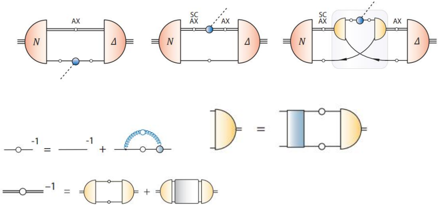
State of art method: Rainbow ladder + quark-di quark approximation (Valentin Mader et al.).

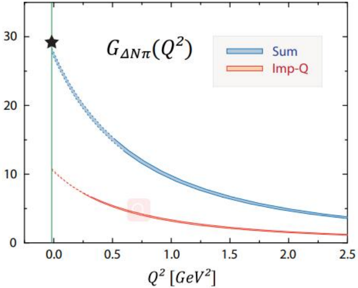


$\rho \to \pi\pi$ transition matrix elements



$\Delta \to N\pi$ transition matrix elements

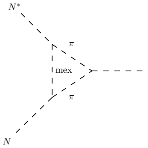




Comparison with 1-loop scalar-triangle

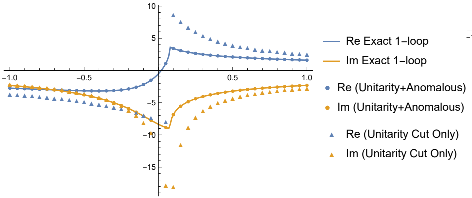
How do we make sure we are right about the analytic structures?

$\rightarrow$ use 1-loop scalar triangle (G. 't Hooft, M. Veltman) as a toy calculation for **double-check**!

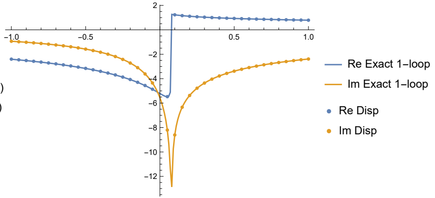


$$= T(s) = \frac{1}{2\pi i}\int_{4m_\pi^2}^{\infty} \frac{disc_{UNI}\,T(z)}{z-c}dz + \frac{1}{2\pi i}\int_{\gamma} \frac{d\gamma}{dt}\frac{disc_{ANOM}\,T((\gamma(t))}{\gamma(t)-s}dt$$

Our dispersive relation for the scalar triangle perfectly matches the analytic results:



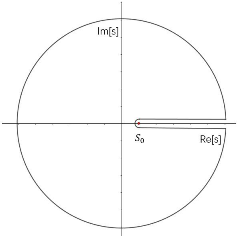
Nucleon exchange



Δ exchange

Dispersion theory in a nutshell

Example: Pion vector form factor



Unitarity cut $[4m_\pi^2, \infty)$

$$\begin{aligned} S &= 1 + iT \\ \text{Unitarity } SS^\dagger &= 1 + i(T - T^\dagger) + |T|^2 = 1 \\ \rightarrow 2ImT &= |T|^2 \\ \rightarrow ImT_{A\rightarrow B} &= \frac{1}{2}\sum_x T_{A\rightarrow x} T^\dagger_{x\rightarrow B} \end{aligned} \tag{1}$$

Simplest example: $A = |\gamma^*\rangle$, $B = \left|\pi^-(p_1)\pi^+(p_2)\right\rangle$.

$$\Rightarrow T_{\gamma^*\rightarrow\pi^-\pi^+} = e\epsilon_\mu \underbrace{\langle\pi^-(p_1)\pi^+(p_2)|j^\mu|0\rangle}_{(p_1^\mu - p_2^\mu)F_v(s)} \tag{2}$$

$$T_{\gamma^*\rightarrow x} = e\epsilon_\mu \langle x|j^\mu|0\rangle \tag{3}$$

$$ImF_v(s)(p_1^\mu - p_2^\mu) = \frac{1}{2}\sum_x \langle\pi^-(p_1)\pi^+(p_2)|x\rangle^* \langle x|j^\mu|0\rangle \tag{4}$$

$|x\rangle = 2pions(s \geq 4m_\pi^2), 4pions(s \geq 16m_\pi^2), 2kaons(s \geq 4m_K^2), \ldots$

Dispersion relation

Cauchy integral formula:

$$F_v(s) = \frac{1}{2\pi i} \int_{s_0=4m_\pi^2}^{\infty} dz \frac{lim_{\epsilon\to 0}[F_v(z+i\epsilon) - F_v(z-i\epsilon)]}{z-s} \tag{5}$$

Schwarz Reflection Principle: $F_v(z-i\epsilon) = F_v(z+i\epsilon)^*$

$$F_v(s) = \frac{1}{\pi} \int_{s_0=4m_\pi^2}^{\infty} dz \frac{Im[F_v(z+i\epsilon)]}{z-s} \text{ Dispersion relation} \tag{6}$$

Consider only the 2 pion contribution

$$2ImF_v(q^2)(p_1^\mu - p_2^\mu) \approx \int d\tau_{2\pi}^{'} \underbrace{\langle \pi^-(p_1)\pi^+(p_2)|\pi^-(p_1^{'})\pi^+(p_2^{'})\rangle^*}_{\text{Pion rescattering amplitude}} \underbrace{\langle \pi^-(p_1^{'})\pi^+(p_2^{'})|j^\mu\,|0\rangle}_{F_v(q^2)(p_1^{'\mu} - p_2^{'\mu})} \tag{7}$$

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