

Elastic nucleon-nucleon scattering in holographic QCD

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Based on:

Wei Xie, AW, and Mei Huang, JHEP 1910, 053 (2019)

Zhibo Liu, Wei Xie, and AW, PRD 107, 014018 (2023)

Yu-Peng Zhang, Xun Chen, Xiao-Hua Li, and AW, PRD 108, 066001 (2023)

14th International Workshop on the Physics of Excited Nucleons (NSTAR2024)

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Outline

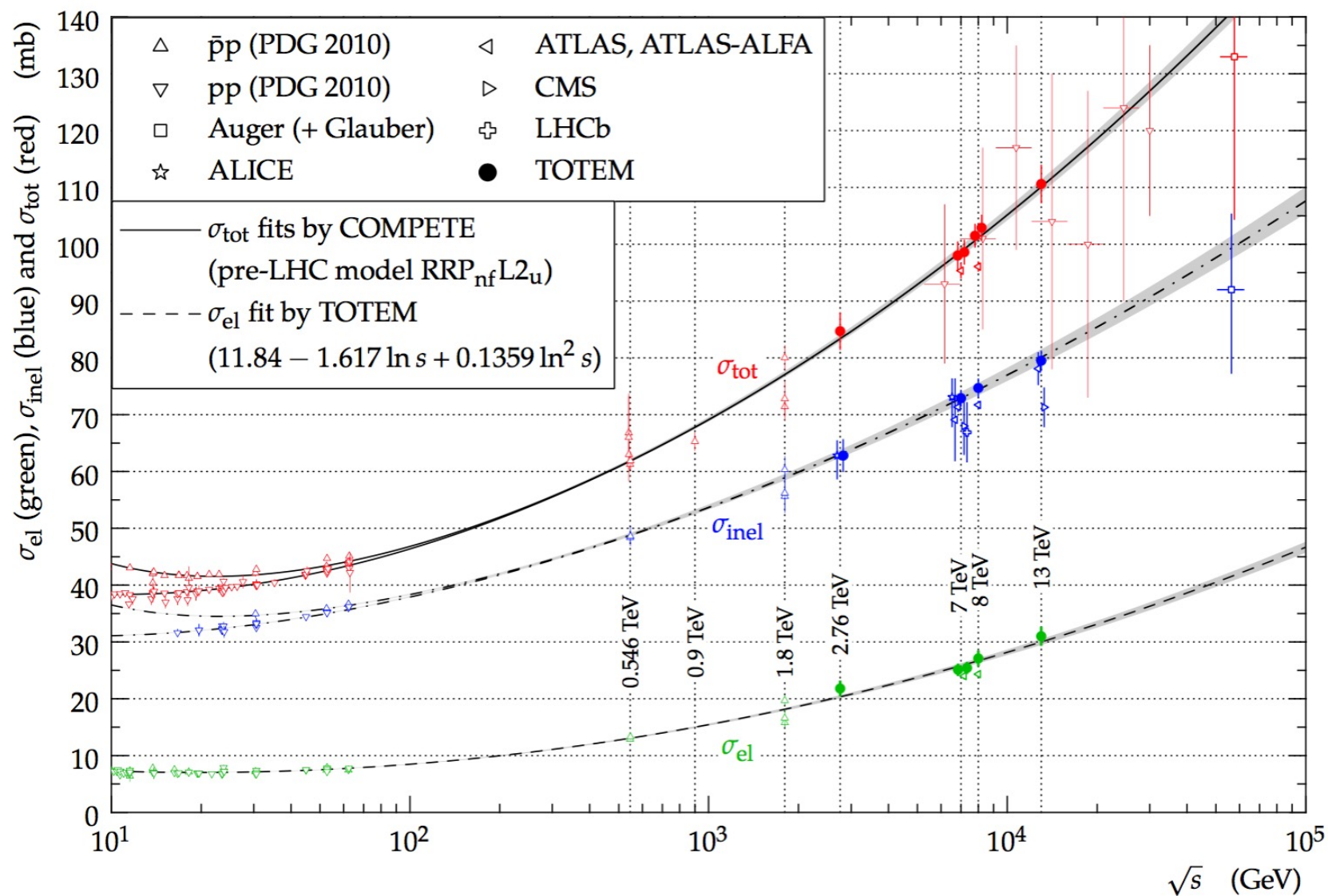
1. Introduction
2. Three attempts for elastic proton-(anti)proton scattering in the Regge regime via holography
 1. Pomeron exchange
(for high energy region)
 2. Adding Reggeon exchange contribution
(for intermediate energy region)
 3. Adding Coulomb interaction contribution
(for very small momentum transfer region)
3. Summary

Motivation

- Understanding the quark-gluon structure of hadrons is one of the most important problems in high energy physics.
- High energy hadron scattering cross sections include information on the internal structure of the involved hadrons, and elastic hadron-hadron scattering is one of the simplest processes.
- Since the cross sections are basically nonperturbative physical quantities, the first principle calculation is almost impossible in practice.
- Improving effective methods is essential.

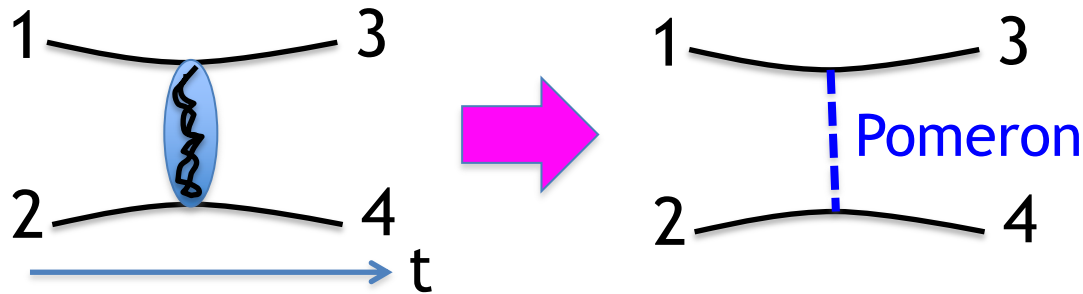
13TeV data from TOTEM

TOTEM Collaboration (2017)



Pomeron exchange picture

- A description of high energy scattering before QCD
- **Pomeron: a color singlet gluonic object**
- It is known that total cross sections of high energy two-body scattering can be well described with this picture



$$\sigma_{tot}(s) \sim s^{\alpha_0 - 1}$$

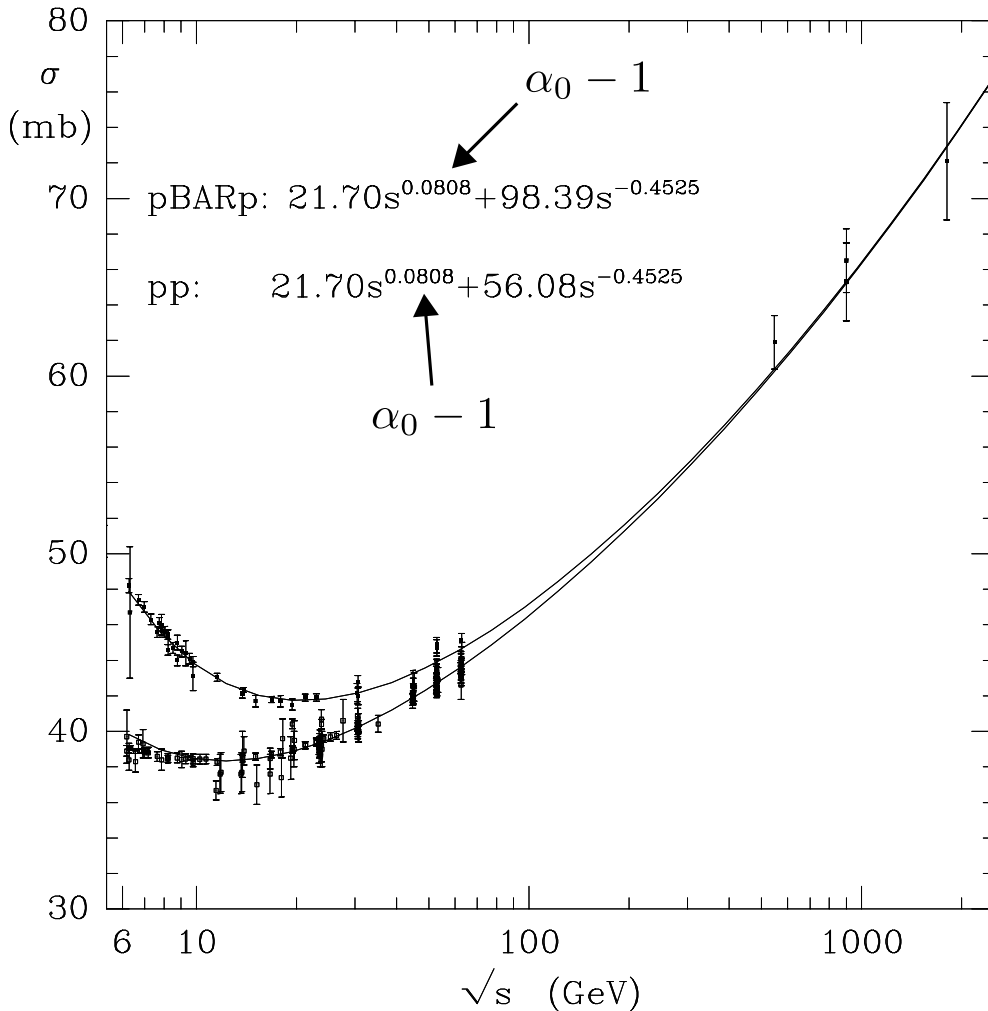
where

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

Total cross sections can be expressed with a single parameter (Pomeron intercept)

Total cross sections via soft Pomeron exchange

proton-(anti)proton total cross section



Donnachie-Landshoff (1992)

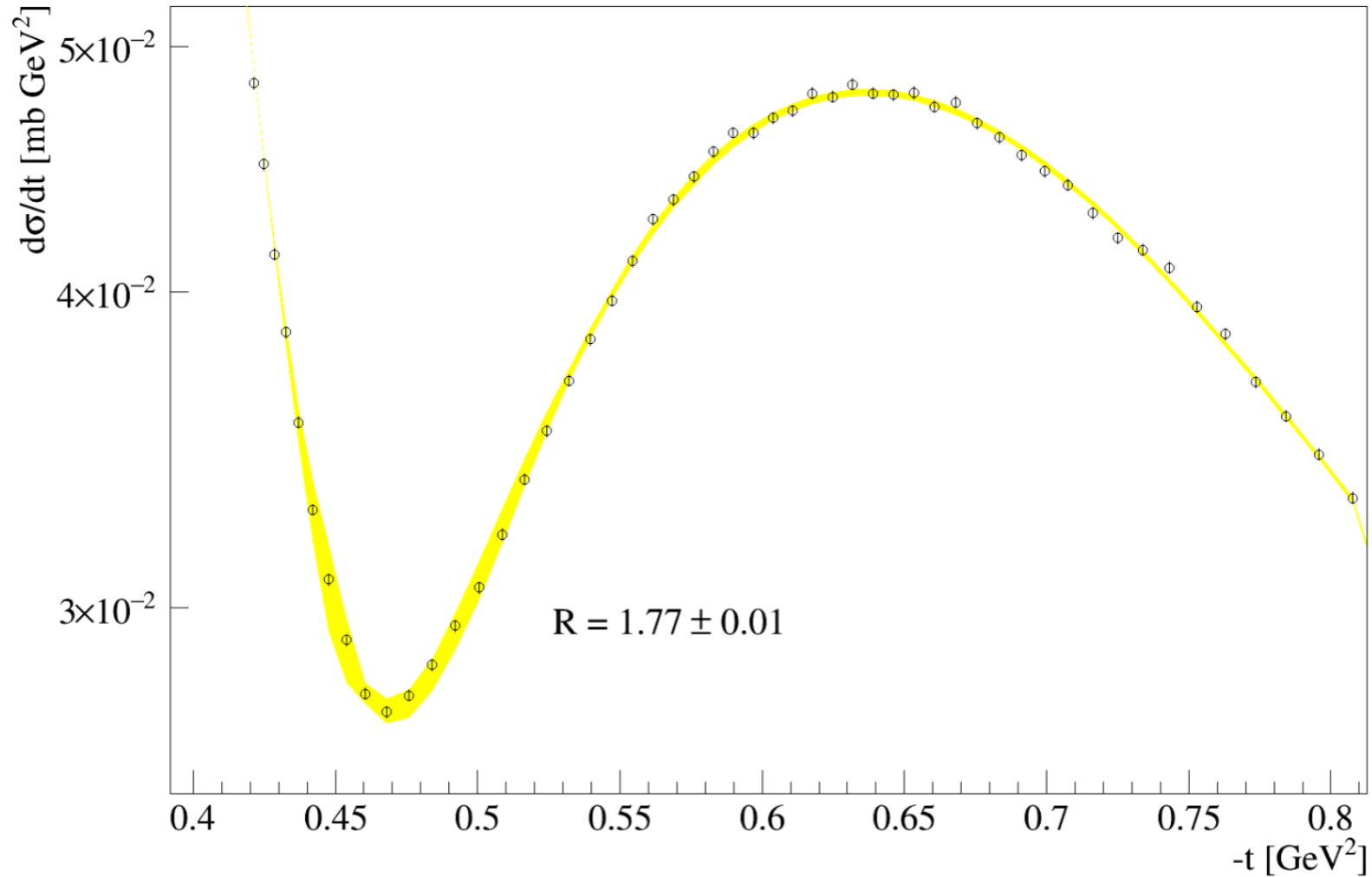
$$\sigma_{tot}(s) \sim s^{\alpha_0 - 1}$$

$\alpha_0 = 1.0808$
(soft Pomeron intercept)

well describing the experimental data

Differential cross section data from TOTEM

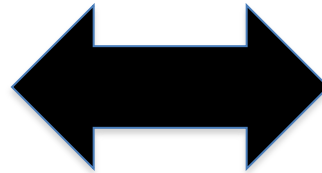
TOTEM collaboration (2018)



Holographic QCD

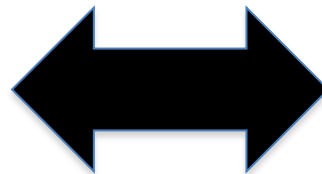
- Holographic QCD, which is constructed based on the AdS/CFT correspondence, has a potential to be a powerful tool for analysis in hadron physics.

type IIB
supergravity theory
on $S^5 \times \text{AdS}_5$



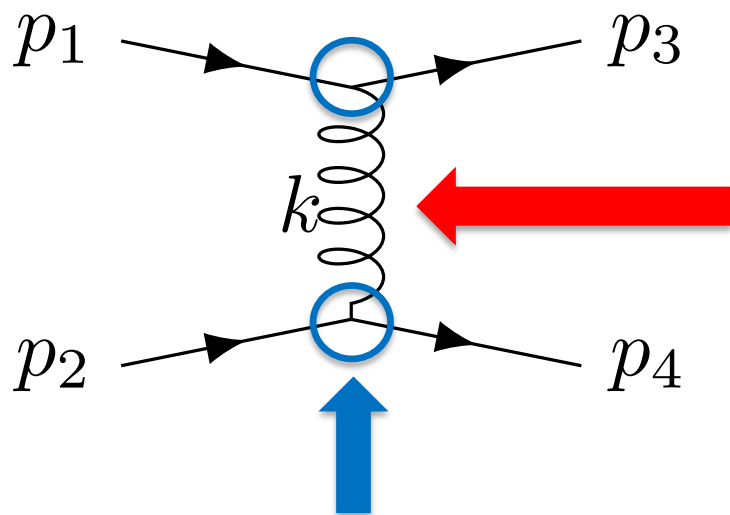
strong coupling 4D N=4
supersymmetric Yang-
Mills (SYM) theory

supergravity theory
(classical theory)
on AdS_5



usual 4D QCD
at strong coupling

Model setup



Domokos-Harvey-Mann (2009)

Reggeized spin-2
particle propagator

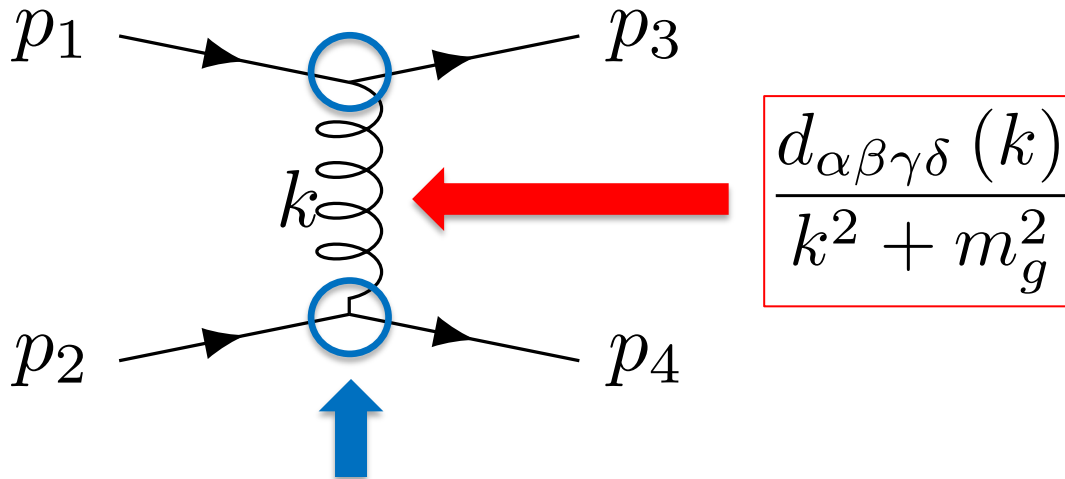
- 3 adjustable parameters are determined with data

Gravitational form factor

- Calculable with the bottom-up AdS/QCD model
- Model parameters are fixed with hadron properties

Applicable for other hadron-hadron scattering processes by replacing the form factors

Spin-2 glueball exchange



$$\langle p', s' | T_{\mu\nu}(0) | p, s \rangle = \bar{u}(p', s') \left[A(t) \frac{\gamma_\alpha P_\beta + \gamma_\beta P_\alpha}{2} + B(t) \frac{i(P_\alpha \sigma_{\beta\rho} + P_\beta \sigma_{\alpha\rho}) k^\rho}{4m_p} + C(t) \frac{(k_\alpha k_\beta - \eta_{\alpha\beta} k^2)}{m_p} \right] u(p, s)$$

Spin-2 glueball exchange

$$\mathcal{M}_g = \frac{\lambda^2 d_{\alpha\beta\gamma\delta}}{4(t - m_g^2)} \left[A(t)(\bar{u}_1 \gamma^\alpha u_3)(p_1 + p_3)^\beta + \frac{iB(t)}{2m_p}(p_1 + p_3)^\beta k_\rho (\bar{u}_1 \sigma^{\alpha\rho} u_3) + \frac{C(t)}{m_p}(\bar{u}_1 u_3)(k^\alpha k^\beta - \eta^{\alpha\beta} t) \right] \\ \times \left[A(t)(\bar{u}_2 \gamma^\gamma u_4)(p_2 + p_4)^\delta + \frac{iB(t)}{2m_p}(p_2 + p_4)^\delta k_\lambda (\bar{u}_2 \sigma^{\gamma\lambda} u_4) + \frac{C(t)}{m_p}(\bar{u}_2 u_4)(k^\gamma k^\delta - \eta^{\gamma\delta} t) \right]$$

where

$$d_{\alpha\beta\gamma\delta} = \frac{1}{2}(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}) - \frac{1}{2m_g^2}(k_\alpha k_\delta \eta_{\beta\gamma} + k_\alpha k_\gamma \eta_{\beta\delta} + k_\beta k_\delta \eta_{\alpha\gamma} + k_\beta k_\gamma \eta_{\alpha\delta}) \\ + \frac{1}{24} \left[\left(\frac{k^2}{m_g^2} \right)^2 - 3 \left(\frac{k^2}{m_g^2} \right) - 6 \right] \eta_{\alpha\beta} \eta_{\gamma\delta} - \frac{k^2 - 3m_g^2}{6m_g^4} (k_\alpha k_\beta \eta_{\gamma\delta} + k_\gamma k_\delta \eta_{\alpha\beta}) + \frac{2k_\alpha k_\beta k_\gamma k_\delta}{3m_g^4}$$

$$\frac{d\sigma}{dt} = \frac{\lambda^4 s^2 A^4(t)}{16\pi(t - m_g^2)^2}$$

Needs to be Reggeized
to include the higher
spin states

$s \gg |t|$

Reggeized Spin-2 particle exchange

$$\frac{d\sigma}{dt} = \frac{\lambda^4 A^4(t) \Gamma^2[-\chi] \Gamma^2\left[1 - \frac{\alpha_c(t)}{2}\right]}{16\pi \Gamma^2\left[\frac{\alpha_c(t)}{2} - 1 - \chi\right]} \left(\frac{\alpha'_c s}{2}\right)^{2\alpha_c(t)-2}$$

$$\sigma_{tot} = \frac{\pi \lambda^2 \Gamma[-\chi]}{\Gamma\left[\frac{\alpha_c(0)}{2}\right] \Gamma\left[\frac{\alpha_c(0)}{2} - 1 - \chi\right]} \left(\frac{\alpha'_c s}{2}\right)^{\alpha_c(0)-1}$$

where

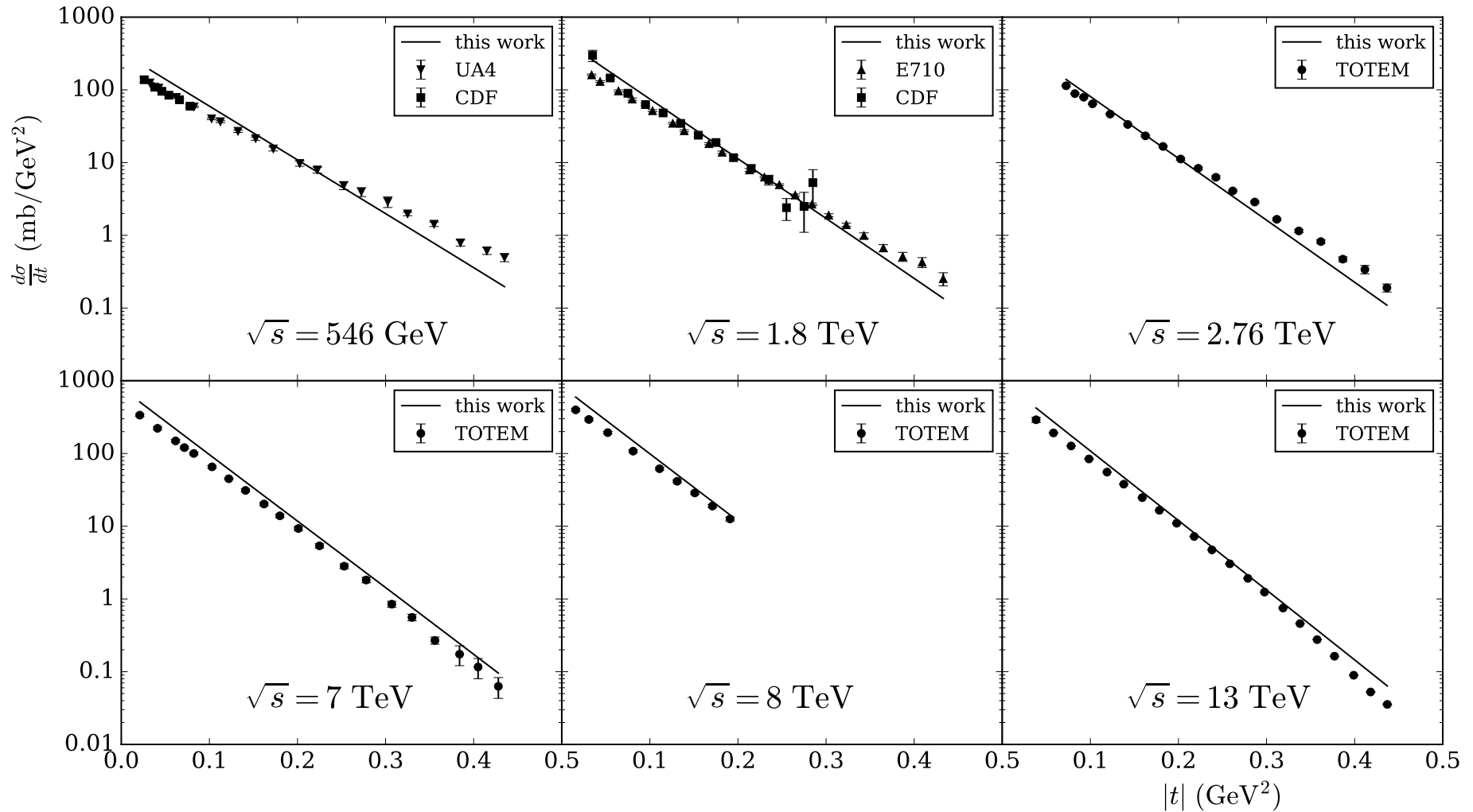
$$\alpha_c(x) = \alpha_c(0) + \alpha'_c x$$

$$\chi = \alpha_c(s) + \alpha_c(t) + \alpha_c(u) = 4\alpha'_c m^2 + 3\alpha_c(0)$$

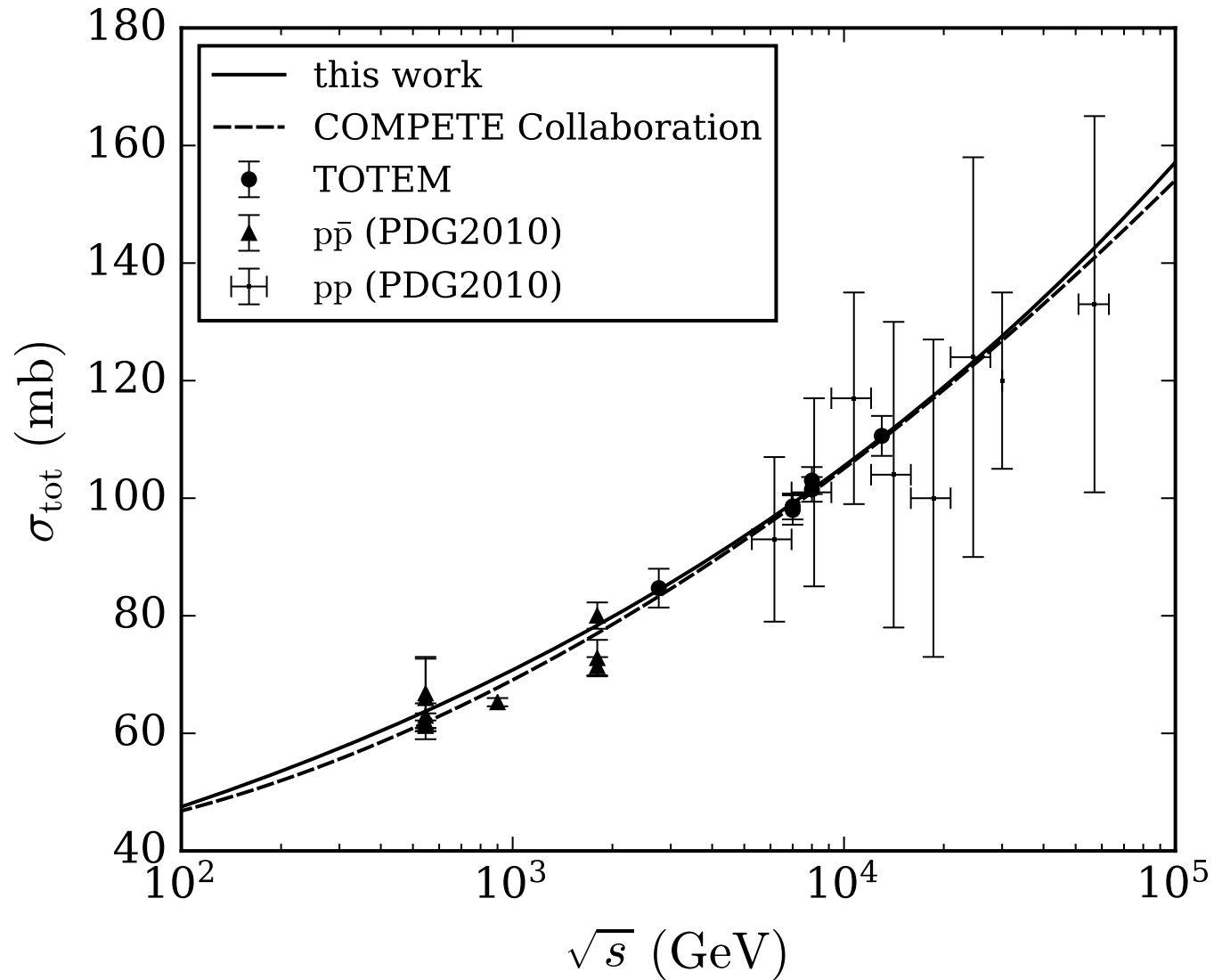
3 parameters: $\lambda, \alpha_c(0), \alpha'_c$

to be determined with experimental data

Differential cross section

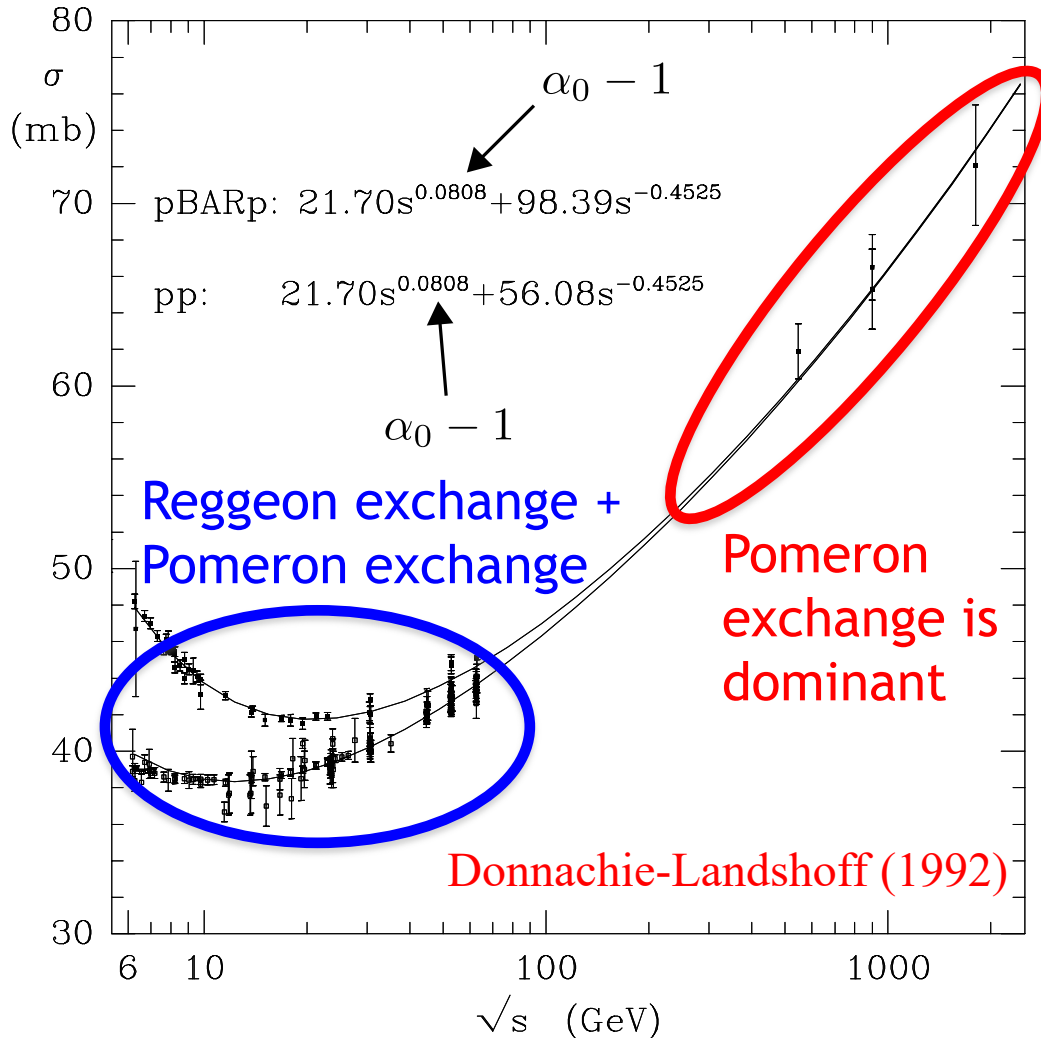


Total cross section



Description with Reggeon + Pomeron

proton-(anti)proton total cross section



$$\sigma_{tot}(s) \sim s^{\alpha_0 - 1}$$

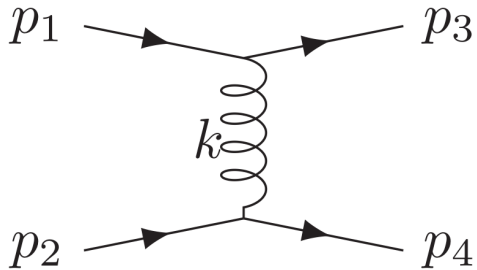
$$\alpha_0 = 1.0808$$

(soft Pomeron intercept)

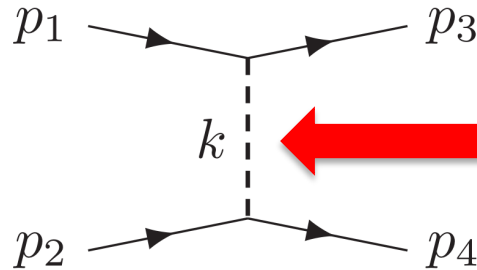
well describing the experimental data

Description with Pomeron + Reggeon

Pomeron exchange



Reggeon exchange



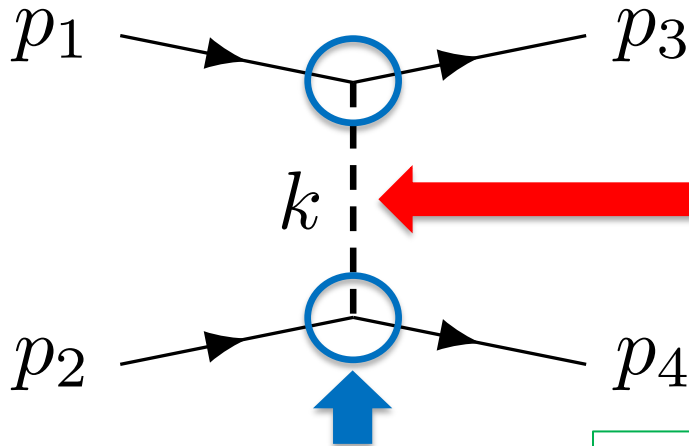
Anderson-Domokos-Mann (2017)

Reggeized vector meson propagator

- Dominant in high energy region
- Same treatment as the previous

- This contribution raises the total cross section and decreases with the energy
- Required to describe the intermediate energy region ($\sqrt{s} < 100\text{GeV}$)

Vector meson exchange



$$D_{\mu\nu}^v(k) = \frac{i}{k^2 + m_v} \eta_{\mu\nu}$$

$$\Gamma_v^\mu = -i\lambda_v \gamma^\mu$$

$$\begin{aligned} \mathcal{A}_v^{pp(p\bar{p})} &= (\bar{u}_1 \Gamma_v^\mu u_3) D_{\mu\nu}^v(k) (\bar{u}_2 \Gamma_v^\nu u_4) \\ &= -\frac{i\lambda_v^2}{k^2 + m_v^2} \eta_{\mu\nu} (\bar{u}_1 \gamma^\mu u_3) (\bar{u}_2 \gamma^\nu u_4) \end{aligned}$$

- For the Pomeron related parameters, we use the previously determined ones.
- The newly introduced parameter λ_v is determined with the total cross section data, and differential cross sections can be predicted without any adjustable parameters.

Total scattering amplitude

$$\begin{aligned} \mathcal{A}_{\text{tot}}^{pp(p\bar{p})} &= \frac{-i\lambda_g^2}{8(t - m_g^2)} [2sA^2(t)(\bar{u}_1\gamma^\alpha u_3)(\bar{u}_2\gamma_\alpha u_4) \\ &\quad + 4A^2(t)p_2^\alpha p_1^\beta (\bar{u}_1\gamma_\alpha u_3)(\bar{u}_2\gamma_\beta u_4)] \\ &\quad + \frac{i\lambda_v^2}{t - m_v^2} \eta_{\mu\nu} (\bar{u}_1\gamma^\mu u_3)(\bar{u}_2\gamma^\nu u_4) \end{aligned}$$

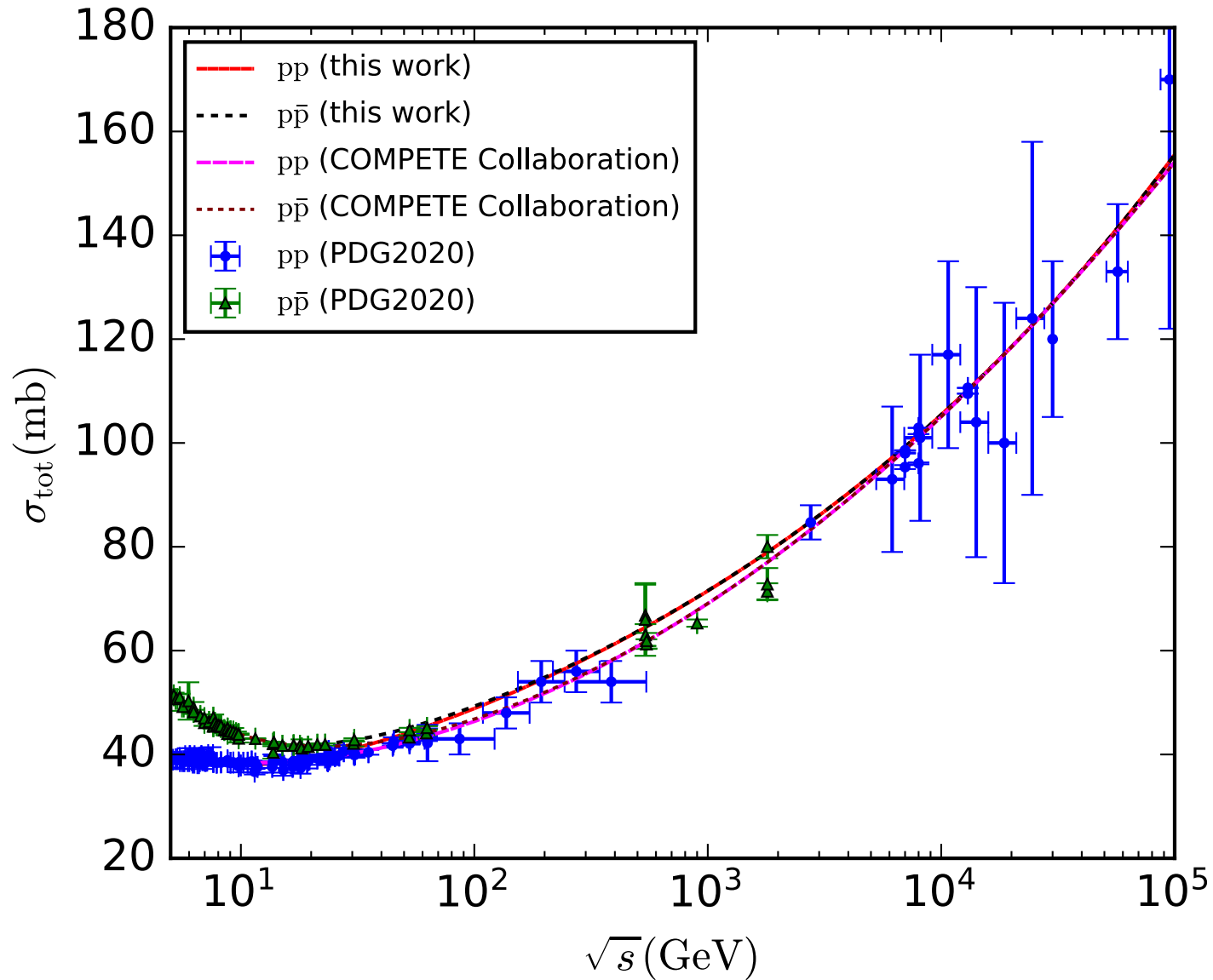
$$\begin{aligned} \frac{d\sigma}{dt} &= \frac{1}{16\pi s^2} |\mathcal{A}_{\text{tot}}|^2 \\ &= \frac{\lambda_g^4 s^2 A^2(t)}{16\pi |t - m_g^2|^2} + \frac{\lambda_g^2 \lambda_v^2 A^2(t) s}{8\pi} \left[\frac{1}{(t - m_g^2)^*} \times \frac{1}{t - m_v^2} \right. \\ &\quad \left. + \frac{1}{(t - m_v^2)^*} \times \frac{1}{t - m_g^2} \right] + \frac{\lambda_v^4}{4\pi |t - m_v^2|^2} \end{aligned}$$

Cross sections after Reggeization

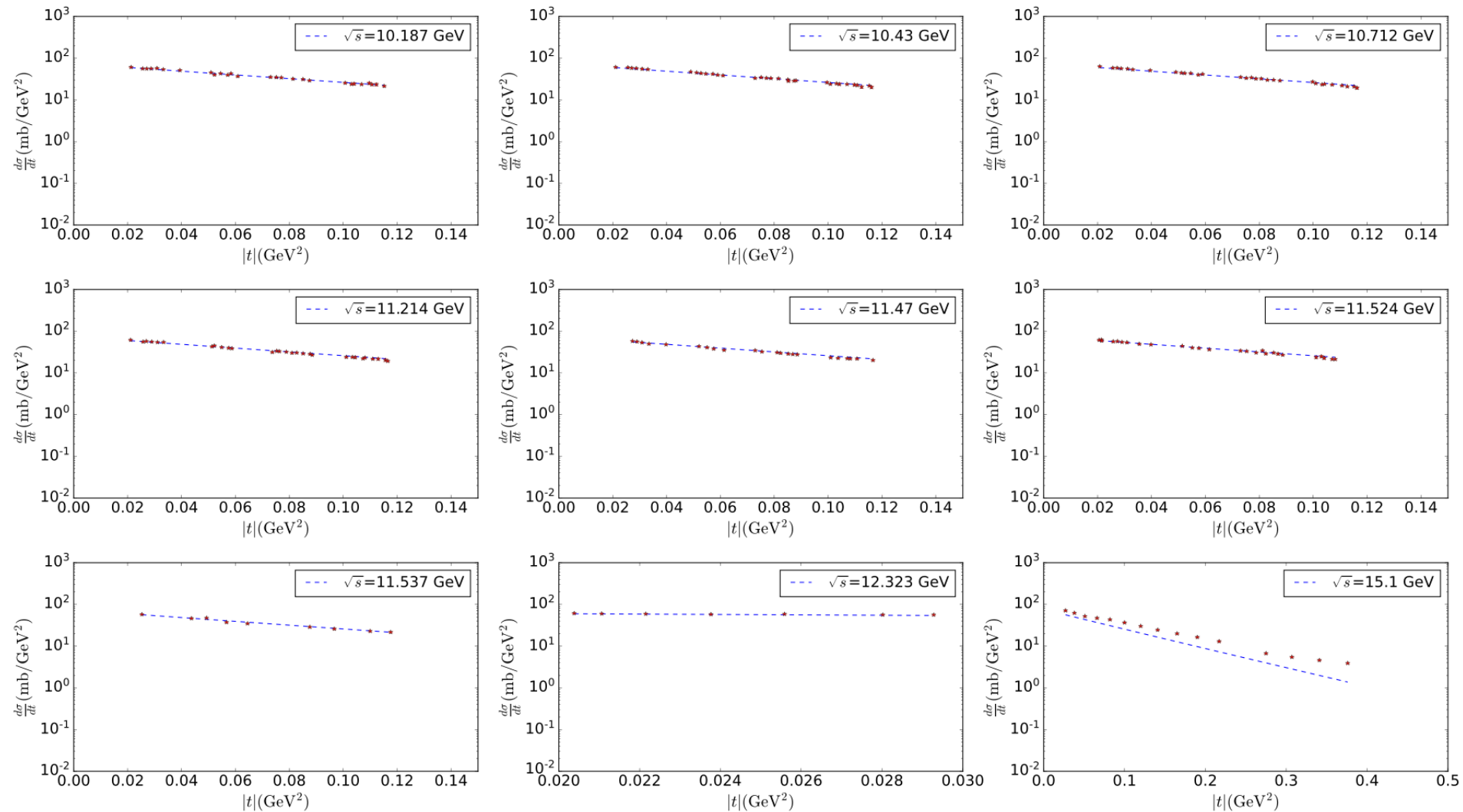
$$\begin{aligned}
 \frac{d\sigma}{dt} = & \frac{\lambda_g^4 s^2 A^4(t)}{16\pi} \left[\frac{\alpha'_g \Gamma \left[3 - \frac{\chi_g}{2} \right] \Gamma \left[1 - \frac{\alpha_g(t)}{2} \right]}{2 \Gamma \left[2 - \frac{\chi_g}{2} + \frac{\alpha_g(t)}{2} \right]} \left(\frac{\alpha'_g s}{2} \right)^{\alpha_g(t)-2} \right]^2 \\
 & - \frac{\lambda_g^2 \lambda_v^2 s A^2(t)}{4\pi} \left[\frac{\alpha'_g \Gamma \left[3 - \frac{\chi_g}{2} \right] \Gamma \left[1 - \frac{\alpha_g(t)}{2} \right]}{2 \Gamma \left[2 - \frac{\chi_g}{2} + \frac{\alpha_g(t)}{2} \right]} \left(\frac{\alpha'_g s}{2} \right)^{\alpha_g(t)-2} \right] \\
 & \times \left[\alpha'_v \sin \left(\frac{\pi \alpha_v(t)}{2} \right) (\alpha'_v s)^{\alpha_v(t)-1} \Gamma[-\alpha_v(t)] \right] \cos \left[\frac{\pi}{2} (\alpha_g(t) - \alpha_v(t)) \right] \\
 & + \frac{\lambda_v^4}{4\pi} \left[\alpha'_v \sin \left(\frac{\pi \alpha_v(t)}{2} \right) (\alpha'_v s)^{\alpha_v(t)-1} \Gamma[-\alpha_v(t)] \right]^2
 \end{aligned}$$

$$\begin{aligned}
 \sigma_{tot} = & \frac{1}{s} \text{Im} \mathcal{A}(s, t=0) \\
 = & \lambda_g^2 \sin \left(\frac{\pi \alpha_g(0)}{2} \right) \frac{\Gamma \left[3 - \frac{\chi_g}{2} \right] \Gamma \left[1 - \frac{\alpha_g(0)}{2} \right]}{\Gamma \left[2 - \frac{\chi_g}{2} + \frac{\alpha_g(0)}{2} \right]} \left(\frac{\alpha'_g s}{2} \right)^{\alpha_g(0)-1} \\
 & - 2 \lambda_v^2 \alpha'_v \sin^2 \left(\frac{\pi \alpha_v(0)}{2} \right) (\alpha'_v s)^{\alpha_v(0)-1} \Gamma[-\alpha_v(0)]
 \end{aligned}$$

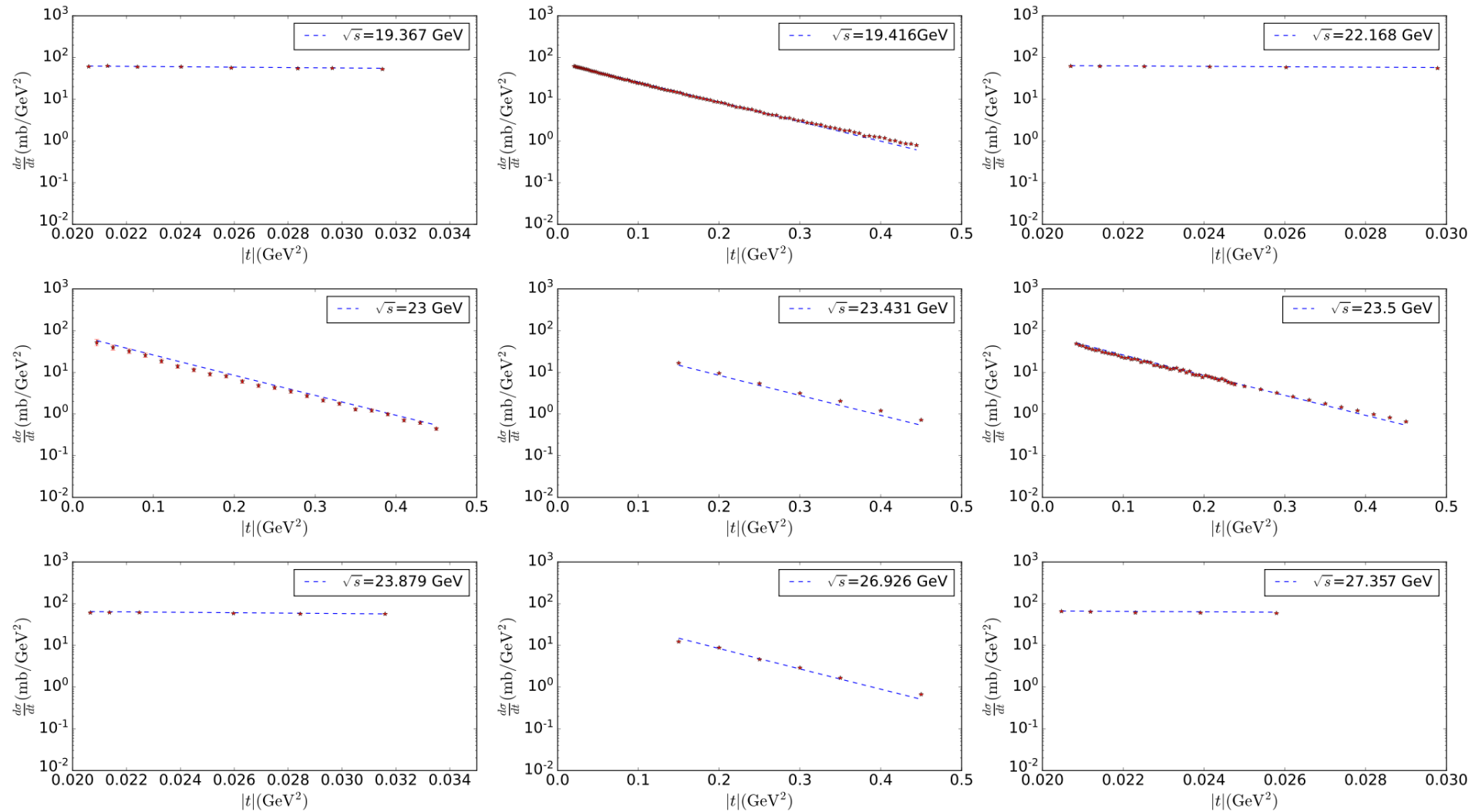
Total cross sections



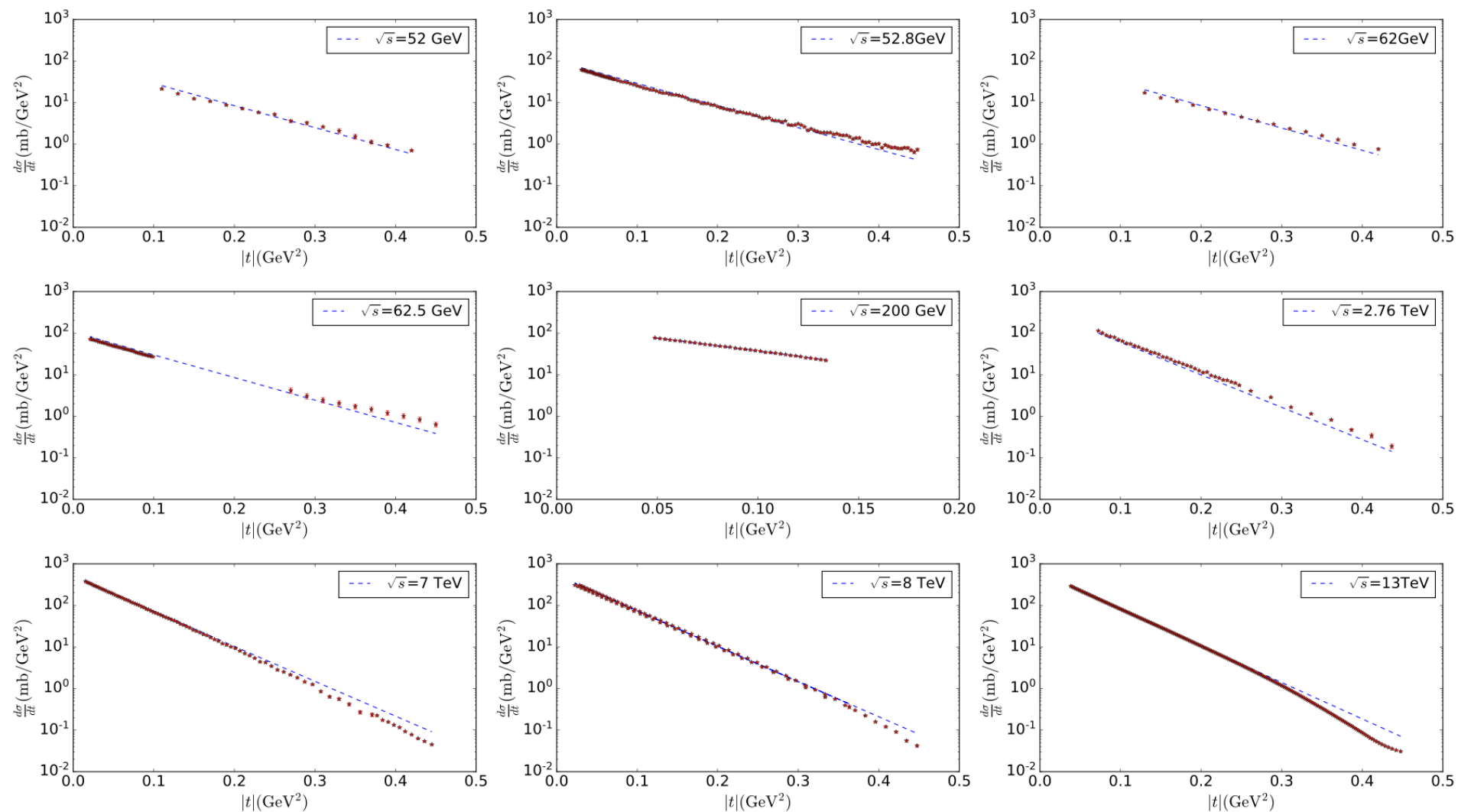
Differential pp cross section (1/3)



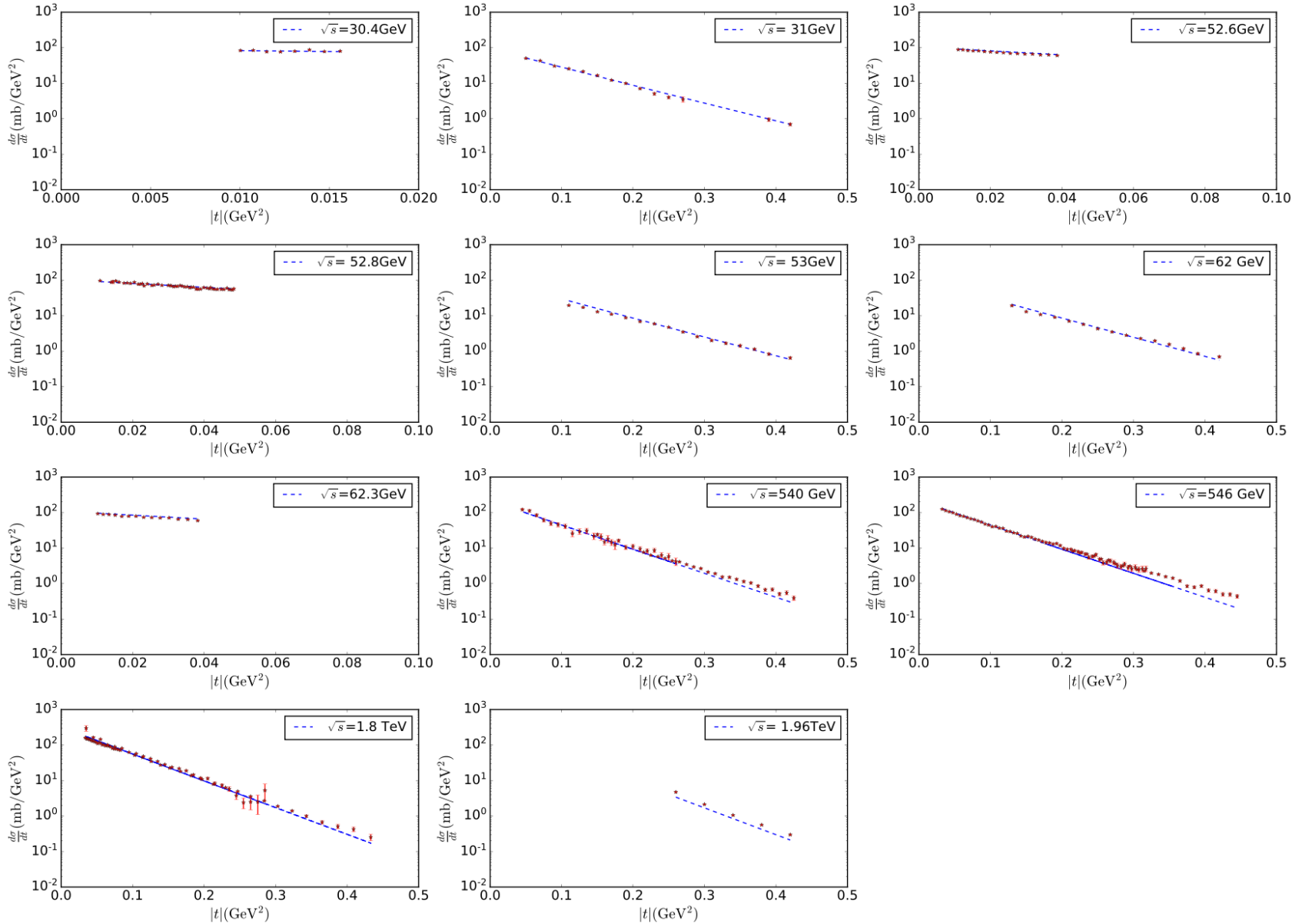
Differential pp cross section (2/3)



Differential pp cross section (3/3)



Differential ppbar cross section



Contribution of Coulomb interaction

Taking into account the Coulomb interaction (one photon exchange), the total scattering amplitude can be expressed as

$$\mathcal{A}_{\text{tot}} = \mathcal{A}_N + e^{i\alpha\phi} \mathcal{A}_C$$

$$\mathcal{A}_C(s, t) = \mp \frac{8\pi\alpha s}{|t|} G_{\text{eff}}^2(t)$$

Negative (positive) sign corresponds to identical (opposite) charges case

For the Coulomb phase, the expression proposed by Cahn is employed:

$$\phi_{\text{Cahn}} = - \left[\ln \left(\frac{B|t|}{2} \right) + \gamma + C \right]$$

Cahn (1982)

where

$$C = \ln \left(1 + \frac{8}{B\Lambda^2} \right) + \frac{4|t|}{\Lambda^2} \ln \left(\frac{4|t|}{\Lambda^2} \right) + \frac{2|t|}{\Lambda^2}$$

The parameter Λ^2 can be determined with the electromagnetic form factor obtained in the bottom-up AdS/QCD model

Total scattering amplitude

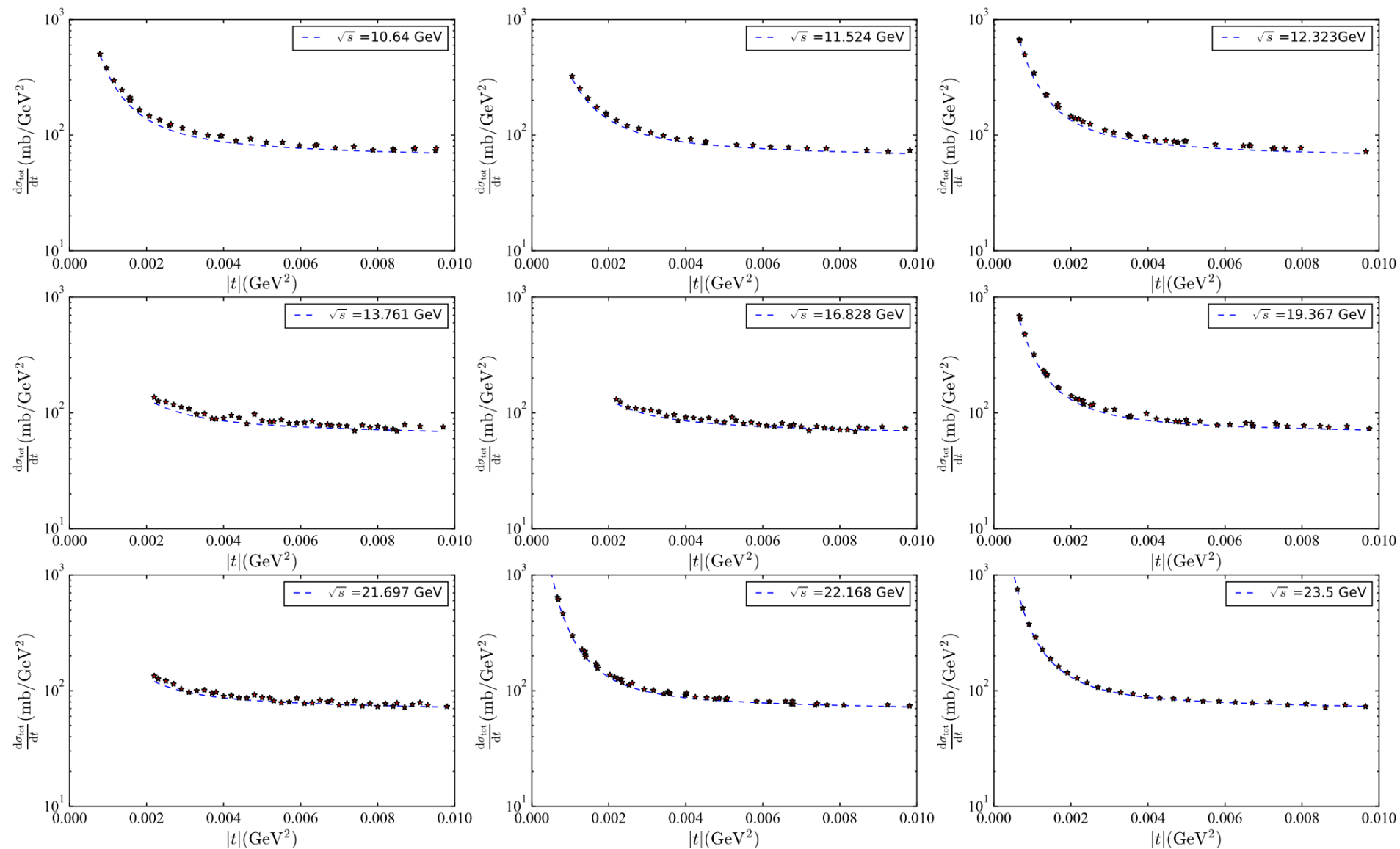
Involving the one photon exchange contribution, the total scattering amplitude is obtained as

$$\begin{aligned} \mathcal{A}_{\text{tot}} = & \mp e^{i\alpha\phi} \frac{8\pi\alpha s}{|t|} G_{\text{eff}}^2(t) \\ & - s\lambda_g^2 A^2(t) e^{-\frac{i\pi\alpha_g(t)}{2}} \frac{\Gamma[-\chi_g] \Gamma\left[1 - \frac{\alpha_g(t)}{2}\right]}{\Gamma\left[\frac{\alpha_g(t)}{2} - 1 - \chi_g\right]} \left(\frac{\alpha'_g s}{2}\right)^{\alpha_g(t)-1} \\ & + s\lambda_v^2 \alpha'_v e^{-\frac{i\pi\alpha_v(t)}{2}} \sin\left[\frac{\pi\alpha_v(t)}{2}\right] (\alpha'_v s)^{\alpha_v(t)-1} \Gamma[-\alpha_v(t)] \end{aligned}$$

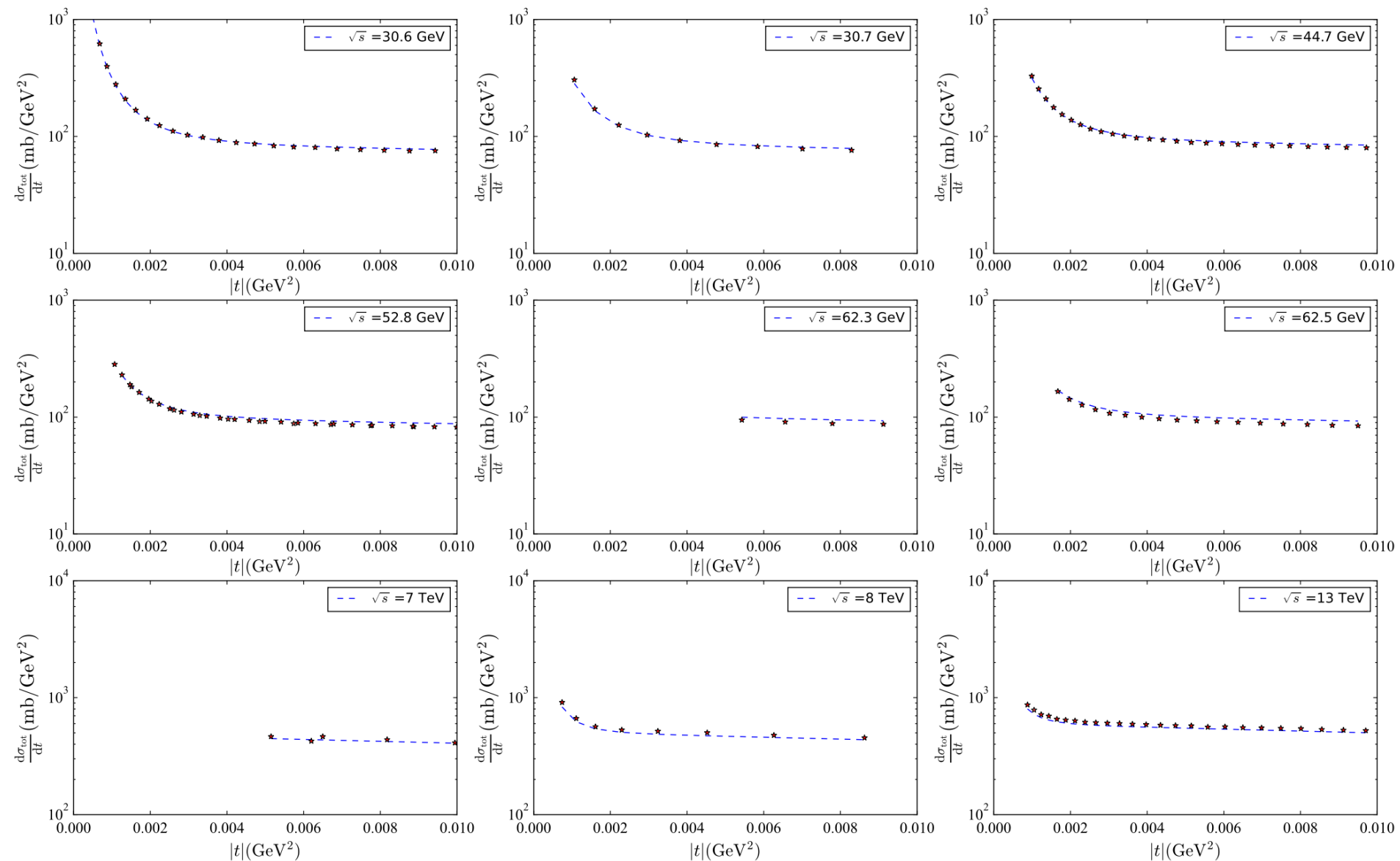
$$\frac{d\sigma_{\text{tot}}}{dt} = \frac{1}{16\pi s^2} |\mathcal{A}_{\text{tot}}(s, t)|^2$$

Cross terms are also important
--> Coulomb-nuclear interference (CNI) effect

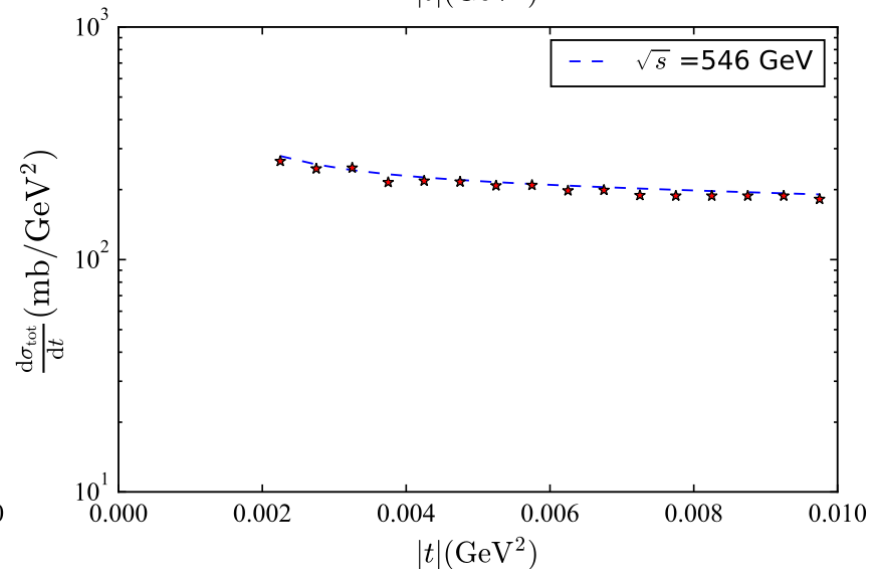
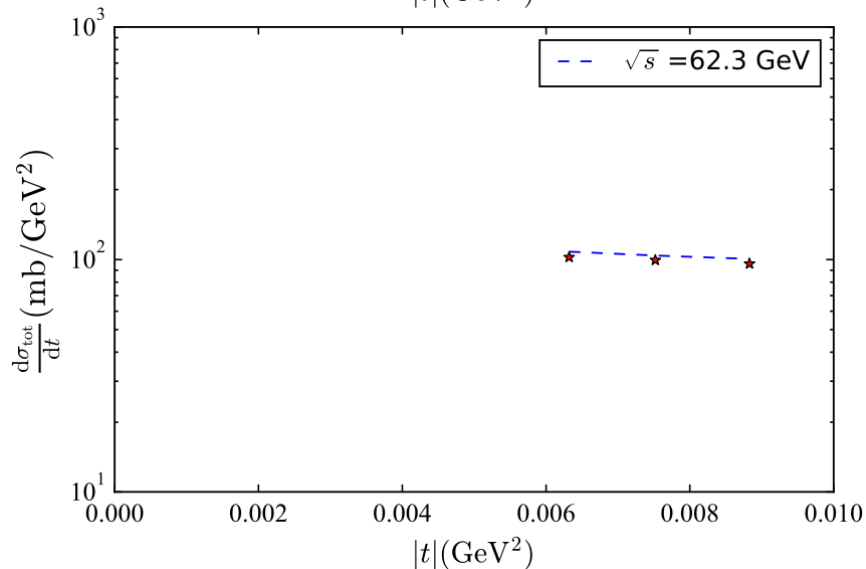
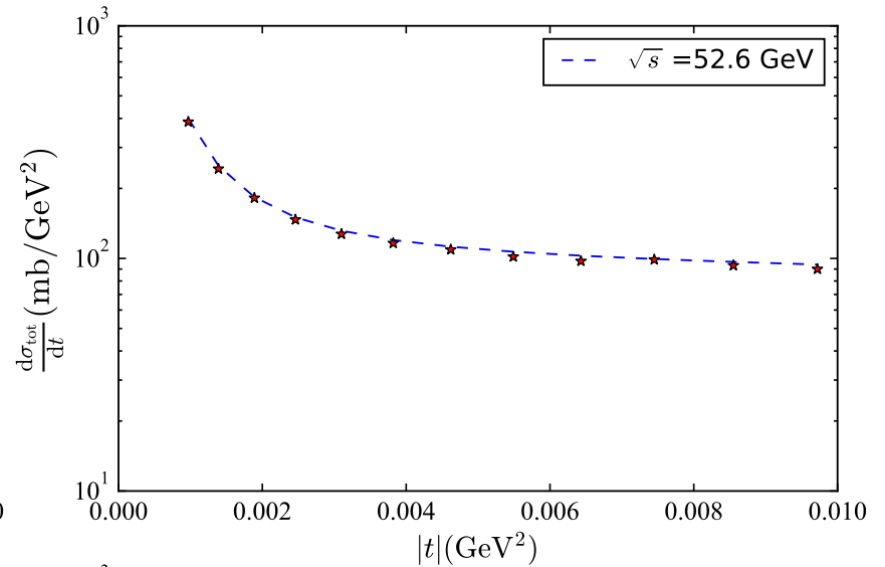
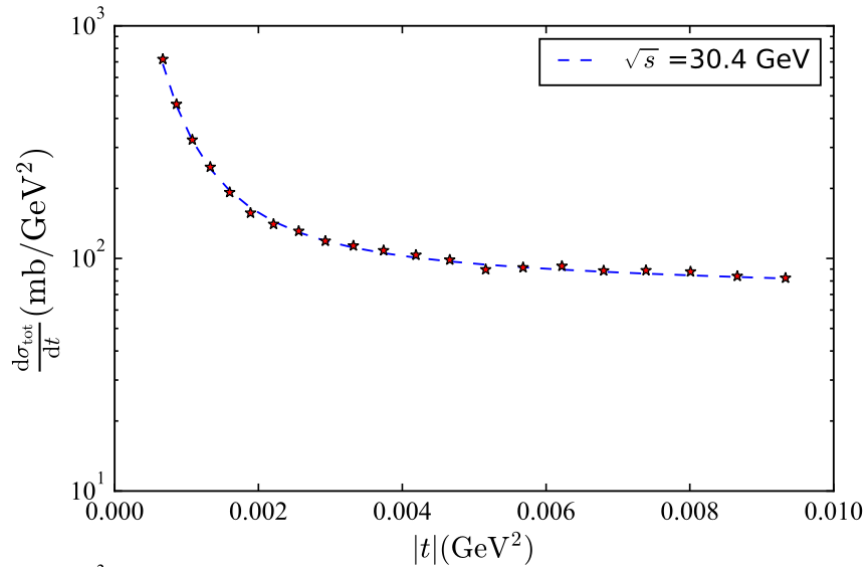
Differential pp cross section (1/2)



Differential pp cross section (2/2)



Differential ppbar cross section



Summary

- We have investigated elastic proton-(anti)proton scattering in the Regge regime in holographic QCD.
- Three contributions (Pomeron + Reggeon + Coulomb interaction) have been taken into account to calculate the cross sections.
- It is found that the experimental data of both the total and differential cross sections can be well described within the model in a wide kinematic region.
- Other hadron-hadron scattering processes can be studied in this framework.