



Exploring Polarization Variables in Two-Pion Photoproduction: Insights from the CLAS Experiment

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NSTAR24

14th International Workshop on the
Physics of Excited Nucleons

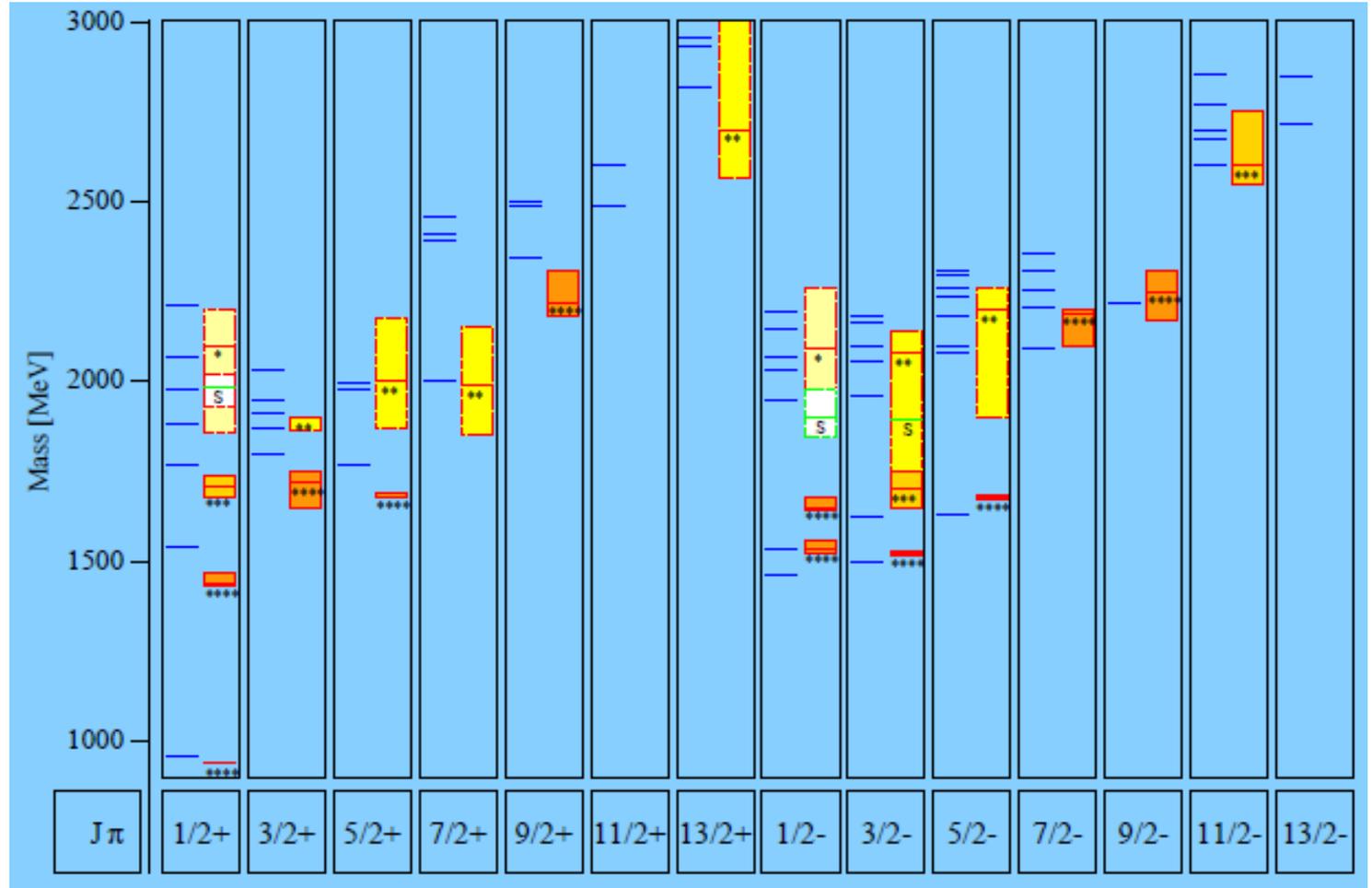
York, UK, June 20, 2024

The light baryon (N^* , Δ) spectrum in the Constituent Quark Model

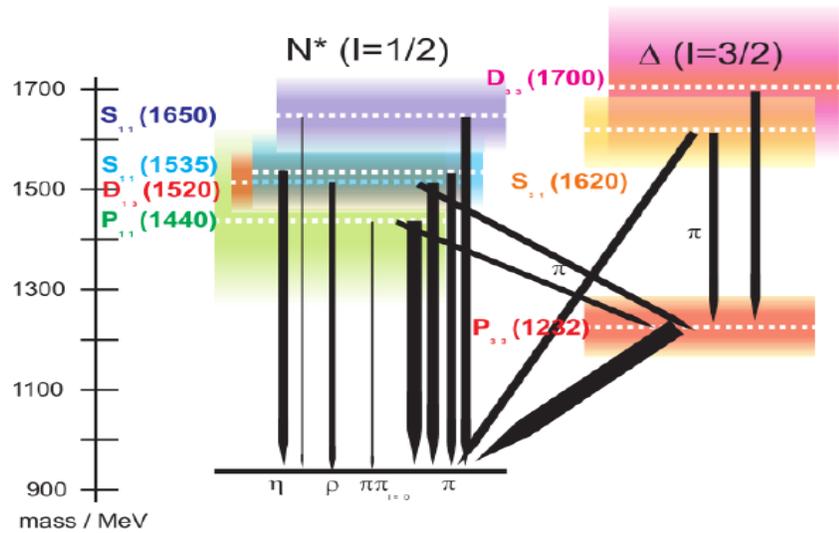
- ▶ Quarks confined into colorless hadrons



- ▶ Description by first principle QCD and constituent Quark Models:
 - Blue lines: expected states
 - ▶ Yellow/orange boxes: observations



The light baryon spectrum: experimental status



(1/2) ⁻	S11(1535)	****	██████
(1/2) ⁻	S11(1650)	****	██████
(3/2) ⁻	D13(1520)	****	██████
(5/2) ⁻	D13(1700)	***	██████
(1/2) ⁺	D15(1675)	****	██████
(1/2) ⁺	P11(1440)	****	██████
(1/2) ⁺	P11(1770)	***	██████
(1/2) ⁺	P11(1880)		██████
(1/2) ⁺	P11(1975)		██████
(3/2) ⁺	P13(1720)	****	██████
(3/2) ⁺	P13(1870)		██████
(3/2) ⁺	P13(1910)		██████
(3/2) ⁺	P13(1950)		██████
(3/2) ⁺	P13(2030)		██████
(5/2) ⁺	F15(1680)	****	██████
(5/2) ⁺	F15(1980)		██████
(5/2) ⁺	F15(2000)	**	██████
(7/2) ⁺	F17(1990)	**	██████

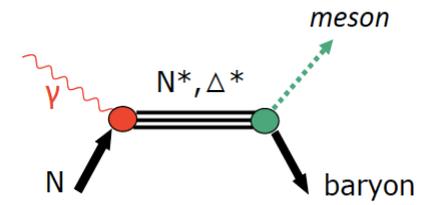
	Nπ
	Nη
	Nη'
	Nω
	Λ K
	Σ K

- ▶ Lowest lying N* and Δ* resonances
 - ▶ 1.3-2 GeV mass range: **second resonant region**
 - ▶ Overlapping states in the same mass region
 - ▶ Broad widths (short lifetimes)
 - ▶ Shared decay modes

- ▶ Most of the available information from pion/kaon beams experiments
 - ▶ Missing states: too small couplings with mesons

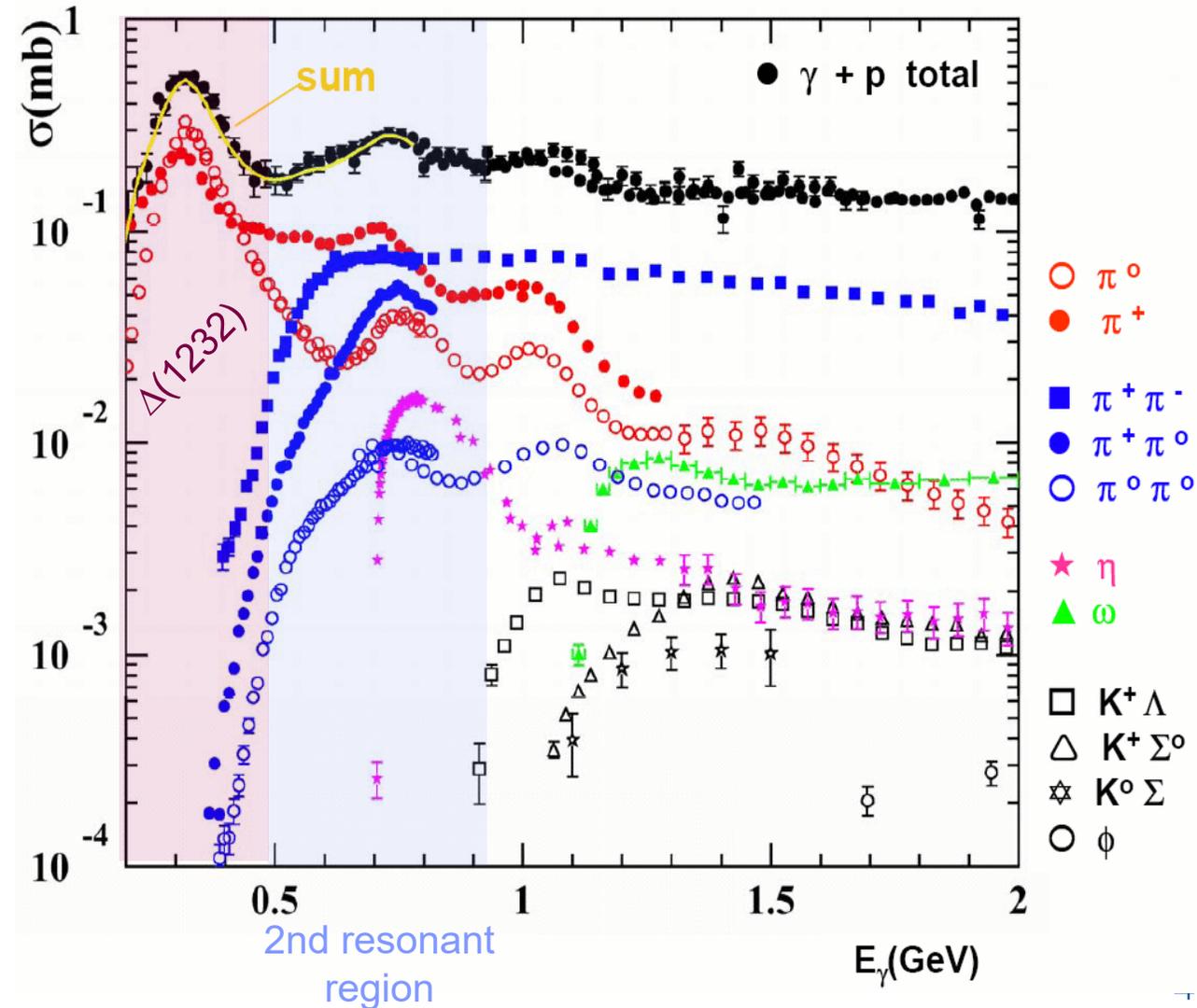
- ▶ How to disentangle each signal and spot missing resonances?
 - ▶ Difficult task if based only on the measurement of cross-sections
 - ▶ **Use new approaches: analysis of polarization observables (additional information: spin)**
 - ▶ Perform precision measurements in as many reactions as possible

N^*/Δ^* in photoproduction reactions



- ▶ Photon induced reaction could favor the formation of missing resonances which might couple strongly to the γN vertex
- ▶ γ reactions not studied extensively in the past - lack of good enough (energy/intensity) photon beams
- ▶ Dominant contributions to the "second resonant region": double-pion and η channels
 - ▶ Double-pion photoproduction: good tool to investigate this mass region

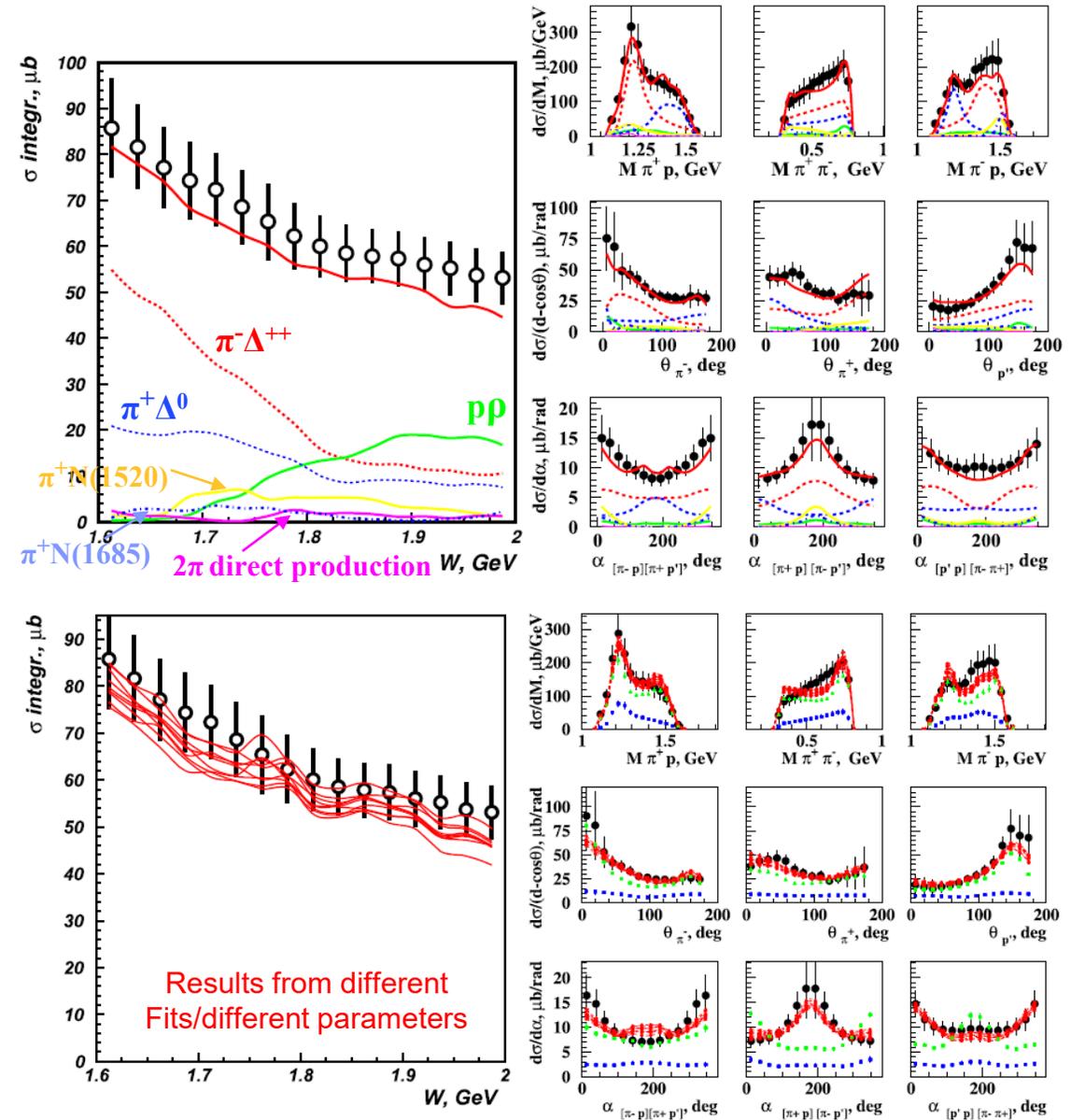
Photonuclear cross sections



Photoproduction of $\pi^+\pi^-$ pairs off protons (unpolarized)

E. Golovatch (CLAS) PL B788 (2019), 371

- ▶ Measurement of 9x 1-fold differential cross sections of the $\gamma p \rightarrow \pi^+\pi^-p$ reaction in the (1.6, 2) GeV range
- ▶ Attempt to reproduce the cross-sections using the JM17 meson-baryon reaction model
 - ▶ Reasonable description
 - ▶ A PWA fit provides the intermediate resonances contributions & parameters
 - Intermediate channels: $\pi\Delta^{++}$, $\pi\Delta^0$, $p\rho^0$, $\pi\pi^+p$ direct production, $\pi^+N(1530) 3/2^-$, $\pi^+N(1685) 5/2^+$
 - Extraction of masses, widths, photocouplings
 - (new) Excited states required in the model:
 - $N(1440) 1/2^+$, $N(1520) 3/2^-$, $N(1535) 1/2^-$, $N(1650) 1/2^-$, $N(1680) 5/2^-$, $N'(1720) 3/2^+$, $N(2190) 7/2^-$
 - $\Delta(1620) 1/2^-$, $\Delta(1700) 3/2^-$, $\Delta(1905) 5/2^+$, $\Delta(1950) 7/2^+$



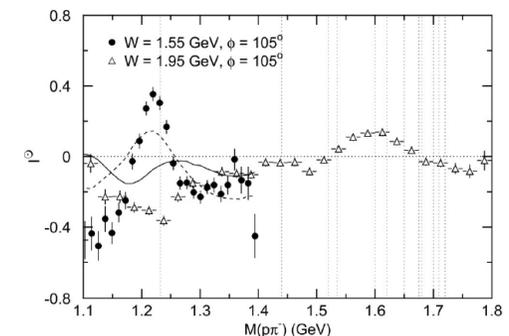
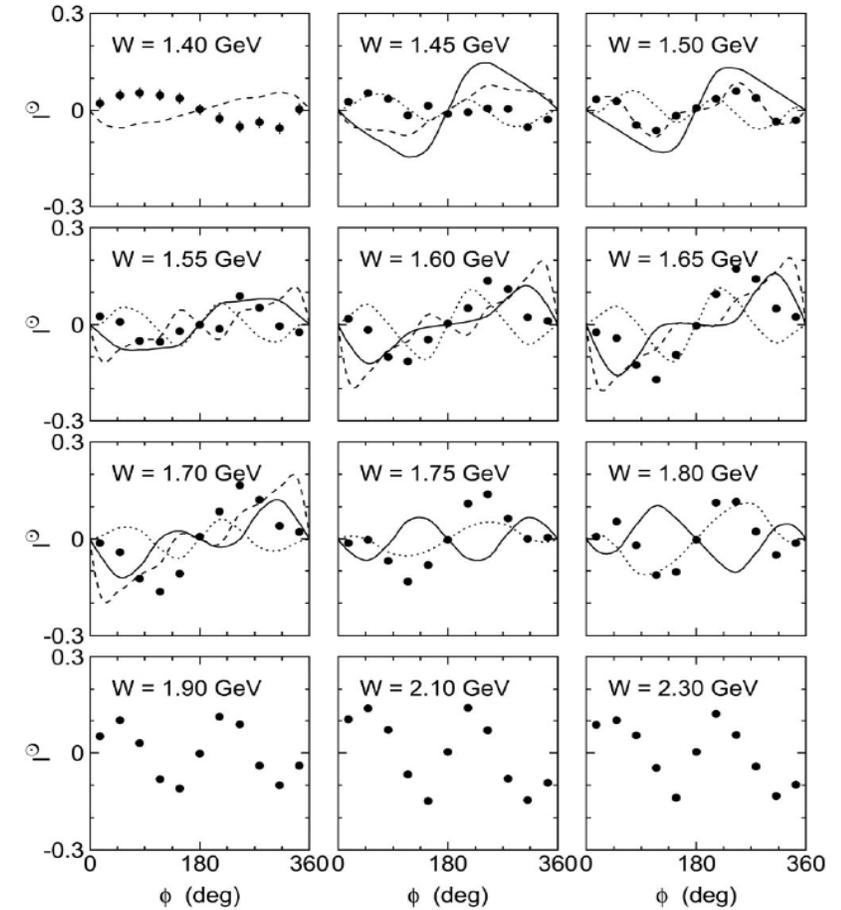
Photoproduction of $\pi^+\pi^-$ pairs from protons with circularly polarized beam

S. Strauch et al. (CLAS) PLR95 (2005), 162003

- ▶ CLAS data: $1.35 < W < 2.30$ GeV
 - ▶ Missing resonances predicted to lie in the region $W > 1.8$ GeV
- ▶ Circularly polarized photon beam, no polarization specified for target and recoil proton
- ▶ First measurement of beam-helicity asymmetry distributions as a function of the helicity angle:

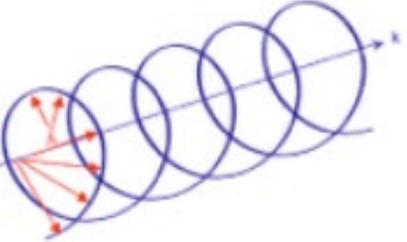
$$I^{\odot} = \frac{1}{P_{\gamma}} \frac{\sigma^{+} - \sigma^{-}}{\sigma^{+} + \sigma^{-}}$$

- ▶ Odd trend in all W sub-ranges
- ▶ Compared with models based on electroproduction of double-charged pions including a set of quasi-two body intermediate states (Moiseev et al.):
 - $\pi\Delta, \rho N, \pi N(1520), \pi N(1680)$ + contributions from $\Delta(1600), N(1700), N(1710), N(1720)$
 - The agreement is not satisfactory, calls for a more detailed description
 - The I^{\odot} observable is critically sensitive to interferences

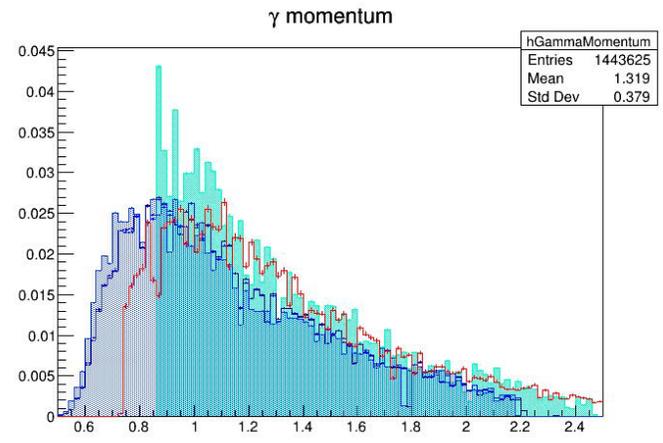


Experimental method – polarized beam and target

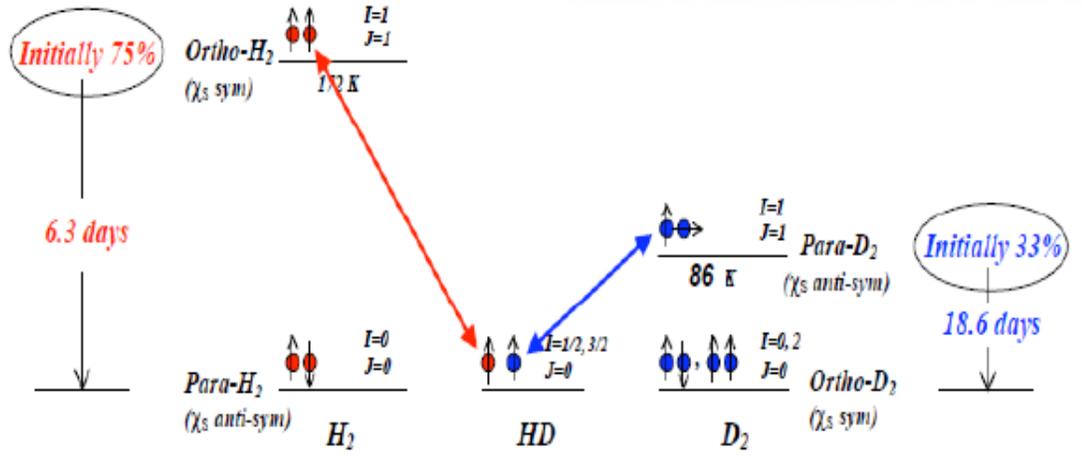
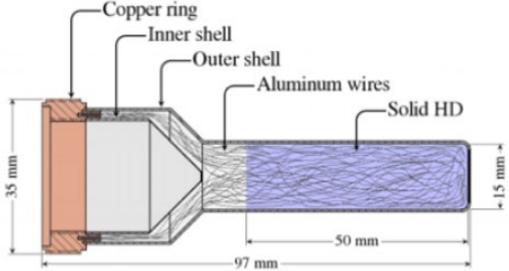
- ▶ CLAS-g14 data taking (2011-2012): *circularly polarized* photon beam with momentum up to 2.5 GeV/c interacting on a *cryogenic HD longitudinally polarized* target
- ▶ **Beam:** circularly polarized photons by bremsstrahlung from a longitudinally polarized electron beam (>85%) through a gold foil radiator
 - Circular: \uparrow/\downarrow (960 Hz flip frequency)
 - Energy dependent γ polarization
- ▶ **Target:** “brute-force + aging” polarization method (< 30%)
 - Longitudinal (along beam direction): \Rightarrow/\Leftarrow
 - Fixed in different data-sets
 - Protons + neutrons



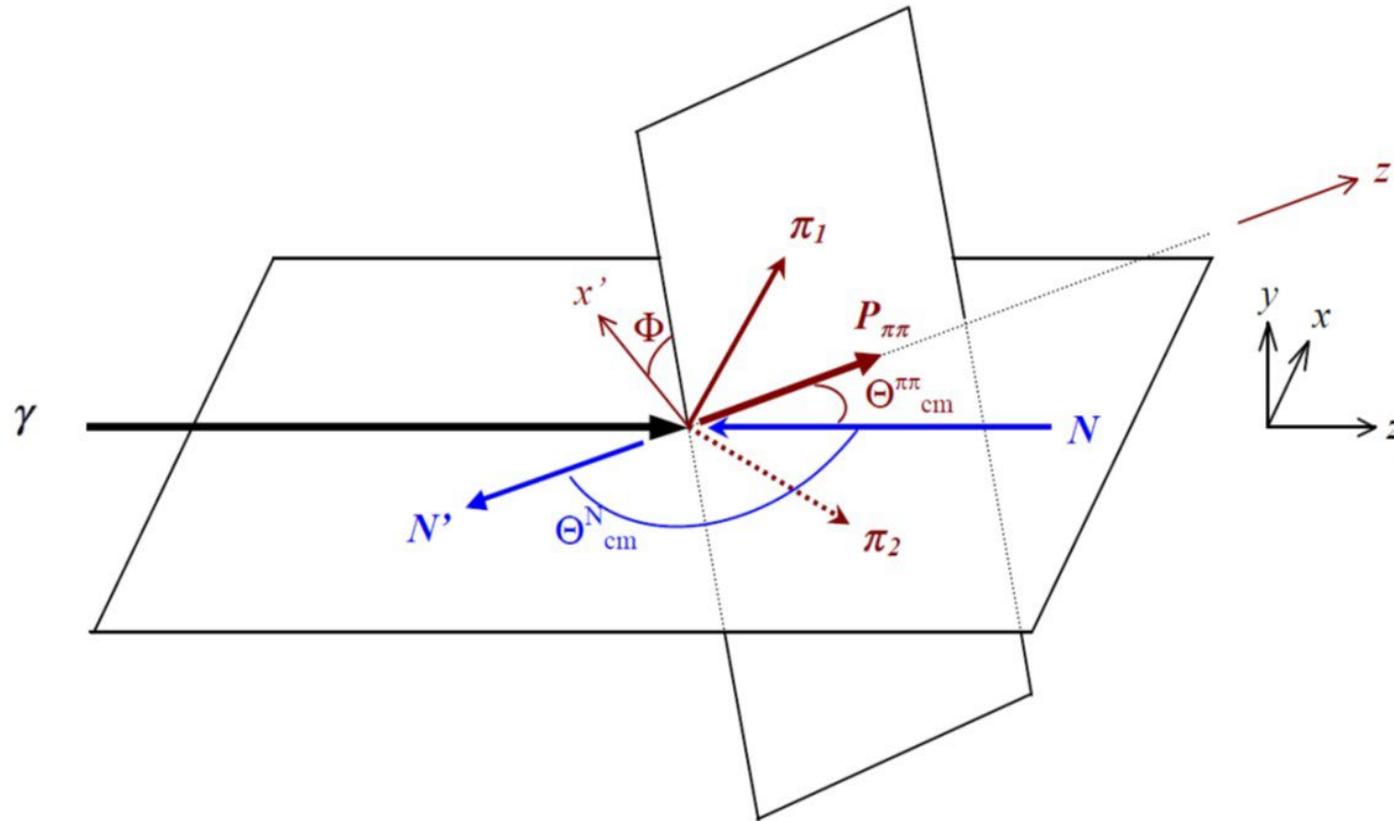
$$x = \frac{E_\gamma}{E_{beam}}$$



$$\delta_\odot = P_{el} \frac{4x - x^2}{4 - 4x + 3x^2}$$



Study of polarization observables in the



▶ Differential cross-section expressed as a function of polarization observables, weighted by the amount of beam δ_{\odot} and/or target Λ polarization

▶ The trend of the polarization observables depends on the resonance content in a given energy range

▶ **Polarization observables are bilinear combinations of partial amplitudes** (Roberts, Oed PRC71 (2005), 0552001): **very sensitive to interference effects**

$$\frac{d\sigma}{dx_i} = \sigma_0 \left\{ (1 + \Lambda_z \cdot \mathbf{P}_z) + \delta_{\odot} (\mathbf{I}^{\odot} + \Lambda_z \cdot \mathbf{P}_z^{\odot}) \right\}$$

Polarization observables extraction

Problem: extract from the number of collected events the I^\odot, P, P^\odot observables as a function of the Φ azimuthal angle in the helicity reference system, in W energy ranges

$$P_z = \frac{1}{\Lambda_z} \frac{[N(\rightarrow\Rightarrow) + N(\leftarrow\Rightarrow)] - [N(\rightarrow\Leftarrow) + N(\leftarrow\Leftarrow)]}{[N(\rightarrow\Rightarrow) + N(\leftarrow\Rightarrow)] + [N(\rightarrow\Leftarrow) + N(\leftarrow\Leftarrow)]}$$

$$I^\odot = \frac{1}{\delta_\odot} \frac{[N(\rightarrow\Rightarrow) + N(\rightarrow\Leftarrow)] - [N(\leftarrow\Rightarrow) + N(\leftarrow\Leftarrow)]}{[N(\rightarrow\Rightarrow) + N(\rightarrow\Leftarrow)] + [N(\leftarrow\Rightarrow) + N(\leftarrow\Leftarrow)]}$$

$$P_z^\odot = \frac{1}{\Lambda_z \delta_\odot} \frac{[N(\rightarrow\Rightarrow) + N(\leftarrow\Leftarrow)] - [N(\rightarrow\Leftarrow) + N(\leftarrow\Rightarrow)]}{[N(\rightarrow\Rightarrow) + N(\leftarrow\Leftarrow)] + [N(\rightarrow\Leftarrow) + N(\leftarrow\Rightarrow)]}$$

- Related to differential cross-section asymmetries
- Depending on the relative beam/target spin configurations
- Two data sets with opposite target (\Rightarrow/\Leftarrow) polarizations needed (with proper normalization)
- Each data-set contains both helicities

Polarization asymmetries in φ_{hel} bins

$$\frac{d\sigma}{dx_i} = \sigma_0 \{ (1 + \Lambda_z \cdot \mathbf{P}_z) + \delta_{\odot} (\mathbf{I}^{\odot} + \Lambda_z \cdot \mathbf{P}_z^{\odot}) \}$$

- ▶ This equation (Roberts et al., PRC 718(2005), 055201) can be split in four depending on the orientation of beam helicity and target polarization (along z)
- ▶ Two data sets with opposite target polarization need to be used (properly normalized)
- ▶ The system of equations can be solved analytically extracting, in every φ bin, I^{\odot} , P_z , P_z^{\odot} and σ_0

$$N_{exp}^{\rightarrow\rightarrow} = \left(\frac{d\sigma}{d\Omega}\right)_0 \mathbf{L} \boldsymbol{\varepsilon} [1 + \Lambda_z P_z + \delta_{\odot} (I_{\odot} + \Lambda_z P_z^{\odot})]$$

$$N_{exp}^{\leftarrow\rightarrow} = \left(\frac{d\sigma}{d\Omega}\right)_0 \mathbf{L} \boldsymbol{\varepsilon} [1 + \Lambda_z P_z - \delta_{\odot} (I_{\odot} + \Lambda_z P_z^{\odot})]$$

$$N_{exp}^{\rightarrow\leftarrow} = \left(\frac{d\sigma}{d\Omega}\right)_0 \mathbf{L} \boldsymbol{\varepsilon} [1 - \Lambda_z P_z + \delta_{\odot} (I_{\odot} - \Lambda_z P_z^{\odot})]$$

$$N_{exp}^{\leftarrow\leftarrow} = \left(\frac{d\sigma}{d\Omega}\right)_0 \mathbf{L} \boldsymbol{\varepsilon} [1 - \Lambda_z P_z - \delta_{\odot} (I_{\odot} - \Lambda_z P_z^{\odot})]$$



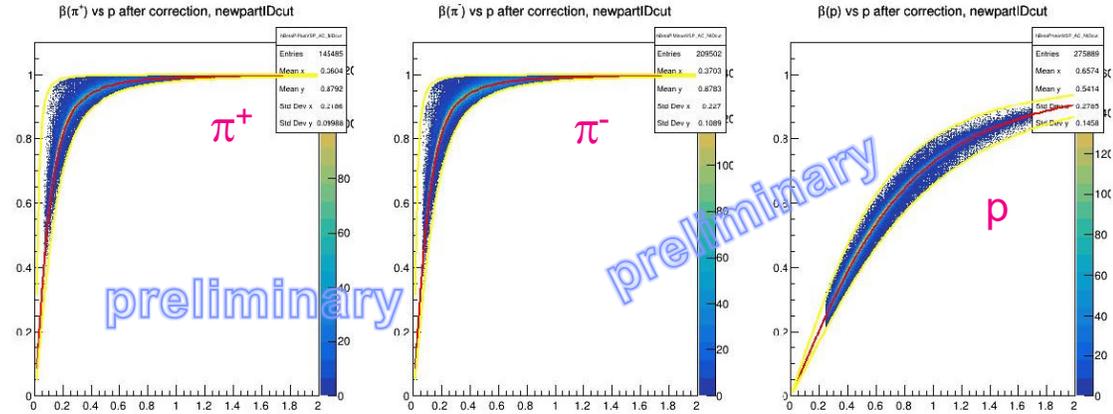
$$I_{\odot} = \frac{\frac{N_1^{\rightarrow\rightarrow} - N_1^{\leftarrow\rightarrow}}{\delta_{\odot 1}} + \frac{\Lambda_{z1}}{\Lambda_{z2}} \cdot \frac{L_{eff1}}{L_{eff2}} \cdot \frac{N_2^{\rightarrow\leftarrow} - N_2^{\leftarrow\leftarrow}}{\delta_{\odot 2}}}{(N_1^{\rightarrow\rightarrow} + N_1^{\leftarrow\rightarrow}) + \frac{\Lambda_{z1}}{\Lambda_{z2}} \cdot \frac{L_{eff1}}{L_{eff2}} (N_2^{\rightarrow\leftarrow} + N_2^{\leftarrow\leftarrow})}$$

$$P_z^{\odot} = \frac{1}{\Lambda_{z2}} \cdot \frac{\frac{N_1^{\rightarrow\rightarrow} - N_1^{\leftarrow\rightarrow}}{\delta_{\odot 1}} - \frac{L_{eff1}}{L_{eff2}} \cdot \frac{N_2^{\rightarrow\leftarrow} - N_2^{\leftarrow\leftarrow}}{\delta_{\odot 2}}}{(N_1^{\rightarrow\rightarrow} + N_1^{\leftarrow\rightarrow}) + \frac{\Lambda_{z1}}{\Lambda_{z2}} \cdot \frac{L_{eff1}}{L_{eff2}} (N_2^{\rightarrow\leftarrow} + N_2^{\leftarrow\leftarrow})}$$

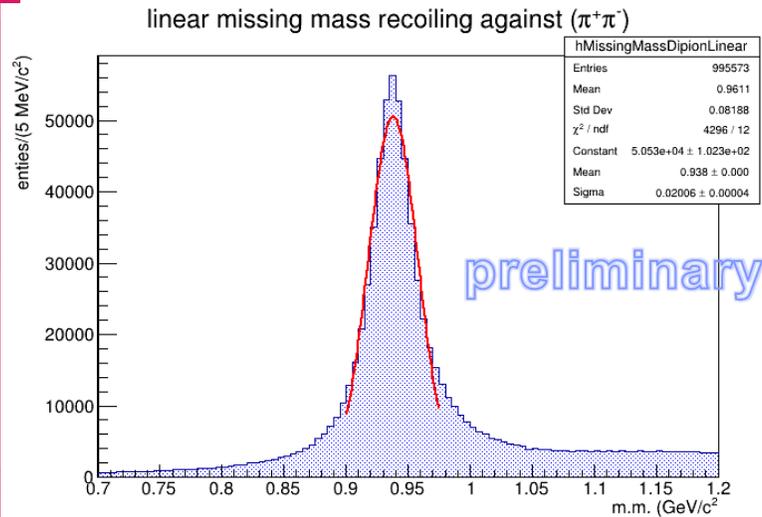
$$P_z = \frac{1}{\Lambda_{z2}} \cdot \frac{(N_1^{\rightarrow\rightarrow} + N_1^{\leftarrow\rightarrow}) - \frac{L_{eff1}}{L_{eff2}} \cdot (N_2^{\rightarrow\leftarrow} + N_2^{\leftarrow\leftarrow})}{(N_1^{\rightarrow\rightarrow} + N_1^{\leftarrow\rightarrow}) + \frac{\Lambda_{z1}}{\Lambda_{z2}} \cdot \frac{L_{eff1}}{L_{eff2}} (N_2^{\rightarrow\leftarrow} + N_2^{\leftarrow\leftarrow})}$$

Data selection – exclusive $\vec{\gamma}\vec{p} \rightarrow \pi^+\pi^-p$ reaction

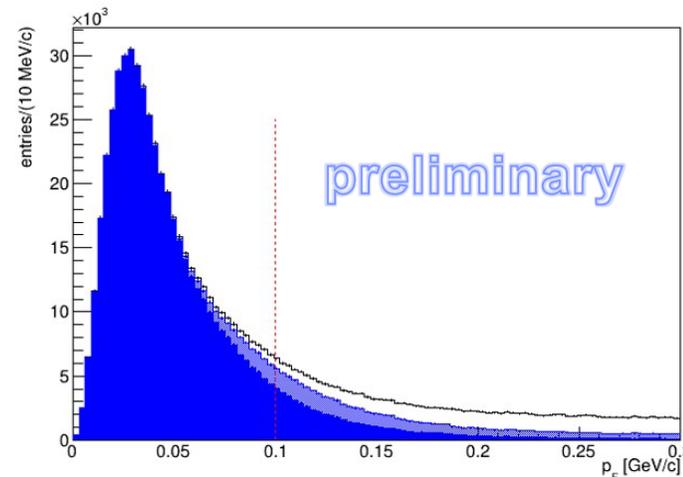
Description	Cut
Particle multiplicity	1 negative, 2 positives
Time coincidence	Time coincidence between: 1 proton, 1 π^+ , 1 π^-
$2\pi p$ z-vertex in HD target	$-9.5 < z_{\text{vertex}} < -5.8$ cm
$2\pi p$ pId: β_{corr}	$p_{\pi^\pm} / \sqrt{p_{\pi^\pm m}^2 + (m_\pi - 80 \text{ [MeV]})^2} \leq \beta_{\pi^\pm}^{\text{corr}} \leq p_{\pi^\pm} / \sqrt{p_{\pi^\pm}^2 + (m_\pi + 80 \text{ [MeV]})^2}$ $p_p / \sqrt{p_p^2 + (m_p - 200 \text{ [MeV]})^2} \leq \beta_p^{\text{corr}} \leq p_p / \sqrt{p_p^2 + (m_p + 200 \text{ [MeV]})^2}$
$2\pi p$ pId: $ \Delta\beta $	$ \Delta(\beta_p) < 0.08$ $p_{\pi^\pm} \leq 500 \text{ [MeV/c]} : \Delta(\beta_{\pi^\pm}) < 0.08$ $p_{\pi^\pm} \geq 500 \text{ [MeV/c]} : \Delta(\beta_{\pi^\pm}) < 0.2$
$2\pi p$ fiducial cuts	π^+ && π^- && p within fiducial volume
Missing mass for proton pId	$0.824 \leq \text{m.m.}(\pi^+\pi^-) \leq 1.052 \text{ [GeV}/c^2]$
Total missing mass	$\text{m.m.}(\pi^+\pi^-p) < 0 \text{ [GeV}/c^2]$
Fermi momentum	$p_F < 100 \text{ MeV}/c$
Coplanarity	$ \text{coplanarity} < 10^\circ$



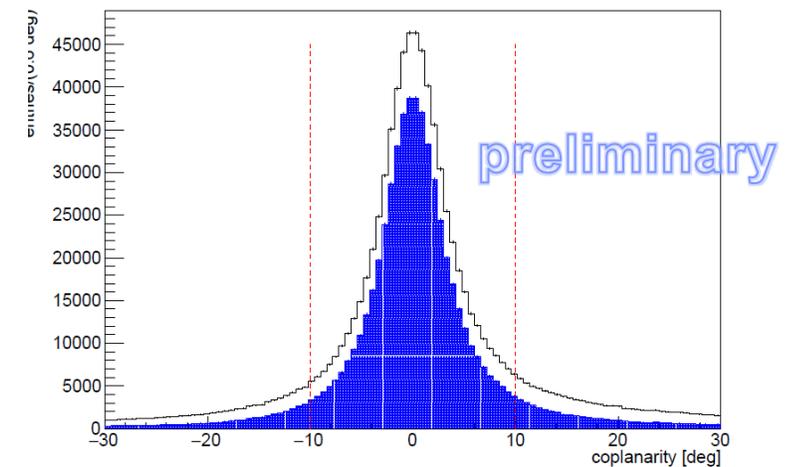
Particle ID for $\pi^+\pi^-$ and p based on TOF
 Further selection on $(\pi^+\pi^-)$ missing mass to identify the proton



Total missing mass cut



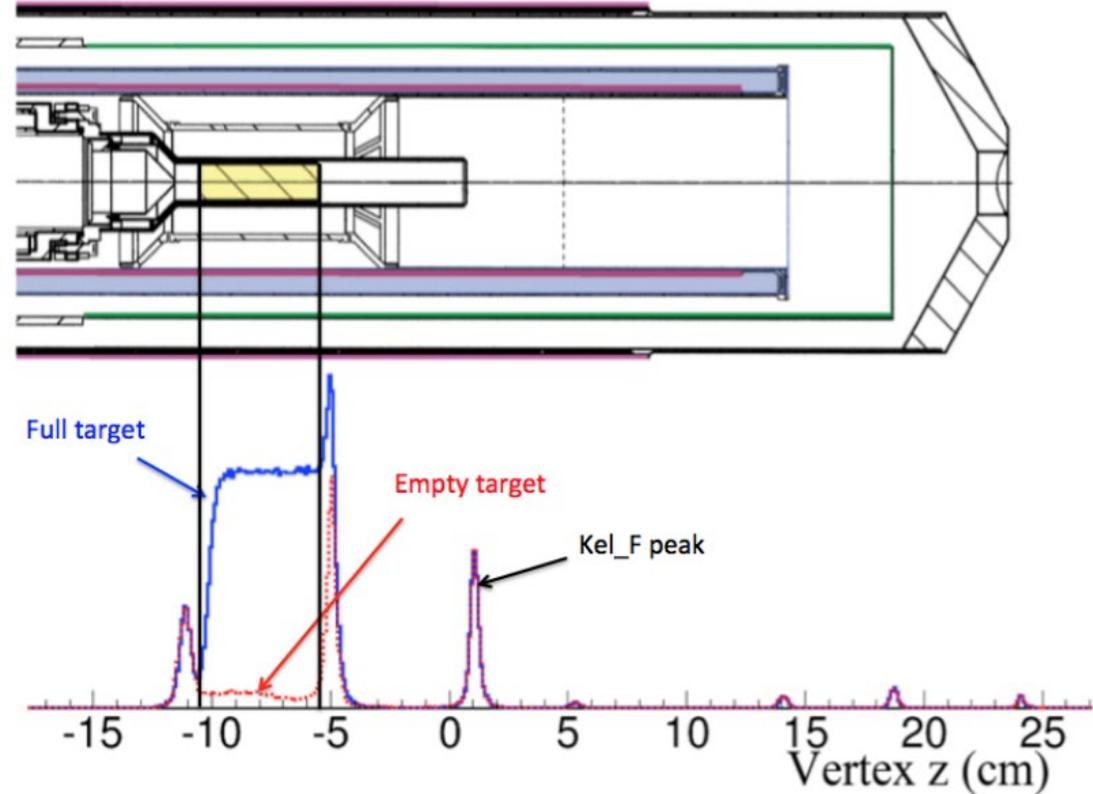
Missing momentum cut: reject reactions without spectator at rest



Coplanarity cut for pion pairs

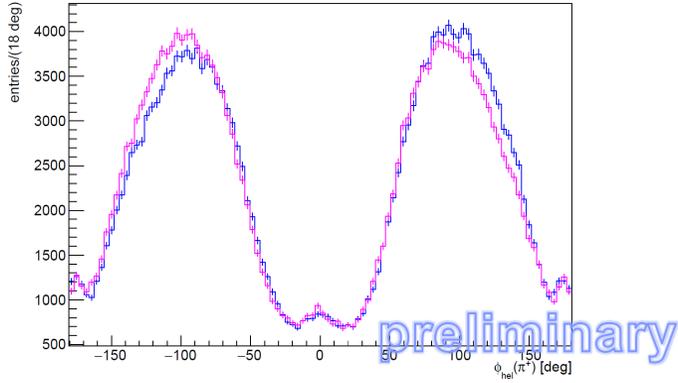
Experimental data: empty target subtraction

- ▶ Selection of events from the HD target: fiducial cut in r and z
- ▶ The events selected in the fiducial volume of the target contain the contribution from the target walls (unpolarized)
 - ▶ Empty target subtraction needed
 - ▶ Relative normalization of different runs: height of Kel-F wall peak
 - ▶ Subtraction with empty-target runs
- ▶ Events in the Kel-F peak also used for relative luminosity normalizations between different data sets

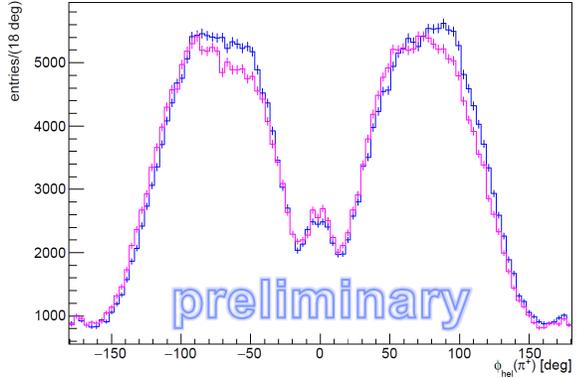


Experimental angular distributions

Set w/ positive target polarization



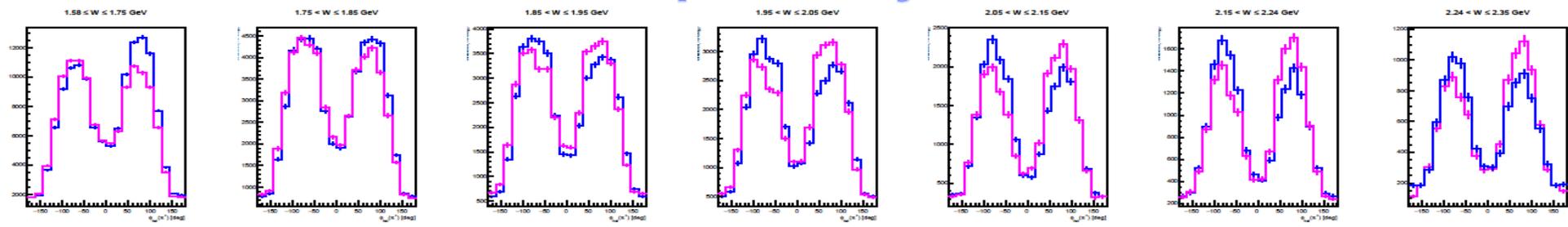
Set w/ negative target polarization



- ▶ Inputs: azimuthal angular distributions (ϕ_{hel})
- ▶ Bin by bin: number of events selected with
 - ▶ given helicity (positive/negative in the same data set)
 - ▶ given target polarization (in different data sets)
 - ▶ selection in W energy ranges (~ 100 MeV wide window)
 - ▶ counts to be properly normalized between different data sets

▶ Slight differences when selecting different combinations of helicities/target polarization: physics!

preliminary



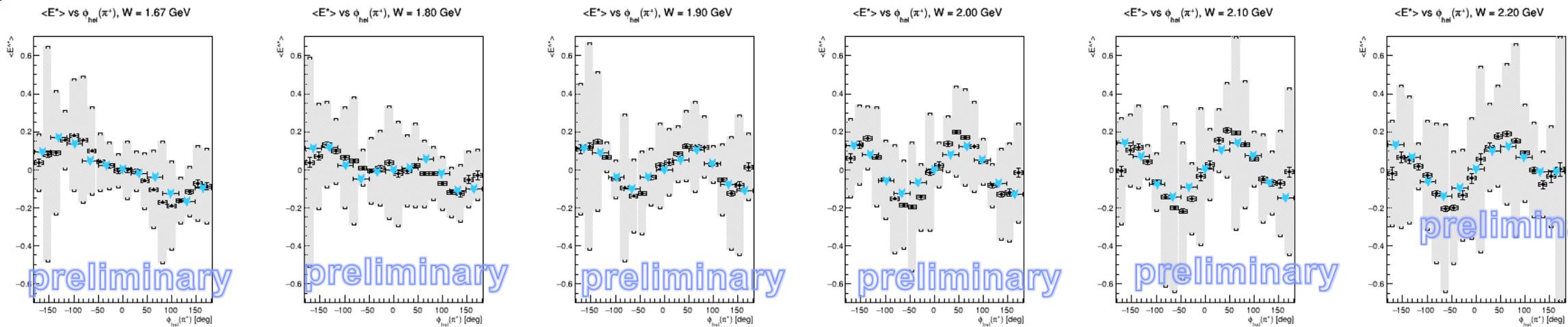
Evaluation of experimental beam-helicity asymmetries E^*

- ▶ E^* can be extracted from all available data samples (with similar experimental conditions)
- ▶ For each data set:

$$E^* = \frac{1}{\delta} \frac{N^+ - N^-}{N^+ + N^-}$$

- ▶ The E^* values agree with previous measurements with polarized beam only (blue points)
- ▶ Gigantic systematic errors (grey bars) from the spread of values obtained with different data sets – to be refined!

Blue points from S. Strauch et al., CLAS Coll., PRL95 (2005), 162003



Again on the observables extraction

- ▶ **Recall:** two data sets needed to extract the polarization observables
 - ▶ Each has its own normalization (i.e. luminosity)
 - ▶ Each data set was acquired with a given trigger (which might have different efficiency)
 - ▶ Each data set is characterized by a different acceptance
- ▶ The L_{eff1}/L_{eff2} factor is crucial

$$I_{\odot} = \frac{\frac{N_1^{\rightarrow\rightarrow} - N_1^{\leftarrow\leftarrow}}{\delta_{\odot 1}} + \frac{\Lambda_{z1}}{\Lambda_{z2}} \cdot \frac{L_{eff1}}{L_{eff2}} \cdot \frac{N_2^{\rightarrow\leftarrow} - N_2^{\leftarrow\leftarrow}}{\delta_{\odot 2}}}{(N_1^{\rightarrow\rightarrow} + N_1^{\leftarrow\leftarrow}) + \frac{\Lambda_{z1}}{\Lambda_{z2}} \cdot \frac{L_{eff1}}{L_{eff2}} (N_2^{\rightarrow\leftarrow} + N_2^{\leftarrow\leftarrow})}$$

$$P_z^{\odot} = \frac{1}{\Lambda_{z2}} \cdot \frac{\frac{N_1^{\rightarrow\rightarrow} - N_1^{\leftarrow\leftarrow}}{\delta_{\odot 1}} - \frac{L_{eff1}}{L_{eff2}} \cdot \frac{N_2^{\rightarrow\leftarrow} - N_2^{\leftarrow\leftarrow}}{\delta_{\odot 2}}}{(N_1^{\rightarrow\rightarrow} + N_1^{\leftarrow\leftarrow}) + \frac{\Lambda_{z1}}{\Lambda_{z2}} \cdot \frac{L_{eff1}}{L_{eff2}} (N_2^{\rightarrow\leftarrow} + N_2^{\leftarrow\leftarrow})}$$

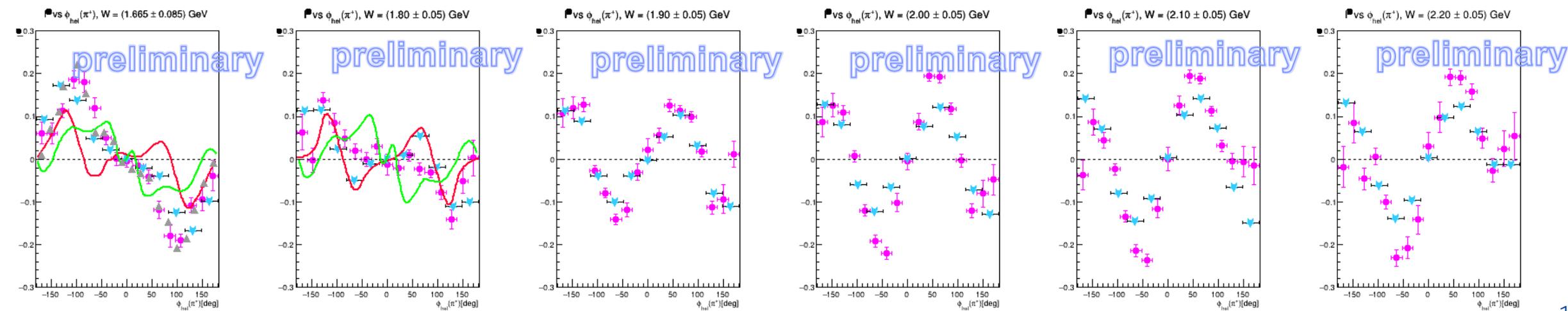
$$P_z = \frac{1}{\Lambda_{z2}} \cdot \frac{(N_1^{\rightarrow\rightarrow} + N_1^{\leftarrow\leftarrow}) - \frac{L_{eff1}}{L_{eff2}} \cdot (N_2^{\rightarrow\leftarrow} + N_2^{\leftarrow\leftarrow})}{(N_1^{\rightarrow\rightarrow} + N_1^{\leftarrow\leftarrow}) + \frac{\Lambda_{z1}}{\Lambda_{z2}} \cdot \frac{L_{eff1}}{L_{eff2}} (N_2^{\rightarrow\leftarrow} + N_2^{\leftarrow\leftarrow})}$$

- ▶ L_{eff1}/L_{eff2} is extracted from the data based on the assumption of the equality of $\left(\frac{d\sigma}{d\Omega}\right)_0$ in all data taking periods

Preliminary results - I^{\odot} on proton

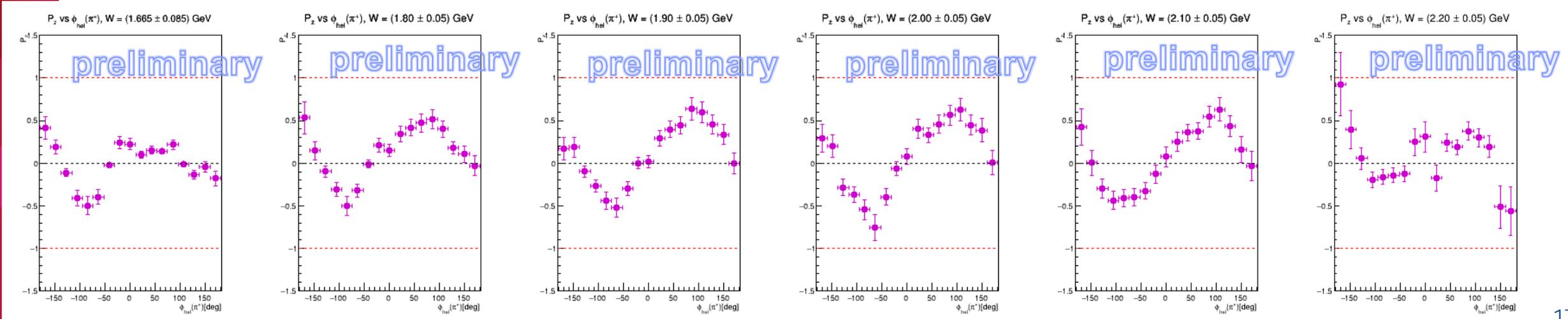
- ▶ According to general symmetry principles I^{\odot} is expected to be an *odd* function of the helicity angle
 - ▶ It depends only on the ratio of target polarization amounts
- ▶ The trend is in reasonable agreement with the earlier observations by CLAS based on a different data-set (E^* with unpolarized target)
- ▶ Counts are acceptance corrected

Blue points from S. Strauch et al., CLAS Coll., PRL **95** (2005), 162003



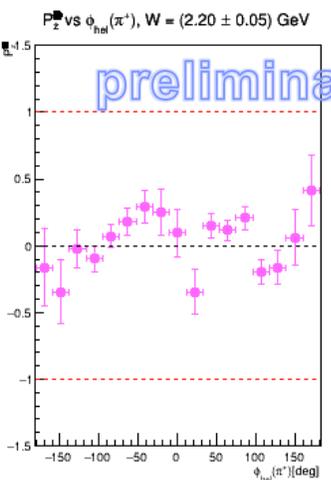
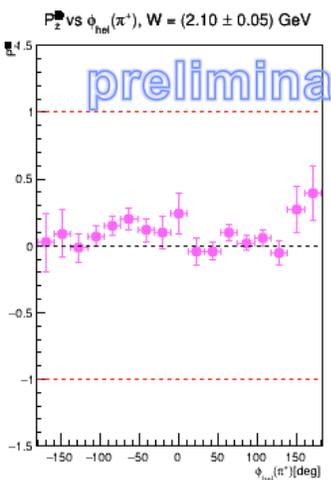
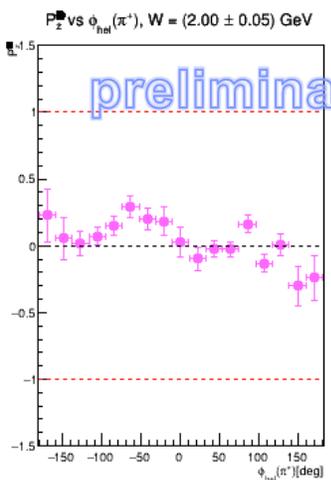
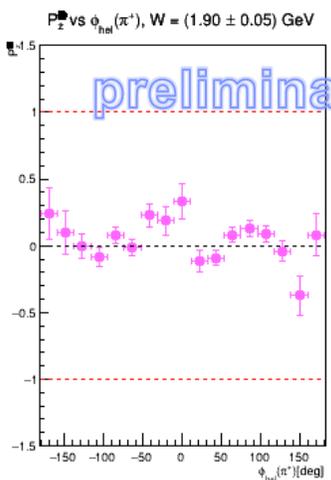
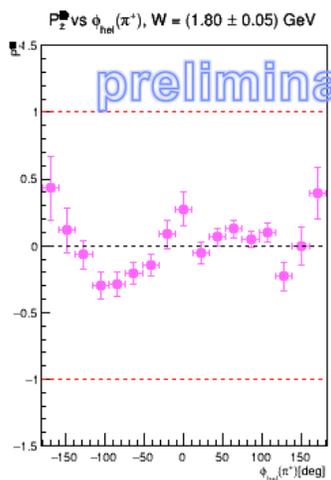
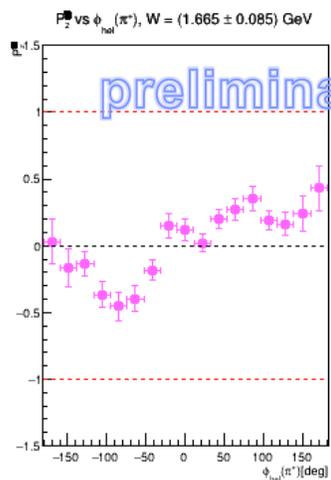
Preliminary results – P_z on proton

- ▶ No other results available for comparisons: first results ever
- ▶ P_z expected to be odd based on partial amplitudes symmetry
 - ▶ Vanishing at zero angle: coplanarity condition
 - ▶ When the helicity angle is oriented in the bottom hemisphere a sign flip occurs in Roberts' equations and, consequently, in the parity of the solutions
- ▶ Improvingly symmetric odd trend with W increase
 - ▶ The lack of left/right symmetry in some bins could be due to instrumental biases (different acceptance for forward/backward hemispheres, unprecise assessment of target polarization, ...)



Preliminary results – P_z^\ominus on proton

- ▶ No other results available for comparisons: first results ever
- ▶ P_z^\ominus expected to be even based on partial amplitudes symmetry
- ▶ P_z^\ominus does not show a clear-cut symmetry
 - ▶ Statistical uncertainties mostly obtained from the error propagation of the system solutions – small extent overall of target polarization (23% max.)
 - ▶ Including systematic uncertainties (work in progress expected <20%) most probably the trend will become consistent with zero



Summary and outlook

- ▶ The study of polarization observables in double-pion photoproduction with polarized beam and target is a novel tool to extract information about the baryonic spectrum
 - ▶ γp channel
 - Extraction of results for all compatible data sets pairs underway, to deliver final averages (problem: data-sets are badly correlated! Only one set with negative target polarization)
 - Final evaluation of systematics in progress
 - ▶ Outlook: γn channel – in progress
 - Same data analysis chain used for γp to be applied to the $\pi^+\pi^-n(p)$ final state
 - Use the same W binning and overall analysis approach
- ▶ The interpretation of results in terms of partial amplitudes contributions calls for new models to reproduce the new observables suitably updating the interference patterns
 - ▶ So far, none of the available reaction models agrees satisfactorily with the extracted asymmetries

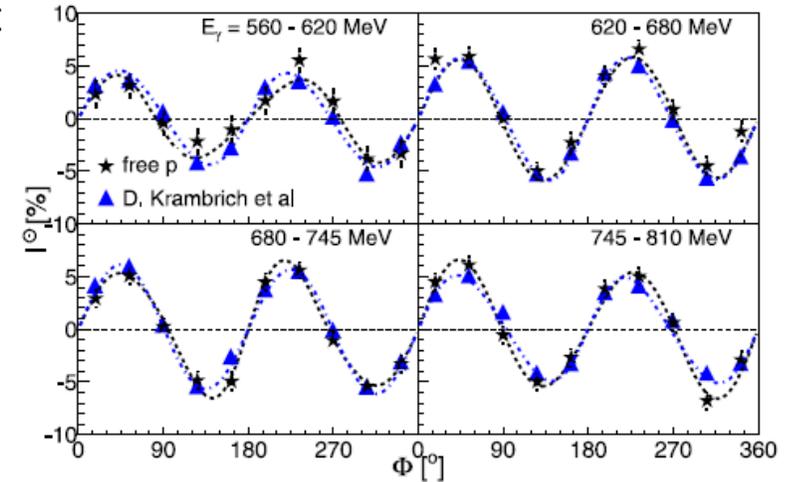


Backup slides

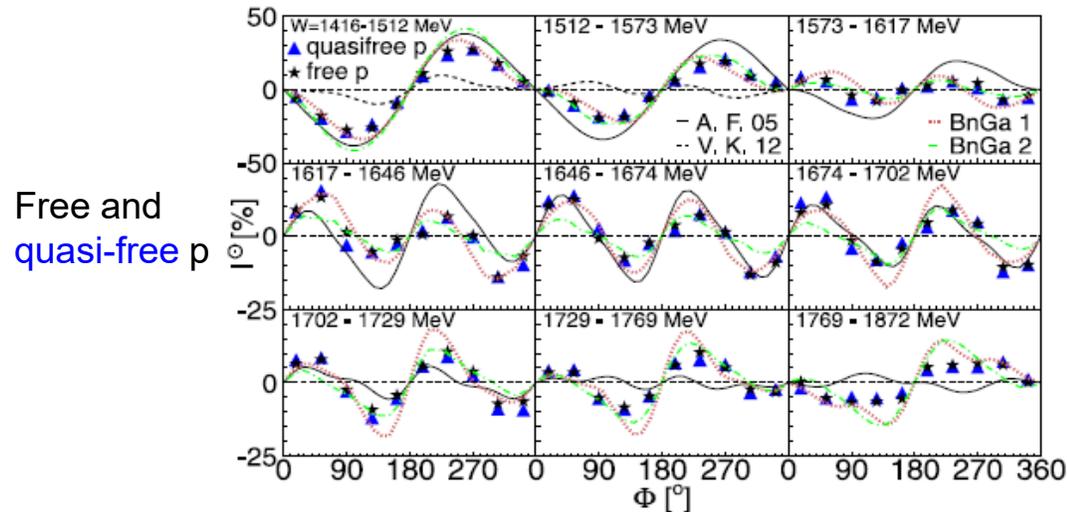
Photoproduction of $\pi^0\pi^0$ pairs from protons and neutrons

M. Oberle et al. (CB, TAPS & A2 @MAMI) PLB271 (2013), 237

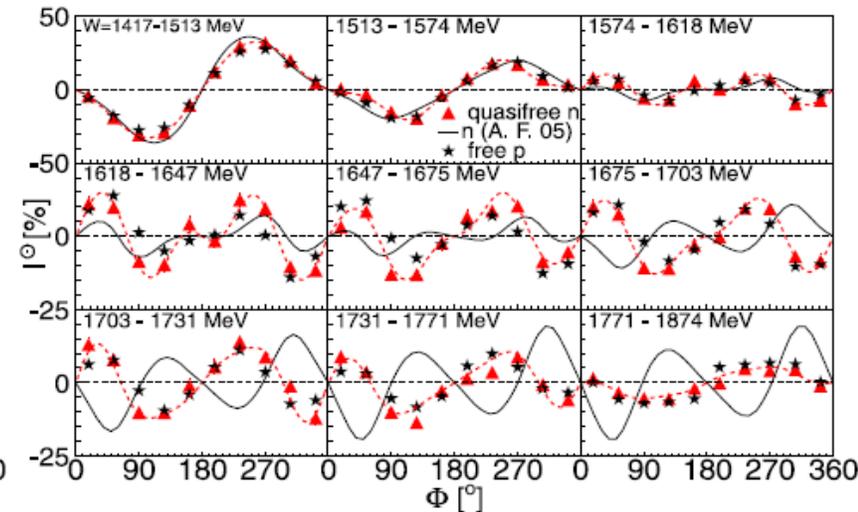
- ▶ Beam-helicity asymmetries in double- π^0 production on LH₂/LD₂ target (free p + quasi-free p & n) with circularly polarized photons up to 1.4 GeV @MAMI
- ▶ I^\odot evaluated through cross-section asymmetries
- ▶ Identical beam-helicity asymmetry measured for free and quasi-free protons; **very similar results from neutrons**
 - ▶ Expected up to the second resonance region ($W < 1.6$ GeV)
 - ▶ Surprising at larger energies due to difference resonances produced
- ▶ Reasonable reproduction of I^\odot trend by Bonn-Gatchina and two-pion MAID models (much worse for Valencia), at least up to the second resonance region



$$I^\odot(\varphi) = \sum_{n=1}^{\infty} A_n \sin(n\varphi)$$



Free and quasi-free p

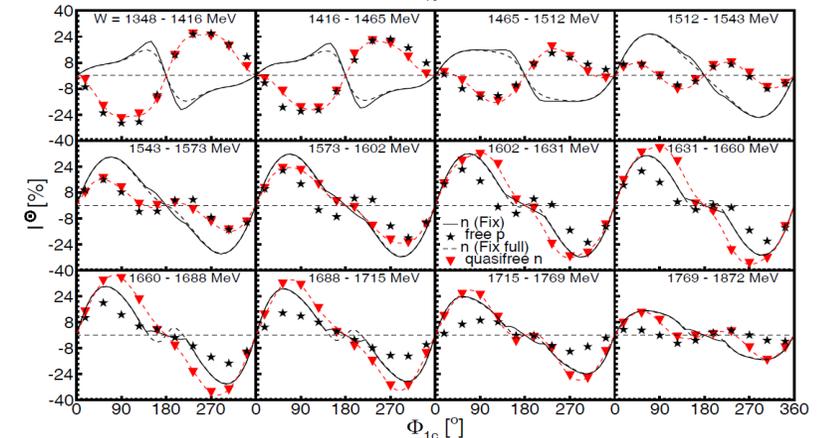
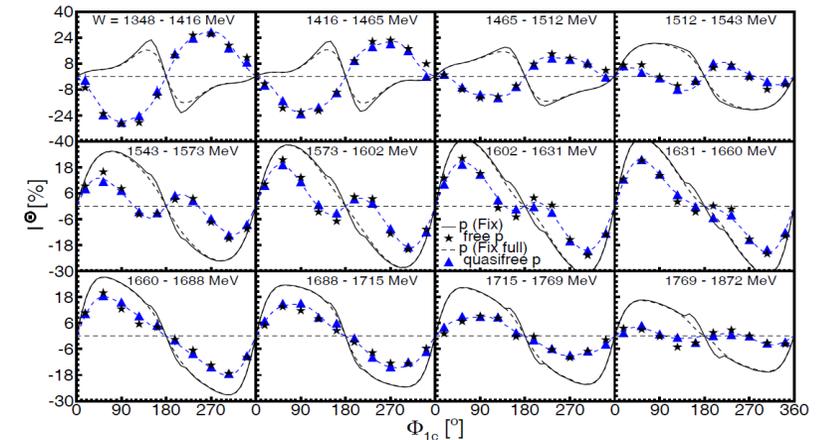
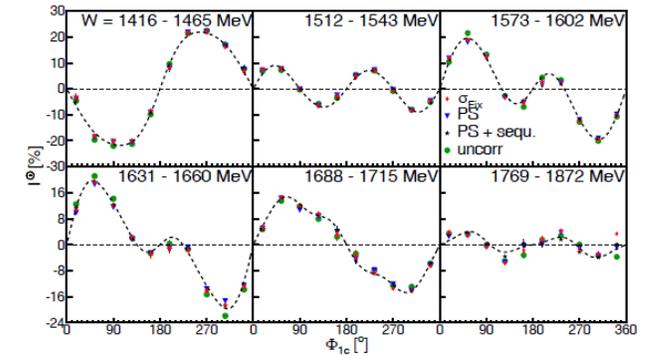


quasi-free n

Photoproduction of $\pi^0\pi^\pm$ pairs from protons and neutrons

M. Oberle et al. (CB, TAPS & A2 @MAMI) EPJ A (2014), 50

- ▶ Beam-helicity asymmetries in double mixed-charge π production on LH_2/LD_2 target (free p + quasi-free p & n) with circularly polarized photons up to 1.4 GeV @MAMI
 - ▶ Sensitive channels to ρ^\pm production effects
 - ▶ More background-populating channels compared to $2\pi^0$
- ▶ I° evaluated through cross-section asymmetries ordering particles by charge and by mass
- ▶ Good agreement between measurements on free and quasi-free proton, reasonable with quasi-free neutrons
- ▶ Worse agreement with models compared to $2\pi^0$, especially at higher energies:
 - ▶ more contributions from mixed charge channels, call to finer tuning of models
 - ▶ Two-pions MAID model behaves better, overall
 - ▶ Beam-helicity asymmetries are very sensitive to interference terms



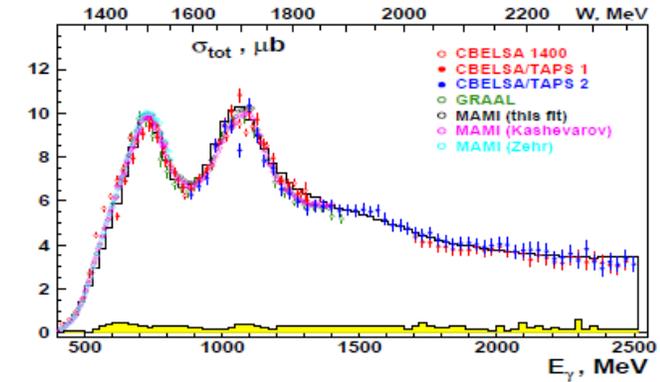
Photoproduction of $\pi^0\pi^0$ pairs off protons

V. Sokhoyan (CB@ELSA/TAPS) EPJ A51 (2015), 95

- ▶ The double- π^0 production is suitable to investigate the $\Delta(1232)\pi$ intermediate channel
 - ▶ Less channels contribute compared to the charged pion channel, especially to the non resonant background
 - Diffractive ρ production
 - Dissociation of the proton into $\Delta^{++}\pi^-$
 - π exchange is not possible

- ▶ Use of real linearly polarized photons (ELSA) from 600 MeV to 2500 MeV: access to the 4th resonance region

- ▶ Extraction of:
 - ▶ total cross section
 - ▶ PWA of the Dalitz plot
 - ▶ Beam-helicity asymmetries for double- π^0 production on the proton



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 \{1 + P_l[I^S(\phi^*) \sin(2\phi) + I^C(\phi^*) \cos(2\phi)]\}$$

