Lattice Insights into Baryon-Baryon Dynamics

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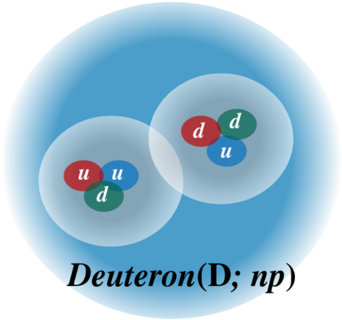
Heavy dibaryons

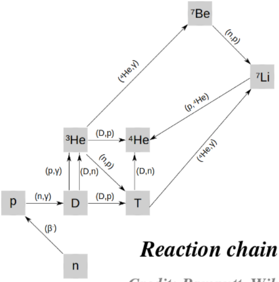
Funding agencies/computational resources




with **D. Chakraborty**, **N. S. Dhindsa**, **P. Junnarkar** and **N. Mathur**.

Deuteron: the longest known dibaryon



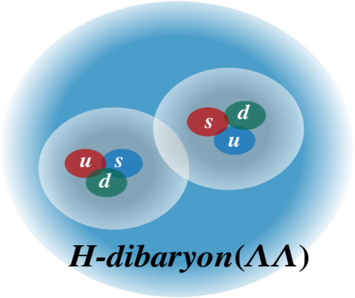


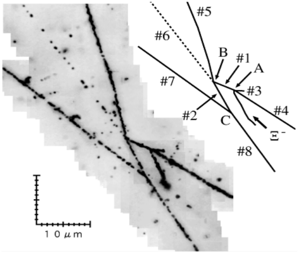
Reaction chain in BBN

Credit: Pamputt, Wikimedia Commons

- Nucleus of Deuterium discovered in 1932. Urey, Brickwedde & Murphy
- A very fine-tuned binding energy $\Delta E = M_D - M_p - M_n = 2.2$ MeV.
- Big bang Nucleosynthesis (BBN) has a deuteron bottleneck:
  Determines the abundances of light nuclei.
- How will the binding energy vary with quark masses?
  Could there be dineutrons or diprotons with heavier light quark masses.

The scalar dihyperon

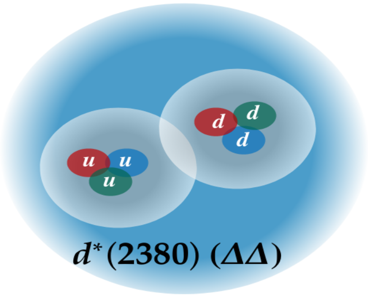


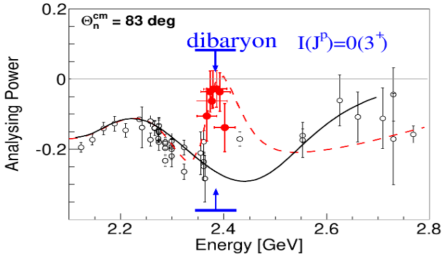


***NAGARA event*:**
Takahashi PRL 87, 212502 (2001)

- Bound *uuddss* flavor-singlet dihyperon with $J^P = 0^+$:
  Perhaps a stable Dihyperon — Jaffe PRL 38 195 (1977)
- NAGARA event: Strongest constraint on binding energy.
  $B_H < B_{\Lambda\Lambda}^{Nagara} = 6.91 \pm 0.16$ MeV — Takahashi *et al.*, PRL 87, 212502 (2001)
- ALICE @ LHC: constraints on $\Lambda\Lambda$ interactions from femtoscopic measurements
  ALICE 1905.07209 PLB

d^* resonance





***d^* dibaryon*:** Clement 1610.05591, WASA@COSY/SAID 1402.6844 PRL

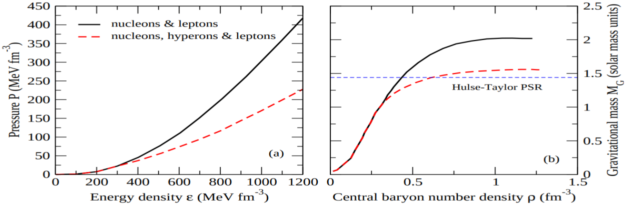
- Prediction for an isoscalar $\Delta\Delta$ configuration with $J^P = 3^+$.
  Assumed SU(6) symmetry. Dyson and Xuong PRL 13 815 (1964)
- Resonance feature at 2.38 GeV with $\Gamma \sim 70$ MeV and $I(J^P) = 0(3^+)$.
  Pole in the coupled ${}^3D_3 - {}^3G_3$ partial waves. Adlarson *et al.*, 1402.6844 PRL
- Whether isosymmetric partner of d^* with maximal isospin exists?
  Other possible nonstrange dibaryon candidates, if any.

Baryon-baryon interactions: Other prospects

- Hyperon formation ⇐ Large nuclear densities in astrophysical objects

Bazavov *et al*, 1404.6511 PRL, 1404.4043 PLB

Chatterjee and Vidaña 1510.06306 EPJA, Vidaña *et al* 1706.09701 PLB



- A handful of experimental efforts using large nuclei reactions.
  Inputs on LECs to EFTs ⇒ nuclear many body calculations.

Epelbaum 2005, INT-NFPNP 2022, $0\nu\beta\beta$ PSWR 2022

- Heavy dibaryons: Relatively free of the light quark chiral dynamics.

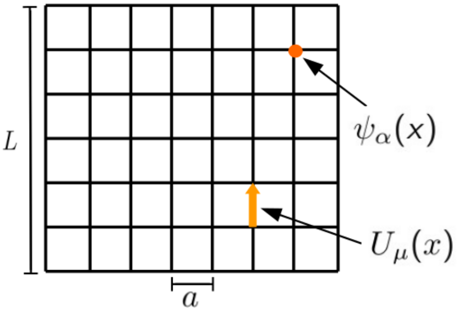
- Heavy dibaryons: no near three or four particle thresholds.
  Simple model studies ($\Omega\Omega$ scattering): widely different inferences.

Richard *et al* 2005.06894 PRL, Liu *et al* 2107.04957 CPL, Huang *et al* 2011.00513 EPJC

Lattice QCD: Basic idea

LQCD : A non-perturbative, gauge invariant regulator for the **QCD** path integrals.

- Quark fields $\psi_\alpha(x)$ on lattice sites
- Gauge fields as parallel transporters U_μ
  Lives in the links. $U_\mu(x) = e^{igaA_\mu(x)}$
- $\bar{\psi}^i_\alpha(x)[U_\mu(x)]_{ij}\psi^j_\alpha(x+a\hat{\mu})$ is gauge invariant.
- Lattice spacing : UV cut off
- Lattice size : IR cut off



Employ MCMC importance sampling methods on a Euclidean metric for numerical studies.

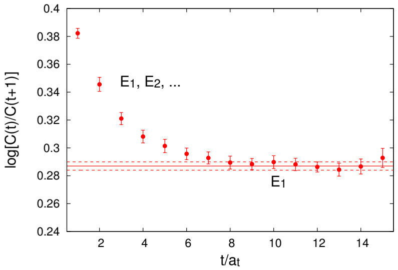
QCD spectrum from Lattice QCD

- Aim : to extract the physical states of QCD.

- Euclidean two point current-current correlation functions

$$C_{ji}(t_f - t_i) = \langle 0|\mathcal{O}_j(t_f)\bar{\mathcal{O}}_i(t_i)|0\rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2m_n} e^{-m_n(t_f - t_i)}$$

where $\mathcal{O}_j(t_f)$ and $\bar{\mathcal{O}}_i(t_i)$ are the desired interpolating operators and $Z_j^n = \langle 0|\mathcal{O}_j|n\rangle$.

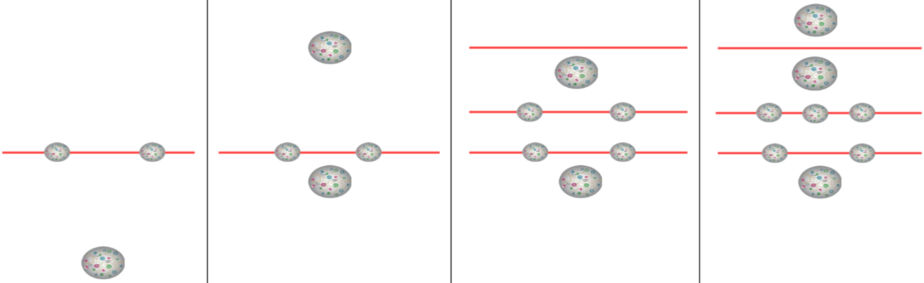


- Effective mass defined as $\log[\frac{C(t)}{C(t+1)}]$

- The ground state : from the exponential fall off at large times.
  Non-linear fitting techniques.

Complexity in Hadron spectroscopy using lattice

Straightforward | ***Relatively Easy*** | ***Difficult*** | ***Quite complex***



Straightforward:
Deeply bound;
Strong decay stable;

$\pi, K, D, p, n, \Lambda, \Xi_{cc}, \ldots$

Relatively Easy:
Shallow bound states;
Elastic resonances;
Only two body decays.

Deuteron,
$\Delta, \rho, D^*, D^*_{s0}, D_{s1}, \ldots$

Difficult:
Inelastic resonances;
Multiple two-body final states;
Most hadrons.

Additionally three-body decays.

Quite complex:
Inelastic resonances;
Multiple final state configs;
Most hadrons.

XYZTPs, glueballs, nuclei, ...

Exponential volume corrections
$[E_\infty - E_L \propto e^{-mL}]$

Power law volume corrections $\Delta E \propto \frac{a_0}{L^3} + O(\frac{1}{L^4})]$
Need a rigorous finite-volume amplitude analysis.

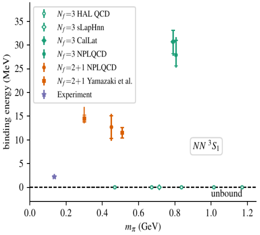
Talks by Leinweber Mon 1000, Mai Wed 1000

Baryon-baryon interactions from lattice QCD

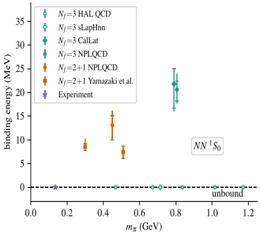
- A handful of lattice QCD efforts on baryon-baryon scattering typically at $m_\pi > m_\pi^{phys}$.

  see works by NPLQCD, HALQCD*, Mainz, CalLat, and others in the past decade.

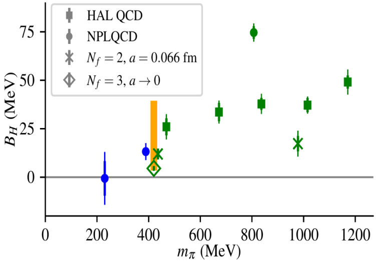
- Focus on light and strange six quark systems: Deuteron, dineutron, H-dibaryon, ...
  Discretization effects could be crucial.



deuteron

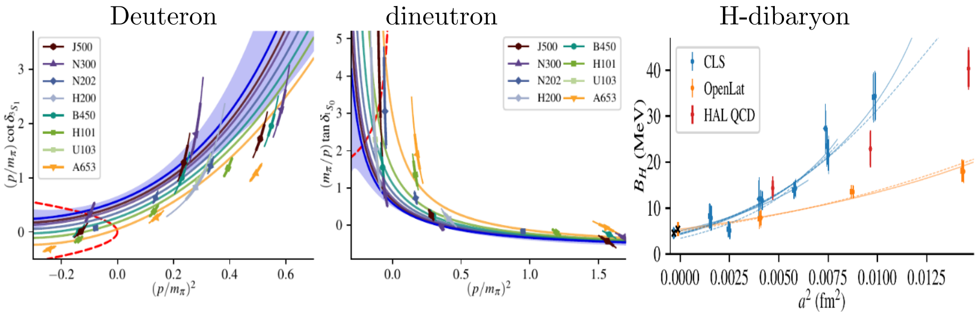


dineutron



Talk by Green @ Santa Fe Workshop 2023, Briceño *et al* Chapter 16 of 2202.01105 FBS

Baryon-baryon interactions from lattice QCD

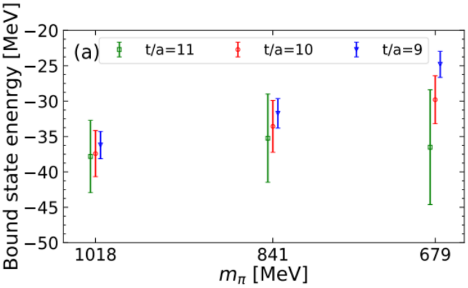


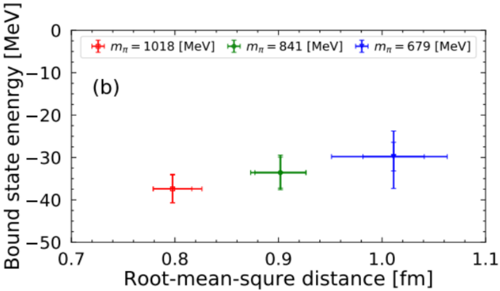
- Discretization effects could be crucial.

Talk by Green @ Santa Fe Workshop 2023

Green @ Liverpool Lattice 2024

- Results at SU(3) point:
  HALQCD @ $m_\pi \sim 840$ MeV and other points @ $m_\pi \sim 420$ MeV.
- Deutron and dineutron potentially a virtual bound pole at $m_\pi \sim 420$ MeV.
  H-dibaryon is a shallow bound state.
- @ the largest lattice spacing Deuteron is nearly a bound state.



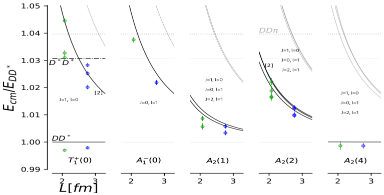


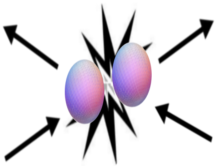
- $\Delta\Delta$ scattering on the lattice. Gongyo *et al*, 2008.00856 PLB
- Results at SU(3) point: HALQCD @ $m_\pi \sim 680$, 840 and 1018 MeV. Stable Δ baryons.
- Lattice spacing $a \sim 0.121$ fm and lattice size $L \sim 3.87$ fm. $d*$ as a quasi-bound state.
- Coarse lattice spacing used.

Scattering on the lattice: Different procedures

- *Bound states and binding energies directly from energy splittings.
- Lüscher's formalism: finite volume level shifts ⇔ infinite volume phase shifts

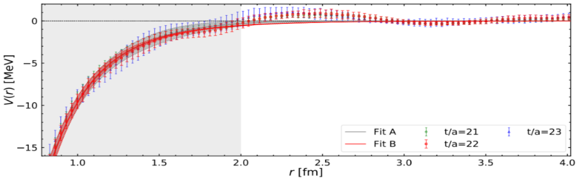
Lüscher 1991 NPB, Briceño 1401.3312 PRD

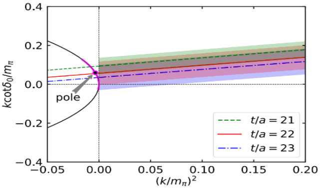




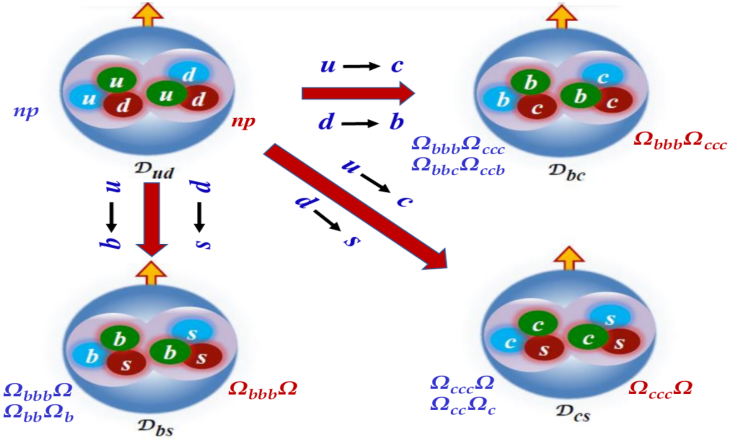
- †Solving QM potentials derived from NBS wave functions

HALQCD 1203.3642 PLB





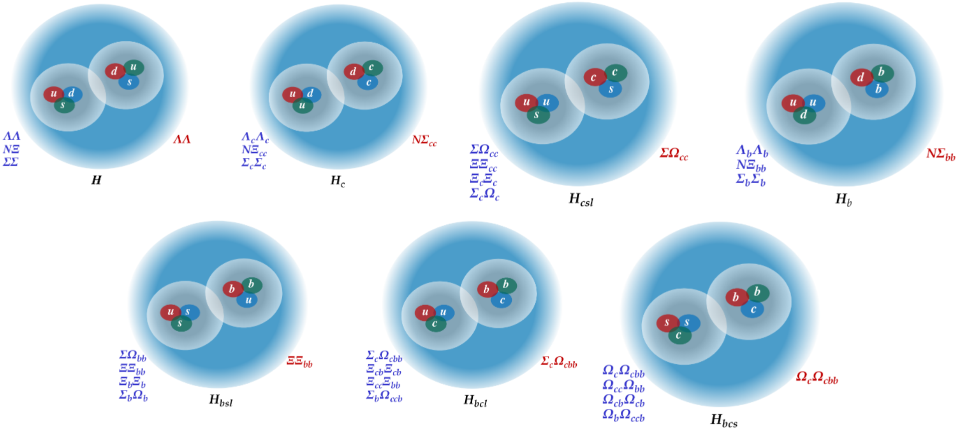
Deuteron-like Heavy dibaryons



Elastic thresholds in red text

Junnarkar and Mathur 1906.06054 PRL

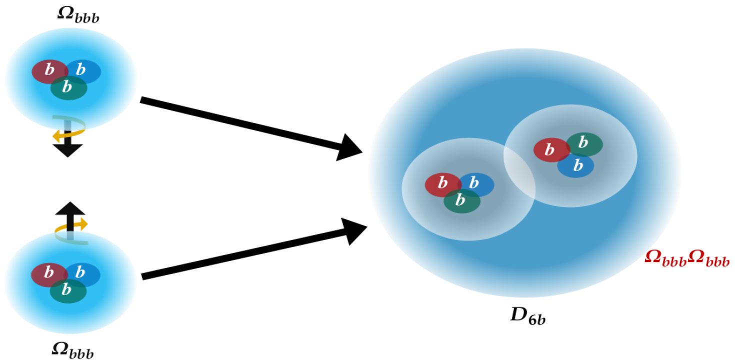
Triply flavored heavy dibaryons



Elastic thresholds in red text

Junnarkar and Mathur 2206.02942 PRD

Single flavored heavy dibaryons ($\mathcal{D}_{6q}$)



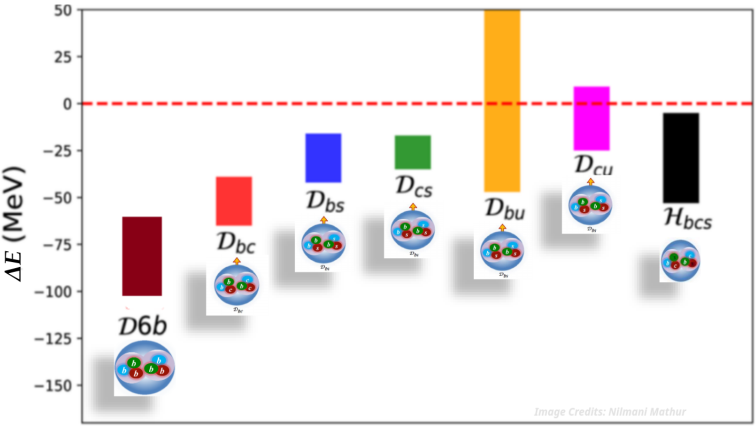
Heavy spin 0 single flavored partner of $d^*(2380)$?? Dyson and Xuong PRL 13 815 (1964)

Leading m_l dependence could arise from pair produced 2π exchanges.

Calculations at m_Q: Relatively cheap calculations with clean signals.

Mathur, MP and Chakraborty 2205.02862 PRL

Heavy dibaryons results summary

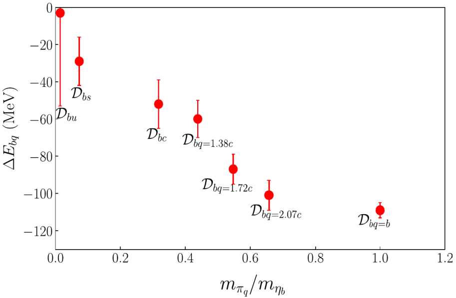


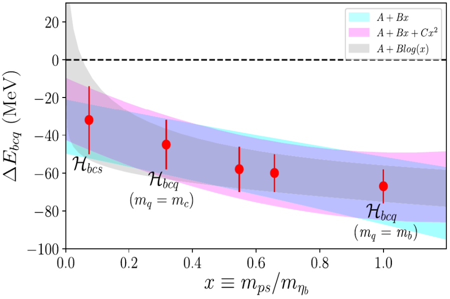
$$\Delta E = E - M_{H_1} - M_{H_2}$$

Junnarkar and Mathur 1906.06054 PRL ($\mathcal{D}_{bc}$, $\mathcal{D}_{bs}$, $\mathcal{D}_{cs}$, $\mathcal{D}_{bu}$, $\mathcal{D}_{cu}$), Mathur, MP, Chakraborty 2205.02862 PRL ($\mathcal{D}_{6b}$),

Junnarkar and Mathur 2206.02942 PRD ($\mathcal{H}_{bcs}$)

Light quark mass dependence



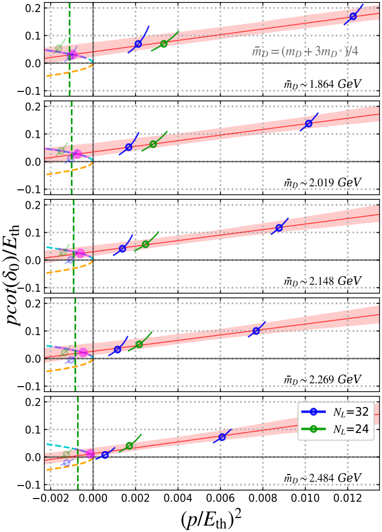


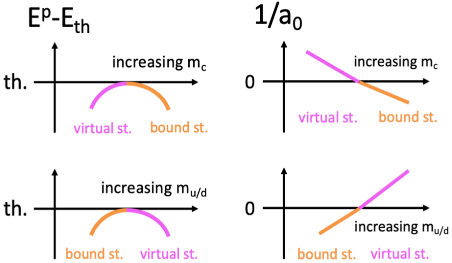
Junnarkar and Mathur 1906.06054 PRL, Junnarkar and Mathur 2206.02942 PRD

Heavier the quark masses, stronger the binding.
Different pattern of binding compared to T_{QQ}

MP, Prelovsek 2202.10110 PRL, Collins, MP, *et al.*, 2402.14715 PRD

m_q dependence of the T_{QQ} pole [ERE]





Collins, Nefediev, MP, Prelovsek 2402:14715 PRD

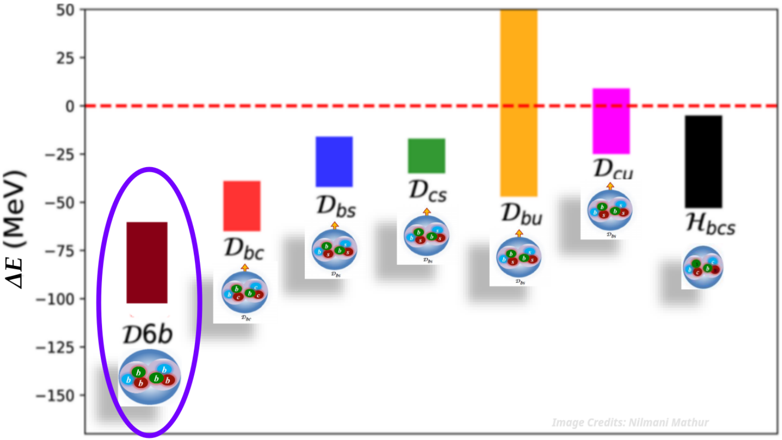
$m_\pi \sim 280$ MeV

- m_q dependence: Purely attractive
  Increasing m_c increasing attraction
  Decreasing m_q increasing attraction

Francis Thu 1100

- ERE: Questionable [OPE interactions and lhc]

Baryon-baryon interactions in heavy sector

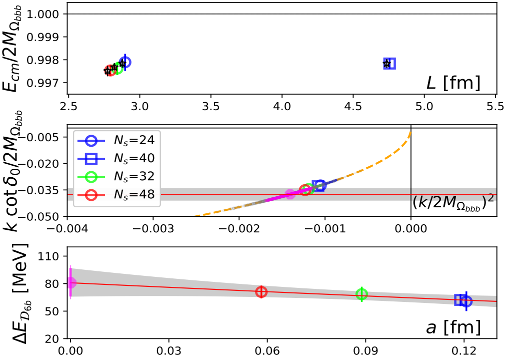


Mathur, MP, Chakraborty 2205.02862 PRL

Not limited to just a finite volume spectrum extraction.

Involved scattering analysis with a zero-range approximation.

Amplitude analysis and binding energy estimate



- Fits with "$-1/a_0^{[0]} - a/a_0^{[1]}$" is found to be the best with $\chi^2/d.o.f. = 0.7/2$

$$a_0^{[0]} = 0.18(^{+0.02}_{-0.02})\ \text{fm},$$

$$a_0^{[1]} = -0.18(^{+0.18}_{-0.11})\ \text{fm}^2$$

- Constraint $k.cot\delta(k) = -\sqrt{-k^2}$ gives us a bound state pole with $\Delta E^{cont}_{\mathcal{D}_{6b}} = -81(^{+14}_{-16})(14)$ MeV.

Using $M^{lphys}_{\Omega_{bbb}} = 14366(7)(9)$ MeV, we compute the mass of this bound state as

$$M^{phys}_{\mathcal{D}_{6b}} = 2M^{lphys}_{\Omega_{bbb}} + \Delta E^{cont}_{\mathcal{D}_{6b}} = 28651(^{+16}_{-17})(15)\ \text{MeV}$$

Other systematic uncertainties

- Lattice QCD configurations: 2+1+1 HISQ, improved to $\mathcal{O}(\alpha_s a^2)$
  b quarks: NRQCD Hamiltonian with pert. imp. coefficients up to $\mathcal{O}(\alpha_s v^4)$
  1S bottomonium hyperfine splitting with an uncertainty < 6 MeV.

- Energy splittings are used as inputs to FV analysis.
  Significantly reduced correlated uncertainties.

- Multiple fitting procedures to identify the correct plateau.
  Statistical and fit-window uncertainties added in quadrature.

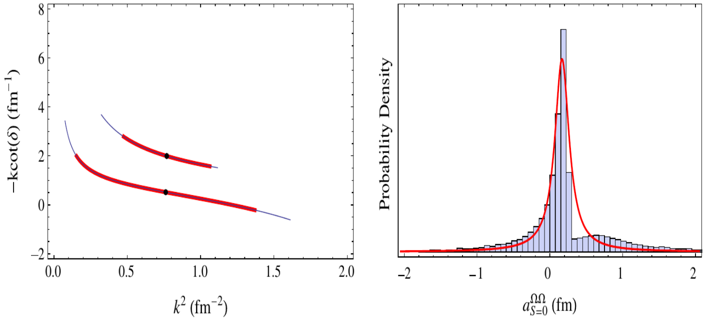
- Convolved through Lüscher's analysis + continuum extrapolation: $(^{+14}_{-16})$ MeV.

- Possible excited state effects using different smearing programs: < 8 MeV.

- Continuum extrapolation fit forms, scale setting, quark mass tuning and EM corrections: < 12 MeV.

$$\Delta E^{cont}_{\mathcal{D}_{6b}} = -81(^{+14}_{-16})(14) \text{ MeV}$$

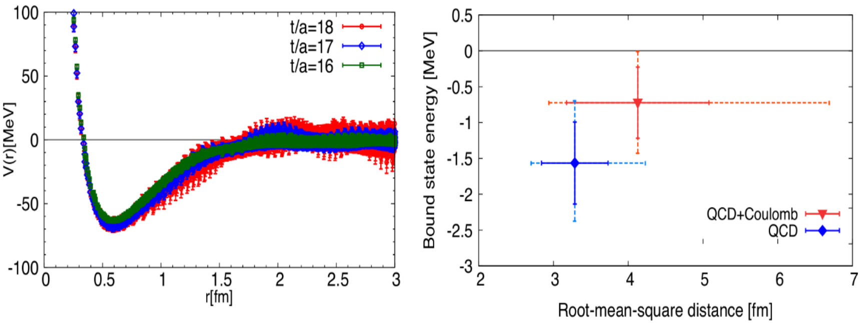
Other existing calculations [$\mathcal{D}_{6s}$]



- Early calculations of S-wave $\Omega\Omega$ scattering using Lüscher's formalism. $m_\pi \sim 390$MeV, $L \sim 2.5$ and 3.9 fm.
- Weakly repulsive interaction observed in the total spin 0. No deeply bound state possible.
- Clear positive energy shifts in the total spin 2 case.

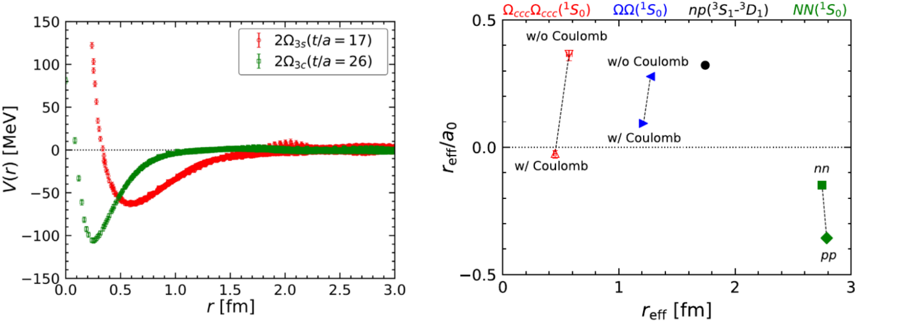
Buchoff, Luu, Wasem 1201.3596 PRD

Other existing calculations [$\mathcal{D}_{6s}$] *contd* ...



- S-wave $\Omega\Omega$ scattering using HALQCD procedure.
  $m_\pi \sim 146$MeV, $L \sim 8.1$ fm
- Weakly attractive interaction observed in the total spin 0. HALQCD 1709.00654 PRL
- Similar interactions at $m_\pi \sim 700$MeV from a $L \sim 2.9$ fm study

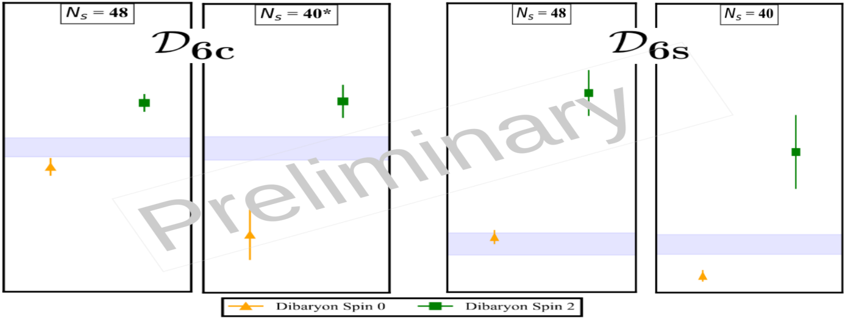
HALQCD 1503.03189 PRD



- S-wave $\Omega\Omega$ scattering using HALQCD procedure.
  $m_\pi \sim 146$MeV, $L \sim 8.1$ fm
- System close to the point where scattering length diverges.

HALQCD 2102.00181 PRL

Follow-up with our setup [$\mathcal{D}_{6q}$]



- S-wave $\Omega\Omega$ scattering in the charm sector.
  Work also in the strange sector in progress.
- Spin 0 ground states below or consistent with the threshold.
  Spin 2 ground states suggest positive shifts and repulsive interactions.
  No bound states possible.

Dhindsa, Mathur, MP, work under progress

Summary

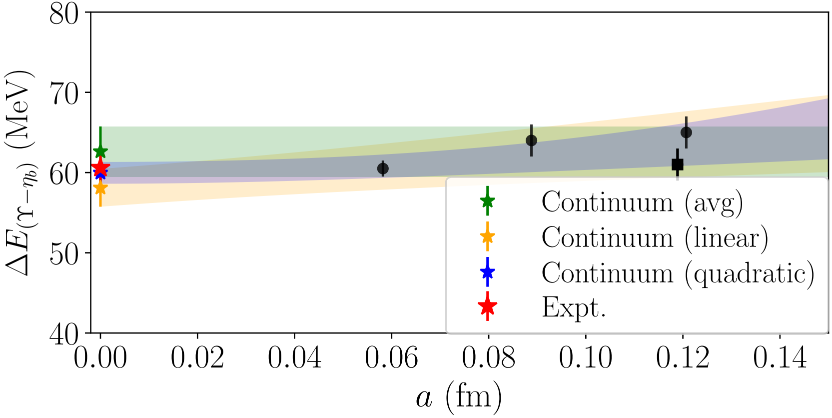
- Baryon-baryon interactions in the light and strange sector:
  Results for Deuteron, dineutron, H-dibaryon.

- Baryon-baryon interactions in the charm and heavy sector:
  Results for $\mathcal{D}_{6Q}$, $\mathcal{D}_{Qq}$, $\mathcal{H}_{Q_1Q_2q}$

- More upcoming lattice studies of dibaryon systems:
  Light, strange as well as in the heavy sector.

Thank you

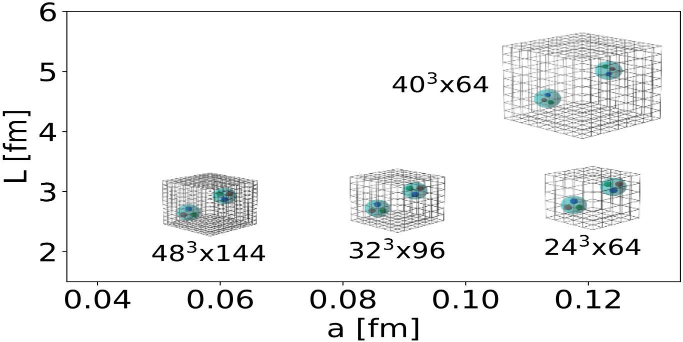
$\overline{1S}$ $\bar{b}b$ hyperfine splitting



Lattice QCD ensembles

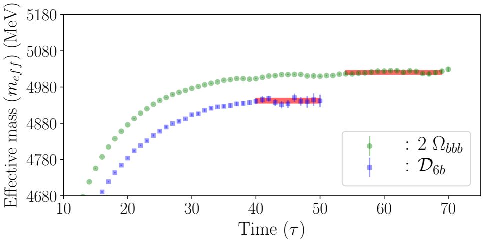
- Ensembles : N_f =2+1+1 HISQ (MILC): $24^3.64$, $32^3.96$, $48^3.144$, & $40^3.64$
  a =0.1207(11), 0.0888(8), 0.0582(5), and 0.1189(7) fm.

MILC Collaboration, arXiv:1212.4768



- Three lattice spacings and two volumes (∼ 2.85fm and 4.76 fm).
  Possible continuum extrapolation and finite-volume scattering analysis.

Lattice results for energy splitting



Ensemble	$\Delta E_{\mathcal{D}_{6b}}$ [MeV]
$24^3 \times 64$	$-61(11)$
$32^3 \times 96$	$-68(9)$
$40^3 \times 64$	$-62(7)$
$48^3 \times 144$	$-71(7)$

- Table presents the energy splitting $\Delta E_{\mathcal{D}_{6b}} = M^L_{\mathcal{D}_{6b}} - 2M_{\Omega_{bbb}}$
- Clear energy gap between the noninteracting and interacting cases.
- Similar energy splittings across all four ensembles.
- Negative energy splittings indicating attractive interaction.

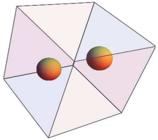
Finite volume spectrum and infinite volume physics

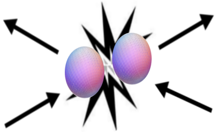
- On a finite volume Euclidean lattice : Discrete energy spectrum
  Cannot constrain infinite volume scattering amplitude away from threshold. Maiani-Testa 1990

- Non-interacting two-hadron levels are given by
  $$E(L) = \sqrt{m_1^2 + \vec{k}_1^2} + \sqrt{m_2^2 + \vec{k}_2^2} \text{ where } \vec{k}_{1,2} = \tfrac{2\pi}{L}(n_x, n_y, n_z).$$

- Switching on the interaction: $\vec{k}_{1,2} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$. *e.g.* in 1D $\quad \vec{k}_{1,2} = \frac{2\pi}{L} n + \frac{2}{L}\delta(k)$.

- Lüscher's formula relates finite volume level shifts ⇔ infinite volume phase shifts.
  Lüscher 1991





- Generalizations of Lüscher's formalism: *c.f.* Briceño 2014

Scattering amplitude parametrization

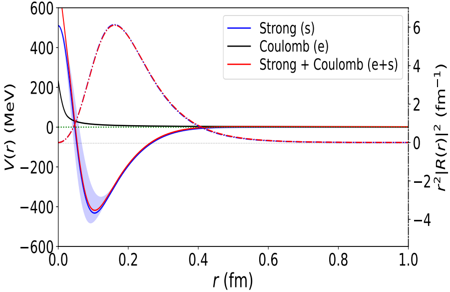
- Scattering amplitude: $S = 1 + i\frac{4k}{E_{cm}t}$
- For the $\Omega_{bbb}\Omega_{bbb}$ system [total spin equals 0], and assuming only S-wave,

$$t^{-1} = \frac{2\tilde{K}^{-1}}{E_{cm}} - i\frac{2k}{E_{cm}}, \quad \text{with} \quad \tilde{K}^{-1} = k.cot\delta(k)$$

  Bound state constraint: $k.cot\delta(k) = -\sqrt{-k^2}$

- Lüscher's prescription: $k.cot\delta(k) = \mathcal{F}(k)$, where $\mathcal{F}(k^2)$ is a known mathematical function.
  k^2 is determined from each extracted energy splitting as
  $k^2 = \frac{\Delta E^L_{D_{6b}}}{4}(\Delta E^L_{D_{6b}} + 4M^{phys}_{\Omega_{bbb}})$
- We parametrize $k.cot\delta(k)$ with a constant (inverse scattering length "$-1/a_0$").
  The remnant lattice spacing "a" dependence: "$-1/a_0^{[0]} - a/a_0^{[1]}$".
  Fits performed with and without "a" dependence.

Possible Coulombic repulsion



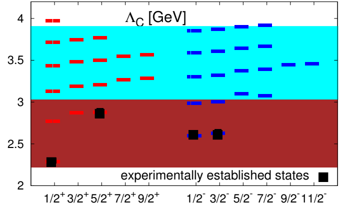
- A heavy object such as $\mathcal{D}_{6b}^{--} \Rightarrow$ compact. Coulombic repulsion could be significant.
- V_s: multi-Gaussian attractive potential such that $\Delta E^{cont}_{\mathcal{D}_{6b}} = -81(^{+14}_{-16})(14)$ MeV.

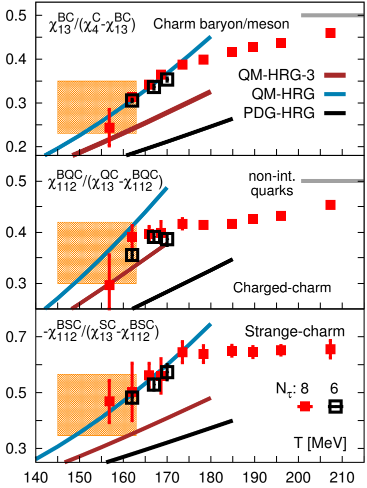
  HALQCD 2021
- Assuming an electric charge distribution as determined in *Can et al 2015*, we determine V_{em} for $\mathcal{D}_{6b}^{--}$.
- Compare the radial probability of the ground state wavefunction for V_s and $V_s + V_{em}$.
- Coulombic potential hardly influences where the probabilities peak. Maximum associated change in $\Delta E^{cont}_{\mathcal{D}_{6b}}$ found to be 10 MeV.

Motivation from finite temperature studies

Ebert *et al.*, PRD, **84**, 014025, 2011

Bazavov *et al.*, PLB, **737**, 210, 2014





✿ **Charm hadron pressure (HRG)** :

$$P(\hat{\mu}_C,\hat{\mu}_B) = P_M cosh(\hat{\mu}_C) + P_{B,C=1} cosh(\hat{\mu}_C+\hat{\mu}_B)$$

$$\chi_{kl}^{BC} = \frac{\partial^{(k+l)}[P(\hat{\mu}_C,\hat{\mu}_B)/T^4]}{\partial\hat{\mu}_B^k\partial\hat{\mu}_C^l}$$

⇒ **Existence of additional charm-light baryons in QGP formed in HIC.**