Dispersion theory for the transition form factors of the nucleon

Application to Delta(1232) and N*(1520)

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Vetenskapsrådet
Swedish Research Council

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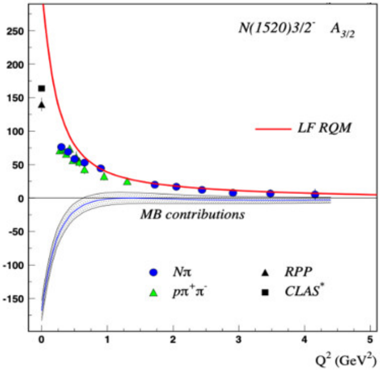

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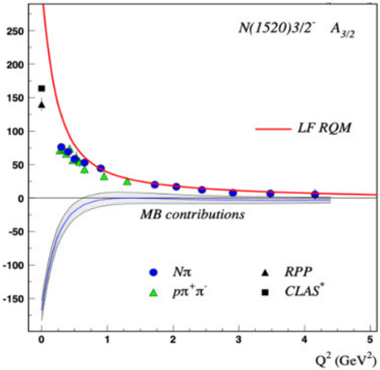

- can one understand the part called “MB contributions” in a model-independent way?
- got funding from Swedish Research Council

Understanding the strong interaction

model-independent methods to explore QCD (and in general QFT):

- perturbative QCD
  - works at high energies where strong interaction is weak
- lattice QCD
  - works best around $\Lambda_{\rm QCD}$, m_s (hadronic scale $\approx 1\,$GeV)
  - light pion sees itself around the torus if volume is too small
  - heavy quark falls through grid
    if grid distance larger than Compton wave length
  - but advantage: quark masses can be varied
- (chiral) effective field theory $\rightsquigarrow$ works at very low energies
- dispersion theory $\rightsquigarrow$ works at low energies (only a few channels)

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- experiment! $\rightsquigarrow$ but quark masses fixed

Unitarity and analyticity

- constraints from local quantum field theory:
  partial-wave amplitudes for reactions/decays must be
  - unitary:

$$S\,S^\dagger = 1\,, \quad S = 1 + iT \quad \Rightarrow \quad 2\,\mathrm{Im}\,T = T\,T^\dagger$$

↪ note that this is a matrix equation:
$\mathrm{Im}\,T_{A\to B} = \sum_X T_{A\to X}\, T^\dagger_{X\to B}$

  - analytic (dispersion relations):

$$T(s) = \frac{1}{\pi}\int\limits_{-\infty}^{\infty} ds'\,\frac{\mathrm{Im}\,T(s')}{s'-s-i\epsilon}$$

⇝ can be used to calculate whole amplitude from imaginary part

- practical limitation: too many states X at high energies

↪ in practice dispersion theory is a low-energy method ($\lesssim 1\,\mathrm{GeV}$)
or use resonance saturation

Form factors (FFs)

- scatter a lepton (electron or neutrino) on a hadron
- and look at final state with a single(!) hadron

$$\ell_1 + h_1 \to \ell_2 + h_2$$

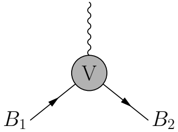
- at high energies one sees the minimal number of quarks that is needed to build $h_{1,2}$ ("quark counting rules")
- at low energies one sees much more

↪ the playground of relativistic many-body physics

- this is opposite to deep inelastic scattering where one sees more particles the higher the energy

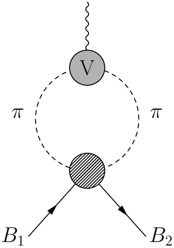
Electromagnetic form factors at low energies

- how to obtain a form factor?



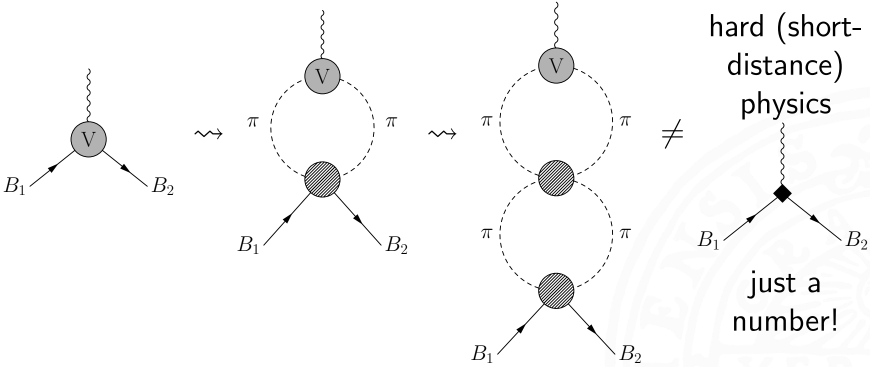
- need to resolve at least the finite size $\lesssim 1\,\mathrm{fm}$
- but inverse size of a hadron is larger than pion mass
- first one probes something universal (independent of $B_{1,2}$):

the "pion cloud":



- now we are in the game with dispersion theory

Deconstruct a form factor

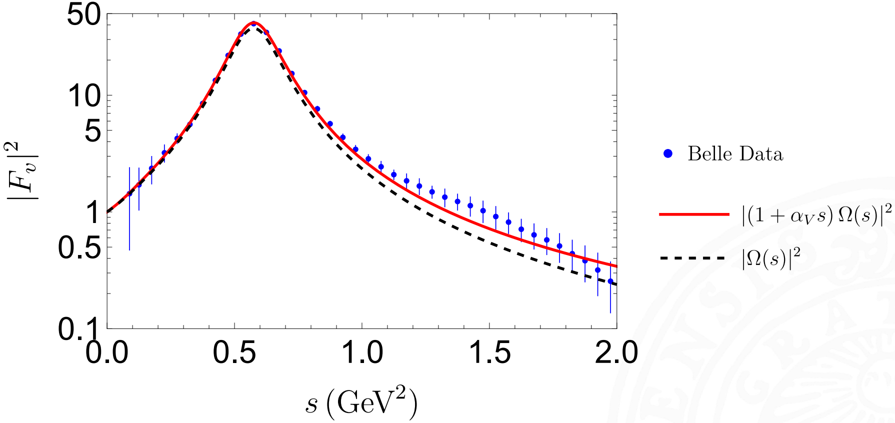


pions interact with each other

what is not contained is hard (short-distance) physics

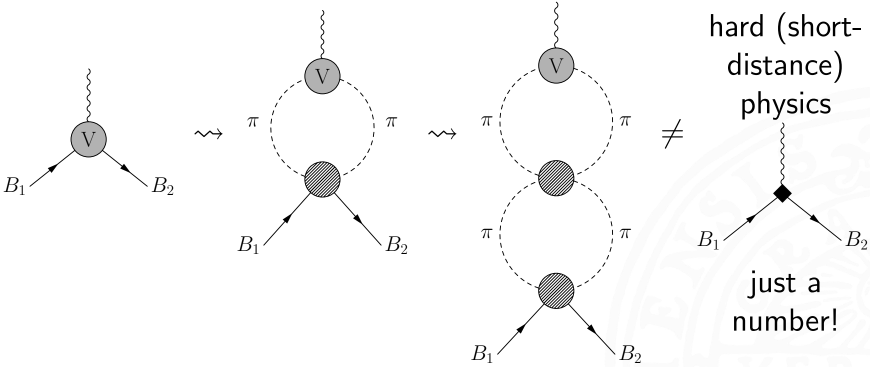
just a number!

Pion vector form factor and data



Alvarado/An/Alvarez-Ruso/SL, Phys.Rev.D 108 (2023) 11, 114021

Deconstruct a form factor



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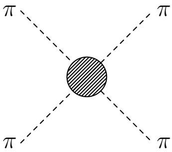
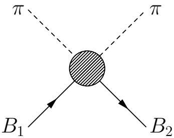
Scattering processes (e.g. for baryons)

from data plus dispersion theory:

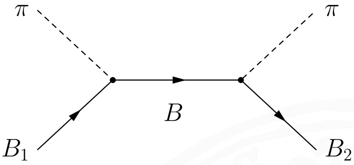


part that is not pion rescattering:

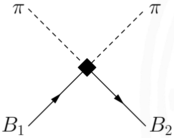


$\rightsquigarrow$

low-energy approximation:

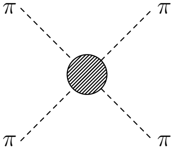


what is not covered is hard physics (contact terms)

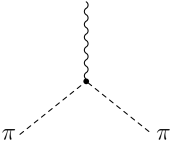


Known input

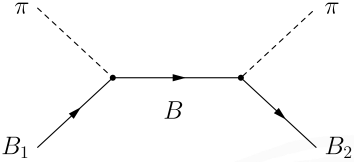
pion scattering



pion charge



low-energy approximation

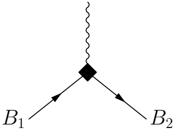


- baryon-pion coupling constants from decay widths

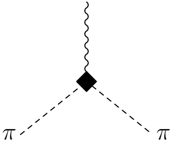
↪ sometimes only moduli known

Unknown: some numbers

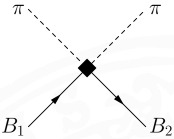
part without two intermediate pions



intermediate state with more than two pions



intermediate state that is not one baryon

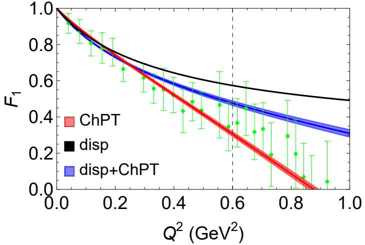


$\hookrightarrow$ fit to data (now) or calculate with quark-gluon based methods (future)

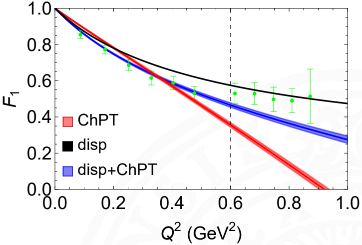
“future”: D. An, G. Eichmann, C. Fischer, SL, work in progress

Results I: nucleon

quark-mass and momentum dependence of nucleon Dirac form factor



$M_\pi = 0.130\,\text{GeV}$



$M_\pi = 0.223\,\text{GeV}$

Alvarado/An/Alvarez-Ruso/SL, Phys.Rev.D 108 (2023) 11, 114021

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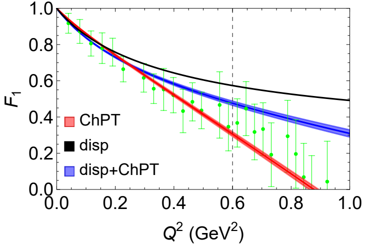

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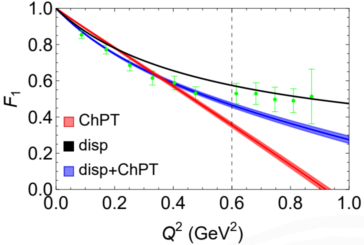

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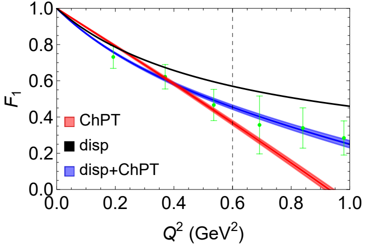
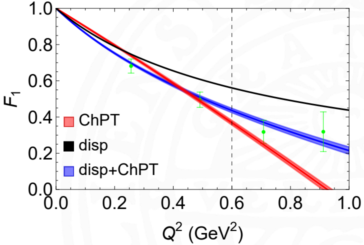


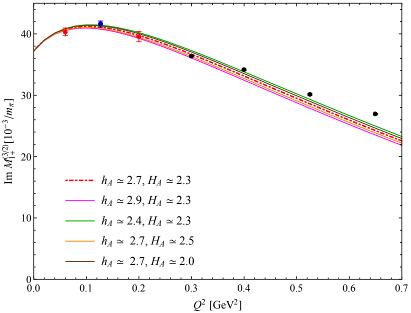
$M_\pi = 0.278\,\mathrm{GeV}$

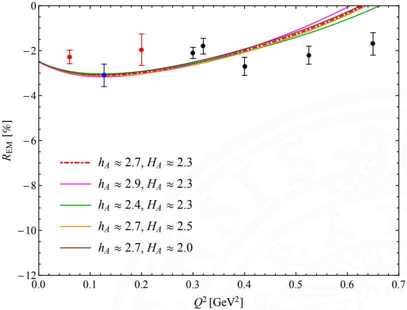


$M_\pi = 0.353\,\mathrm{GeV}$

Results II: $\Delta(1232)$

transition form factors from nucleon to Δ





M. M. Aung, S. Leupold, E. Perotti and Y. Yan, arXiv:2401.17756 [hep-ph]

Results III: $N^*(1520)$

- transition form factors from nucleon to $N^*(1520)$ (one example)

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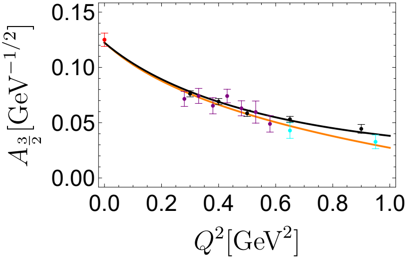
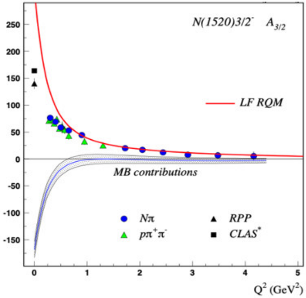
- yeah



An Di/SL, in preparation



Aznauryan/Burkert, Prog. Part. Nucl. Phys. 67, 1

What can we learn here?

- the long-distance part is universal

↪ needs to be understood once, not always new for each process

↪ a lot is fixed by (chiral) symmetries

- the short-distance part is process dependent
  and sensitive to the details (dynamics) of QCD

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- the short-distance part is process dependent
  and sensitive to the details (dynamics) of QCD
- outlook:
  - hadron based effective field theories + dispersion theory

  ↪ allow parametrization of short-distance physics
  - quark-based methods

  ↪ should allow determination of parameter values

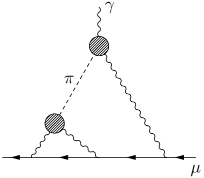
↪ combine methods

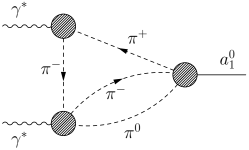
An Di, G. Eichmann, C. Fischer, SL, work in progress

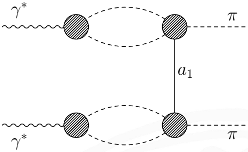
Present and future applications

- standard model prediction for magnetic moment of the muon

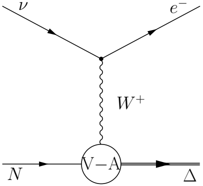
  Hoferichter/Hoid/Kubis/SL/Schneider, Phys.Rev.Lett. 121 (2018) 11, 112002

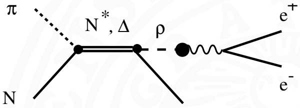






- weak form factors for neutrino-matter scattering





- hadronic input for electromagnetic radiation from hot/dense strongly interacting matter

figure bottom right: Friman, Pirner, Nucl.Phys.A 617 (1997) 496

Many thanks to my collaborators

- Di An; PhD Uppsala (UU) ongoing (supported by VR)
- Fernando Alvarado; PhD Valencia ongoing
- Moh Moh Aung; PhD SUT (Thailand), now postdoc
  (supported by International Science Programme ISP)

- Carlos Granados; postdoc UU, now researcher Vienna, Virginia
- Elisabetta Perotti; PhD UU, now postdoc Boulder, Colorado
- Olov Junker; physics master UU, now TSL, Uppsala
- Timea Vitos; physics master UU, now VR postdoc, Budapest
- Luis Alvarez-Ruso; professor Valencia
- Yupeng Yan; professor SUT

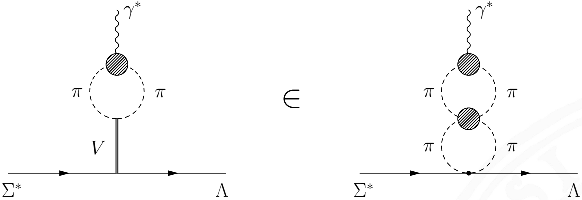
Publications

- C. Granados, S. Leupold and E. Perotti,
  Eur. Phys. J. A **53**, no.6, 117 (2017)
- S. Leupold, Eur. Phys. J. A **54**, no.1, 1 (2018)
- O. Junker, S. Leupold, E. Perotti and T. Vitos,
  Phys. Rev. C **101**, no.1, 015206 (2020)
- F. Alvarado, D. An, L. Alvarez-Ruso and S. Leupold,
  Phys. Rev. D **108**, no.11, 114021 (2023)
- M. M. Aung, S. Leupold, E. Perotti and Y. Yan,
  arXiv:2401.17756 [hep-ph].
- D. An, S. Leupold, in preparation

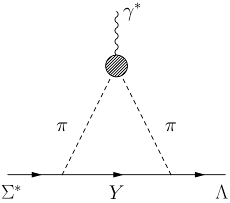
Spare slides

Form factors in $\Sigma^{*0}(1385) \to \Lambda\, e^+ e^-$ (not measured yet)

- our method: dispersion relation (unsubtracted in lack of data)
  O. Junker, SL, E. Perotti, T. Vitos, Phys. Rev. C 101 (2020) 1, 015206
- ρ meson is included via pion phase shift (model independent)

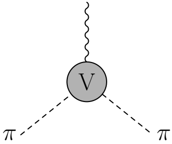


- “our” triangles with baryons are beyond vector-dominance model



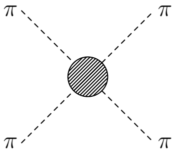
How to get the pion vector form factor?

apply same logic to pion vector FF

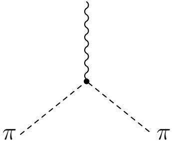


input:

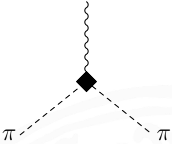
pion scattering



pion charge



hard part of pion vector FF



$$F_V(s) = (1 + \alpha_V\, s)\, \exp\left\{ s \int\limits_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\delta(s')}{s'\,(s' - s - i\epsilon)} \right\}$$

with pion phase shift δ
and $\alpha_V \approx 0.12\,\text{GeV}^{-2}$ (from fit to FF data)