3-PARTICLE INTERACTIONS ON THE LATTICE

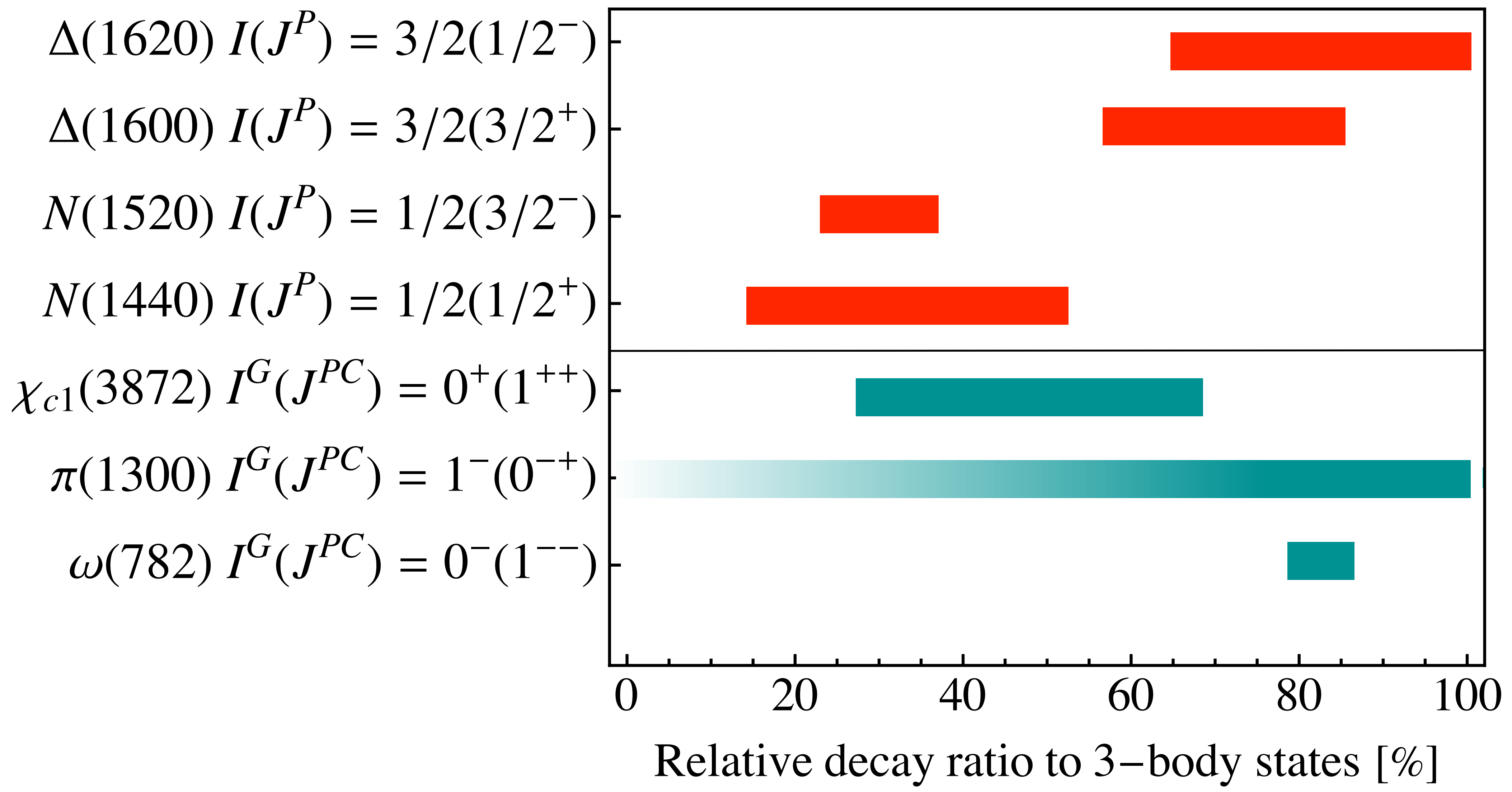
MAXIM MAI

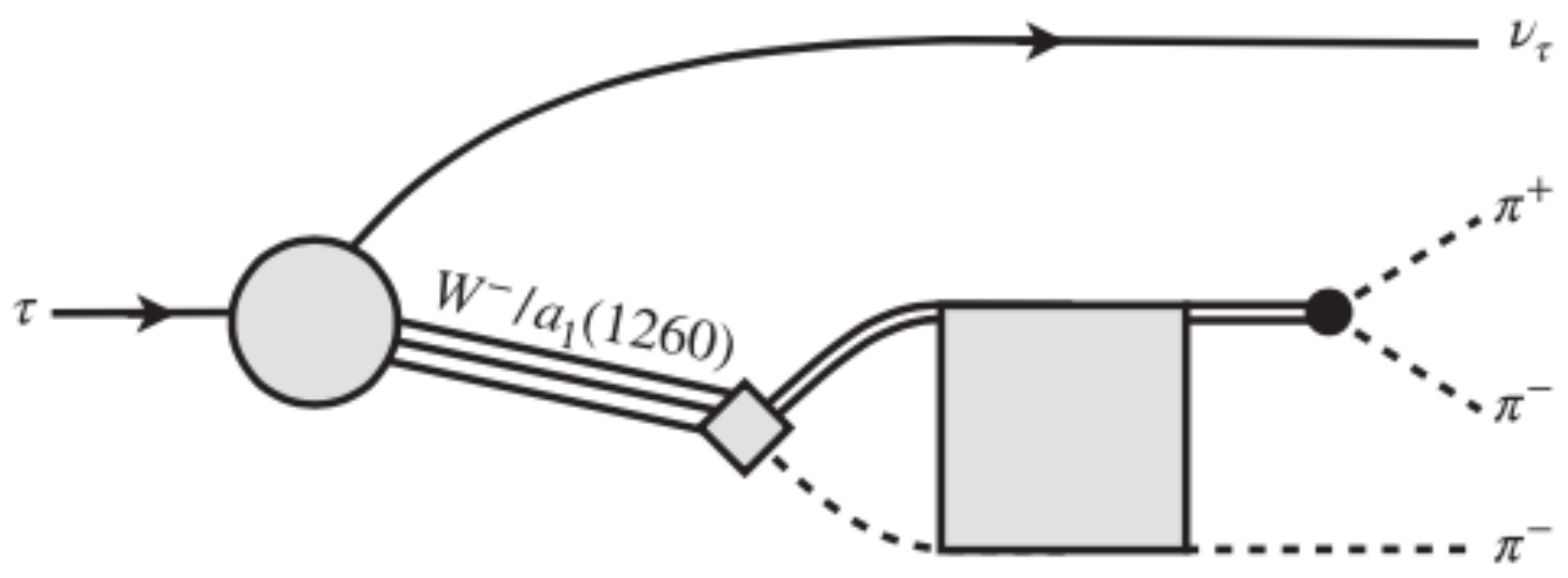
University of Bonn | The George Washington University

NSF: PHY-2012289 — DOE: DE-SC0016582/83 — DFG: CRC 110, MA 7156/3-1

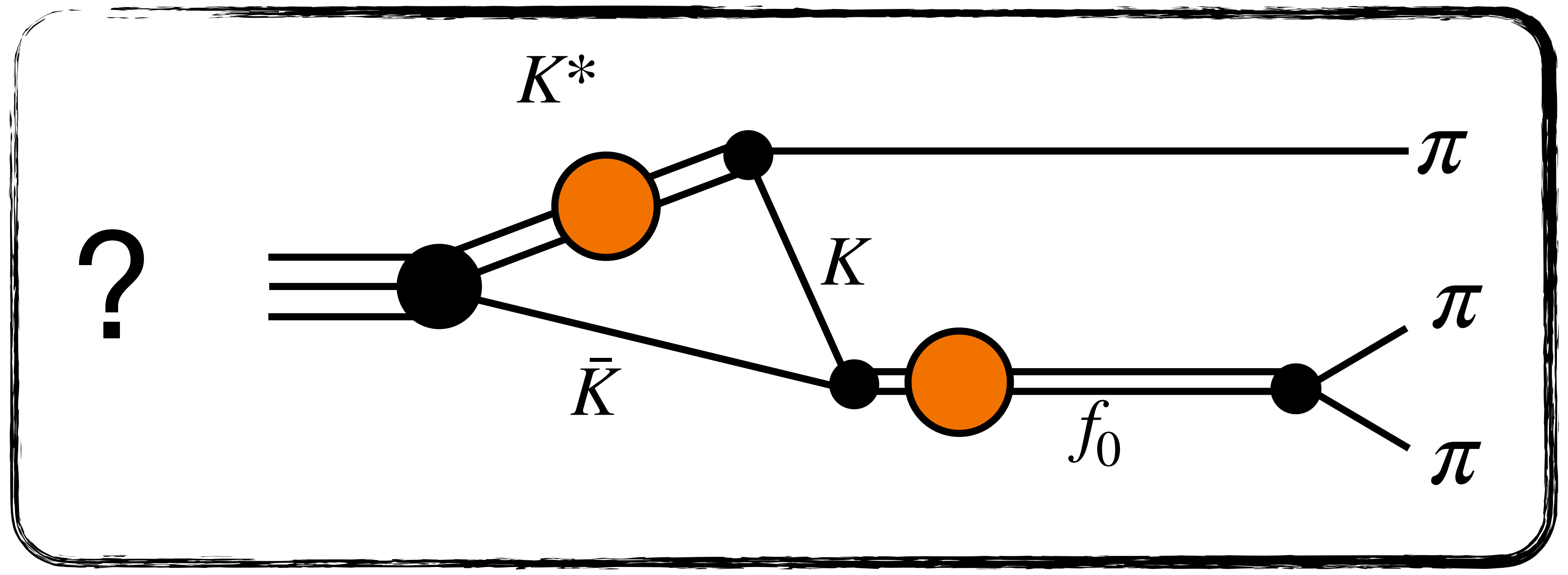
HADRON SPECTRUM

- Many known states have large 3-body content
  - $N(1440)$
  - $a_1(1260)$, $a_1(1420)$?
  - $X(3872)$
- Beyond Standard Model searches (τ-EDM/…)
- Exotic states of matter[1]



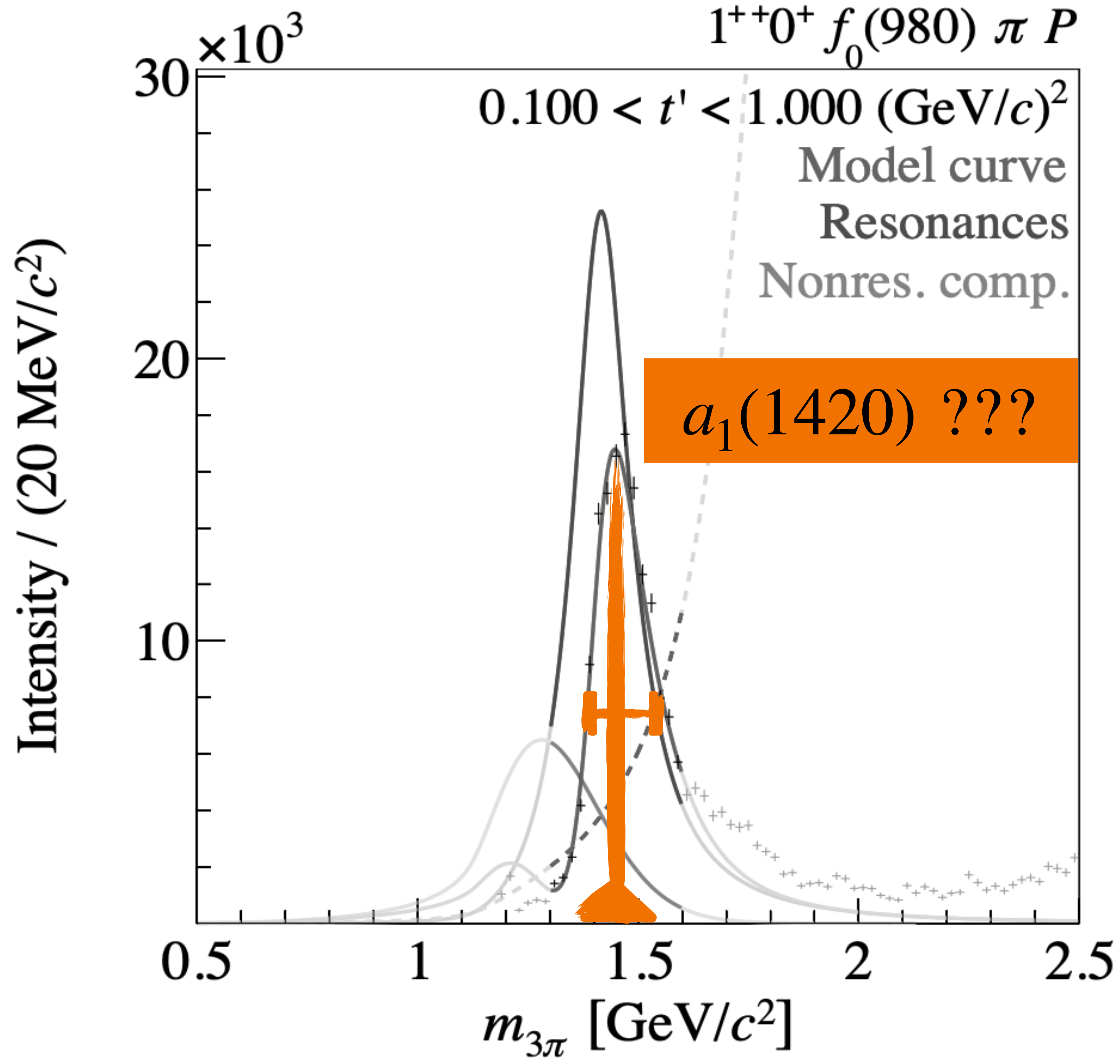


[1] Experimental programs: GlueX@JLAB; COMPASS@CERN;
[FIG] Data from Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

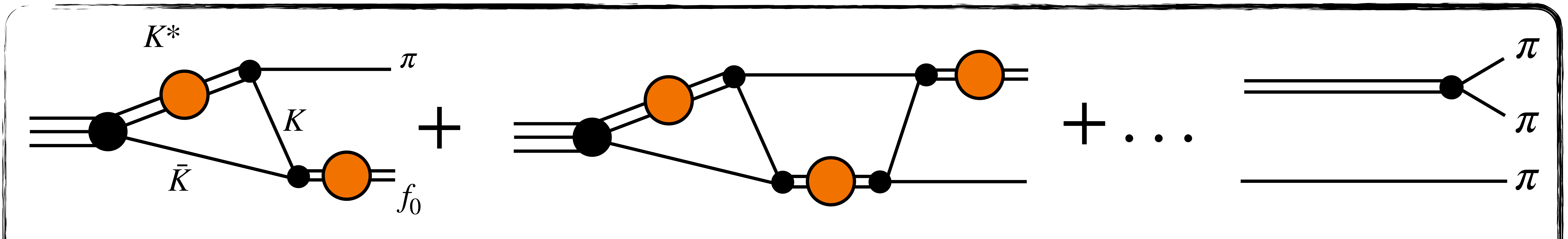


Experimental input

- many high-precision experiments[2] → line-shapes
  - resonances ⇜⇝ increased interaction rates
  - modulo reaction-type
  - modulo kinematic singularities[3]

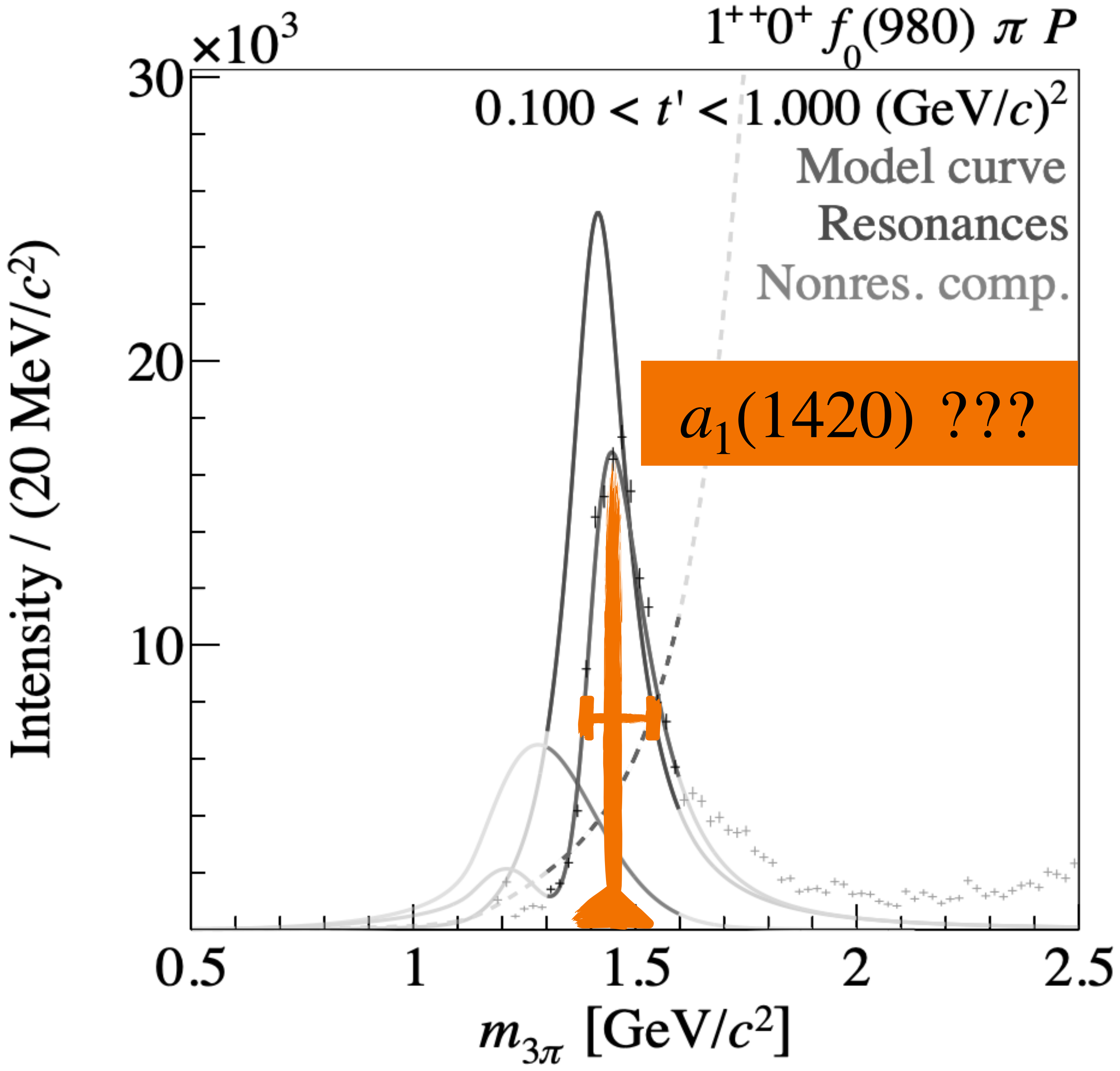


[2] CLAS12, GlueX, ...
[3,FIG] [COMPASS] Phys.Rev.Lett. 115 (2015) 8. Review: Ketzer/Grube/Ryabchikov Prog.Part.Nucl.Phys. 113 (2020) 103755



Experimental input

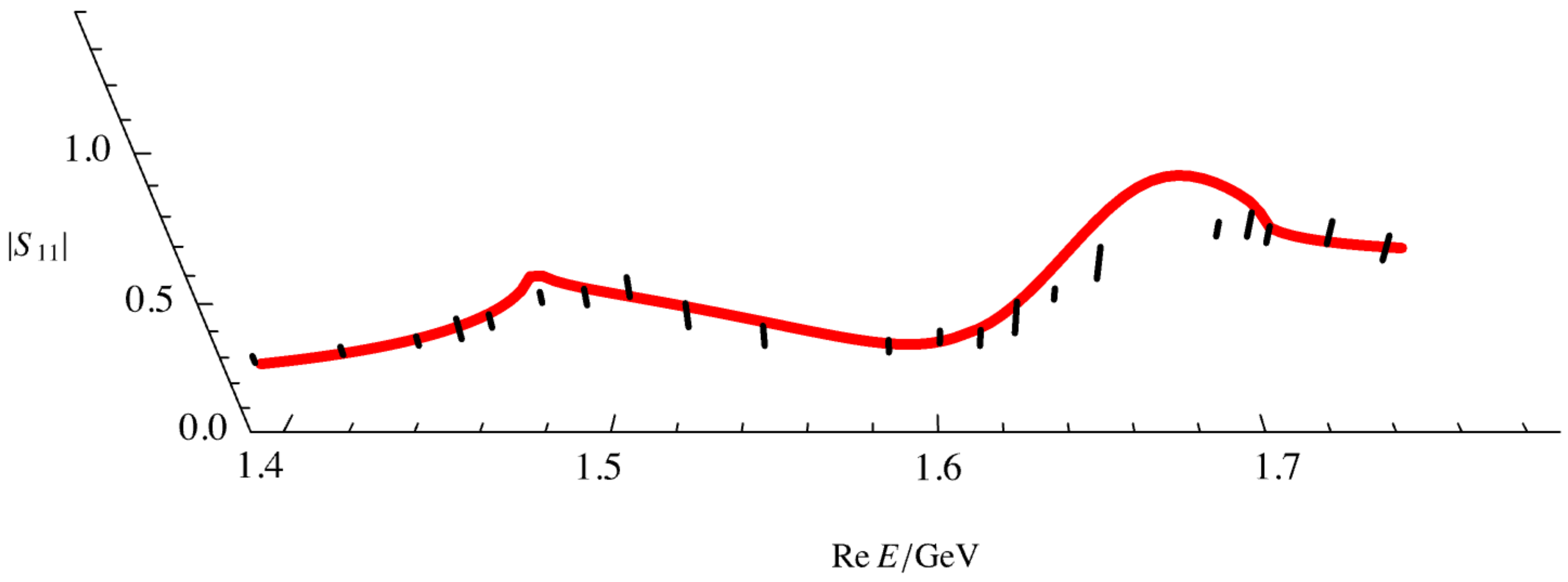
- many high-precision experiments[2] → line-shapes
  - resonances ⇜⇝ increased interaction rates
  - modulo reaction-type
  - modulo kinematic singularities[3]



[2] CLAS12, GlueX, ...
[3,FIG] [COMPASS] Phys.Rev.Lett. 115 (2015) 8. Review: Ketzer/Grube/Ryabchikov Prog.Part.Nucl.Phys. 113 (2020) 103755

Universal resonance parameters

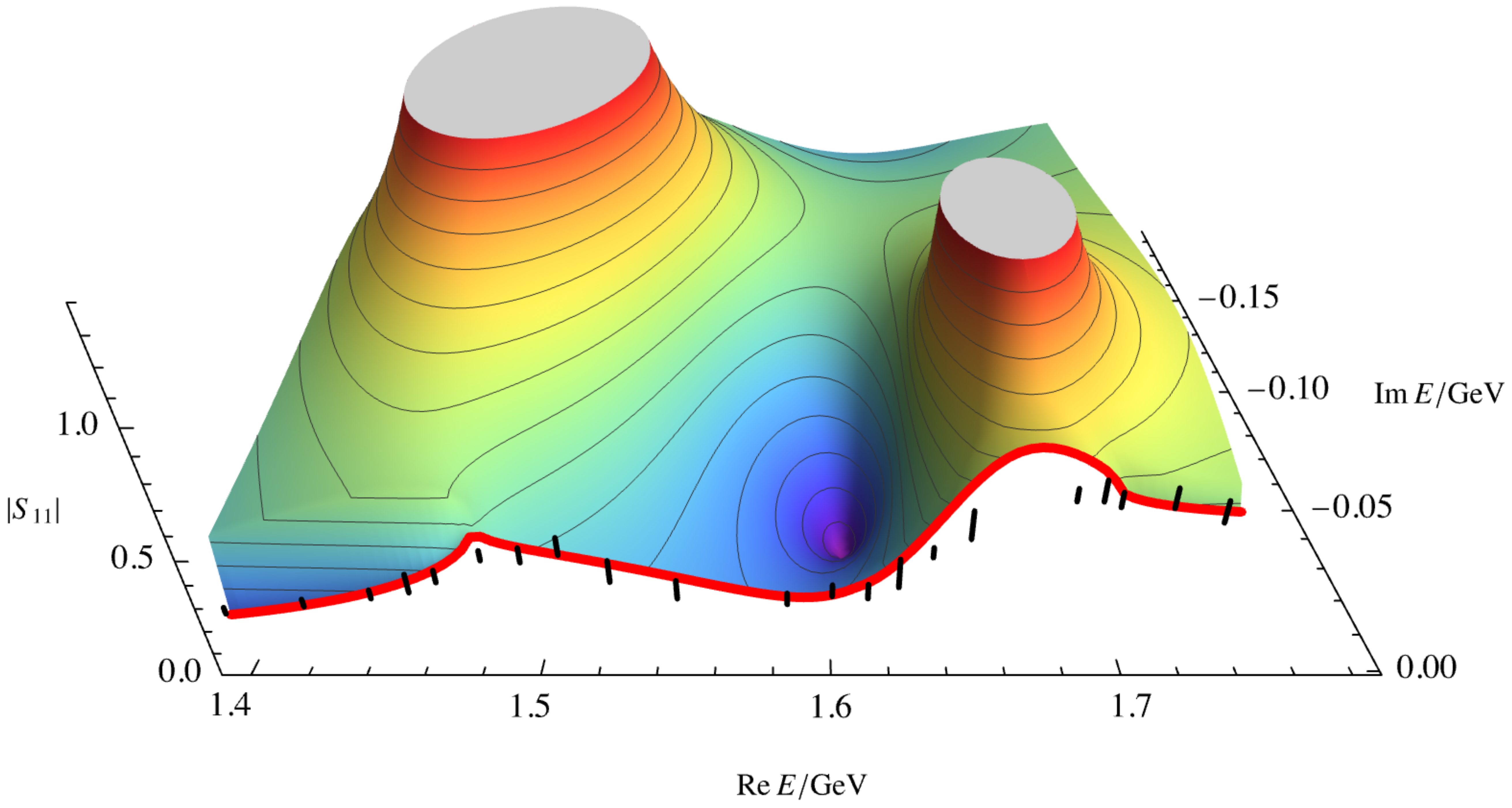
- S-matrix theory: ***transition amplitude*** $T(E \in \mathbb{C})$
  - Unitarity/Analyticity/Crossing symmetry
  - Poles on unphysical Riemann Sheets



[1] Fig: Data: SAID: Phys. Rev. C 74 (2006) 045205 Model: MM et al. Phys.Rev.D 86 (2012) 094033

Universal resonance parameters

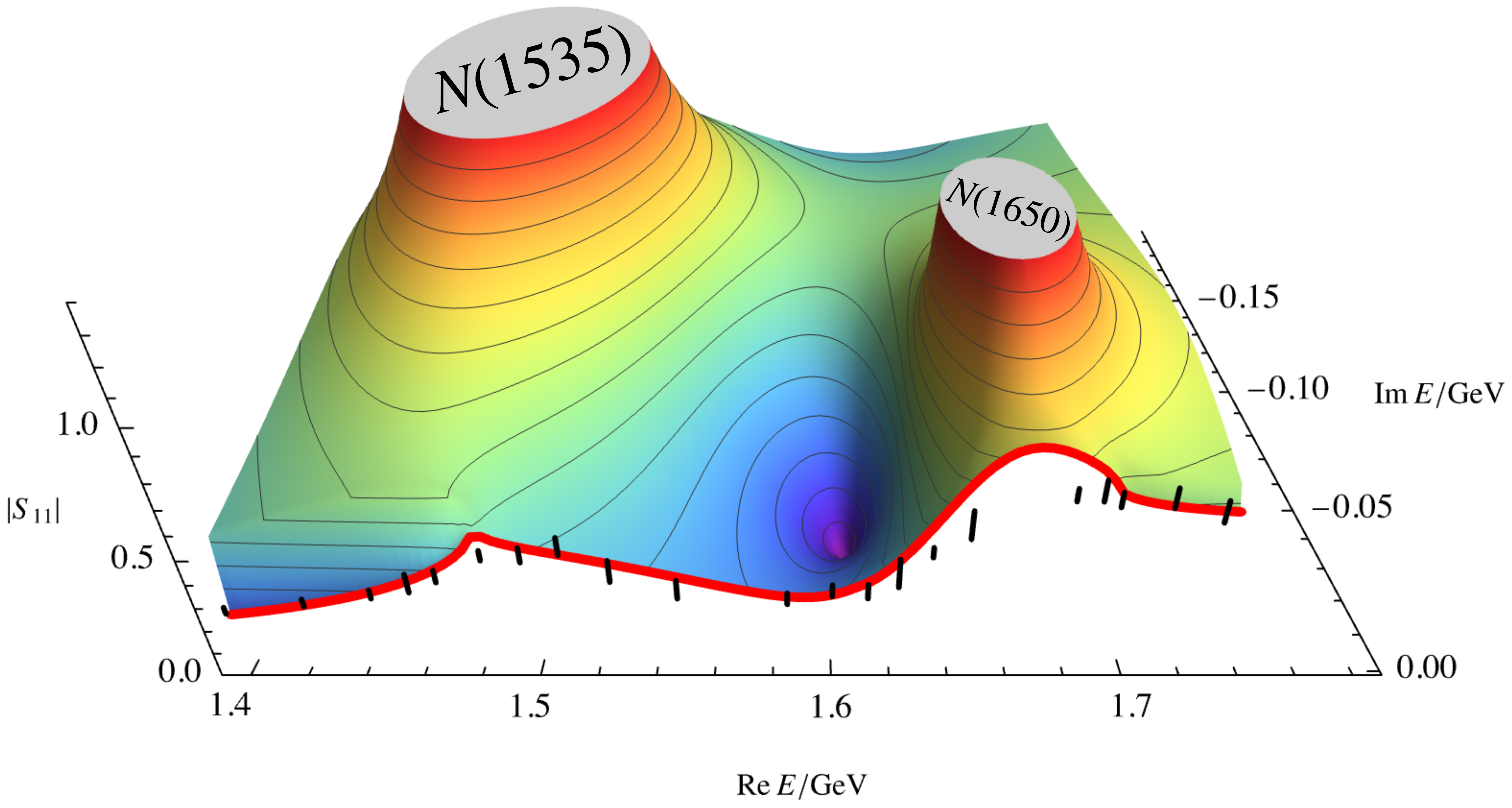
- S-matrix theory: ***transition amplitude*** $T(E \in \mathbb{C})$
  - Unitarity/Analyticity/Crossing symmetry
  - Poles on unphysical Riemann Sheets



[1] Fig: Data: SAID: Phys. Rev. C 74 (2006) 045205 Model: MM et al. Phys.Rev.D 86 (2012) 094033

Universal resonance parameters

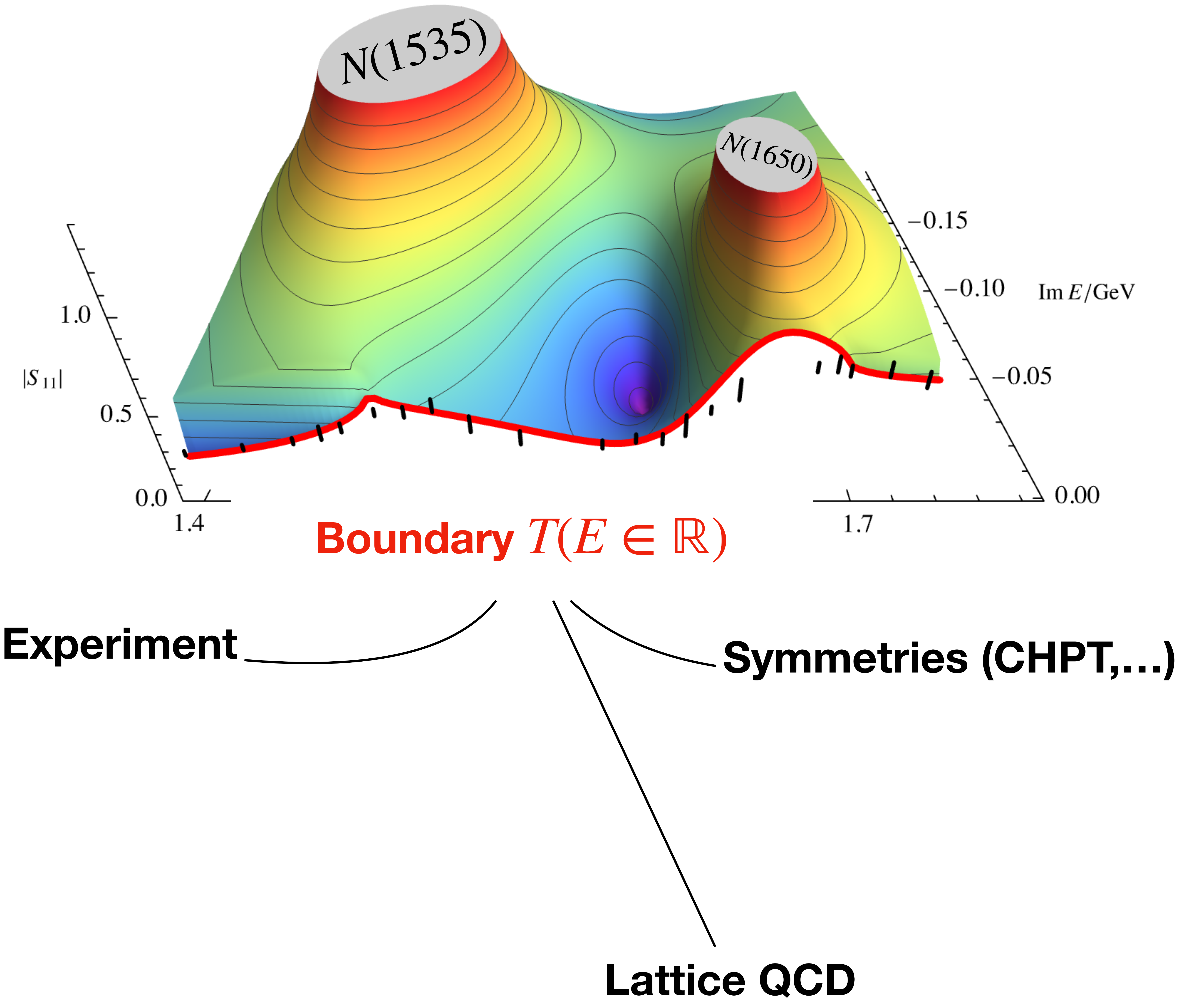
- S-matrix theory: ***transition amplitude*** $T(E \in \mathbb{C})$
  - Unitarity/Analyticity/Crossing symmetry
  - Poles on unphysical Riemann Sheets



[1] Fig: Data: SAID: Phys. Rev. C 74 (2006) 045205 Model: MM et al. Phys.Rev.D 86 (2012) 094033

Universal resonance parameters

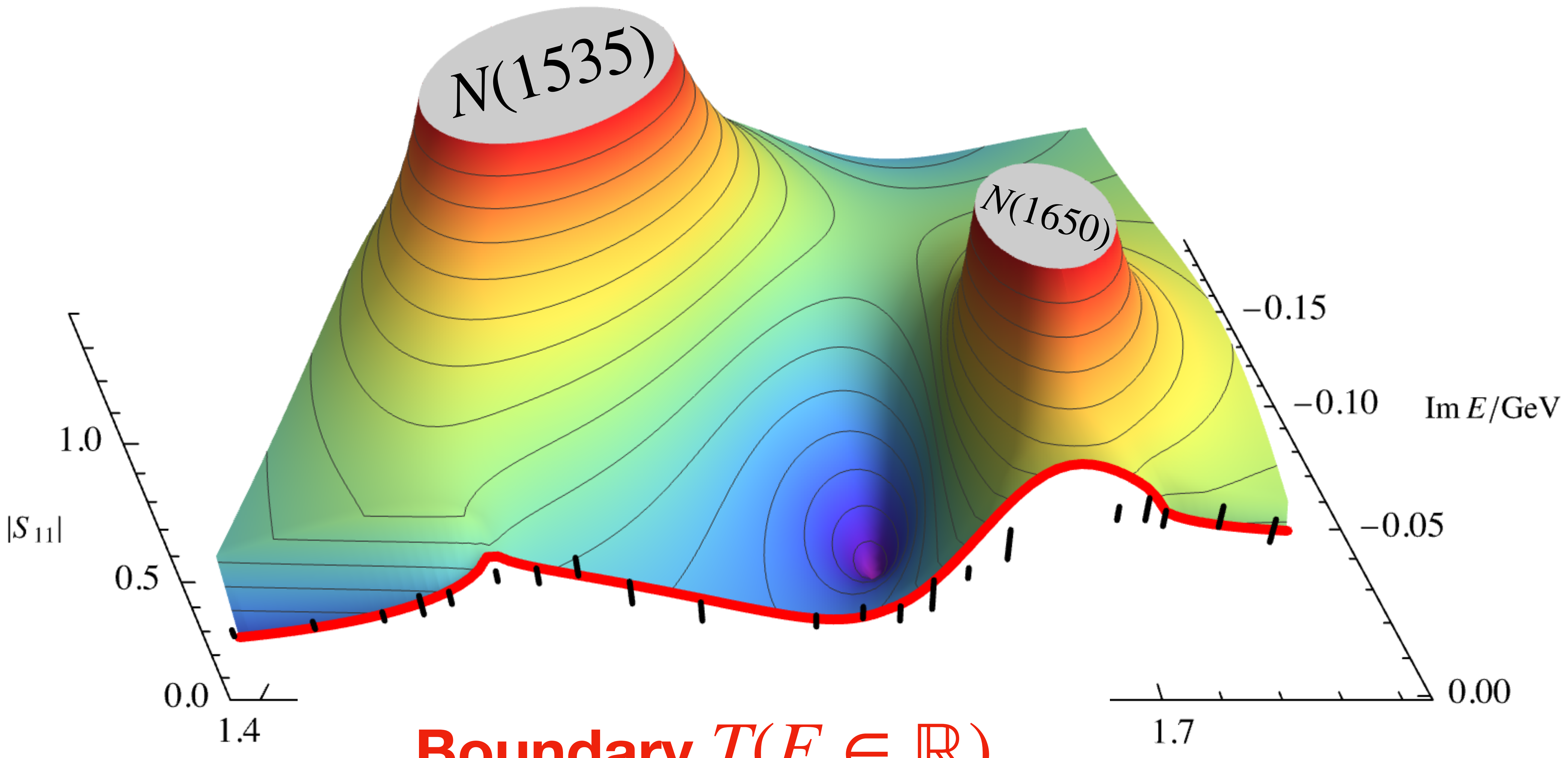
- S-matrix theory: ***transition amplitude*** $T(E \in \mathbb{C})$
  - Unitarity/Analyticity/Crossing symmetry
  - Poles on unphysical Riemann Sheets



[1] Fig: Data: SAID: Phys. Rev. C 74 (2006) 045205 Model: MM et al. Phys.Rev.D 86 (2012) 094033

Universal resonance parameters

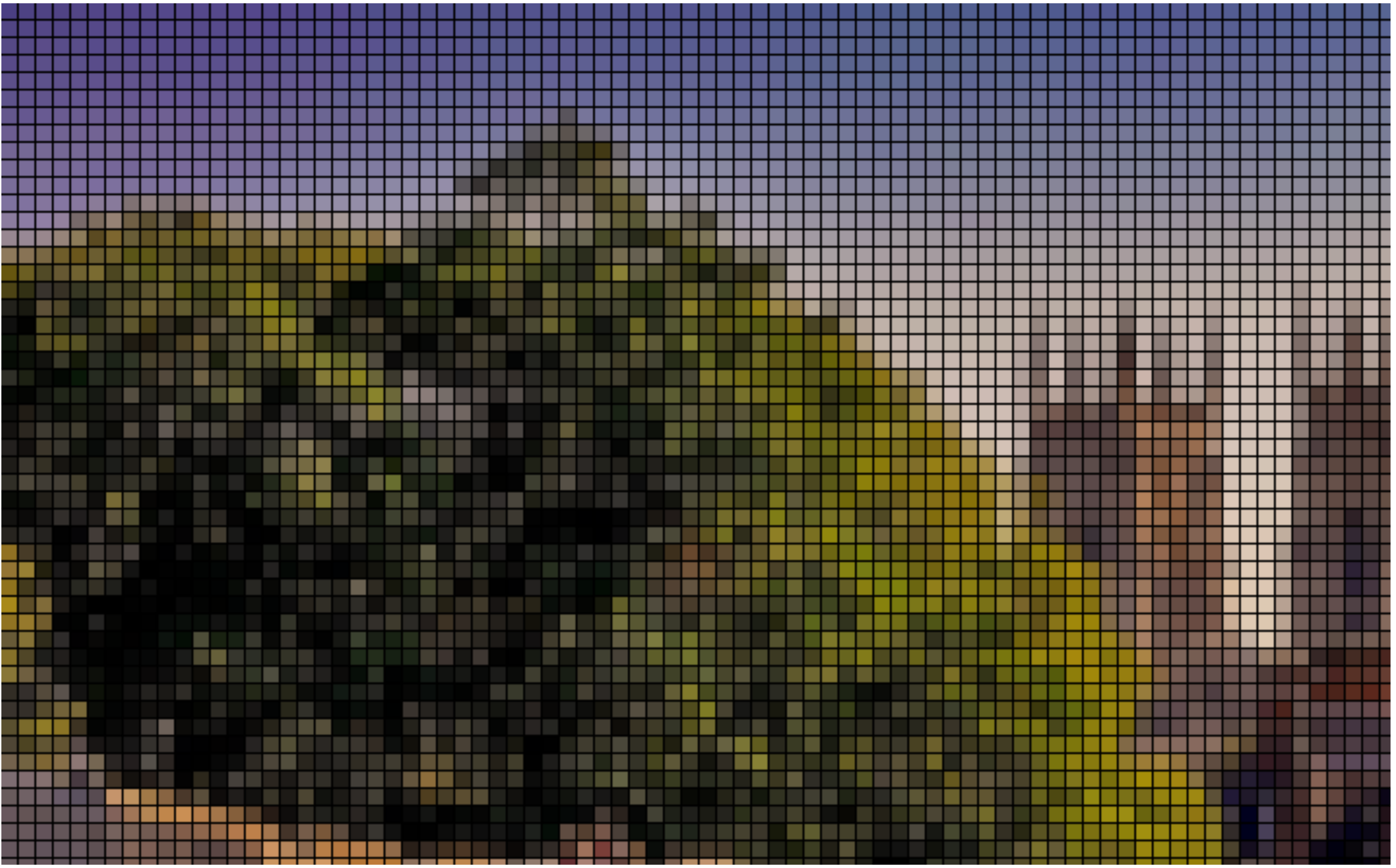
- S-matrix theory: ***transition amplitude*** $T(E \in \mathbb{C})$
    - Unitarity/Analyticity/Crossing symmetry
    - Poles on unphysical Riemann Sheets



Boundary $T(E \in \mathbb{R})$

Tridge (Midland, MI/USA)

[1] Fig: Data: SAID: Phys. Rev. C 74 (2006) 045205 Model: MM et al. Phys.Rev.D 86 (2012) 094033



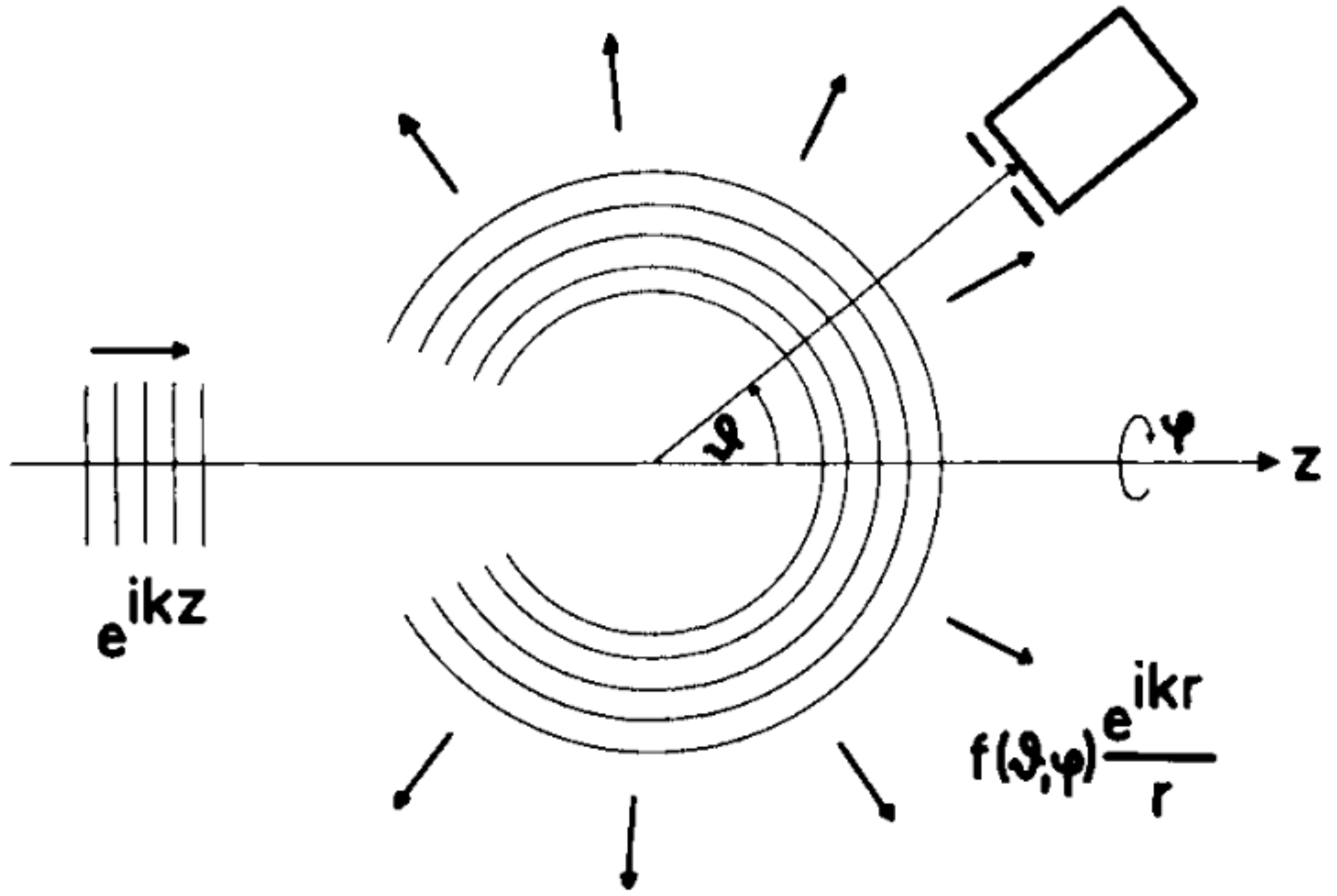
TRANSITION AMPLITUDE

INTRICACIES OF THE H3BP

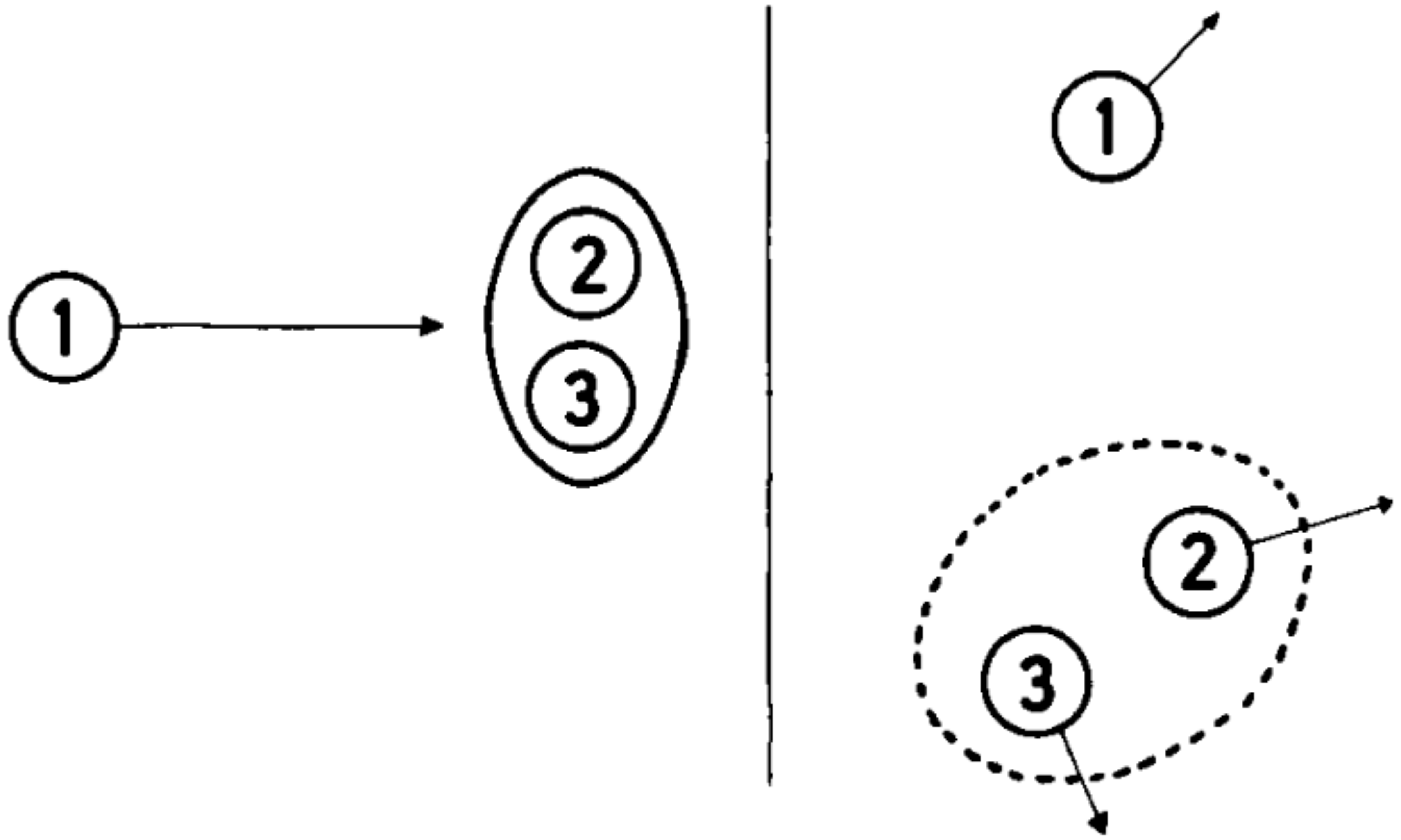
Hadronic 3-body problem

- goal: transition probabilities ↔ $T(E \in \mathbb{C})$
- challenges:
  - continuum of two-body scattering states[1]
  - 8 kinematic degrees of freedom
  - complex branch cuts[2]

    triangle singularities, left hand cuts, …



$2 \rightarrow 2$

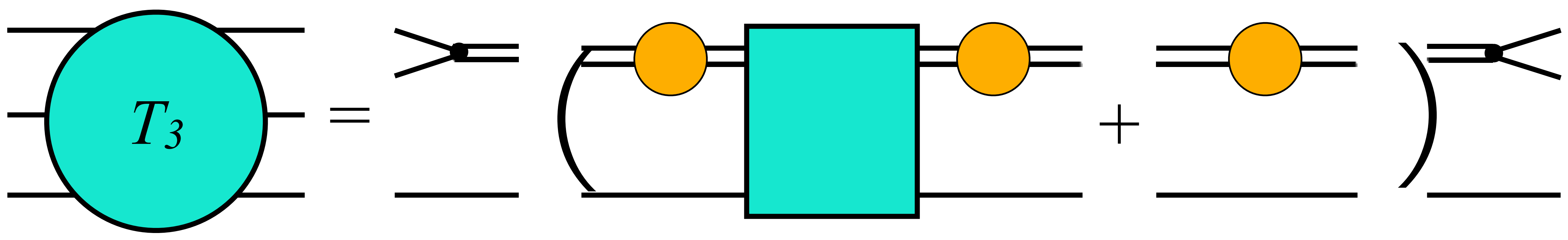


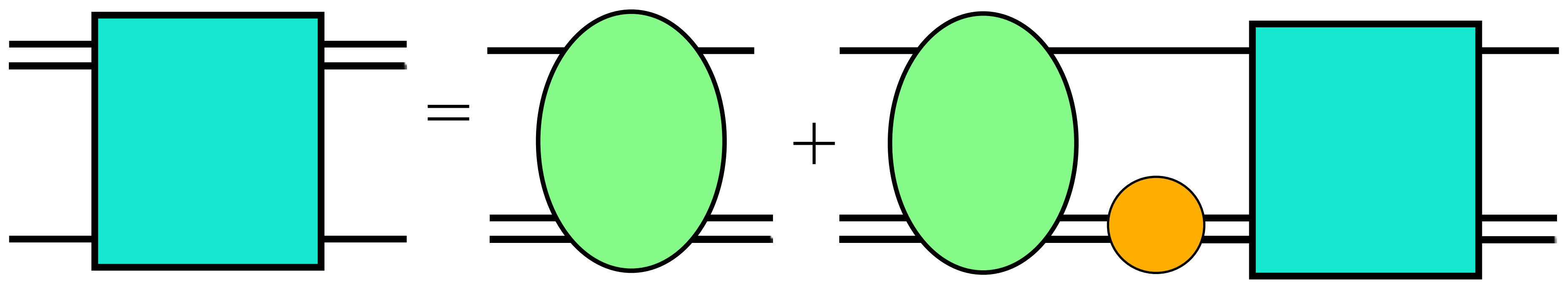
$3 \rightarrow 3$

[1] Faddeev, ...
[FIG] Schmid/Ziegelmann Pergamon Press 1974
[2] Hetherington/Schick/Coleman ... e.g. Lutz et al. PRD 92 (2015) 1, Du et al PRL. 131 (2023) 13, ...

"Infinite Volume Unitarity"[1]

- express 3-body through a 2+1 system[2]
- generic building blocks



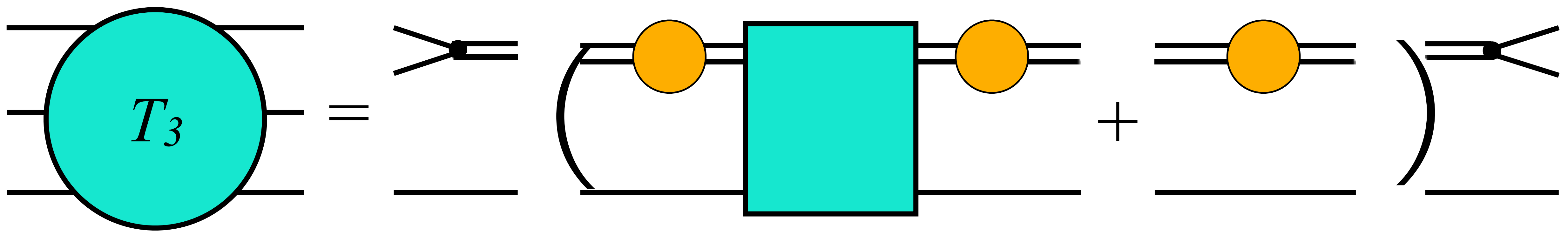


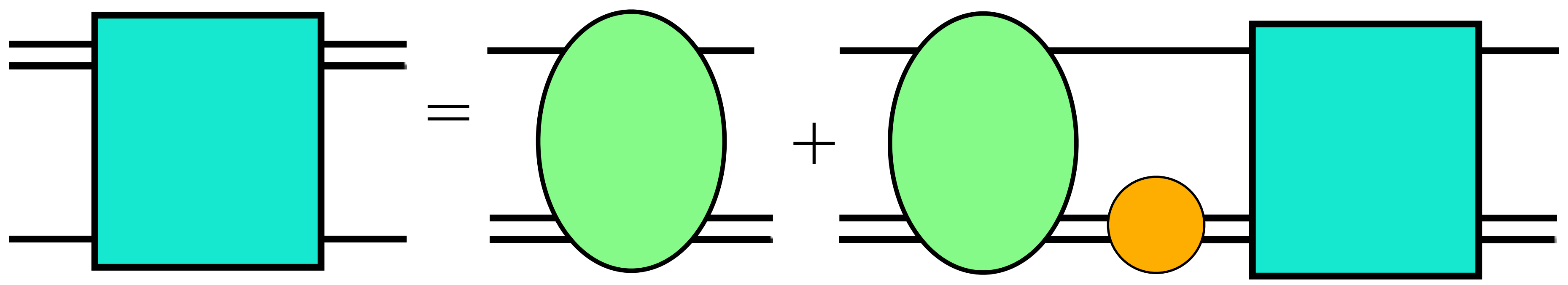
[1] MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)
[2] Related approaches: Hansen/Sharpe(2014)....; Wunderlich et al. JHEP 08 (2019); Jackura et al. Eur.Phys.J.C 79 (2019); .... Weinberg Phys.Rev. 131 (1963)

IVU FORMALISM

"Infinite Volume Unitarity"[1]

- express 3-body through a 2+1 system[2]
- generic building blocks

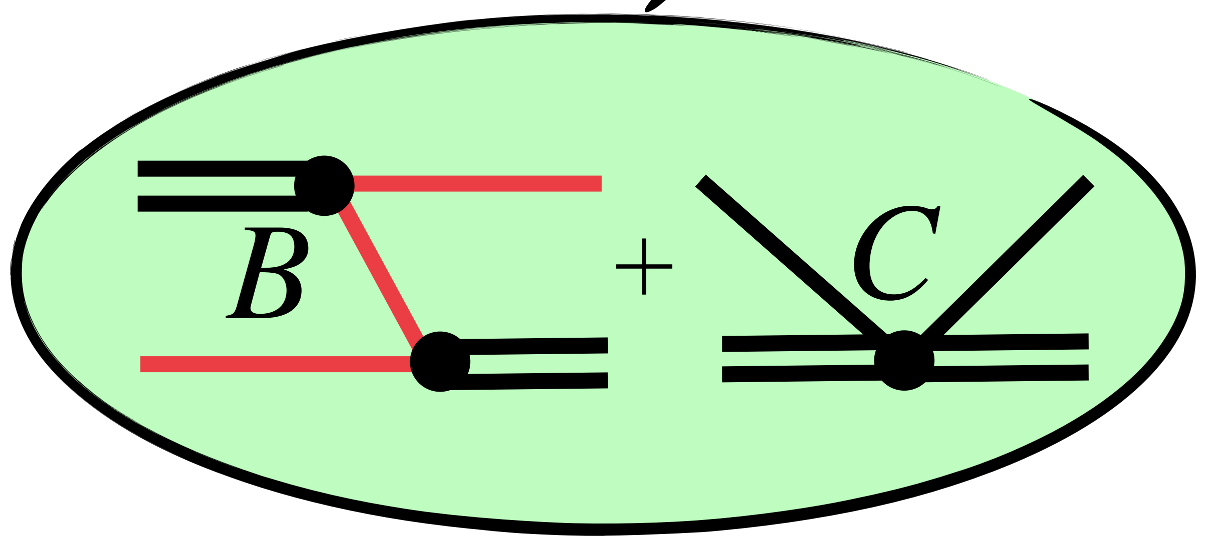


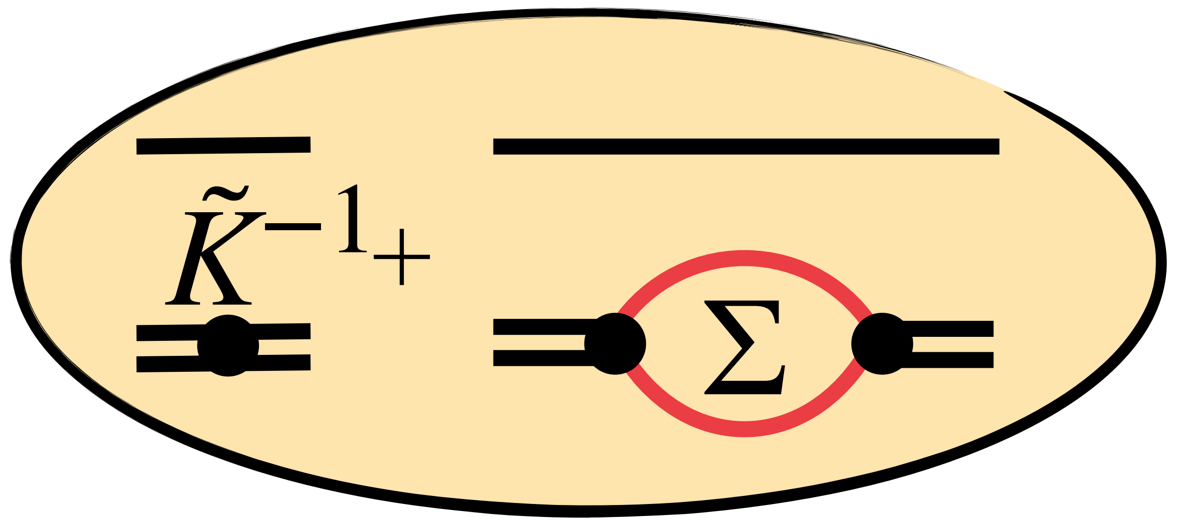


Three/two-body unitarity

$$\langle \{q\} | T_3 - T_3^\dagger | \{p\} \rangle = \int_{\mathrm{PS}} \langle \{q\} | T_3^\dagger | \{k\} \rangle \langle \{k\} | T_3 | \{p\} \rangle$$

- two classes of 3particle on-shell configurations $B, \Sigma \in \mathbb{C}$
- … up to real functions $C, \tilde{K} \in \mathbb{R}$ — dynamics of the system

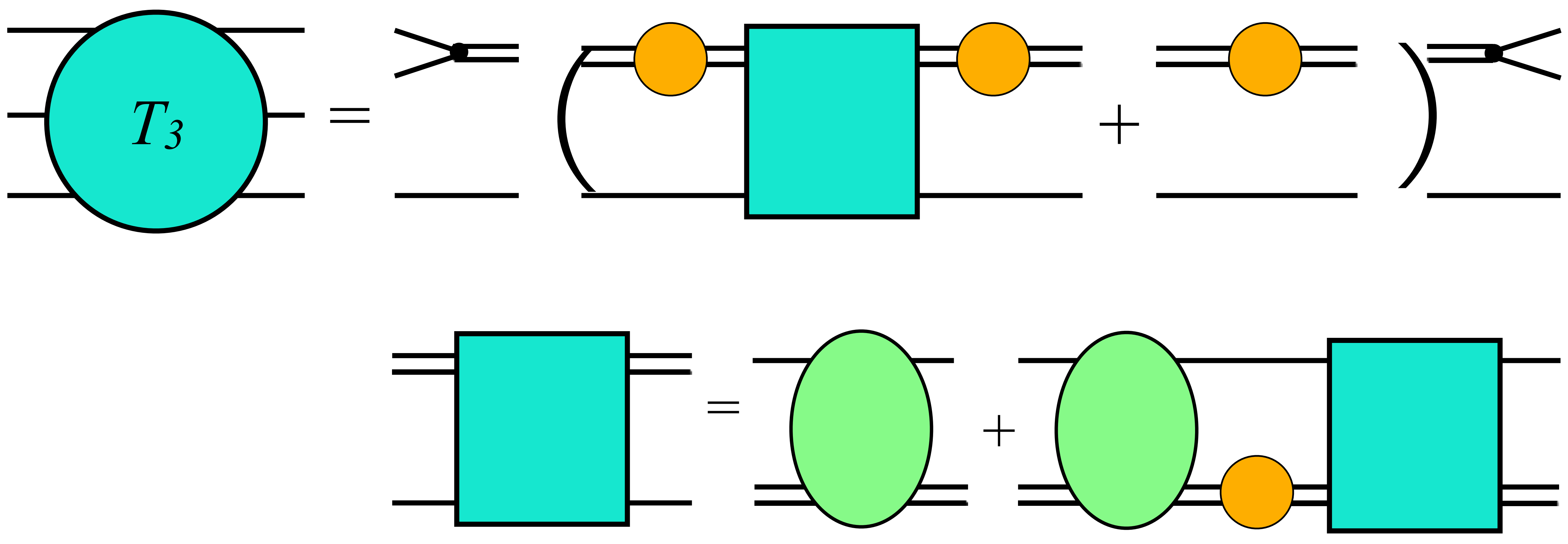




[1] MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)
[2] Related approaches: Hansen/Sharpe(2014)….; Wunderlich et al. JHEP 08 (2019); Jackura et al. Eur.Phys.J.C 79 (2019); …. Weinberg Phys.Rev. 131 (1963)

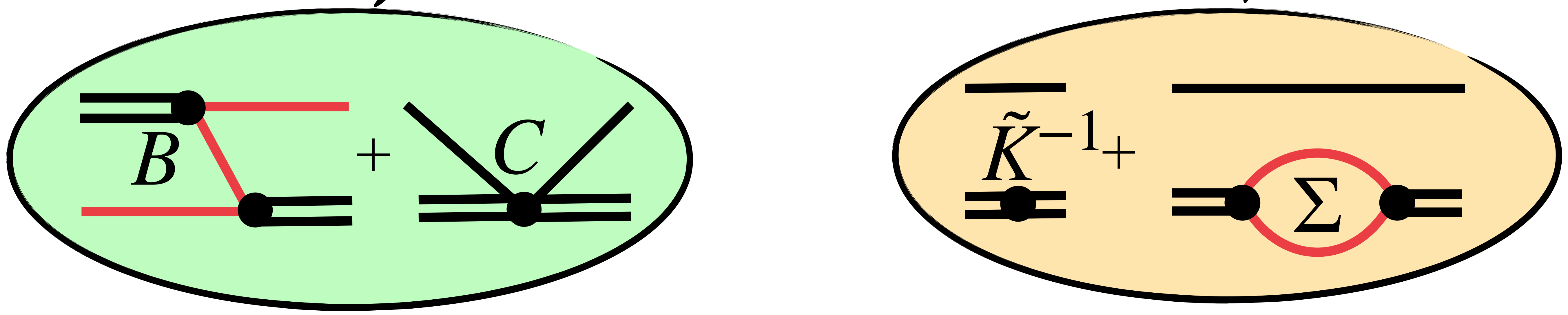
"Infinite Volume Unitarity"[1]

- express 3-body through a 2+1 system[2]
- generic building blocks



IVU

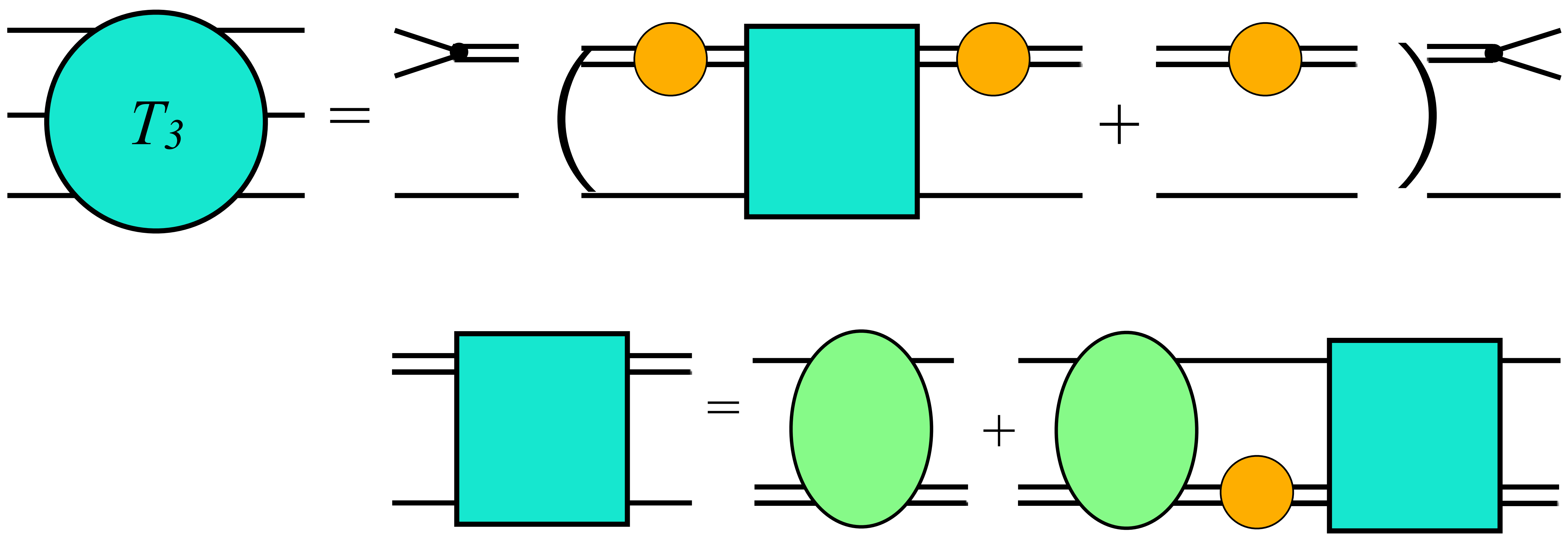
$$T = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B + C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma} T$$



[1] MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)
[2] Related approaches: Hansen/Sharpe(2014)....; Wunderlich et al. JHEP 08 (2019); Jackura et al. Eur.Phys.J.C 79 (2019); .... Weinberg Phys.Rev. 131 (1963)

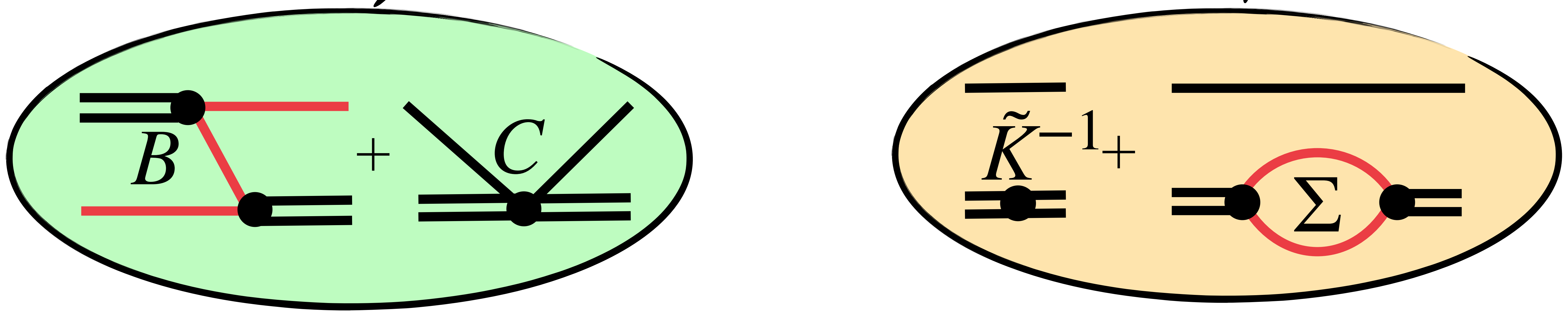
“Infinite Volume Unitarity”[1]

- express 3-body through a 2+1 system[2]
- generic building blocks



IVU

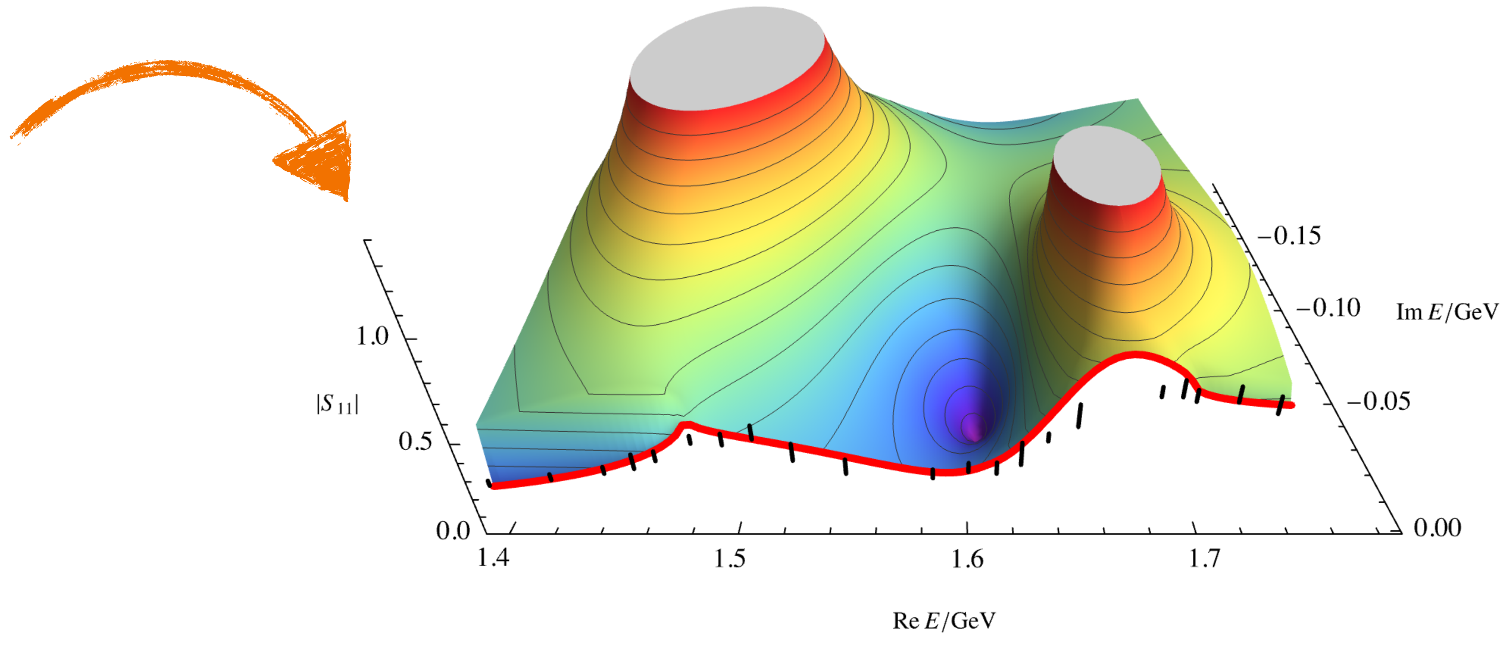
$$T = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B + C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma} T$$



🧐 ***what does it have to do with Lattice QCD?***

[1] MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)
[2] Related approaches: Hansen/Sharpe(2014)....; Wunderlich et al. JHEP 08 (2019); Jackura et al. Eur.Phys.J.C 79 (2019); .... Weinberg Phys.Rev. 131 (1963)

S-matrix, phenomenology, experiment…



IVU[1]

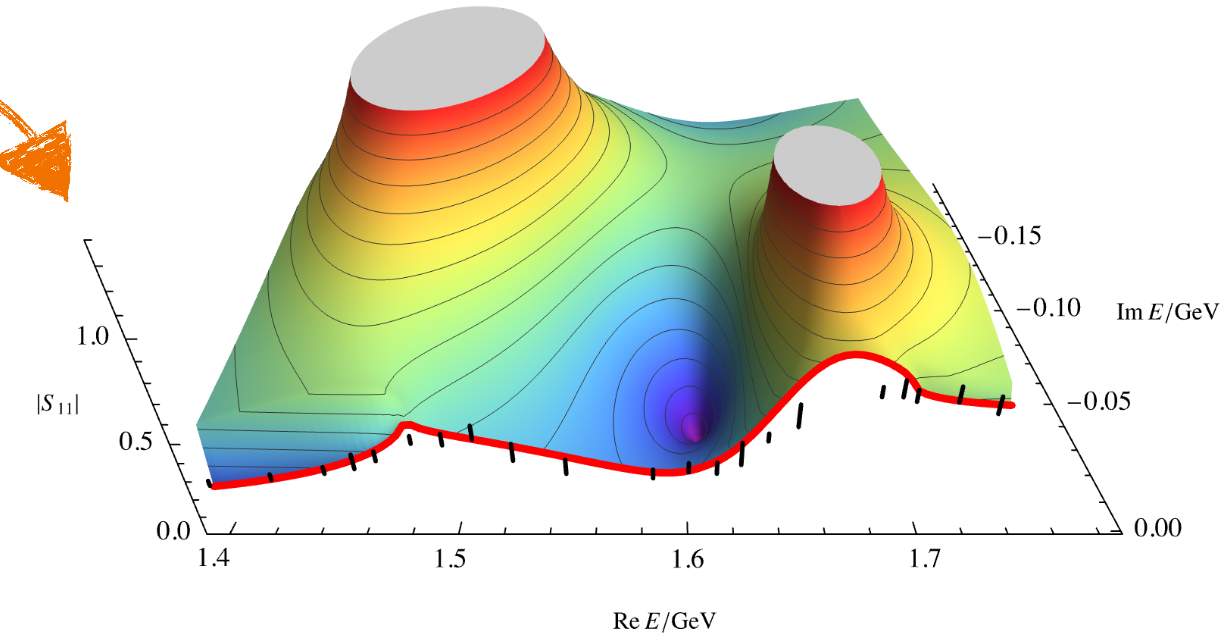
$$T = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B + C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma} T$$

3-body scattering amplitude

[1] MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)
[2] Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, …
Reviews: Briceno/Dudek/Young Rev.Mod.Phys. 90 (2018) 2 Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);
[3] MM/Döring Eur.Phys.J.A 53 (2017) 12, 240

FINITE-VOLUME SPECTRUM

S-matrix, phenomenology, experiment…



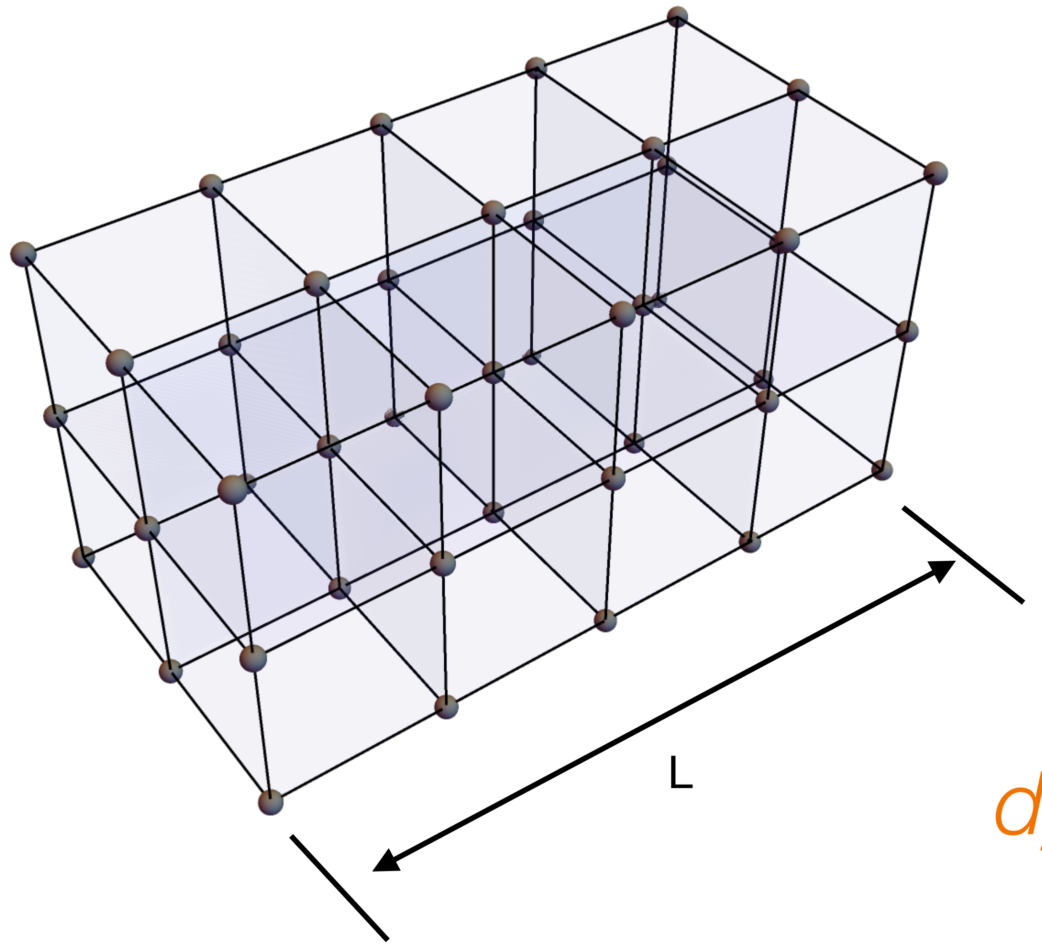
IVU[1]

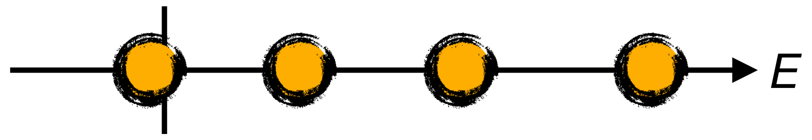
$$T = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma} T$$

3-body scattering amplitude

Lattice QCD: numerical access to QCD Green's functions:

Euclidean space-time / unphysical pion mass / **finite-volume**





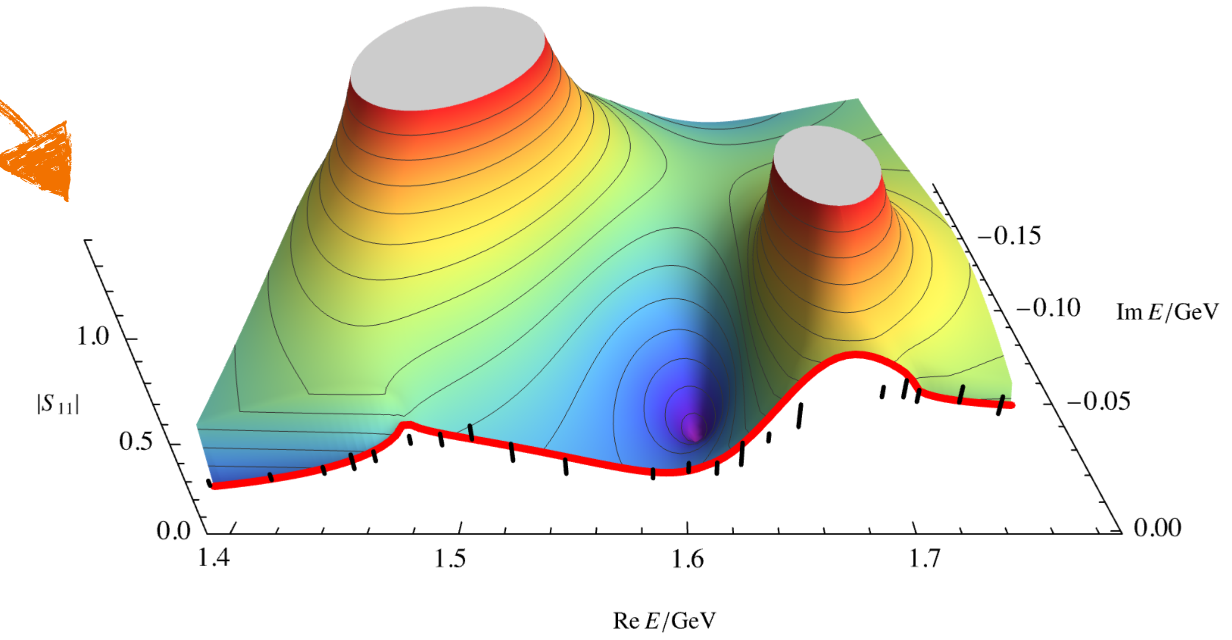
discrete spectrum

dynamics from volume dependence[2]

[1] MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)
[2] Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, …
Reviews: Briceno/Dudek/Young Rev.Mod.Phys. 90 (2018) 2 Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);
[3] MM/Döring Eur.Phys.J.A 53 (2017) 12, 240

FINITE-VOLUME SPECTRUM

S-matrix, phenomenology, experiment…



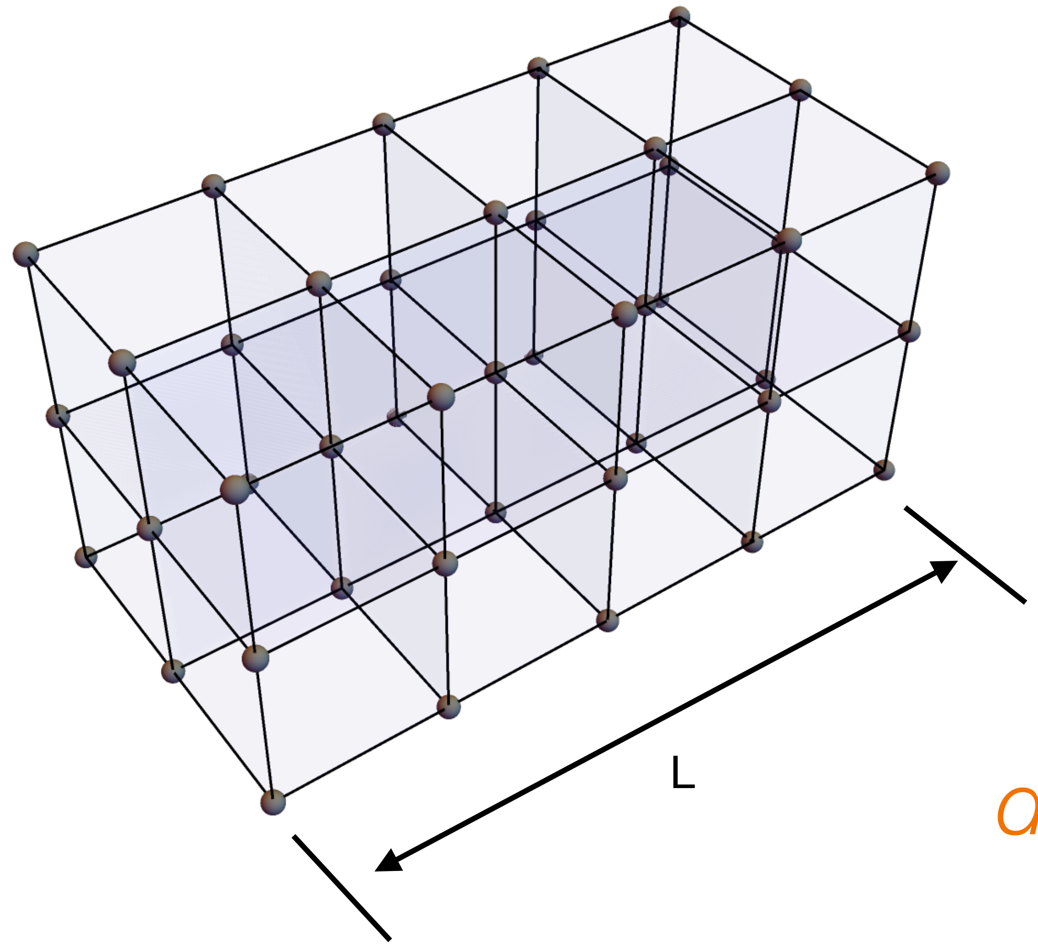
IVU[1]

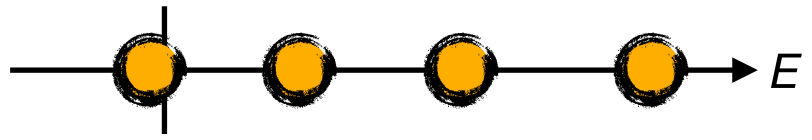
$$T = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B + C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma} T$$

3-body scattering amplitude

Lattice QCD: numerical access to QCD Green's functions:

Euclidean space-time / unphysical pion mass / **finite-volume**





discrete spectrum

dynamics from volume dependence[2]

FVU[3]

$$\det \left[2L^3 E_{\mathbf{p}} \left(\tilde{K}_2^{-1} - \Sigma^L \right) - B - C \right]^\Gamma = 0$$

3-body quantization condition

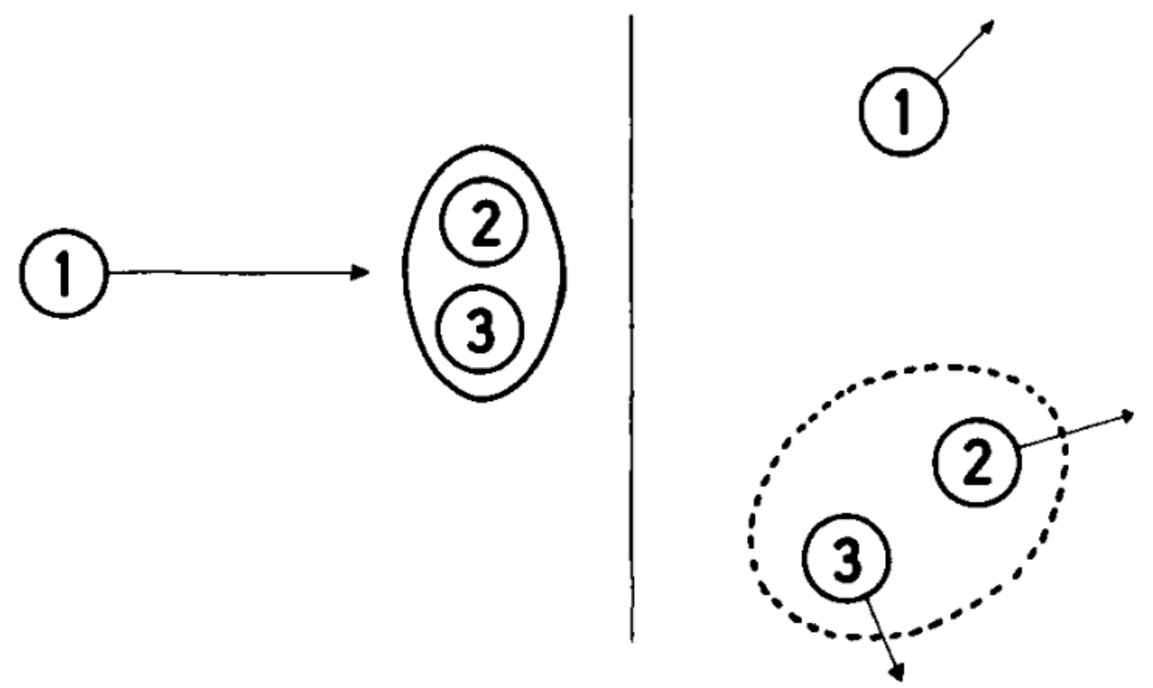
On-shell particles "feel" the box size — B, Σ

Volume-independent quantities — $C, \tilde{K}^{-1}$

[1] MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)
[2] Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, …
Reviews: Briceno/Dudek/Young Rev.Mod.Phys. 90 (2018) 2 Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);
[3] MM/Döring Eur.Phys.J.A 53 (2017) 12, 240

3-body amplitude/3-body quantization conditions

- spectator can carry arbitrary momentum away
- integral/determinant equation
- cutoff required (form factors, hard cutoff,…)

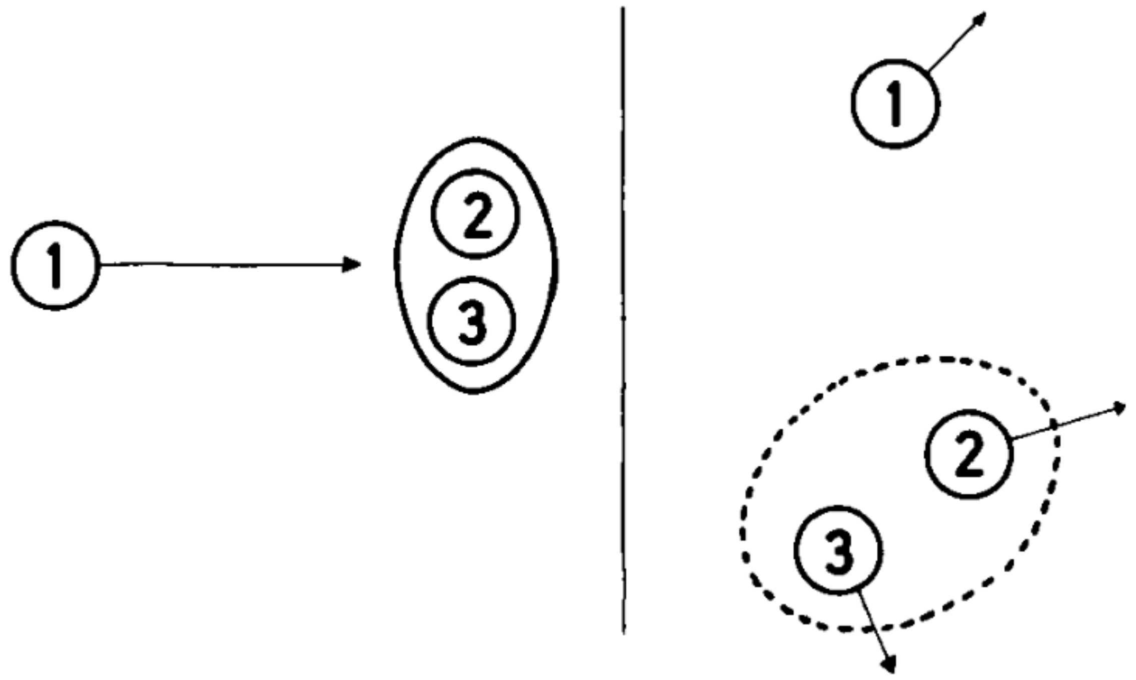


$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

[1] **in preparation …**
[2] Bedaque/Hammer/van Kolck, Phys. Rev. Lett. 82 (1999) 463; Bedaque/Hammer/van Kolck, Nucl.Phys. A 646 (1999) 444

3-body amplitude/3-body quantization conditions

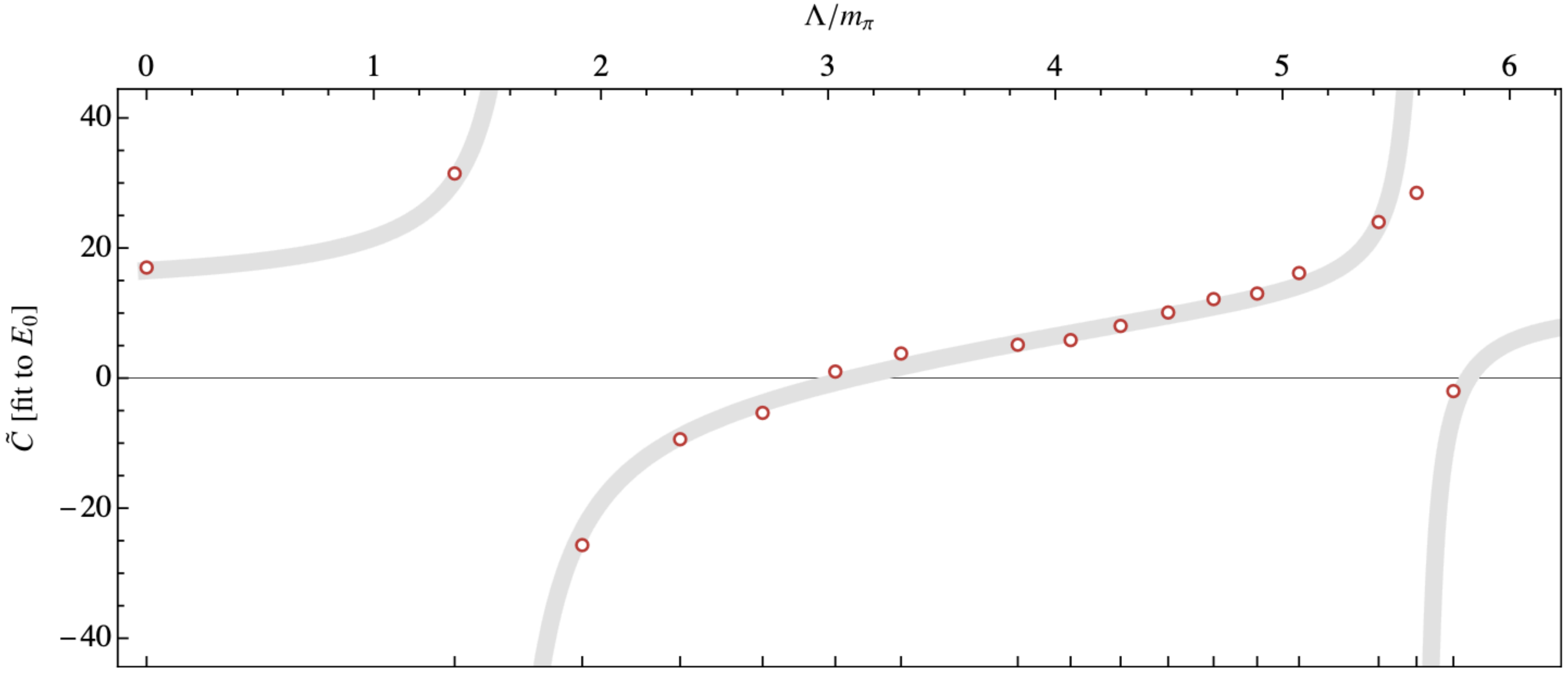
- spectator can carry arbitrary momentum away
- integral/determinant equation
- cutoff required (form factors, hard cutoff,…)



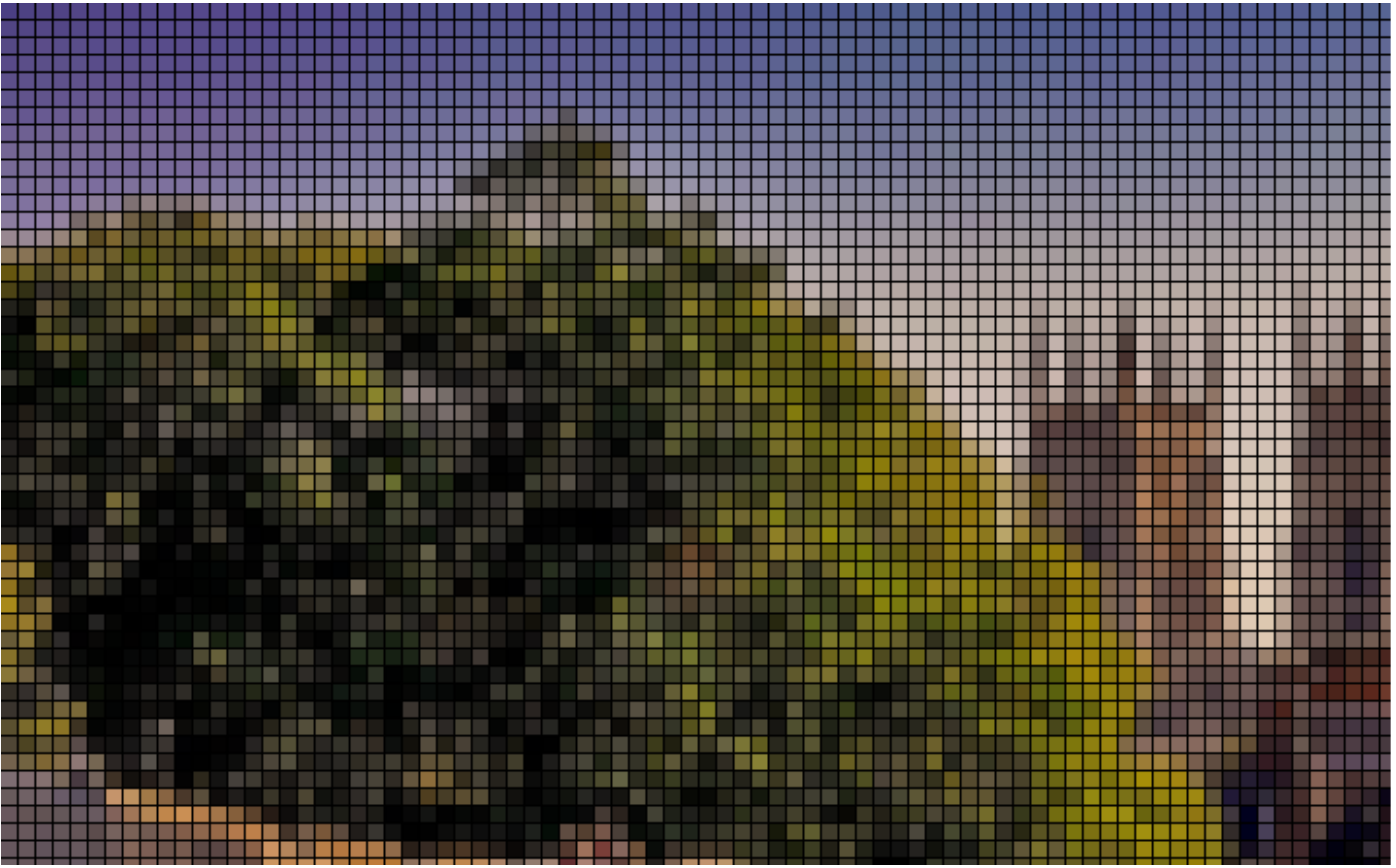
$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

Example: $\pi\rho/\pi(\pi\pi)_2$ system[1]

- change cutoff & refit C to a fixed LQCD spectrum
- $C(\Lambda)$ shows cyclic behaviour[2]
- ***3-body force is not an observable***



[1] **in preparation …**
[2] Bedaque/Hammer/van Kolck, Phys. Rev. Lett. 82 (1999) 463; Bedaque/Hammer/van Kolck, Nucl.Phys. A 646 (1999) 444



APPLICATIONS

3-MESON SYSTEMS

Maximal isospin

- Formalism development / feasibility studies[1]
- several LQCD calculations[2]
- 3-body force extraction (vs CHPT…) [3]

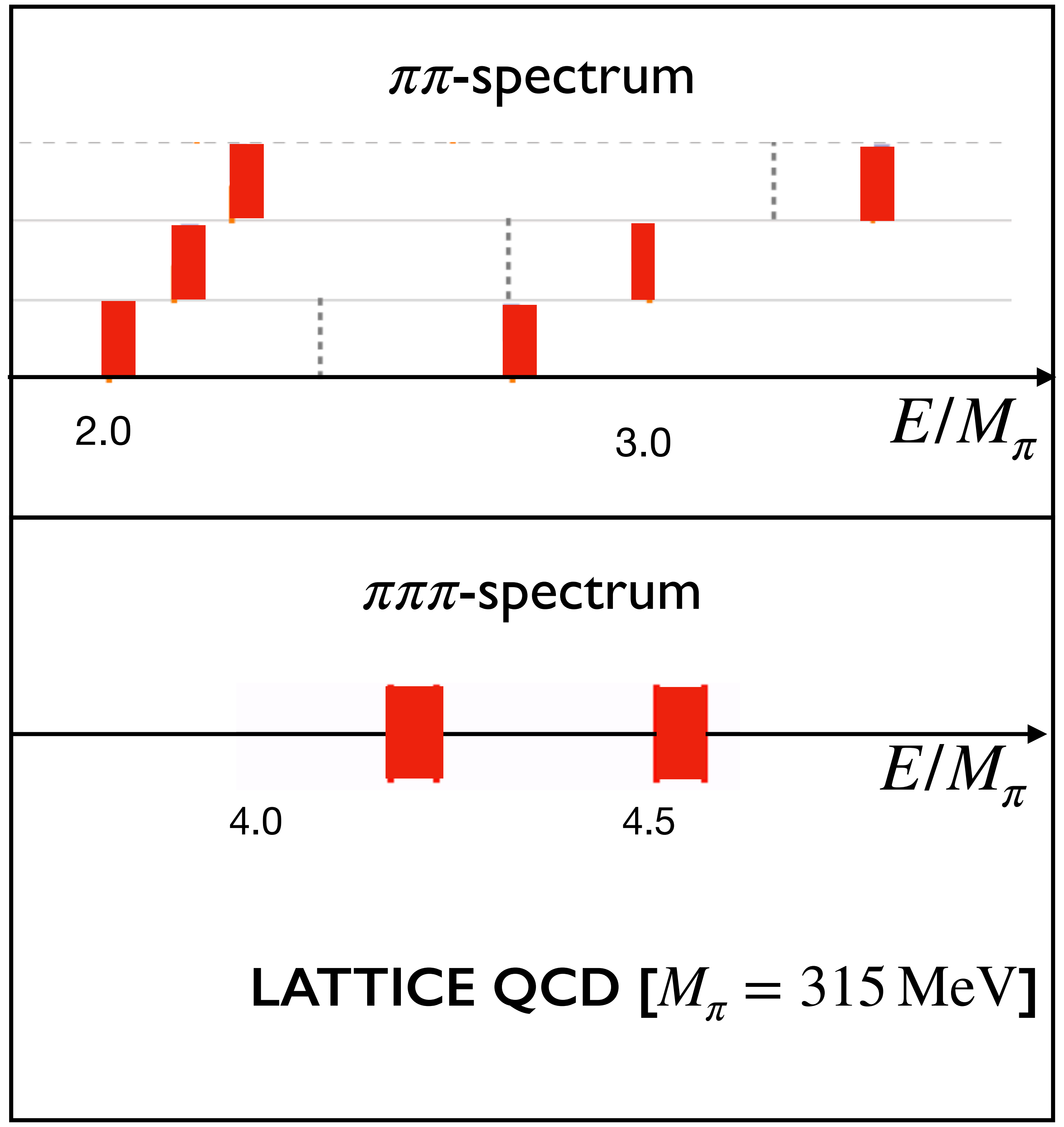
[1] Blanton, Draper, Briceño, Döring, Guo, Hammer, Hansen, MM, Meißner, Müller, Pang, Polejaeva, Romero-López, Rusetsky, Sharpe ...
[2] NPLQCD/GWQCD/Horz-Hanlon/HadSpec
[3] MM/Döring PRL122 (2019) Romero-López et al. PRL 124(2020) Culver et al PRD 101 (2020) Alexandru et al. PRD 102 (2020) Hansen et al. PRL 126(2021) Blanton et al. JHEP (2022) Draper JHEP 05 (2023)
Reviews: Hansen/Sharpe(2019) MM/Döring/Rusetsky(2021)
[4] Garofalo et al. JHEP 02(2023)
[5] MM, Alexandru, Brett, Culver, Döring, Lee, Sadasivan PRL127, 222001 (2021)

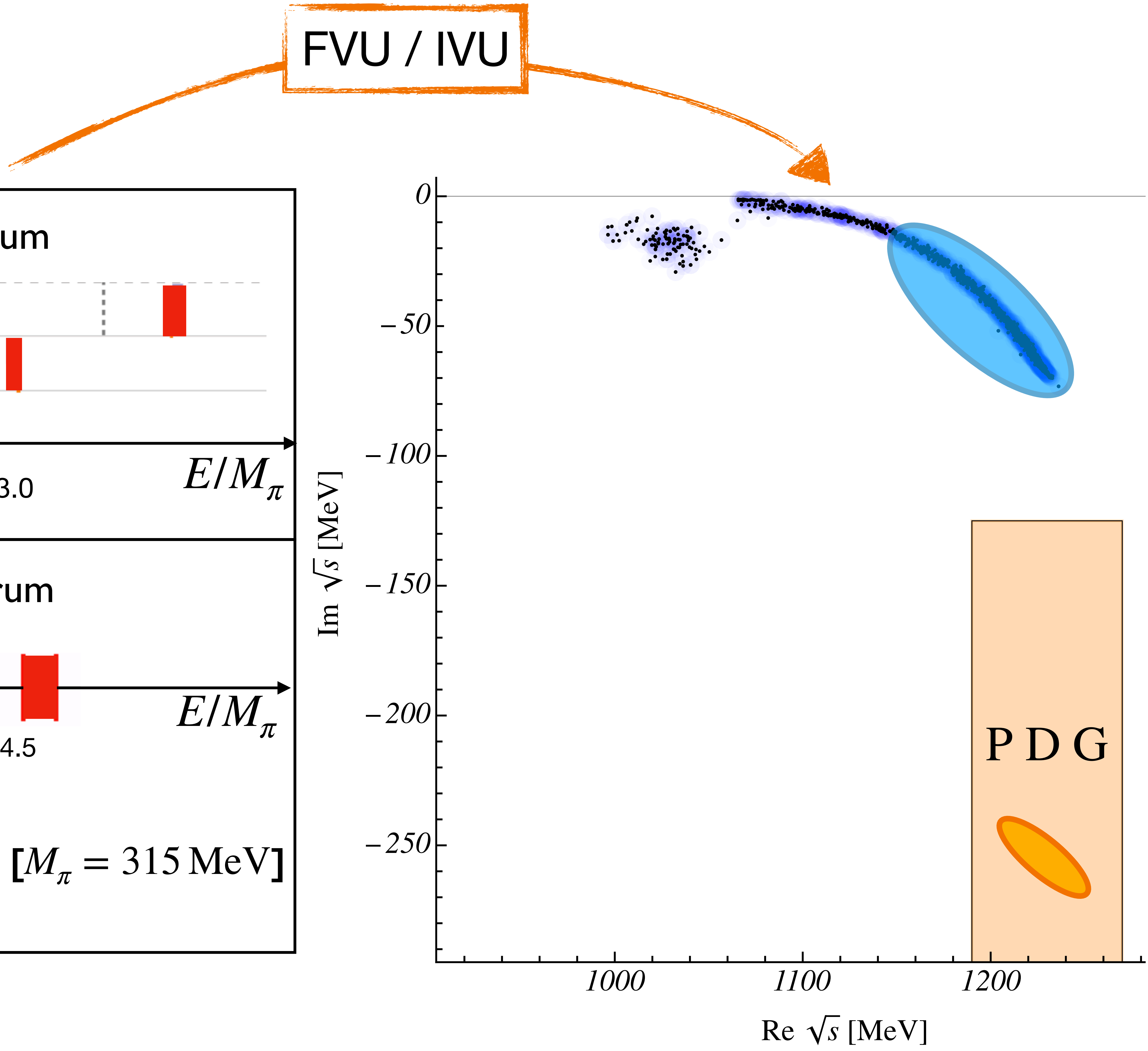
Maximal isospin

- Formalism development / feasibility studies[1]
- several LQCD calculations[2]
- 3-body force extraction (vs CHPT…) [3]

Resonant system

- Formalisms comparison on ϕ^4-theory [4]
- $a_1(1260)$ from Lattice QCD [5]



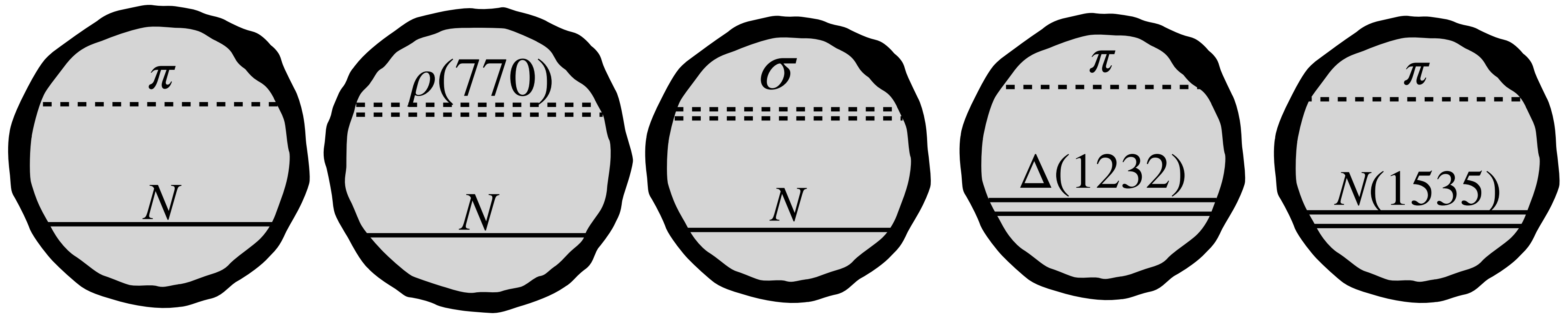


[1] Blanton, Draper, Briceño, Döring, Guo, Hammer, Hansen, MM, Meißner, Müller, Pang, Polejaeva, Romero-López, Rusetsky, Sharpe …
[2] NPLQCD/GWQCD/Horz-Hanlon/HadSpec
[3] MM/Döring PRL122 (2019) Romero-López et al. PRL 124(2020) Culver et al PRD 101 (2020) Alexandru et al. PRD 102 (2020) Hansen et al. PRL 126(2021) Blanton et al. JHEP (2022) Draper JHEP 05 (2023)
Reviews: Hansen/Sharpe(2019) MM/Döring/Rusetsky(2021)
[4] Garofalo et al. JHEP 02(2023)
[5] MM, Alexandru, Brett, Culver, Döring, Lee, Sadasivan PRL127, 222001 (2021)

ROPER $N(1440)$ — FINITE VOLUME

Talks: U. Thoma/D. Leinweber

CHANNELS



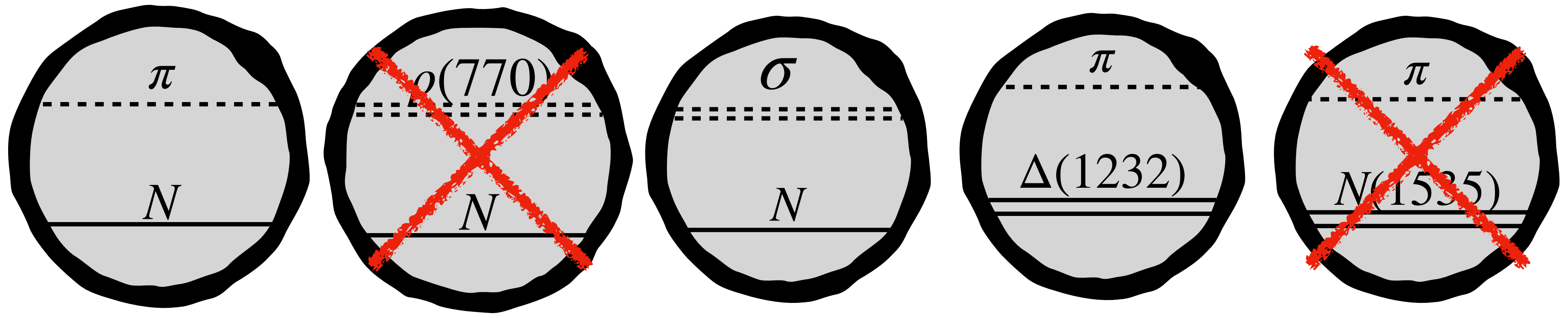
... and more in SU(3)

[1] Severt, MM, Ulf-G.Meißner JHEP 04 (2023) 100
[2] Lattice values (black dots) Lang et al. Phys.Rev.D 95 (2017) 1

ROPER $N(1440)$ — FINITE VOLUME

Talks: U. Thoma/D. Leinweber

CHANNELS



~~... and more in SU(3)~~

Simplified pilot study[1]

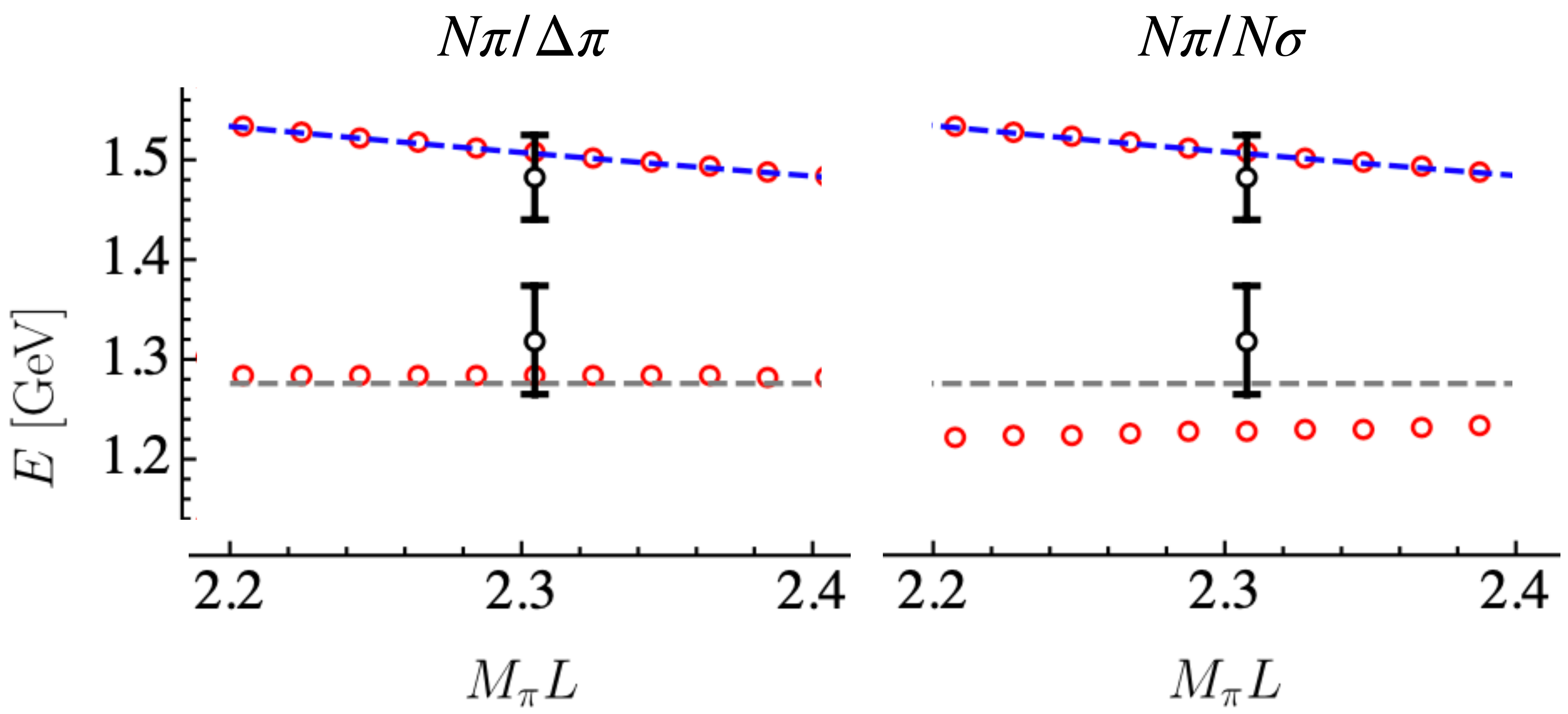
◉ self-energy formalism via particle-dimer Lagrangian

⚠️ *no particle-exchange diagrams*

Predict finite-volume spectrum for fixed parameters

➡ tiny energy shifts (opposing effects $N\sigma \leftrightarrow \pi\Delta$ channels)

➡ phenomenological input necessary

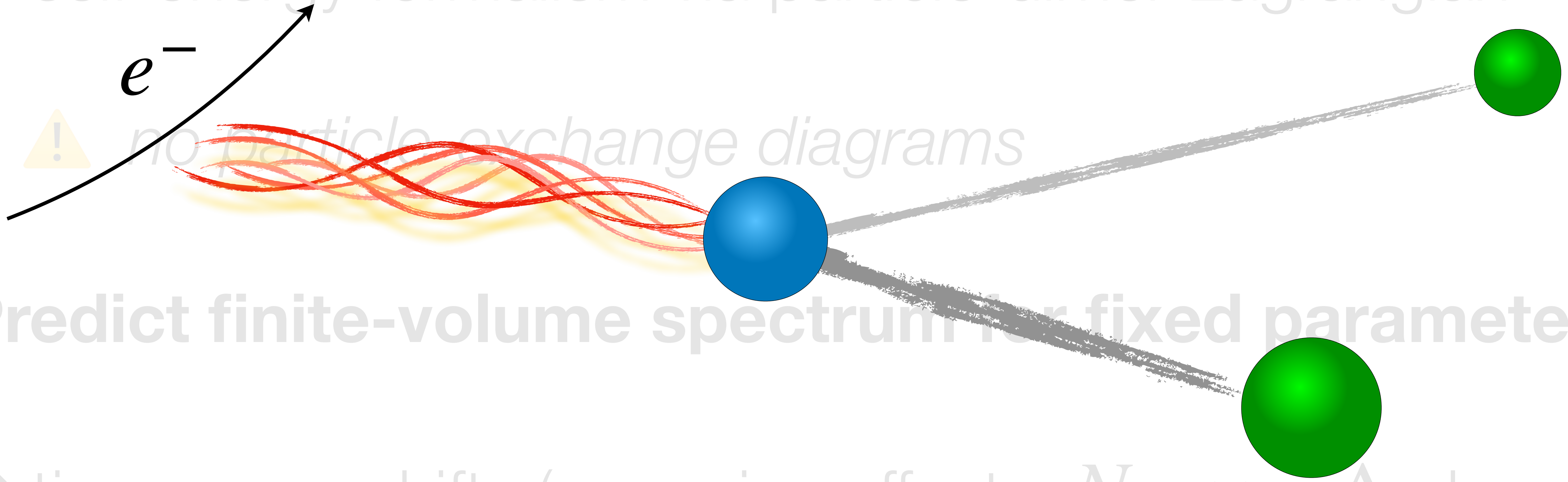


[1] Severt, MM, Ulf-G.Meißner JHEP 04 (2023) 100
[2] Lattice values (black dots) Lang et al. Phys.Rev.D 95 (2017) 1

Simplified pilot study[1]

Meson electroproduction off the proton

- self-energy formalism via particle-dimer Lagrangian



- ⚠️ *no particle exchange diagrams*

Predict finite-volume spectrum for fixed parameters

➡ tiny energy shifts (opposing effects $N\sigma \leftrightarrow \pi\Delta$ channels)

➡ phenomenological input necessary

[1] Severt, MM, Ulf-G.Meißner JHEP 04 (2023) 100
[2] Lattice values (black dots) Lang et al. Phys.Rev.D 95 (2017) 1

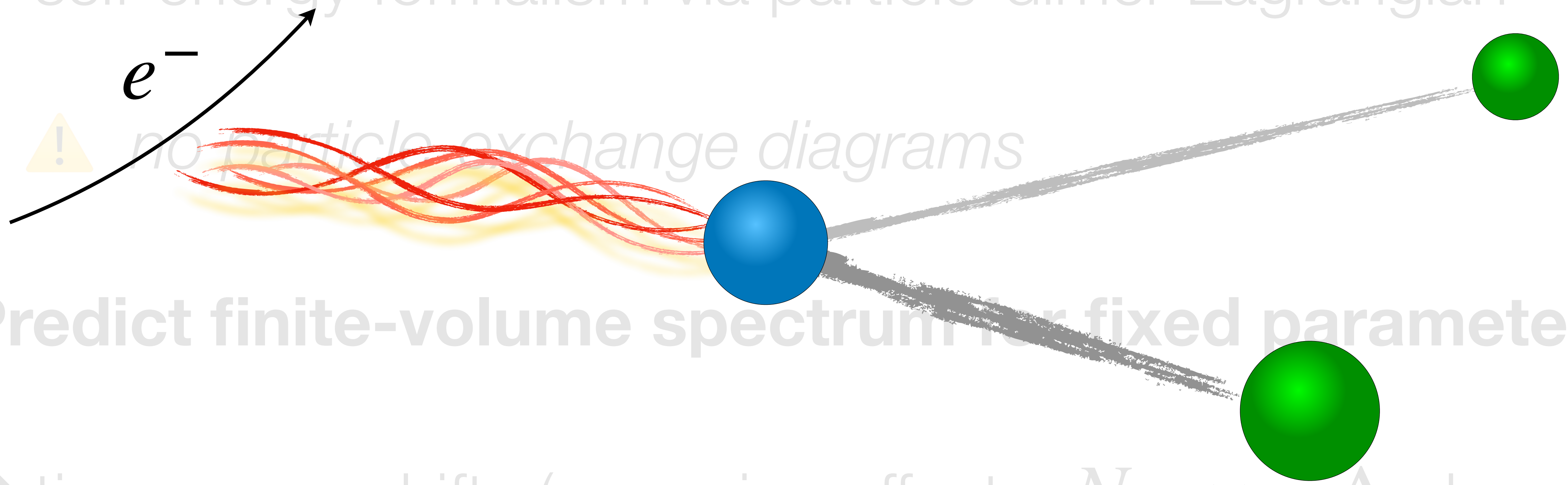
Simplified pilot study[1]

- self-energy formalism via particle-dimer Lagrangian
- ⚠️ no particle exchange diagrams

Predict finite-volume spectrum for fixed parameters

➡ tiny energy shifts (opposing effects $N\sigma \leftrightarrow \pi\Delta$ channels)

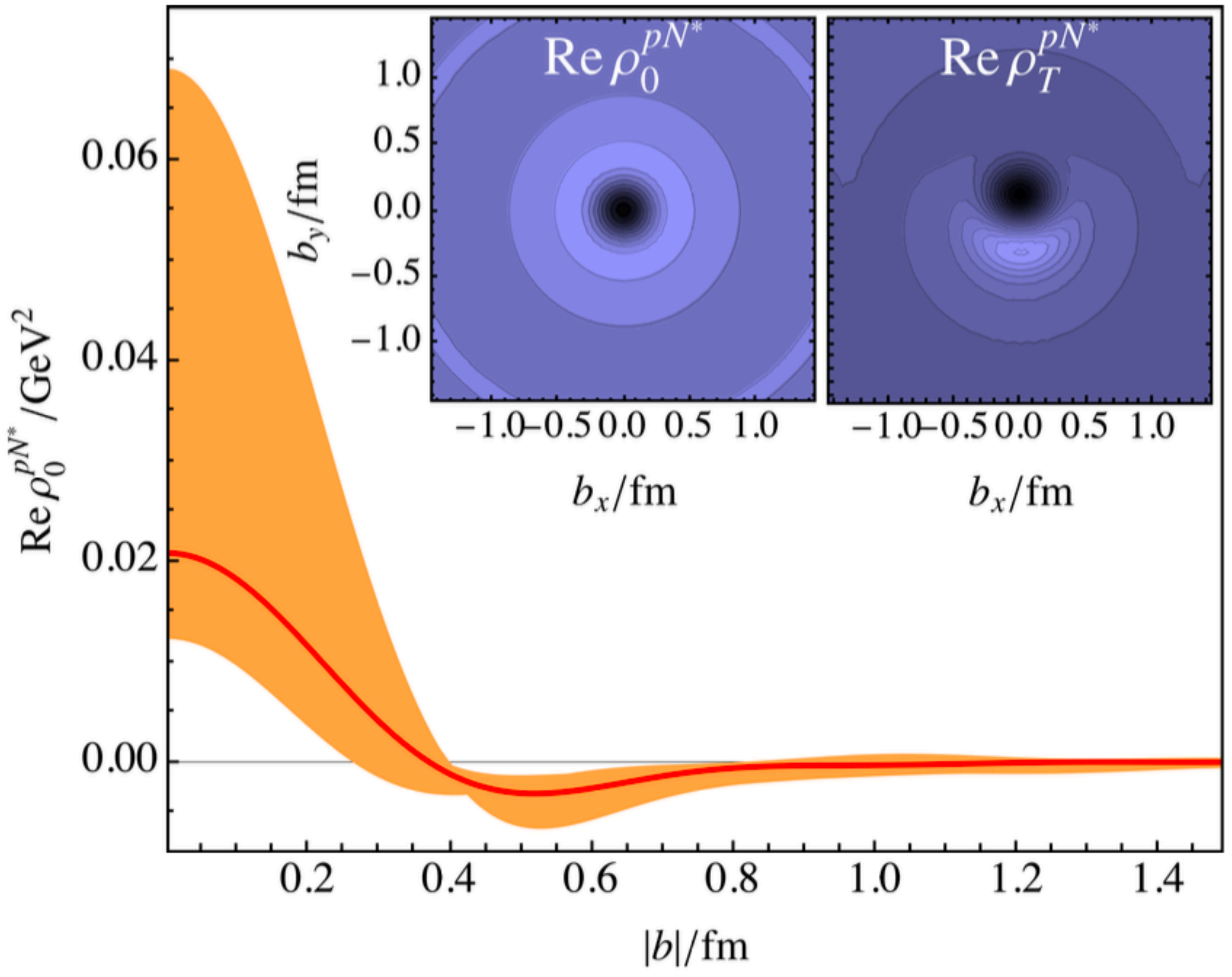
Meson electroproduction off the proton



➡ phenomenological input necessary

- Plenty of data (also upcoming)
- Formalism for extracting resonance parameters

Jülich-Bonn-Washington (jbw.phys.gwu.edu/)



ArXiv:2404.17444v2

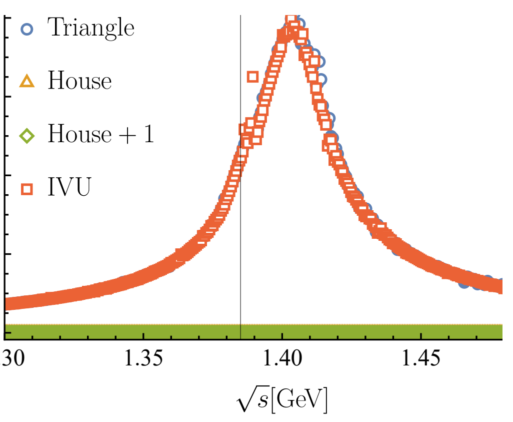
→ **Talk by M. Döring (Thursday)**

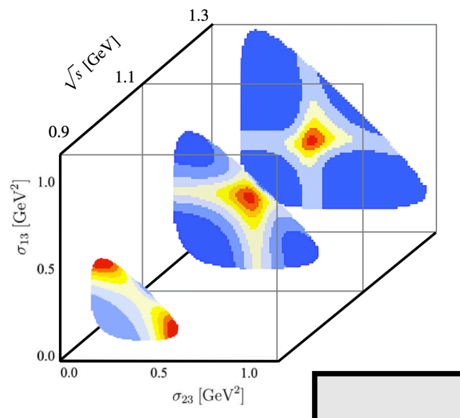
[1] Severt, MM, Ulf-G.Meißner JHEP 04 (2023) 100
[2] Lattice values (black dots) Lang et al. Phys.Rev.D 95 (2017) 1

SUMMARY

IVU

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$





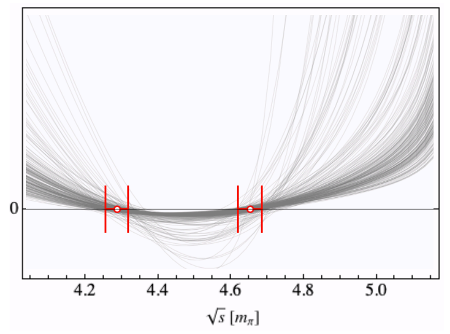
Infinite volume three-body formalism

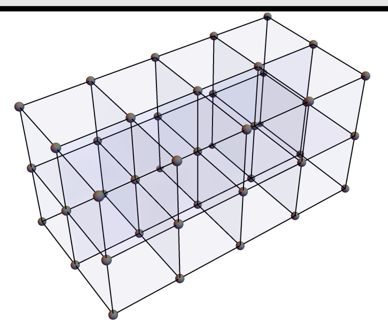
- ◉ Unitarity induced analytic structure
- ◉ universal resonance parameter
- ◉ Analytic structure

THANK YOU

FVU

$$\det\left[\, 2L^3 E_{\mathbf{p}} \left(\tilde{K}_2^{-1} - \Sigma_2^L\right) - B - C \,\right]^{T_{1g}}$$





Finite-volume three-body formalism

- ◉ 3b quantization condition
- ◉ several applications
- ◉ first chiral trajectories of 3b-resonances

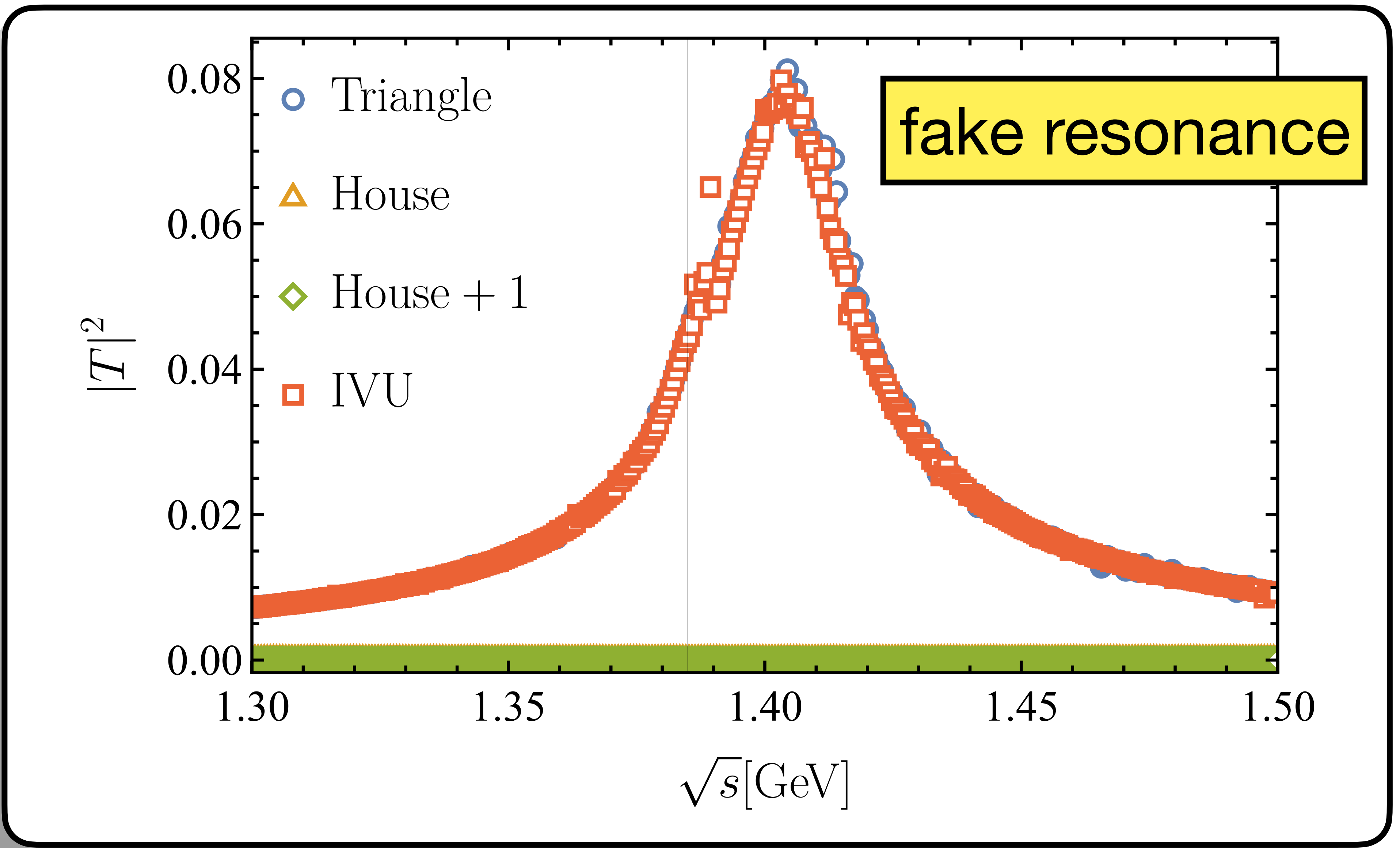
Tridge (Midland, MI/USA)

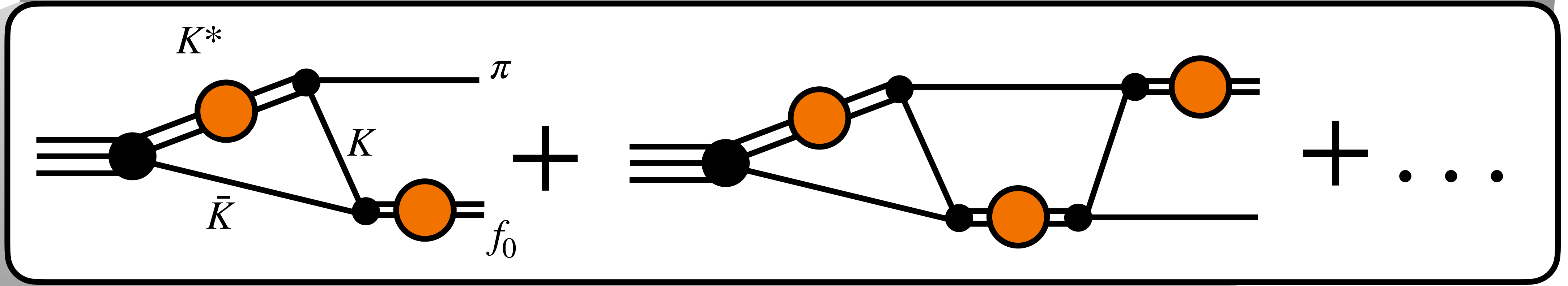
OUTLOOK

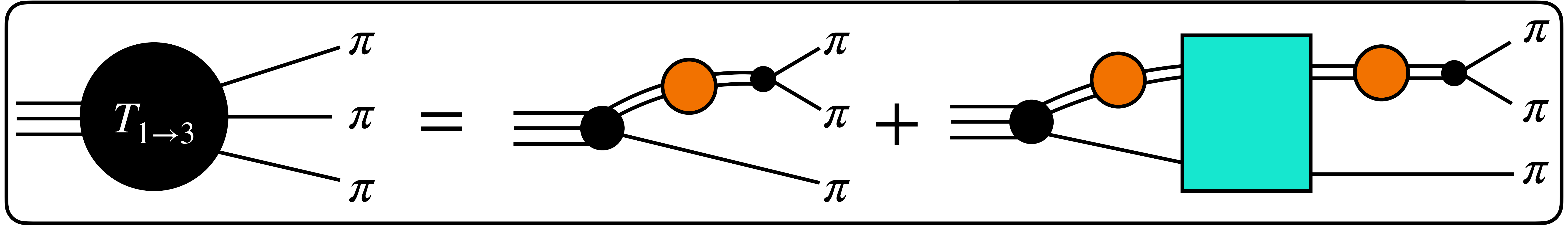
- ◉ $\pi\pi N$ content of Roper-resonance
  … connections to DCC global studies
- ◉ $\pi\pi\Lambda$ and strangeness resonances (?)
- ◉ $\bar{K}d$ scattering
- ◉ …

HILBERT'S HOTEL

https://www.ias.edu/ideas/2016/pires-hilbert-hotel



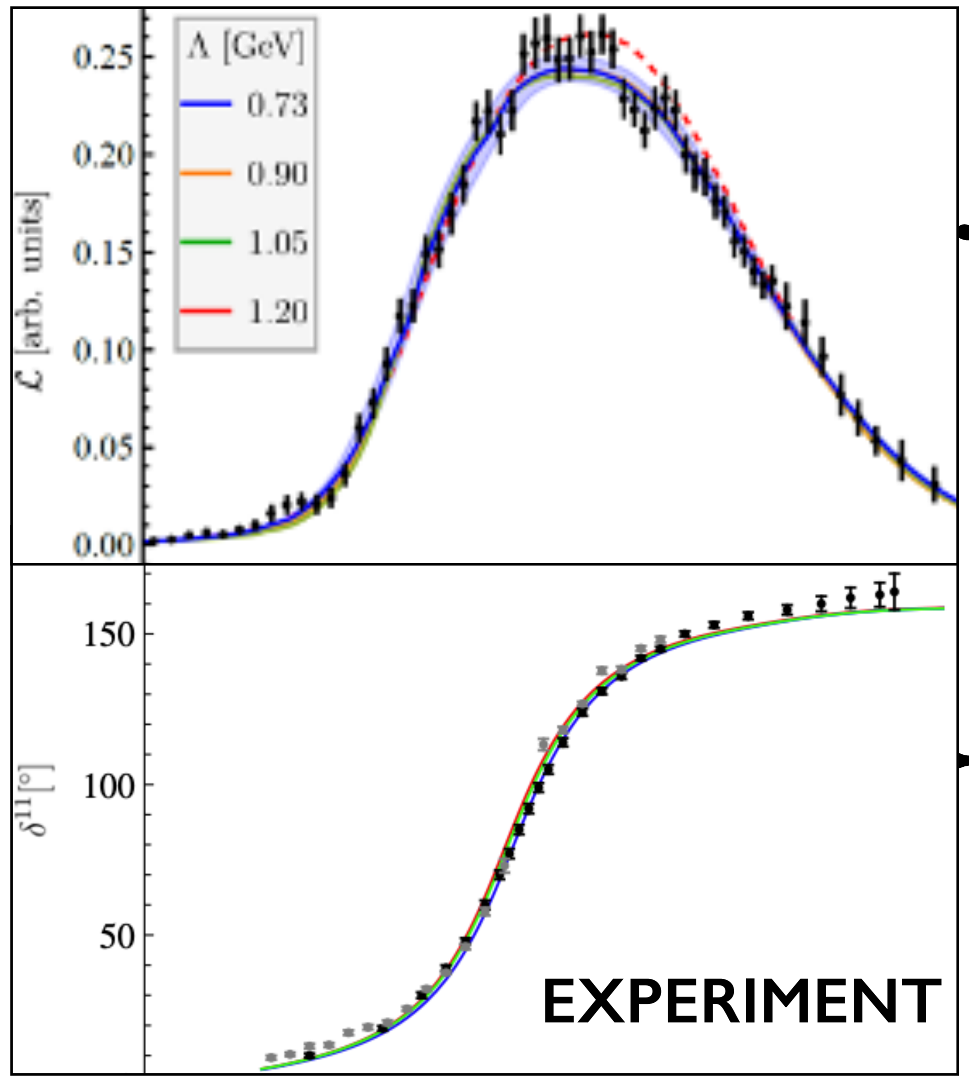




[1] Du et al. Phys.Rev.Lett. 131 (2023) 13; Hansen et al. 2401.06609 [hep-lat]
[2] Korpa/Lutz/Guo/Heo Phys.Rev.D 107 (2023) 3; Isken et al. 2309.09695; ... Ketzer/Mikhashenko/Aceti/Dai/Oset/Bayar/Guo...
[3] Sakhtivasan/MM in preparation

BLUEPRINT — $a_1(1260)$

INPUT[1]



TRANSITION AMPLITUDES

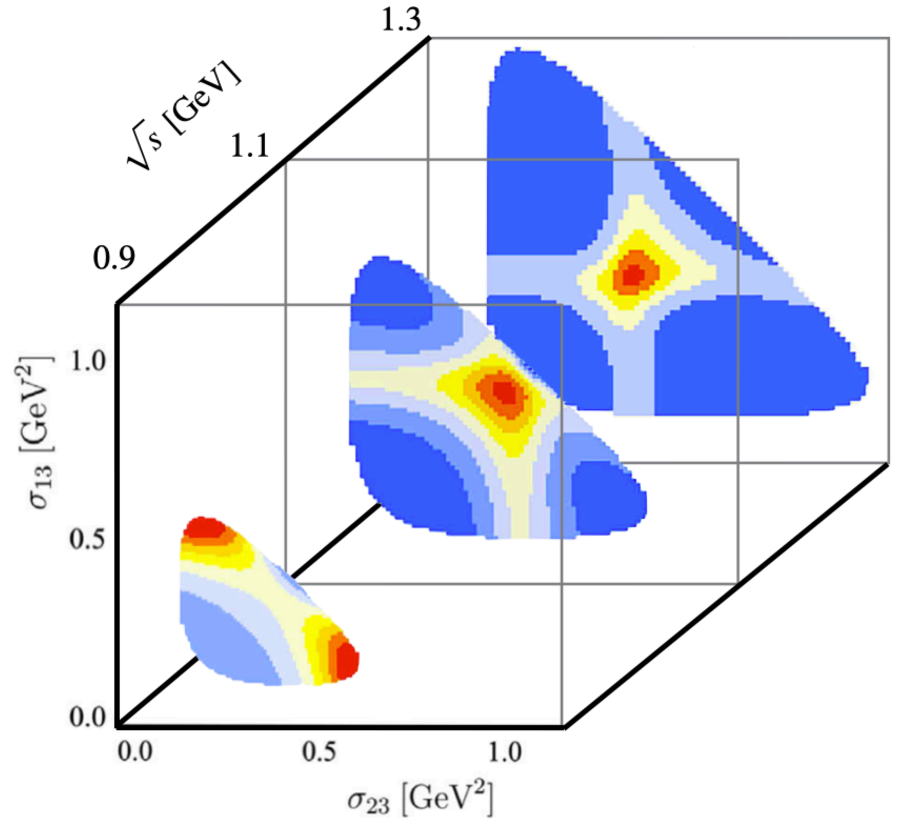
IVU

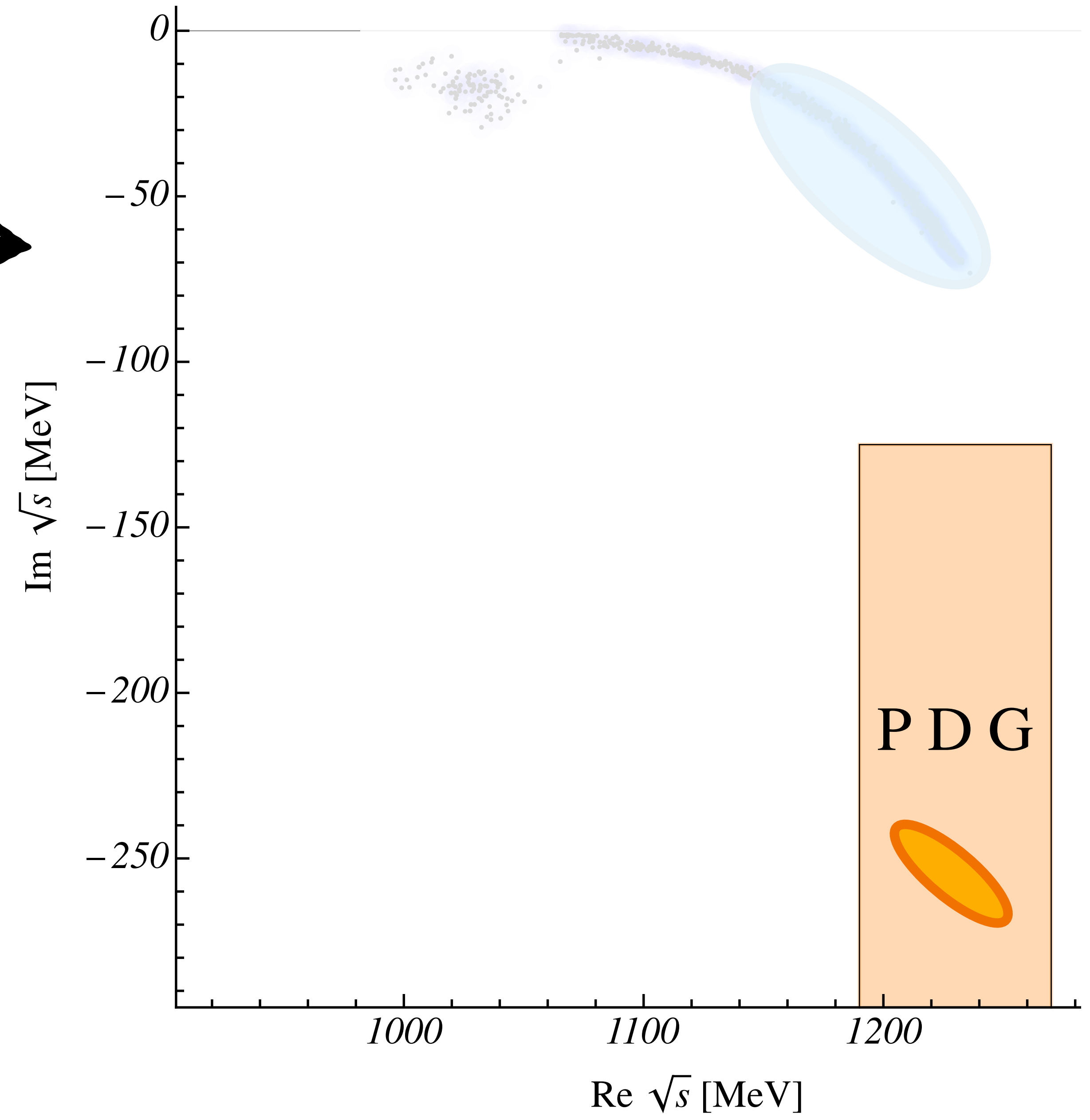
$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$

FVU

$$\det\left[2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1} - \Sigma_2^L\right) - B - C\right]^{T_{1g}}$$

OUTPUT[2]

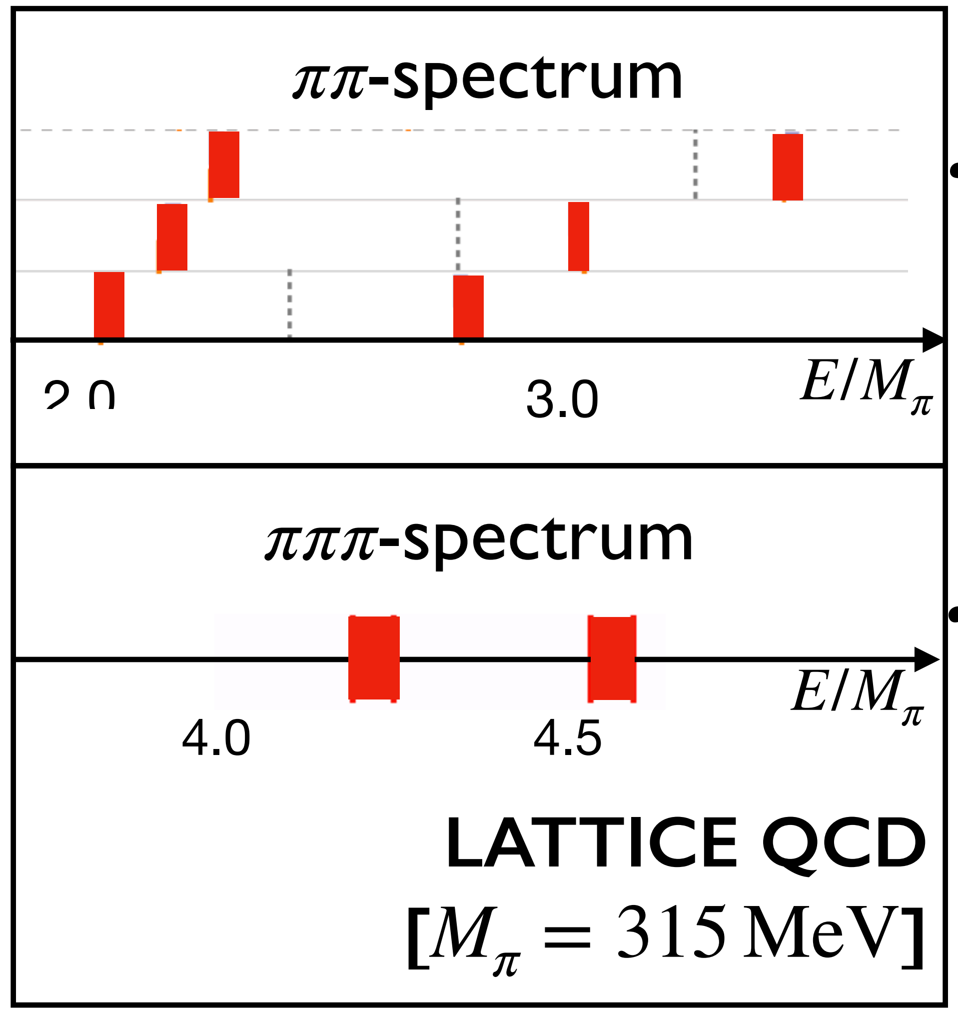




[1] Schael [ALEPH] Phys.Rept. 421 (2005); Nucl.Phys.B 79; Phys.Rev.D 7; [GWQCD] PRD94(2016) PRD98 (2018) PRD 100(2019)
[2] Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020); MM/Culver/Sadasivan/Brett/Döring/Alexandru/Lee [GWQCD] PRL 127 (2021)
other phenomenological determinations: JPAC/....

BLUEPRINT — $a_1(1260)$

INPUT[1]



TRANSITION AMPLITUDES

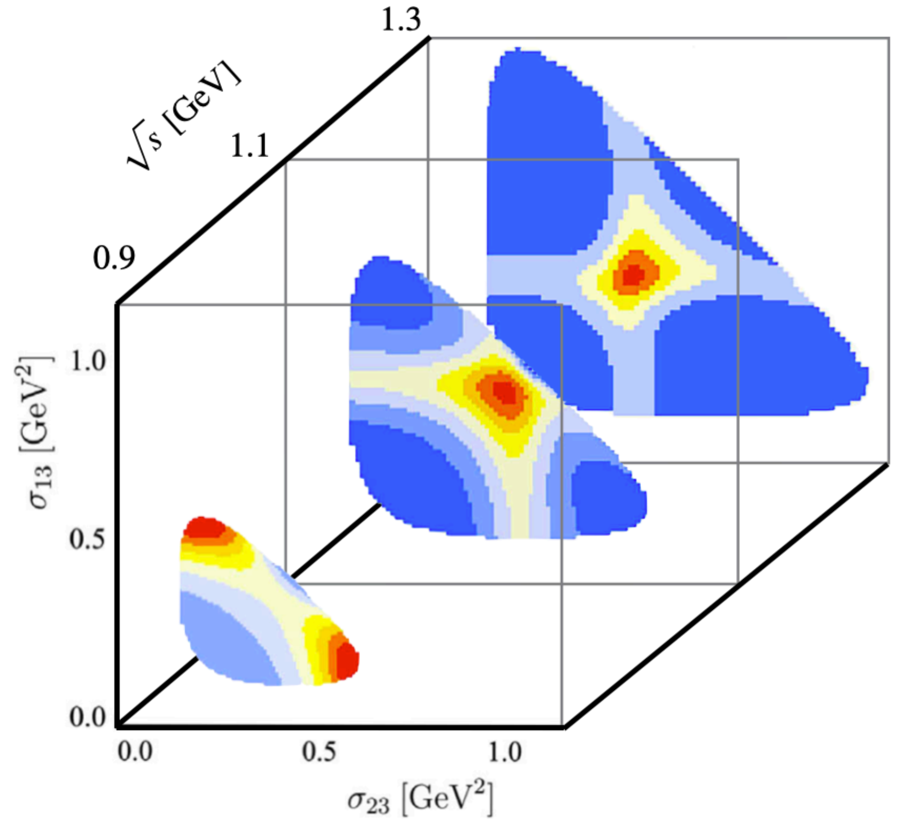
IVU

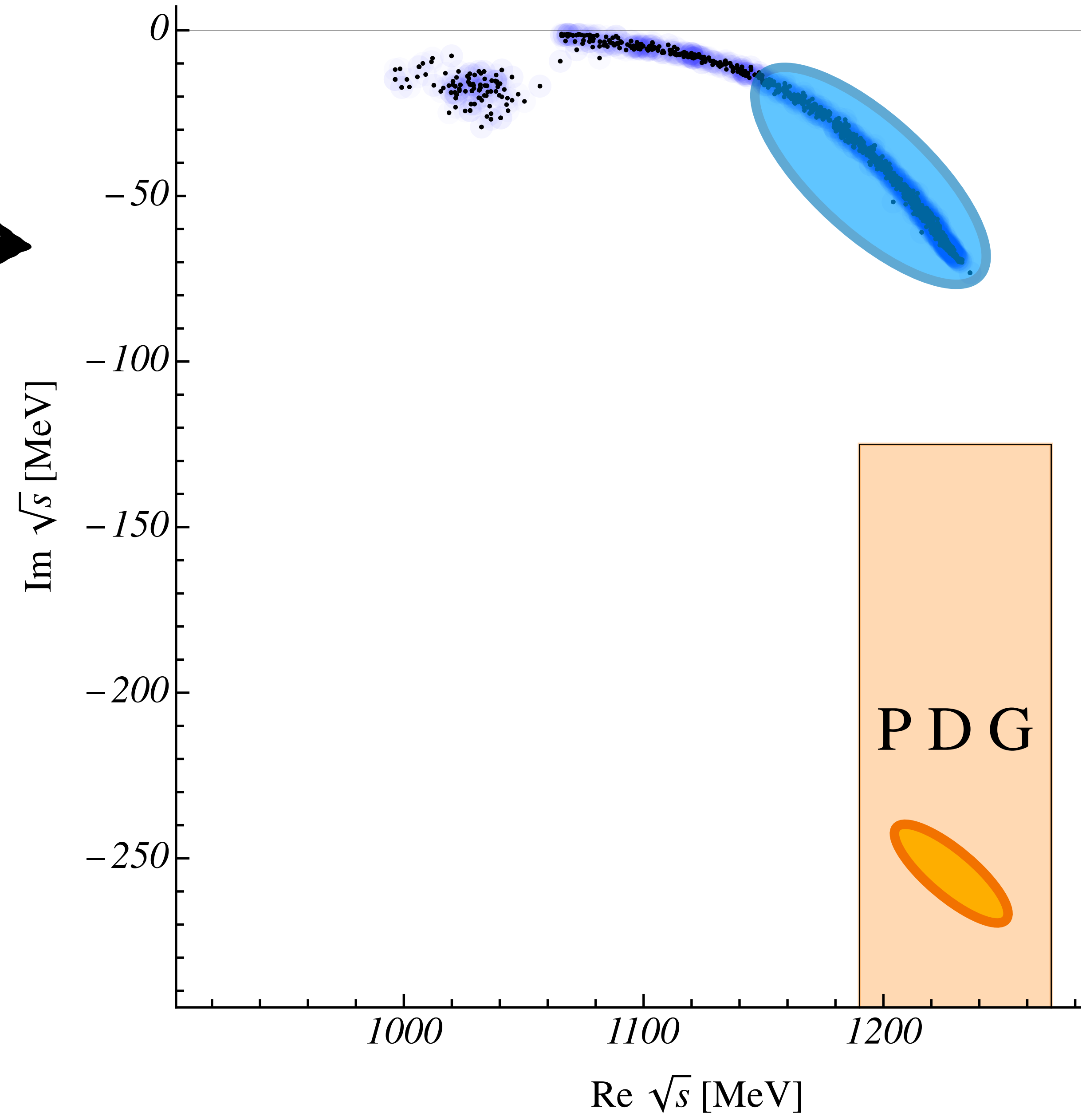
$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$

FVU

$$\det\left[\, 2L^3 E_{\mathbf{p}} \left(\tilde{K}_2^{-1} - \Sigma_2^L\right) - B - C \,\right]^{T_{1g}}$$

OUTPUT[2]

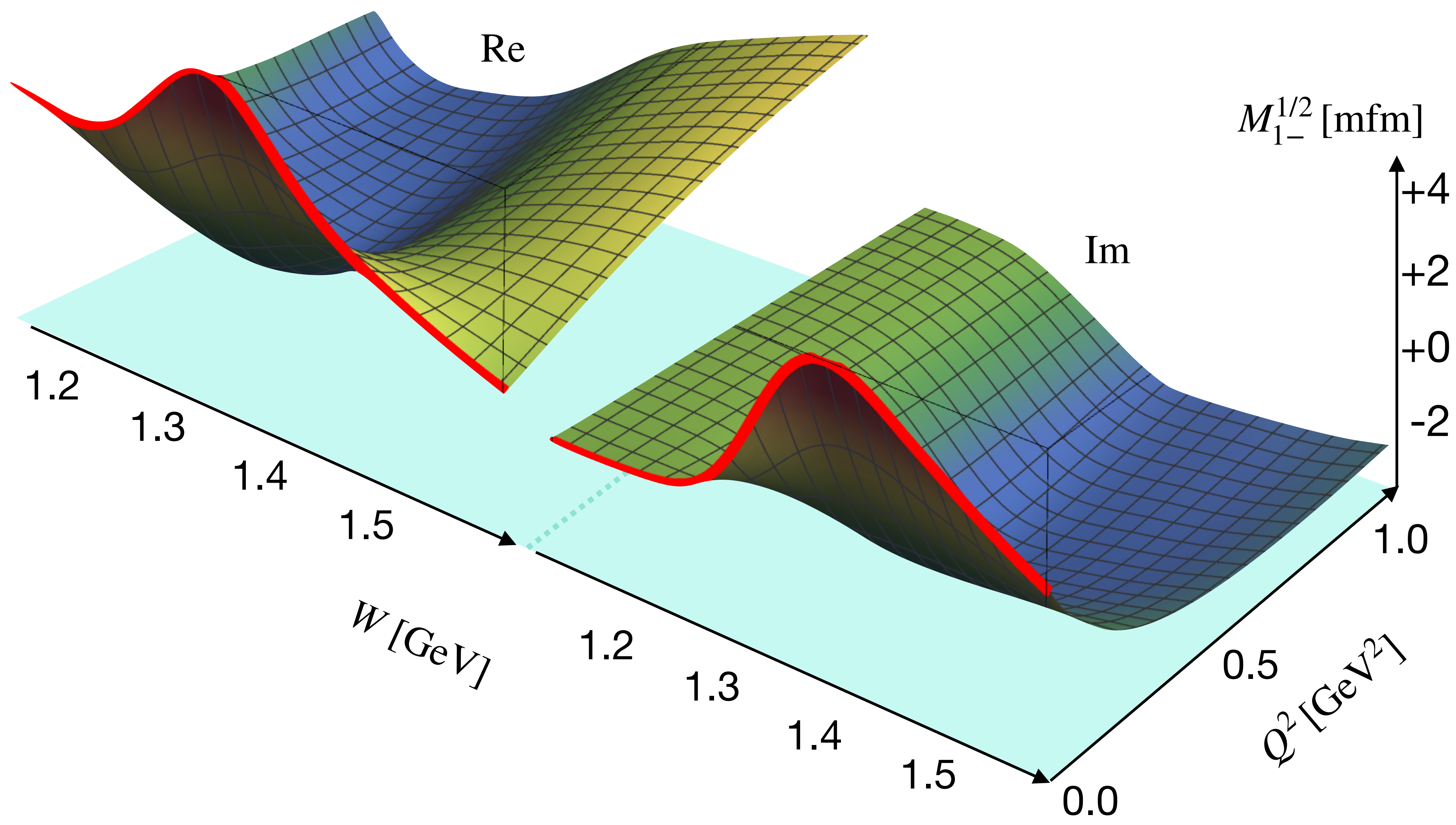


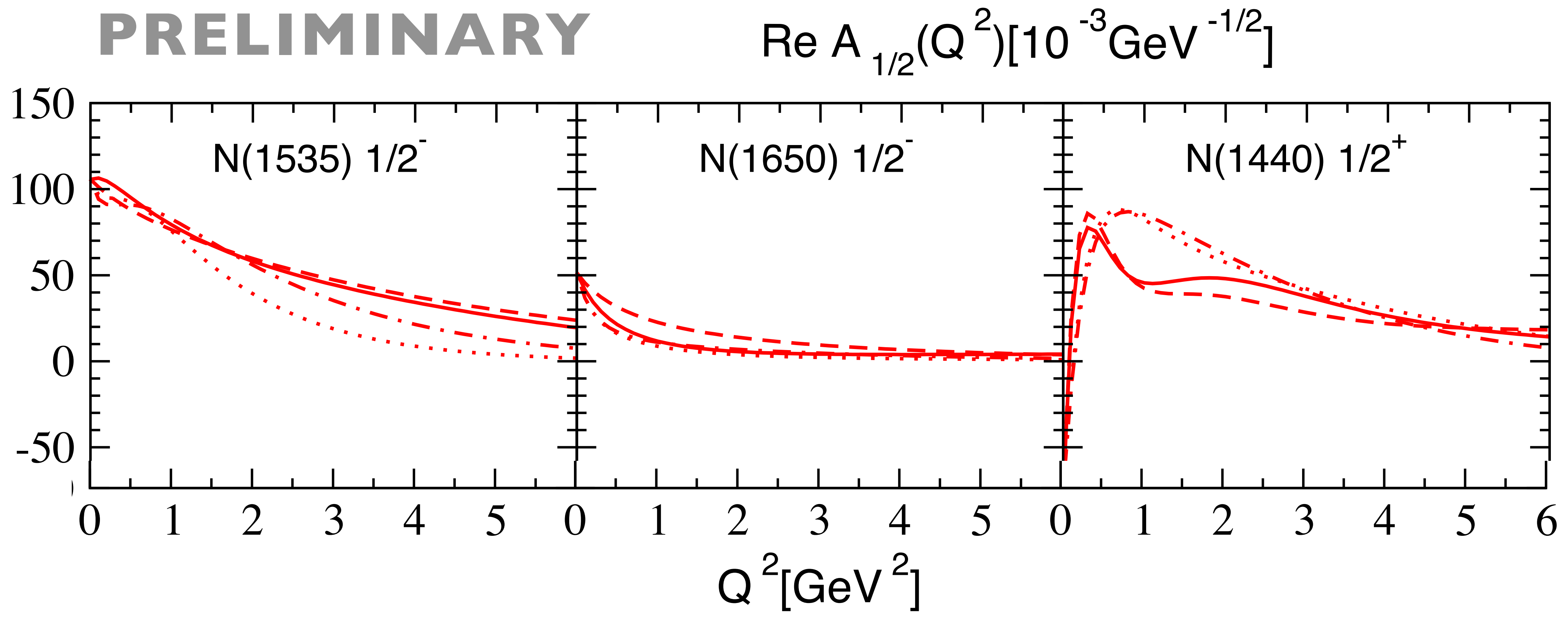


[1] Schael [ALEPH] Phys.Rept. 421 (2005); Nucl.Phys.B 79; Phys.Rev.D 7; [GWQCD] PRD94(2016) PRD98 (2018) PRD 100(2019)
[2] Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020); MM/Culver/Sadasivan/Brett/Döring/Alexandru/Lee [GWQCD] PRL 127 (2021)
other phenomenological determinations: JPAC/....

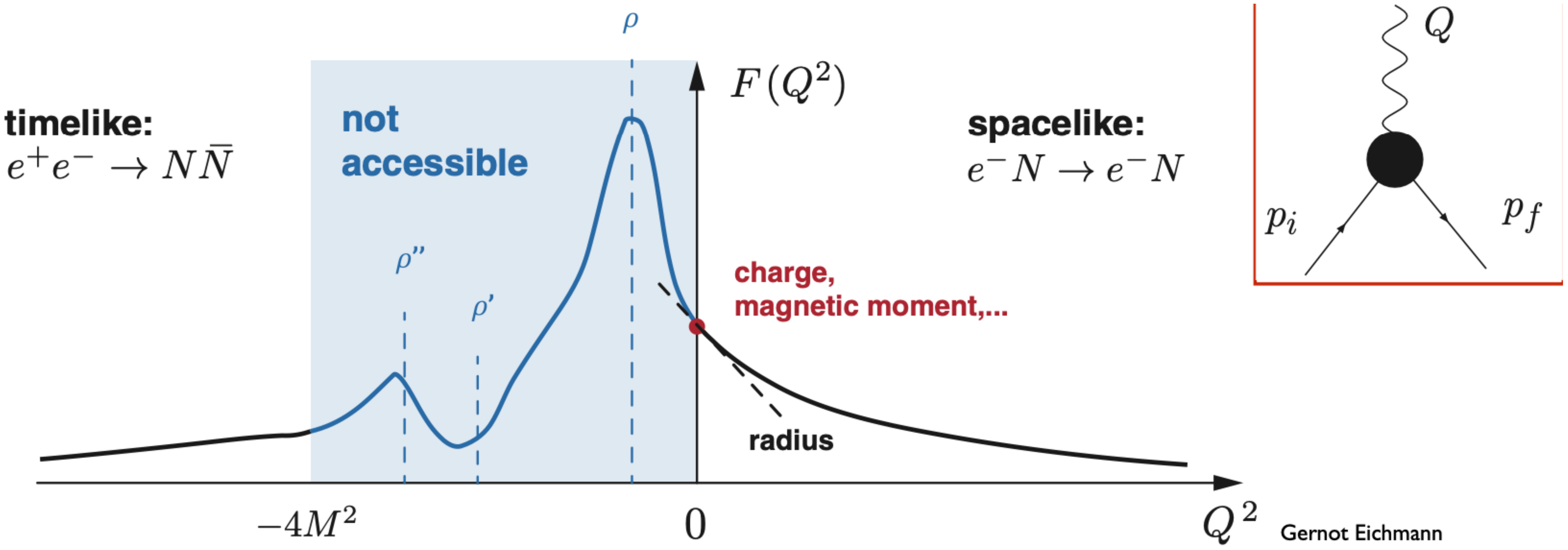
Global analysis (bird's view)

- many experimental data & ongoing experiments
  - @MAMI, @ELSA, @JLAB,…
  - $\gamma N \to \pi(\pi)N, \eta N, K\Lambda\ldots$
- Jülich-Bonn-Washington[1,2] DCC
  - → Roper has very unusual $f(W, Q^2)$: $\pi\pi N$ effect(?)
  - → Transition form-factors[3]



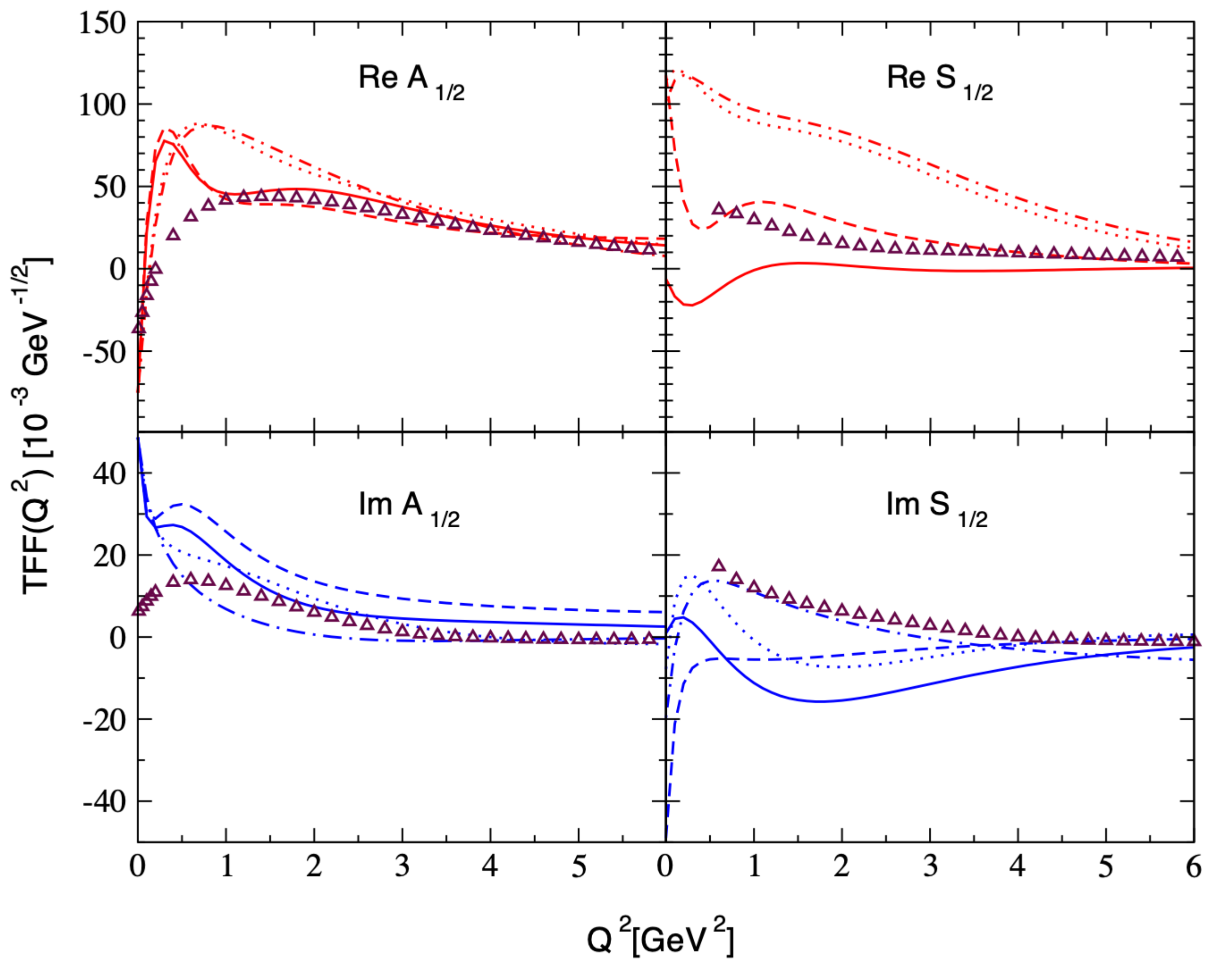


[1] [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201; Eur.Phys.J.A 59 (2023) 12; jbw.phys.gwu.edu/
[2] Related approaches MAID/SAID/Gent/ANLOsaka
[3] Wang/MM/… in progress



timelike:
$e^+e^- \to N\bar{N}$
not
accessible
ρ
ρ''
ρ'
$F(Q^2)$
spacelike:
$e^-N \to e^-N$
charge,
magnetic moment,...
radius
$-4M^2$
0
Q^2
Gernot Eichmann
Q
p_i
p_f

PRELIMINARY



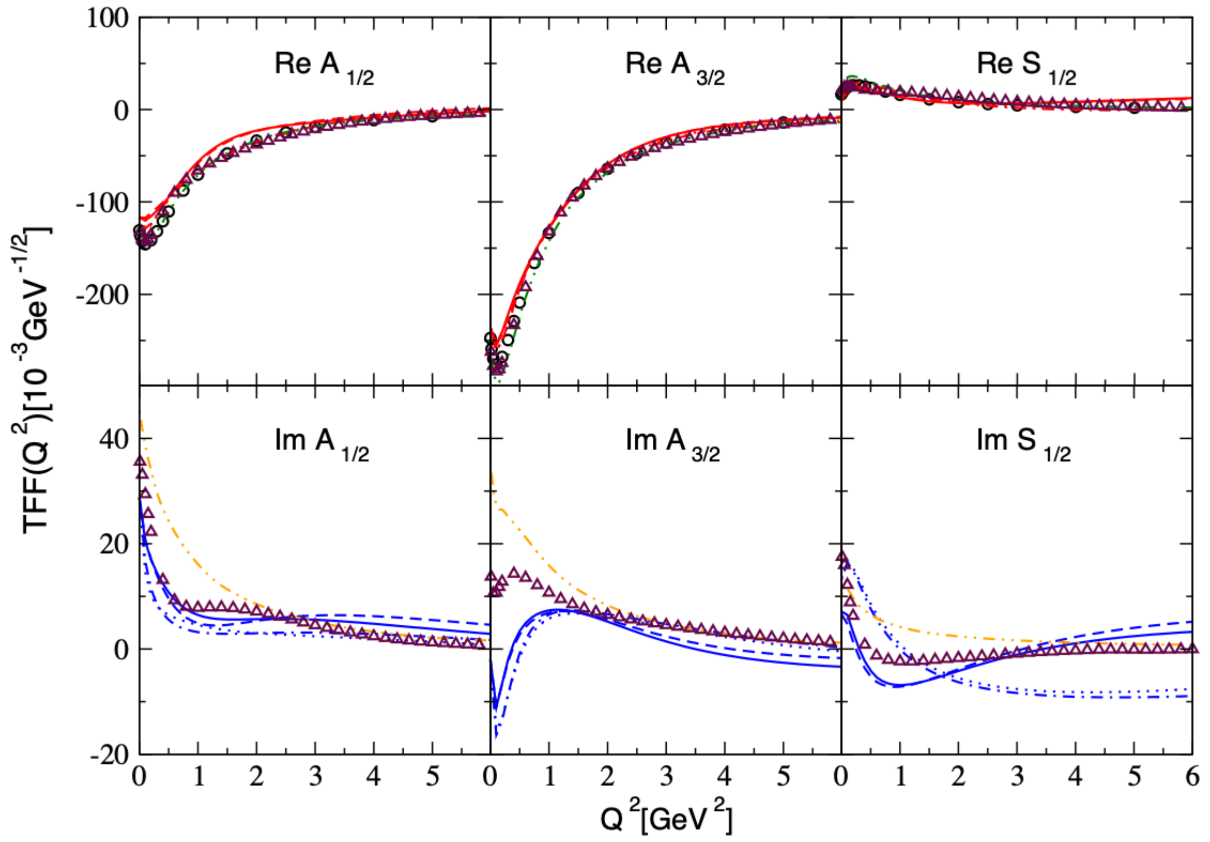
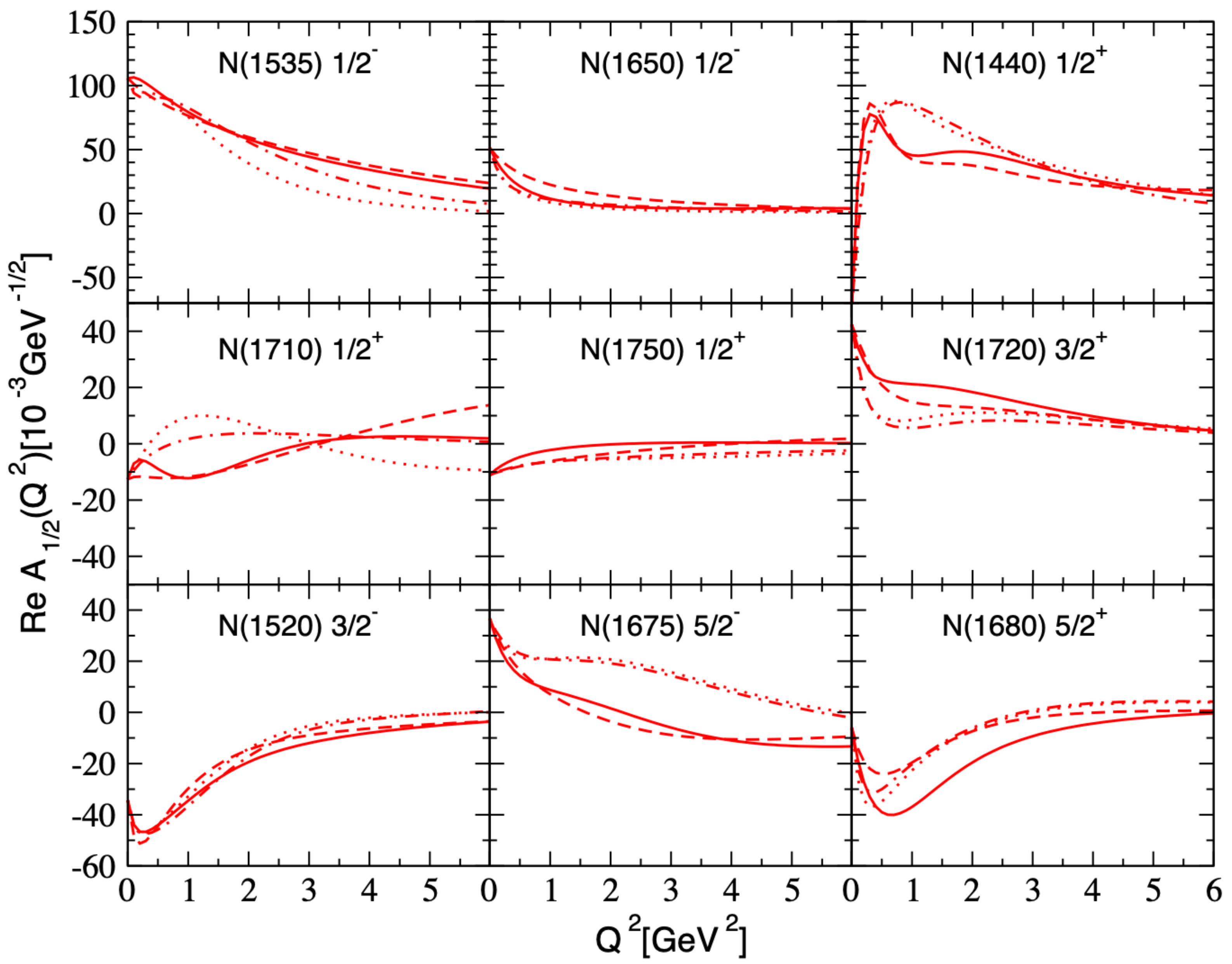
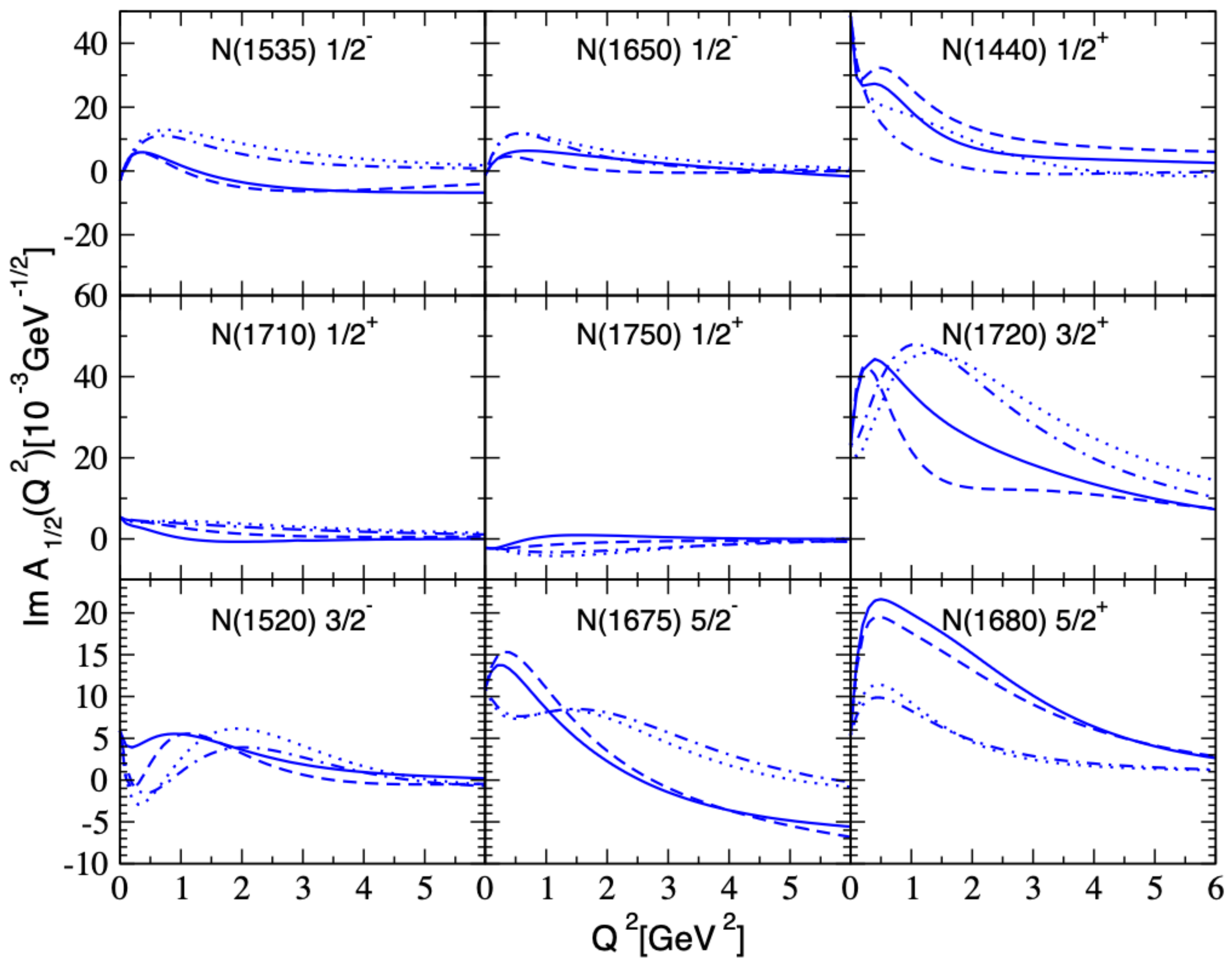


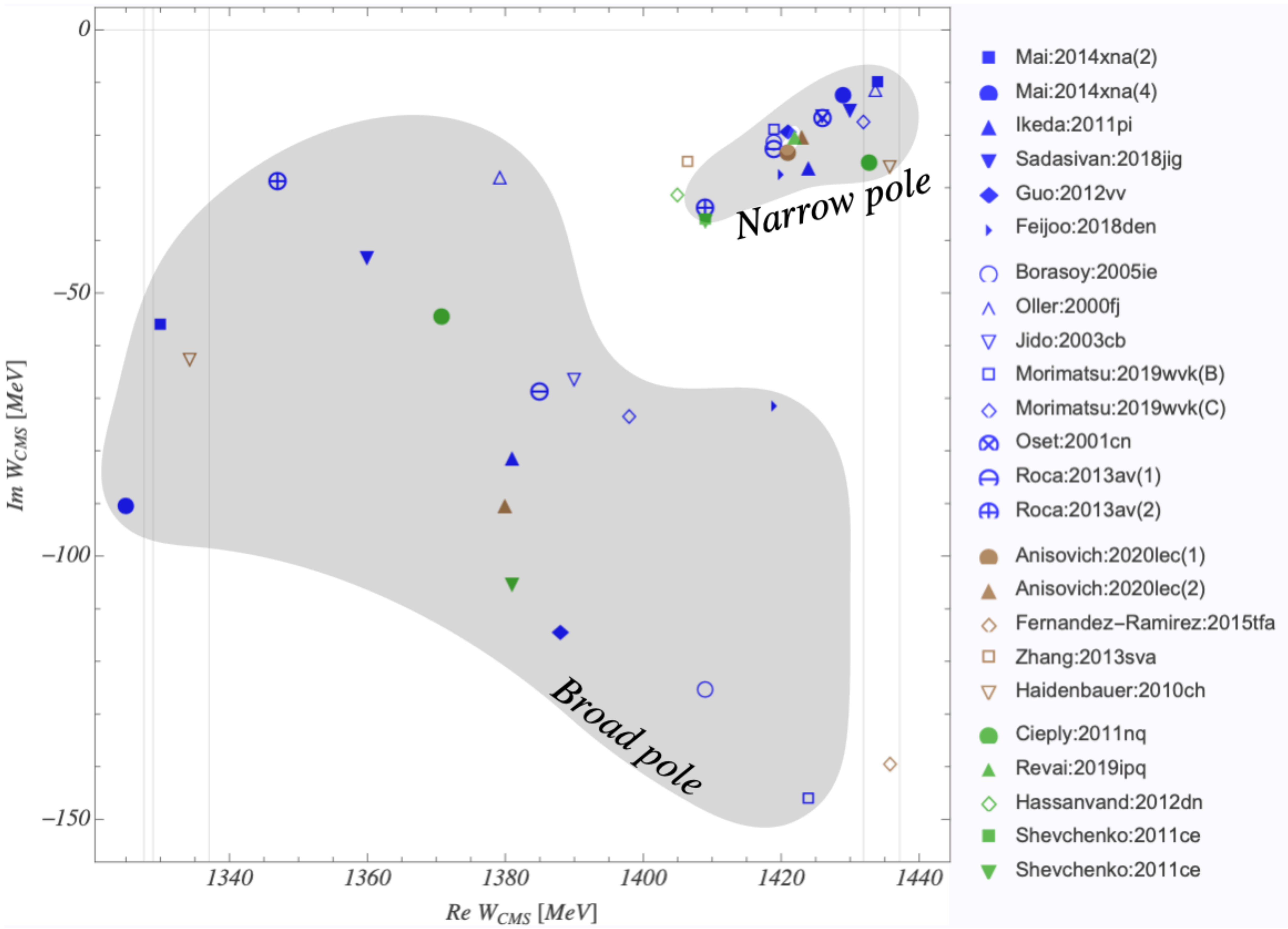
FIG. 1: The TFFs of $\Delta(1232)$. Solid, dashed, dotted, dash-dotted lines: results of this work, corresponding to fits 1 to 4 in Ref. [34]. Double-dotted lines: results from Ref. [30] based on the MAID results. Black circles: MAID results from the unitary isobar model (real-valued). Triangles: preliminary results of the ANL-Osaka model.



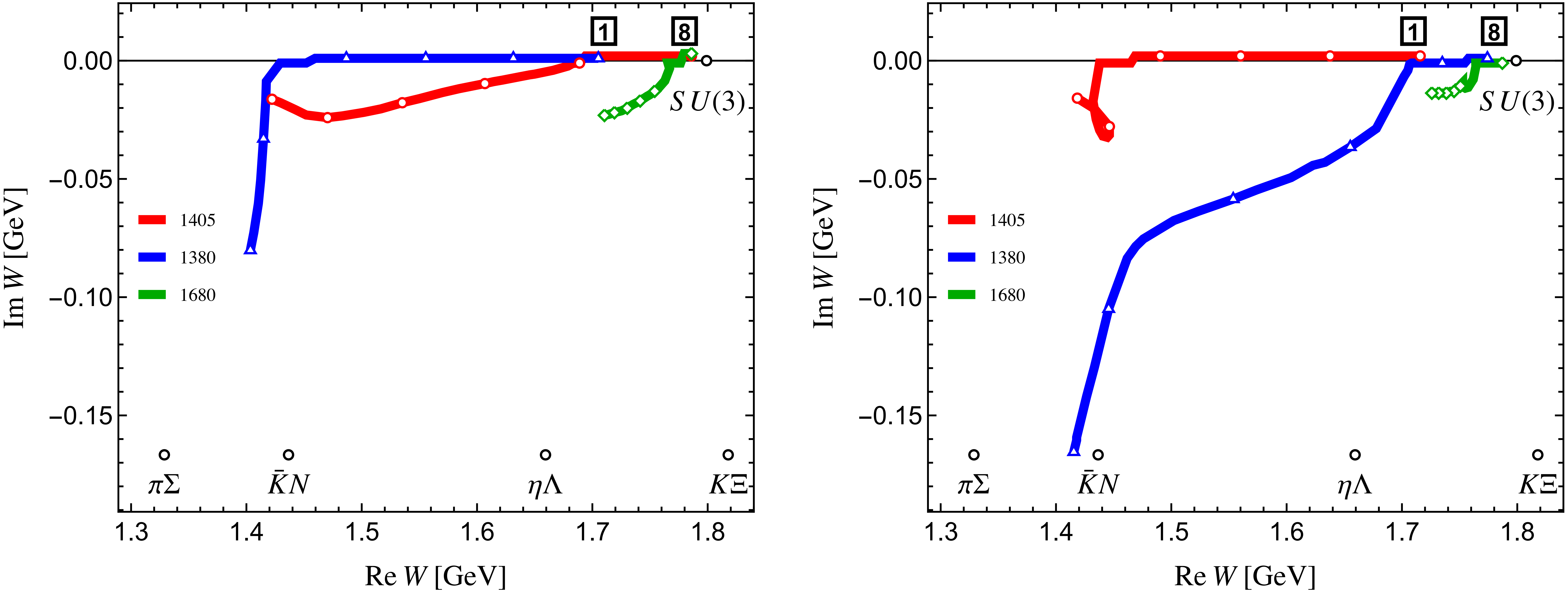


…

- Theory frontier: NNLO UCHPT determination[1]
- Consistently two poles, but the second pole is less well known
  - second pole below KbarN threshold
  - line-shape only through $\gamma p \to K\pi\Sigma$ [2]



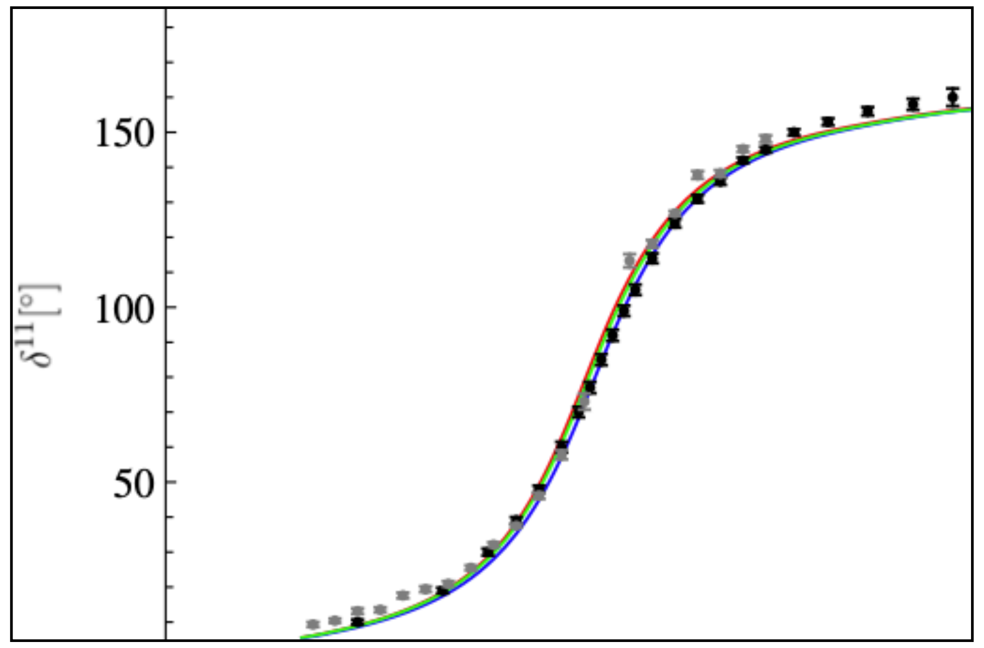
[1] Lu/Geng/Döring/MM Phys.Rev.Lett. 130 (2023)
[2] [CLAS] Moriya et al (2013)

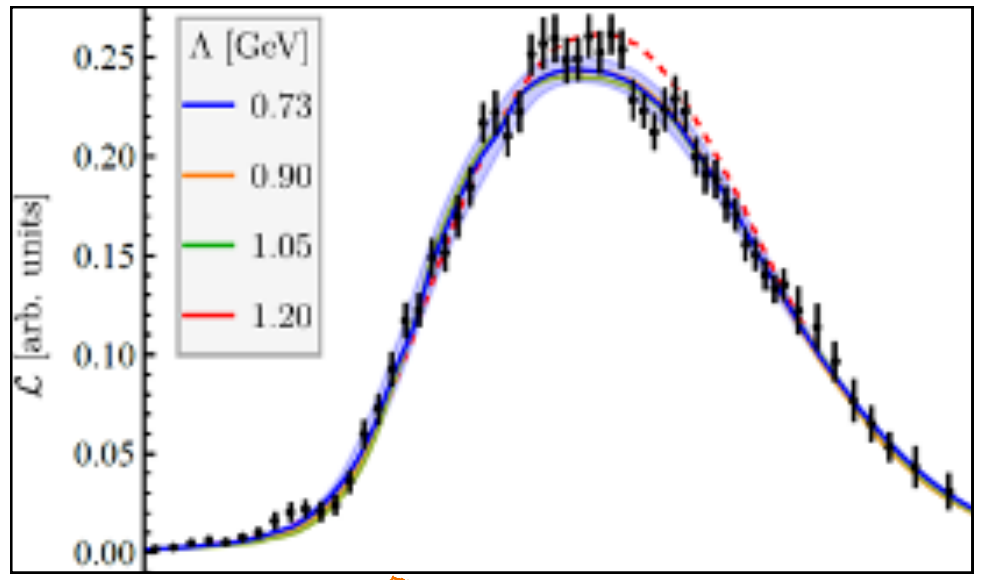


Im W [GeV]
Re W [GeV]
1405
1380
1680
1
8
SU(3)
πΣ
K̄N
ηΛ
KΞ
0.00
−0.05
−0.10
−0.15
1.3
1.4
1.5
1.6
1.7
1.8

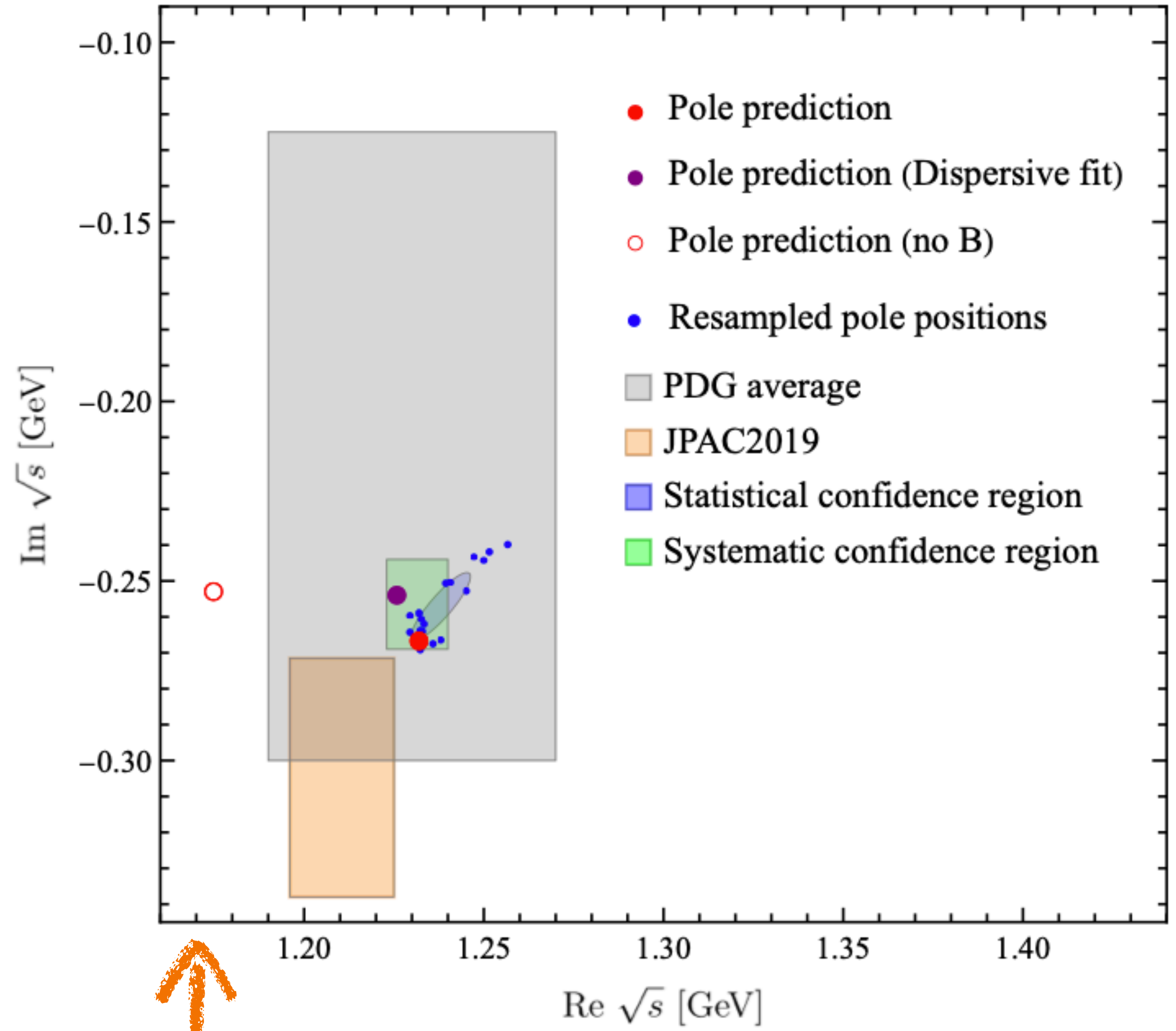
APPLICATION: $a_1(1260)$

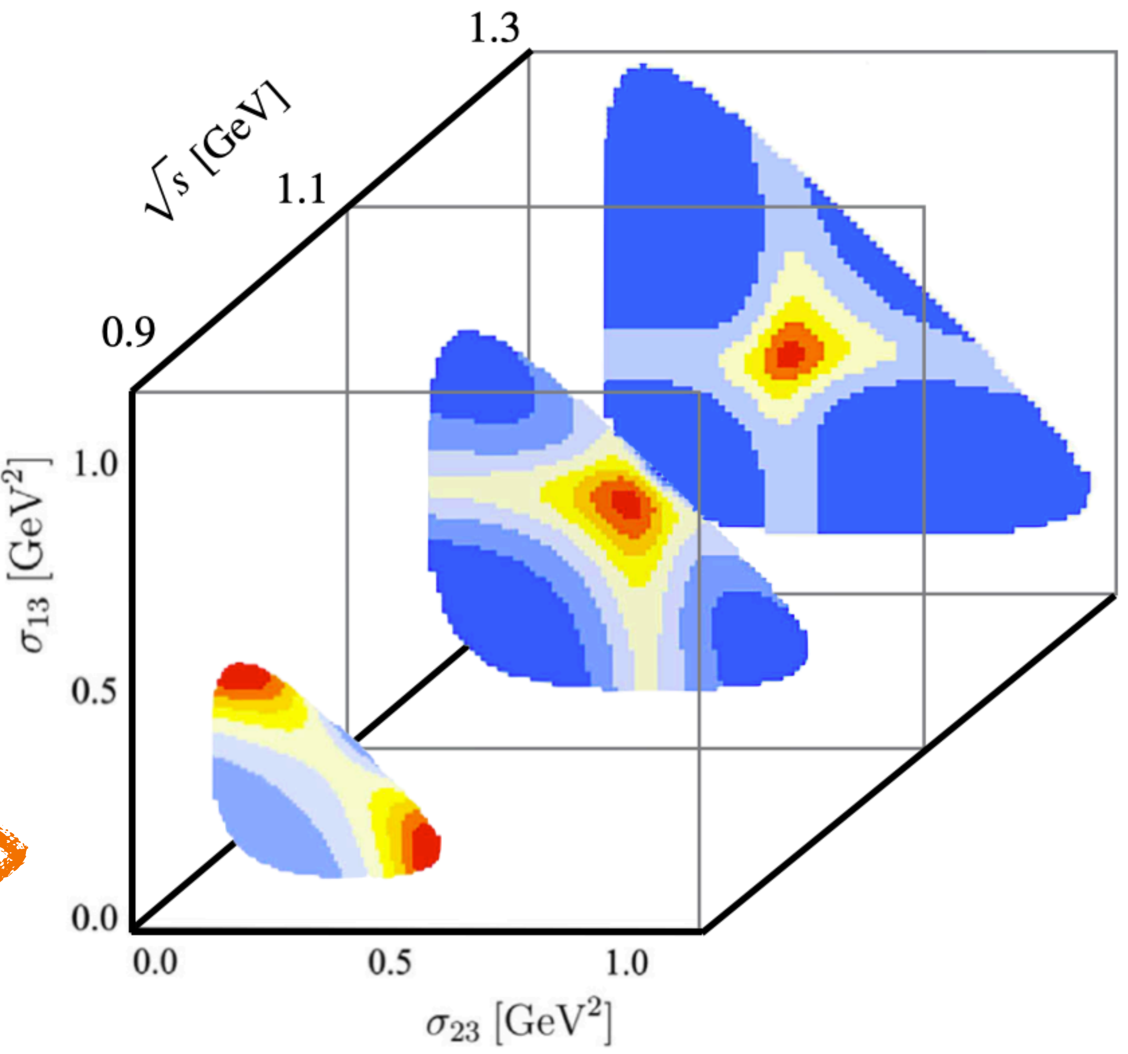
- $\pi\rho$ dynamics dominates the $1^-(1^{++})$ system
- Integral equation solved
  - Helicity formalism
  - complex momentum mapping
- $\pi\rho/\pi\sigma/\pi(\pi\pi)_2$ extended…





$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\,\frac{(B+C)}{2E_l}\,\frac{1}{\tilde{K}_n^{-1}-\Sigma_n}\,T^c$$



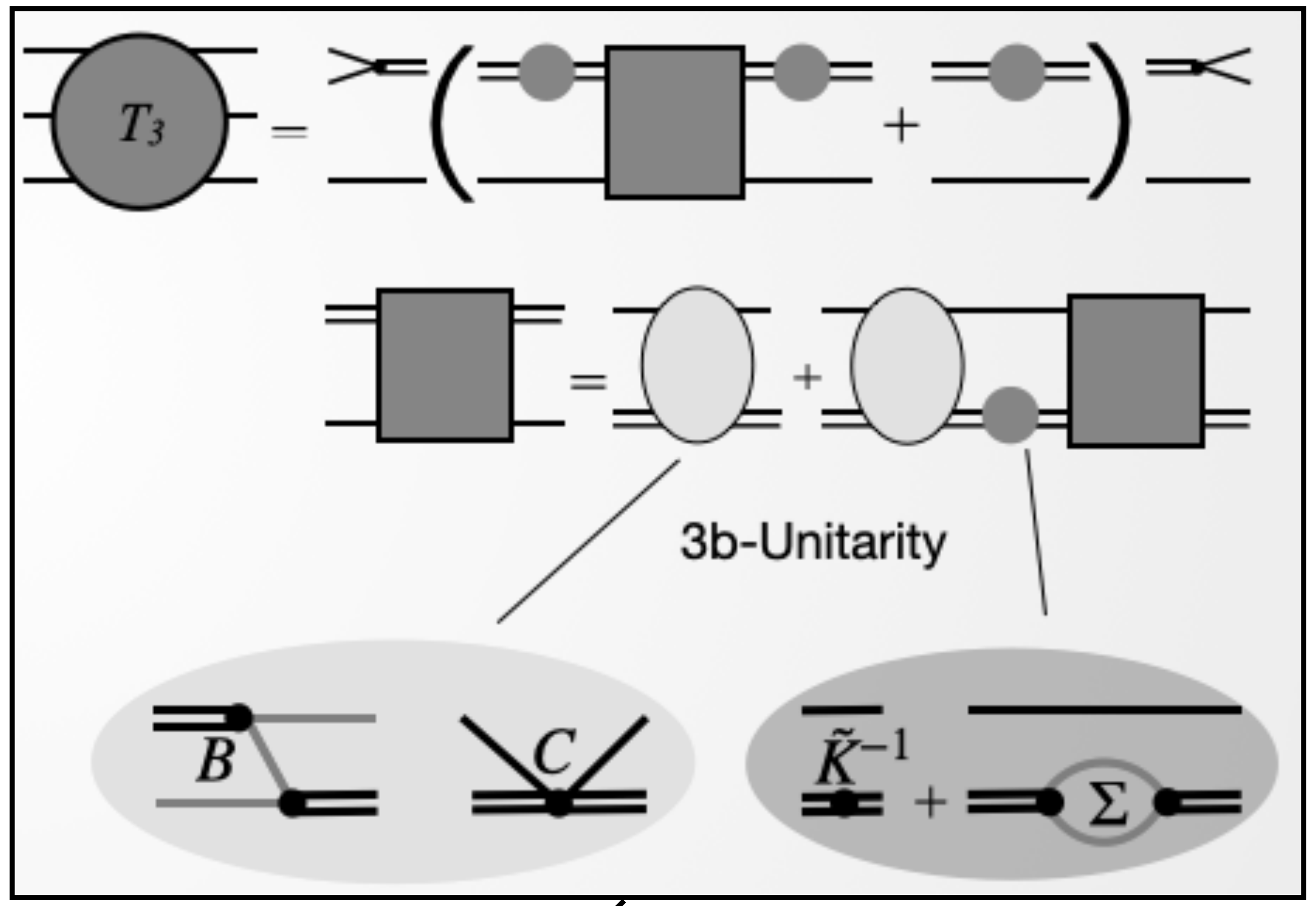


Data: Schael [ALEPH] Phys.Rept. 421 (2005); Nucl.Phys.B 79; Phys.Rev.D 7;
Pole extraction: Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020)

3-BODY QUANTIZATION CONDITION (FVU)

- **Finite-volume unitarity (FVU[1])**
  - heavily simplified:
    - on-shell particle-configurations: $\mathbf{\Delta}E \sim mL$
    - off-shell particle-configurations: $\mathbf{\Delta}E \sim e^{-mL}$
  - *<u>Unitary</u> 3-body amplitude separates these effects*
  - unknown volume independent quantities (K, C)

reminder

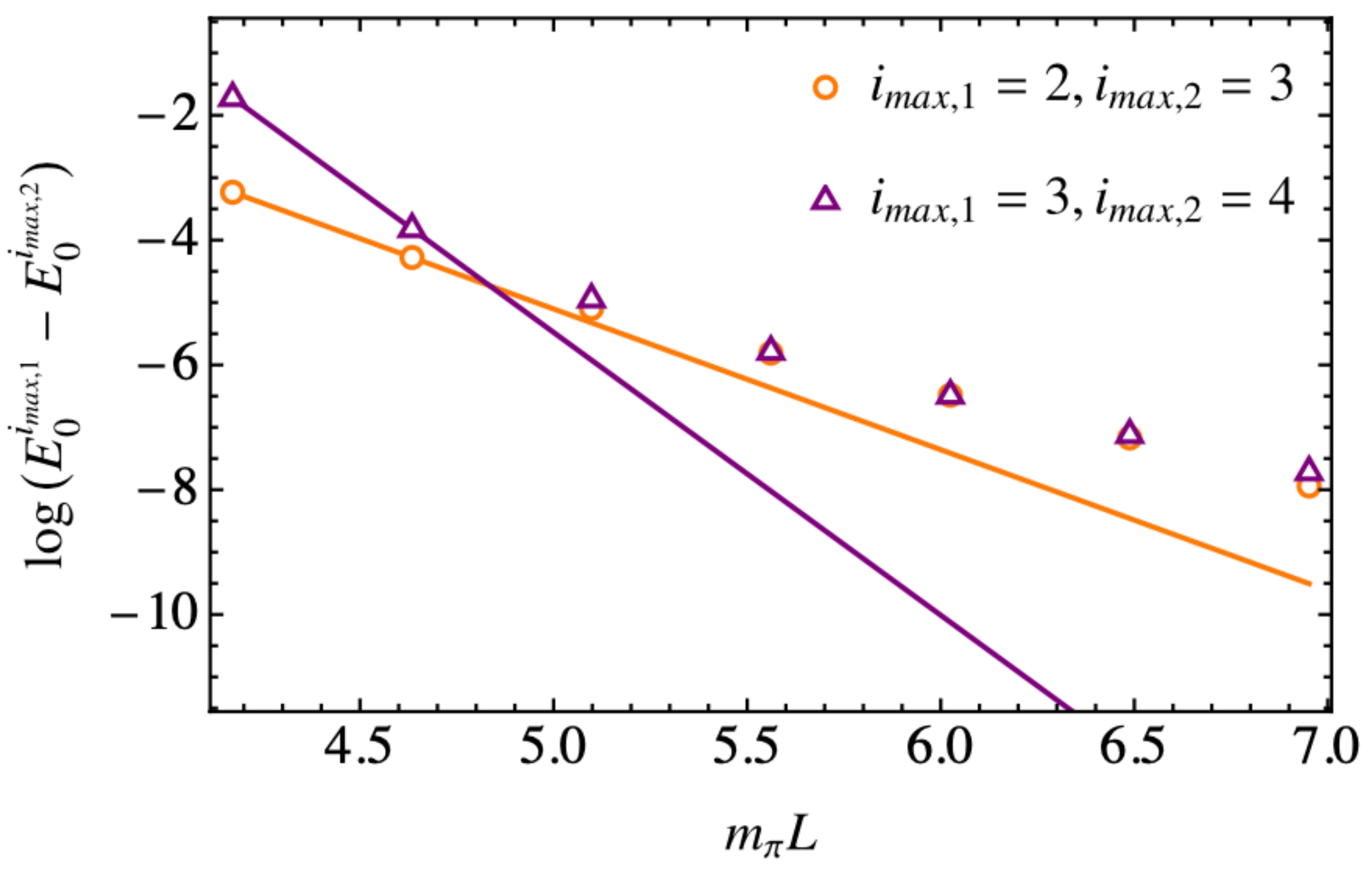


$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

[1] MM/Döring Phys.Rev.Lett. 122 (2019) 6
Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);

CUTOFF DEPENDENCE[1]

Consider fixed C, K then increase <u>hard</u> cutoff



- 3-body amplitude = genuine integral equation
  - spectator can carry arbitrary momentum away
  - cutoff required (form factors, hard cutoff,…)

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

$$B(\sqrt{s}) = \frac{1}{\sqrt{s} - \sqrt{s_{\rm on}} + i\epsilon}$$

- energy eigenvalues change slower than $\mathbf{\Delta}E \sim e^{-mL}$
- one-particle exchange falls off not rapidly enough

[1] paper in preparation

CUTOFF DEPENDENCE[1]

- 3-body amplitude = genuine integral equation
  - spectator can carry arbitrary momentum away
  - cutoff required (form factors, hard cutoff,…)

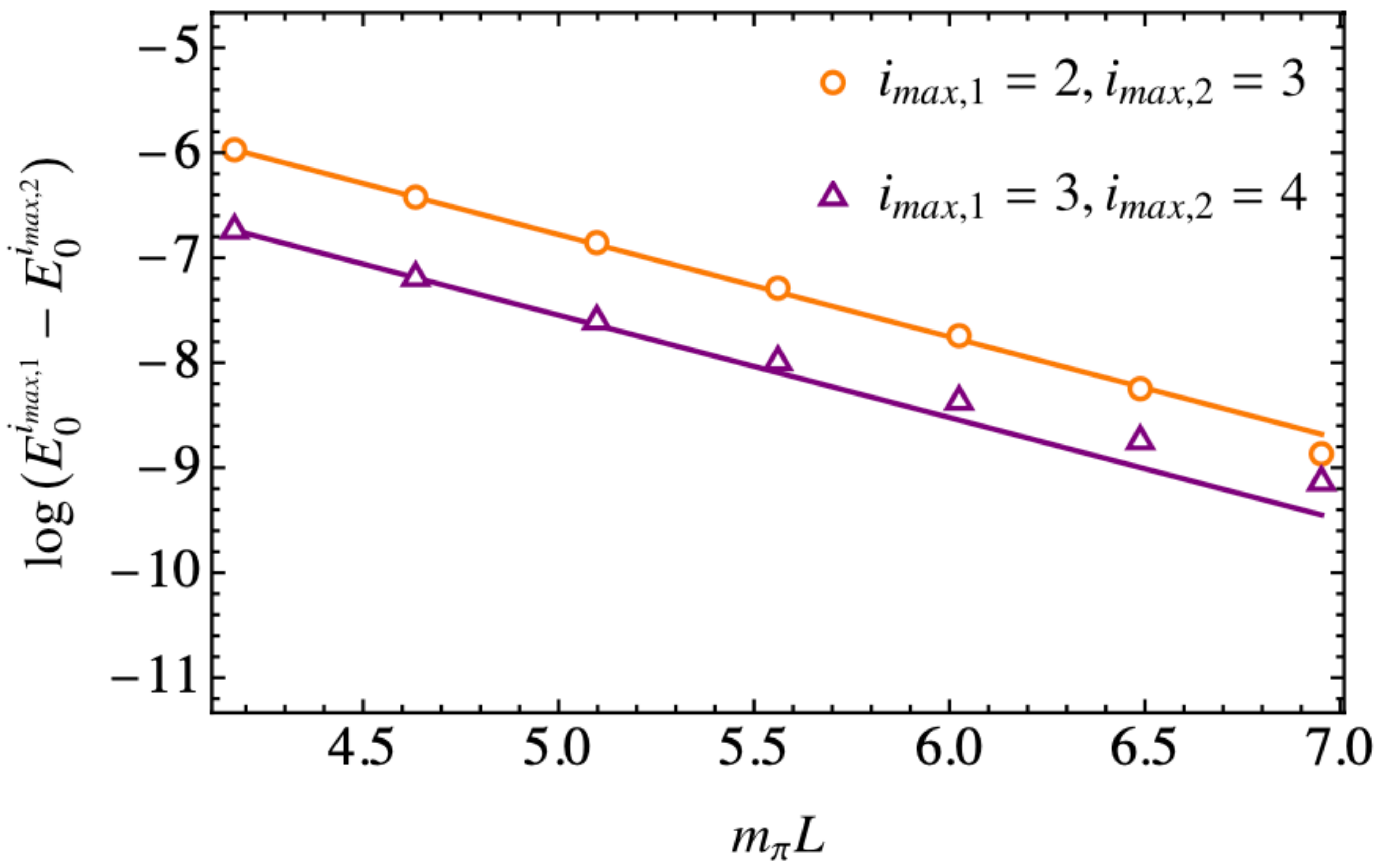
$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

$$B(\sqrt{s}) = \frac{1}{\sqrt{s} - \sqrt{s_{\rm on}} + i\epsilon}$$

Consider fixed C, K then increase <u>hard</u> cutoff

… over-subtract OPE

$$\overline{B}(\sqrt{s}) = B(0) + B'(0)\sqrt{s} + \frac{s}{s_{\rm on}}\frac{N}{2E_{p+p'}}\frac{1}{\sqrt{s} - \sqrt{s_{\rm on}} + i\epsilon}$$

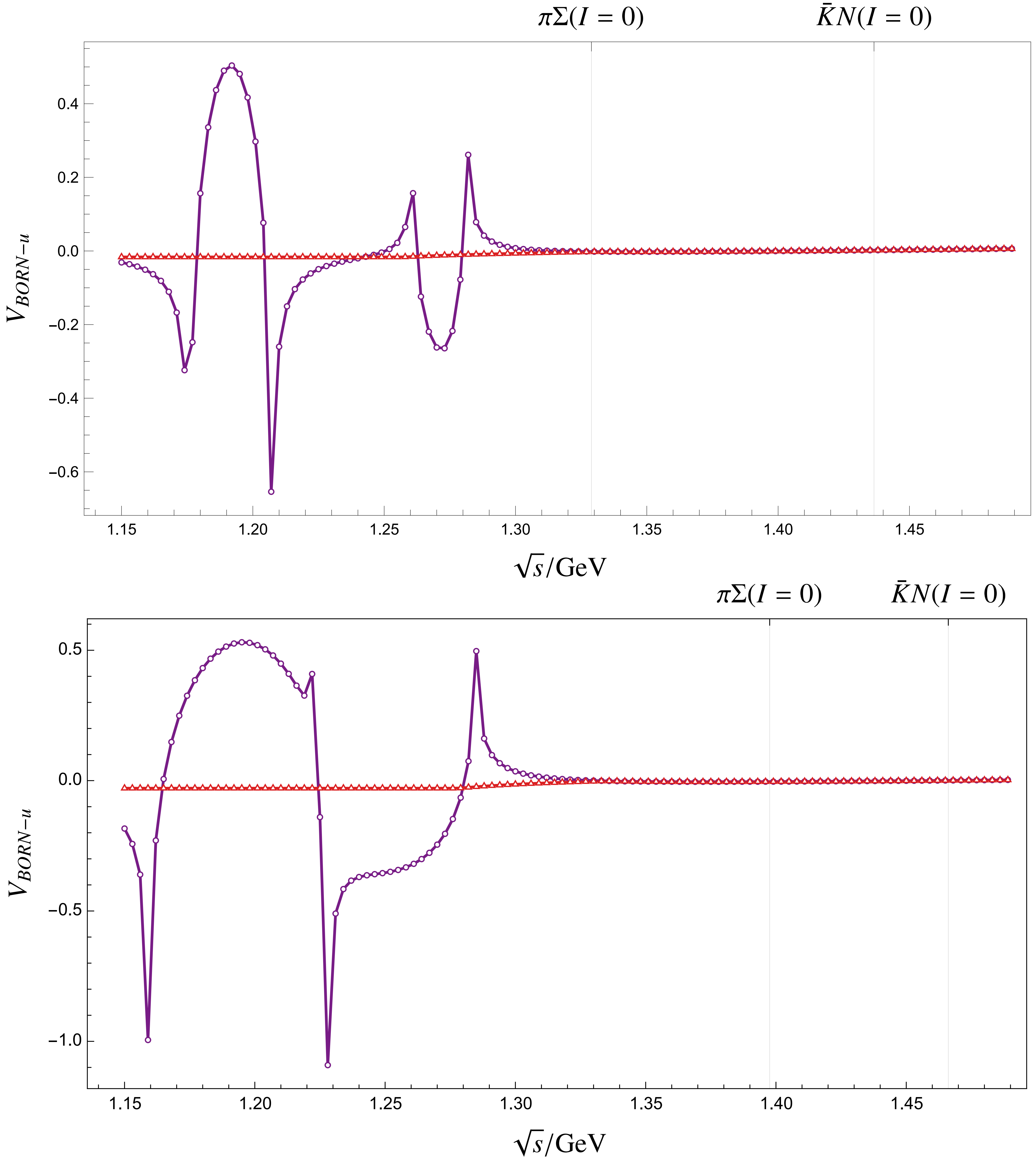


- energy eigenvalues change as $\mathbf{\Delta}E \sim e^{-mL}$

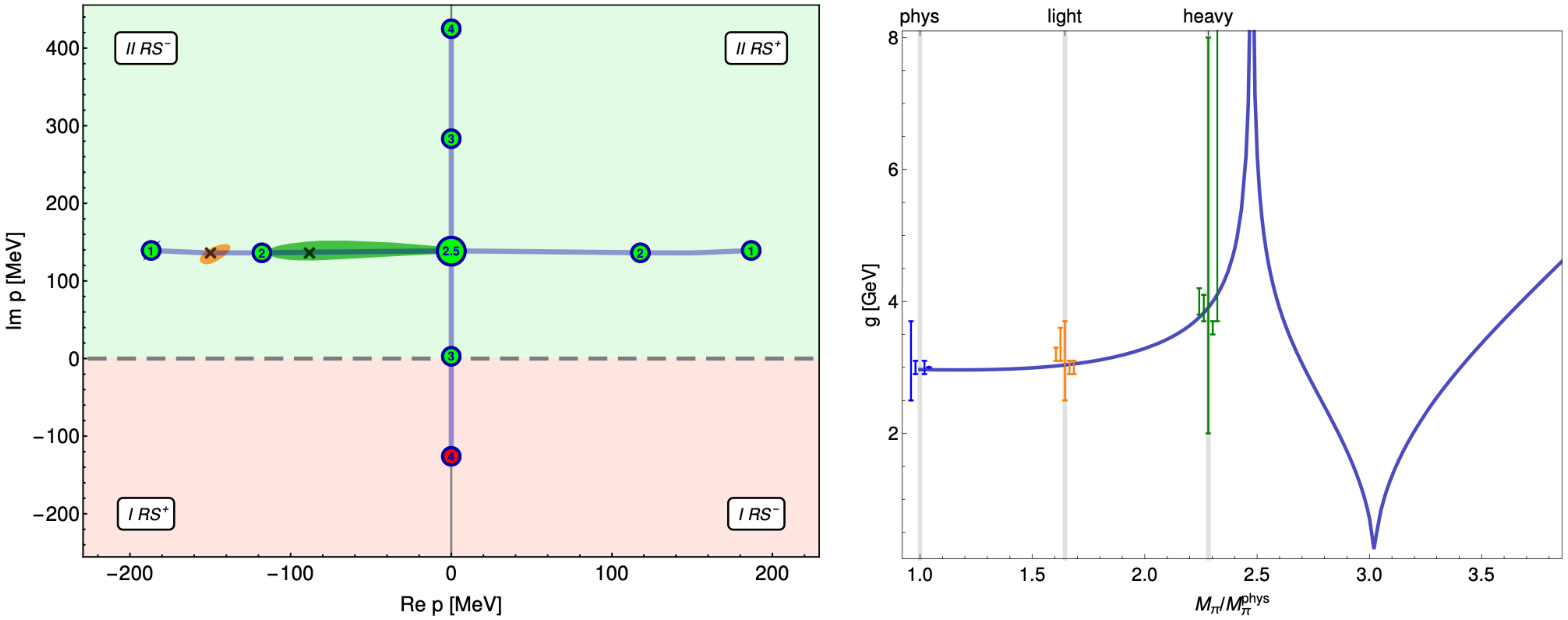
[1] paper in preparation

U-CHANNEL IN THE Λ(1405)

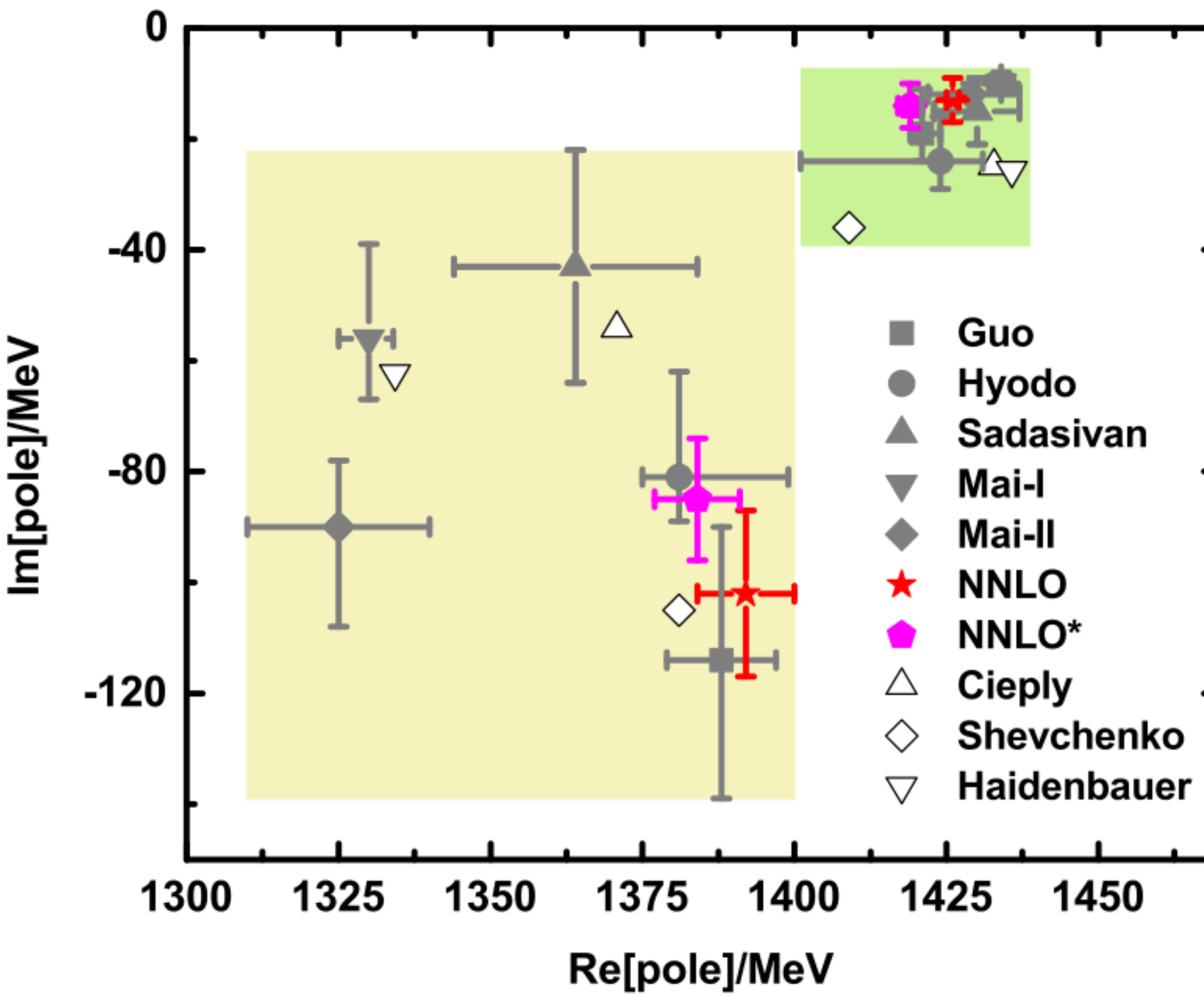
- New insights[1] from LQCD [next talk]
  - confirming two-pole scenario
- Chiral extrapolations (through UCHPT)[2]
  - u-channel baryon exchange may complicate the picture (3-body)
  - sub-leading effect



[1] [BaSc] Bulava et al. 2307.10413; 2307.13471
[2] Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)



II RS⁻
II RS⁺
I RS⁺
I RS⁻
Im p [MeV]
Re p [MeV]
phys
light
heavy
g [GeV]
$M_\pi/M_\pi^{\rm phys}$



0
-40
-80
-120
Im[pole]/MeV
1300
1325
1350
1375
1400
1425
1450
Re[pole]/MeV
Guo
Hyodo
Sadasivan
Mai-I
Mai-II
NNLO
NNLO*
Cieply
Shevchenko
Haidenbauer

$$\{1, 8_s, 8_a, 10, \overline{10}, 27\}$$

$$\begin{pmatrix} |\pi\Sigma\rangle \\ |\bar{K}N\rangle \\ |\eta\Lambda\rangle \\ |K\Xi\rangle \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} \sqrt{15} & -\sqrt{24} & 0 & -1 \\ -\sqrt{10} & -2 & \sqrt{20} & -\sqrt{6} \\ -\sqrt{5} & -\sqrt{8} & 0 & 3\sqrt{3} \\ \sqrt{10} & 2 & 2\sqrt{5} & \sqrt{6} \end{pmatrix} \begin{pmatrix} |1\rangle \\ |8\rangle \\ |8'\rangle \\ |27\rangle \end{pmatrix},$$

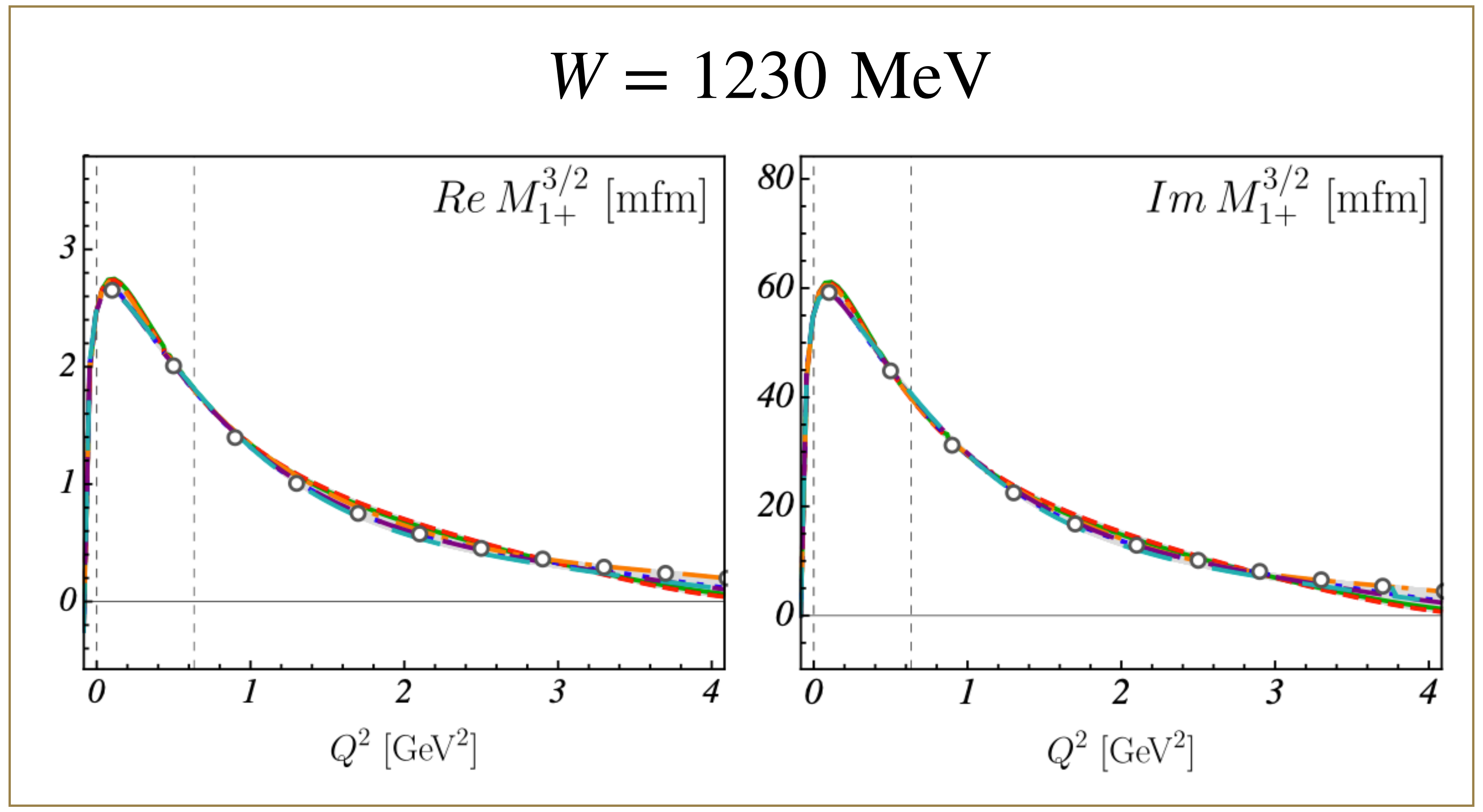
$$C_{\alpha\beta} = \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad \text{for} \quad \alpha, \beta \in \{1, 8, 8', 27\}.$$

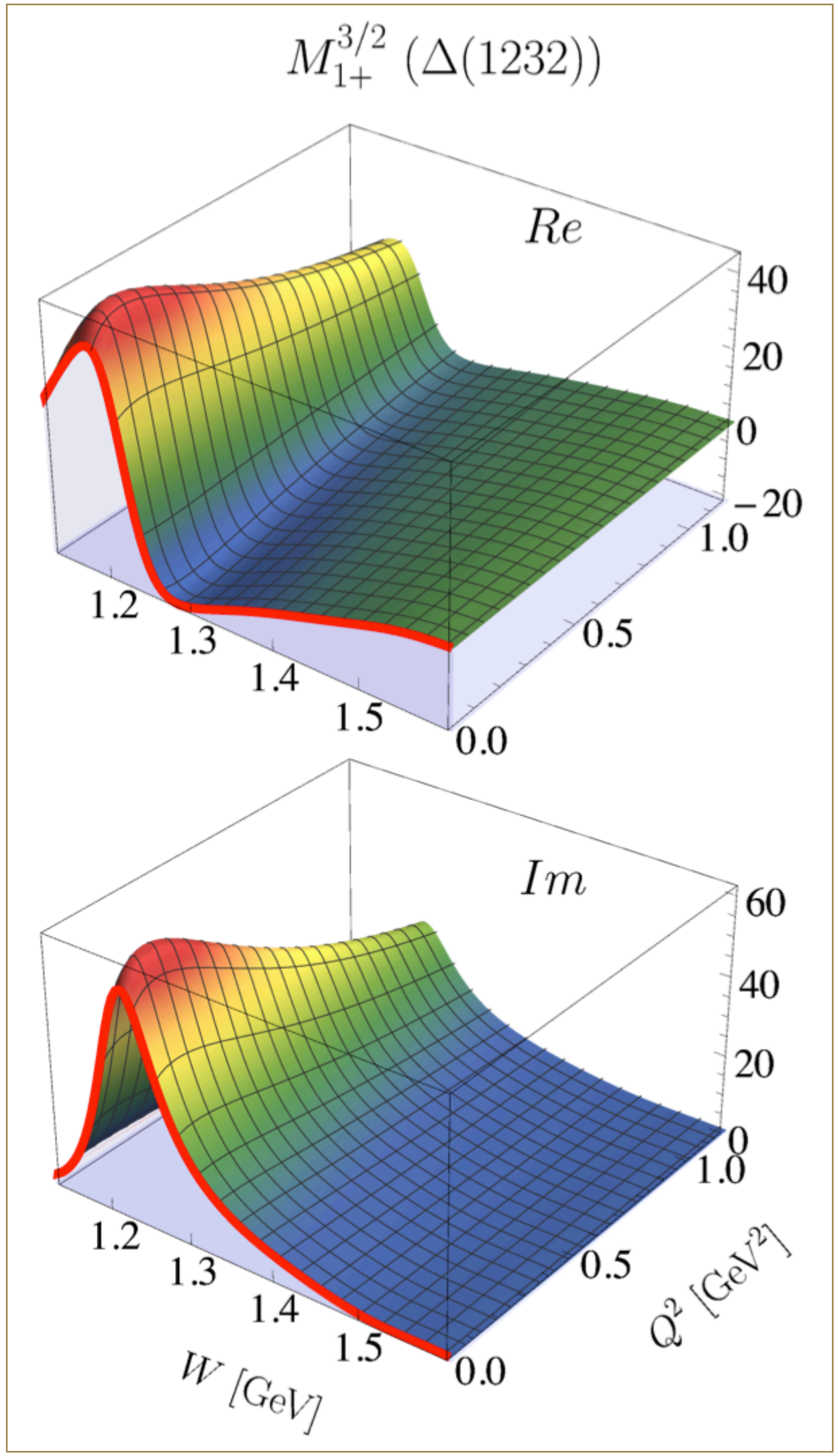
$$C^{\mathrm{NLO1}}_{\alpha\beta} = \begin{pmatrix} \frac{4}{3}(3b_0 + 7b_D)m_q & 0 & 0 & 0 \\ 0 & \frac{2}{3}(6b_0 + b_D)m_q & -\sqrt{20}b_F m_q & 0 \\ 0 & -\sqrt{20}b_F m_q & 2(2b_0 + 3b_D)m_q & 0 \\ 0 & 0 & 0 & 4(b_0 + b_D)m_q \end{pmatrix},$$

$$C^{\mathrm{NLO2}}_{\alpha\beta} = \begin{pmatrix} -3d_2 + \frac{9}{2}d_3 + d_4 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(-3d_2 + d_3 + 2d_4) & -\frac{\sqrt{5}}{2}d_1 & 0 \\ 0 & -\frac{\sqrt{5}}{2}d_1 & \frac{1}{2}(9d_2 - d_3 + 2d_4) & 0 \\ 0 & 0 & 0 & \frac{1}{2}(2d_2 + d_3 + 2d_4) \end{pmatrix}.$$

NLO breaks accidental octet symmetry

RESULTS





Delta(1232):

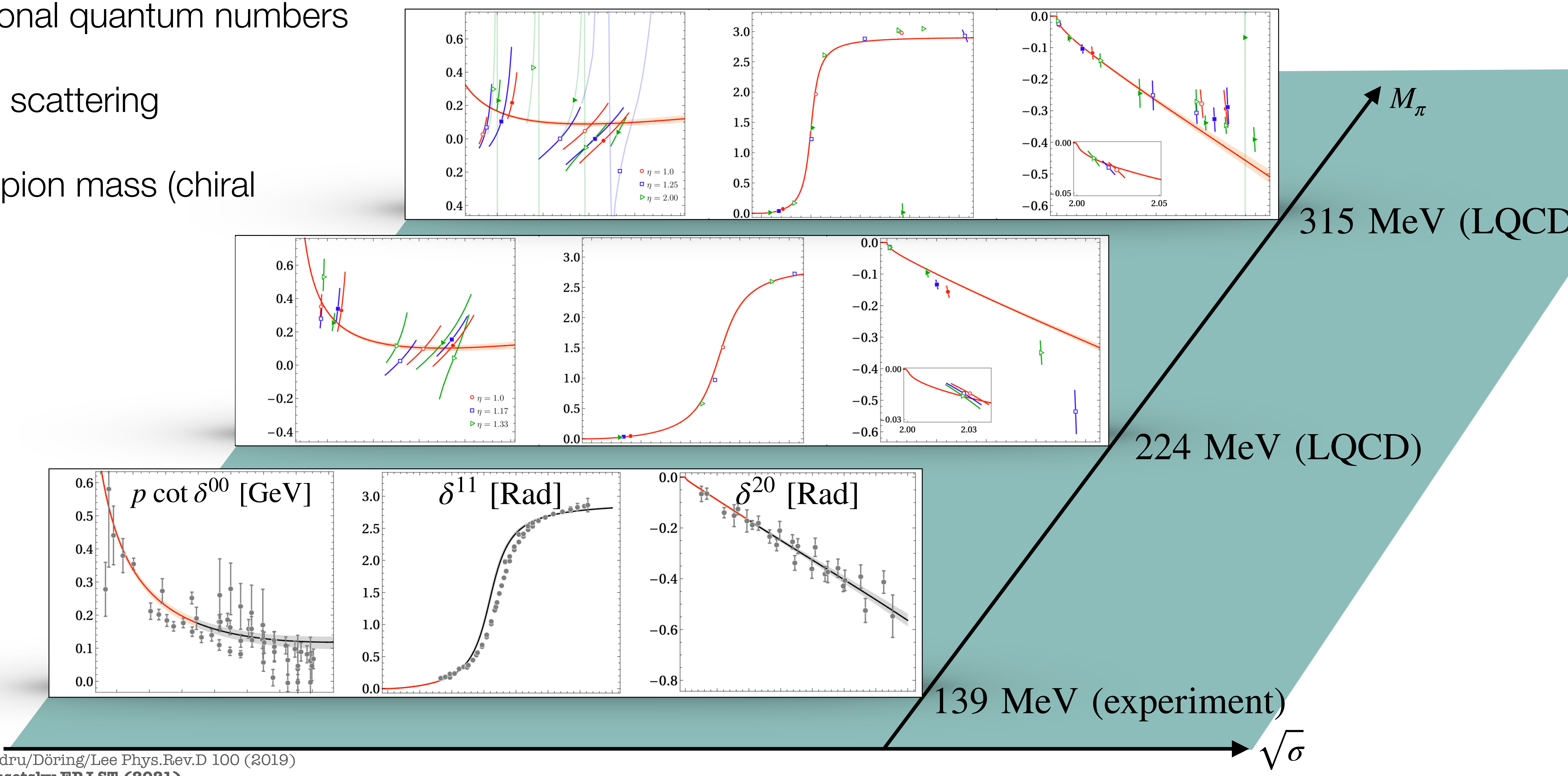
- Large multipoles well determined
- simple Q^2 dependence

[JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

LATTICE HADRON SPECTROSCOPY

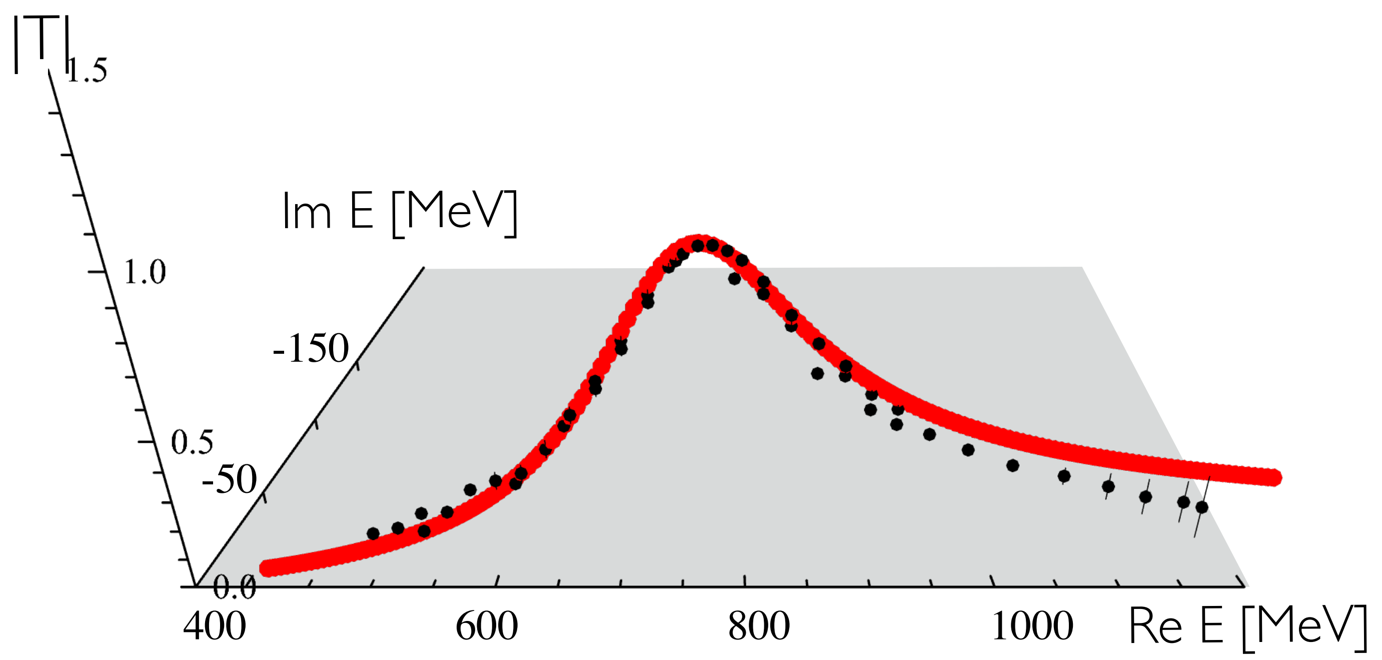
- Experimentally inaccessible scenarios:
- Unconventional quantum numbers
- Three-body scattering
- Unphysical pion mass (chiral trajectories)

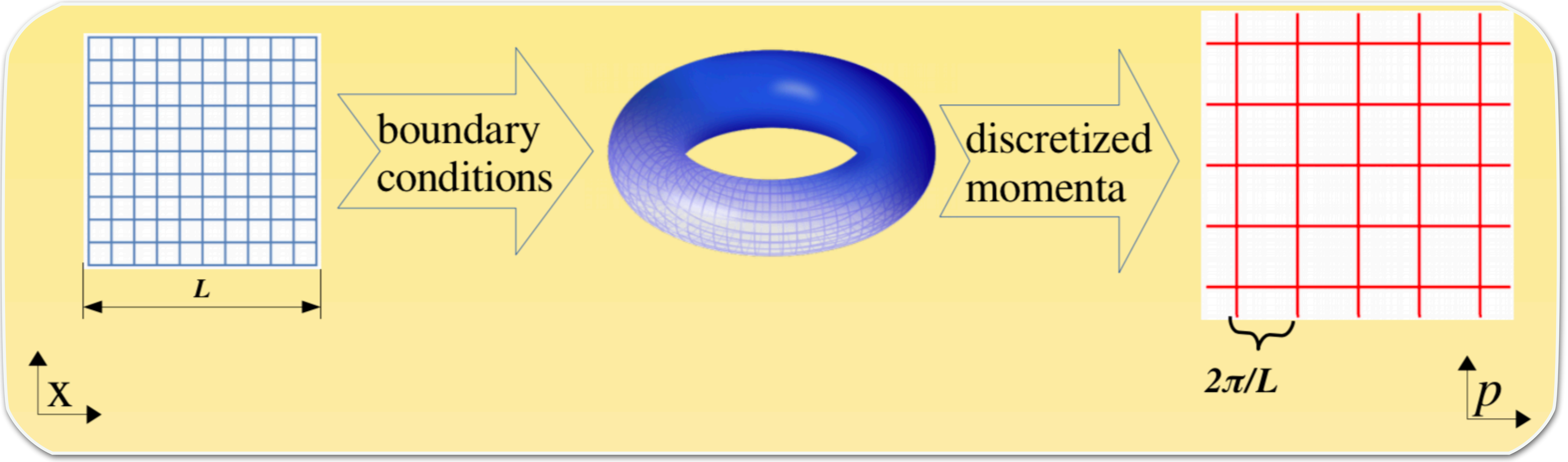
…

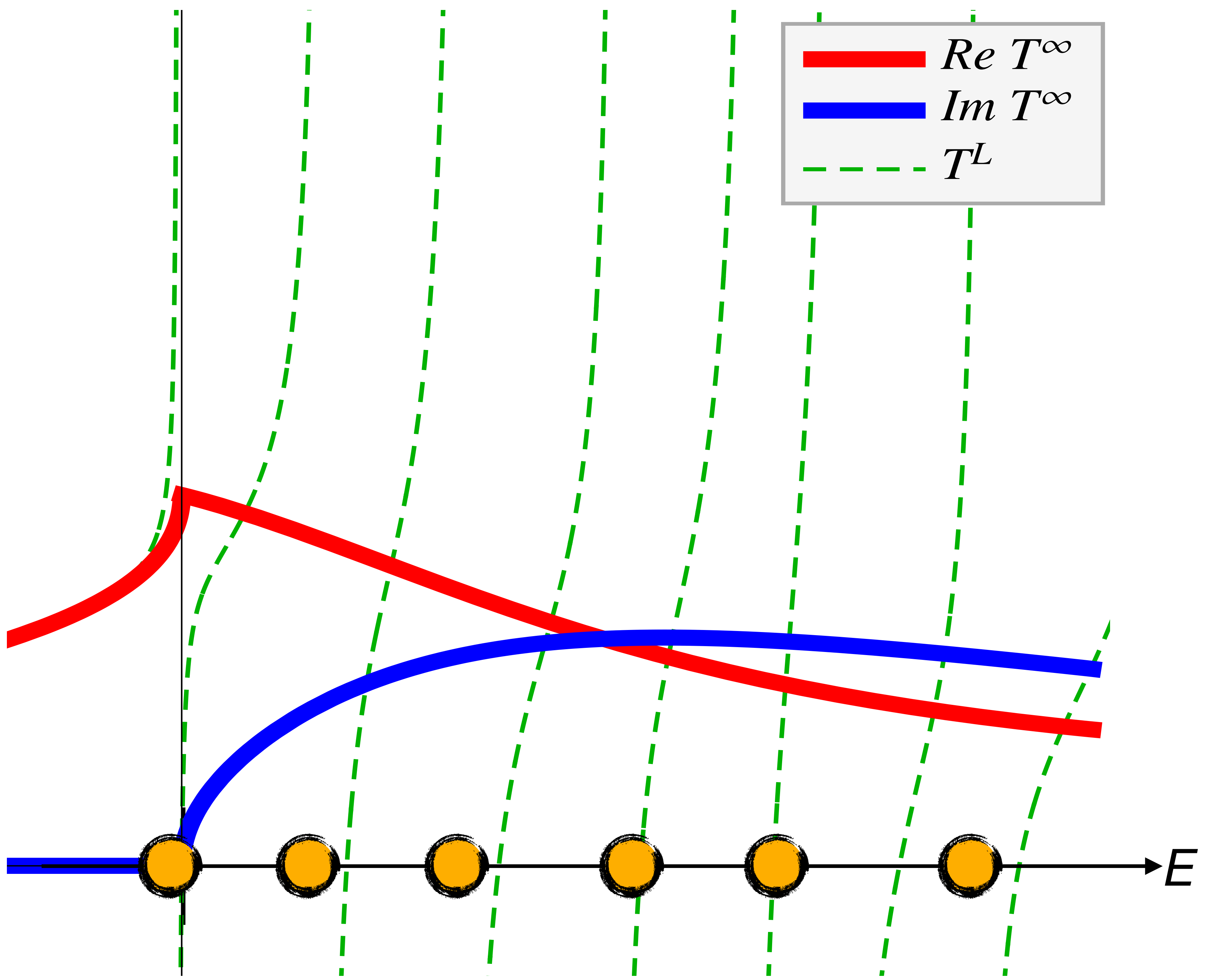


MM/Culver/Brett/Alexandru/Döring/Lee Phys.Rev.D 100 (2019)
Review: MM/Döring/Rusetsky EPJ ST (2021)

HADRONS IN A BOX





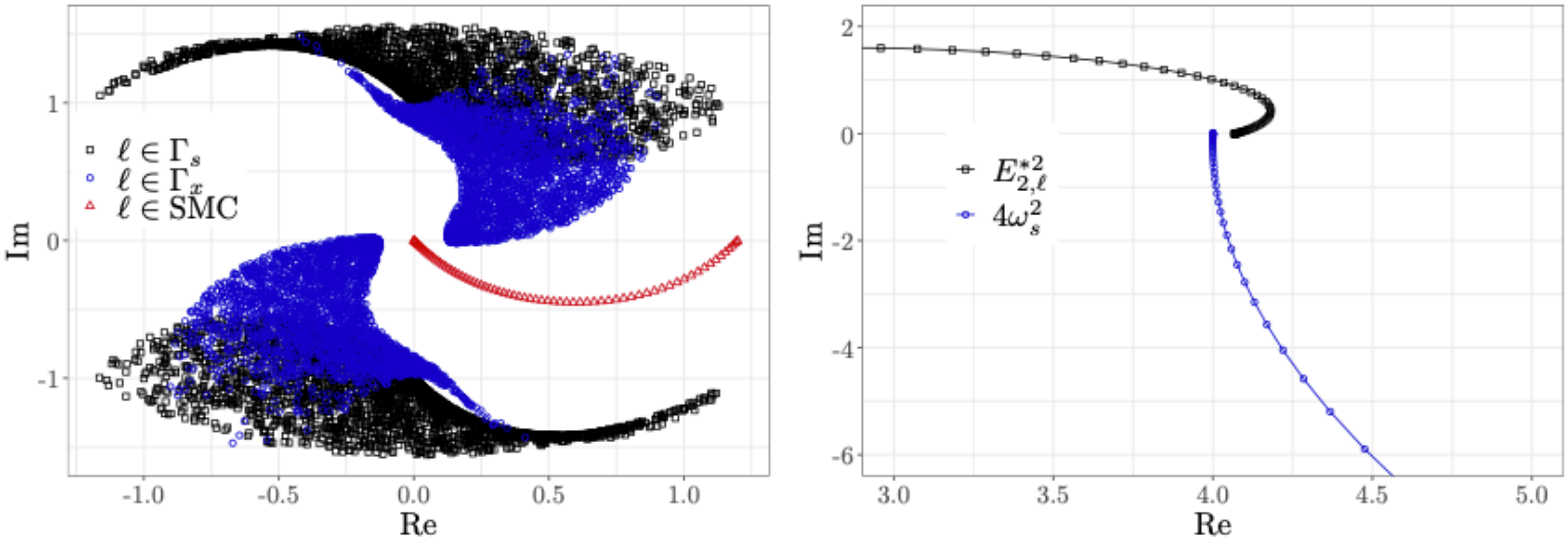


🤗 Heavily simplified:

on-shell particle-configurations: $\Delta E \sim mL$

off-shell particle-configurations: $\Delta E \sim e^{-mL}$

1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, …
2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);



$\ell \in \Gamma_s$
$\ell \in \Gamma_x$
$\ell \in$ SMC
Im
Re
1
0
-1
-1.0
-0.5
0.0
0.5
1.0
$E_{2,\ell}^{*2}$
$4\omega_s^2$
2
0
-2
-4
-6
3.0
3.5
4.0
4.5
5.0

Current frontier: 3-body dynamics from LQCD

➥ 3-body Quantization Conditions[1]

➥ RFT / FVU / NREFT

➥ many perturbatively interacting systems are studied[2]

$$0 = \det\left(L^3\left(\tilde{F}/3 - \tilde{F}(\tilde{K}_2^{-1} + \tilde{F} + \tilde{G})^{-1}\tilde{F}\right)^{-1} + K_{\mathrm{df},3}\right)$$ RFT

$$0 = \det\left(B_0 + C_0 - E_L\left(K^{-1}/(32\pi) + \Sigma_L\right)\right)$$ FVU

3-body force

2-body interaction

one-particle exchange

2-body self-energy

1) Rusetsky, Bedaque, Grießhammer, Sharpe, Meißner, Döring, Hansen, Davoudi, Guo....

Reviews:

Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019);

MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

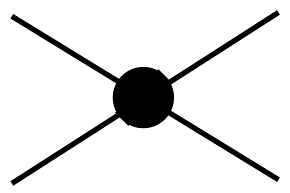
2) MM/Döring PRL122(2019); Blanton et al. PRL 124 (2020); Hansen et al. PRL 126 (2021); ....

AVOIDED LEVEL CROSSING

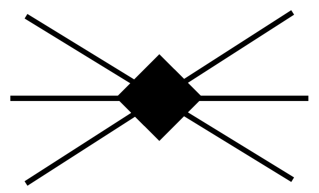
Variate g($\varphi_1 \to \varphi_0\varphi_0\varphi_0$) coupling:

- avoided level crossing becomes wider
- RFT and FVU

$$q^* \cot\delta = \frac{1}{aM_0}$$



$$C = \frac{c_0}{E_3^3 - m_1^2} + c_1$$



g		a	m_1	c_0	c_1	m'_1	c'_0	c'_1	$\chi^2_{\rm dof}$
5	FVU	−0.1512(9)	3.0229(1)	−0.0188(35)	–	–	–	–	2.9
	RFT	−0.1522(12)	–	–	–	3.0232(2)	31.6(8.4)	–	2.5
	FVU	−0.1569(12)	3.0233(2)	−0.0297(57)	2.29(38)	–	–	–	1.5
	RFT	−0.1571(10)	–	–	–	3.0237(2)	37.6(9.0)	2789(540)	1.5
10	FVU	−0.1521(11)	3.0205(2)	−0.0475(66)	–	–	–	–	1.7
	RFT	−0.1531(13)	–	–	–	3.0212(3)	80(14)	–	1.6
	FVU	−0.1549(16)	3.0205(2)	−0.0595(99)	0.93(41)	–	–	–	1.5
	RFT	−0.1563(27)	–	–	–	3.0213(3)	97(16)	1773(980)	1.4
20	FVU	−0.1444(11)	3.0184(2)	−0.1136(77)	–	–	–	–	1.6
	RFT	−0.1450(17)	–	–	–	3.0199(2)	178(17)	–	1.6
	FVU	−0.1464(14)	3.0183(2)	−0.1363(148)	0.84(39)	–	–	–	1.3
	RFT	−0.1484(16)	–	–	–	3.0200(2)	210(23)	2227(600)	1.2

... same fit quality

... observables determined consistently

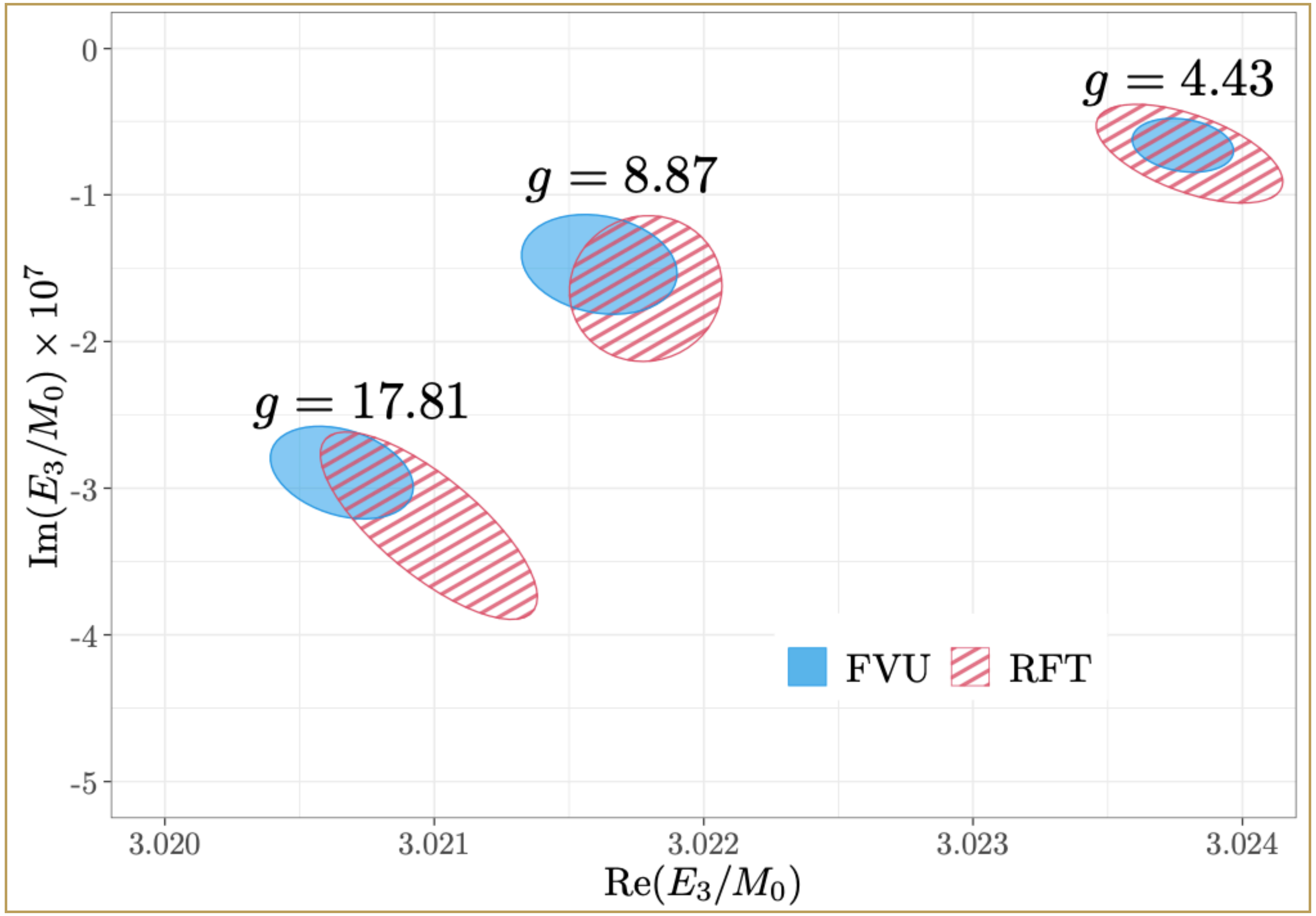
Pole positions

- FVU: complex energy-plane analysis[1]

  -- resonance width grows ~ ***g²***

  -- avoided level crossing gap >> width

- Similarly from RFT with Breit-Wigner like approximation



1) Sadasivan/MM/.. Phys.Rev.D 101 (2020)