

SBS-GMN Analysis Overview and Status (NOT “results”)

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Elastic Electron-Nucleon Scattering and Form Factors

- The Dirac (F_1) and Paul (F_2) form factors describe the most general form of the virtual photon-nucleon vertex function consistent with the symmetries of QED; namely, Lorentz invariance, parity conservation and gauge invariance/current conservation
- They are real-valued functions of the (space-like) squared four-momentum transfer $q^2 = (k - k')^2 < 0$.
- Experimental observables sensitive to form factors include differential cross sections and double-spin asymmetries involving polarized e^- beams and/or targets

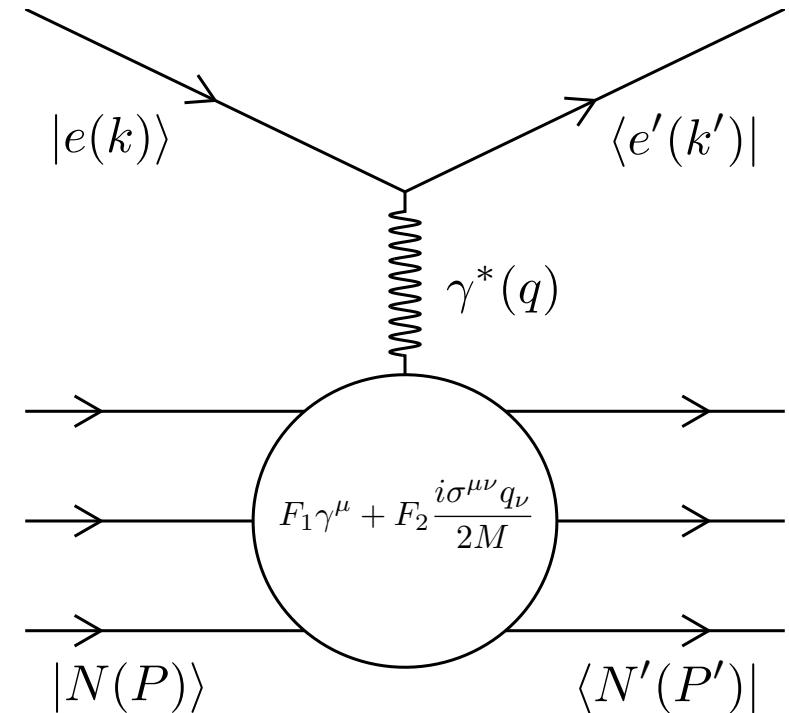
$$\text{Invariant amplitude: } \mathcal{M} = 4\pi\alpha \bar{u}(k')\gamma^\mu u(k) \left(\frac{g_{\mu\nu}}{q^2} \right) \bar{u}(P')\Gamma^\nu u(P)$$

$$\gamma^* N \text{ vertex function: } \Gamma^\mu = F_1(q^2)\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2(q^2)$$

$$\text{Sachs FF: } G_E = F_1 - \tau F_2$$

$$G_M = F_1 + F_2$$

$$\text{Rosenbluth Formula: } \frac{d\sigma}{d\Omega_e} = \left(\frac{d\sigma}{d\Omega_e} \right)_{\text{Mott}} \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1 + \tau)}$$

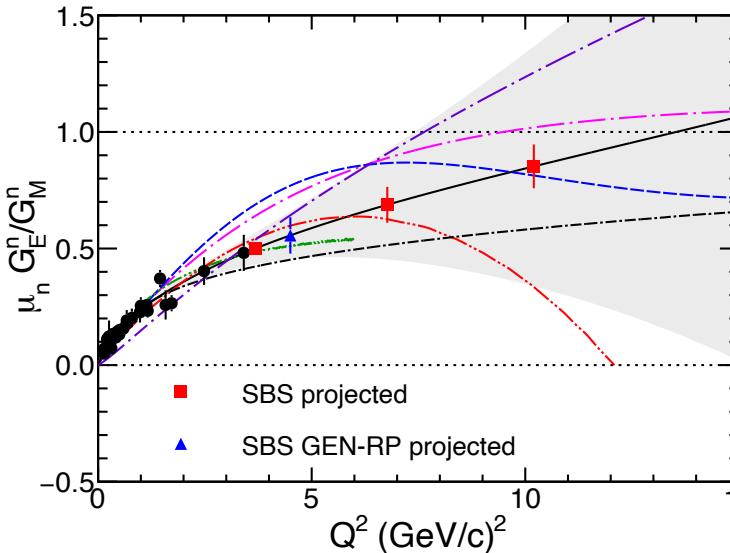
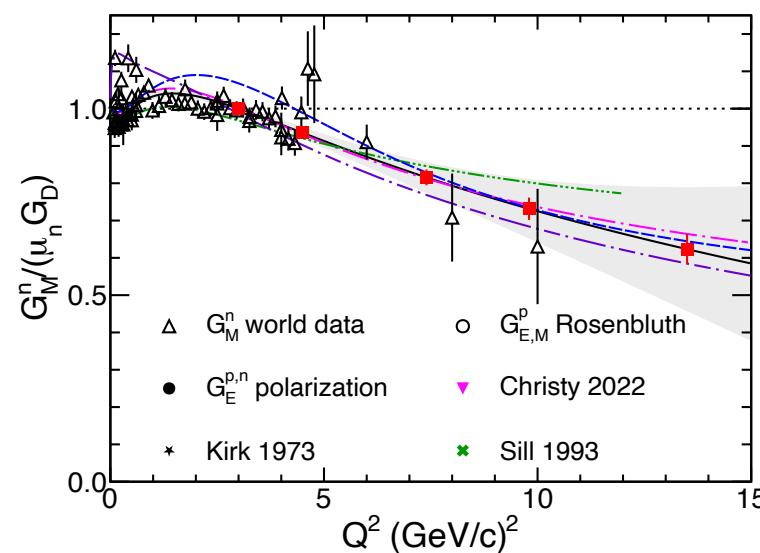
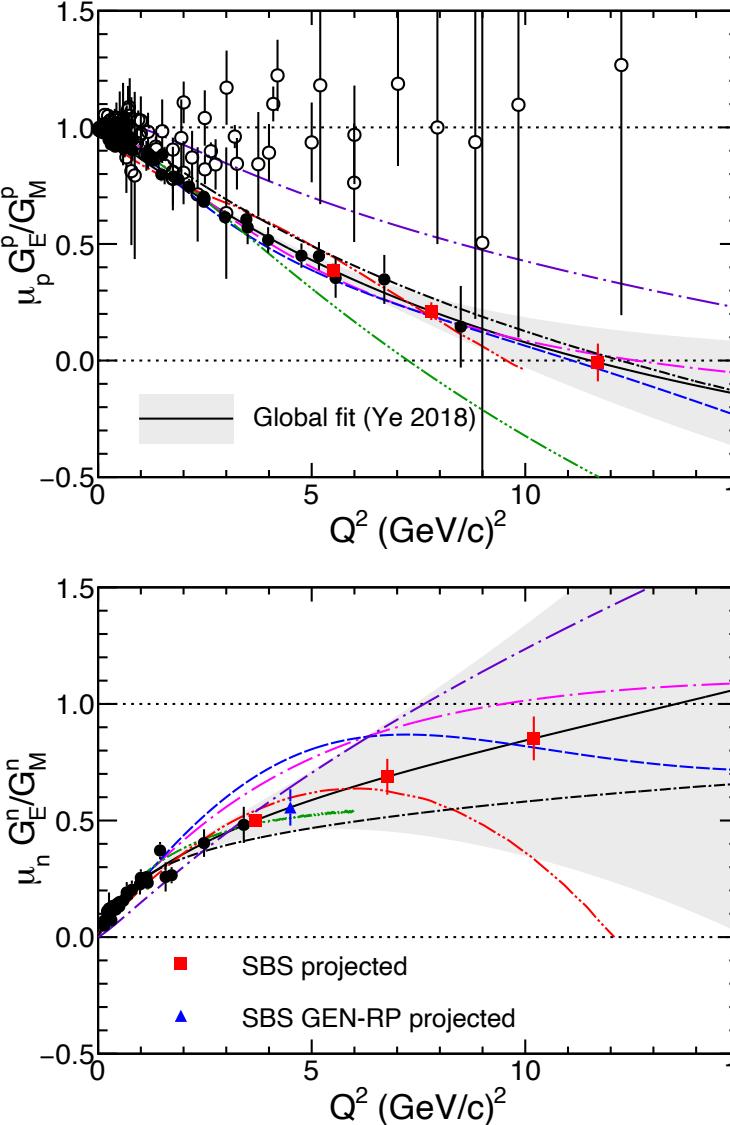
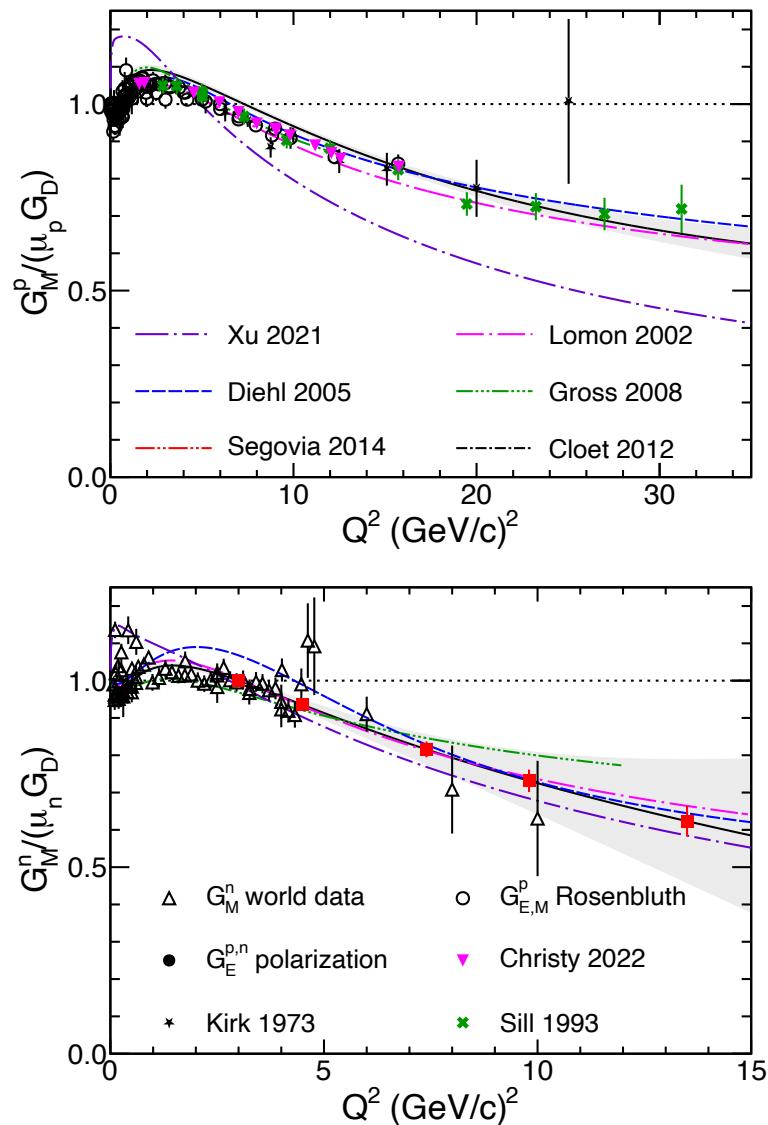


Feynman Diagram for elastic $eN \rightarrow eN$ scattering in OPE approximation

$$\tau \equiv \frac{Q^2}{4M^2}$$

$$\epsilon \equiv \left[1 + 2(1 + \tau) \tan^2 \left(\frac{\theta_e}{2} \right) \right]^{-1}$$

The SBS high- Q^2 Form Factor Program in Hall A



- Figure from “50 Years of QCD”: : <https://arxiv.org/abs/2212.11107>
- GMN/nTPE (E12-09-019/E12-20-010): Completed Oct. 2021-Feb. 2022
- GEN Helium-3: ~70% complete as of this writing
- GEN-RP: Projected run 2024
- GEP: Projected run 2024-2025

The Neutron Form Factors

- More difficult to measure than the proton due to lack of free neutron targets
- Far less accurately known than the proton FFs over a far more limited Q^2 range
- Cross section dominated by G_M^n over most of measured Q^2 range
- Most reliable G_E^n data come from polarization observables
- Most reliable G_M^n data come from “ratio” method on deuterium: first proposed by Durand, [Phys. Rev. 115, 1020 \(1959\)](#)
 - Some extractions also exist from absolute cross section and polarization measurements

“Ratio” method for G_M^n

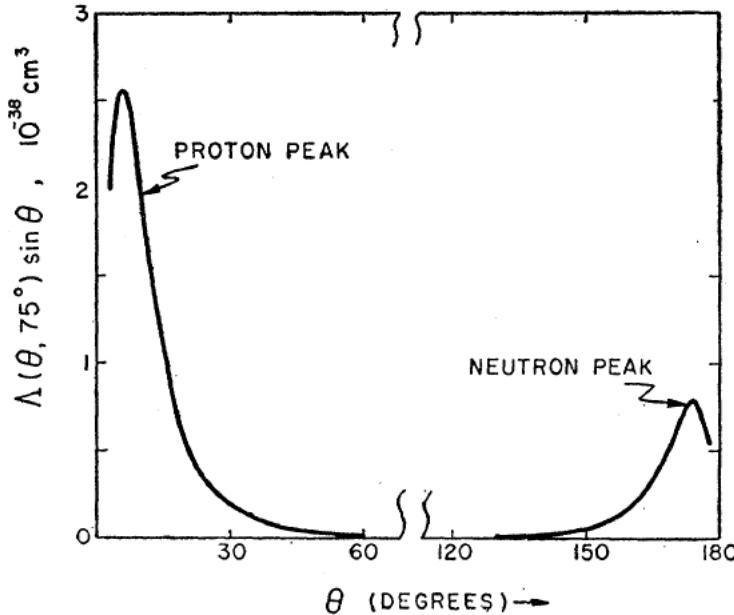
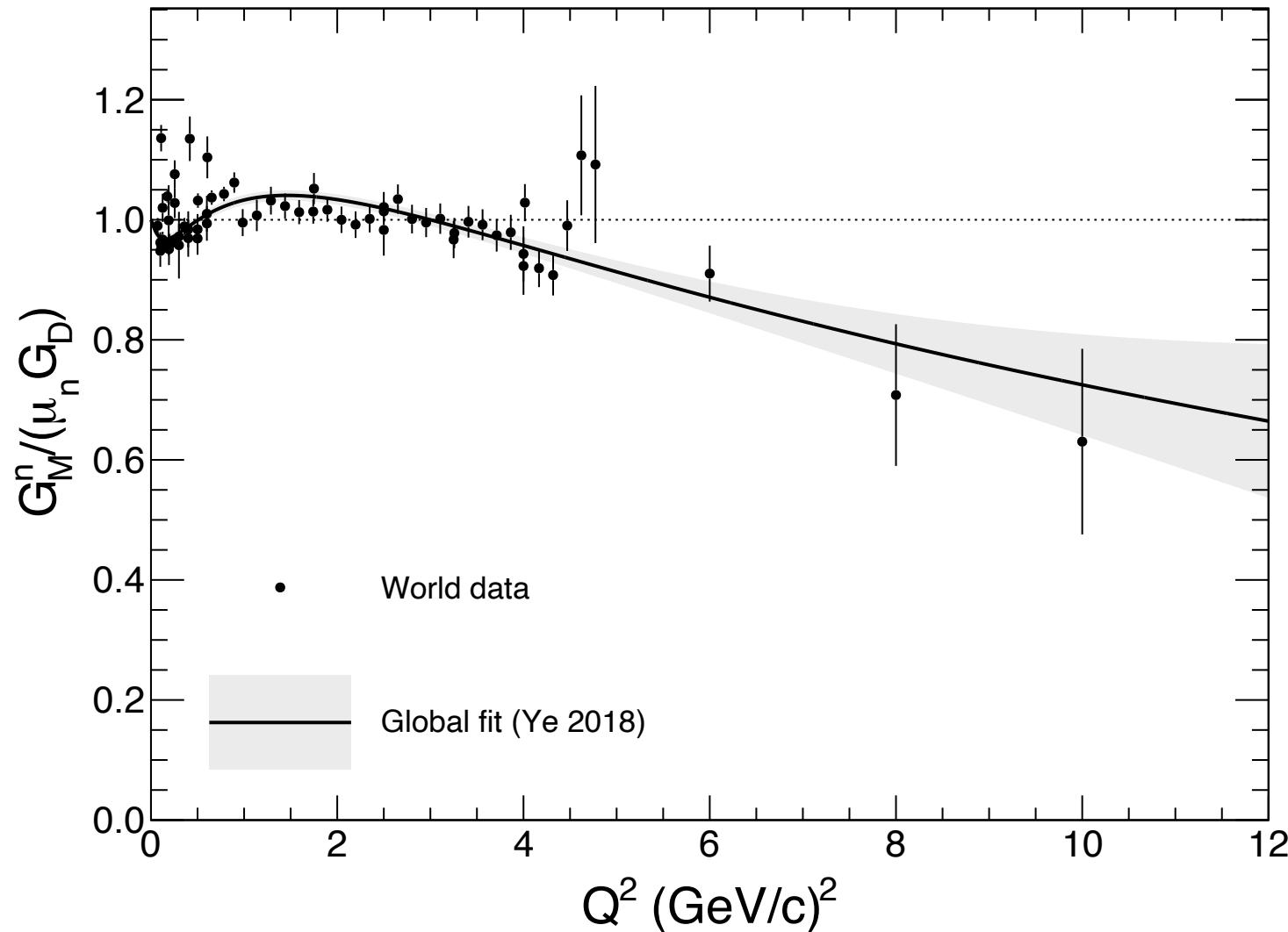


FIG. 1. The angular distribution function $\Lambda(\theta, \vartheta) \sin \theta$ in the absence of final-state interactions is plotted as a function of the proton scattering angle in the nucleon center-of-mass system [$\cos \theta = \hat{p} \cdot \hat{q}$] for the scattering of 500-Mev electrons through an angle $\vartheta = 75^\circ$ with a momentum transfer giving $p = \frac{1}{2}q = 1.3 \times 10^{13} \text{ cm}^{-1}$. $\Lambda(\theta, \vartheta)$ is defined in Eq. (11.2); the function $F(\theta)$ entering the definition was evaluated using a Hulthén wave function for the deuteron. The cross section $d^3\sigma / (d\theta d\Omega_e dE_{e'})$ is given by $(4.71 \times 10^5 \text{ cm}^{-1} \text{ rad}^{-1} \text{ sterad}^{-1} \text{ Mev}^{-1}) \Lambda(\theta, \vartheta) \sin \theta$. No nucleon form factors have been introduced into the results.

- Idea: simultaneous measurement of $d(e, e'n)p$ and $d(e, e'p)n$ in quasi-elastic kinematics
- Cancels many sources of experimental systematic uncertainty (electron acceptance/detection efficiency, luminosity, detector and DAQ livetime, etc).
- Small nuclear model dependence—nuclear (and radiative) corrections are similar/nearly identical for $(e, e'n)$ and $(e, e'p)$ cross sections
- Combine with existing knowledge of free proton cross section to extract free neutron cross section
- **Major remaining source of systematic uncertainty is the relative acceptance/efficiency between protons and neutrons! → SBS-HCAL was designed to minimize this!**

Figure from Durand, 1959 (see previous slide for reference)

G_M^n World Data Summary



- References for “world data” are the same as given in the caption to Figure 17 of [Puckett *et al.*, PRC 85, 045203 \(2012\)](#)
- “Global Fit” is from [Ye *et al.*, Phys.Lett.B 777 \(2018\) 8-15](#)
- Measurements to 10 GeV^2 exist with very large uncertainties (Rock *et al.*, PRL 49, 1139 (1982) and PRD 46, 24 (1992))
- Most precise data with widest Q^2 coverage are from CLAS Collaboration: [J. D. Lachniet *et al.*, PRL 102, 192001 \(2009\)](#) from 1-4.8 GeV^2 using ratio method
- Other measurements used either inclusive quasi-elastic double-spin asymmetry on polarized Helium-3, or absolute cross section measurements on inclusive $d(e,e')$ or coincidence $d(e,e'n)$

High- Q^2 G_M^n and quark flavor FFs

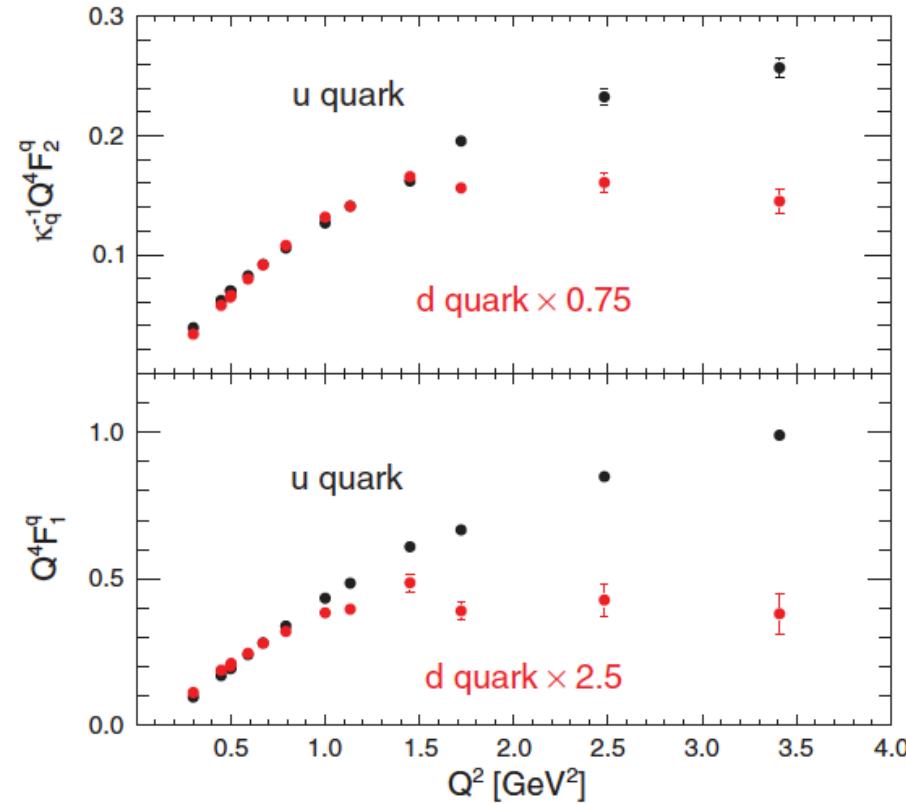
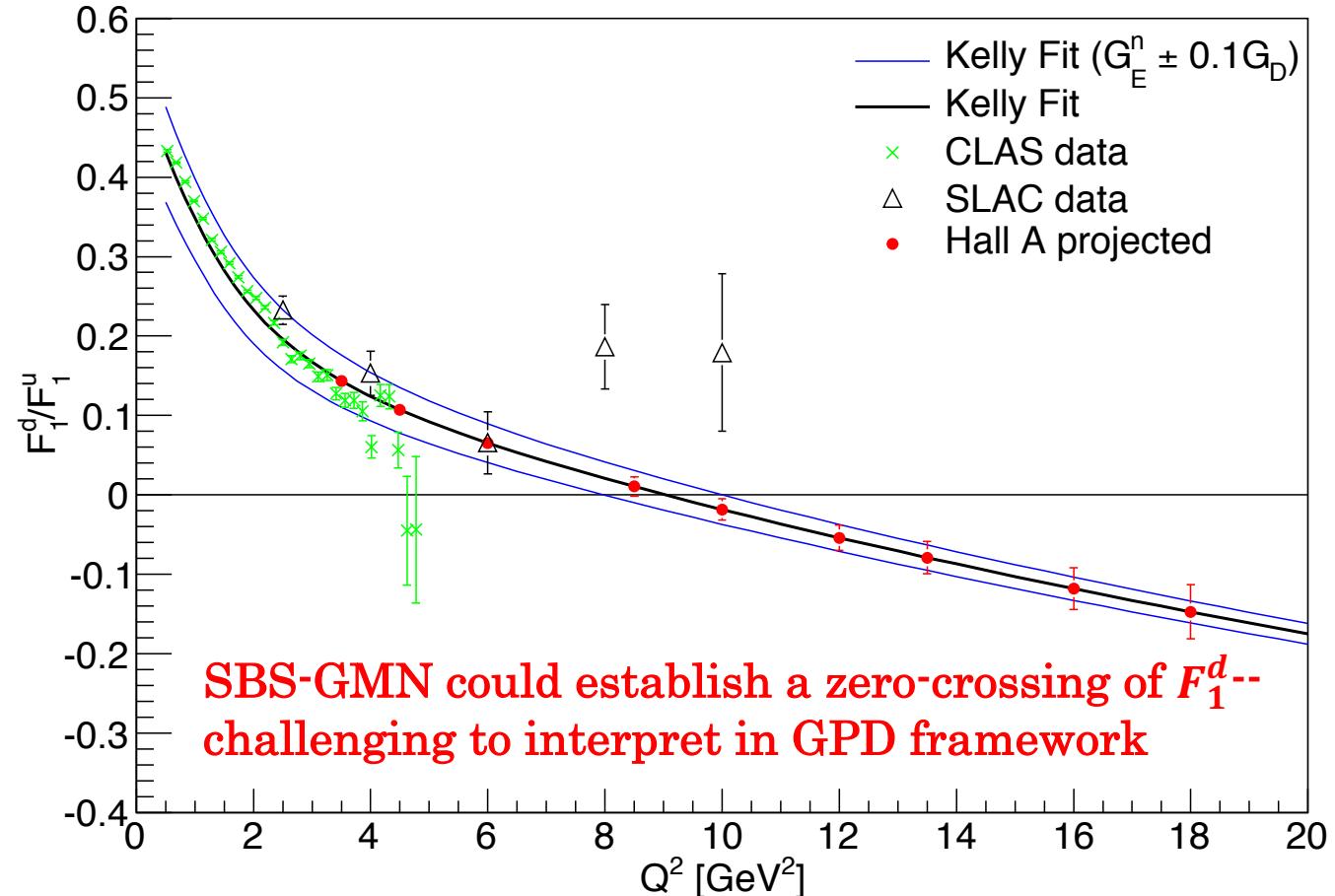


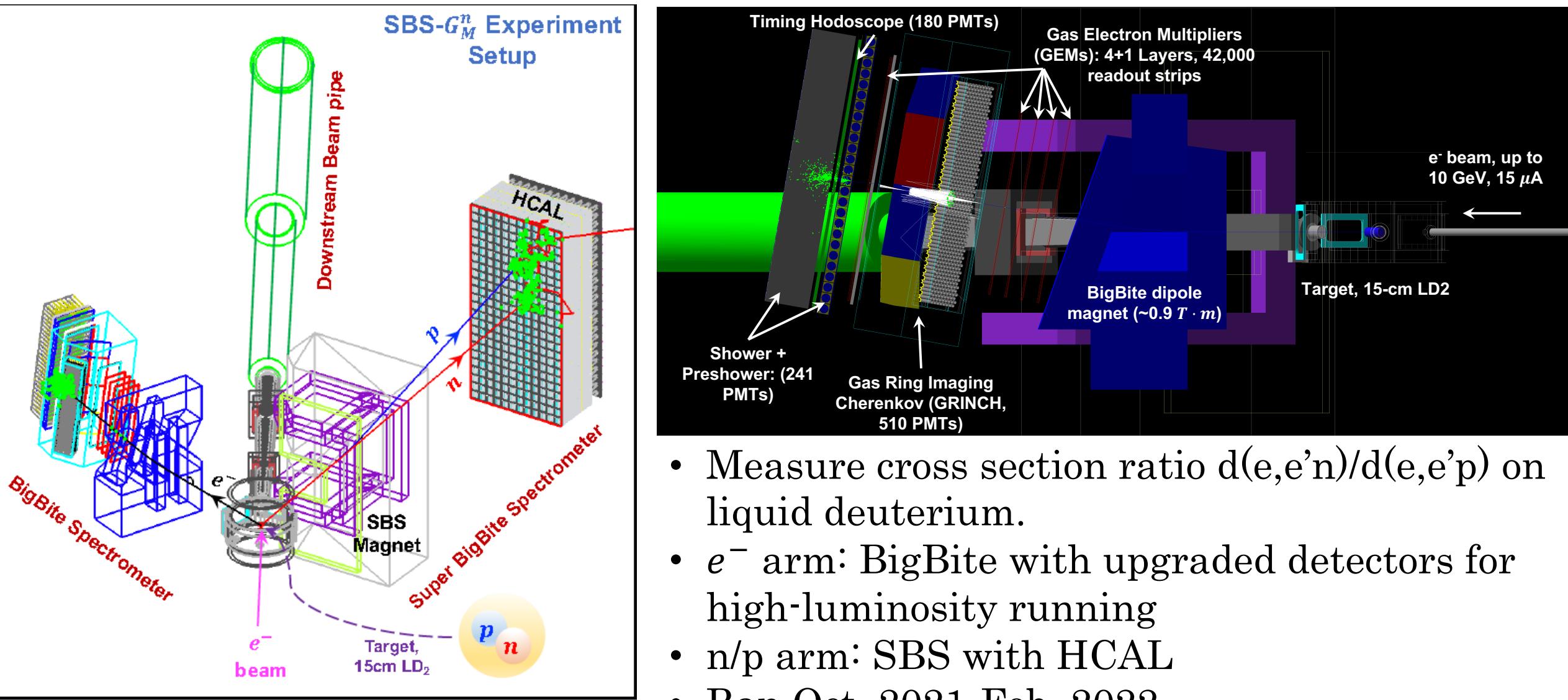
FIG. 3 (color). The Q^2 dependence for the u and d contributions to the proton form factors (multiplied by Q^4). The data points are explained in the text.

Cates *et al.*, PRL 106, 252003 (2011)



- Notable behaviors: d and u quark FFs show dramatically different Q^2 dependence.
- Flavor FF ratios F_2^q/F_1^q almost constant for both u and d above 1 GeV 2

GMN/nTPE Overview

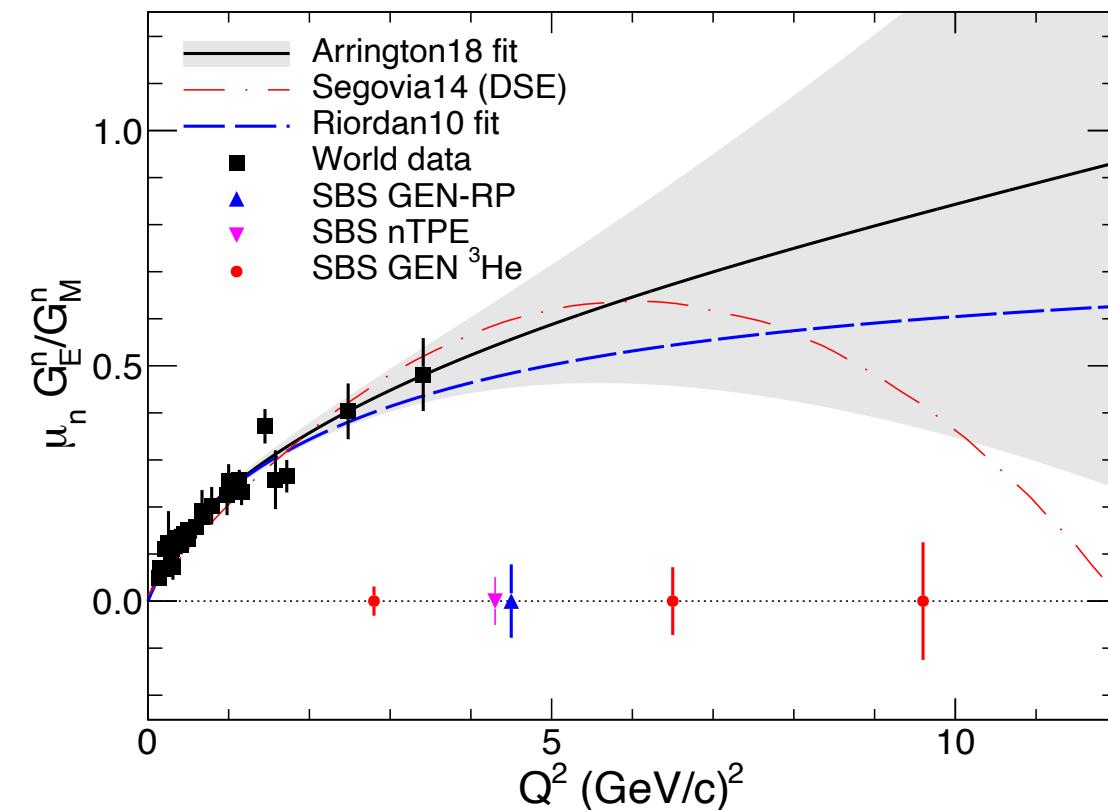
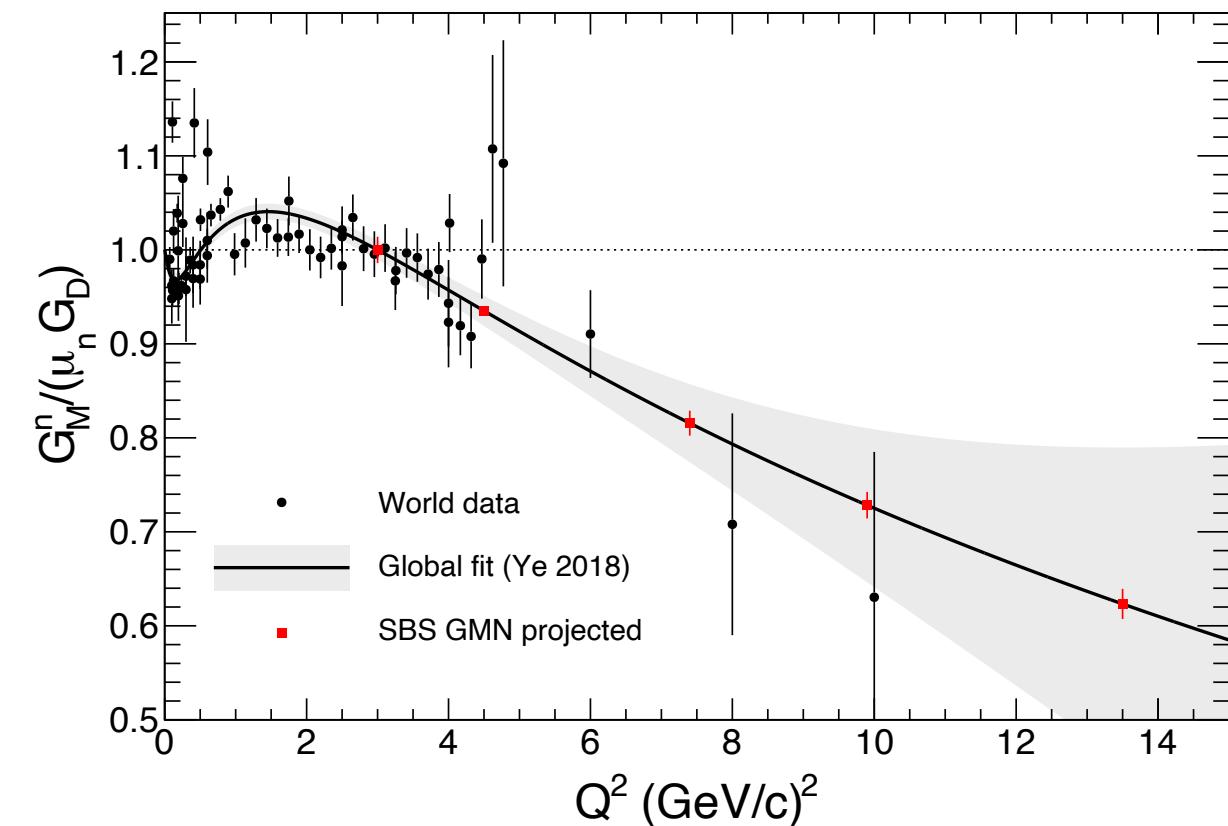


GMN/nTPE Kinematics

Name	Ebeam (GeV)	BigBite angle (deg)	BigBite Distance (m)	SBS angle (deg)	SBS distance (m)	HCAL “angle” (deg)	HCAL distance (m)	Q^2 (GeV 2)	ϵ	Electron P (GeV)	Nucleon P (GeV)
SBS-1 (comm.)	1.9217	51.0	1.85	33.5	2.25	34.5	13.5	1.55	0.61	1.09	1.5
SBS-4	3.7393	36.0	1.80	31.9	2.25	31.9	11.0	3.0	0.72	2.12	2.4
SBS-7	7.9308	40	1.85	16	2.25	16	14	9.8	0.51	2.66	6.1
SBS-11	9.8890	42.0	1.55	13.3	2.25	13.3	14.5	13.5	0.42	2.67	8.1
SBS-14	5.9828	46.5	1.85	17.3	2.25	17.3	14.0	7.4	0.47	2.00	4.81
SBS-8	5.9826	26.5	2.00	29.9	2.25	29.4	11.0	4.5	0.80	3.58	3.2
SBS-9	4.0268	49.0	1.55	22.5	2.25	22.0	11.0	4.5	0.51	1.6	3.2

- Yellow Highlights indicate points where HCAL “angle” differs from SBS central angle.
- Green Highlights indicate nTPE/Rosenbluth separation kinematics

GMN/nTPE Projected Physics Results and Uncertainties



- LEFT: expected uncertainties for GMN versus Q^2 , based on actually collected data and actually reconstructed statistics in first analysis pass, w/proposal systematics (**all points will be systematics-limited!**)
- RIGHT: projected accuracy of nTPE Rosenbluth slope, compared to existing and planned data for $\mu_n G_E^n / G_M^n$
- Note:** SBS G_M^n points are projected uncertainties plotted arbitrarily along the global fit curve; i.e., not data

GMN/nTPE thesis students and subsystem responsibility/expertise

- John Boyd, UVA: GEMs
- Provakar Datta, UConn: BBCAL
- Nathaniel Lashley-Colthirst, Hampton: Beamline
- Ralph Marinaro, Glasgow: BigBite timing hodoscope
- Anuruddha Rathnayake, UVA: GEMs
- Maria Satnik, W&M: BigBite GRINCH
- Sebastian Seeds, UConn: HCAL
- Ezekiel Wertz, W&M: GEMs

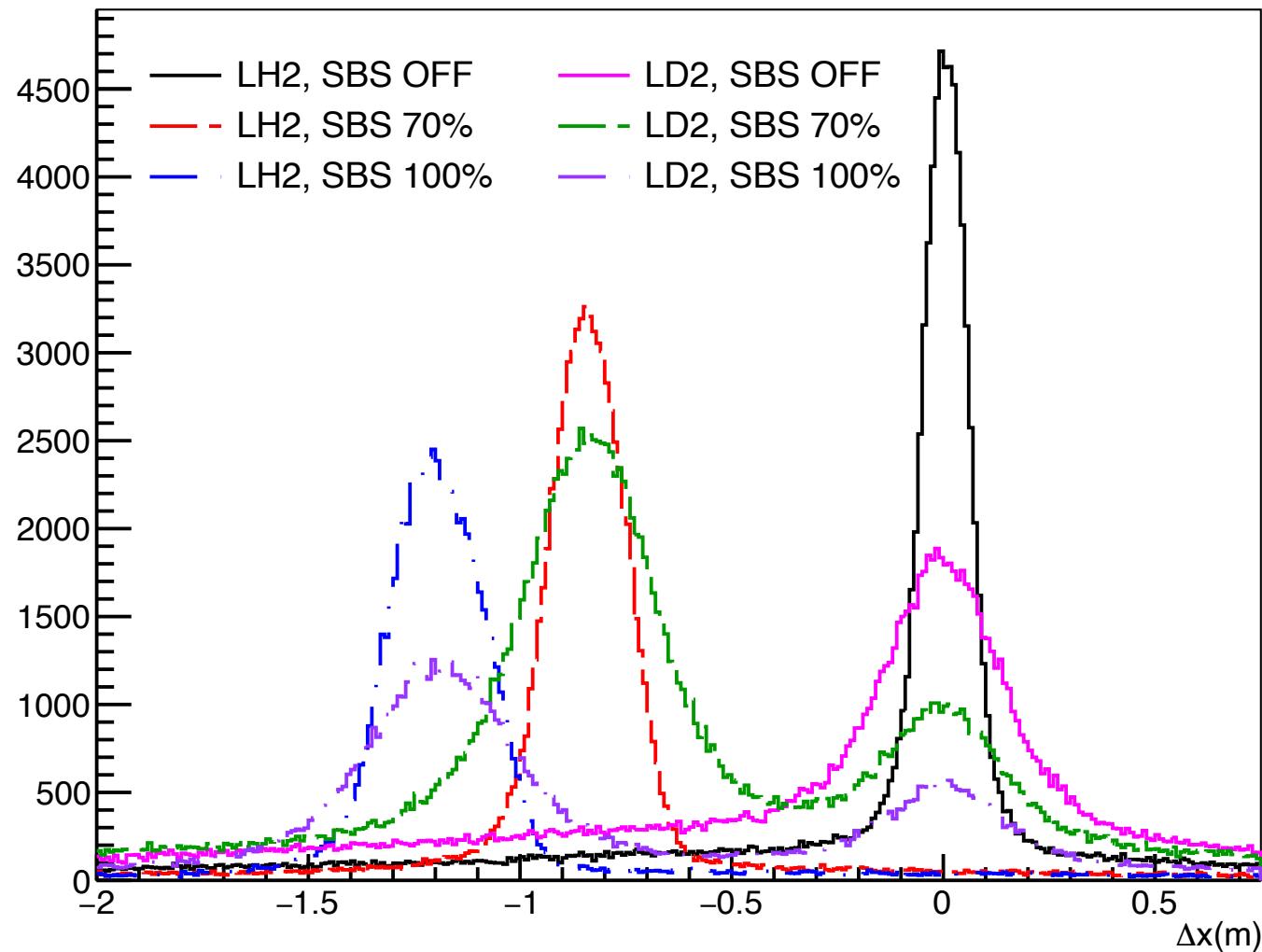
GMN analysis basics

- Goal is to extract σ_n/σ_p in quasi-elastic kinematics with small uncertainties.
- Nuclear and radiative effects are expected to (mostly) cancel in the ratio, especially at high Q^2
- Electron acceptance, efficiency, luminosity/etc also cancel
- Most important known sources of systematic uncertainty:
 - Relative acceptance/efficiency between neutrons and protons (if any)
 - Inelastic contamination (and other backgrounds, e.g., accidentals, fake GEM tracks/etc)
- SBS HCAL was designed to minimize n/p acceptance/efficiency difference!
 - Large acceptance
 - High (and very similar) efficiencies for p, n (by design)

$$\begin{aligned} R_{np} \equiv \frac{\sigma_{d(e,e'n)p}}{\sigma_{d(e,e'p)n}} &\approx \frac{\sigma_{en \rightarrow en}}{\sigma_{ep \rightarrow ep}} \\ &\approx \frac{\epsilon G_E^n{}^2 + \tau G_M^n{}^2}{\epsilon G_E^p{}^2 + \tau G_M^p{}^2} \\ \implies G_M^n &\approx \sqrt{\frac{R_{np} \sigma_R^p - \epsilon G_E^n{}^2}{\tau}} \end{aligned}$$

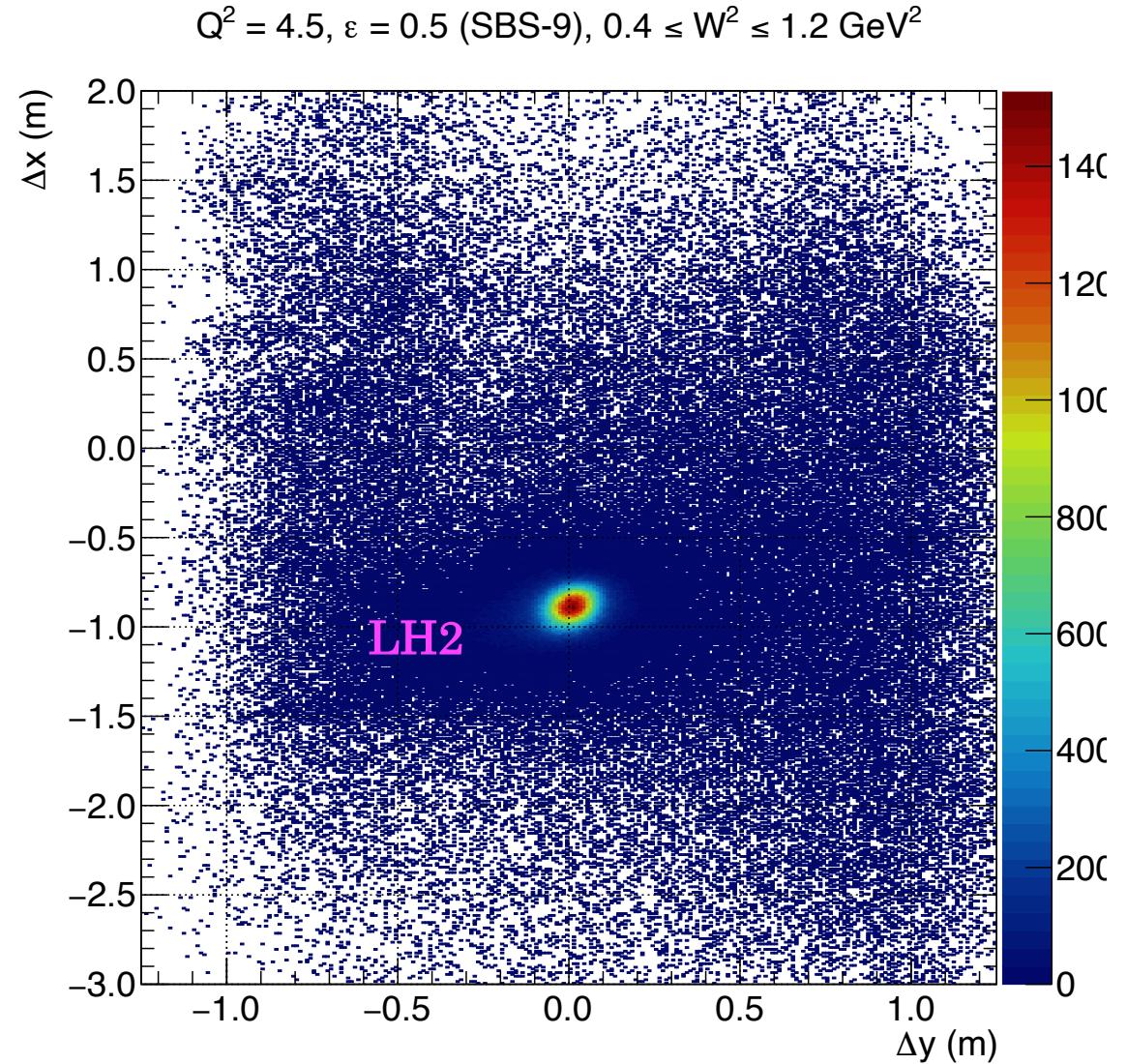
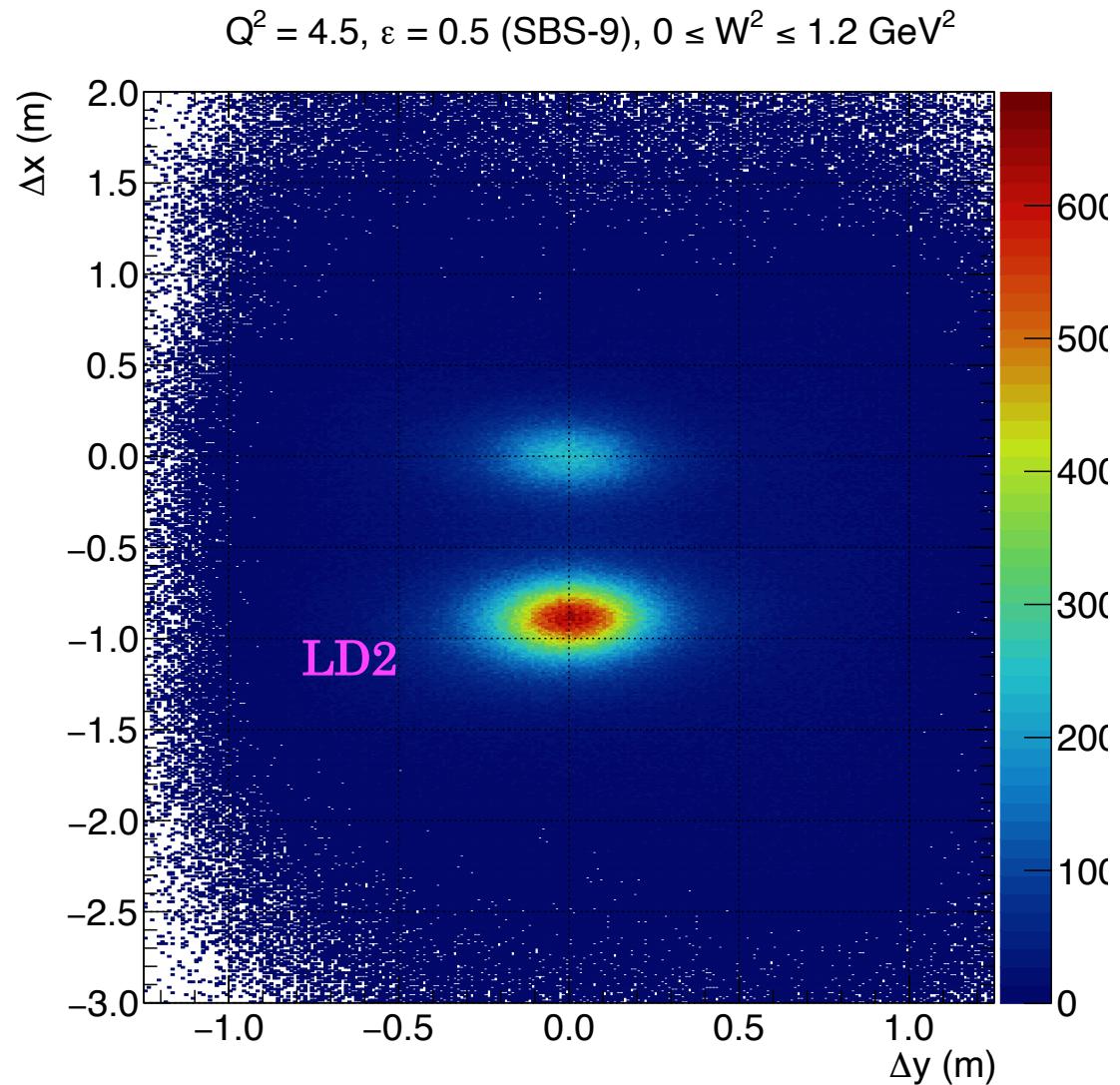
- BigBite gives \vec{q} vector and interaction vertex
- Project to the surface of HCAL and compare to detected nucleon position/energy/time.

Neutron/proton separation

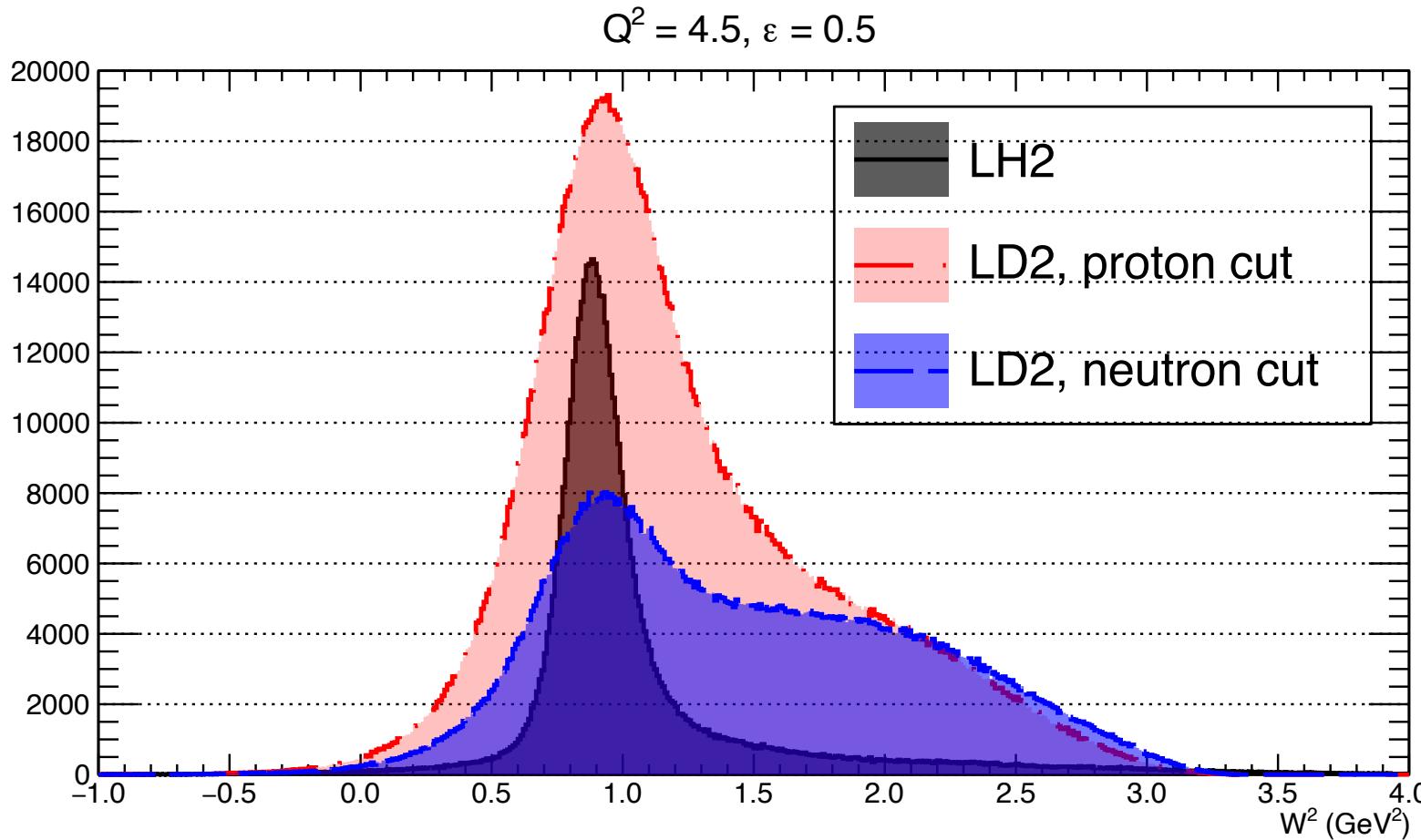


- Nucleon charge ID is accomplished by a small vertical deflection of protons in SBS magnet
- Optimal deflection is that which gives "clean" n/p separation while minimizing acceptance/efficiency difference between neutrons and protons
- "Fiducial cut" is calculated based on reconstructed *electron* kinematics—requires that both proton and neutron in quasi-elastic kinematics would hit HCAL active area with a safety margin equivalent to ~ 100 MeV Fermi smearing

Elastic event selection, $Q^2 = 4.5, \epsilon = 0.5$ (SBS-9)



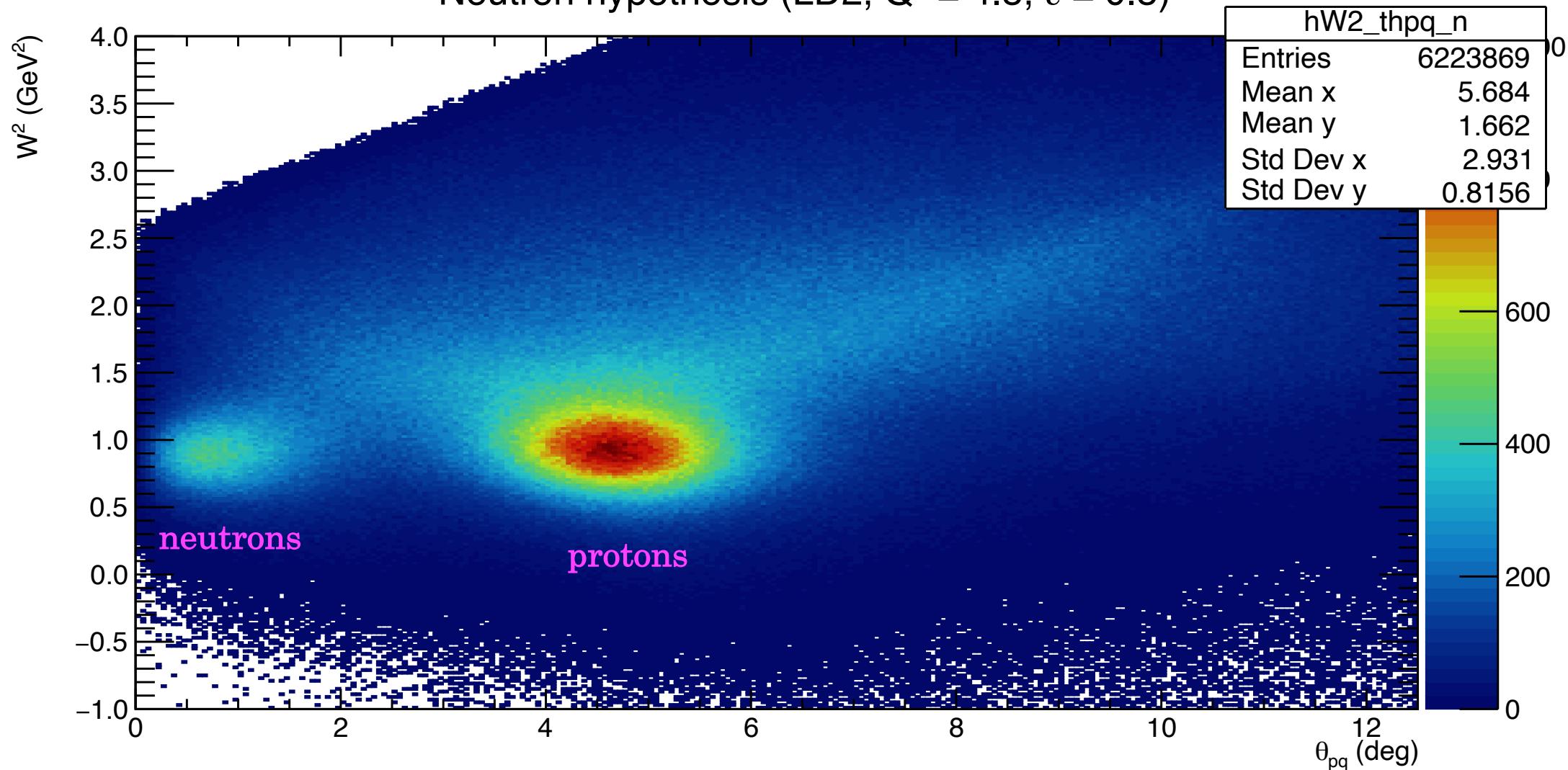
W^2 distributions with HCAL correlation cuts



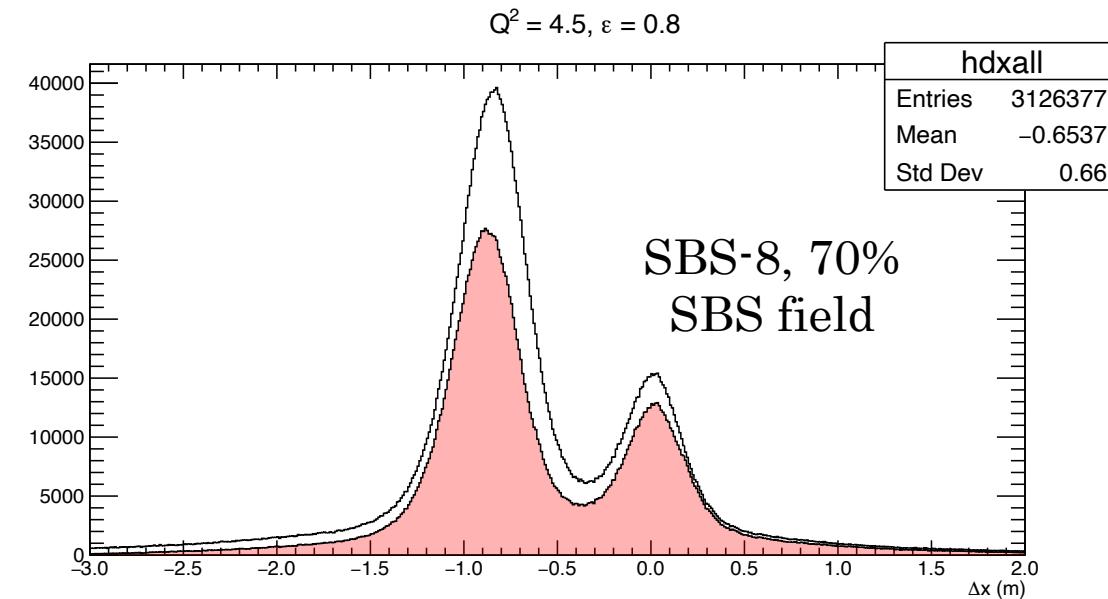
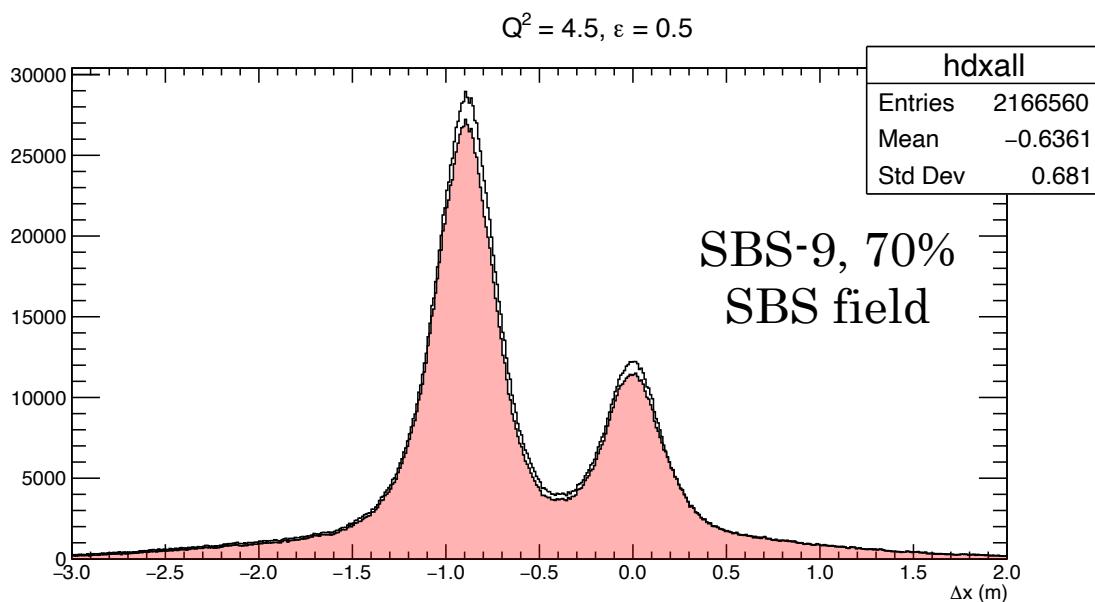
Invariant mass squared distributions obtained with 3σ elliptical cut around p, n “spots” in previous slide (also loose coincidence time cut and HCAL energy cut)

W^2 versus θ_{pq}

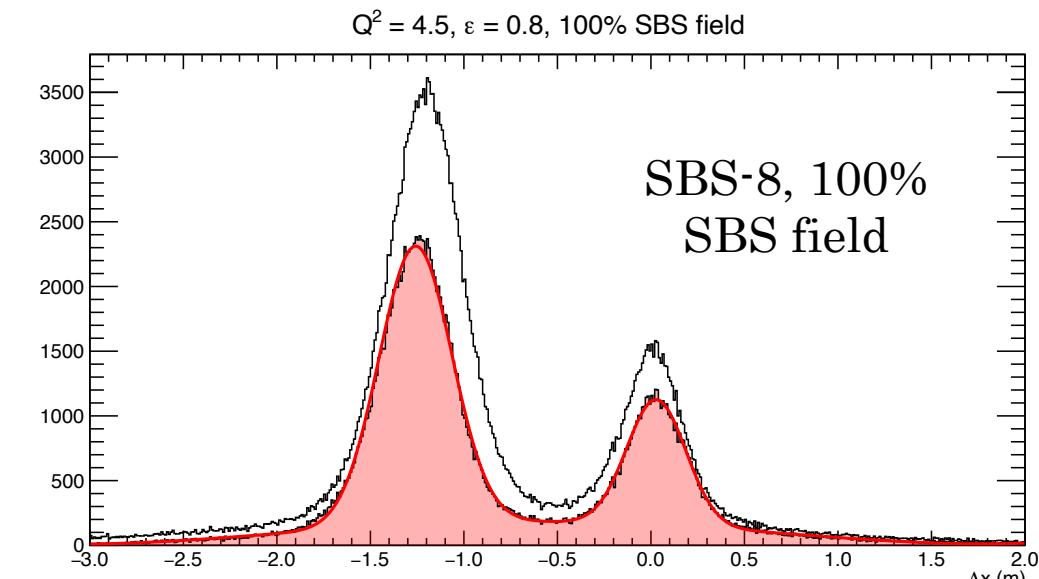
Neutron hypothesis (LD2, $Q^2 = 4.5$, $\varepsilon = 0.5$)



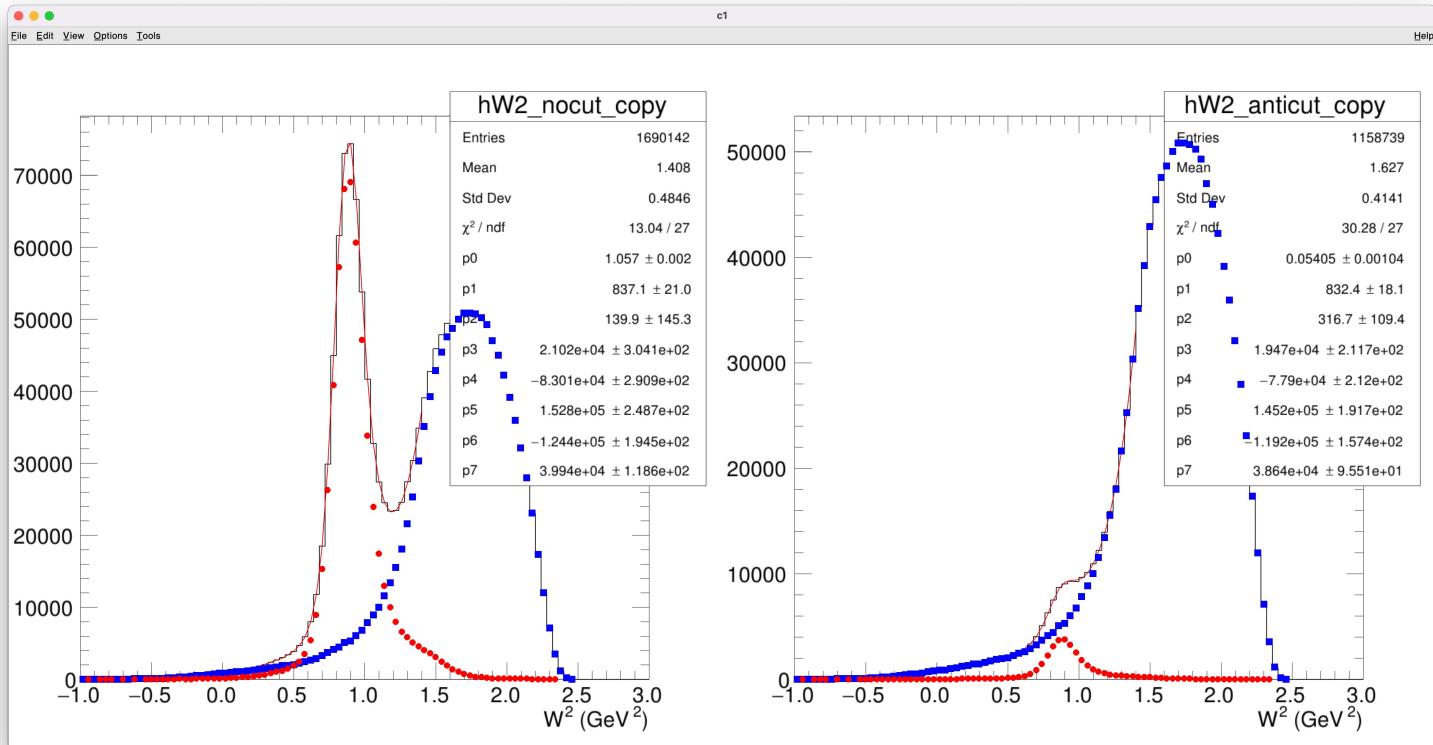
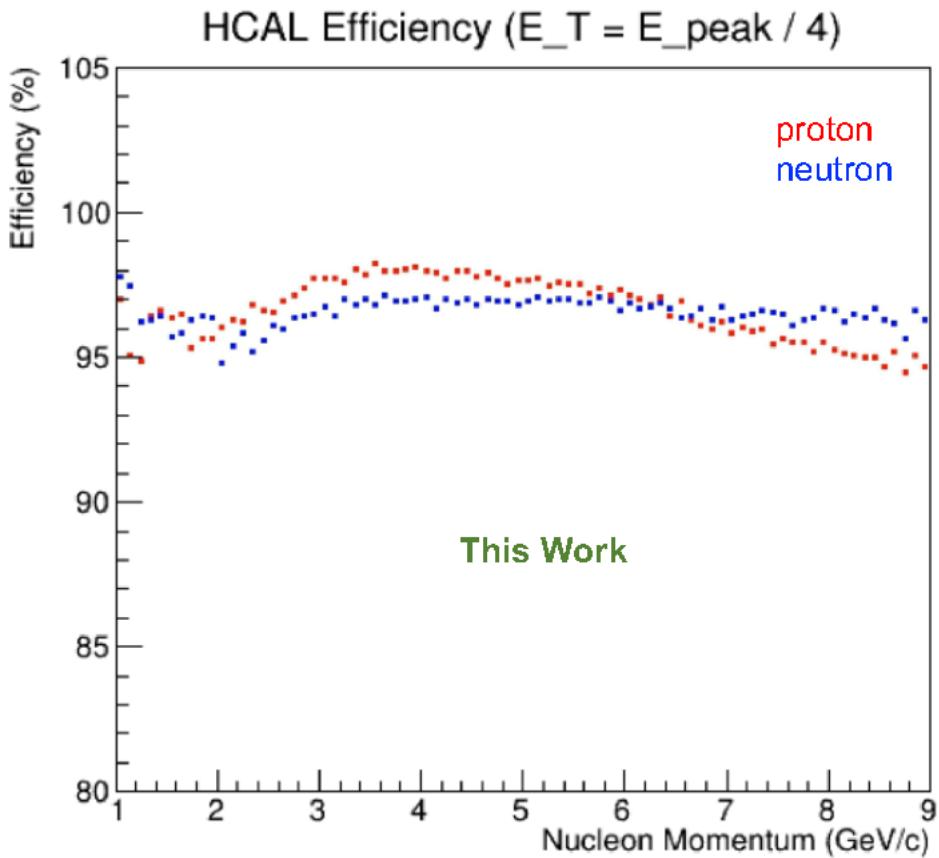
Rough n and p estimates from Δx projection, effect of fiducial cuts



- Δx projection **with (without)** fiducial cut shows effect of this cut for the two different ϵ points at 4.5 GeV^2
- Envelope of elastic events on HCAL is much smaller for low ϵ , effect of fiducial cut is much greater for high ϵ
- *Raw n/p* ratios from crude fit method are in good qualitative agreement across field settings AFTER applying fiducial cut
- Interpretation of ϵ dependence requires more precise analysis, inclusion of electric form factors, bin centering, etc



HCAL efficiencies from MC and data



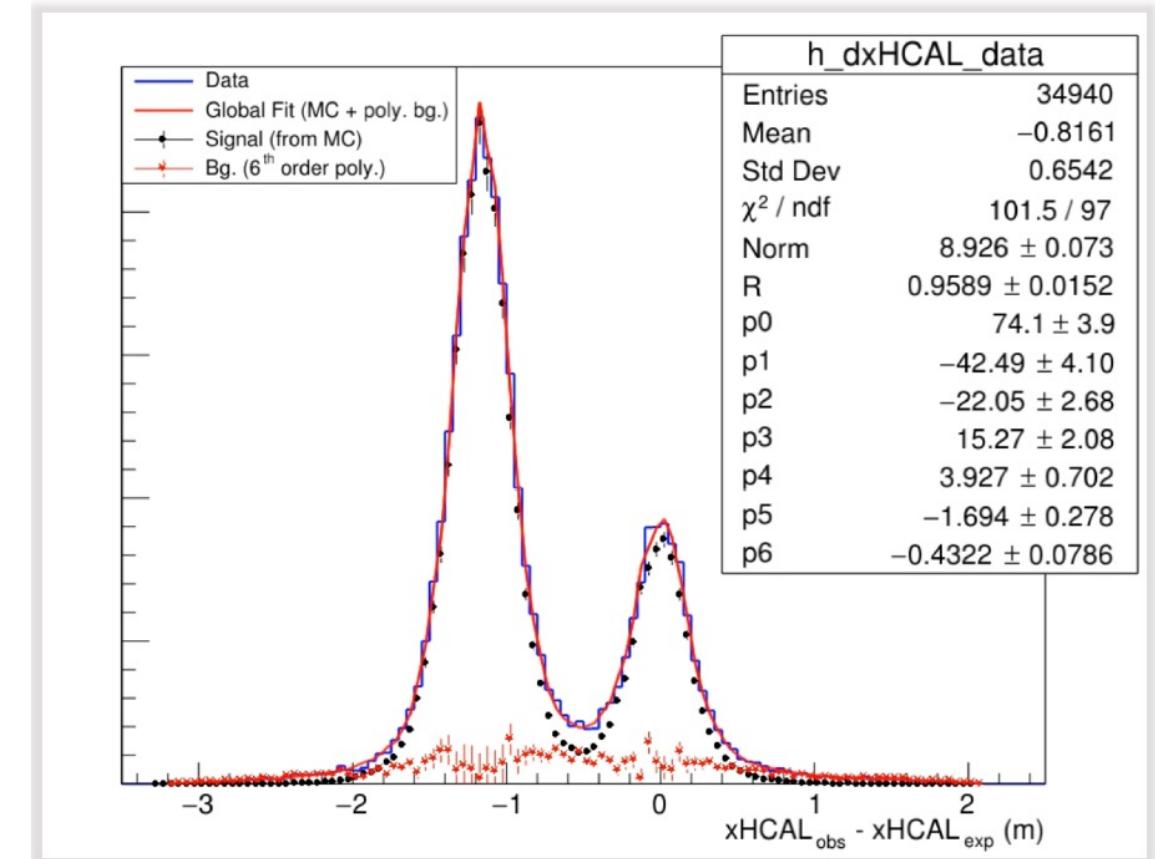
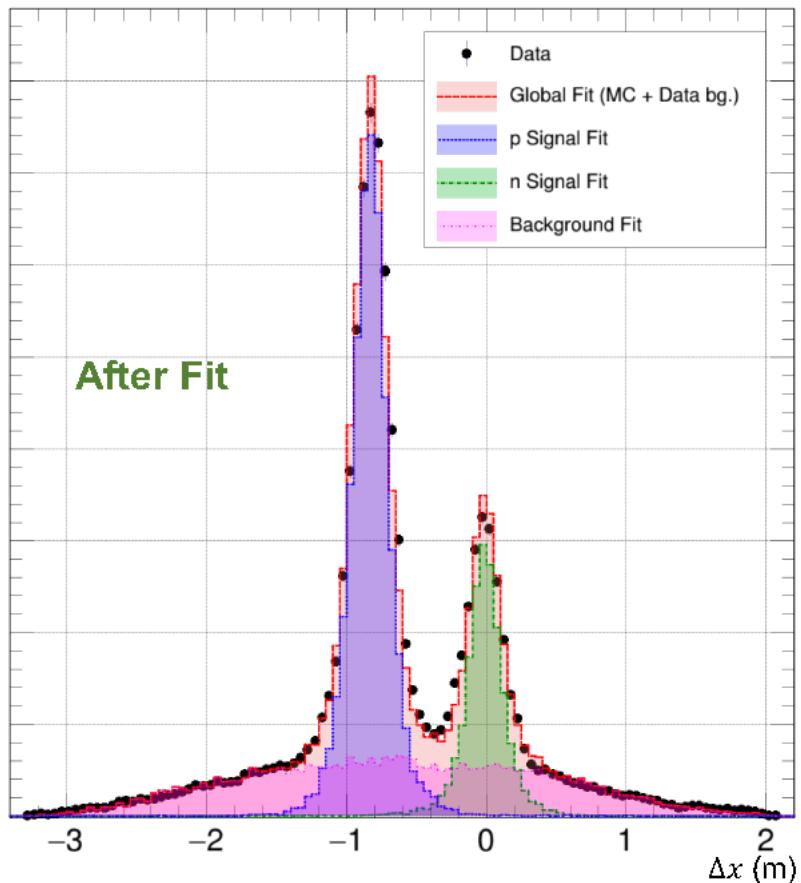
- Expected efficiencies are ~95-98% from simulation, almost independent of nucleon momentum
- Effectively realized efficiency is highly sensitive to real detector performance and analysis cuts, backgrounds, pileups, best cluster selection, reconstruction algorithm, etc.
- Use LH2 elastic data to benchmark proton efficiency
- **We presently rely on MC for neutrons! Undesirable!**

Toward real physics analysis: Data/MC comparisons

$Q^2 = 3 \text{ GeV}^2, 0.49 \leq W^2 \leq 1.44 \text{ GeV}^2, \text{Fiducial Cuts}$

$Q^2 = 7.4 \text{ GeV}^2$

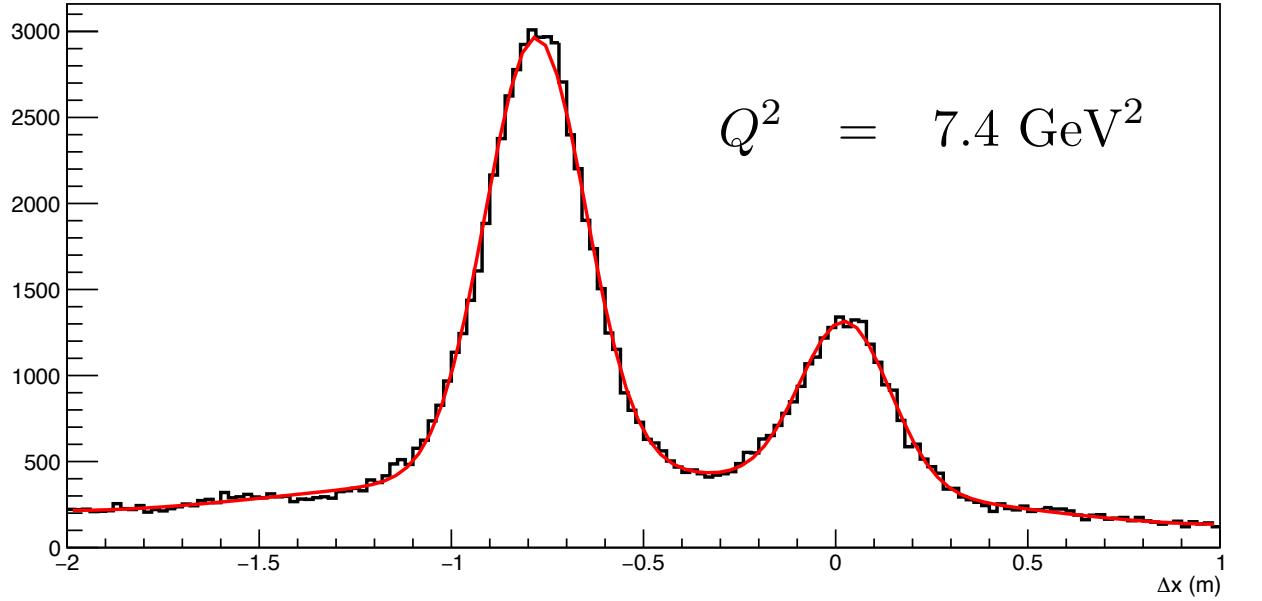
$0.5 \leq W^2 \leq 1.3 \text{ GeV}^2$
 $|\Delta y| < 0.3 \text{ m}$
Fiducial Cuts



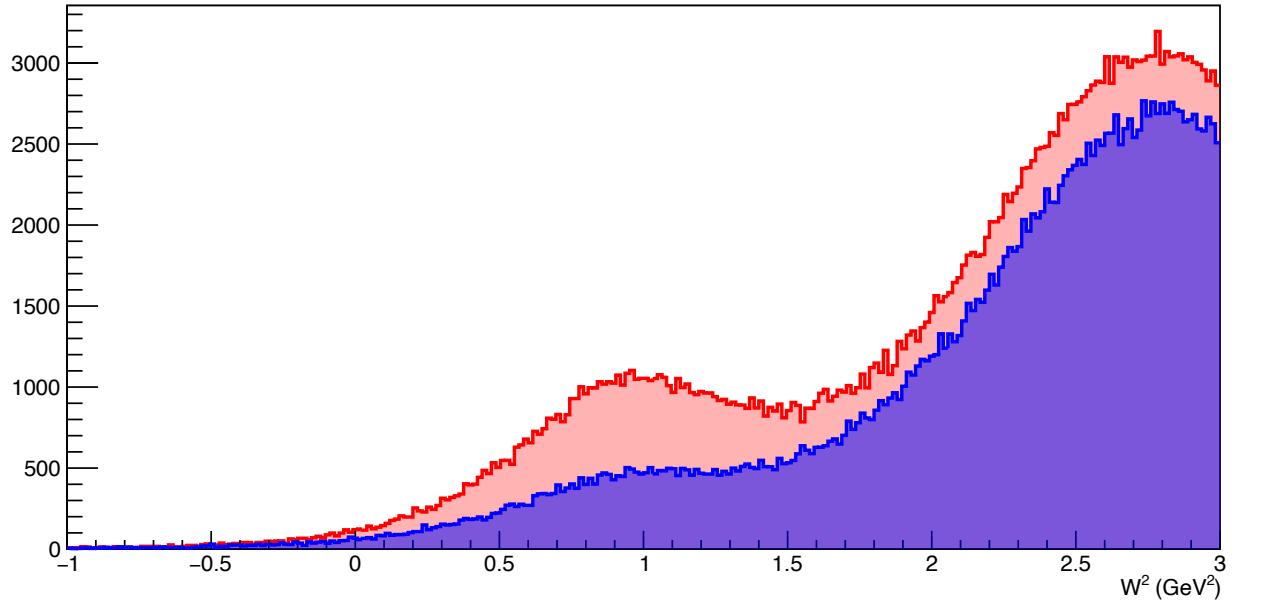
- See Provakar's talk for progress on SIMC modeling of deuteron Fermi motion/binding and radiative effects

Initial look at high- Q^2 data

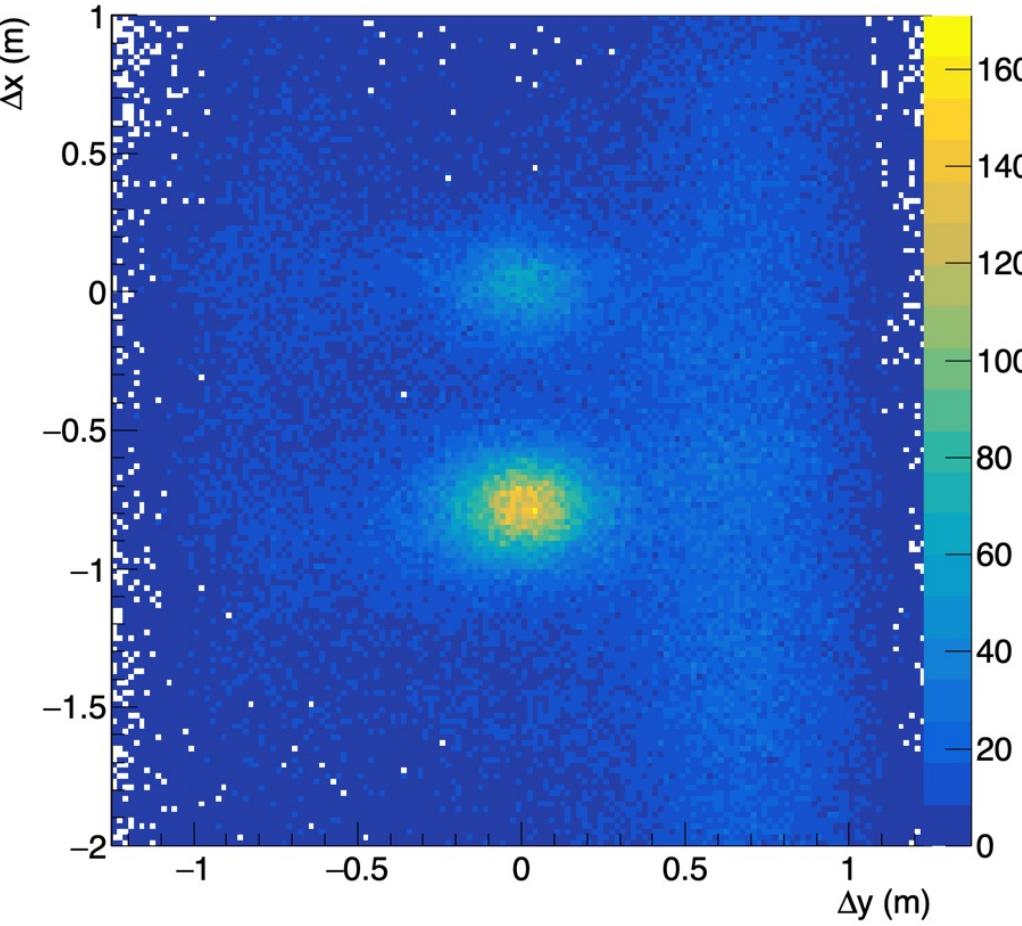
SBS-14 ($Q^2 = 7.4$), $|W^2 - 0.88| < 0.5 \& |\Delta y| < 0.3$ m



SBS-14 ($Q^2 = 7.4$ GeV 2)

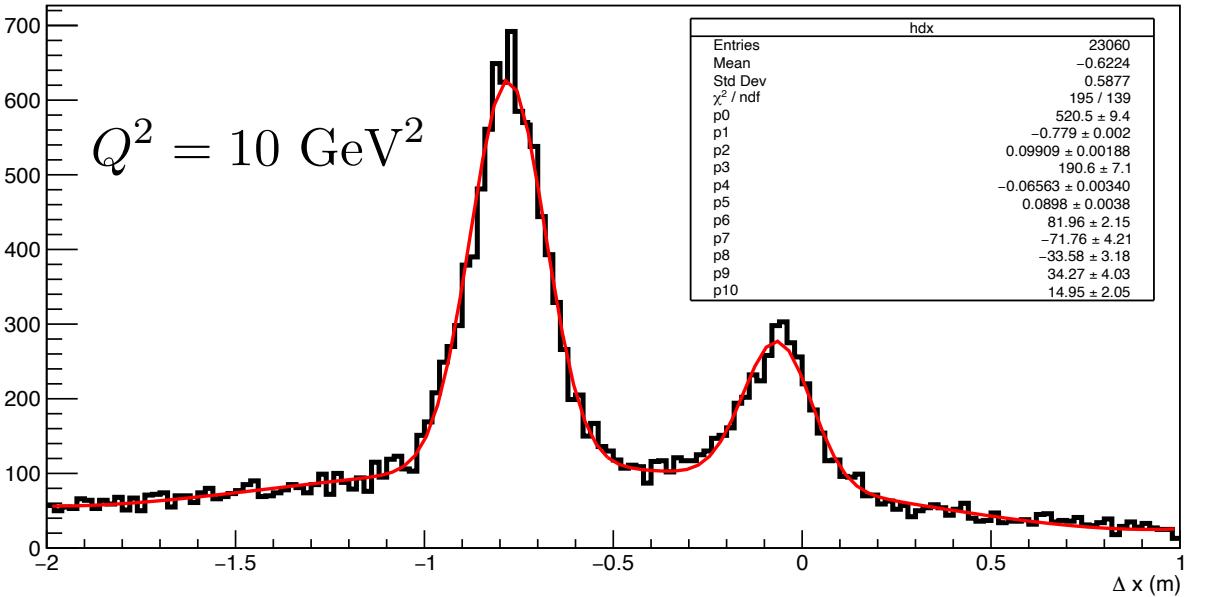


SBS-14 ($Q^2 = 7.4$), $|W^2 - 0.88| < 0.5$

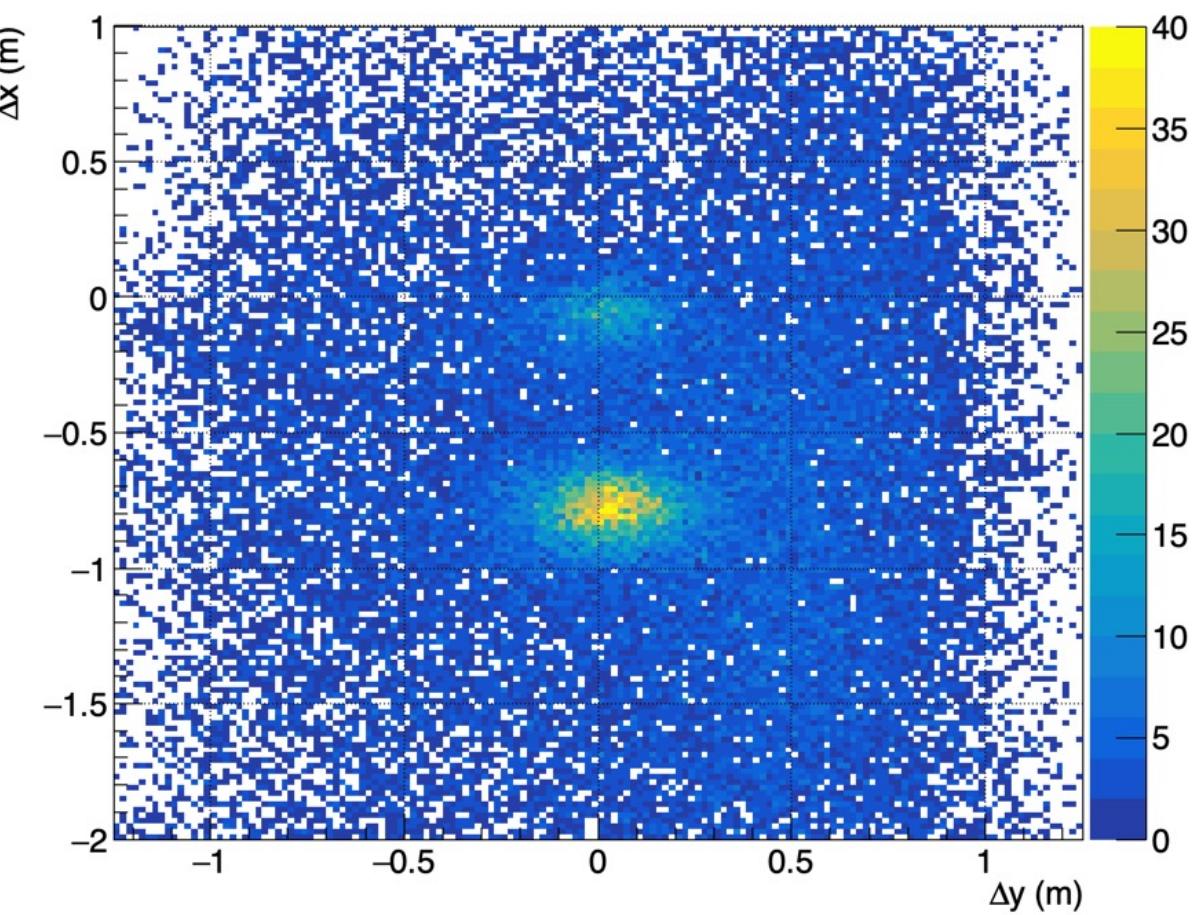


- SBS-14 ($Q^2 = 7.4$ GeV 2): "deltax" (top left), "dx vs dy" (top right), W^2 (bottom left):
- **Proton cut ($\theta_{apq} < 0.025$ under proton hypothesis)**
- **Neutron cut ($\theta_{apq} < 0.025$ under neutron hypothesis)**

SBS-7 ($Q^2 = 10$), $|W^2 - 0.88| < 0.5$, $|\Delta y| < 0.3$ m

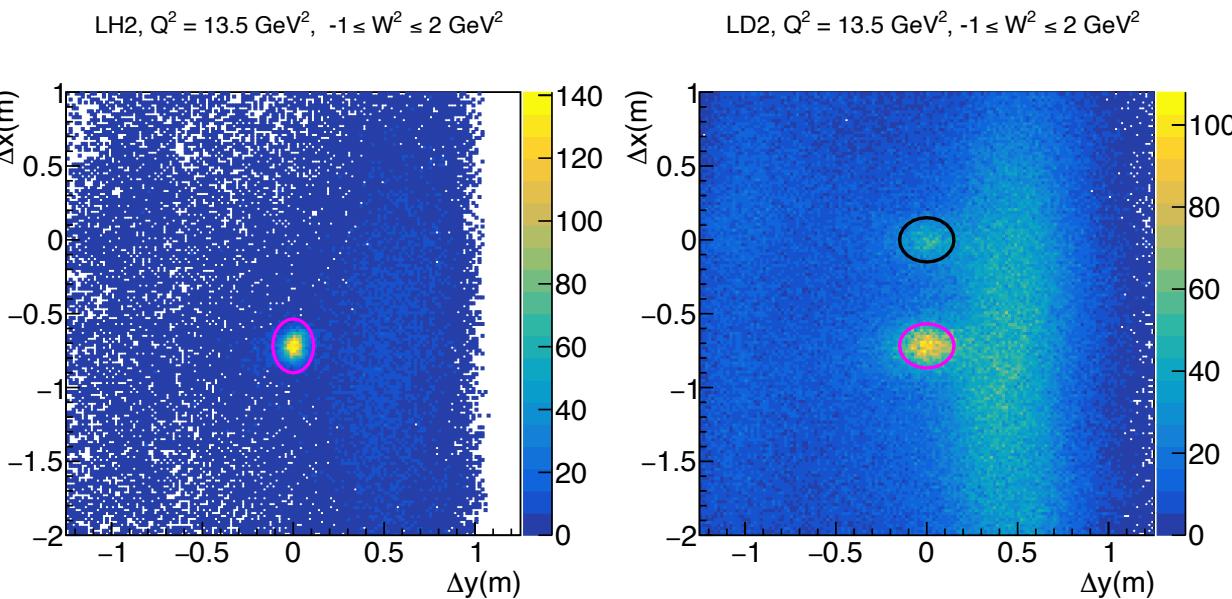
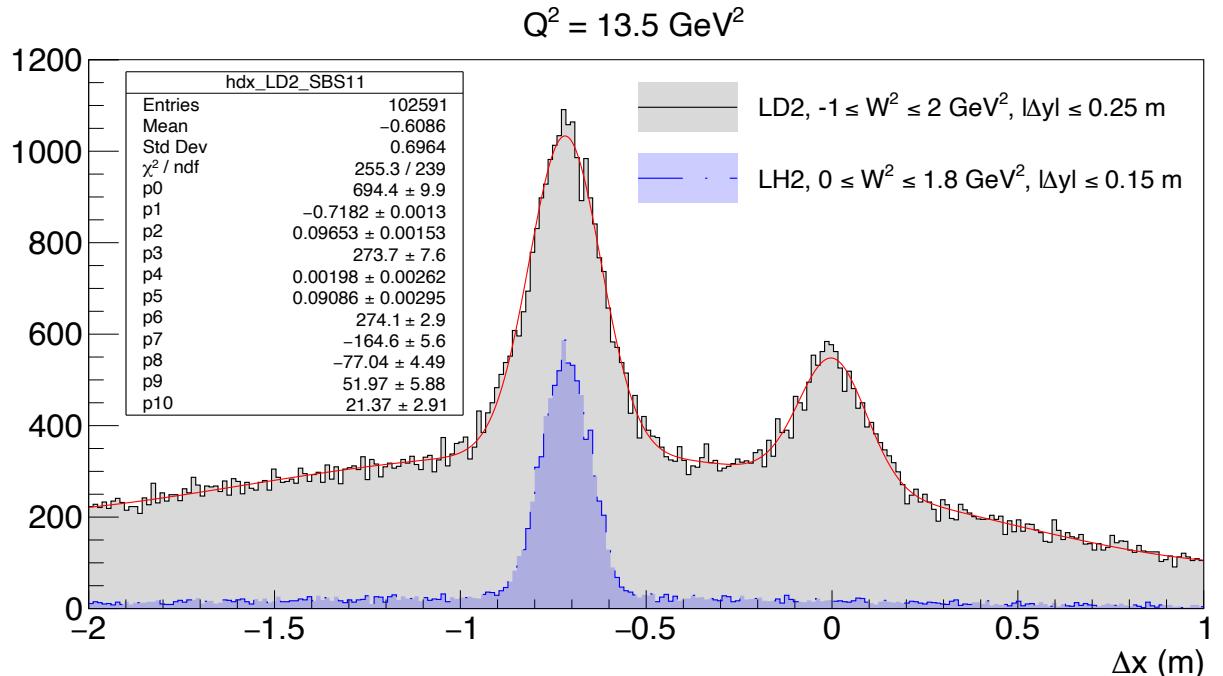


SBS-7 ($Q^2 = 10 \text{ GeV}^2$), $|W^2 - 0.88| < 0.5 \text{ GeV}^2$

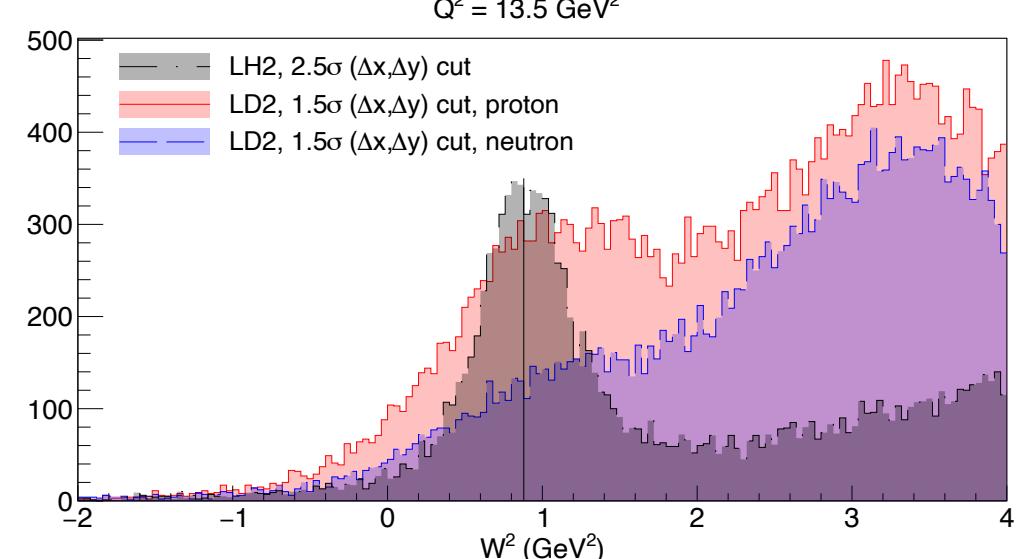


- SBS-7 ($Q^2 = 10 \text{ GeV}^2$): "deltax" (top left), "dx vs dy" (top right), W^2 (bottom left):
- **Proton cut ($\text{thetapq} < 0.02$ under proton hypothesis)**
- **Neutron cut ($\text{thetapq} < 0.02$ under neutron hypothesis)**

GMN Quasi-Elastic Event Selection, $Q^2 = 13.5 \text{ GeV}^2$



- Above, right: $(\Delta x, \Delta y)$ distribution for LH2 and LD2
- Above, left: Δx distributions for LH2, LD2
- Bottom, right: W^2 distributions for LH2, LD2 with proton, neutron cuts



SBS GMN analysis challenges—toward reliable physics results

- Neutron efficiency calibration?
 - Attempt to obtain a “tagged neutron” sample from analysis of downbending π^+ tracks in BigBite from $\gamma p \rightarrow \pi^+ n$ for several kinematics favorable for “endpoint” method → SBS-4 and SBS-9 look potentially viable (this is being investigated by Anuruddha from UVA)
- Do our HCAL proton efficiencies from H₂ elastic data REALLY agree with MC to the required accuracy?
- Is our methodology for estimating HCAL proton efficiency from LH₂ elastic data valid/reliable?
- HCAL energy calibration uncertainties and data/MC mismatch in HCAL energy spectrum
- HCAL cluster multiplicity per event and best cluster selection—maximizing reconstruction efficiency for QE while avoiding any charge-dependent bias from cuts
- Improved data/MC comparisons in W^2 , Δx , Δy , etc. for signal and inelastic background systematics. How much fine-tuning of the signal MC is needed? We need a background MC. How sophisticated does it need to be for our purposes? How to model/estimate non-physics background (fake GEM tracks, accidentals, etc)?
- Need to deal with inefficiency of SIMC quasi-elastic event generators relative to MC statistics needed for GMN analysis (nTPE in particular)

SBS GMN/nTPE analysis challenges—continued

- Improvement of timing analysis for all detectors—understanding the logic of signal timing during GMN:
 - Calibrate the timing hodoscope well enough for it to be useful for analysis!
 - Reconstruct nucleon momentum from time-of-flight
- Understanding (and improving) GEM tracking efficiency and GEM hardware performance during GMN. Not super critical for GMN physics analysis, but important for future SBS and Hall A experiments
- Finishing next round of detector calibrations and quality checks and completing a 2nd reconstruction pass for GMN/nTPE
- Generating a common set of “official” simulated, digitized, reconstructed quasi-elastic events for data/MC comparisons and R_{np} extractions
- Developing (and automating?) multiple redundant cross checks of R_{np} extractions, evaluation of systematics, and control of nTPE systematics to the required level to achieve physics goal(s).

SBS-GMN analysis status—Summary and Conclusions

- The bad news: as of today, we are far from having even preliminary physics results
 - Analysis momentum has (temporarily) slowed, notably since SBS GEN started
 - We (software/analysis team) need to redouble our analysis efforts to keep on track toward timely publication
 - The next steps of analysis are the most fun (but also the most difficult)
- The good news:
 - First cooking pass confirms the SBS GMN/nTPE dataset is of sufficient quality (and quantity!) to achieve the precision/accuracy/ Q^2 goals for this physics
 - Rapid analysis progress was made in the first year after the experiment
 - After “pass 2”, essential event reconstruction and detector calibrations for the GMN/nTPE dataset will be mature enough to extract (preliminary) physics results.
 - The SBS GMN result at 13.5 GeV^2 will be unrivaled in terms of precision and Q^2 reach for years (if not decades) to come (so let’s get it done, but take the time to get it right)!

