The Gerasimov-Drell-Hearn (GDH) Sum Rule in Hall D

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Sum Rules

Generally derived by combining dispersion relations and the optical theorem.
Many also have alternative derivations, OPE or QCD on the LC

$$\int_{\nu_0}^{\infty} \frac{\Delta\sigma(\nu)}{\nu} \mathrm{d}\nu = \frac{4\pi^2 S\alpha\kappa^2}{M^2}$$

GDH Sum Rule

$$\int_{\nu_0}^{\infty} \frac{\sigma_P + \sigma_A}{\nu^2} \mathrm{d}\nu = 4\pi^2(\alpha_E + \beta_M)$$

Baldin sum rule, electric and magnetic polarizabilities

$$\int_{\nu_0}^{\infty} \frac{\Delta\sigma(\nu)}{\nu^3} \mathrm{d}\nu = -4\pi^2\gamma_0$$

forward spin polarizability

$$\begin{aligned}\Delta\sigma &= \sigma_{3/2} - \sigma_{1/2} \\ &= \sigma_P - \sigma_A \\ &= \sigma^{\overset{\rightarrow}{\Rightarrow}} - \sigma^{\overset{\leftarrow}{\Rightarrow}}\end{aligned}$$

This sign convention gives proton and neutron positive GDH values.

GDH Sum Rule

Spin-dependent photoproduction cross-sections

$$\int_{\nu_{thr}}^{\infty}(\sigma^{3/2}-\sigma^{1/2})\frac{d\nu}{\nu} = \frac{4\alpha S\pi^2\kappa^2}{M^2}$$

anomalous magnetic moment

spin

Photon energy

Mass

Fundamental Quantum Field Theory prediction. Applicable to any type of target.

Links the anomalous magnetic moment κ of a particle to its helicity-dependent photoproduction cross-sections

Conditions for the sum rule to be valid:

  Spin-dependent forward Compton amplitude f2(v) must vanish at large v (no-subtraction hypothesis).

  Imaginary part of f_2, $(\sigma^{3/2}-\sigma^{1/2})$ must decrease with v faster than ~1/ln(v) (for the integral to converge).

Experimentally verified on the proton to ~10% but not yet for the neutron.

Derivation

Mode than 1 method:
- 1966 Gerasimov, Drell and Hearn: dispersion theoretic approach
- 1966 Hosoda and Yamamoto: current algebra formalism
- 1972 Dicus and Palmer: light-cone

Dispersion theory derivation uses the following fundamental principles.

Lorentz invariance
Gauge invariance
Crossing symmetry
Rotational invariance
Causality
Unitarity (the optical theorem) - Connect elastic scattering (here, Compton scattering) to the total cross section

Derivation (dispersion relation)

forward real Compton scattering amplitude $F(\nu)$

$$F(\nu) = f_1(\nu)\,\boldsymbol{\epsilon_2}^* \cdot \boldsymbol{\epsilon_1} + f_2(\nu)\,\boldsymbol{\sigma}(\boldsymbol{\epsilon_2}^* \times \boldsymbol{\epsilon_1})$$

Baldin (unpolarized) GDH (polarized)

$\boldsymbol{\epsilon_1}, \boldsymbol{\epsilon_2}$ polarization of photon in and out, $\boldsymbol{\sigma}$ Pauli matrices

Real photons only have 2 terms

$f_2(\nu)$ is analytic in the Complex plane (causality), Cauchy relation

$$\begin{aligned} f_2(\nu) &= \frac{1}{2i\pi} \oint \frac{f_2(\varepsilon)}{\varepsilon - \nu}\,\mathrm{d}\varepsilon \\ &= \frac{1}{2i\pi} \int_{-\infty}^{+\infty} \frac{f_2(\varepsilon)}{\varepsilon - \nu}\,\mathrm{d}\varepsilon \end{aligned}$$

if Jordan Lemmas hold: $f_2(\nu) \to 0$ as $\nu \to \infty$

$$\Re e\big(f_2(\nu)\big) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{\Im m\big(f_2(\varepsilon)\big)}{\varepsilon - \nu}\,\mathrm{d}\varepsilon$$

Kramer-Kronig relation

Derivation (dispersion relation)

Crossing symmetry implies $f_2(\varepsilon) = -f_2(-\varepsilon)^*$

$$\Re\left(f_2(\nu)\right) = \frac{2\nu}{\pi} P \int_0^{+\infty} \frac{\Im\left(f_2(\varepsilon)\right)}{\varepsilon^2 - \nu^2} d\varepsilon$$

$\mathrm{Im}\, f_2(\varepsilon) = \dfrac{1}{8\pi}(\sigma_P(\varepsilon) - \sigma_A(\varepsilon))$ optical theorem for forward Compton scattering

A low energy theorem (using Lorentz and gauge invariance and crossing symmetry) used to expand f_2 in ν

$$f_2(\nu) = -\frac{\alpha\kappa^2}{2M^2}\nu + \gamma\nu^3 + \mathcal{O}(\nu^5)$$

Take derivative and substitute in

$$\left.\frac{df_2(\nu)}{d\nu}\right|_{\nu=0} = \frac{\alpha\kappa^2}{2M^2} = \frac{1}{4\pi^2}\int_{\nu_0}^{\infty} (\sigma_P - \sigma_A)\frac{\mathrm{d}\nu}{\nu}$$

Extension to virtual photons

Due to its connection with the Bjorken sum rule, the extension of the integral to finite Q provides a bridge from the non-perturbative region to the perturbative region of QCD.

well defined over the entire Q2-range

$$I_{\mathrm{GDH}}(Q^2) = \int_{\nu_0}^{\infty} \frac{\Delta\sigma(\nu, Q^2)}{\nu}$$

in pQCD

$$I_{\mathrm{GDH}}(Q^2) = \frac{8\pi^2\alpha}{M} \int_0^{x_0} \frac{g_1(x, Q^2) - \gamma^2 g_2(x, Q^2)}{K} \frac{dx}{x}$$

$$\int_0^1 g_1(x, Q^2)dx = \frac{Q^2 S_1(0,Q^2)}{8}$$

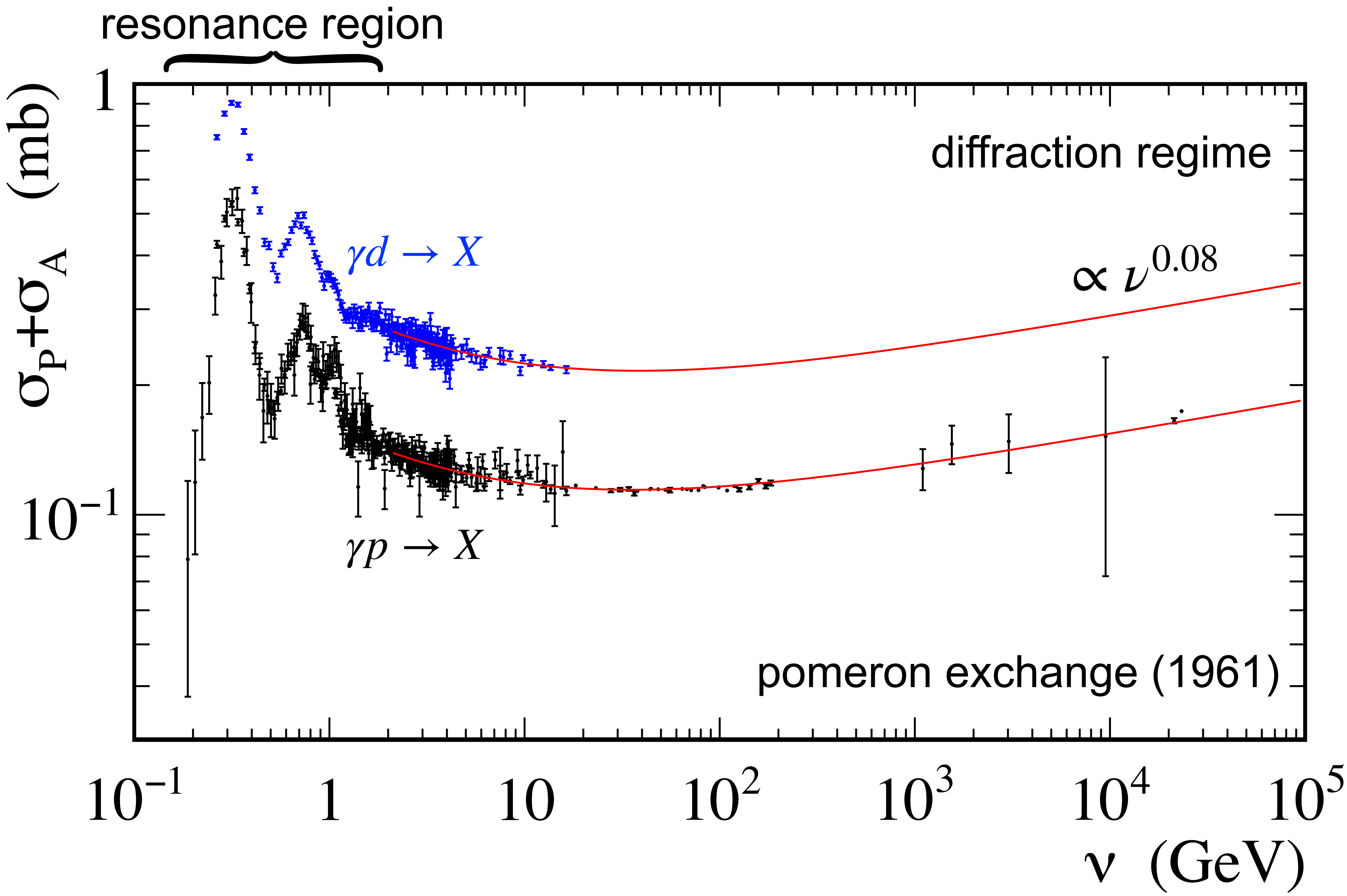
$S_1(\nu, Q^2)$ first forward virtual Compton scattering amplitude

Measurement

Current situation and existing data

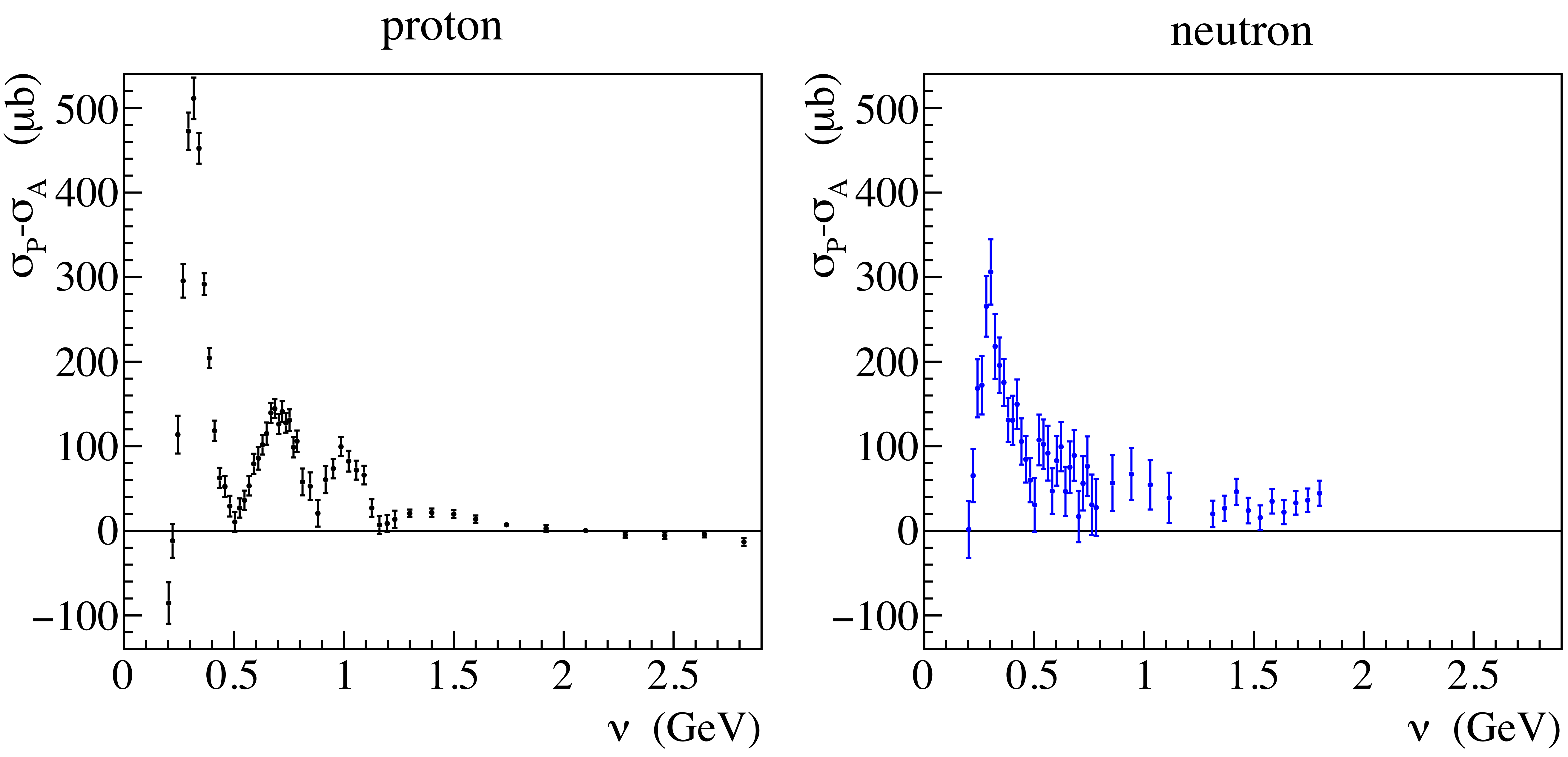
Photoproduction

Unpolarized version of GDH integral $\int(\sigma^{3/2} + \sigma^{1/2})d\nu$ does not converge.



Helicity dependent photoabsorption

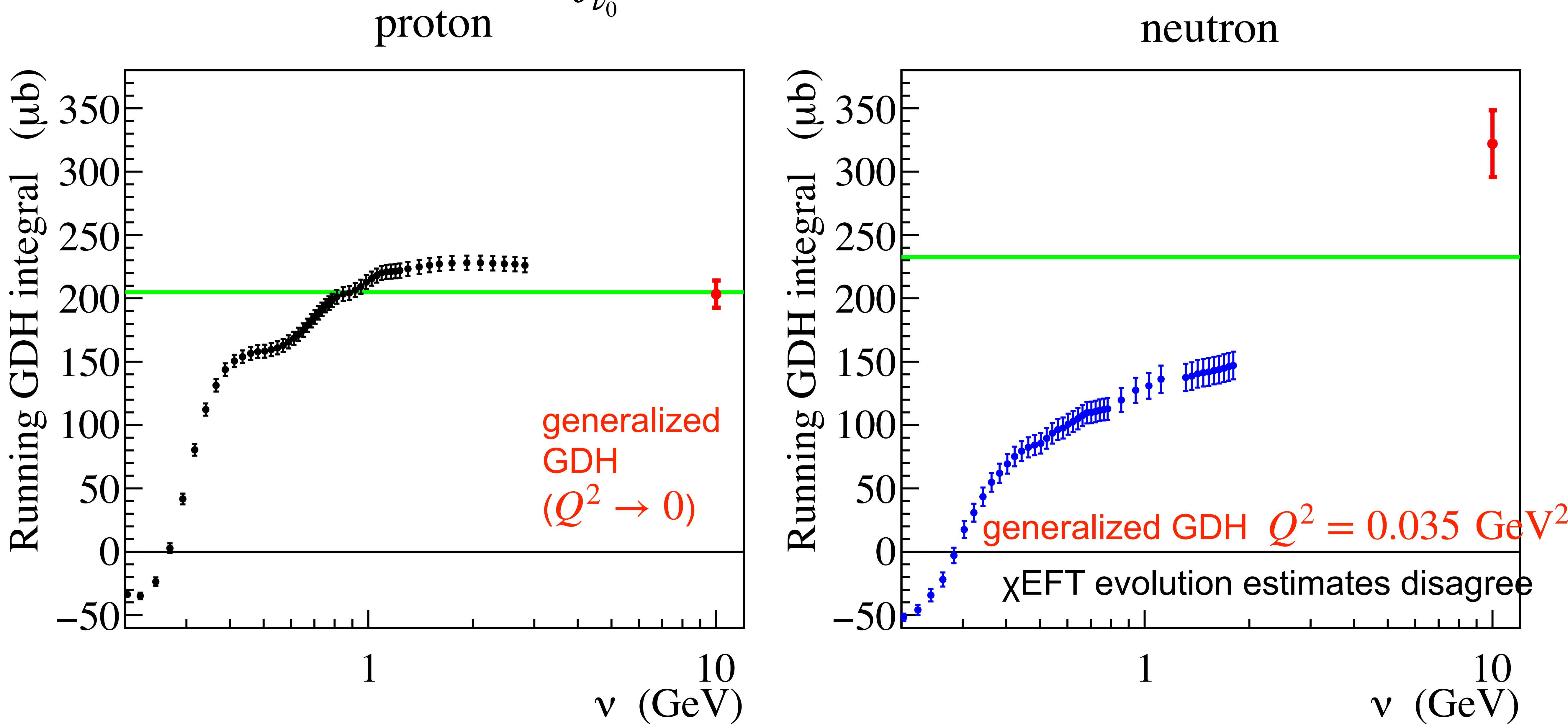
Existing data from MAMI and ELSA. Partial contributions from LEGS and CLAS.



Threshold and high energy regions cannot be measured,
need models like MAID/SAID and Regge phonomenology.

GDH Integral

$$\int_{\nu_0}^{\infty} \frac{\Delta\sigma(\nu)}{\nu}\,\mathrm{d}\nu = \frac{4\pi^2 S\alpha\kappa^2}{M^2}$$



Unmeasured part estimated using Regge model. Dominates uncertainty.

Has not converged yet

Contributions below 0.2 GeV: ≈ -28 μb (proton), ≈ -41 μb (neutron)

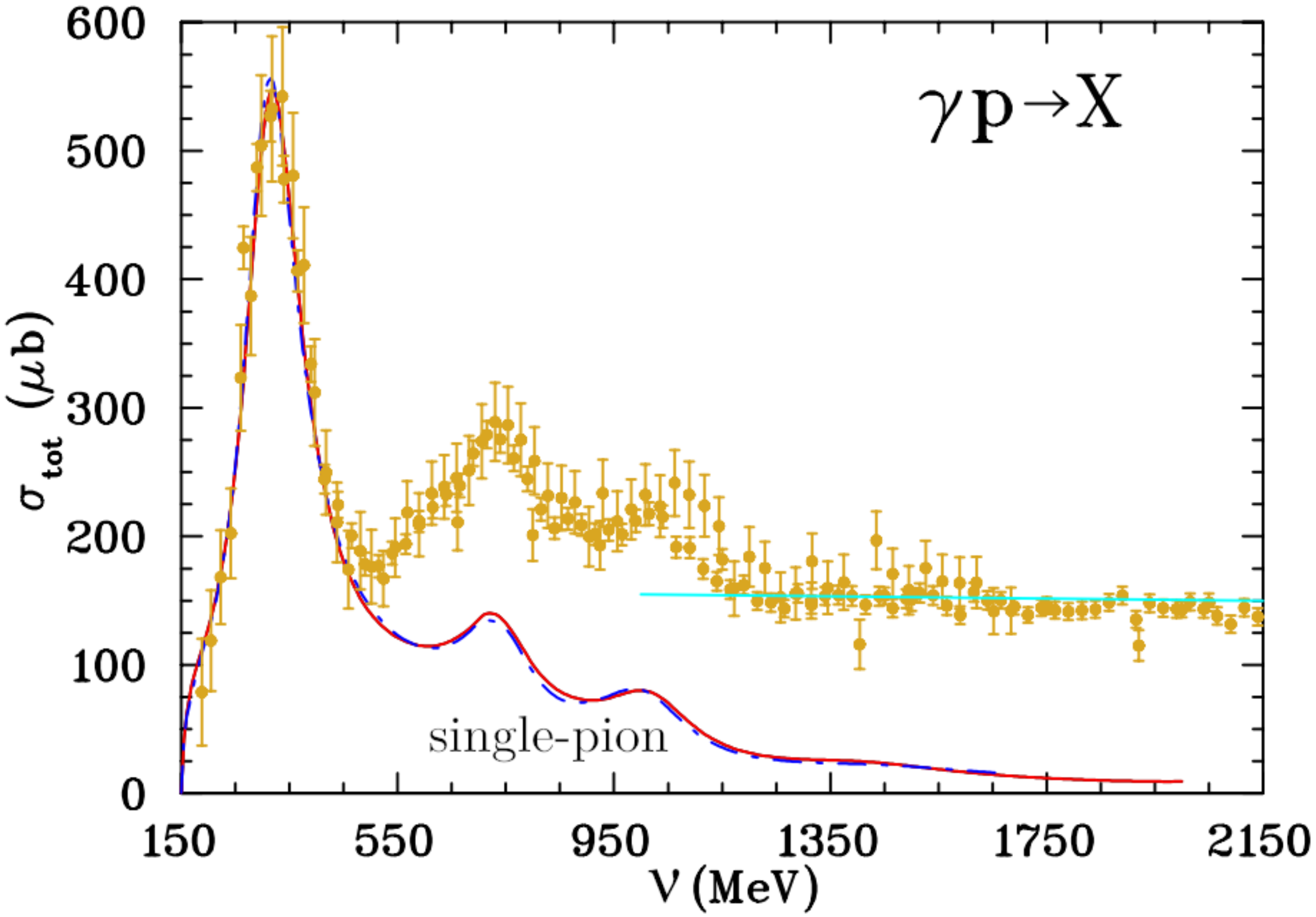
Threshold Region

Very important due to $1/\nu$ weight

MAID and SAID both give $I^{p}_{\mathrm{GDH}}(\nu \leqslant 0.2 \text{ GeV}) \approx -28\ \mu\mathrm{b}$

Dynamical Models for Pion Photo- and Electroproduction on the Nucleon

Based on fits to large amounts of low energy scattering data.



Regge Phenominology

Regge theory at high ν: $\Delta\sigma(\nu) \propto (\nu + M/2)^{\alpha_0 - 1}$

α_0 is Regge intercept

Isovector part: $\Delta\sigma^{p-n} \equiv \Delta\sigma^p - \Delta\sigma^n$

determined by $a_1(1260)$

Isoscaler part: $\Delta\sigma^{p+n} \equiv \Delta\sigma^p + \Delta\sigma^n$

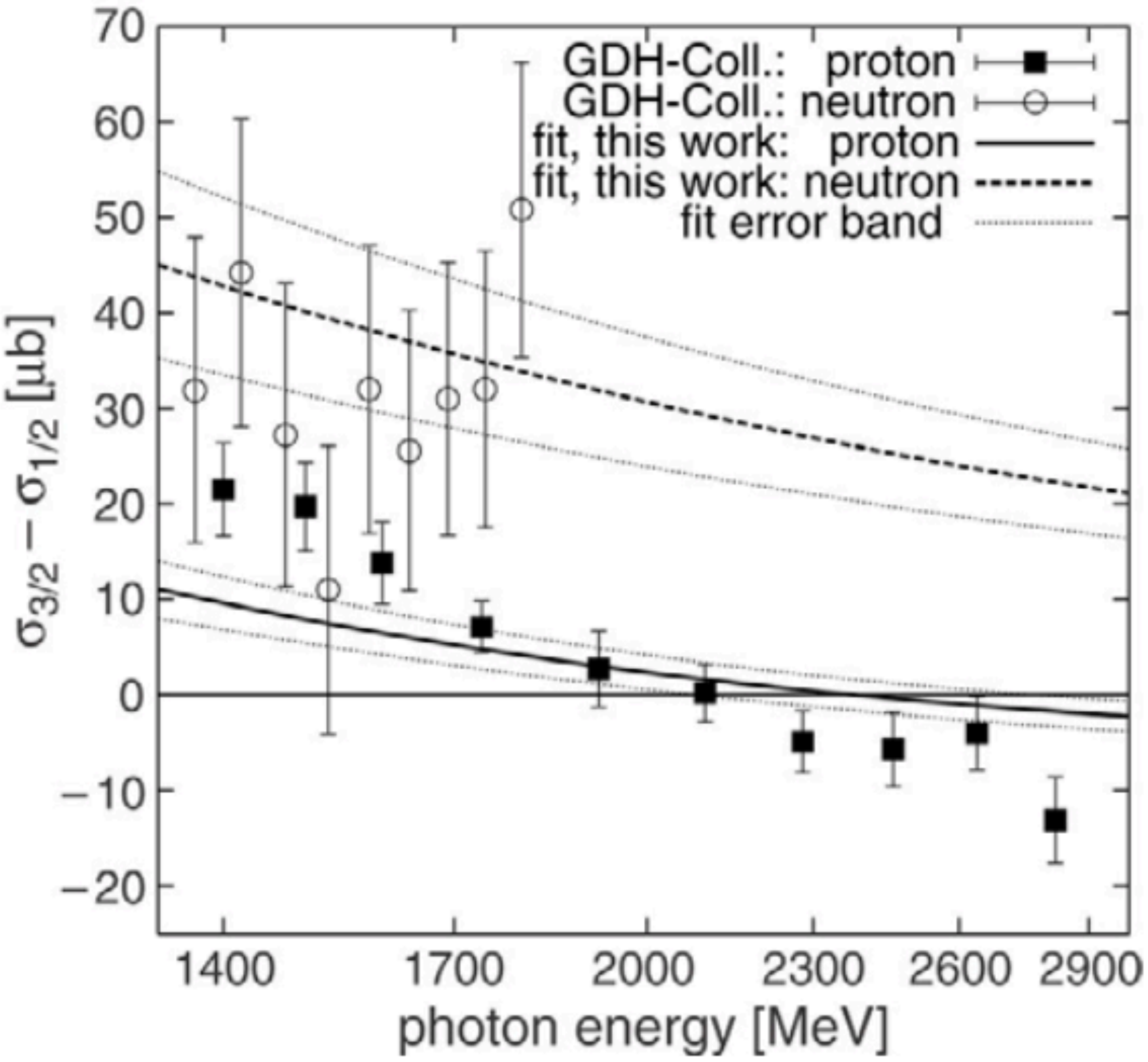
determined by $f_1(1285)$

$$\sigma_P - \sigma_A = Ic_1 s^{\alpha_{a_1} - 1} + c_2 s^{\alpha_{f_1} - 1}$$

isovector isoscaler

Value of c_2 unknown and assumed zero in some analyses since existing polarization measurements on deuteron in diffractive regime consistent with 0.

Regge Phenominology



Fits on proton only give a Regge contribution of $-20\ \mu\text{b}$ to $-35\ \mu\text{b}$

combined fits give $-14\ \mu\text{b}$ but don't agree well with the data.

The current uncertainty from this is

$$I_p = 226 \pm \overset{\substack{\text{stat}\\ \downarrow}}{5} \pm \overset{\substack{\text{syst}\\ \downarrow}}{12} \pm \underset{\substack{\uparrow\\ \text{large-}\nu\text{ projection}}}{10}\ \mu\text{b}$$

$$I_p = \frac{4\pi^2\alpha\kappa^2}{M^2} = 204.8\ \mu\text{b}$$

Violation of the Sum Rule

Unknown high energy phenomena
  eg quark substructure (quark anomalous magnetic moment)

J=1 pole of the nucleon Compton amplitude

Chiral anomaly (anomalous nonconservation of a chiral current)

other, more exotic possibilities heave been proposed

REGGE Real Gamma GDH Experiment

Measure the high energy behavior of $\Delta\sigma(\nu)$

Verify **convergence** of integral
  $\Delta\sigma(\nu)$ must decrease faster than $1/\log\nu$

Test **validity** of sum rule for
  neutron (first time)
  proton improve from

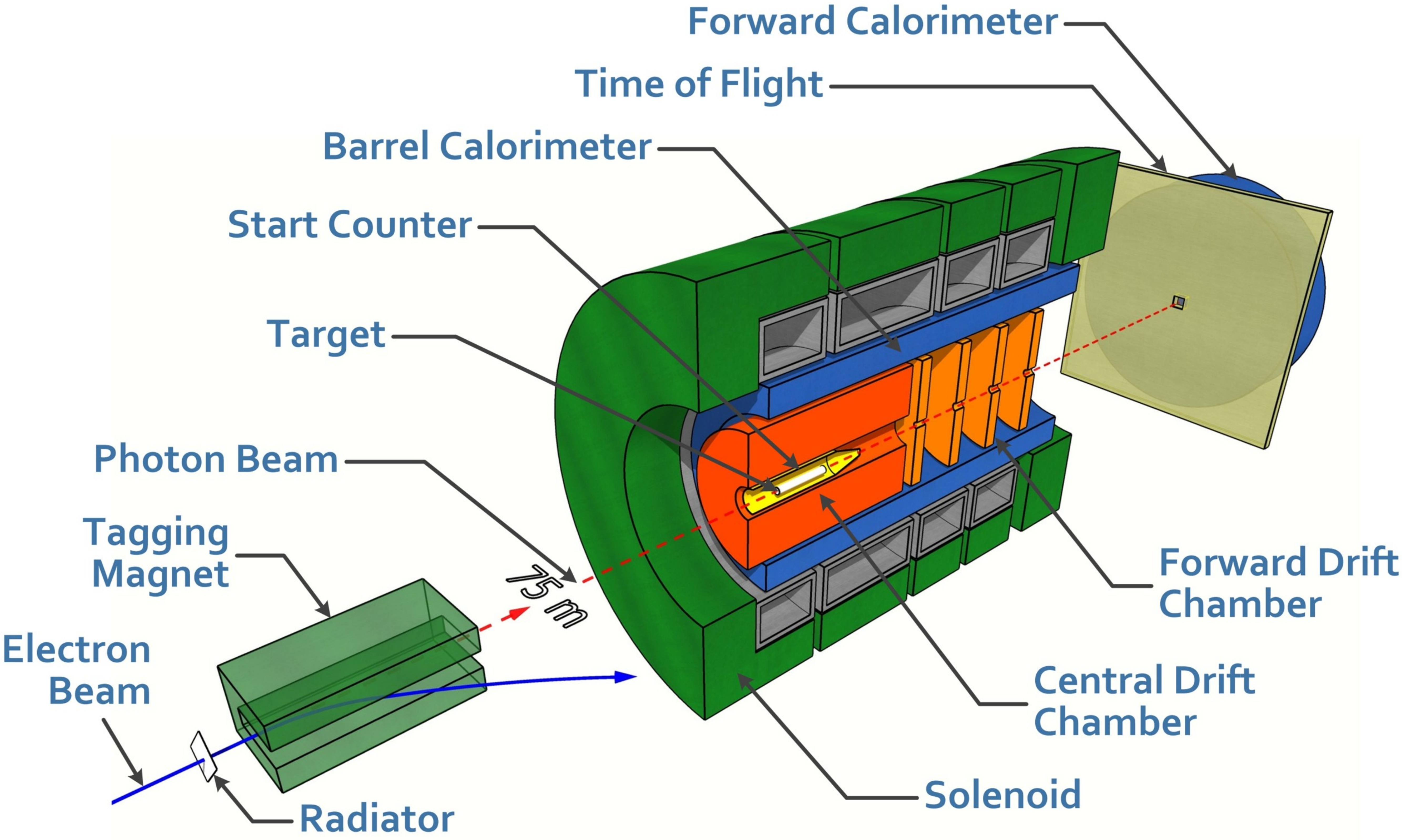
Improve sensitivity to physics that would cause a real (or apparent $\nu \neq \infty$) violation

Failure of sum rule would occur at high energy

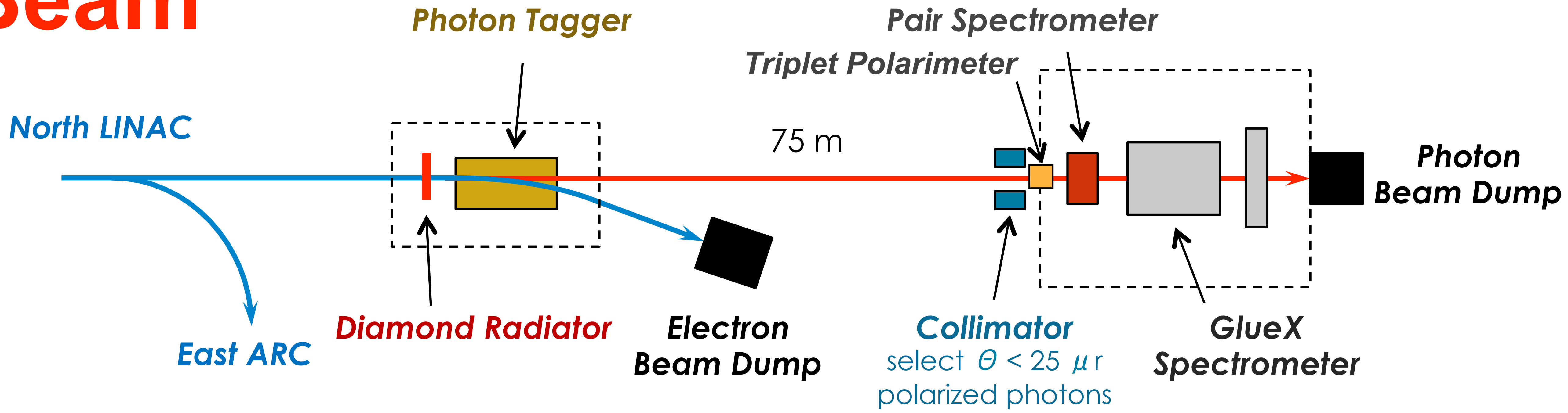
Resolve discrepancy in Regge parameter determination

Isospin decomposition

The GDH Experiment in Hall D



Beam



Photon Tagger

Pair Spectrometer

Triplet Polarimeter

North LINAC

75 m

Photon Beam Dump

East ARC

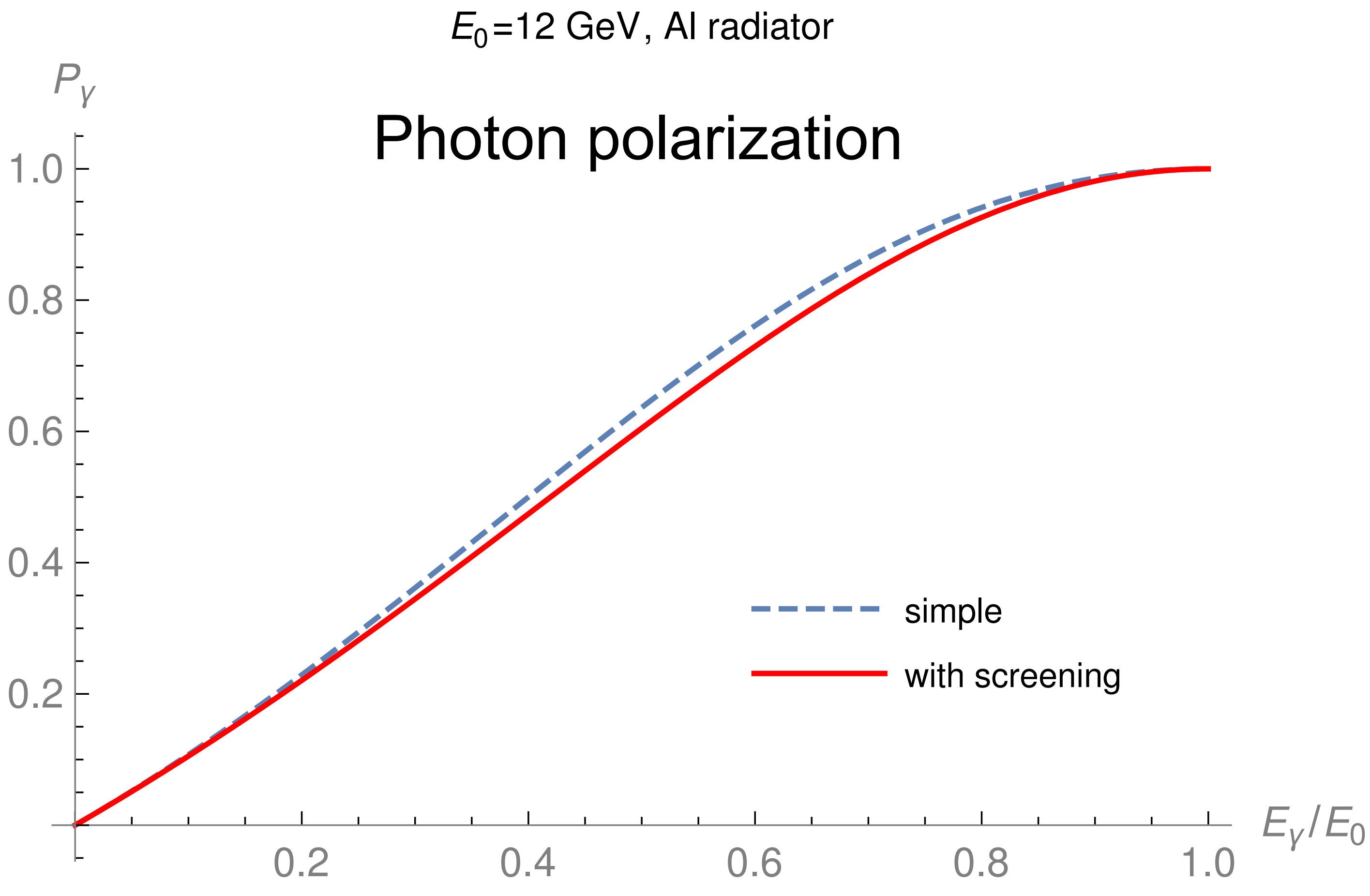
Diamond Radiator

Electron Beam Dump

Collimator
select $\theta < 25\ \mu r$
polarized photons

GlueX Spectrometer

Requires longitudinally polarized electrons.
Electron polarization transferred to photon depending on energy.



Target

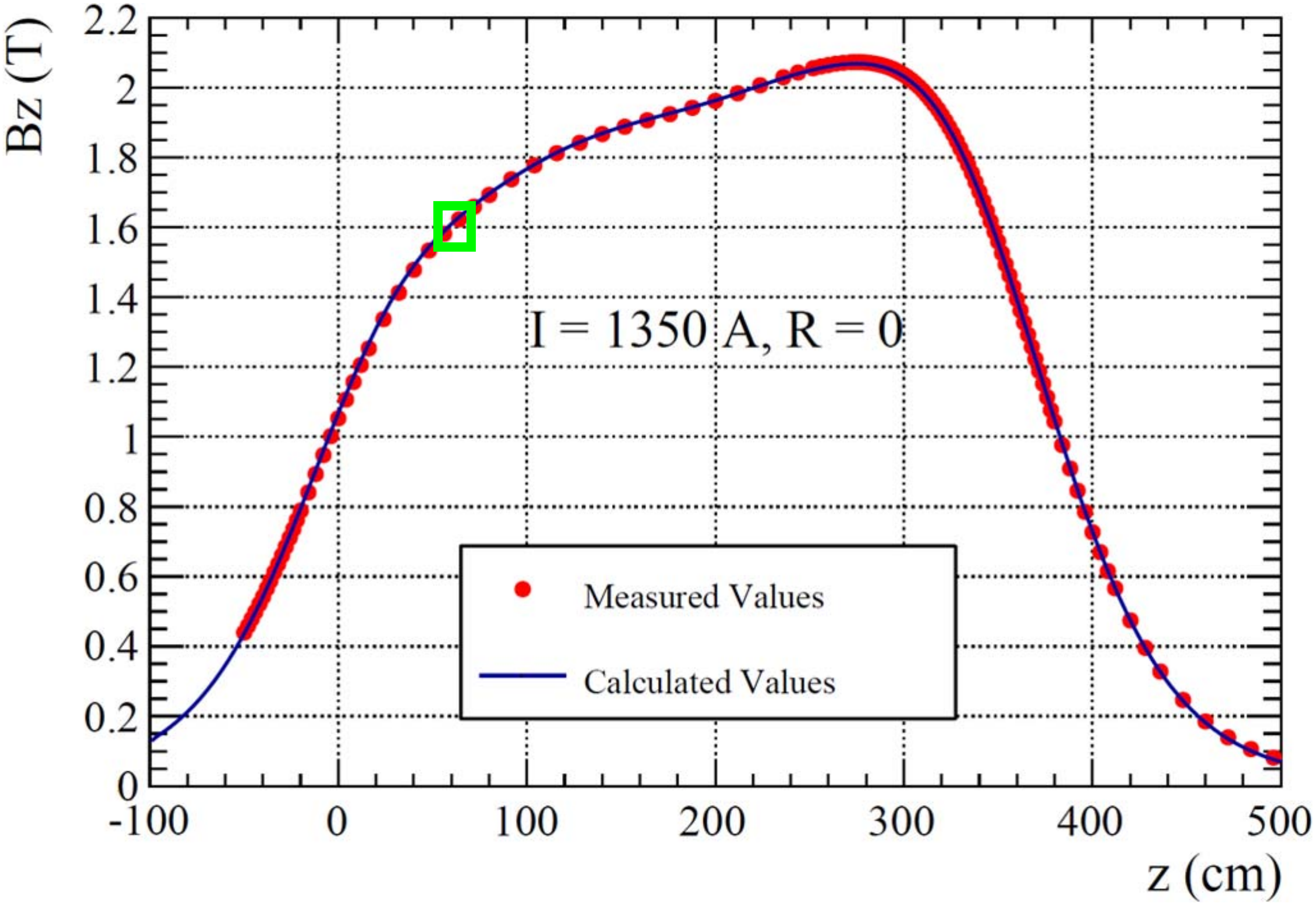
A new polarized target for Hall D
Dynamic Nuclear Polarization (Continuously polarize target in place)
on butanol (C_4H_9OH), *p* and *d* polarizations up to 90 %

- Requires high (2.5 T) and very uniform (300 ppm) magnetic field
- Requires very low temperatures (300 mK)
- Requires paramagnetic impurities at about 10^{-4} level
- At 300 mK and 2.% T, unpaired electrons are polarized >99.9%

Microwaves induce spin-flip transitions transferring polarization to the nuclear spins.

Currently region has field ~1.60 to ~1.65 T.

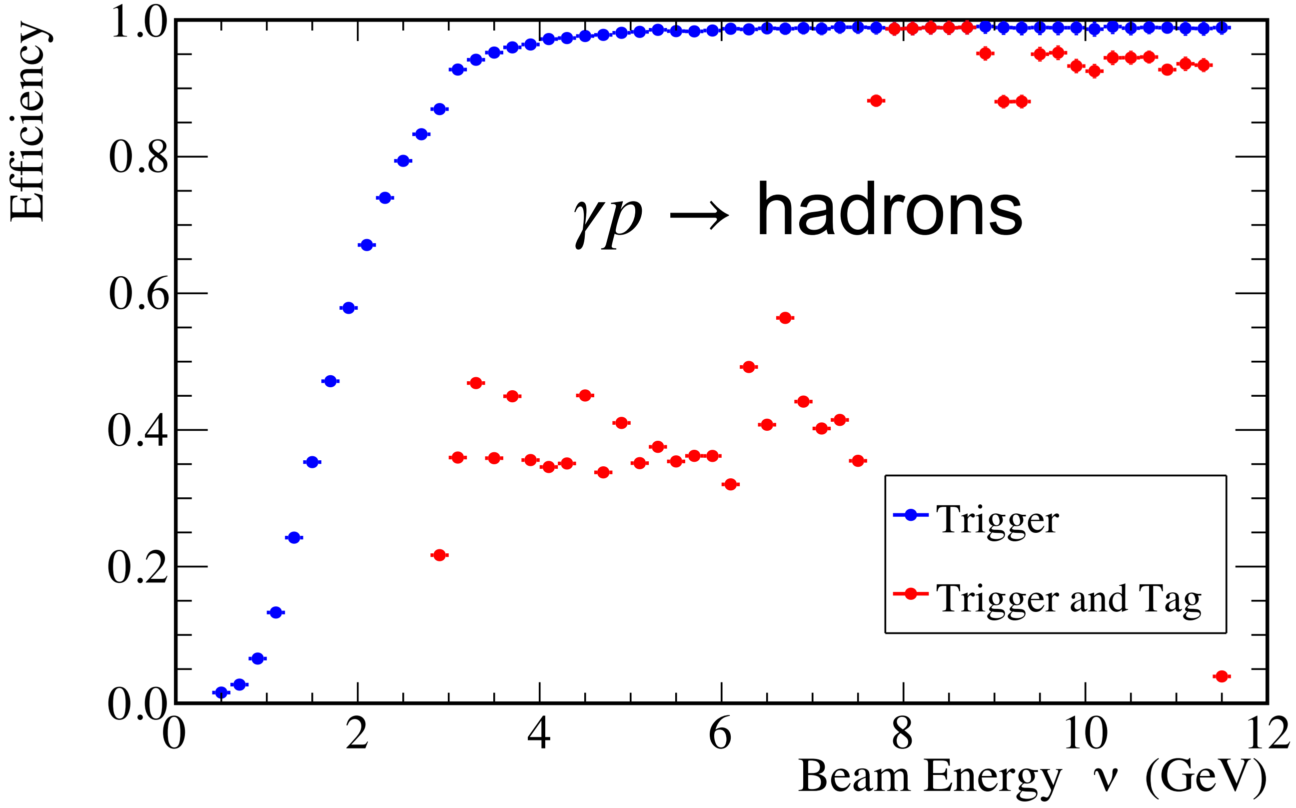
Superconducting coils installed in target will raise field to 2.5 T and make it more homogenous.

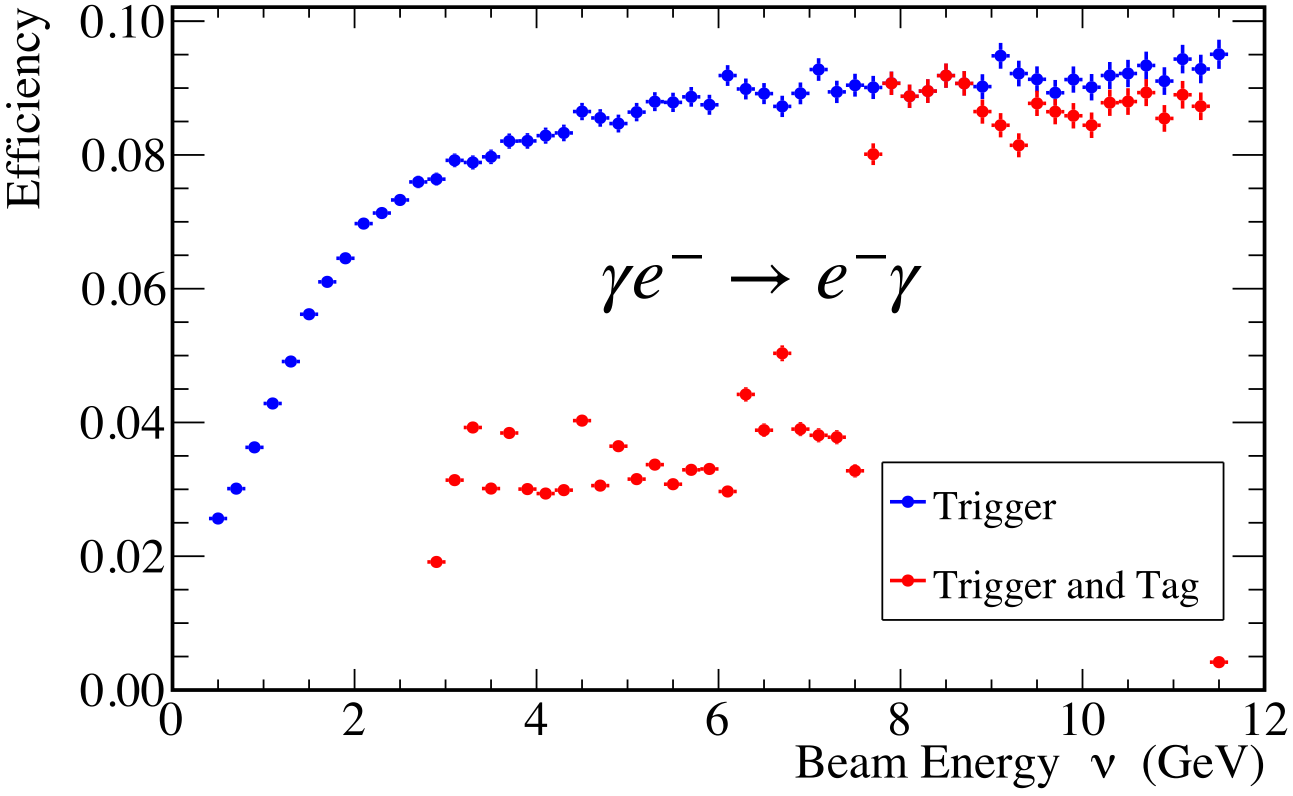


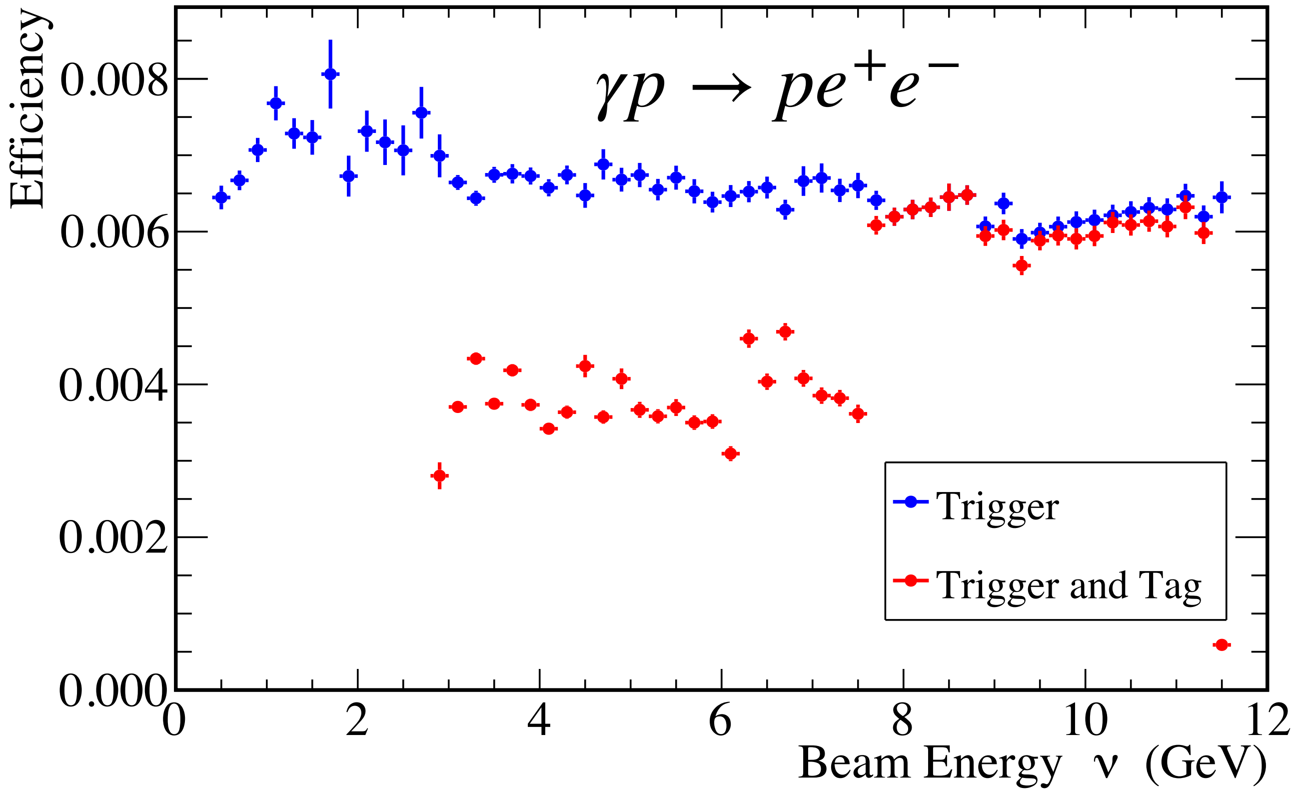
Signal and Background

Using a trigger which requires a high energy deposited in the calorimeters
$$E_{\mathrm{BCAL}} + 2E_{\mathrm{FCAL}} > 1\ \mathrm{GeV}$$

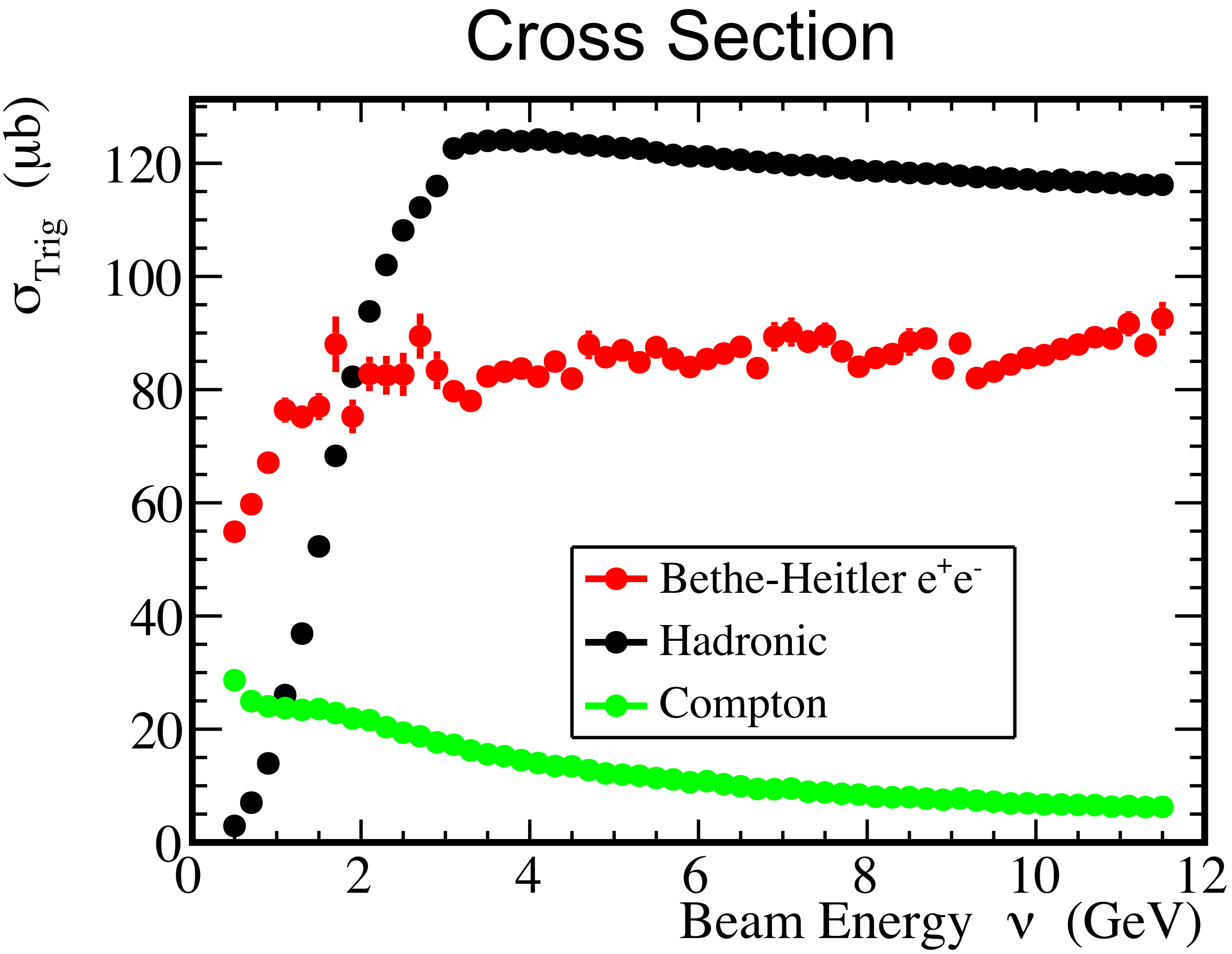
but cuts out very small angles, first 3 blocks of FCAL $\theta \gtrsim 2^\circ$

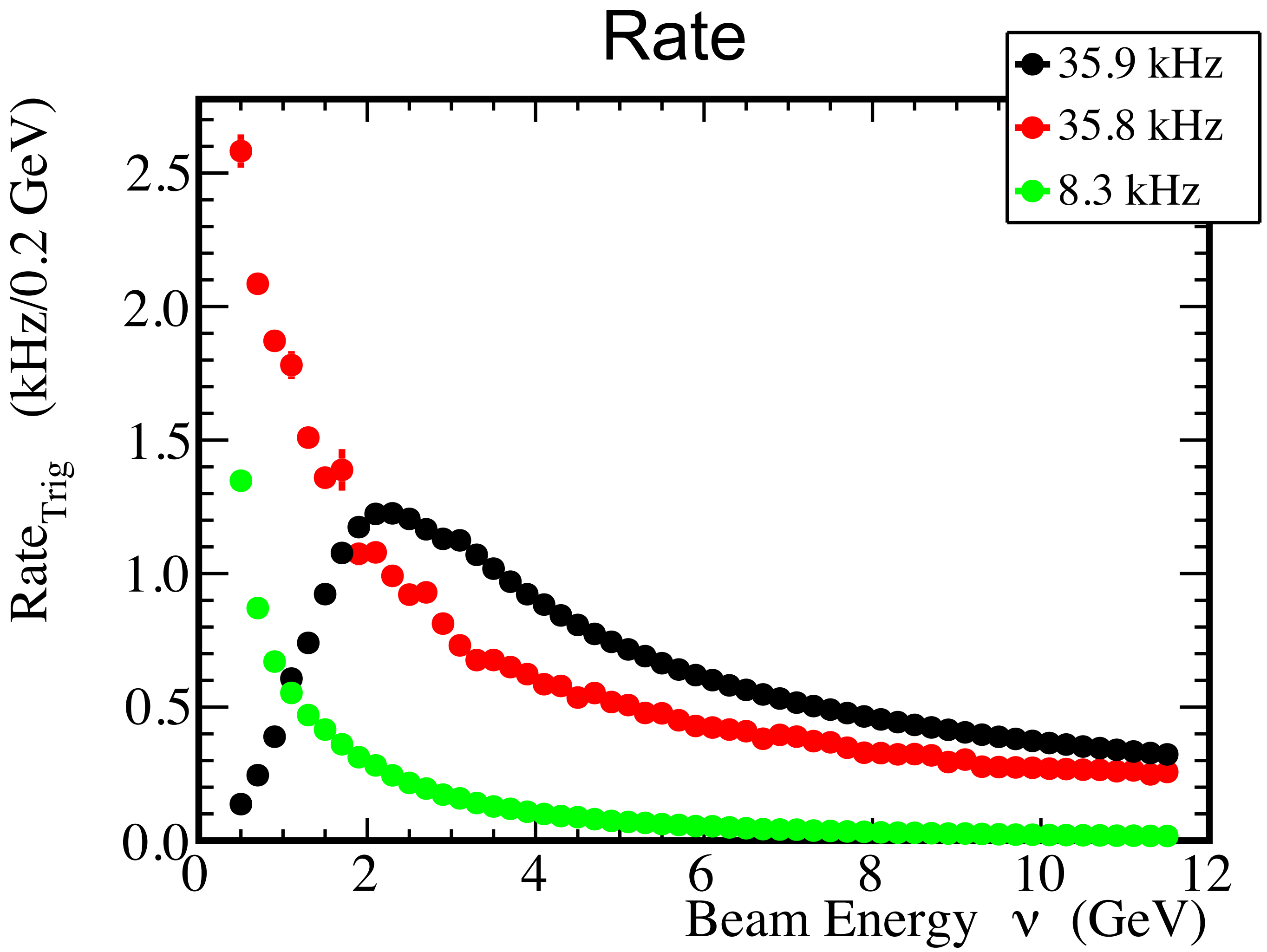






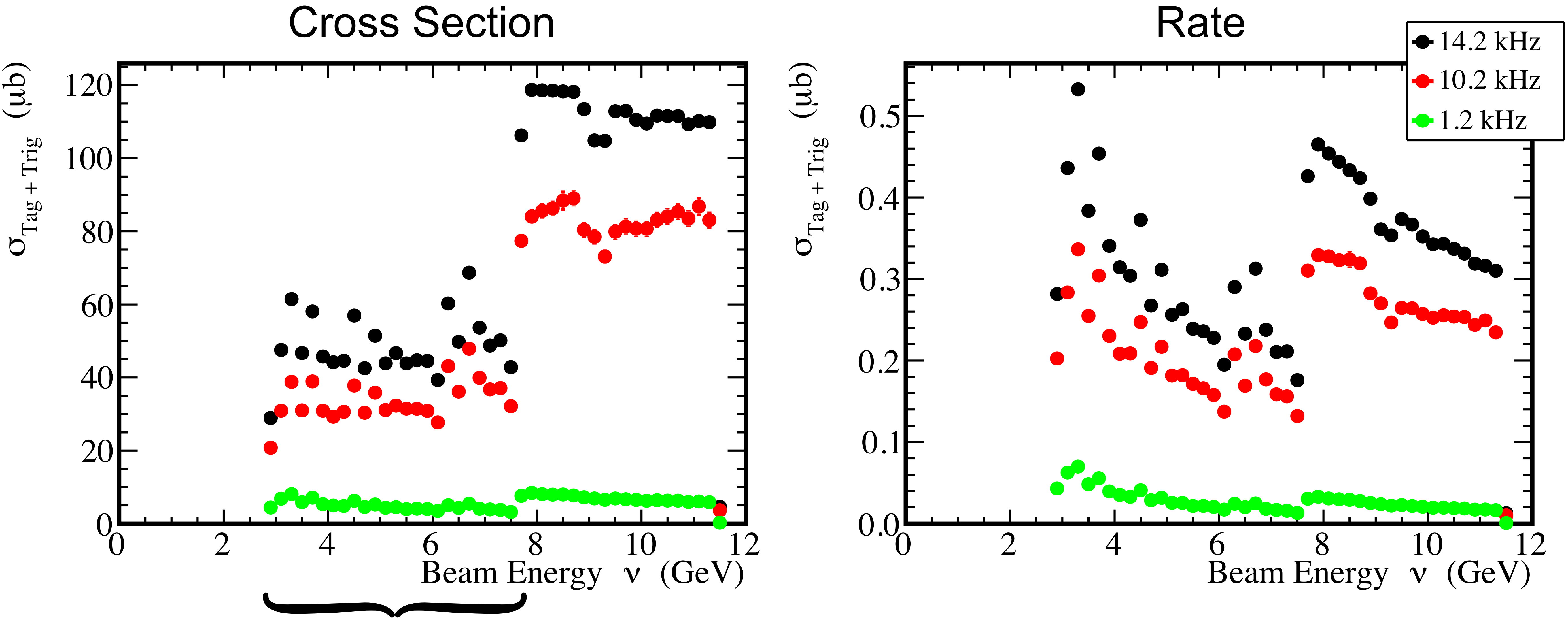
Signal and Background





Data Acquisition has limit of ~80 kHz.
Backgrounds eat up a half of this rate.
Bethe Heitler has cross section of 12-15 mb.

Signal and Background

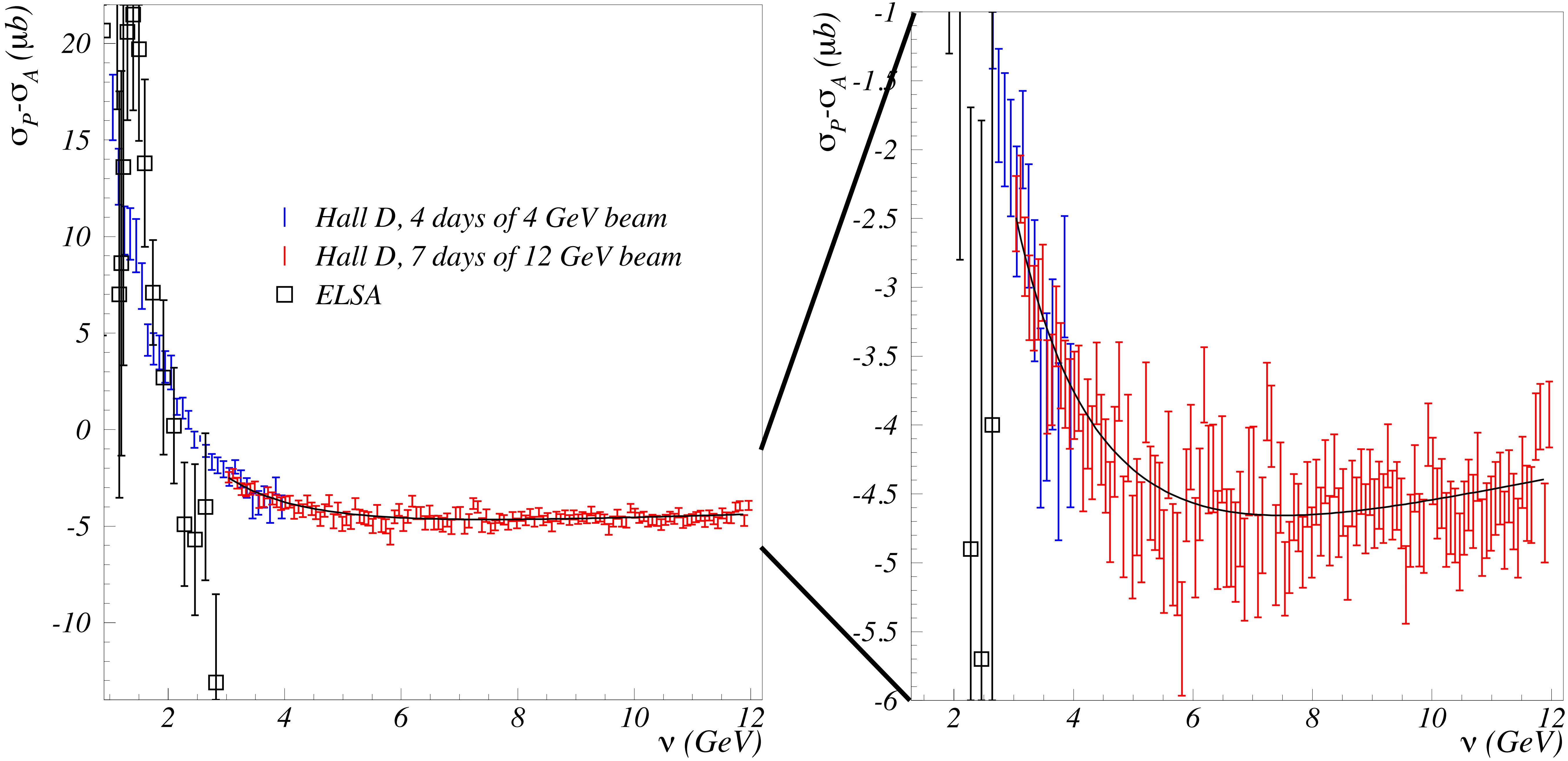


45% coverage

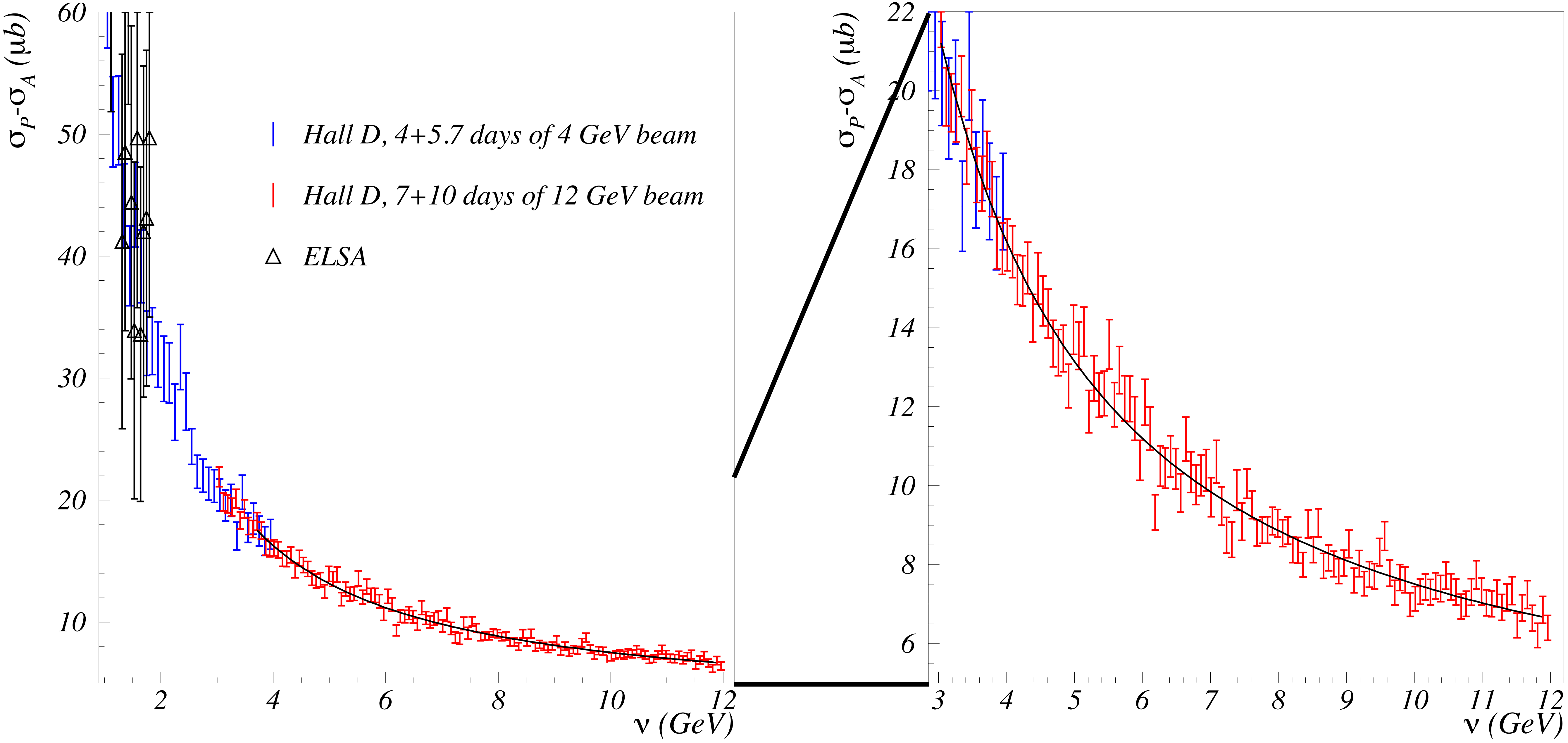
Tagging photons reduces the efficiency.
Depends on geometrical acceptance of tagger.

Projected Results (Proton)

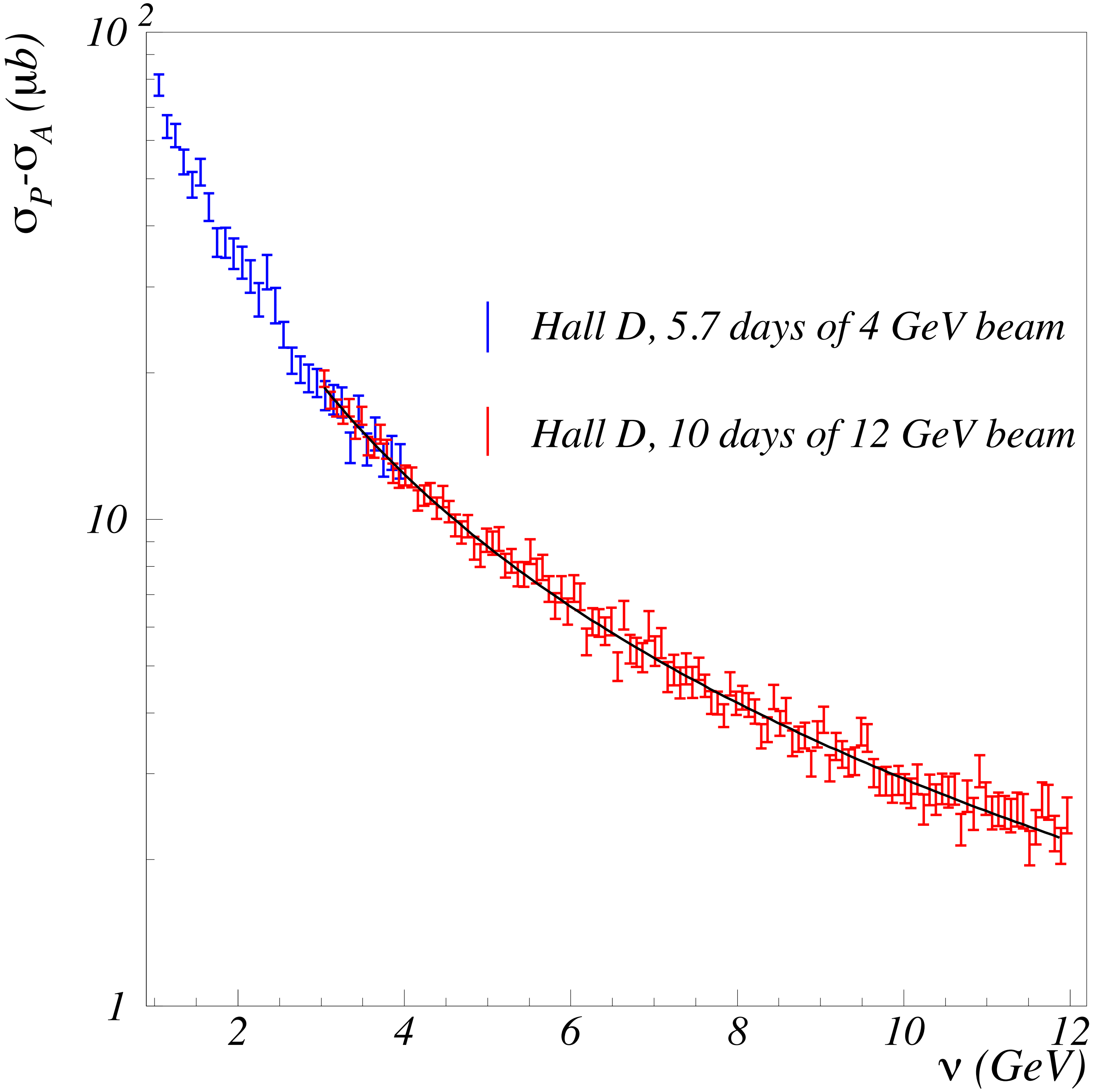
Projection using $\alpha_{a_1} = 0.412, \alpha_{f_1} = -0.629$



Projected Results (Neutron)



Projected Results (Deuteron)



Interpretation of REGGE data

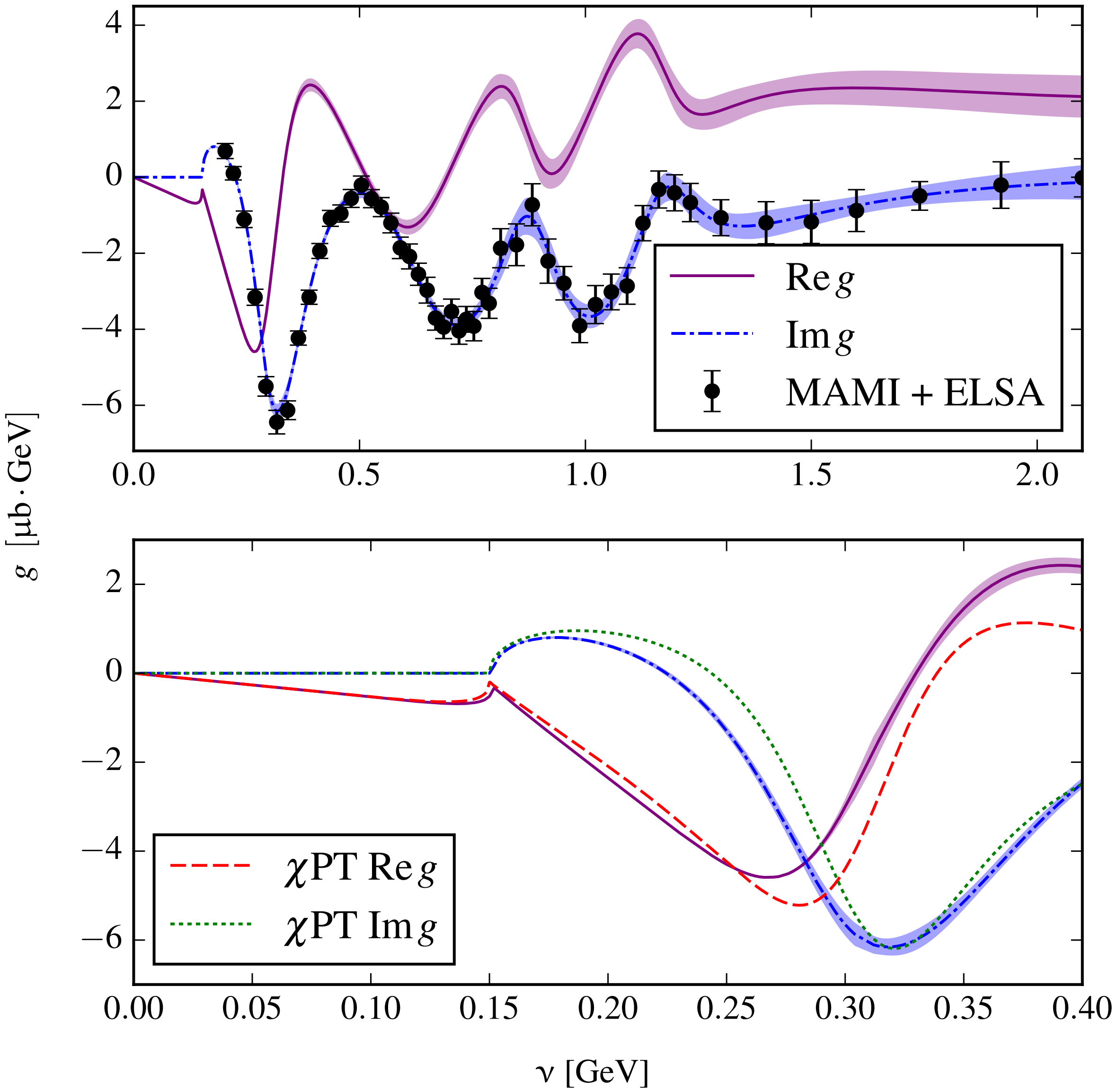
Spin-dependent Compton Amplitude

$$\mathfrak{I}\left(f_2(\varepsilon)\right) = \frac{\varepsilon}{8\pi}\Delta\sigma$$

Directly measure imaginary part of the amplitude

Access real part by dispersion relation

$$\mathfrak{R}\left(f_2(\varepsilon)\right) = \frac{2\nu}{\pi} P \int_0^{+\infty} \frac{\mathfrak{I}\left(f_2(\varepsilon)\right)}{\varepsilon^2 - \nu^2} d\varepsilon$$



Gryniuk++ Phys. Rev. D 94 (2016)

Extend existing data by factor of 6 in energy

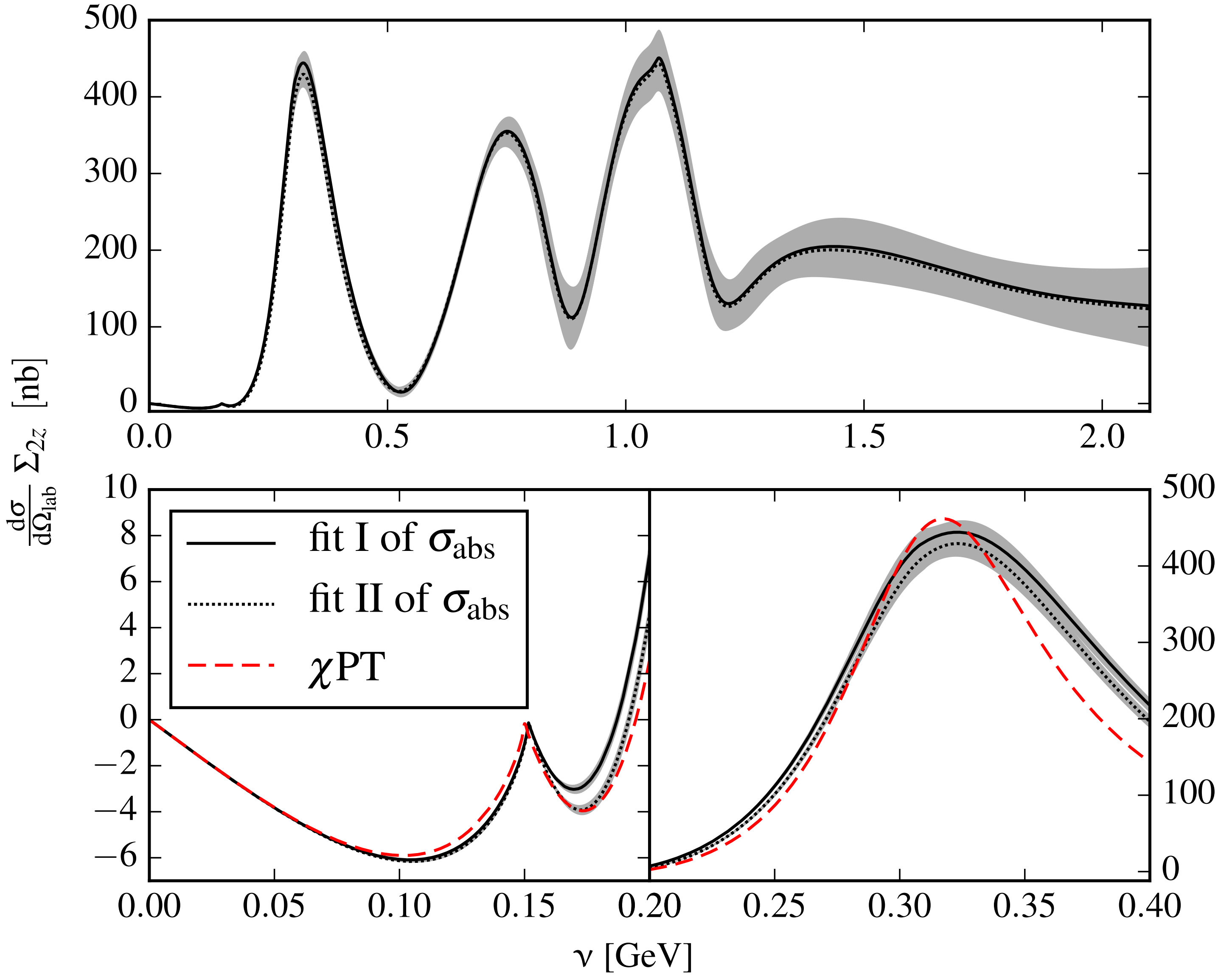
Study Compton scattering without doing a dedicated Compton scattering experiment

Noticeable difference between data and NNLO χEFT calculation at ~0.25 GeV.

Spin-dependent Compton Amplitude

Since unpolarized amplitude f_1 is well measured

Can determine cross section and beam-target asymmetry in forward limit.



$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=0} = |f_1|^2 + |f_2|^2$$

$$\left.\Sigma_{2z}\right|_{\theta=0} = \frac{2\Re(f_1 f_2^*)}{|f_1|^2 + |f_2|^2}$$

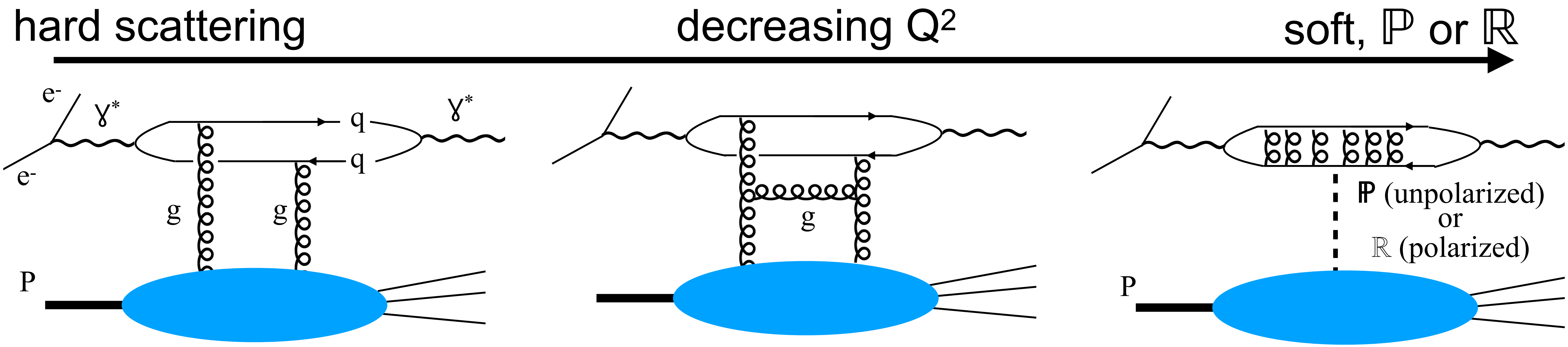
Expand analysis to neutron and deuteron

Describing spin observables from JLab low-Q^2 has been a challenge for χEFT. Data in a different regime is valuable.

Gryniuk++ Phys. Rev. D 94 (2016)

Transition to diffractive regime

Explore transition between polarized DIS and diffraction regimes



hard scattering

decreasing Q^2

soft, ℙ or ℝ

Diquark picture of low-x ep scattering. Coherence length $\propto x^{-1}M^{-1}$

Pomeron ℙ: unpolarized diffractive scattering

Reggeon ℝ: doubly polarized diffractive scattering (will be measured at EIC)

Will provide $Q^2 = 0$ baseline for these transition studies.

From Nucleons to Nuclei

The GDH Sum on Nuclei

REGGEON: REGGE on Nuclei

Magnetic moment of a particle with charge Qe, mass $M$and spin $\vec{S}$:

$$\vec{\mu} = \frac{e}{M}(Q + \kappa)\vec{S}$$

For a nucleus of mass $M \approx AM_p$ and charge Ze

$$\vec{\mu} = \frac{e}{AM_p}(Z + \kappa)\vec{S} \implies \kappa = \frac{A}{2|\vec{S}|}\frac{\mu}{\mu_N} - Z$$

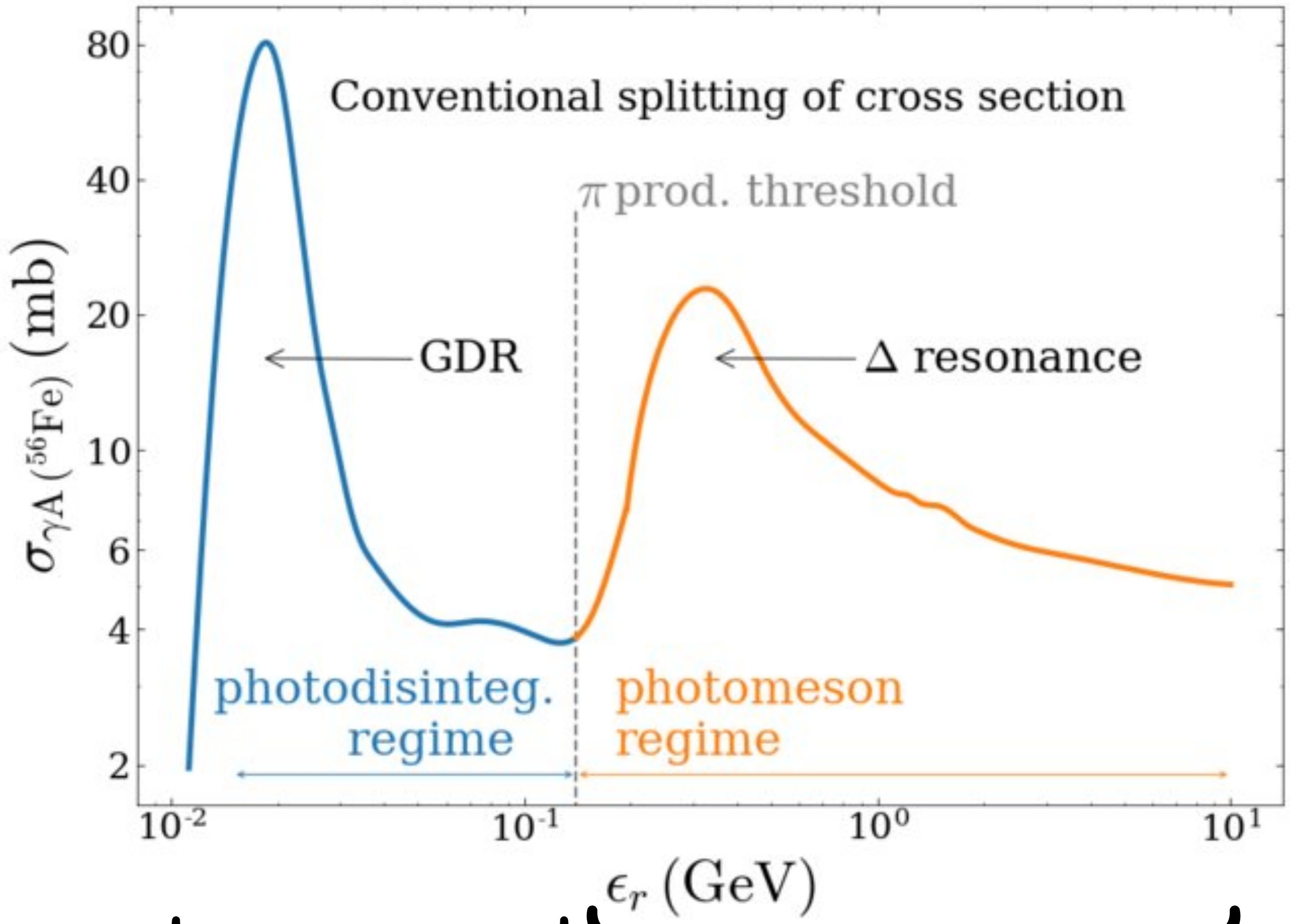
This allows us to calculate κ for all stable nuclei with spin and compute the static part of the GDH sum rule.

Nuclear spectrum

No data on $\Delta\sigma$ exists for A>3

photo excitation of nucleus
properties of nucleus

photoproduction of hadrons
properties of nucleon



negative

$I^{p*}_{\mathrm{GDH}} \approx 270\ \mu\mathrm{b}$

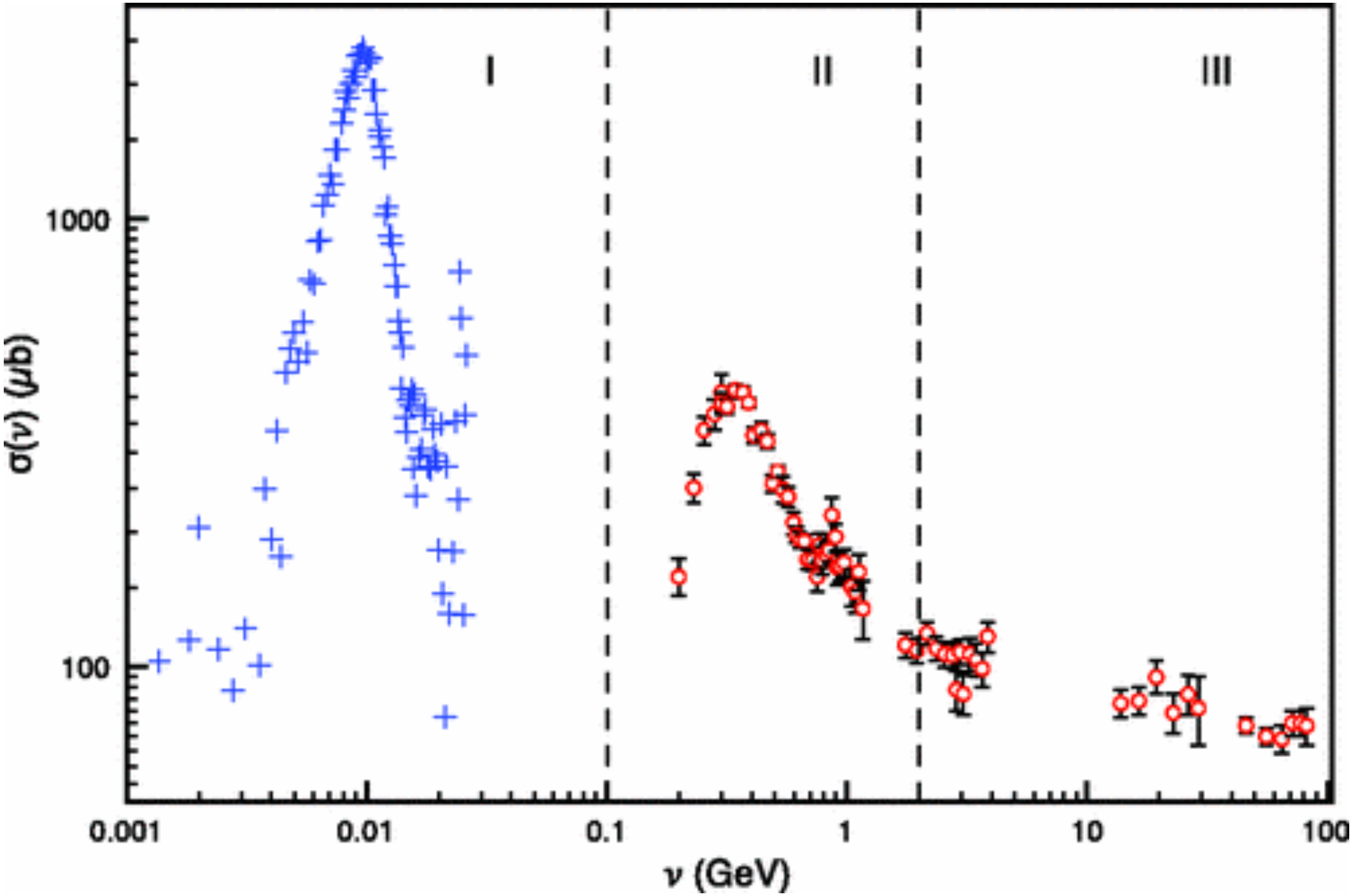
example for ^{7}Li

$I^{^7\mathrm{Li}}_{\mathrm{GDH}} \approx 83\ \mu\mathrm{b}$

Morejon++ JCAP 11, 007 (2019)

Nuclear spectrum

Photoabsorption cross-section data for a ^{207}Pb target



Gorchtein++ Phys. Rev. C 84 (2011)

Candidate Nuclei

Choice will depend on target feasibility and FOM

The strongest candidate is ^{7}Li:

- Also the subject of unpolarized (E12–10–008) and polarized (E12–14–001: $Q^2 > 1$ GeV2) EMC experiments at JLab
- A GDH measurement will provide the Q2 → 0 limit …
- … and help to establish which of the two competing explanations of the EMC effect (MF or SRC) is most likely

	J^{π}	μ	κ	M	I_{GDH}
$^{1}\mathrm{H}$	$1/2^{+}$	2.793	1.793	0.9383	204.8
$^{2}\mathrm{H}$	1^{+}	0.857	-0.1426	1.875	0.6484
$^{3}\mathrm{He}$	$1/2^{+}$	-2.128	-8.383	2.808	499.9
$^{7}\mathrm{Li}$	$3/2^{-}$	3.256	4.598	6.532	83.39
$^{13}\mathrm{C}$	$1/2^{-}$	0.702	3.131	12.11	3.753
$^{17}\mathrm{O}$	$5/2^{+}$	-1.894	-14.44	15.83	233.4
$^{19}\mathrm{F}$	$1/2^{+}$	2.628	40.94	17.69	300.5

Modification of bound nucleons

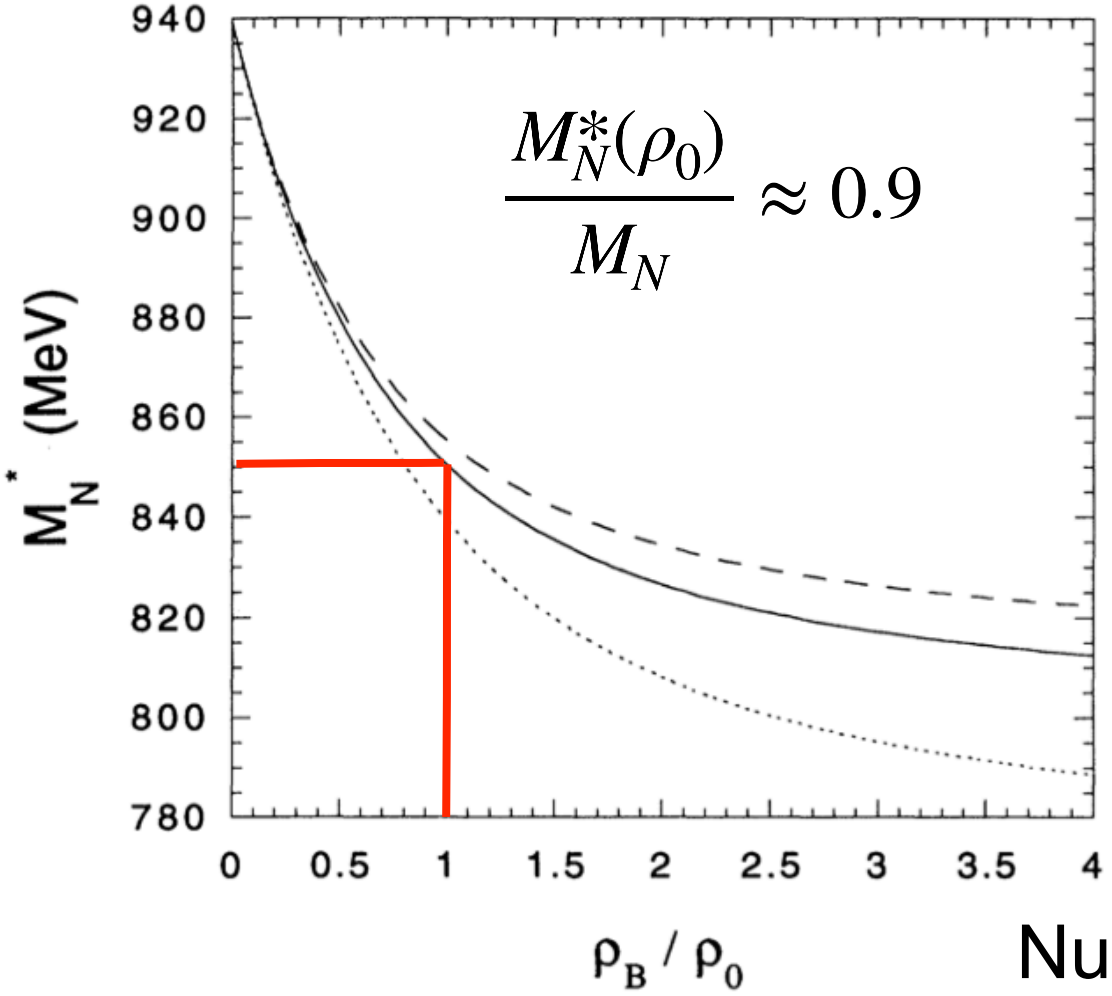
A nucleon in the nuclear medium will be modified
⇒ modification of both sides of the nucleon sum rule

Bass, Acta Phys. Pol. B 52, 42 (2021)
Bass++, arXiv:2212.04795 [nucl-th]

Static Side

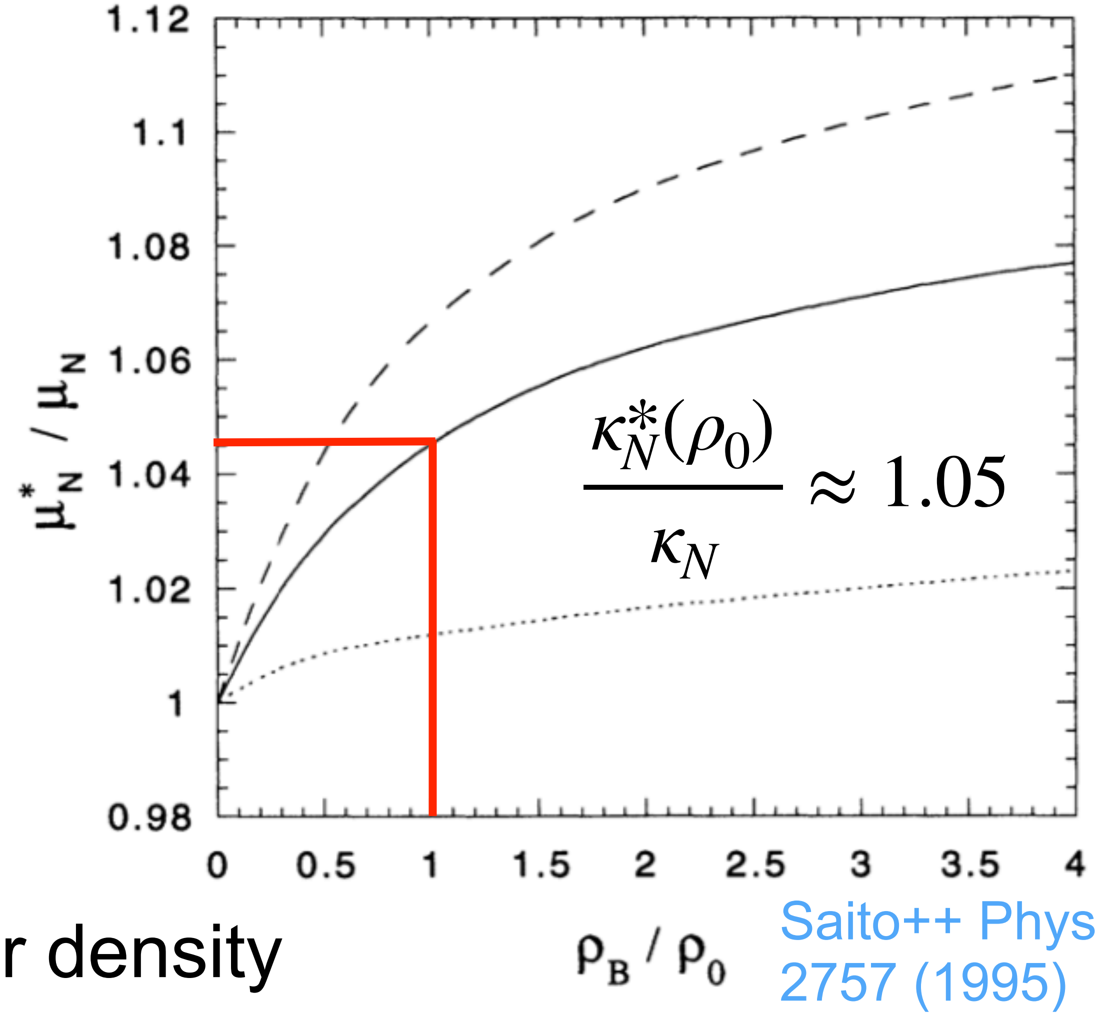
Quark Meson Coupling (QMC) model predicts modification of mass and anomalous magnetic moment.

$$\left(\frac{\kappa_N^*(\rho_0)}{M_N^*(\rho_0)}\right)^2 \Big/ \left(\frac{\kappa_N}{M_N}\right)^2 \approx 1.3$$



$$\frac{M_N^*(\rho_0)}{M_N} \approx 0.9$$

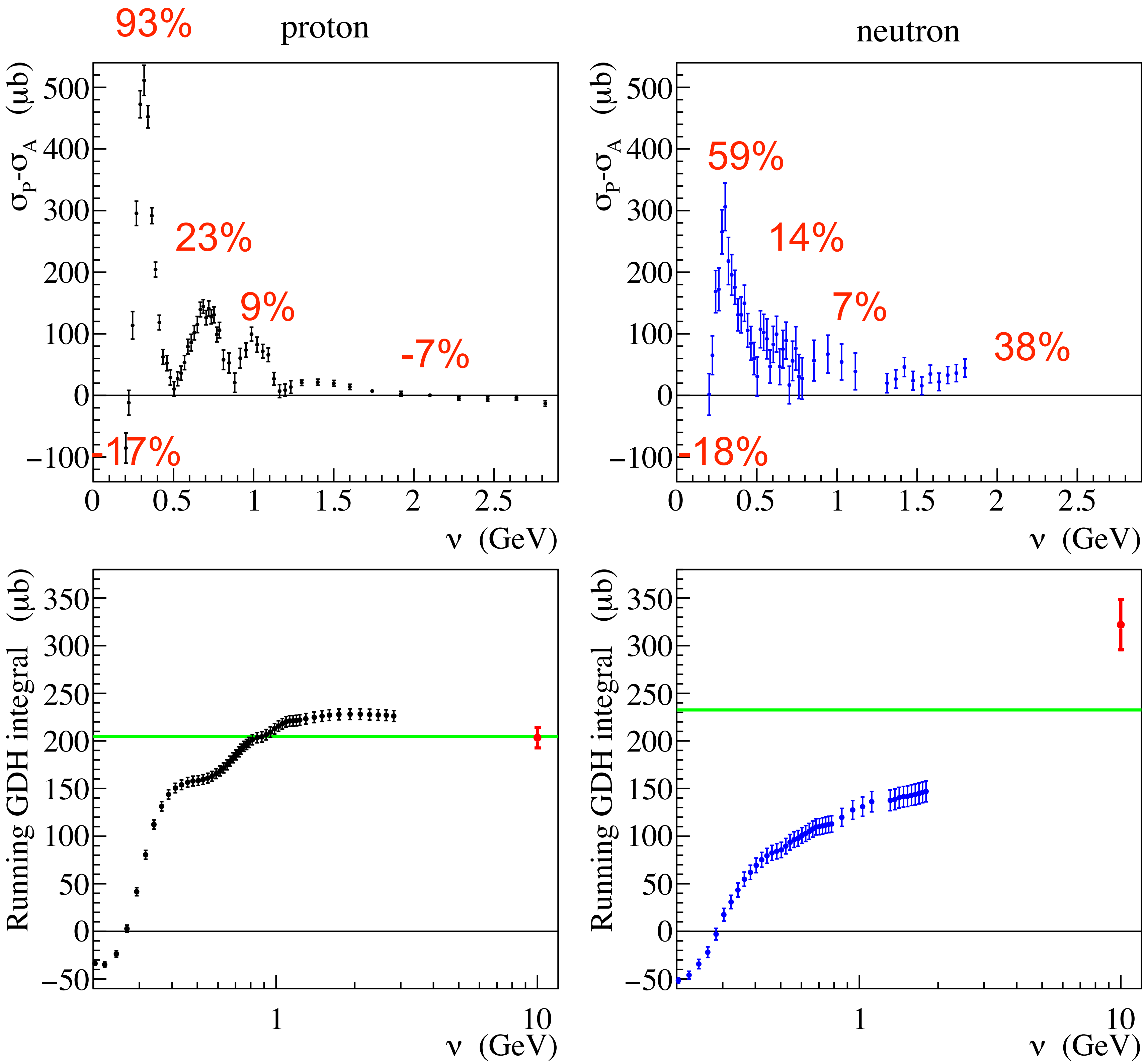
Nuclear density



$$\frac{\kappa_N^*(\rho_0)}{\kappa_N} \approx 1.05$$

Saito++ Phys. Rev. C 51, 2757 (1995)

Modification of bound nucleons



Dynamic (integral) Side

$$\int_{\nu_0}^{\infty} \frac{\Delta\sigma(\nu)}{\nu}\, d\nu = \frac{4\pi^2 S\alpha\kappa^2}{M^2}$$

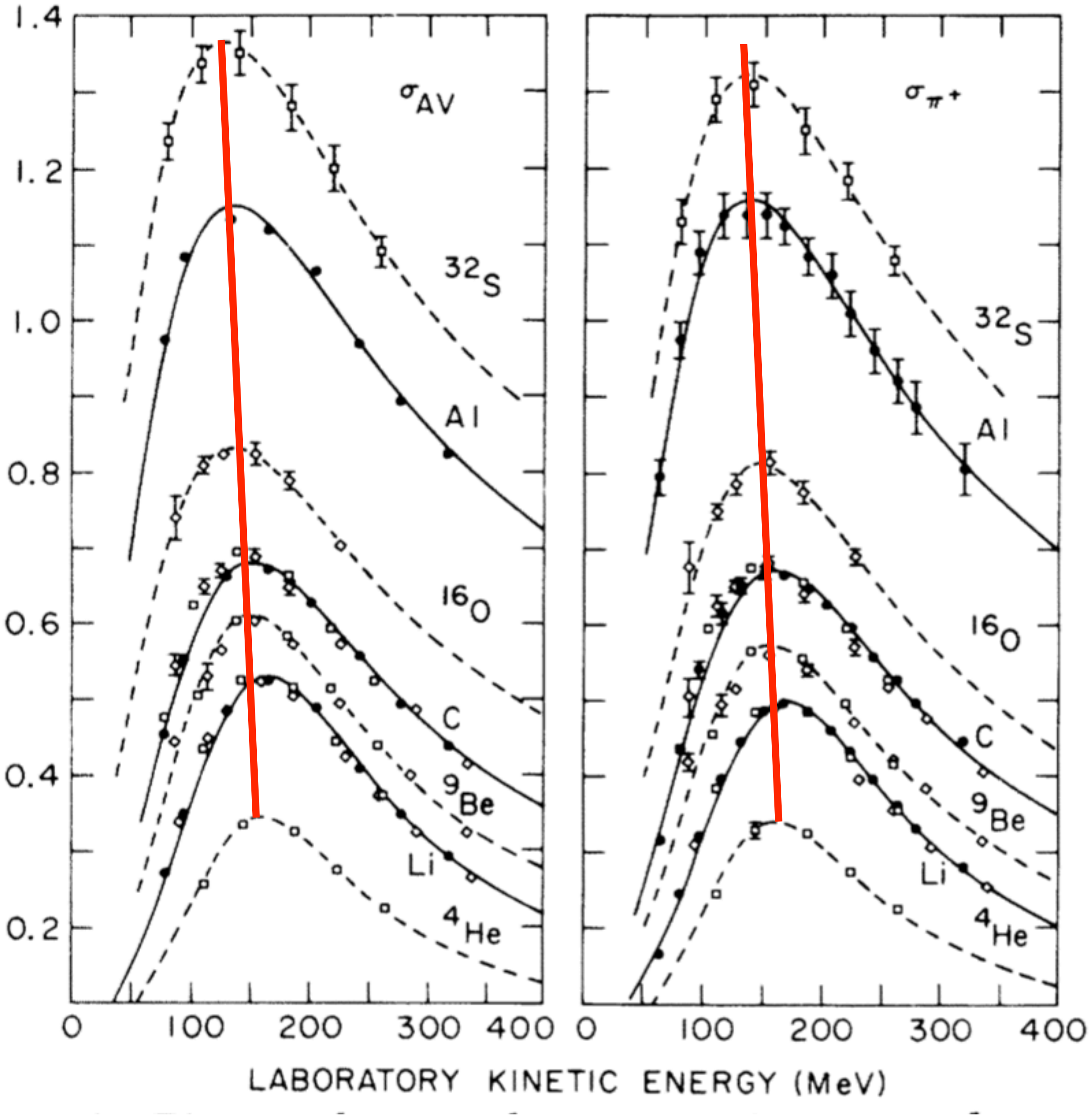
In-medium shift of resonance mass, $1/\nu$ dependence (slower than $1/M^2$)

Δ(1232)

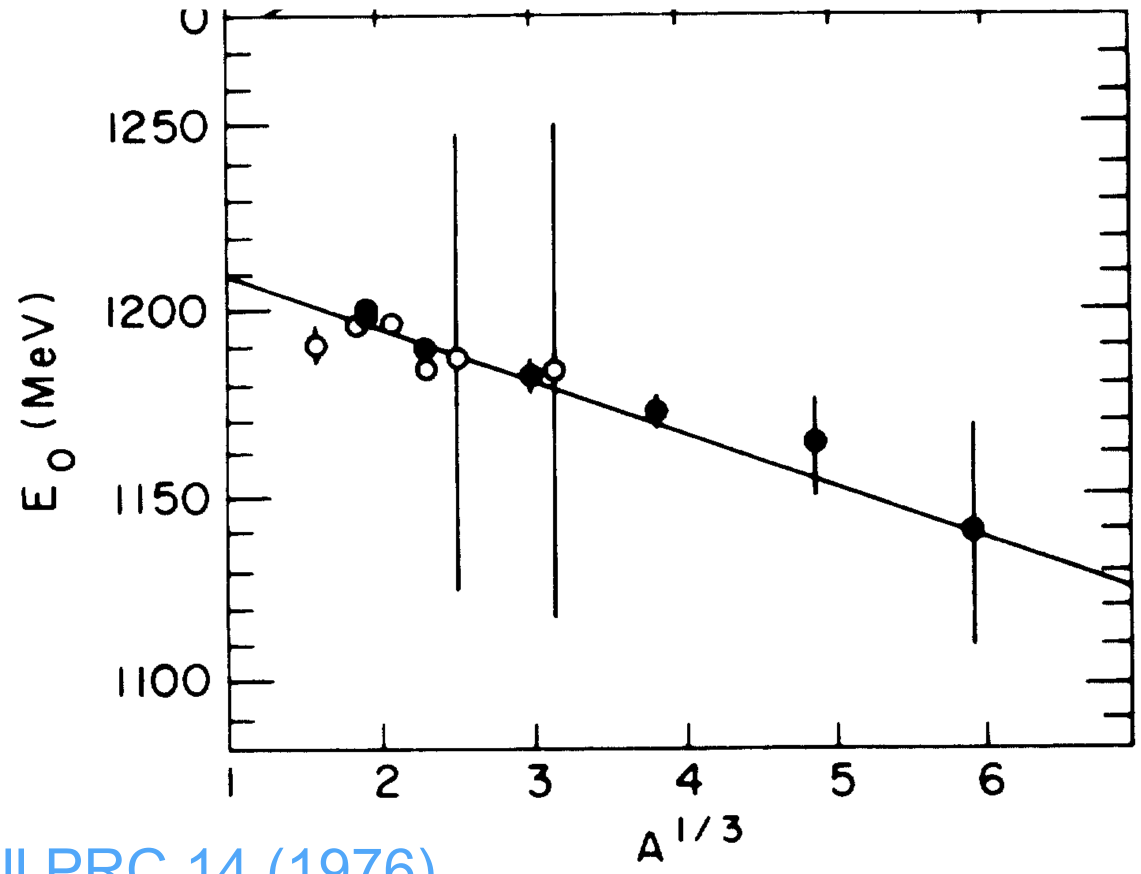
$D_{13}(1520)$, $S_{11}(1535)$ expected to be small

3rd resonance and Regge modification

Δ(1232) Mass Modification



$$M^*_\Delta \approx 1228 - 16A^{1/3} \text{ MeV}$$



Carroll PRC 14 (1976)

Summary

- Fundamental Quantum Field Theory prediction. Applicable to any type of target.
- First measurement of the high-ν behavior of GDH integrant $(\sigma^{3/2}-\sigma^{1/2})/\nu$
- High-ν is where a failing of the sum rule would be revealed. Unpolarized version of GDH integral does not converge. Data at $\nu<3$ GeV fail to see divergence of unpolarized cross-section.
- Primary goal: map yield difference $N^{3/2} - N^{1/2}$ for the proton and neutron. This will determine whether the integral converges or not.
- 17-days measurement + assuming Regge behavior provide α_{f1} and α_{a1} at 2% level (present uncertainties: 50%)
- Secondary goals (regardless of the convergence and sum rule validity):
  - Verify proton GDH sum rule within 6%. (Need point-to-point uncorrelated uncertainties and combine with LEGS/MAMI/ELSA data).
  - Solve discrepancy between DIS data and Regee theory prediction.
  - Provide first non zero data on $\sigma^{3/2}-\sigma^{1/2}$ for the deuteron.
  - Allow extraction of complex Compton amplitude f_2 and new test of χpT.
  - Improve knowledge of hyperfine splitting in Hydrogen. Connection with proton radius puzzle.
  - Data teach us about diffractive QCD: phenomenology essentially unknown when spin degrees of freedom are explicit. Helpful for EIC: determination of α_{a1} and α_{f1} will provide a $Q^2=0$ baseline for g_1 for EIC. $\Longrightarrow$ study of the transition between DIS and diffractive regimes.

Summary Continued

Studying the GDH sum rule on nuclei might be very interesting too.

Application of the sum rule and Mean Field nuclear theory suggest there will be a significant difference from free protons or neutrons in the Regge region.