HUGS2023

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3D partonic structure of pions and nucleons

Patrick Barry, Jefferson Lab

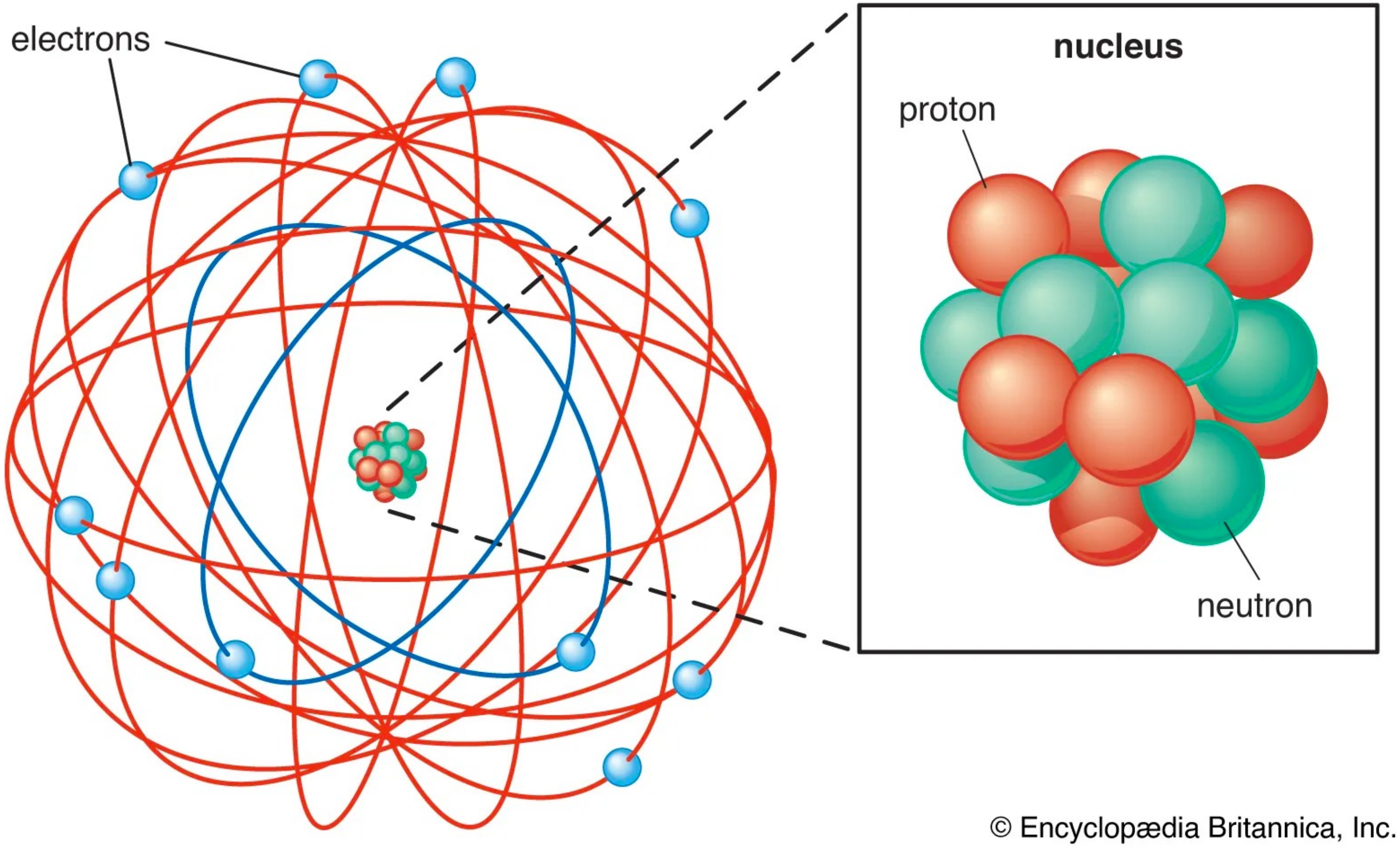
HUGS 2023 summer school, June 12, 2023

Jefferson Lab
Exploring the Nature of Matter

Theory
JAM
Computer Science
Experiment

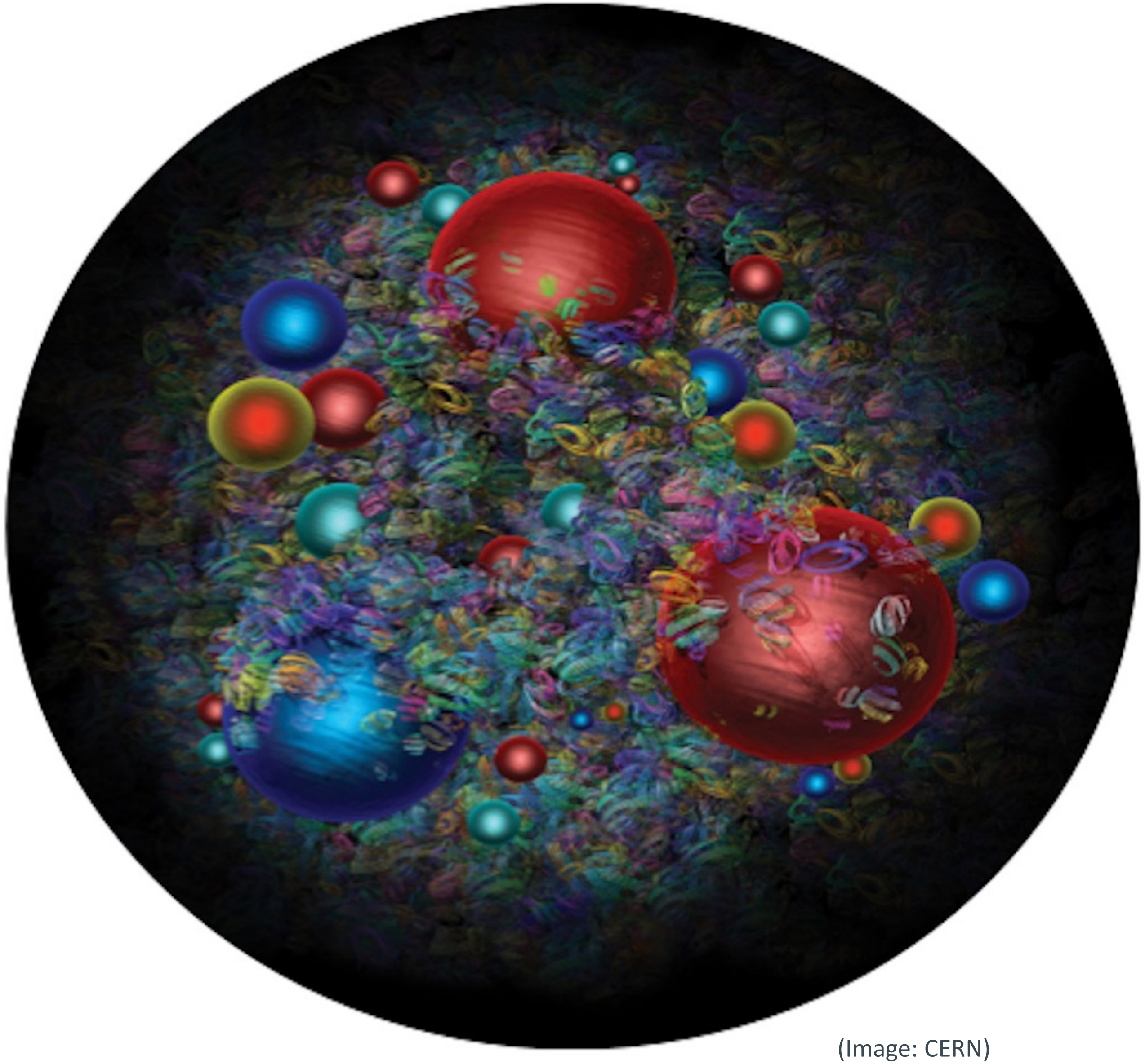
Motivation

- All visible matter is made up of atoms
- The mass of these atoms are largely from the nucleus
- The nucleus is made up of protons and neutrons



Motivation

- In turn, these protons and neutrons are made of quarks and gluons
- We want to study the structure of the nuclear matter



(Image: CERN)

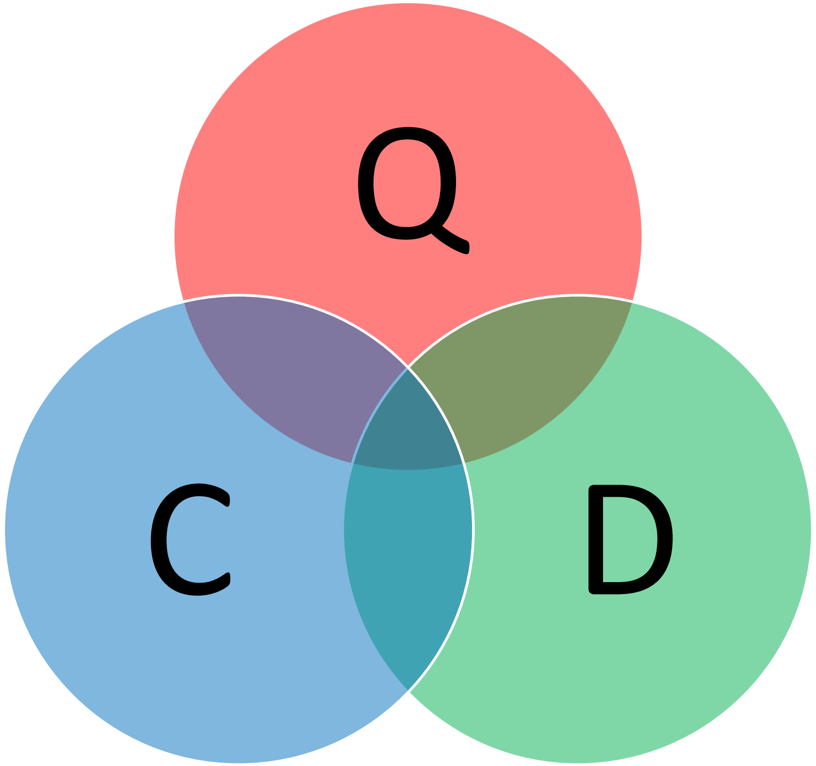
What's the problem?

Quarks and gluons are not directly measurable because of color confinement!

Have to be inferred from experimental data

How to handle this

- We make use of QCD, which allows us to study the structure of hadrons in terms of partons (quarks, antiquarks, and gluons)
- Use factorization theorems to separate hard partonic physics out of soft, non-perturbative objects to quantify structure



Game plan

What to do:

- Define a structure of hadrons in terms of quantum field theories
- Identify physical observables that can be theoretically factorized with controllable approximations, or factorizable lattice QCD observables
- Perform global QCD analysis as structures are universal and are the same in all processes

Complicated Inverse Problem

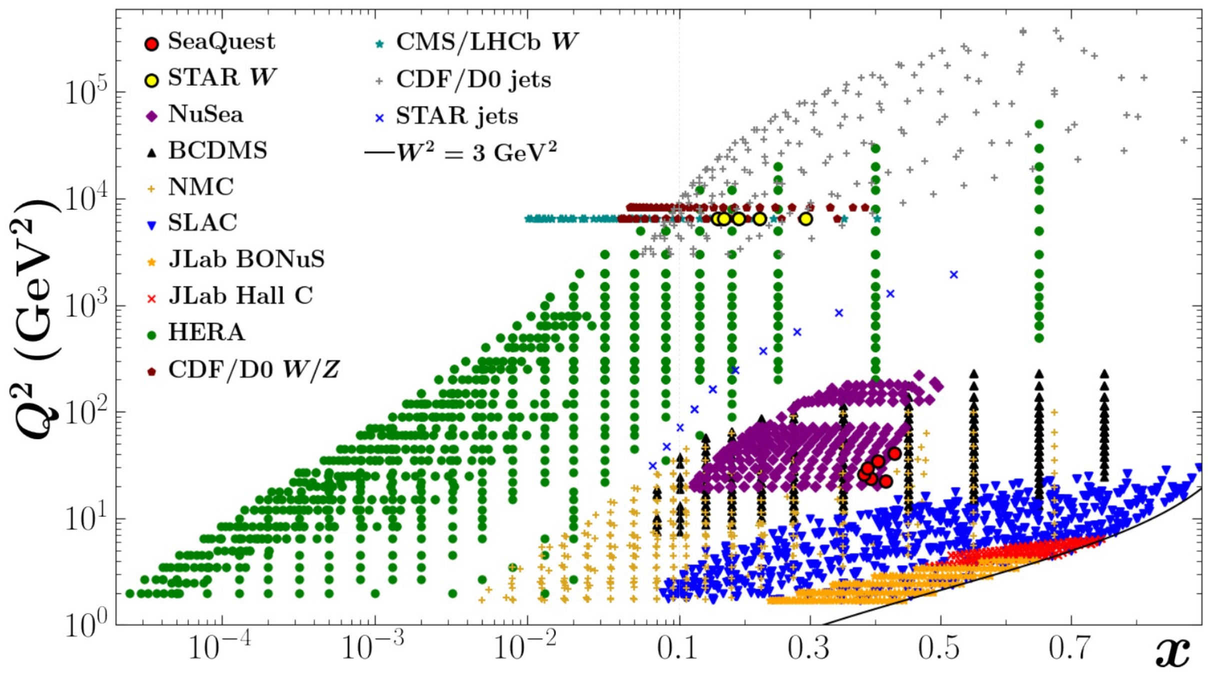
- Factorization theorems involve convolutions of hard perturbatively calculable physics and non-perturbative objects

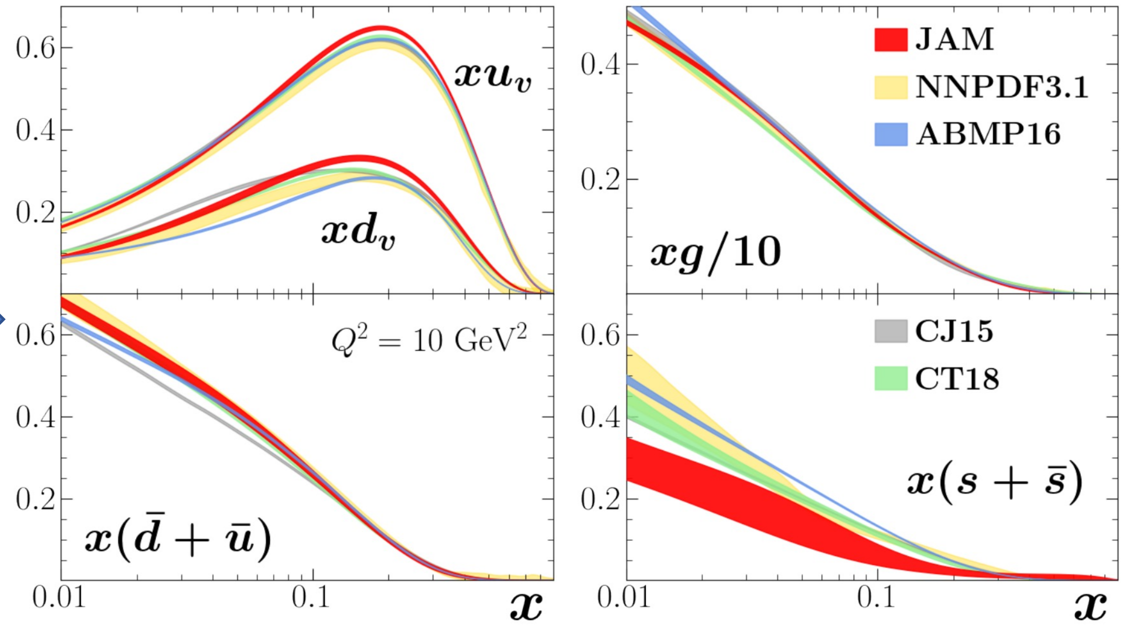
$$\frac{d\sigma}{d\Omega} \propto \mathcal{H} \otimes f = \int_x^1 \frac{d\xi}{\xi} \mathcal{H}\left(\frac{x}{\xi}\right) f(\xi)$$

- Parametrize the non-perturbative objects and perform global analysis

What do we know about structures?

- Most well-known structure is through longitudinal structure of hadrons, particularly protons

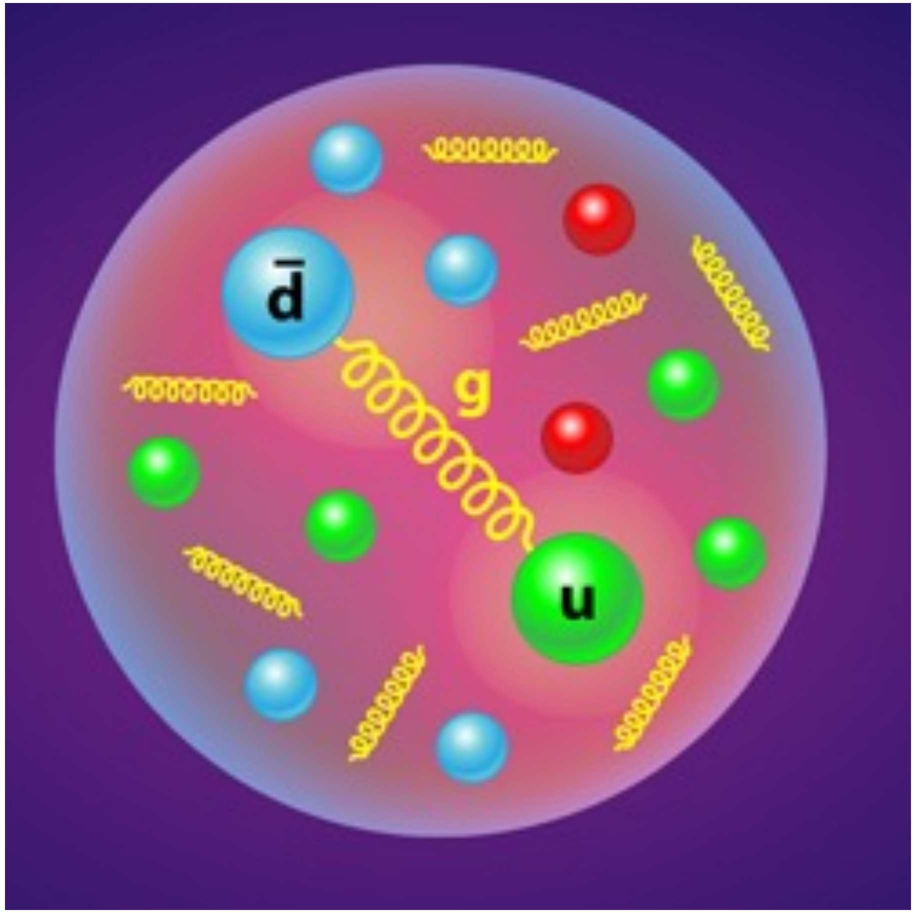




C. Cocuzza, et al., Phys. Rev. D **104**, 074031 (2021) (*many* other groups working on this!)

Other structures?

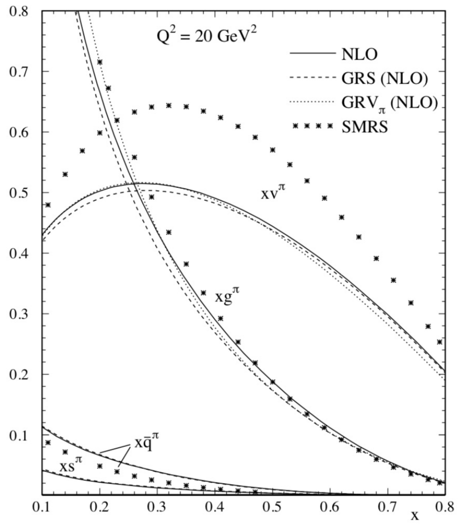
- To give deeper insights into color confined systems, we shouldn't limit ourselves to proton structures
- Pions are also important because of their Goldstone-boson nature while also being made up of quarks and gluons



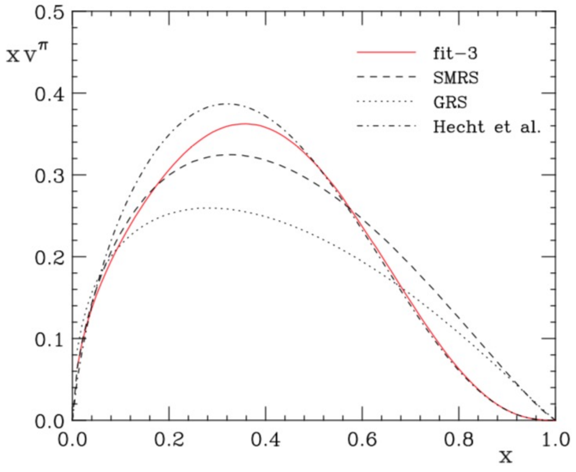
Pion structure in phenomenology

- Historically, pion distributions have been extracted from fixed target πA data
  - Drell-Yan (DY) $\pi A \to \mu^+ \mu^- X$
  - Prompt photon $\pi A \to \gamma X$

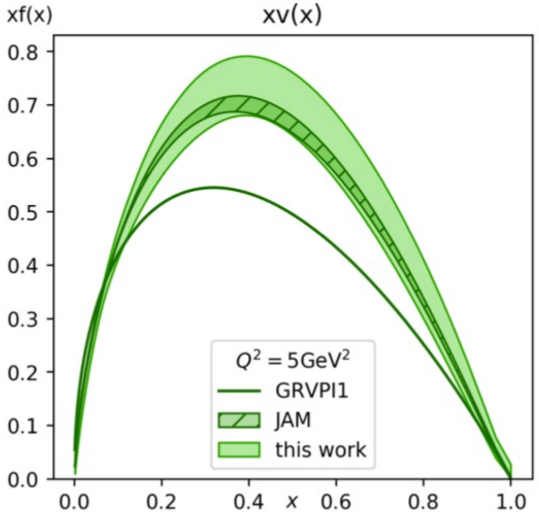
Owens attempted to use J/ψ production



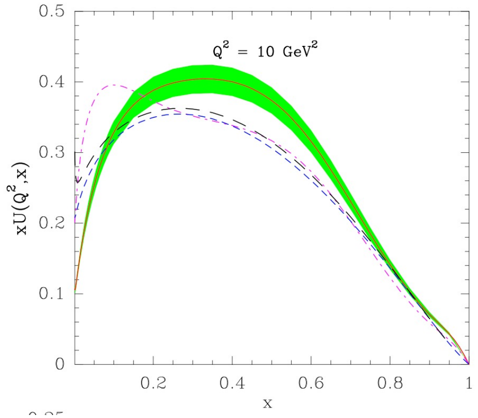
GRS, GRV, and SMRS



ASV valence PDF



xFitter



Statistical modeling

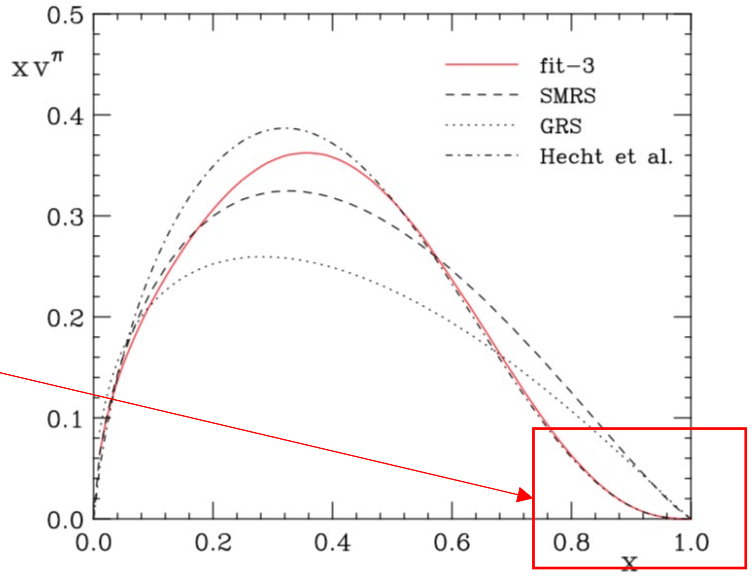
Large momentum fraction behavior

- Many theoretical papers have studied the behavior of the valence quark distribution as $x \rightarrow 1$ and
- Debate whether $q_v^\pi(x \rightarrow 1) \sim (1-x)$ or $(1-x)^2$

R. J. Holt and C. D. Roberts, Rev. Mod. Phys. **82**, 2991 (2010).
W. Melnitchouk, Eur. Phys. J. A **17**, 223 (2003).
G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. **43**, 246 (1979).
E. L. Berger and S. J. Brodsky, Phys. Rev. Lett. **42**, 940 (1979).
M. B. Hecht, C. D. Roberts, and S. M. Schmidt, Phys. Rev. C **63**, 025213 (2001).
Z. F. Ezawa, Nuovo Cimento A **23**, 271 (1974).
P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. **B53**, 473 (1973).
J. F. Gunion, S. J. Brodsky, and R. Blankenbecler, Phys. Rev. D **8**, 287 (1973).
T. Shigetani, K. Suzuki, and H. Toki, Phys. Lett. B **308**, 383 (1993).
A. Szczepaniak, C.-R. Ji, and S. R. Cotanch, Phys. Rev. D **49**, 3466 (1994).
R. M. Davidson and E. Ruiz Arriola, Phys. Lett. B **348**, 163 (1995).
S. Noguera and S. Scopetta, J. High Energy Phys. 11 (2015) 102.
P. T. P. Hutauruk, I. C. Cloët, and A. W. Thomas, Phys. Rev. C **94**, 035201 (2016).
T. J. Hobbs, Phys. Rev. D **97**, 054028 (2018).
K. D. Bednar, I. C. Cloët, and P. C. Tandy, Phys. Rev. Lett. **124**, 042002 (2020).
G. de Téramond, T. Liu, R. S. Sufian, H. G. Dosch, S. J. Brodsky, and A. Deur, Phys. Rev. Lett. **120**, 182001 (2018).
J. Lan, C. Mondal, S. Jia, X. Zhao, and J. P. Vary, Phys. Rev. Lett. **122**, 172001 (2019).
J. Lan, C. Mondal, S. Jia, X. Zhao, and J. P. Vary, Phys. Rev. D **101**, 034024 (2020).
L. Chang, K. Raya, and X. Wang, Chin. Phys. C **44**, 114105 (2020).
A. Kock, Y. Liu, and I. Zahed, Phys. Rev. D **102**, 014039 (2020).
Z. F. Cui, M. Ding, F. Gao, K. Raya, D. Binosi, L. Chang, C. D. Roberts, J. Rodríguez-Quintero, and S. M. Schmidt, Eur. Phys. J. C **80**, 1064 (2020).

Large-x_π behavior

- Generally, the parametrization lends a behavior as $x \to 1$ of the valence quark PDF of $q_v(x) \propto (1-x)^\beta$
- For a fixed order analysis, analyses find $\beta \approx 1$
- Aicher, Schaefer Vogelsang (ASV) found $\beta = 2$ with threshold resummation

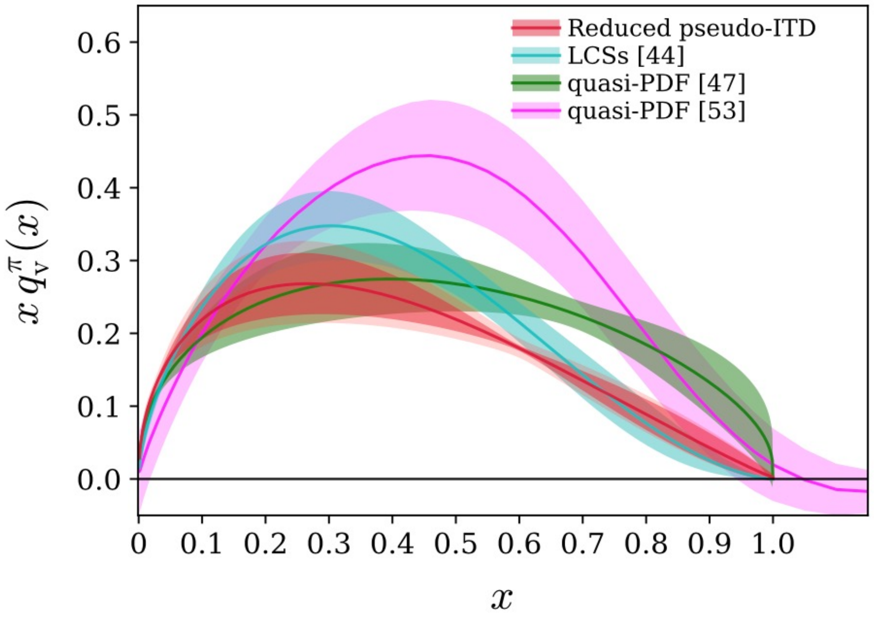


ASV valence PDF

Phys. Rev. Lett. **105**, 114023 (2011).

Lattice QCD Activity

- Simulations on the lattice have been done to investigate this structure



Phys. Rev. D **100**, 114512 (2019).

Subset of pion lattice QCD analyses

J.-H. Zhang, J.-W. Chen, L. Jin, H.-W. Lin, A. Schäfer, and Y. Zhao, Phys. Rev. D **100**, 034505 (2019), arXiv:1804.01483 [hep-lat].

Z.-Y. Fan, Y.-B. Yang, A. Anthony, H.-W. Lin, and K.-F. Liu, Phys. Rev. Lett. **121**, 242001 (2018), arXiv:1808.02077 [hep-lat].

R. S. Sufian, J. Karpie, C. Egerer, K. Orginos, J.-W. Qiu, and D. G. Richards, Phys. Rev. D **99**, 074507 (2019), arXiv:1901.03921 [hep-lat].

J.-W. Chen, H.-W. Lin, and J.-H. Zhang, Nucl. Phys. B **952**, 114940 (2020), arXiv:1904.12376 [hep-lat].

T. Izubuchi, L. Jin, C. Kallidonis, N. Karthik, S. Mukherjee, P. Petreczky, C. Shugert, and S. Syritsyn, Phys. Rev. D **100**, 034516 (2019), arXiv:1905.06349 [hep-lat].

B. Joó, J. Karpie, K. Orginos, A. V. Radyushkin, D. G. Richards, R. S. Sufian, and S. Zafeiropoulos, Phys. Rev. D **100**, 114512 (2019), arXiv:1909.08517 [hep-lat].

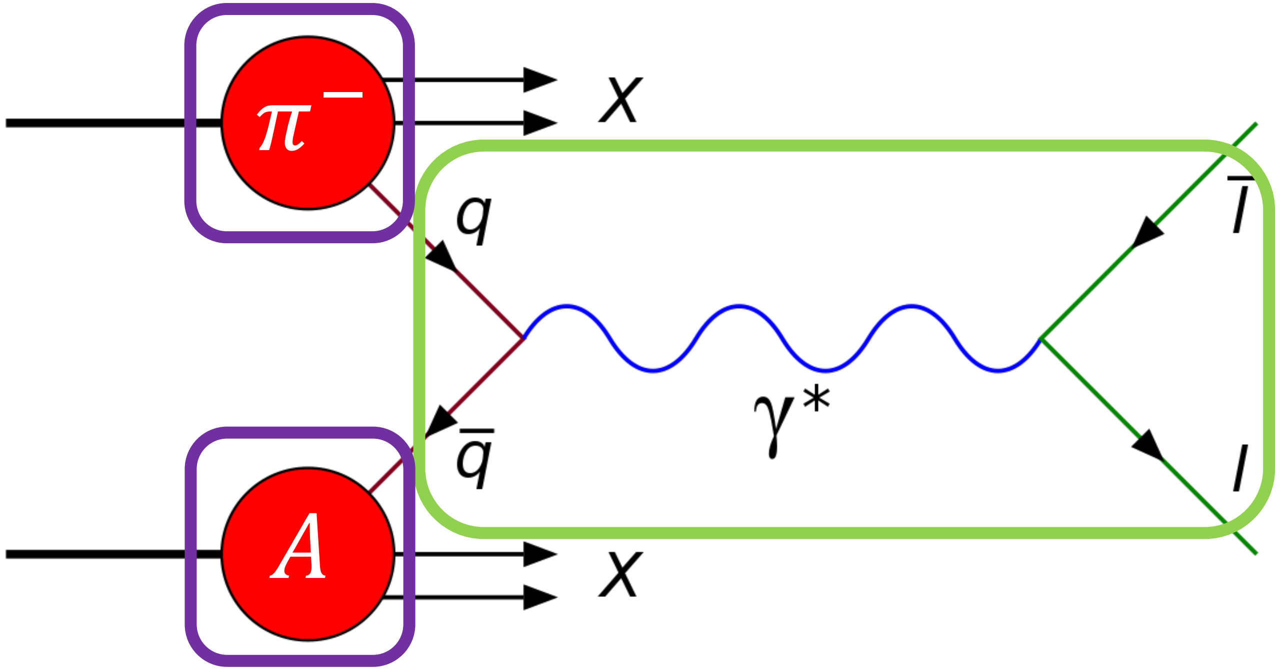
H.-W. Lin, J.-W. Chen, Z. Fan, J.-H. Zhang, and R. Zhang, Phys. Rev. D **103**, 014516 (2021), arXiv:2003.14128 [hep-lat].

R. S. Sufian, C. Egerer, J. Karpie, R. G. Edwards, B. Joó, Y.-Q. Ma, K. Orginos, J.-W. Qiu, and D. G. Richards, Phys. Rev. D **102**, 054508 (2020), arXiv:2001.04960 [hep-lat].

N. Karthik, Phys. Rev. D **103**, 074512 (2021), arXiv:2101.02224 [hep-lat].

Z. Fan and H.-W. Lin, Phys. Lett. B **823**, 136778 (2021), arXiv:2104.06372 [hep-lat].

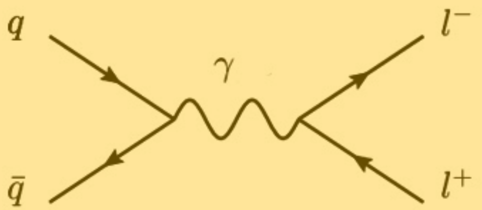
Drell-Yan (DY)



$$\sigma \propto \sum_{i,j} f_i^{\pi}(x_\pi, \mu) \otimes f_j^{A}(x_A, \mu) \otimes C_{i,j}(x_\pi, x_A, Q/\mu)$$

Fixed Order Up to NLO

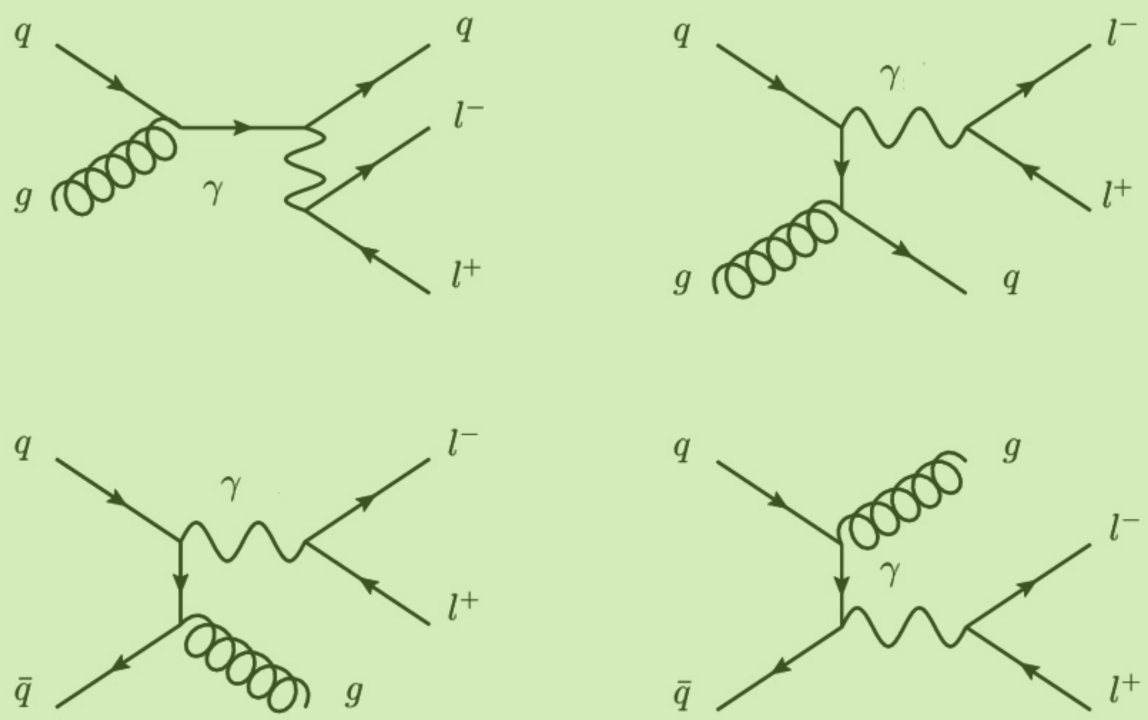
LO: $\mathcal{O}(1)$



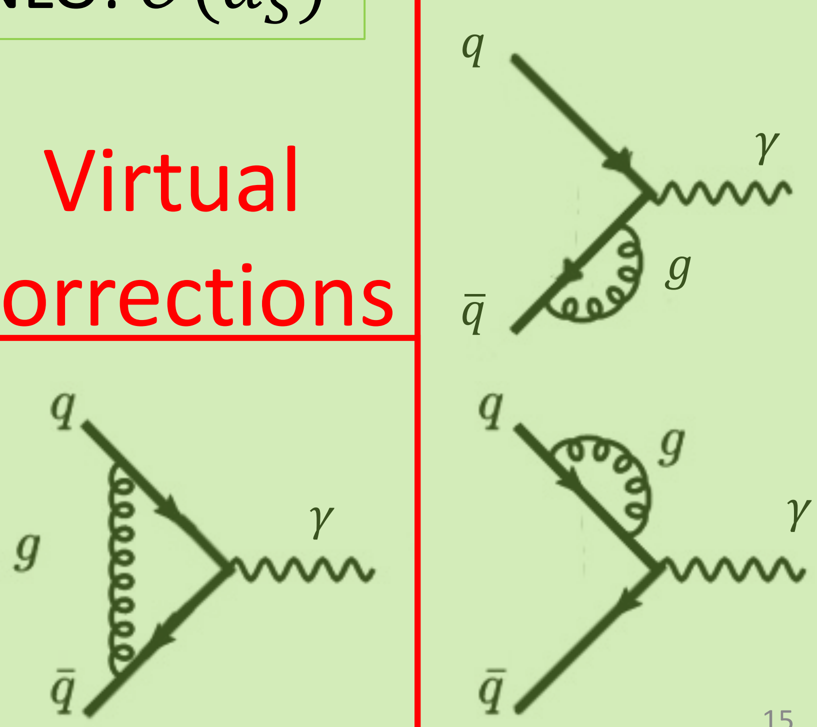
Feynman diagrams for DY amplitudes in collinear factorization

NLO: $\mathcal{O}(\alpha_S)$

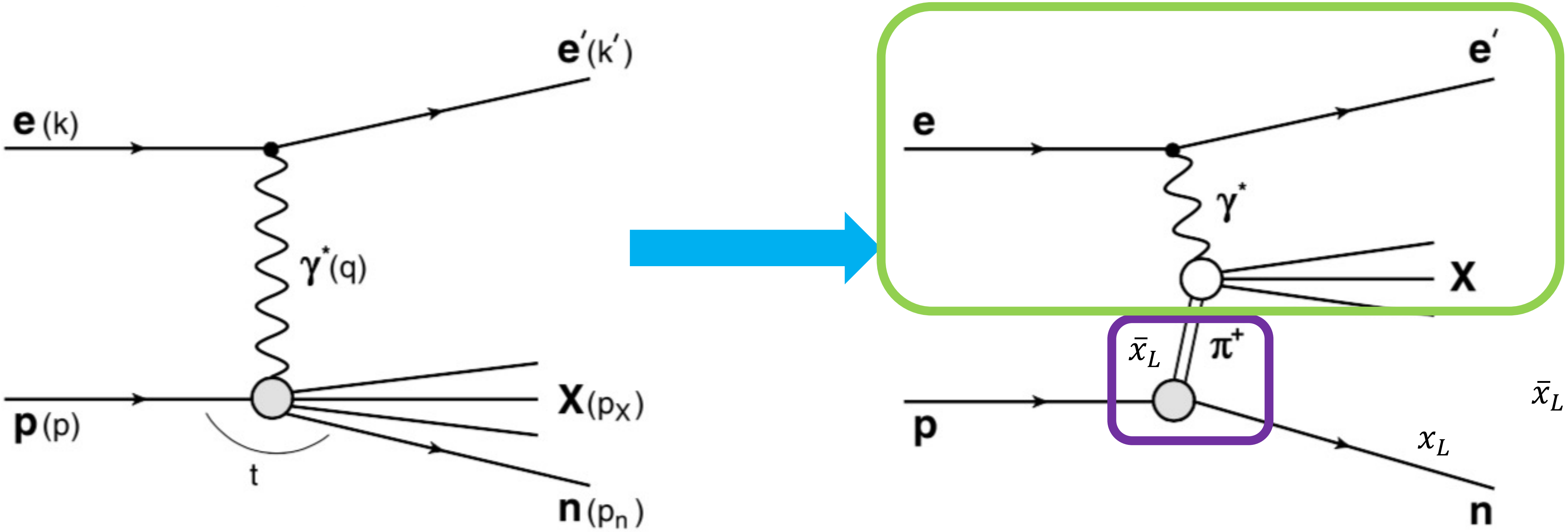
Real emissions



Virtual Corrections



Leading Neutron (LN)

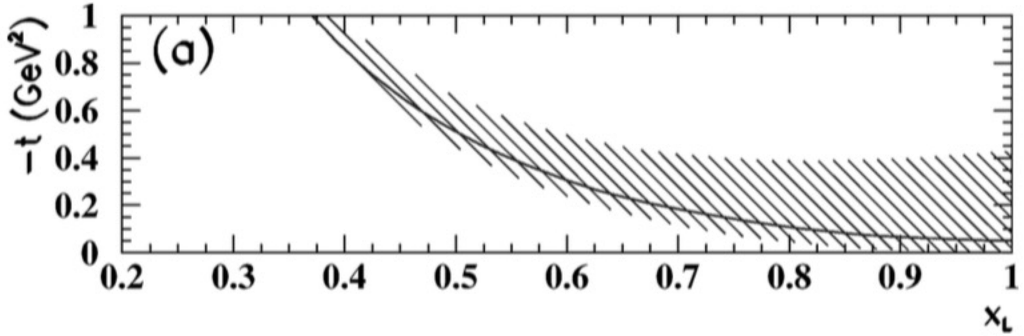


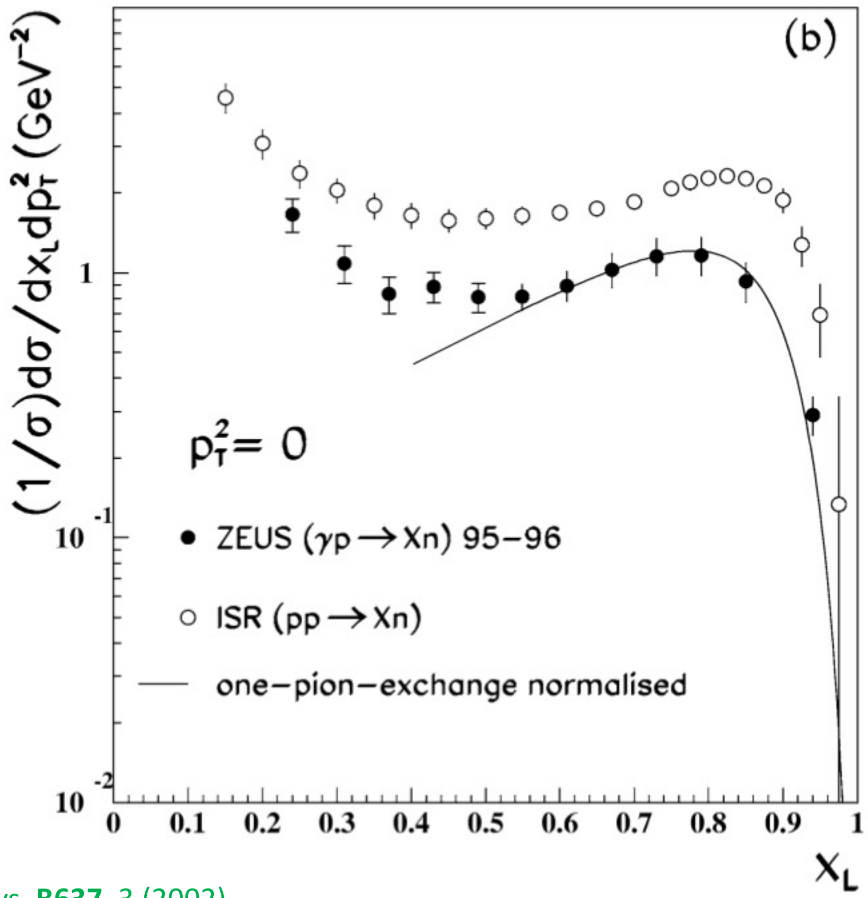
$\bar{x}_L = 1 - x_L$

$$\frac{d\sigma}{dx\,dQ^2\,d\bar{x}_L} \propto f_{\pi N}(\bar{x}_L) \sum_i \int_{x/\bar{x}_L}^{1} \frac{d\xi}{\xi} C\left(\frac{x/\bar{x}_L}{\xi}\right) f_i(\xi, \mu^2)$$

Large x_L

- x_L is fraction of longitudinal momentum carried by neutron relative to initial proton
- For t to be close to pion pole, has to go near 0 – happens at large x_L
- In this region, one pion exchange dominates





Nucl. Phys. **B637**, 3 (2002).

How to relate PDFs with lattice observables?

- Make use of good lattice cross sections and appropriate matching coefficients

$$\Sigma_{n/h}(\nu, z^2) \equiv \langle h(p)|T\{\mathcal{O}_n(z)\}|h(p)\rangle$$
$$= \sum_i f_{i/h}(x, \mu^2) \otimes \mathcal{C}_{n/i}(x\nu, z^2, \mu^2)$$
$$+\mathcal{O}(z^2\Lambda^2_{\mathrm{QCD}})$$

- Structure just like experimental cross sections – good for global analysis

Fitting the Data and Systematic Corrections

Valence quark distribution in pion

Wilson coefficients for matching

$$\mathrm{Re}\,\mathfrak{M}(\nu, z^2) = \int_0^1 \mathrm{d}x\, q_v(x, \mu_{\mathrm{lat}})\, \mathcal{C}^{\mathrm{Rp-ITD}}\left(x\nu, z^2, \mu_{\mathrm{lat}}\right) + z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi (L-z)} F_1(\nu) + \ldots$$

Integration lower bound is 0

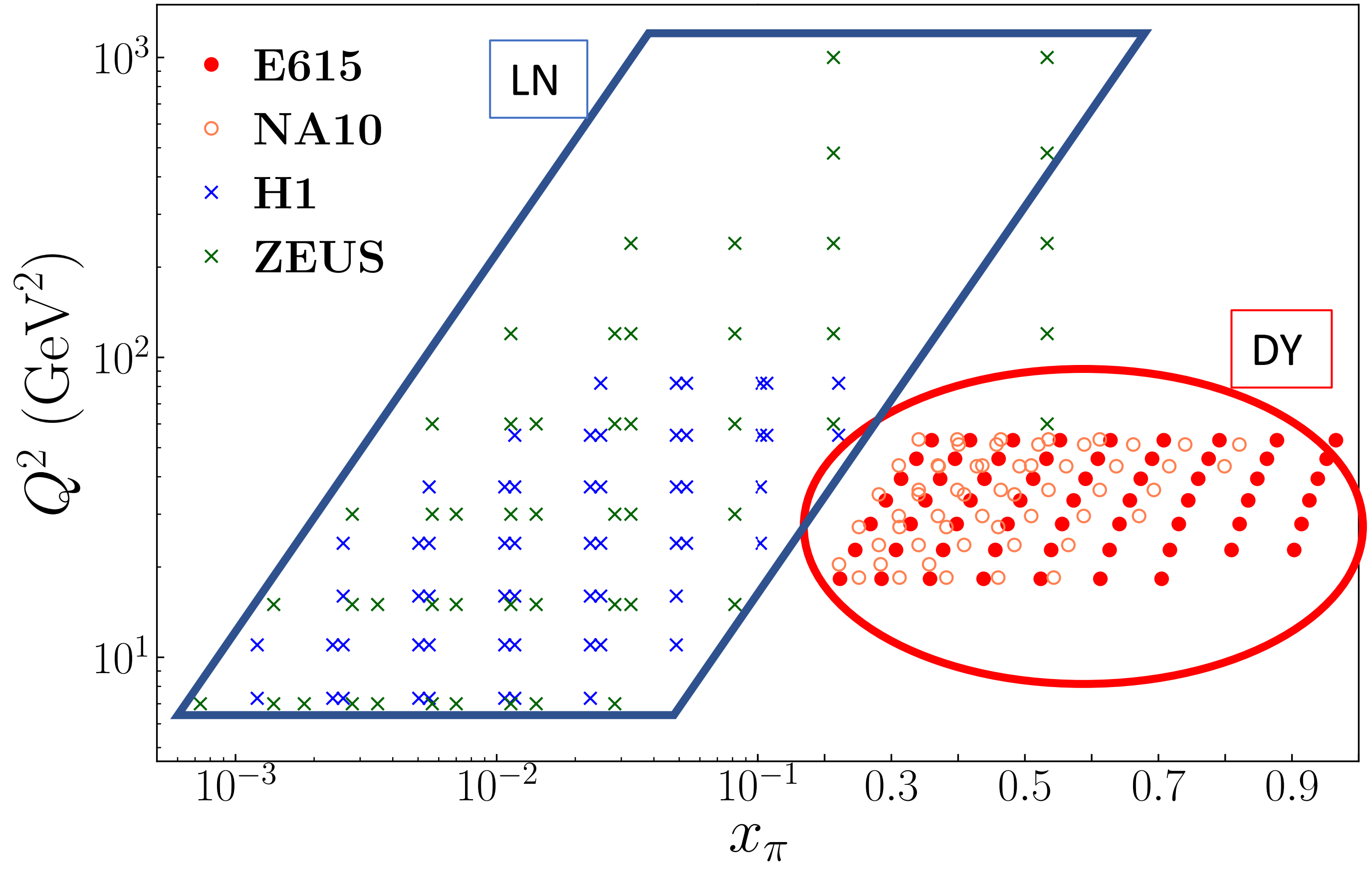
Other potential systematic corrections the data is not sensitive to

Systematic corrections to parametrize

- $z^2 B_1(\nu)$: power corrections
- $\frac{a}{|z|} P_1(\nu)$: lattice spacing errors
- $e^{-m_\pi (L-z)} F_1(\nu)$: finite volume corrections

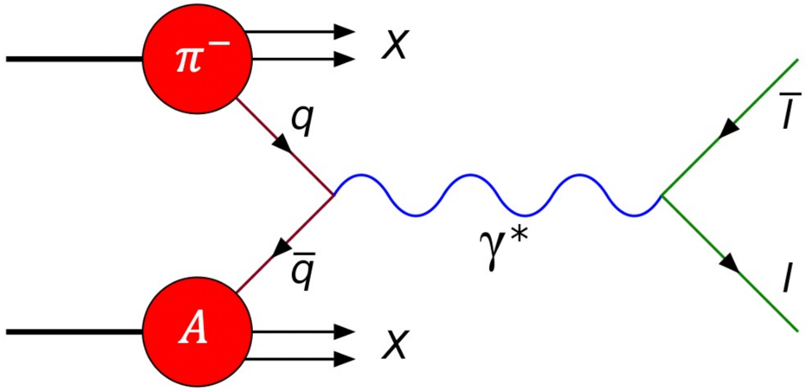
barryp@jlab.org

Datasets -- Kinematics

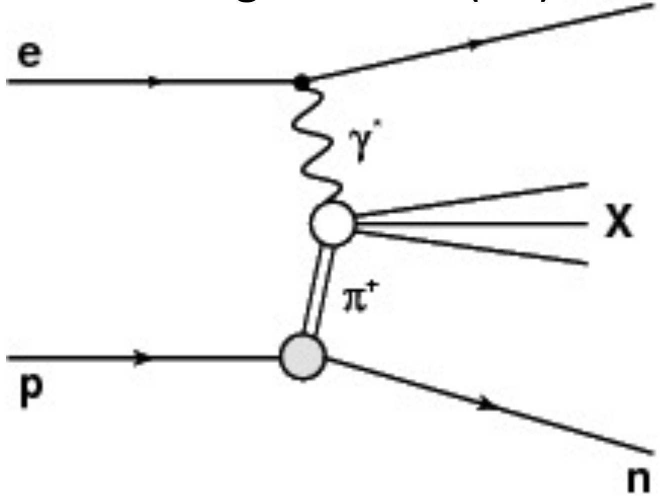


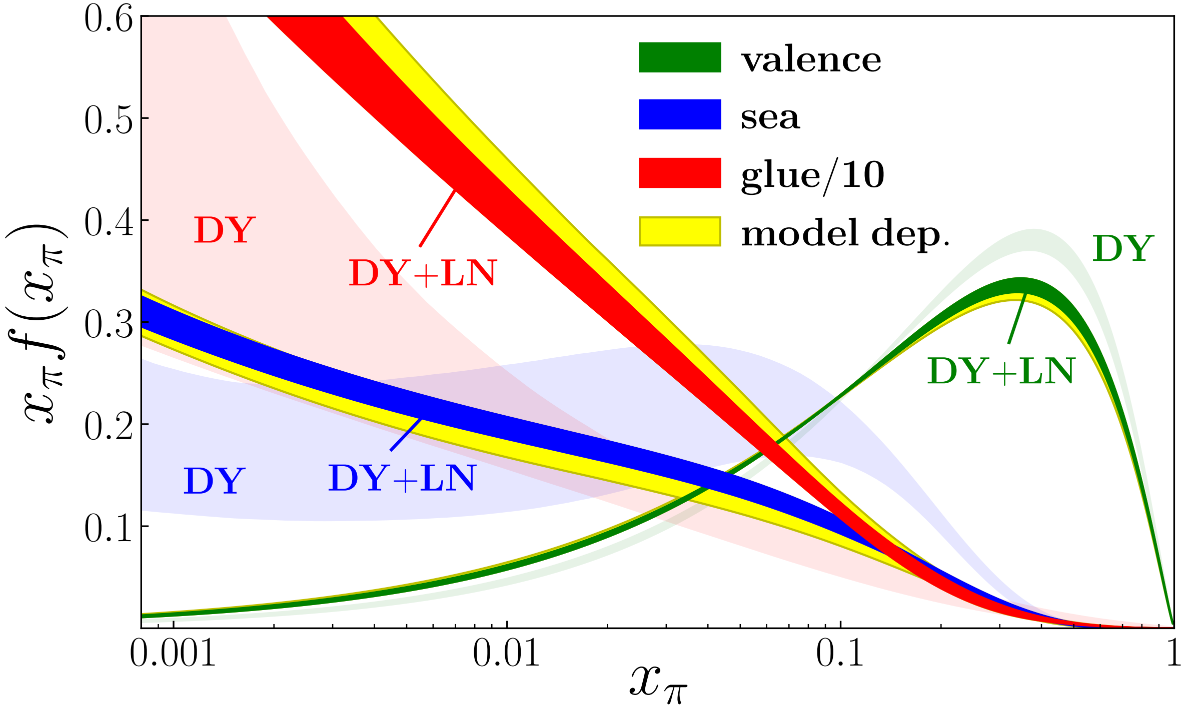
Experiments to probe pion structure

Drell-Yan (DY)



Leading Neutron (LN)





PHYSICAL REVIEW LETTERS **121,** 152001 (2018)

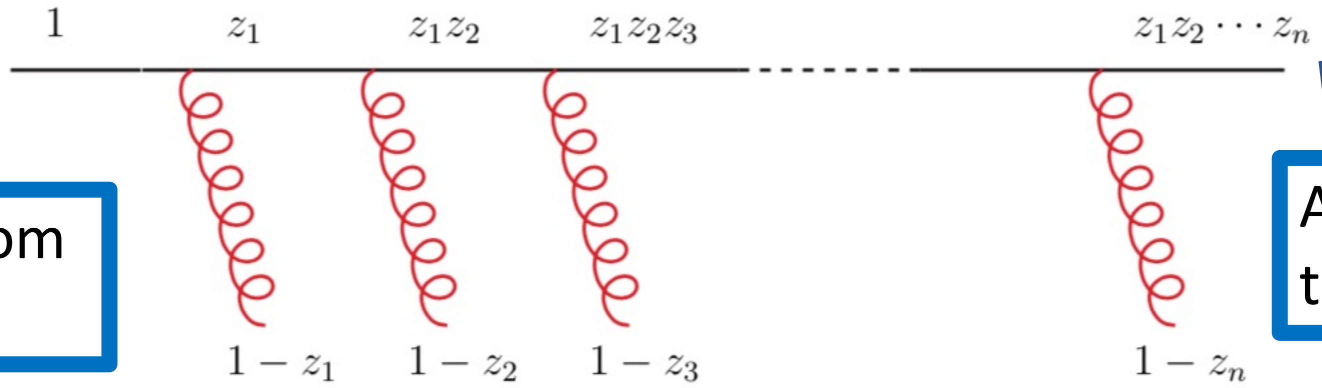
Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry,[1] N. Sato,[2] W. Melnitchouk,[3] and Chueng-Ryong Ji[1]

Threshold Resummation

Initial quark line from hadron



Annihilates with antiquark to produce virtual photon

Significant contributions to cross section occur in soft gluon emissions and follow the pattern

$$d\hat{\sigma}^{q\bar{q}}_{N^k LO} \propto \alpha_S^k \frac{\ln^{2k-1}(1-z)}{1-z} + \ldots$$

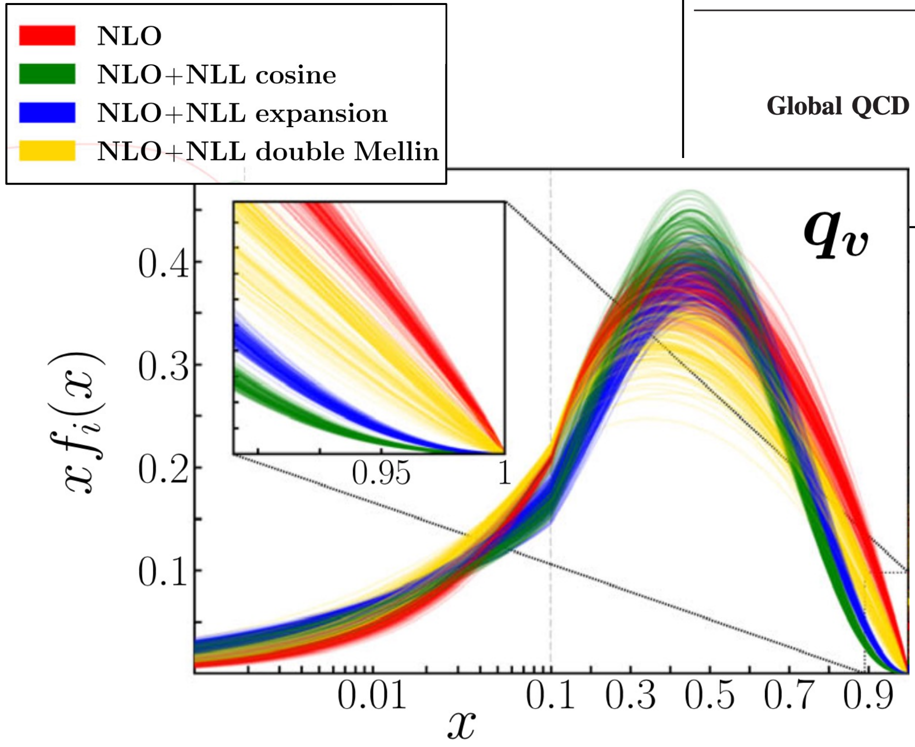
JAM analysis with threshold resummation

PHYSICAL REVIEW LETTERS **127,** 232001 (2021)

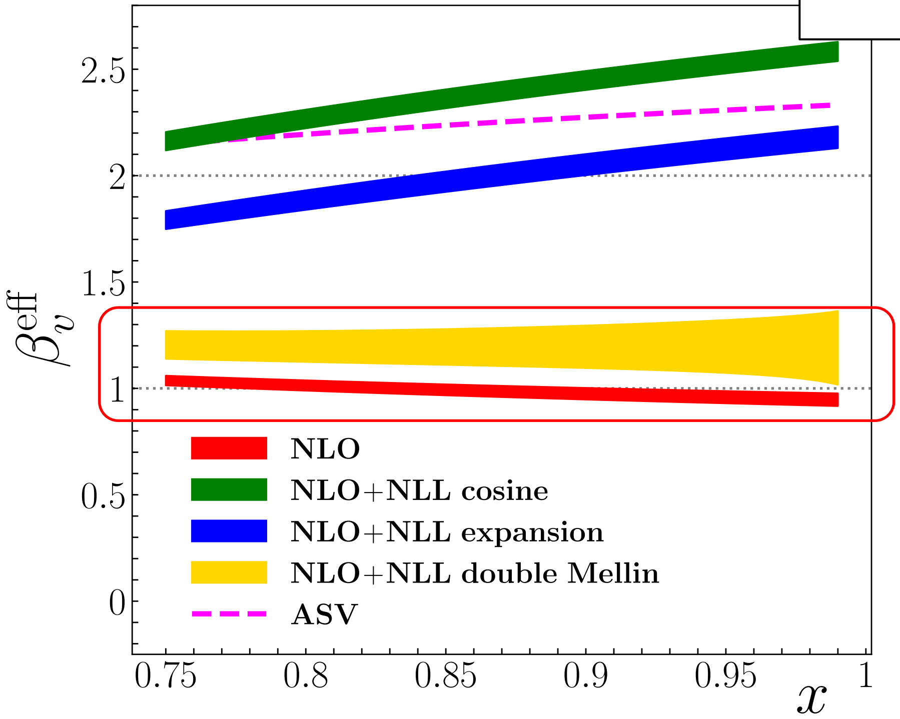
Global QCD Analysis of Pion Parton Distributions with Threshold Resummation

P. C. Barry,[1] Chueng-Ryong Ji,[2] N. Sato,[1] and W. Melnitchouk[1]

(JAM Collaboration)



$$\beta_{\text{eff}}(x,\mu) = \frac{\partial \log |q_v(x,\mu)|}{\partial \log(1-x)}$$



- Highly dependent on perturbative approach
- NLO and NLO+NLL double Mellin methods better on theoretical grounds

Including lattice QCD data from HadStruc

- Can we learn more about pion PDFs with the inclusion of lattice QCD data?

PHYSICAL REVIEW D **105,** 114051 (2022)

Complementarity of experimental and lattice QCD data on pion parton distributions

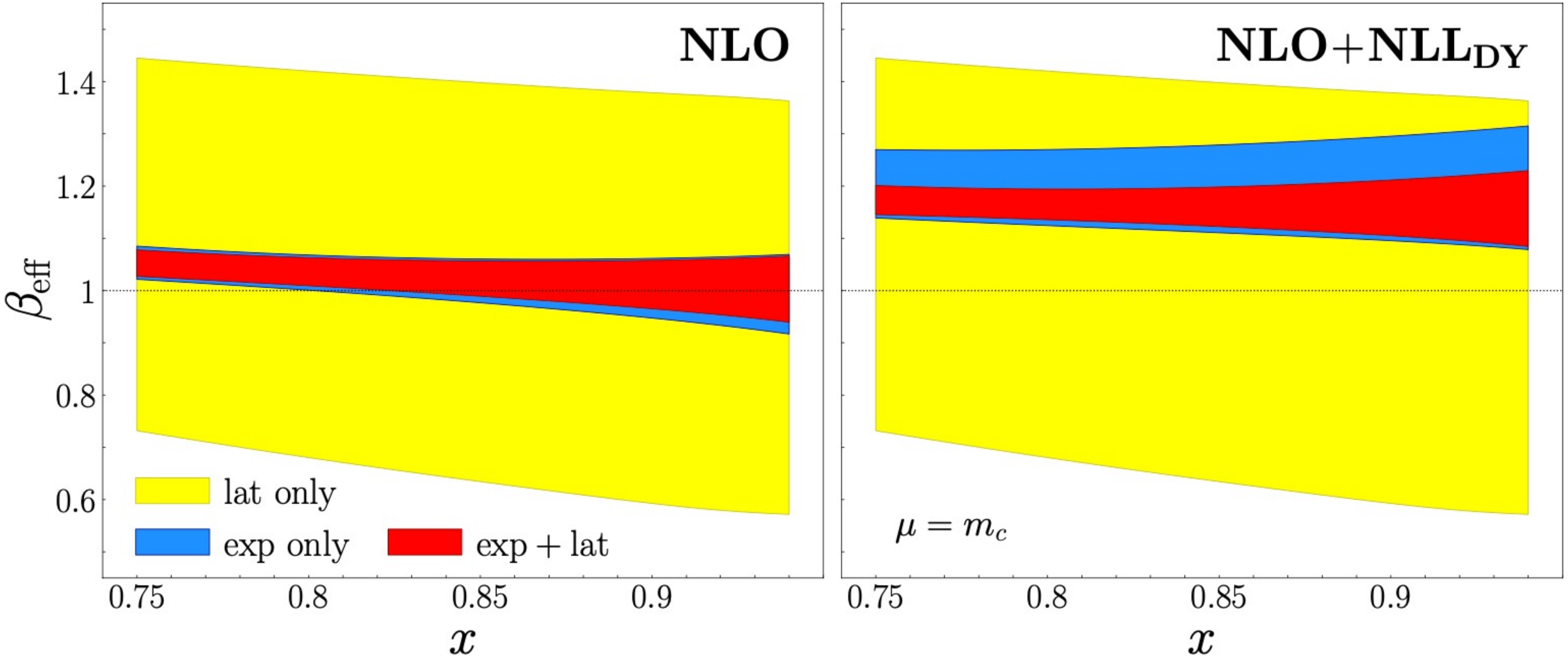
P. C. Barry,[1] C. Egerer,[1] J. Karpie,[2] W. Melnitchouk,[1] C. Monahan,[1,3] K. Orginos,[1,3] Jian-Wei Qiu,[1,3] D. Richards,[1] N. Sato,[1] R. S. Sufian,[1,3] and S. Zafeiropoulos[4]

(Jefferson Lab Angular Momentum (JAM) and HadStruc Collaborations)

Effective β from $(1-x)^{\beta_{\text{eff}}}$

$$\beta_{\text{eff}}(x,\mu) = \frac{\partial \log |q_v(x,\mu)|}{\partial \log(1-x)}$$

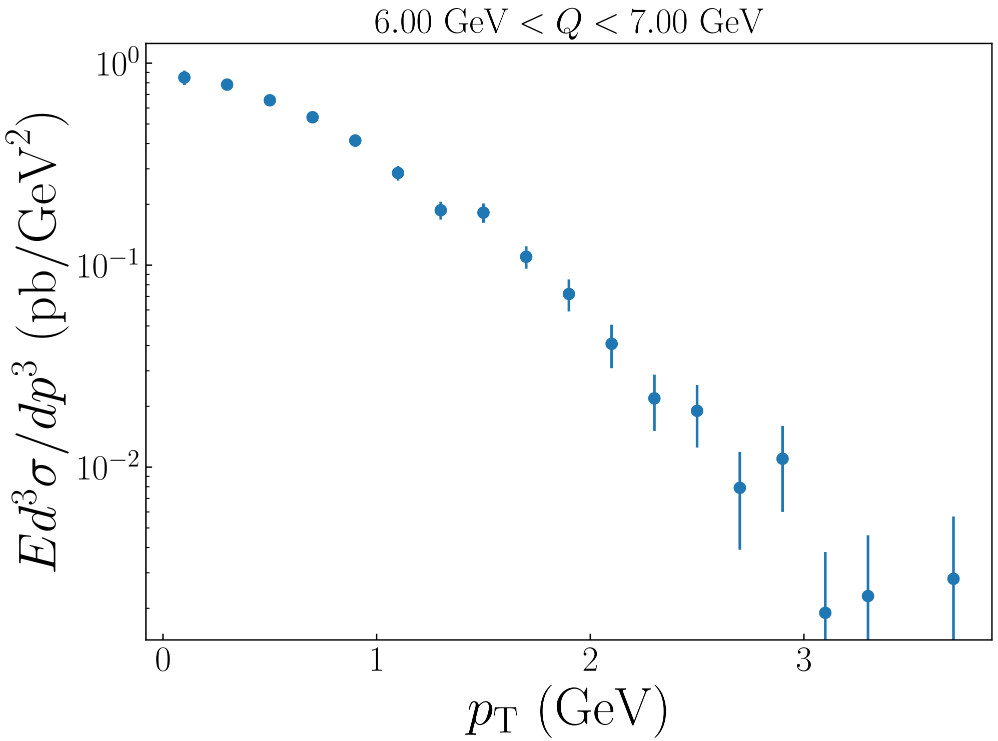
Calculations from QCD do not predict $\beta_{\text{eff}} = 2$



What about the transverse momentum dependence?

p_{T}-dependent spectrum in the nucleon

- Small-p_{T} data – TMD factorization – partonic transverse momentum
- Large-p_{T} data – collinear factorization – recoil transverse momentum

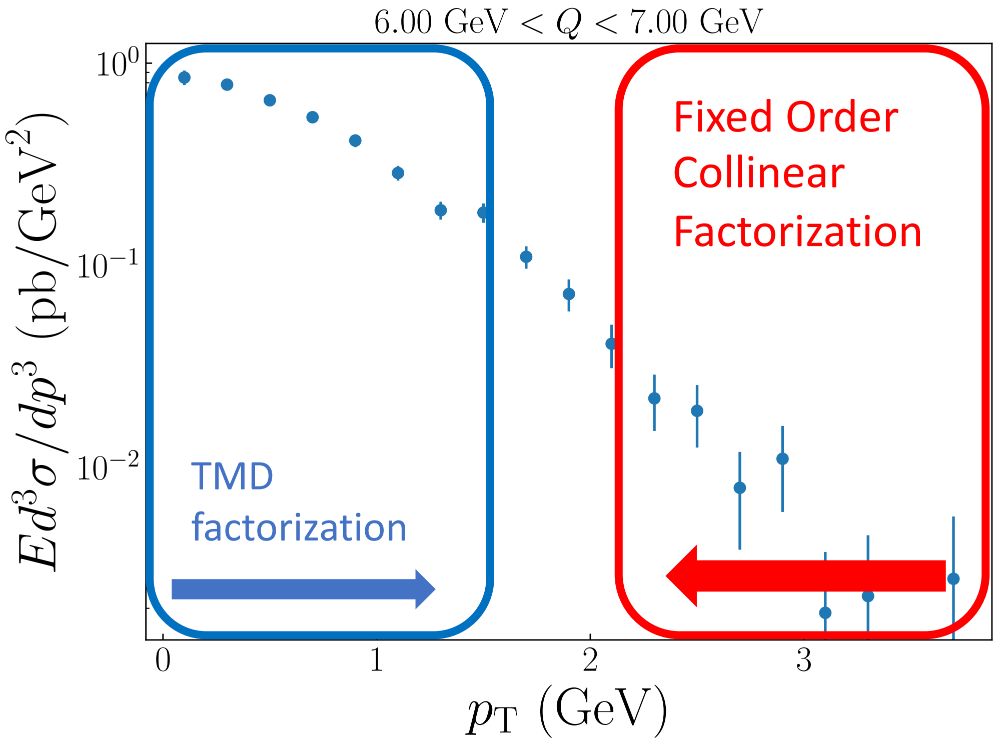


E288 pp Drell-Yan

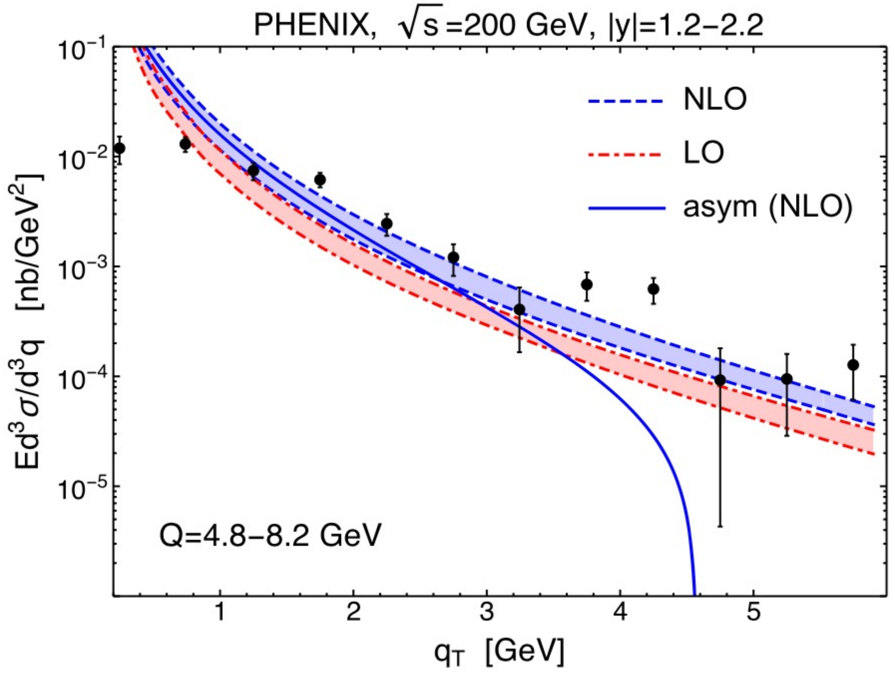
Phys. Rev. D **39**, 92 (1989).

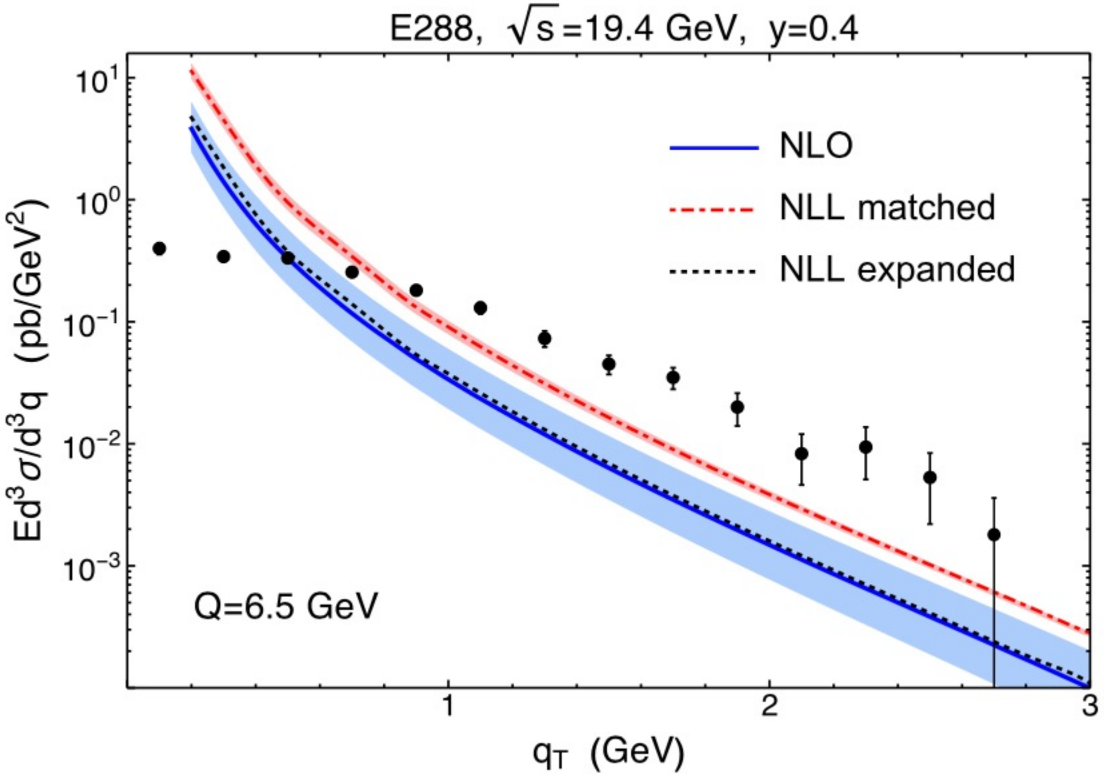
p_T-dependent spectrum in the nucleon

- Various factorization theorems break down in certain regions of p_T
- Errors are related with $\mathcal{O}(p_\mathrm{T}/Q)$ (low-p_T) or $\mathcal{O}(m/p_\mathrm{T})$ (large-p_T)



Large p_{T} Drell-Yan in the nucleon





Collider (PHENIX) data are well described by $\mathcal{O}(\alpha_s^2)$ in collinear factorization

Phys. Rev. D **100**, 014018 (2019).

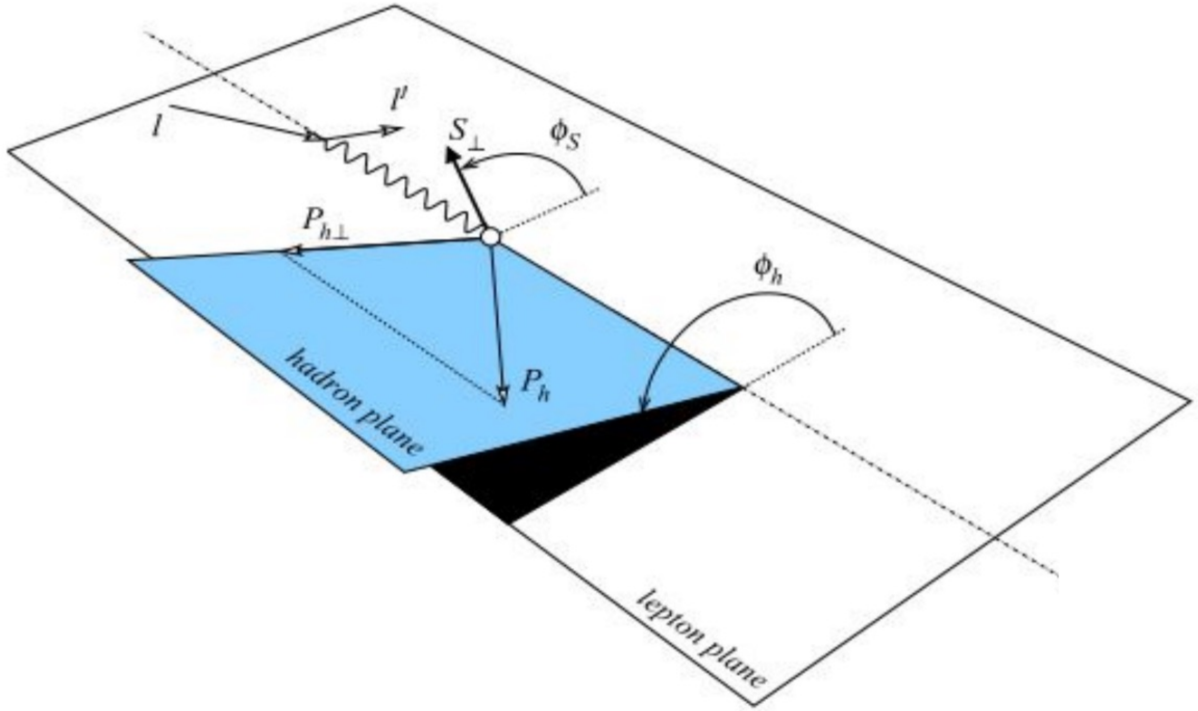
Fixed target (Fermilab E288) data are challenging to describe at $\mathcal{O}(\alpha_s^2)$ or even with resummation (NLL)

Phys. Rev. D **100**, 014018 (2019).

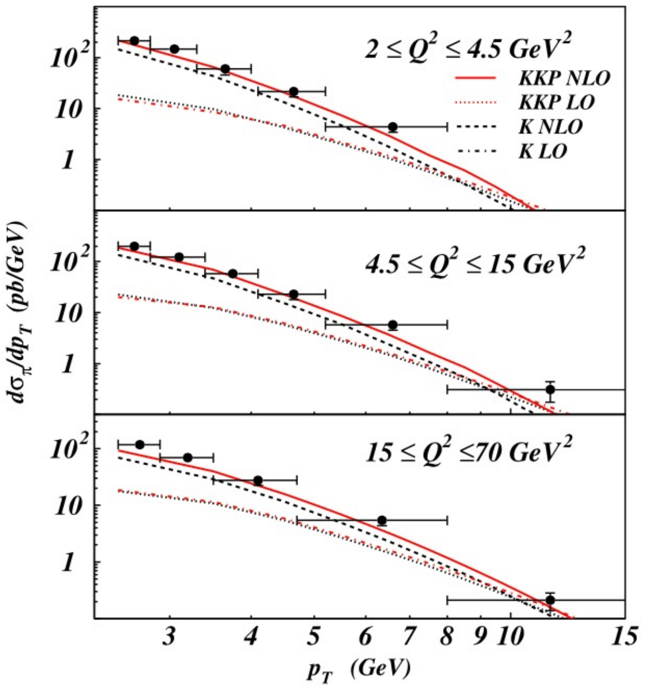
Semi-Inclusive DIS (SIDIS)

- Incoming electron beam emits a virtual photon
- Breaks up the target proton
- Another hadron (like a pion) is detected (fragmentation function)
- Measure the p_T dependence of the detected hadron

$$l + P \rightarrow l' + h + X$$

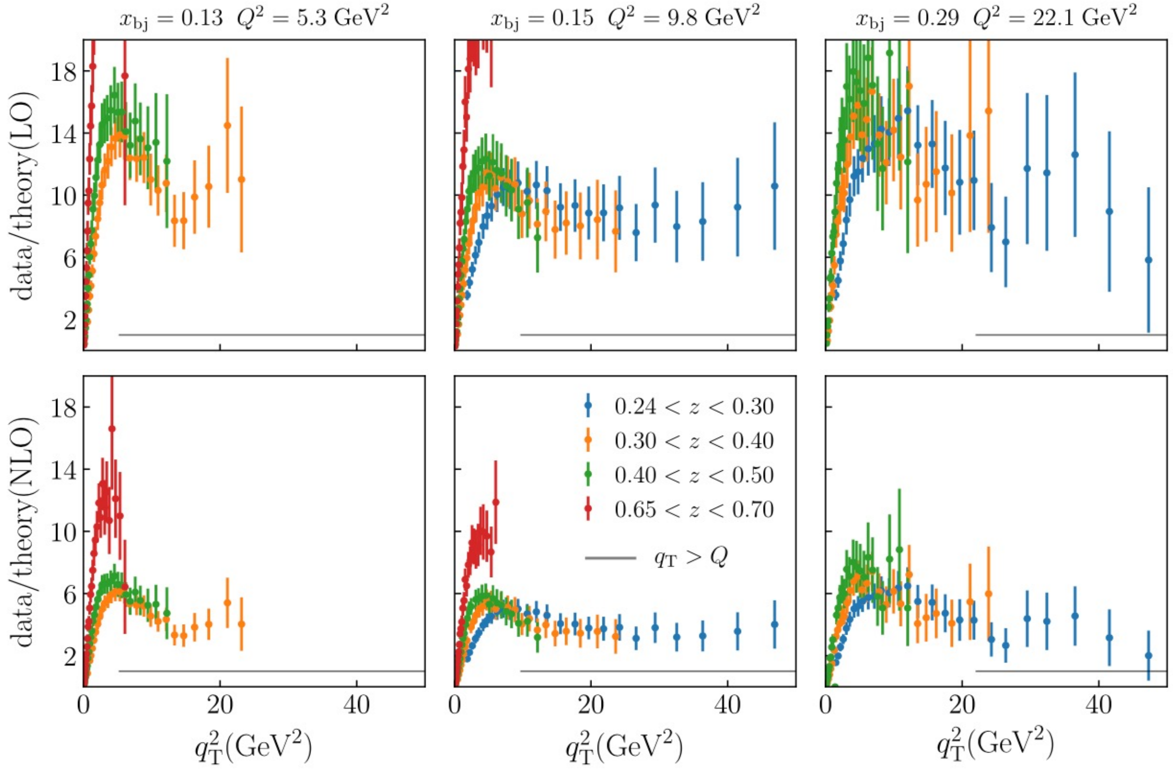


Large p_{T} SIDIS in the nucleon



Collider (H1) data are well described by $\mathcal{O}(\alpha_s^2)$ in collinear factorization

Phys. Rev. D **71**, 034013 (2005).

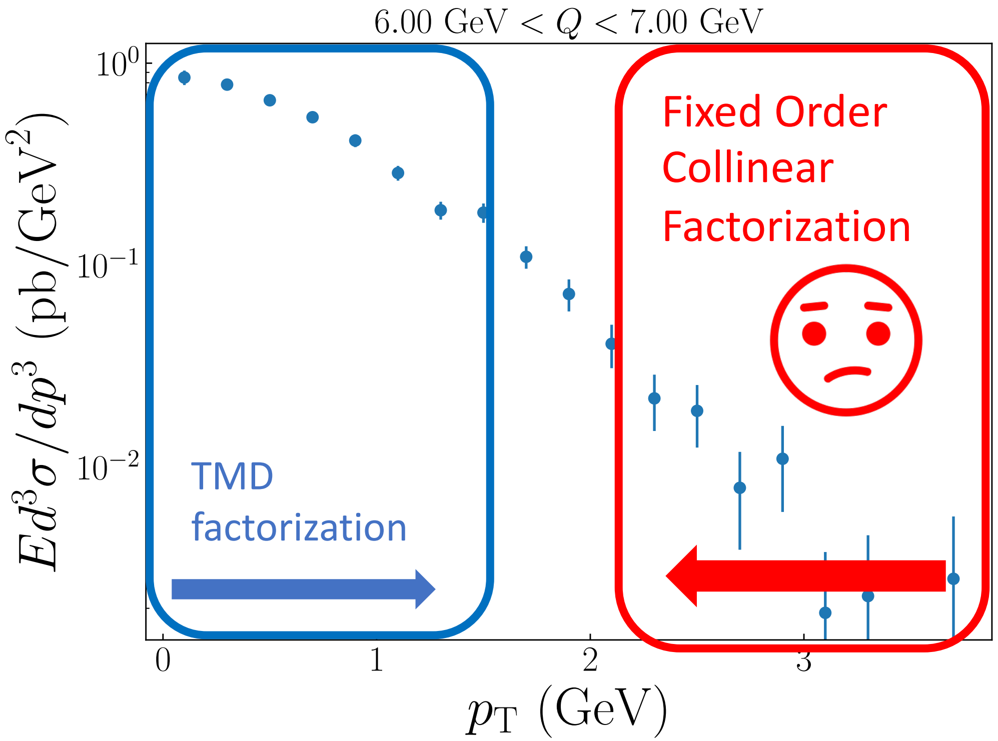


Fixed target (COMPASS 17) data are challenging to describe at $\mathcal{O}(\alpha_s)$ (top) or $\mathcal{O}(\alpha_s^2)$ (bottom)

Phys. Rev. D **98**, 114005 (2018).

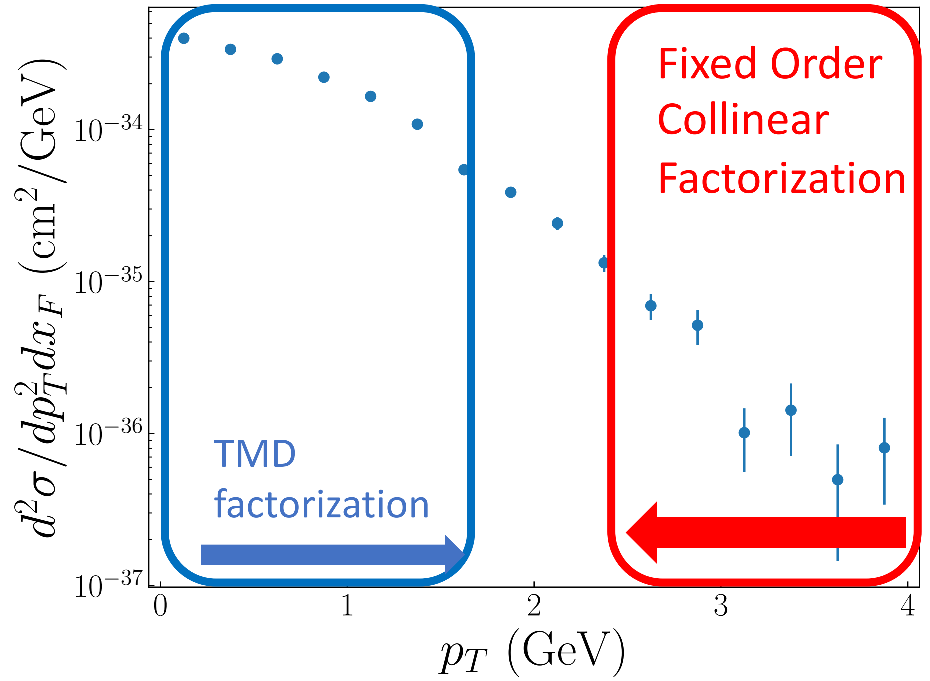
p_{T}-dependent spectrum in the nucleon

- Various factorization theorems break down in certain regions of p_{T}
- Errors are related with $\mathcal{O}(p_{\mathrm{T}}/Q)$ (low-p_{T}) or $\mathcal{O}(m/p_{\mathrm{T}})$ (large-p_{T})



What about the pion?

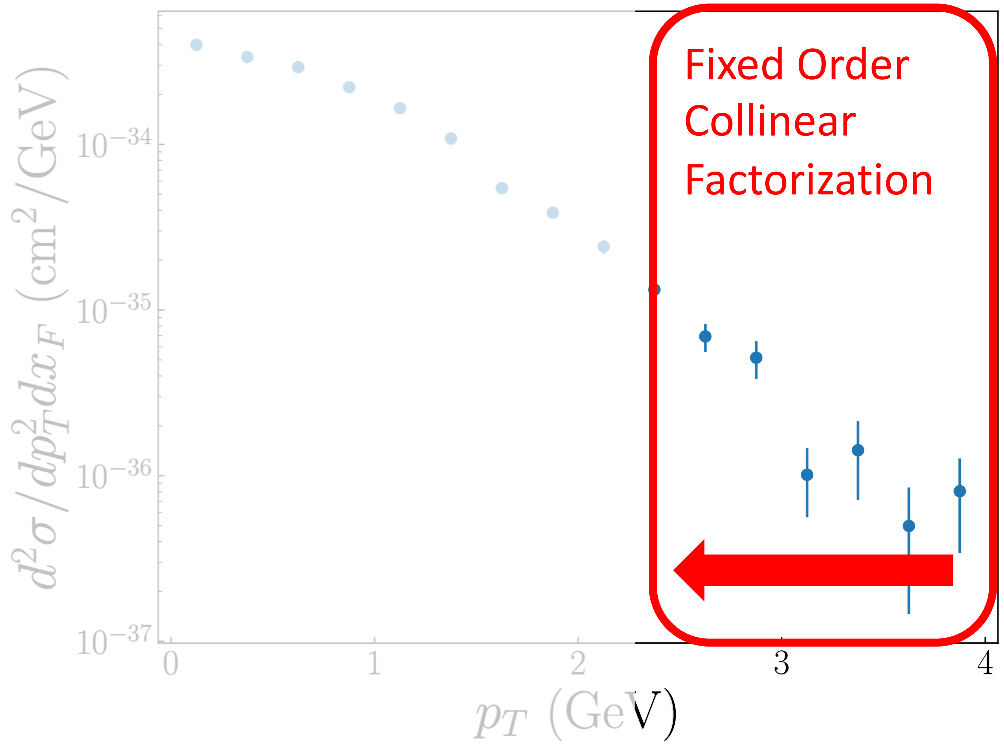
- Available p_{T}-dependent Drell-Yan data from E615 Phys. Rev. D **39**, 92 (1989).
- Fixed Target data (no collider pion data)

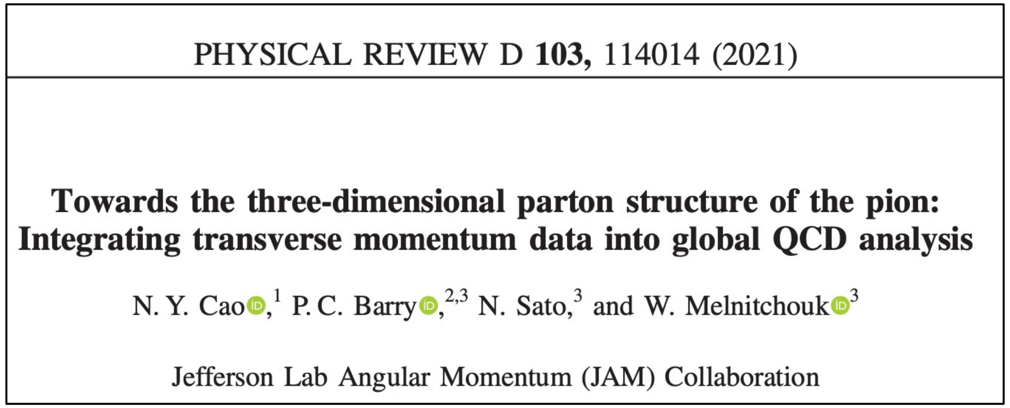


πA data

p_{T}-dependent Pion Drell-Yan spectra

- First, we examine the large p_{T} spectrum





PHYSICAL REVIEW D **103,** 114014 (2021)

Towards the three-dimensional parton structure of the pion: Integrating transverse momentum data into global QCD analysis

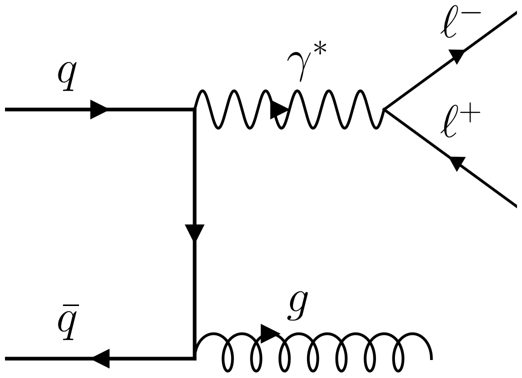
N. Y. Cao,[1] P. C. Barry,[2,3] N. Sato,[3] and W. Melnitchouk[3]

Jefferson Lab Angular Momentum (JAM) Collaboration

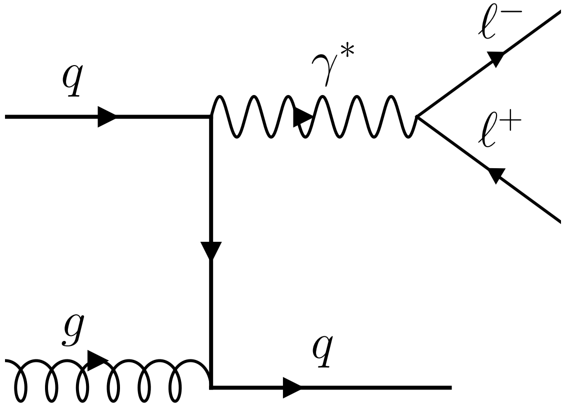
Drell-Yan (DY)

- p_T dependent DY in collinear factorization

$$\frac{d\sigma}{dQ^2 dY dp_T^2} = \frac{4\pi\alpha^2}{3N_C Q^2 S} \sum_{i,j} e_q^2 \int_{x_\pi^0}^1 dx_\pi f_i^\pi(x_\pi, \mu) f_j^A(x_A, \mu) \times \frac{d\hat{\sigma}_{i,j}}{dQ^2 d\hat{t}}$$



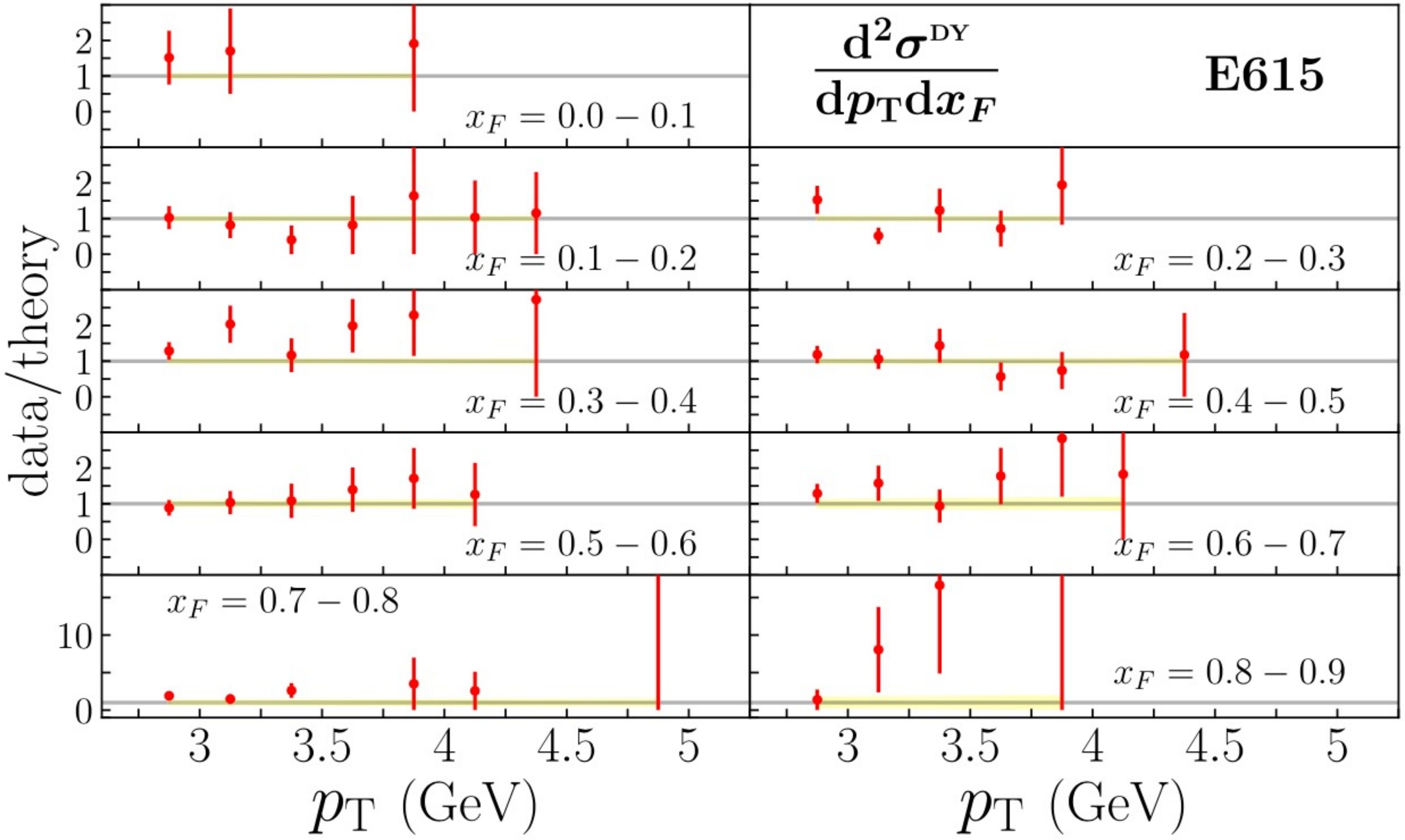
$q\bar{q}$ channel example



qg channel example

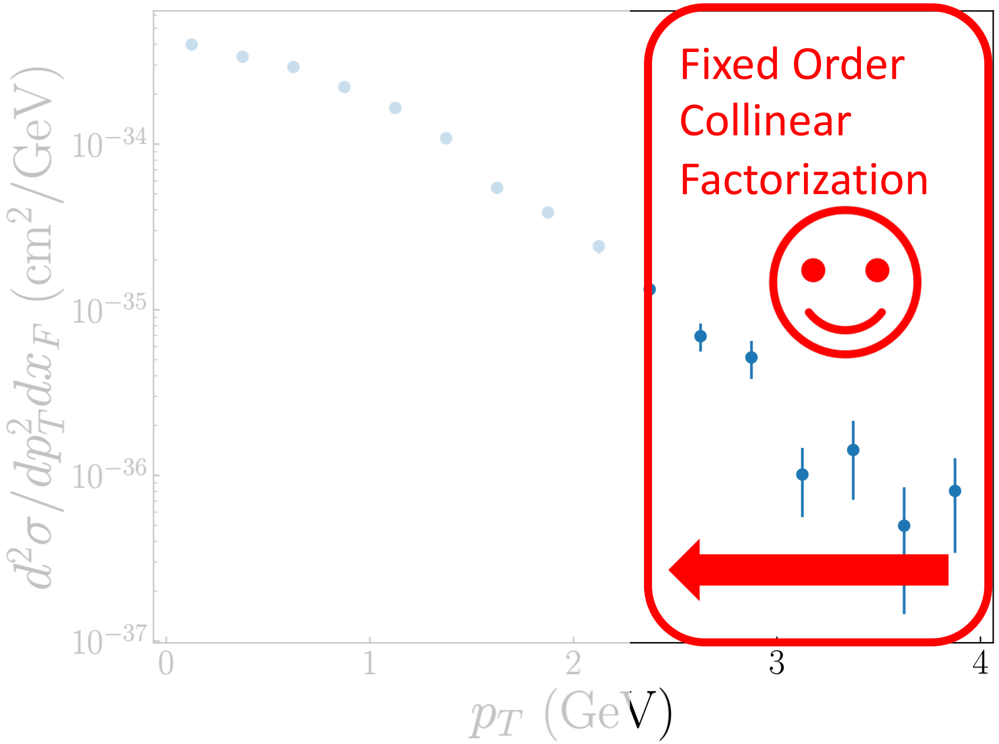
Data and theory comparsion

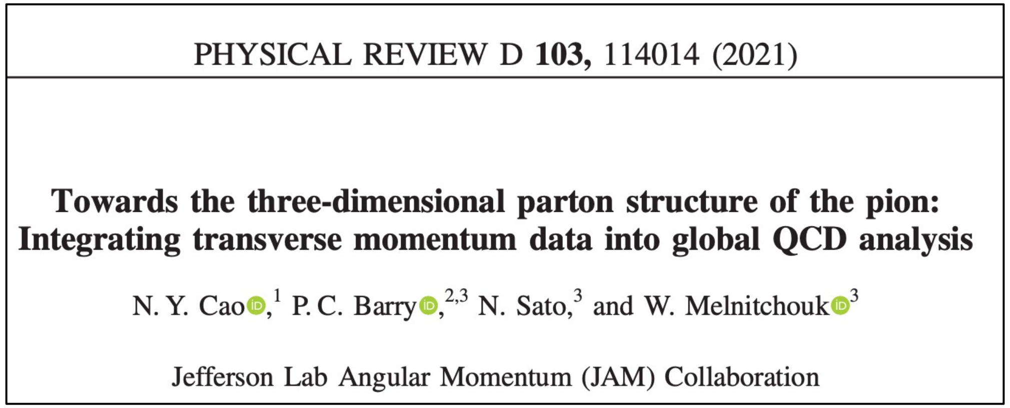
- Data are rather noisy
- Pion's smaller gluon component than in the proton may lead to easier description
- Large normalization uncertainty here



p_{T}-dependent Pion Drell-Yan spectra

- First, we examine the large p_{T} spectrum





PHYSICAL REVIEW D **103,** 114014 (2021)

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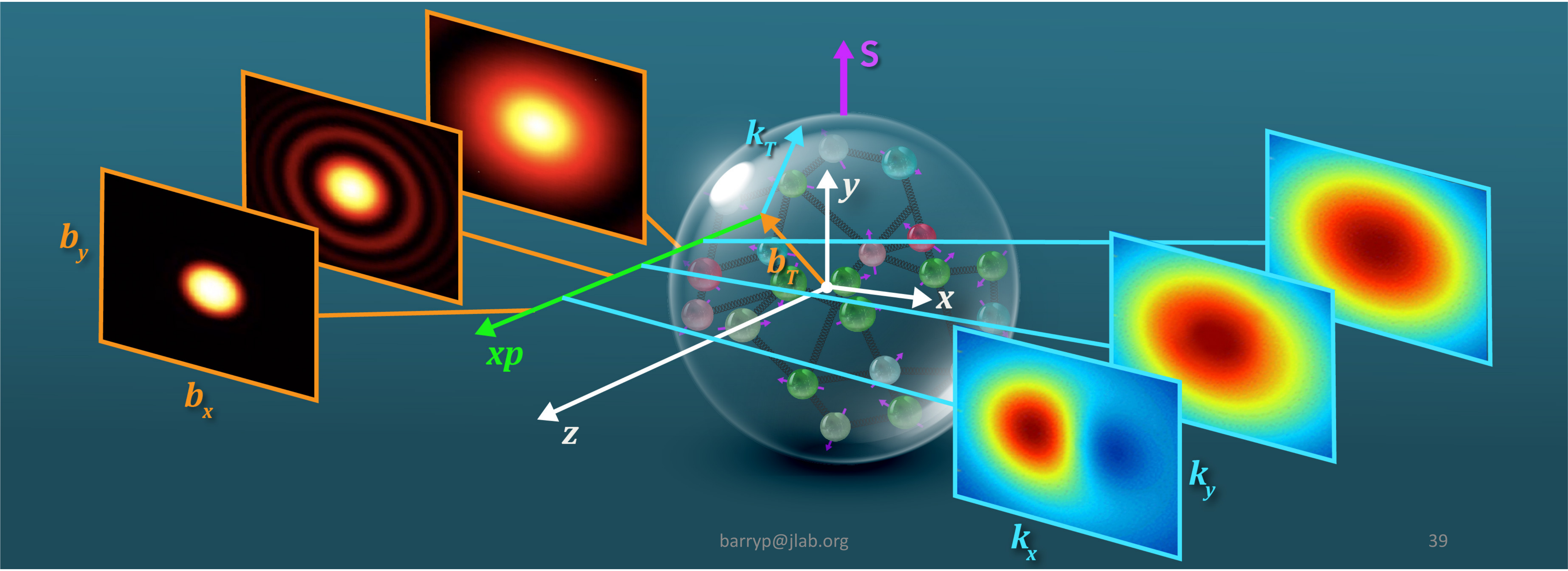
What’s next?

- We now have a good understanding of collinear structures of pions and protons
- Not all non-perturbative momentum structure is in the longitudinal direction

3D structures of hadrons

- Even more challenging is the 3d structure through GPDs and TMDs



First, a few nice references from theory standpoint

TMD handbook: arXiv:2304.03302

- This list is in no way complete

J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. **B250**, 199 (1985).

J. Collins, (2011), *Foundations of Perturbative QCD*, (Cambridge University Press, Cambridge, England, 2011).

A. Kotzinian, Nucl. Phys. B **441**, 234 (1995), arXiv:hep-ph/9412283.

R. D. Tangerman and P. J. Mulders, Phys. Rev. D **51**, 3357 (1995), arXiv:hep-ph/9403227.

P. J. Mulders and R. D. Tangerman, Nucl. Phys. B **461**, 197 (1996), [Erratum: Nucl.Phys.B 484, 538–540 (1997)], arXiv:hep-ph/9510301.

D. Boer and P. J. Mulders, Phys. Rev. D **57**, 5780 (1998), arXiv:hep-ph/9711485.

J. C. Collins and D. E. Soper, Nucl. Phys. **B193**, 381 (1981).

J. C. Collins and D. E. Soper, Nucl.Phys. **B197**, 446 (1982).

S. Aybat and T. C. Rogers, Phys.Rev. **D83**, 114042 (2011), arXiv:1101.5057 [hep-ph].

J. Collins and T. Rogers, Phys.Rev. **D91**, 074020 (2015), arXiv:1412.3820 [hep-ph].

X.-d. Ji, J.-P. Ma, and F. Yuan, Phys. Lett. B **597**, 299 (2004), arXiv:hep-ph/0405085.

M. G. Echevarria, A. Idilbi, and I. Scimemi, JHEP **07**, 002 (2012), arXiv:1111.4996 [hep-ph].

J. Collins and T. C. Rogers, Phys. Rev. D **96**, 054011 (2017), arXiv:1705.07167 [hep-ph].

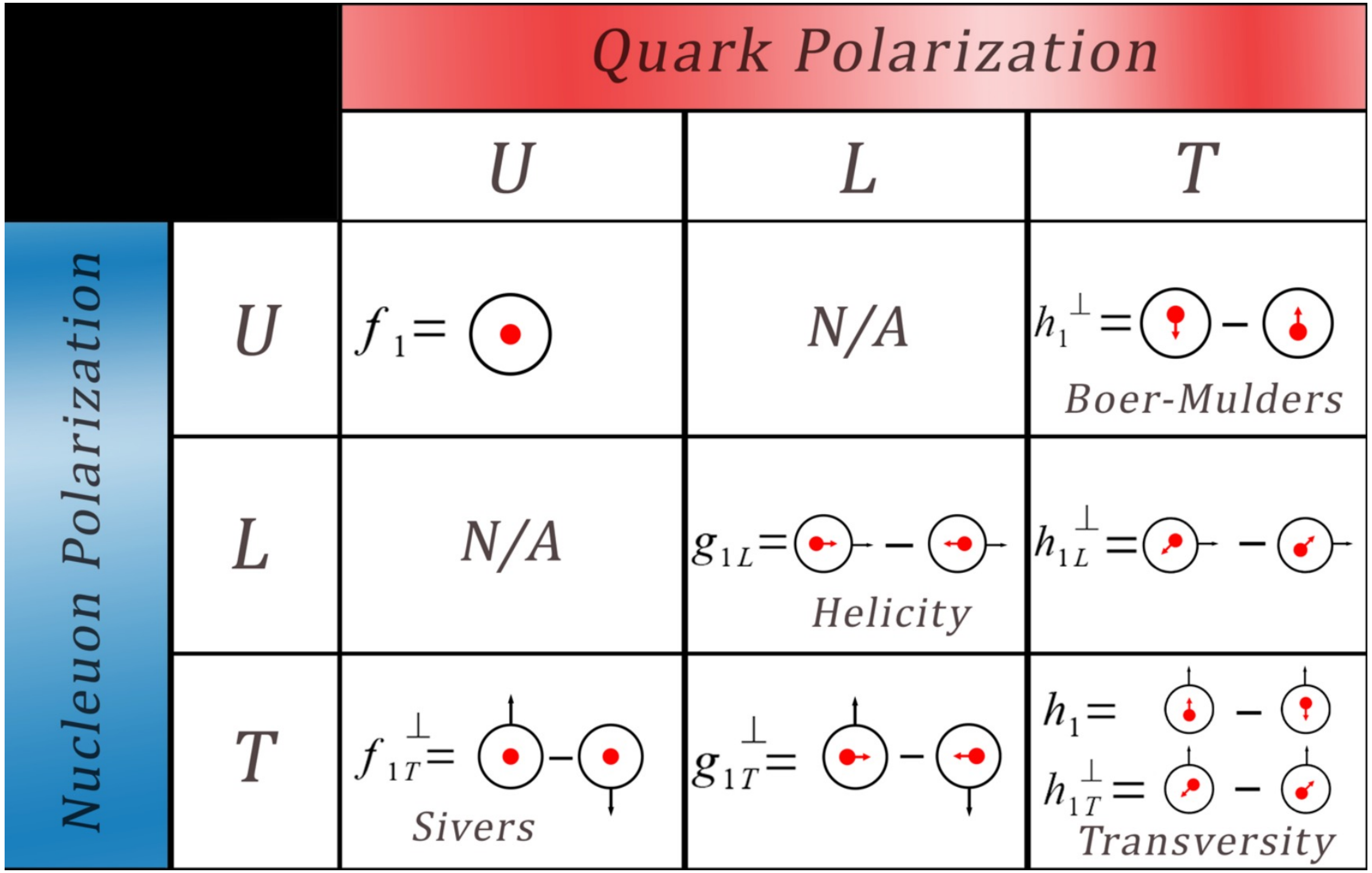
T. Becher and M. Neubert, Eur. Phys. J. C **71**, 1665 (2011), arXiv:1007.4005 [hep-ph].

J. O. Gonzalez-Hernandez, T. C. Rogers, and N. Sato, Phys. Rev. D **106**, 034002 (2022), arXiv:2205.05750 [hep-ph].

M. A. Ebert, J. K. L. Michel, I. W. Stewart, and Z. Sun, JHEP **07**, 129 (2022), arXiv:2201.07237 [hep-ph].

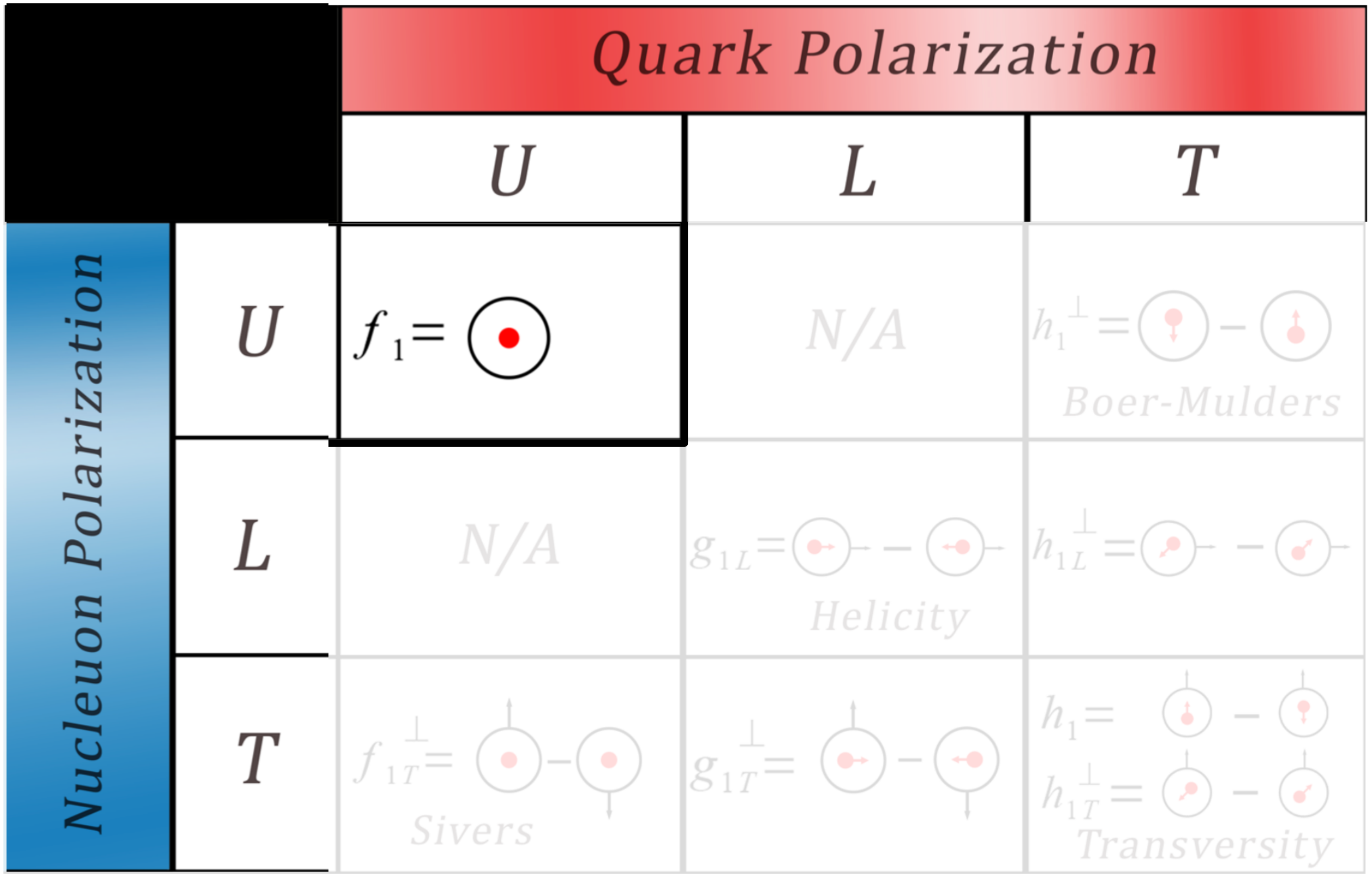
Types of TMDs

- 8 types of TMDs described by the polarization of quarks and hadron



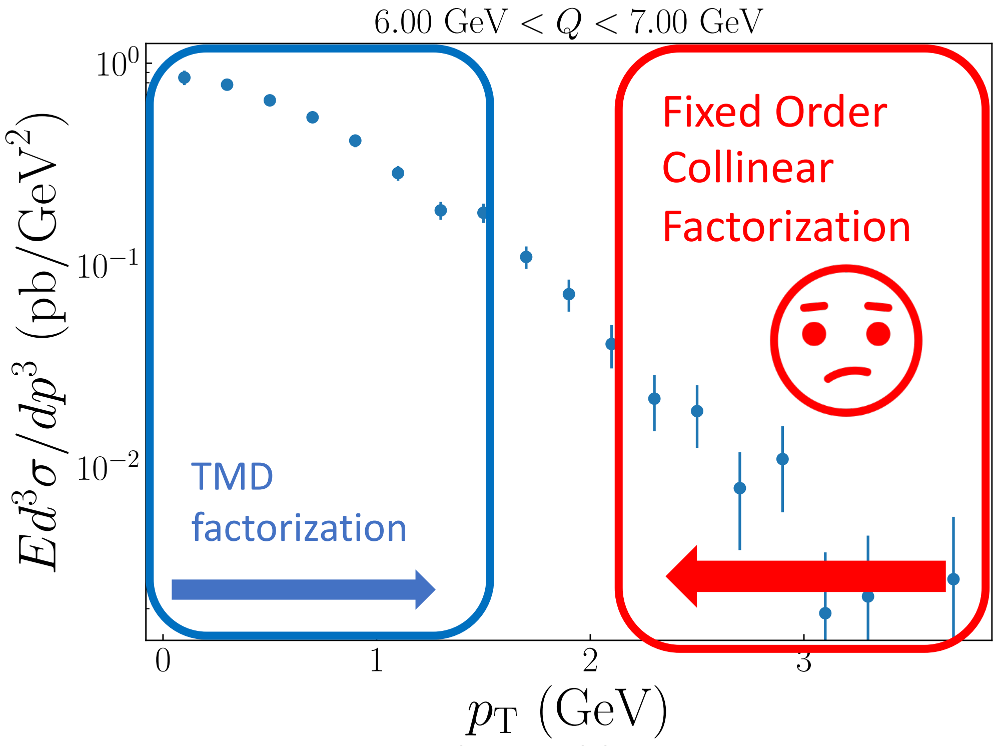
Types of TMDs

- 8 types of TMDs described by the polarization of quarks and hadron
- Focus here only on the unpolarized TMDs



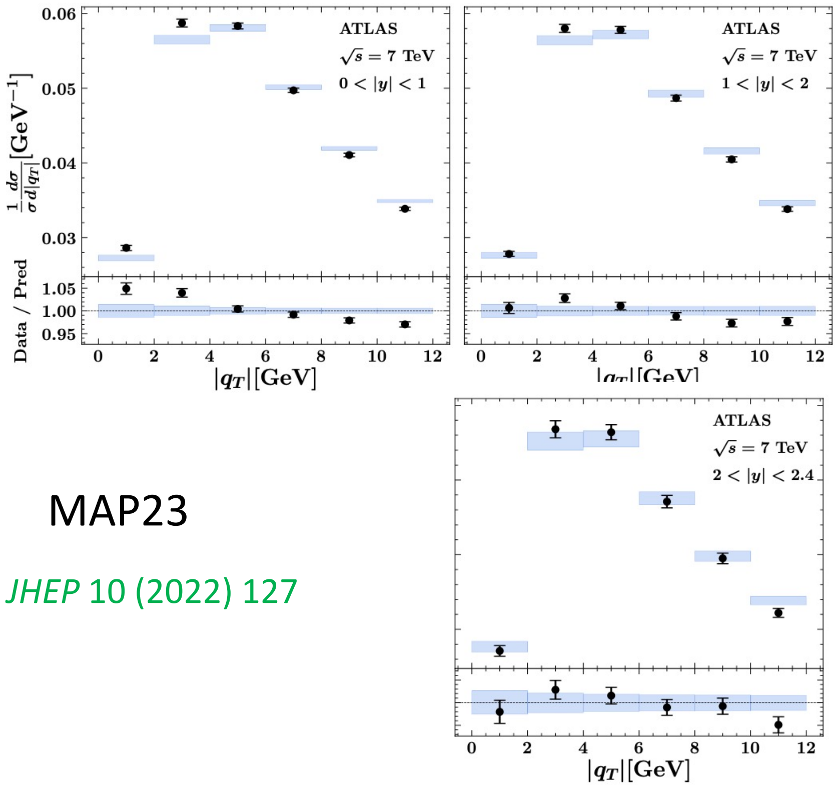
p_{T}-dependent spectrum in the nucleon

- Various factorization theorems break down in certain regions of p_{T}
- Errors are related with $\mathcal{O}(p_{\mathrm{T}}/Q)$ (low-p_{T}) or $\mathcal{O}(m/p_{\mathrm{T}})$ (large-p_{T})



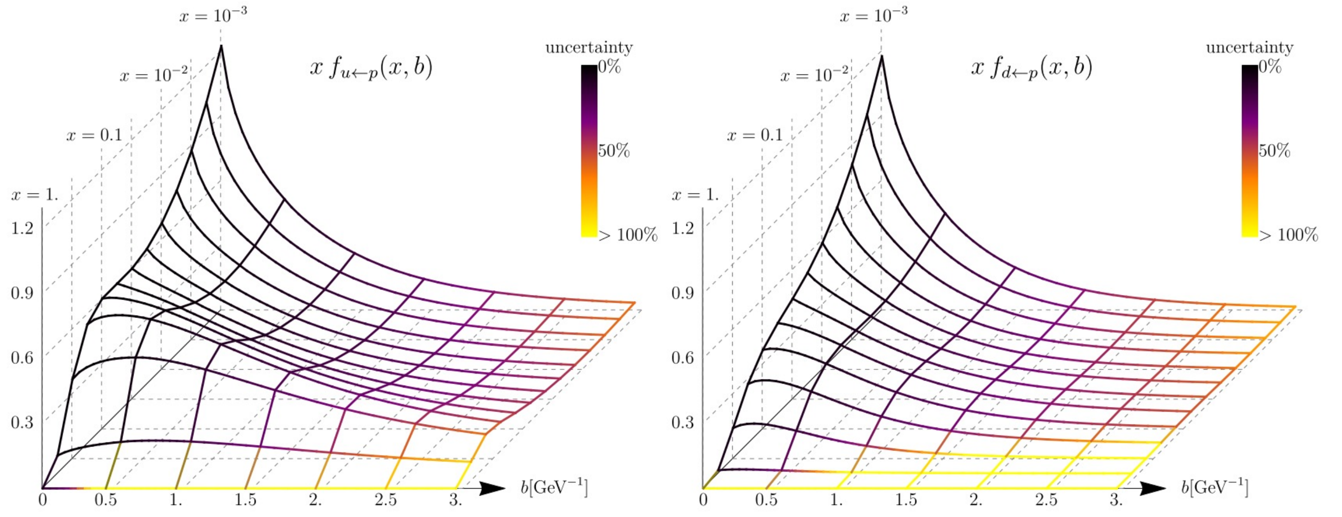
Success at small-p_{T} in nucleon

- MAP and Artemide groups have fit TMDs to low-p_{T} collider and fixed target Drell-Yan (and sometimes SIDIS) data



MAP23

JHEP 10 (2022) 127

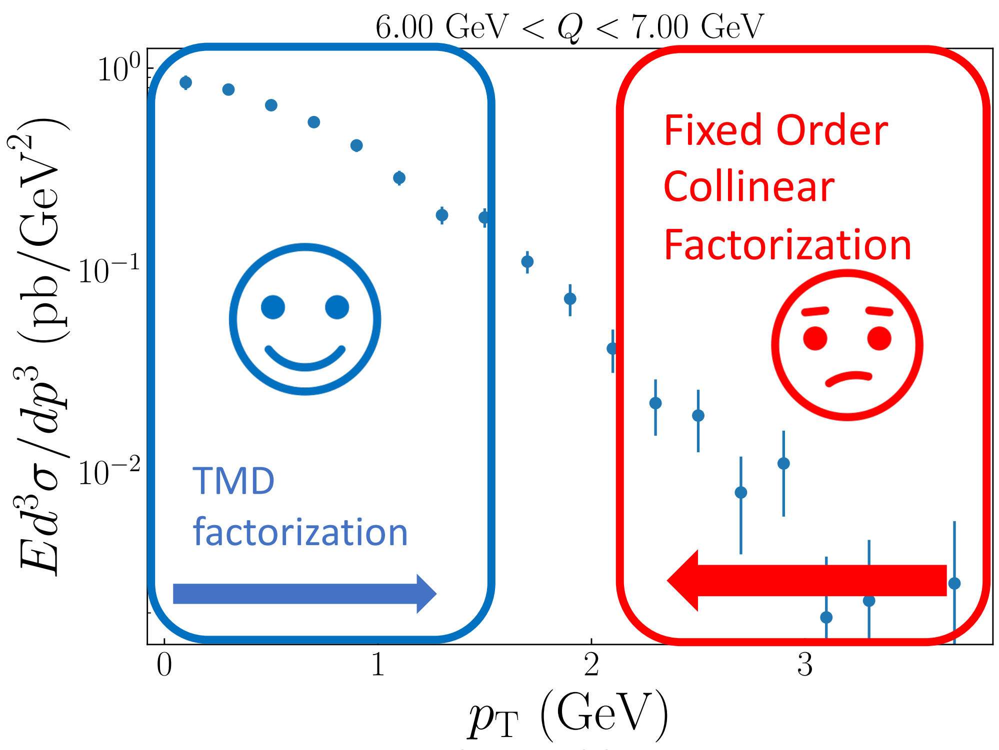


ART23

arXiv:2305.07473

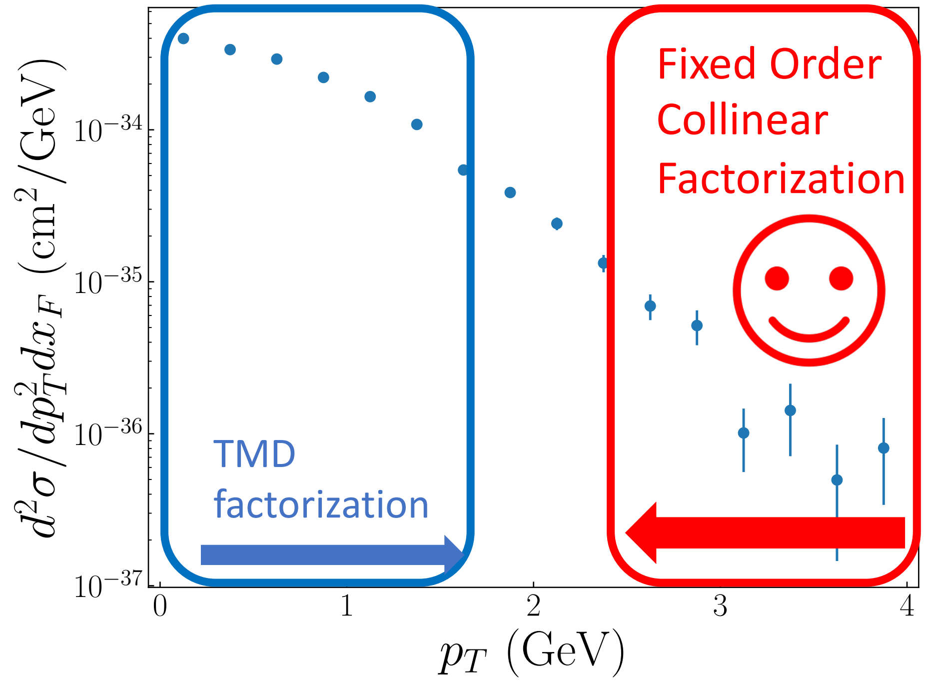
p_{T}-dependent spectrum in the nucleon

- Various factorization theorems break down in certain regions of p_{T}
- Errors are related with $\mathcal{O}(p_{\mathrm{T}}/Q)$ (low-p_{T}) or $\mathcal{O}(m/p_{\mathrm{T}})$ (large-p_{T})



What about the pion?

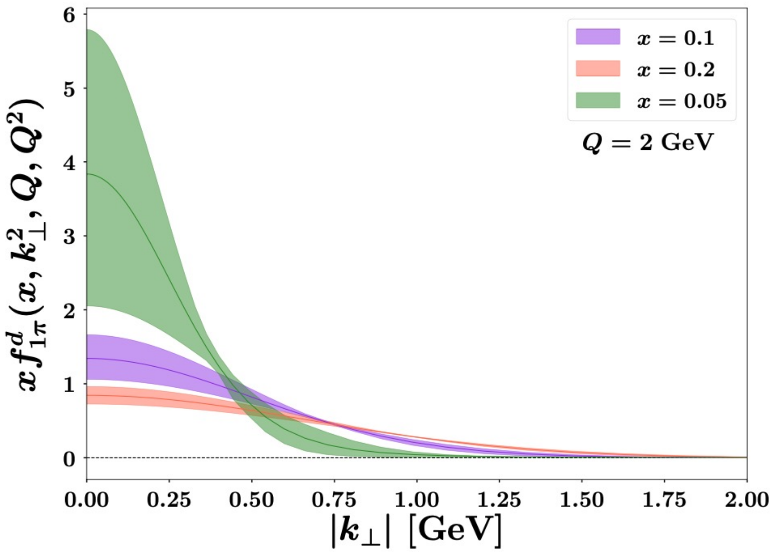
- Available p_{T}-dependent Drell-Yan data from E615 Phys. Rev. D **39**, 92 (1989).
- Fixed Target data (no collider pion data)



πA data

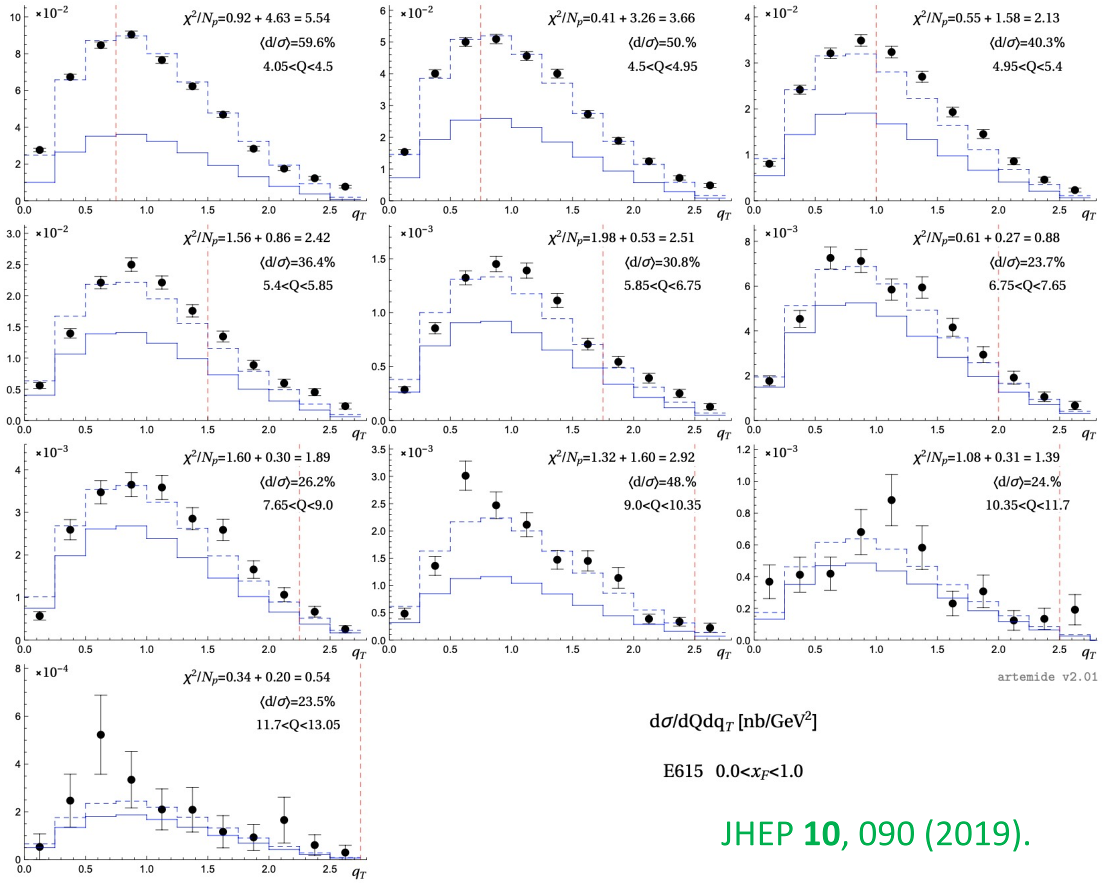
Historical pion TMDs

MAP pions



Phys. Rev. D **107**, 104014 (2023).

A. Vladimirov



JHEP **10**, 090 (2019).

Our analysis in JAM

For the remainder of the talk, I will outline:

- Theoretical structure for TMDs
  - Are collinear distributions related?
- How we implement TMD observables in a global analysis
- Results of the analysis
  - First of its kind in some ways – will explain along the way
- Interesting avenues for the future

Our analysis in JAM

- We are interested in the non-perturbative structure, with a motivation for studying pion structure
- Only available data that we have are from low-energy fixed target πA DY experiments
- We must also understand the nuclear environment
- Perform a simultaneous extraction of pion and proton (nuclear) TMDs

Unpolarized TMD PDF

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{\mathrm{d}b^-}{4\pi} e^{-ixP^+b^-} \mathrm{Tr}\left[\langle \mathcal{N} \,|\, \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \,|\, \mathcal{N} \rangle\right]$$

$$b \equiv (b^-, 0^+, \boldsymbol{b_T})$$

- $\boldsymbol{b_T}$ is the Fourier conjugate to the intrinsic transverse momentum of quarks in the hadron, $\boldsymbol{k_T}$
- We can learn about the coordinate space correlations of quark fields in hadrons
- Modification needed for UV and rapidity divergences; acquire regulators: $\tilde{f}_{q/\mathcal{N}}(x, b_T) \to \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)$

Factorization for low-q_T Drell-Yan

- Like collinear observable, a hard part with two functions that describe structure of beam and target
- So called "W"-term, valid only at low-q_T

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\tau \mathrm{d}Y \mathrm{d}q_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2,\mu) \int \mathrm{d}^2 b_T\, e^{i b_T \cdot q_T} \times \tilde{f}_{q/\pi}(x_\pi, b_T, \mu, Q^2)\, \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2),$$

Evolution equations for the TMD PDF

$$\frac{\partial \ln \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

Collins-Soper (CS) kernel

Rapidity scale

Has its own renormalization group equation

$$\frac{\mathrm{d}\tilde{K}(b_T; \mu)}{\mathrm{d}\ln \mu} = -\gamma_K\left(\alpha_S(\mu)\right)$$

Anomalous dimension of CS kernel

$$\frac{\mathrm{d}\ln \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\mathrm{d}\ln \mu} = \gamma_f(\alpha_S(\mu)) - \frac{1}{2}\gamma_K(\alpha_S(\mu)) \ln \frac{\zeta}{\mu^2}$$

Renormalization scale

Anomalous dimension of TMDPDF

Small b_T operator product expansion

- At small b_T, the TMD PDF can be described in terms of its OPE:

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{\mathcal{C}}_{q/j}(x/\xi, b_T; \mu, \zeta_F) f_{q/\mathcal{N}}(\xi; \mu) + \mathcal{O}((\Lambda_{\rm QCD} b_T)^a)$$

- where $\tilde{C}$ are the Wilson coefficients, and $f_{q/\mathcal{N}}$ is the collinear PDF
- Breaks down when b_T gets large

b_* prescription

- A common approach to regulating large b_T behavior

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1+b_T^2/b_{\max}^2}}.$$

Must choose an appropriate value; a transition from perturbative to non-perturbative physics

- At small b_T, $b_*(b_T) = b_T$
- At large b_T, $b_*(b_T) = b_{\max}$

Introduction of non-perturbative functions

- Because $b_* \neq b_T$, have to non-perturbatively describe large b_T behavior

Completely general – independent of quark, hadron, PDF or FF

$$g_K(b_T; b_{\max}) = -\tilde{K}(b_T, \mu) + \tilde{K}(b_*, \mu)$$

Non-perturbative function dependent in principle on flavor, hadron, etc.

$$e^{-g_{q/\mathcal{N}(A)}(x,b_T)} = \frac{\tilde{f}_{q/\mathcal{N}(A)}(x, b_T; \mu, \zeta)}{\tilde{f}_{q/\mathcal{N}(A)}(x, b_*; \mu, \zeta)} e^{g_K(b_T;b_{\max}) \log\left(\sqrt{\zeta}/Q_0\right)}$$

TMD PDF within the b_* prescription

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}.$$

Low-b_T: perturbative
high-b_T: non-perturbative

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = (C \otimes f)_{q/\mathcal{N}(A)}(x; b_*)$$
$$\times \exp\left\{ -g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T) \ln\frac{Q}{Q_0} - S(b_*, Q_0, Q, \mu_Q) \right\}$$

Relates the TMD at small-b_T to the **collinear** PDF
$\Rightarrow$ TMD is sensitive to collinear PDFs

$g_{q/\mathcal{N}(A)}$: intrinsic non-perturbative structure of the TMD
g_K: universal non-perturbative Collins-Soper kernel

Controls the perturbative evolution of the TMD

Collins, Soper, Sterman, NPB **250**, 199 (1985).

TMD factorization in Drell-Yan

- In small-q_T region, use the Collins-Soper-Sterman (CSS) formalism and b_* prescription

$$\frac{d\sigma}{dQ^2\,dy\,dq_\mathrm{T}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s}\sum_{j,j_A,j_B} H_{j\bar{j}}^{\mathrm{DY}}(Q,\mu_Q,a_s(\mu_Q))\int \frac{d^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}$$

$$\times\, e^{-g_{j/A}(x_A,b_\mathrm{T};b_{\max})}\int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\,\tilde{C}_{j/j_A}^{\mathrm{PDF}}\left(\frac{x_A}{\xi_A},b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$

$$\times\, e^{-g_{\bar{j}/B}(x_B,b_\mathrm{T};b_{\max})}\int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\,\tilde{C}_{\bar{j}/j_B}^{\mathrm{PDF}}\left(\frac{x_B}{\xi_B},b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$

$$\times\exp\left\{-g_K(b_\mathrm{T};b_{\max})\ln\frac{Q^2}{Q_0^2}+\tilde{K}(b_*;\mu_{b_*})\ln\frac{Q^2}{\mu_{b_*}^2}+\int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma_j(a_s(\mu'))-\ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

Can these data constrain the pion collinear PDF?

Perturbative pieces

Non-perturbative pieces

Non-perturbative piece of the CS kernel

Nuclear TMD PDFs

- The TMD factorization allows for the description of a quark inside a nucleus to be $\tilde{f}_{q/A}$
- However, the intrinsic non-perturbative structure will in-principle change from nucleus-to-nucleus
- Want to model these in terms of protons and neutrons as we don't have enough observables to separately parametrize different nuclei

Nuclear TMD PDFs – working hypothesis

- We must model the nuclear TMD PDF from proton

$$\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A}\tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A}\tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

- Each object on the right side independently obeys the CSS equation
  - **Assumption** that the bound proton and bound neutron follow TMD factorization
- Make use of isospin symmetry in that $u/p/A \leftrightarrow d/n/A$, etc.

Nuclear TMD parametrization

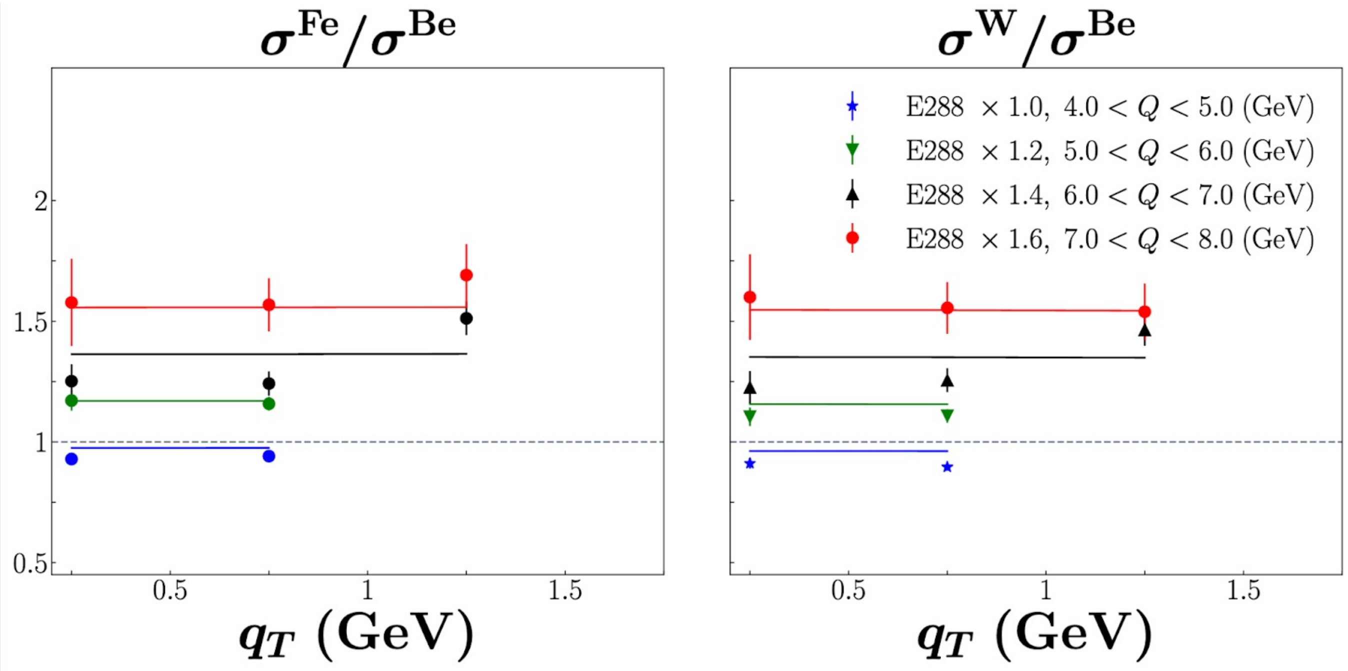
- Specifically, we include a parametrization similar to Alrashed, et al., Phys. Rev. Lett **129**, 242001 (2022).

$$g_{q/\mathcal{N}/A} = g_{q/\mathcal{N}} \left(1 - a_{\mathcal{N}} \left(A^{1/3} - 1\right)\right)$$

- Where $a_{\mathcal{N}}$ is an additional parameter to be fit

A few words on nuclear dependence

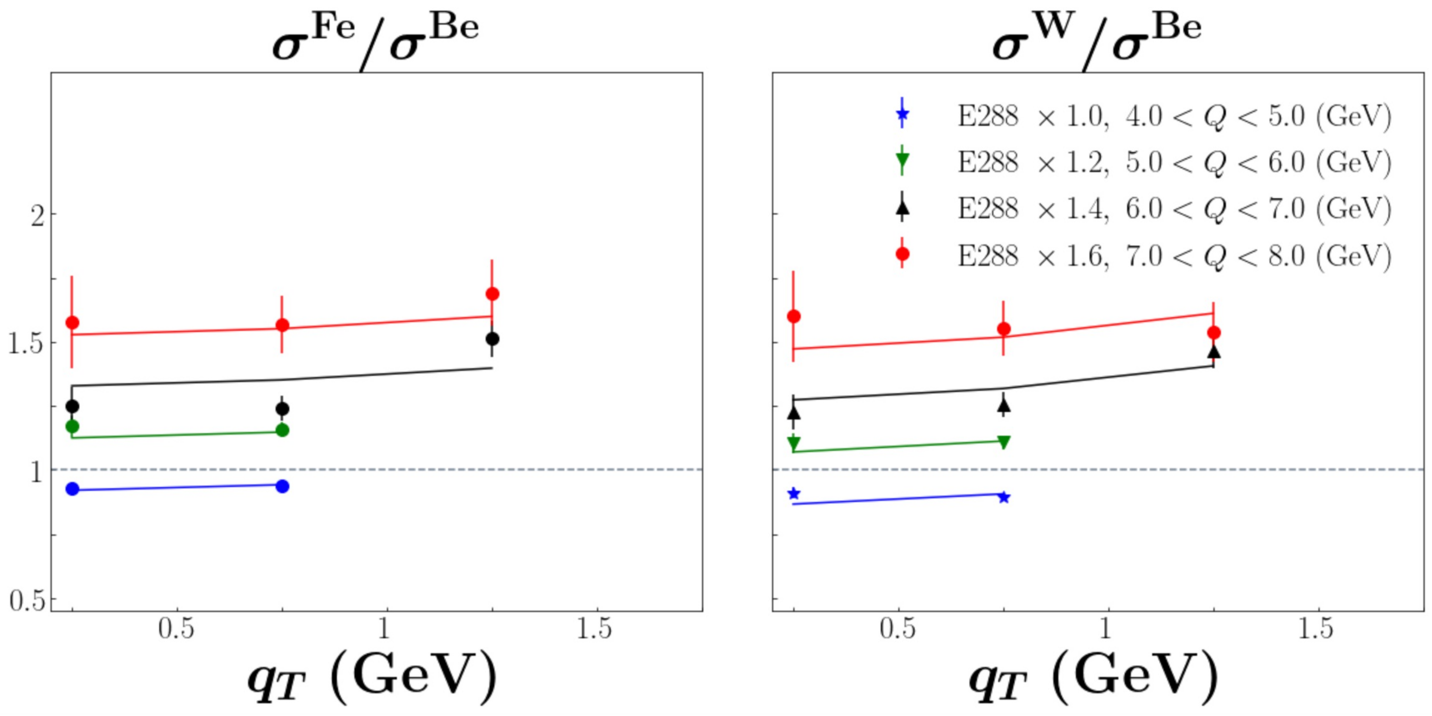
- The ratios from the E866 experiment provided a look to nuclear effects in TMDs as well as the importance of nuclear collinear effects
- Ignoring any nuclear corrections in TMDs and collinear PDFs



col	obs	tar	npts	chi2/npts	Z-score
E866	ratio	Fe/Be	10	2.2	2.16
E866	ratio	W/Be	10	3.51	3.67

Including nuclear dependence

- Better description when including the nuclear dependence in the collinear PDF and TMD



col	obs	tar	npts	chi2/npts	Z-score
E866	ratio	Fe/Be	10	1.11	0.4
E866	ratio	W/Be	10	0.92	0.04

Datasets in the q_T-dependent analysis

Expt.	$\sqrt{s}$ (GeV)	Reaction	Observable	Q (GeV)	x_F or y	$N_{\text{pts.}}$
E288 [39]	19.4	$p + Pt \to \ell^+\ell^- X$	$Ed^3\sigma/d^3\mathbf{q}$	$4-9$	$y = 0.4$	38
E288 [39]	23.8	$p + Pt \to \ell^+\ell^- X$	$Ed^3\sigma/d^3\mathbf{q}$	$4-12$	$y = 0.21$	48
E288 [39]	24.7	$p + Pt \to \ell^+\ell^- X$	$Ed^3\sigma/d^3\mathbf{q}$	$4-14$	$y = 0.03$	74
E605 [40]	38.8	$p + Cu \to \ell^+\ell^- X$	$Ed^3\sigma/d^3\mathbf{q}$	$7-18$	$x_F = 0.1$	49
E772 [41]	38.8	$p + D \to \ell^+\ell^- X$	$Ed^3\sigma/d^3\mathbf{q}$	$5-15$	$0.1 \le x_F \le 0.3$	61
E866 [50]	38.8	$p + Fe \to \ell^+\ell^- X$	R_{FeBe}	$4-8$	$0.13 \le x_F \le 0.93$	10
E866 [50]	38.8	$p + W \to \ell^+\ell^- X$	R_{WBe}	$4-8$	$0.13 \le x_F \le 0.93$	10
E537 [38]	15.3	$\pi^- + W \to \ell^+\ell^- X$	$d^2\sigma/dx_F dq_T$	$4-9$	$0 < x_F < 0.8$	48
E615 [4]	21.8	$\pi^- + W \to \ell^+\ell^- X$	$d^2\sigma/dx_F dq_T^2$	$4.05-8.55$	$0 < x_F < 0.8$	45

- Total of 383 number of points
- All fixed target, low-energy data
- We perform a cut of $q_T^{\max} < 0.25\, Q$

Parametrizations of the TMDs

- First perform single fits of these data to explore various aspects
- Many types of parametrizations have been used in the past
- For the “intrinsic” non-perturbative TMD, we perform fits with each of the following

Gaussian

$$\exp(-g_{q/N}(x,b_T)) = \exp\left(-g_q(x,A)\, b_T^2\right) ,$$

Exponential

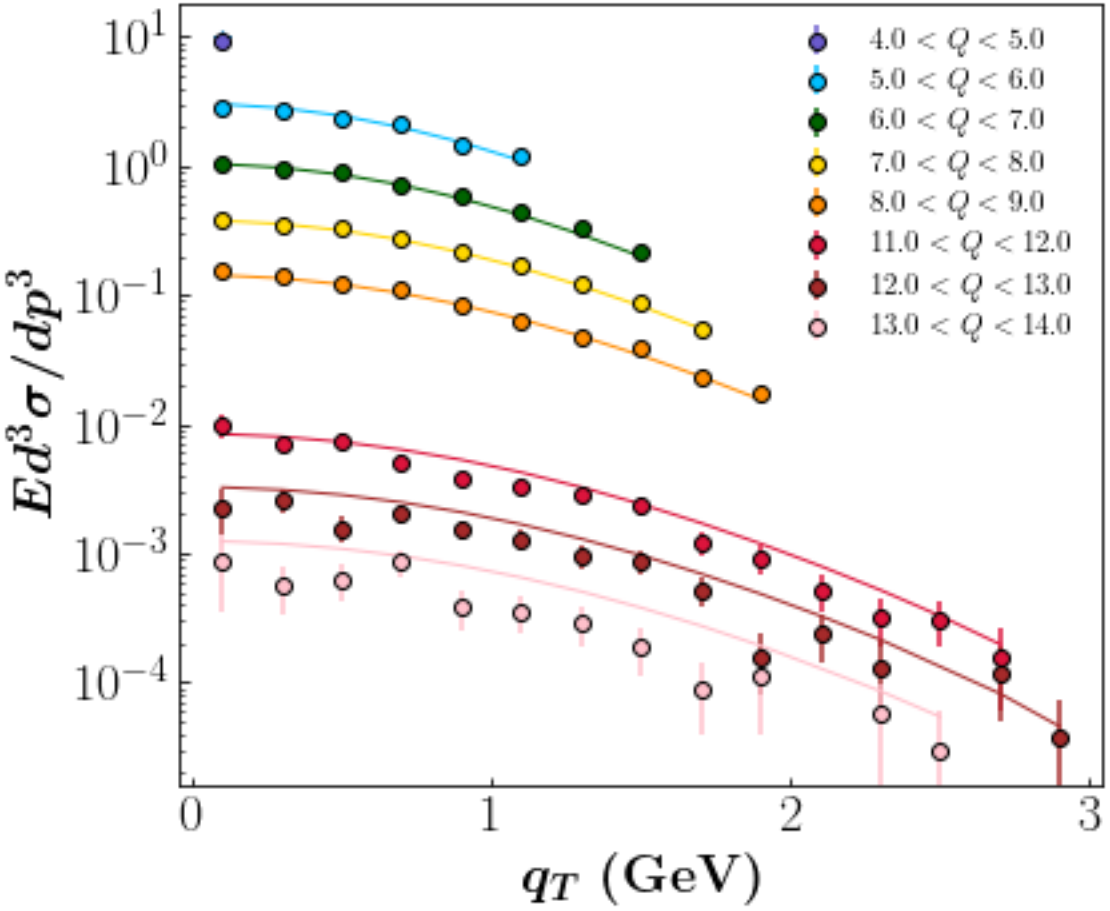
$$\exp(-g_{q/N}(x,b_T)) = \exp\left(-g_q(x,A)\, b_T\right) ,$$

Gaussian-to-Exponential

$$\exp(-g_{q/N}(x,b_T)) = \exp\left(-g_q(x,A)\, \frac{b_T^2}{\sqrt{1+B_{NP}(x) b_T^2}}\right) ,$$

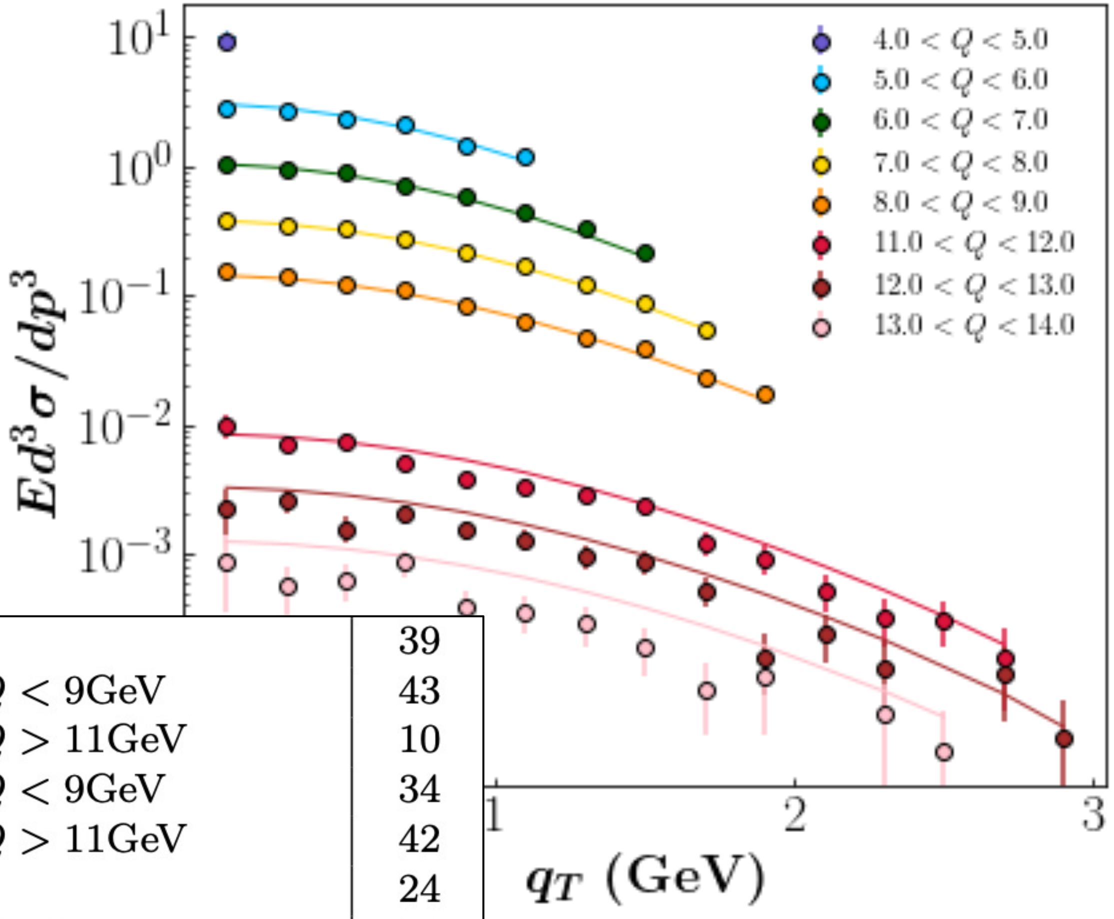
Problem describing data

- The E288 400 GeV data are difficult to describe the same above and below the Υ resonance
- Theory overpredicts data when $Q > 11\text{GeV}$



Problem describing data

- The E288 400 GeV data are difficult to describe the same above and below the Υ resonance
- Theory overpredicts data when $Q > 11\text{GeV}$
- Could treat as separate datasets – separate normalizations:



E228-200	39
E228-300 $Q < 9\text{GeV}$	43
E228-300 $Q > 11\text{GeV}$	10
E228-400 $Q < 9\text{GeV}$	34
E228-400 $Q > 11\text{GeV}$	42
E772	24
E605 $Q < 9\text{GeV}$	21
E605 $Q > 11\text{GeV}$	32

MAP parametrization

- A recent work from the MAP collaboration (Phys. Rev. D **107**, 014014 (2023).) used a complicated form for the non-perturbative function

$$f_{1\,NP}(x, \boldsymbol{b}_T^2; \zeta, Q_0) = \frac{g_1(x)\, e^{-g_1(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda^2\, g_{1B}^2(x)\left[1 - g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}\right] e^{-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda_2^2\, g_{1C}(x)\, e^{-g_{1C}(x)\frac{\boldsymbol{b}_T^2}{4}}}{g_1(x) + \lambda^2\, g_{1B}^2(x) + \lambda_2^2\, g_{1C}(x)} \left[\frac{\zeta}{Q_0^2}\right]^{g_K(\boldsymbol{b}_T^2)/2}, \quad (38)$$

$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}}(1-x)^{\alpha^2_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}}(1-\hat{x})^{\alpha^2_{\{1,2,3\}}}},$$

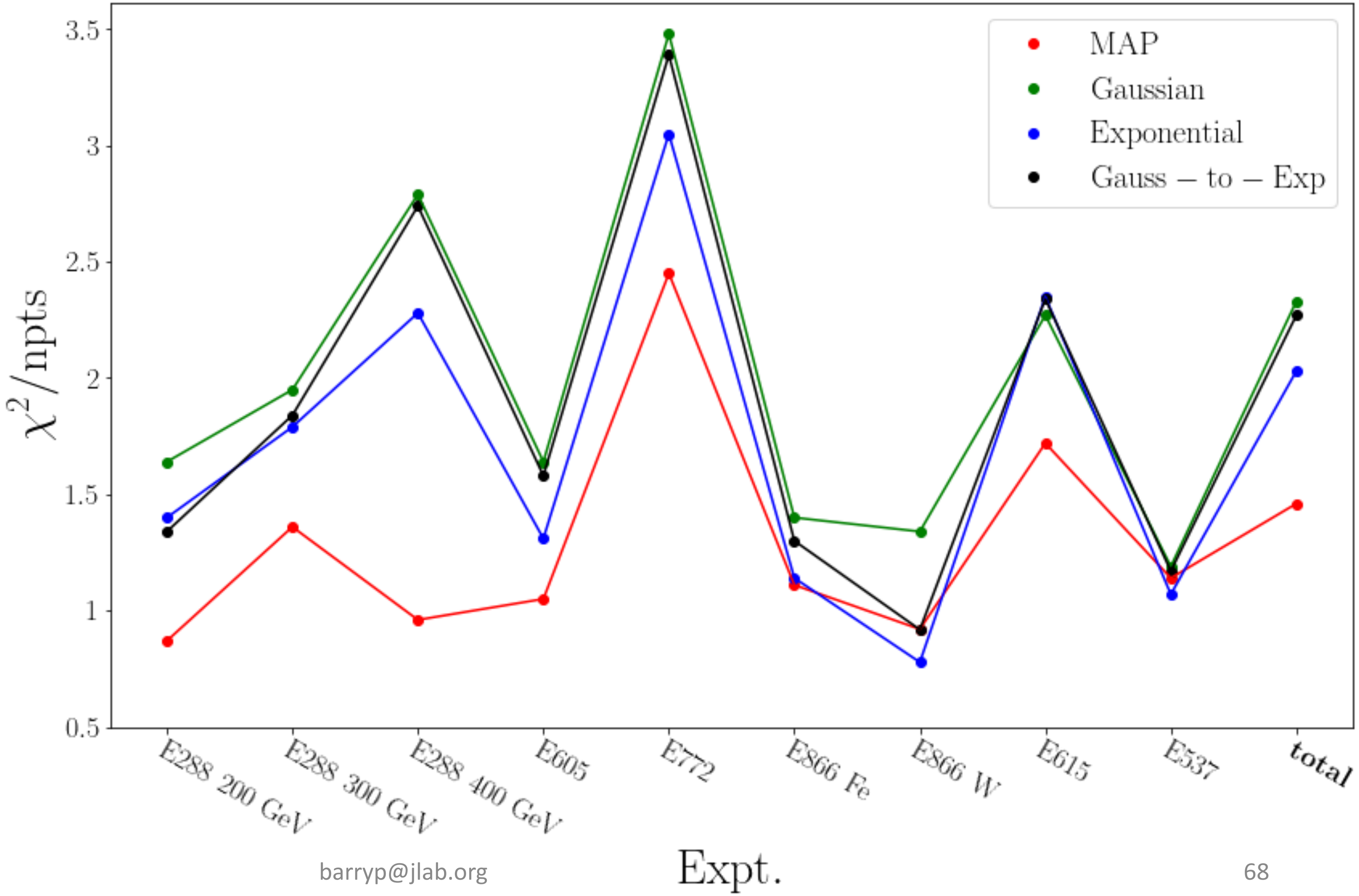
$$g_K(\boldsymbol{b}_T^2) = -g_2^2 \frac{\boldsymbol{b}_T^2}{2}$$

Universal CS kernel

- 11 free parameters for each hadron! (flavor dependence not necessary) (12 if we include the nuclear TMD parameter)

Resulting χ^2 for each parametrization

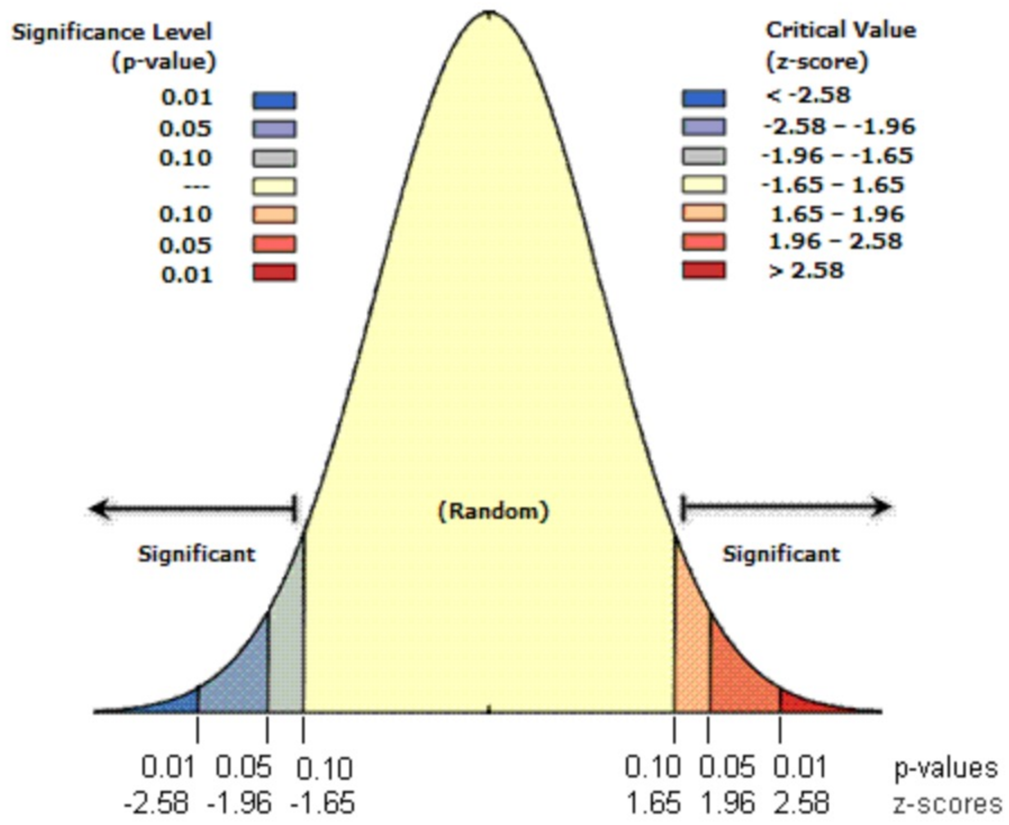
- MAP gives best overall
- How significant?



Z-scores

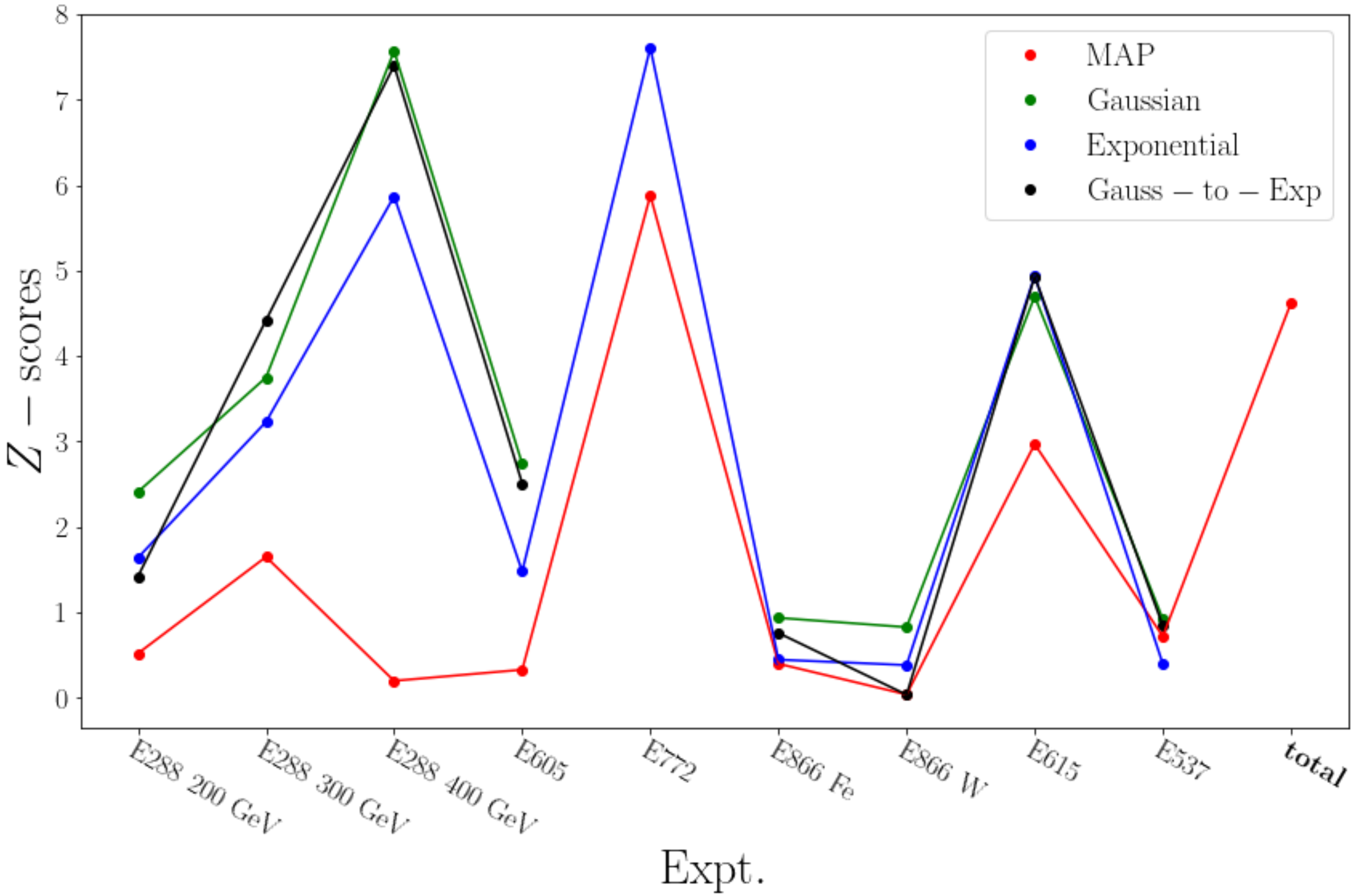
- A measure of significance with respect to the normal distribution
- Null hypothesis is the expected χ^2 distribution

$$Z = \Phi^{-1}(p) \equiv \sqrt{2}\,\mathrm{erf}^{-1}(2p-1).$$



Z-scores

- Example of significance of the χ^2 values with respect to the expected χ^2 distribution
- Those that are absent - Z is effectively infinite



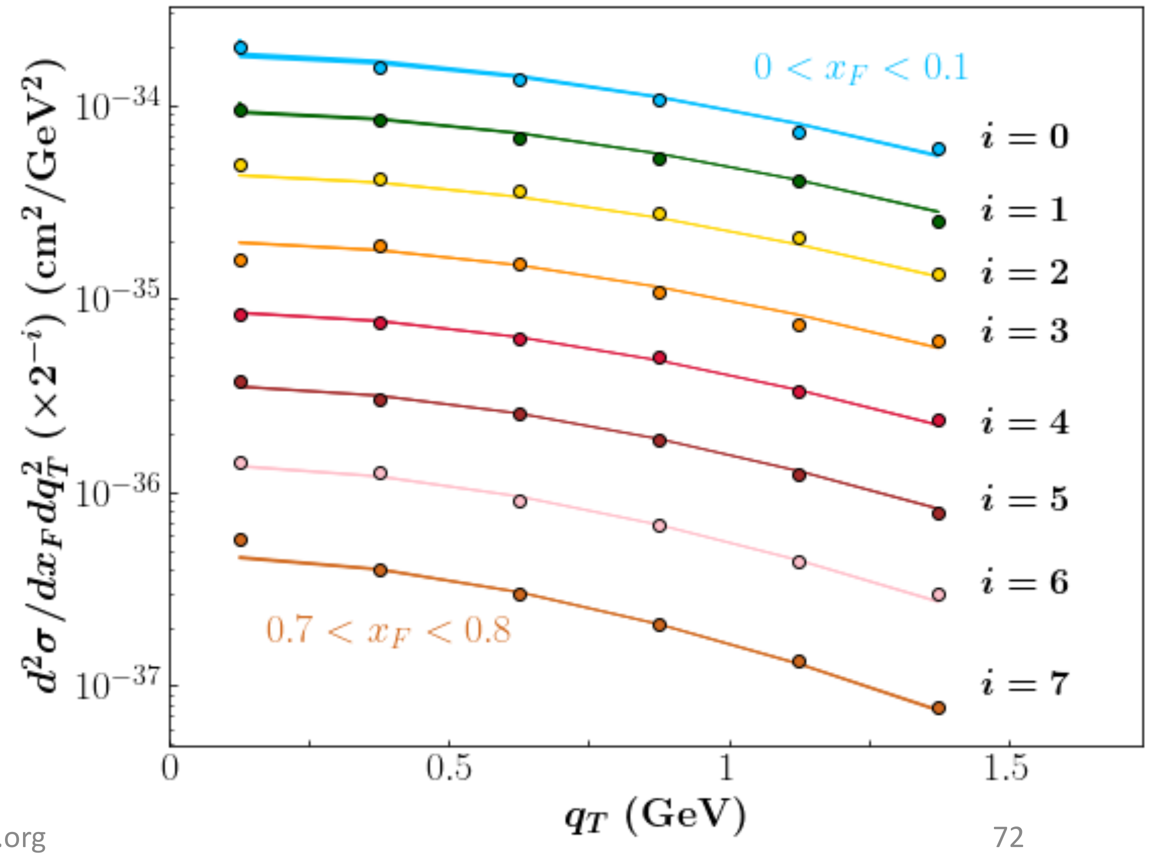
Perform the Monte Carlo

- We use the MAP parametrization
- Now, we can include the pion collinear PDF and its collinear datasets
- Include an additional 225 collinear data points
- Simultaneously extract
  1. Pion TMD PDFs
  2. Pion collinear PDFs
  3. Proton TMD PDFs
  4. Nuclear dependence
  5. Non-perturbative CS kernel

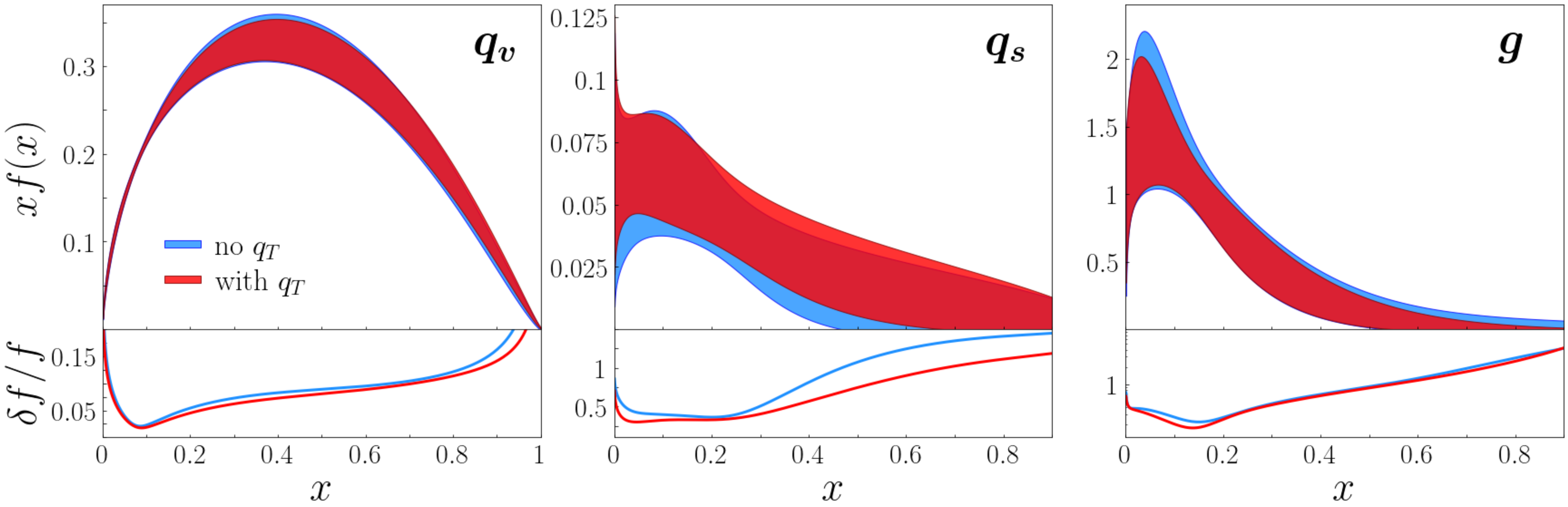
Data and theory agreement

- Fit both pA and πA DY data and achieve good agreement to both

Process	Experiment	$\sqrt{s}$ GeV	χ^2/np	Z-score
q_T-integr. DY $\pi W \to \mu^+\mu^- X$	E615 [37]	21.8	0.86	0.76
	NA10 [38]	19.1	0.54	2.27
	NA10 [38]	23.2	0.91	0.18
Leading neutron $ep \to e'nX$	H1 [73]	318.7	0.36	4.61
	ZEUS [74]	300.3	1.48	2.16
q_T-dep. pA DY $pA \to \mu^+\mu^- X$	E288 [67]	19.4	0.93	0.25
	E288 [67]	23.8	1.33	1.54
	E288 [67]	24.7	0.95	0.23
	E605 [68]	38.8	1.07	0.39
	E772 [69]	38.8	2.41	5.74
	E866 (Fe/Be) [70]	38.8	1.07	0.29
	E866 (W/Be) [70]	38.8	0.89	0.11
q_T-dep. πA DY $\pi W \to \mu^+\mu^- X$	E615 [37]	21.8	1.61	2.58
	E537 [71]	15.3	1.11	0.57
Total			1.15	2.55



Extracted pion PDFs



- The small-q_T data do not constrain much the PDFs

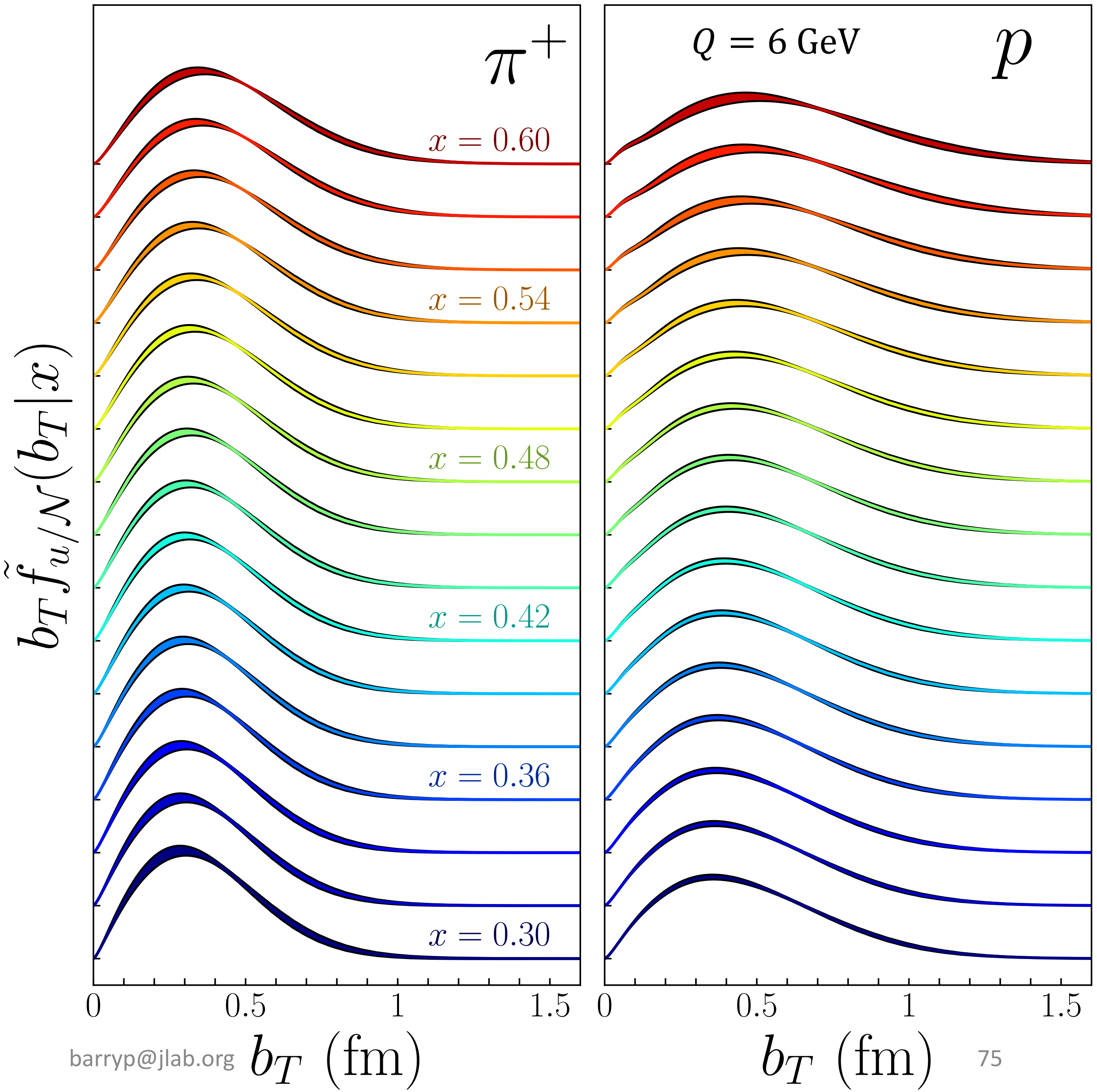
Conditional density

- We define a quantity in which describes the ratio of the 2-dimensional density to the integrated, b_T-independent number density, dependent on "b_T given x"

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int \mathrm{d}^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)} \,.$$

Resulting TMD PDFs of proton and pion

- Shown in the range where pion and proton are both constrained
- Broadening appearing as x increases
- Up quark in pion is narrower than up quark in proton



Average b_T

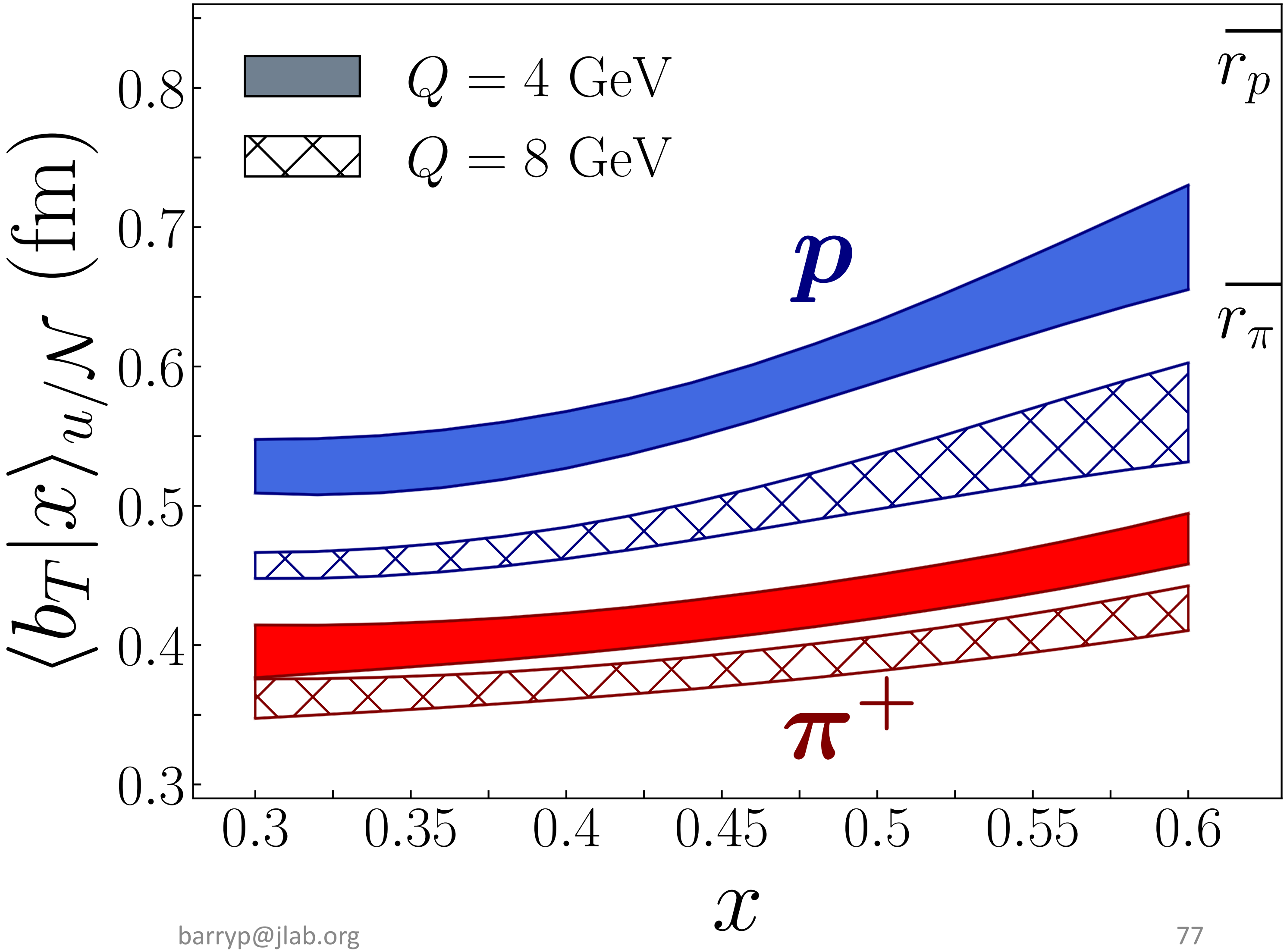
- The conditional expectation value of b_T for a given x

$$\langle b_T|x\rangle_{q/\mathcal{N}} = \int \mathrm{d}^2\boldsymbol{b}_T\, b_T\, \tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2)$$

- Shows a measure of the transverse correlation in coordinate space of the quark in a hadron for a given x

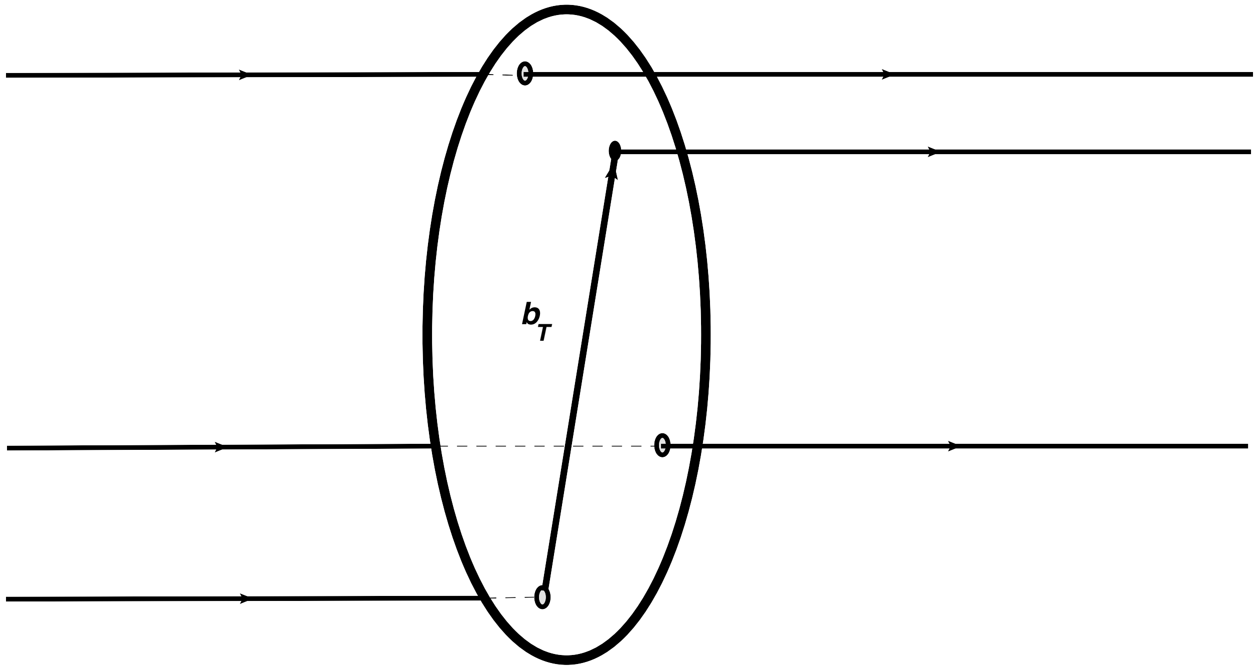
Resulting average b_T

- Pion's $\langle b_T | x \rangle$ is $5.3 - 7.5\sigma$ smaller than proton in this range
- Decreases as x decreases



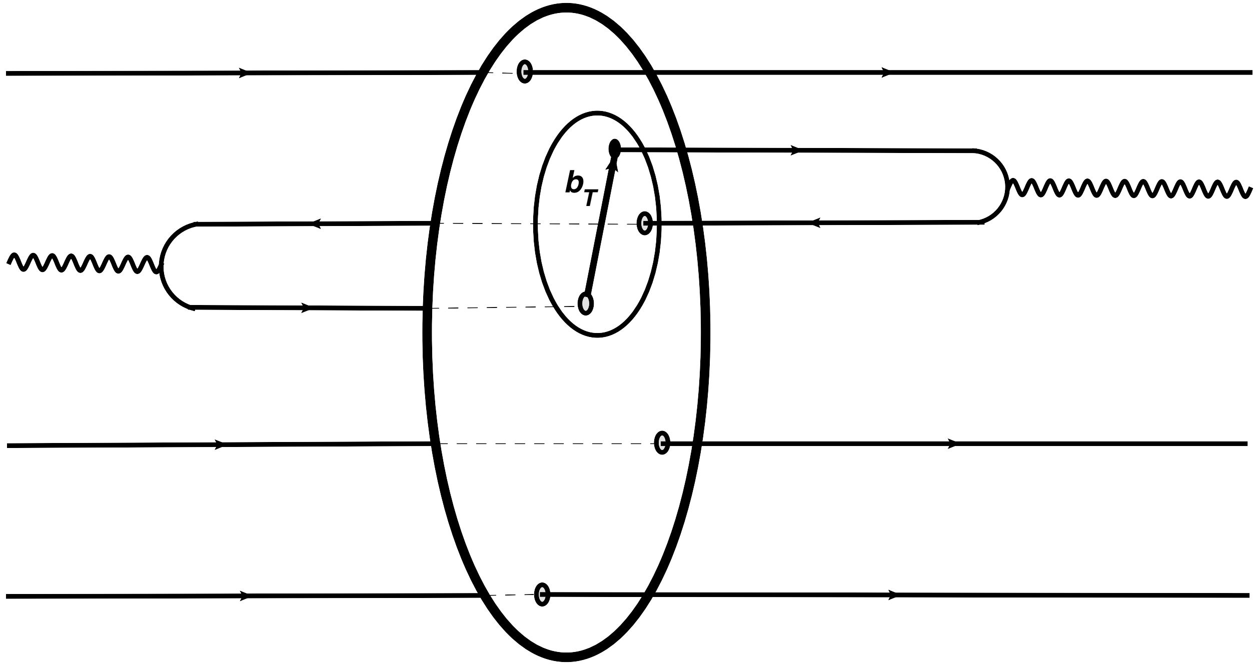
Possible explanation

- At large x, we are in a valence region, where only the valence quarks are populating the momentum dependence of the hadron



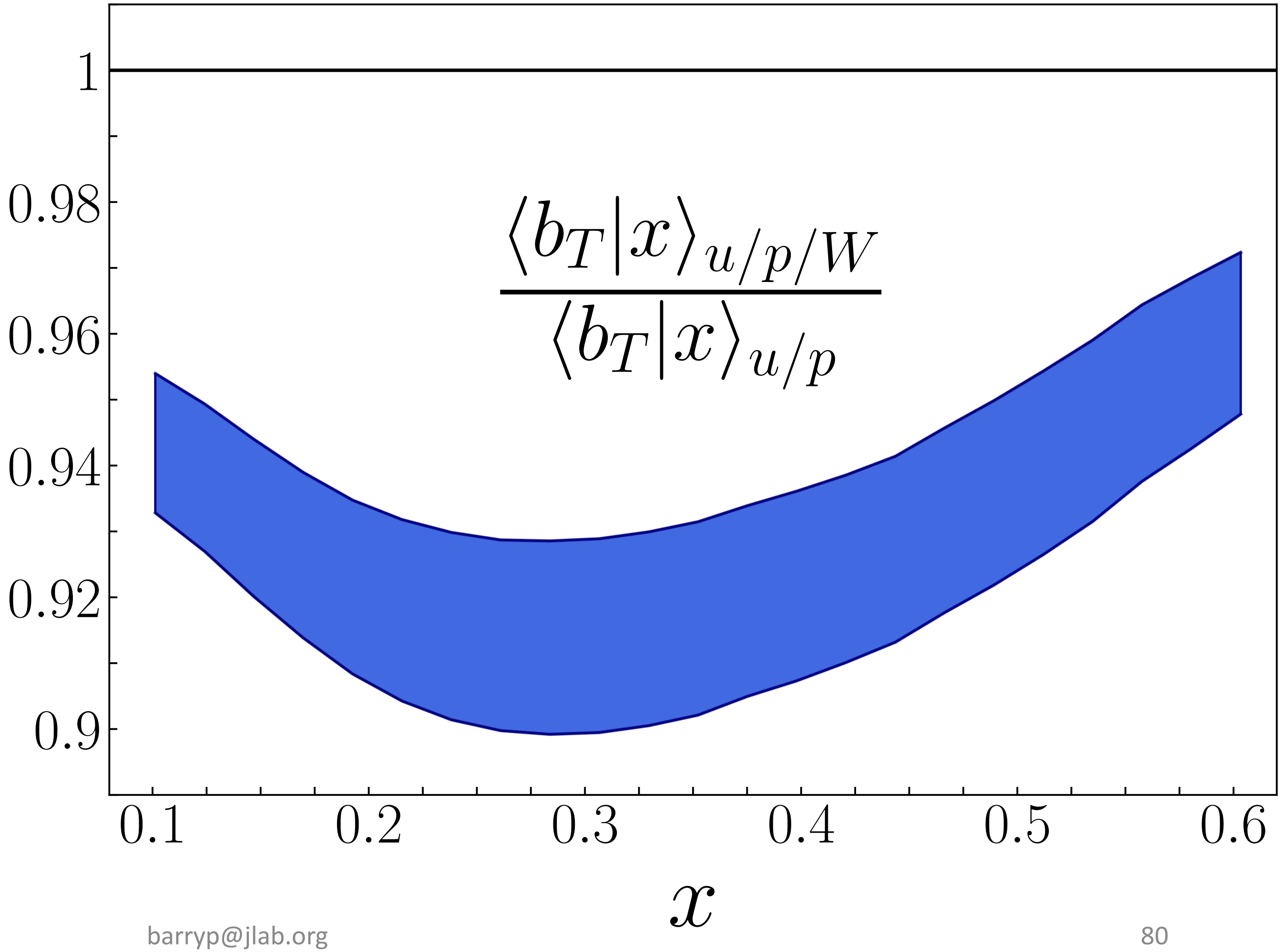
Possible explanation

- At small x, sea quarks and potential $q\bar{q}$ bound states allowing only for a smaller bound system



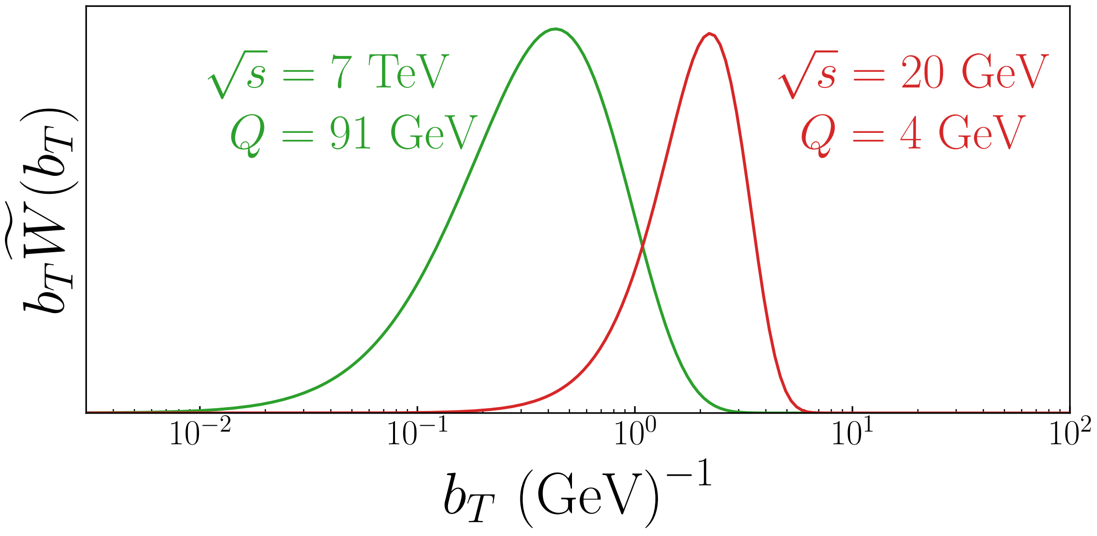
Transverse EMC effect

- Compare the average b_T given x for the up quark in the bound proton to that of the free proton
- Less than 1 by $\sim 5 - 10\%$ over the x range



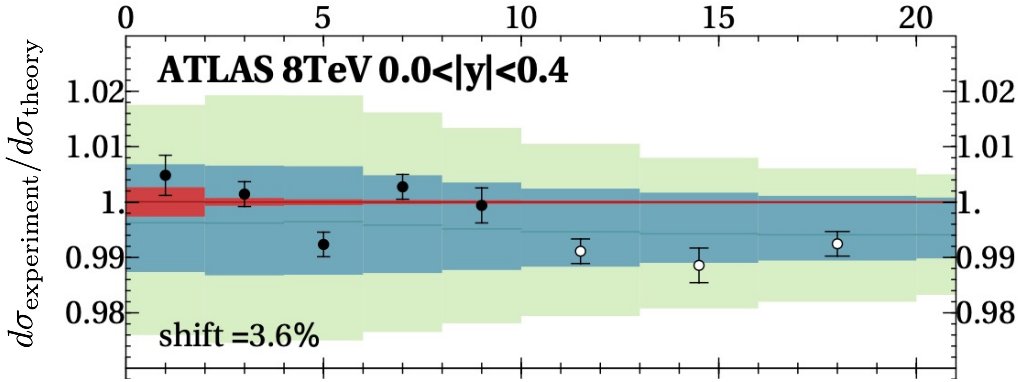
What about LHC energies?

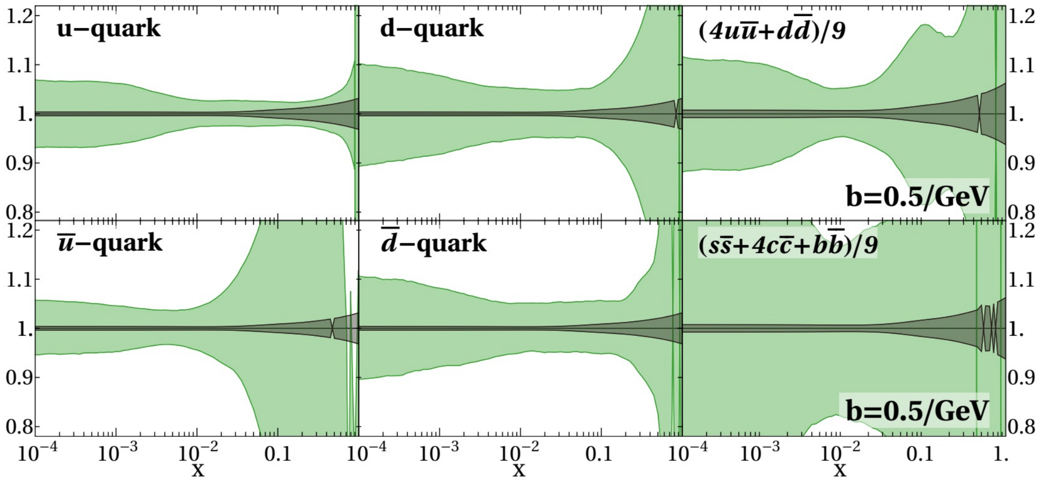
- Fixed-target energies: sensitive to non-perturbative TMD structures
  - Large portion of $\widetilde{W}$ spectrum in large-b_T region
- LHC energies: sensitive to perturbative calculations
  - Have opportunity to study collinear distributions



High energy PDF uncertainties

- From Bury, et al. JHEP **118** (2022).



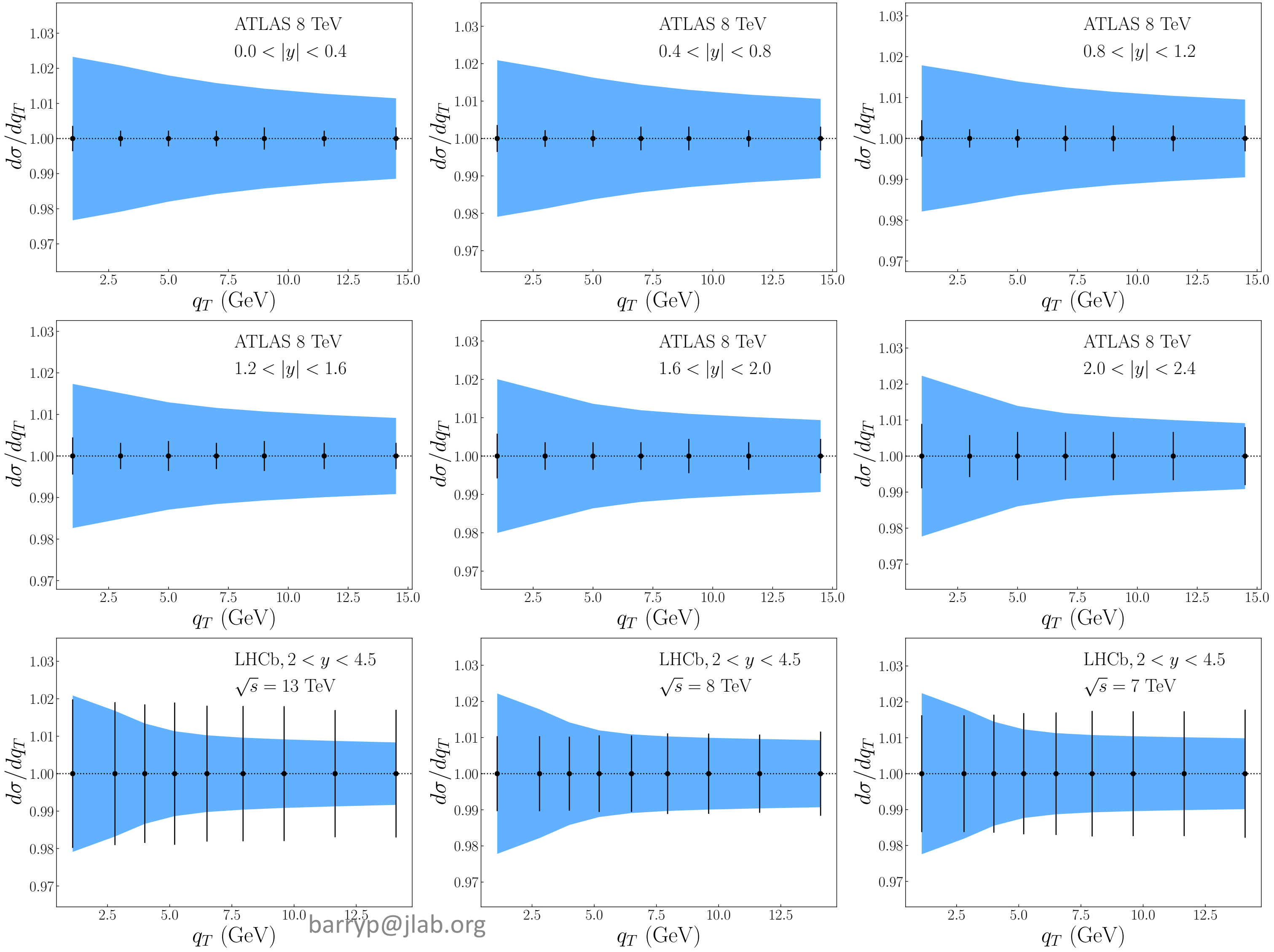


Moos, Scimemi, Vladimirov, Zurita, arXiv:2305.07473

- Studies about the uncertainties of the PDFs relative to data

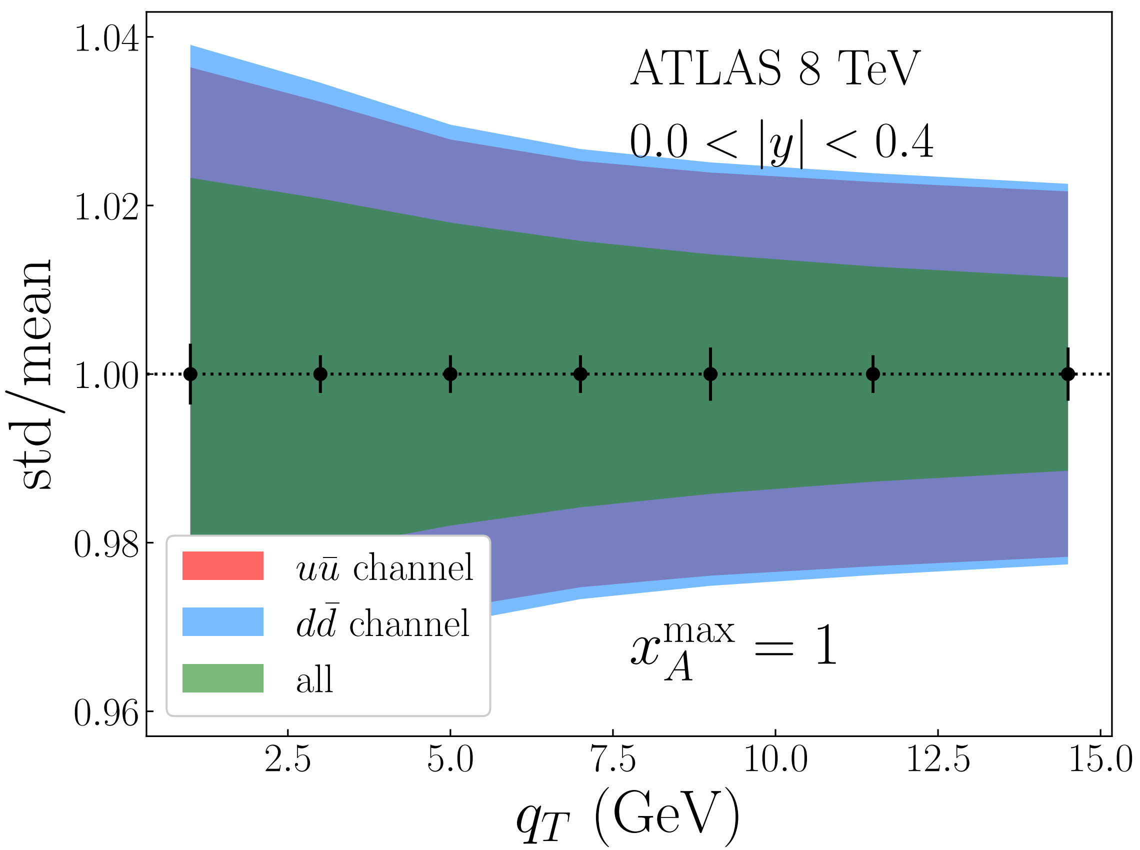
Uncertainties from JAM PDFs only

- Bands come from varying only the collinear PDFs
- High precision in ATLAS and LHCb data indicate potential constraining power



Individual quarks

- Green: full contributions
- Red (looks purple): contribution when u in beam PDF and $\bar{u}$ in target
- Blue: corresponding $d\bar{d}$



Points not (or only briefly) mentioned in this talk

There are additional ways to implement the TMD phenomenologically

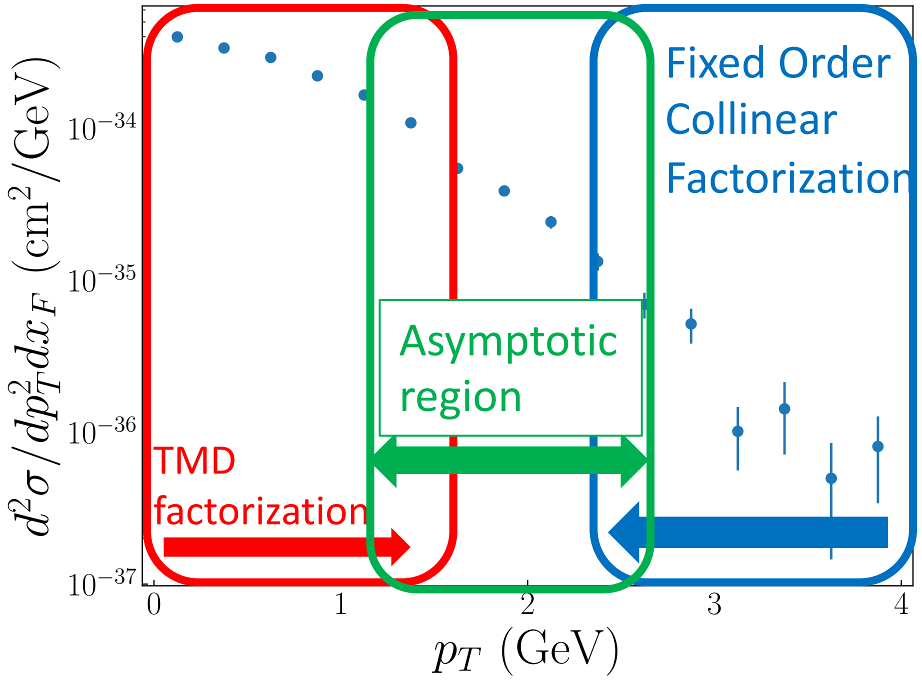
- Qiu-Zhang method: Qiu, Zhang, PRD **63**, 114011 (2001).
- ζ-prescription: see e.g. SV17: Eur. Phys. J. C **78**, 89 (2018).
- Hadron structure oriented (HSO) approach: Phys. Rev. D **106**, 034002 (2022).

Full q_T-spectrum described by

$$\frac{d\sigma}{dq_T} = \mathrm{W} + \mathrm{Y}, \qquad \mathrm{Y} = \mathrm{FO} - \mathrm{ASY}$$

Entire q_T range

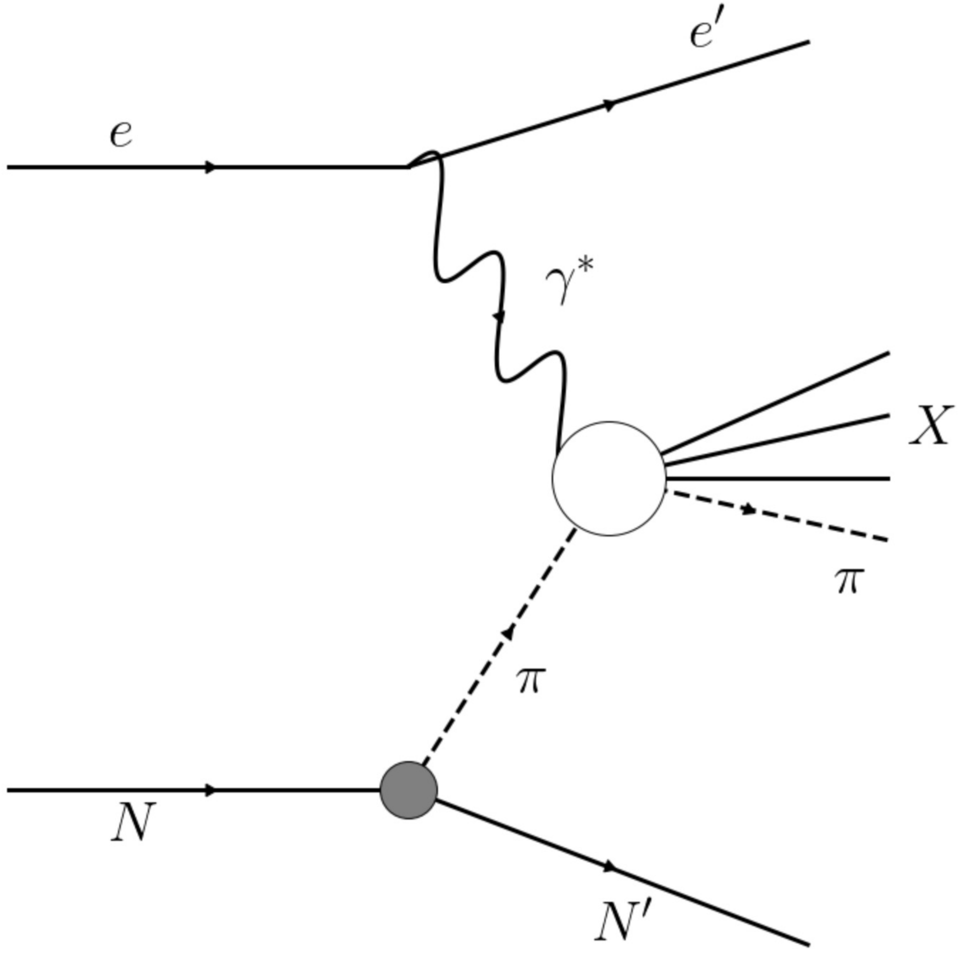
- Describing the entire spectrum has *never been done in phenomenology*
- We have shown the ability to perform a global analysis separately of the large-q_T and small- q_T regions in the pion
- Tackle the challenging "asymptotic region"
- Can we combine these analyses in the π-sector?



Future experiment – pion SIDIS

$$eN \to e'N'\pi X$$

- Measure an outgoing pion in the TDIS experiment
- Gives us another observable sensitive to pion TMDs
  - Needed for tests of universality



Outlook

- Future studies needed for theoretical explanations of these phenomena
- Important to study various hadronic systems to provide a more complete picture of strongly interacting quark-gluon systems emerging from QCD
- Lattice QCD can in principle calculate any hadronic state – look to kaons, rho mesons, etc.
- Future tagged experiments such as at EIC and JLab 22 GeV can provide measurements for neutrons, pions, and kaons

Backup

Issues with Perturbative Calculations

$$\hat{\sigma} \sim \delta(1-z) + \alpha_S(\log(1-z))_+$$

$$\downarrow$$

$$\hat{\sigma} \sim \delta(1-z)[1 + \alpha_S \log(1-\tau)]$$

$$\tau = \frac{Q^2}{S}$$

$$z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{\hat{x}_\pi \hat{x}_A}$$

$\hat{S}$ is the center of mass momentum squared of incoming partons

- If τ is large, can potentially spoil the perturbative calculation
- Improvements can be made by resumming $\log(1-z)_+$ terms

Methods of resummation – Mellin-Fourier

- Threshold resummation is done in conjugate space

$$\sigma_{\mathrm{MF}}(N,M) \equiv \int_0^1 \mathrm{d}\tau \tau^{N-1} \int_{\log\sqrt{\tau}}^{\log\frac{1}{\sqrt{\tau}}} \mathrm{d}Y e^{iMY} \frac{\mathrm{d}^2\sigma}{\mathrm{d}\tau \mathrm{d}Y},$$

Two choices occur when isolating the hard part

$$\hat{\sigma}_{\mathrm{MF}}(N,M) = \int_0^1 \mathrm{d}z z^{N-1} \cos\left(\frac{M}{2}\log z\right) \frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}\tau \mathrm{d}Y}(z)$$

Keep cosine intact – "cosine" method

Keep the first order term in the expansion – $\cos\left(\frac{M}{2}\log z\right) \approx 1$ "expansion" method

Method of resummation – double Mellin

- Alternatively, perform a double Mellin transform

$$\sigma_{\rm DM}(N,M) \equiv \int_0^1 {\rm d}x_\pi^0\,(x_\pi^0)^{N-1} \int_0^1 {\rm d}x_A^0\,(x_A^0)^{M-1} \frac{{\rm d}^2\sigma}{{\rm d}\tau{\rm d}Y}.$$

where $x_\pi^0 = \sqrt{\tau}e^{Y}, \quad x_A^0 = \sqrt{\tau}e^{-Y}$

- Double Mellin transform is theoretically cleaner and sums up terms appropriately

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Make sure only counted once!
- Subtract the matching

	LL	NLL	...	N^pLL
LO	1	--	...	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	...	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2 (\log(N)^2, \log(N)^3)$	...	--
...	...	...	...	...
N^kLO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k \left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	...	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

Reduced Ioffe time pseudo-distribution (Rp-ITD)

- Lorentz-invariant Ioffe time pseudo-distribution:

$$\mathcal{M}(\nu, z^2) = \frac{1}{2p^0} \langle p|\bar{\psi}(0)\gamma^0 \mathcal{W}(z;0)\psi(z)|p\rangle$$

Quark and antiquark fields

Gauge link

"Ioffe time"

$$\nu = p \cdot z$$

$$z = (0,0,0,z_3)$$

Observable is the *reduced* Ioffe time pseudo-distribution (Rp-ITD)

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$$

Ratio cancels UV divergences

Deriving resummation expressions – MF

$$z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{\hat{x}_\pi \hat{x}_A}$$

$$y = \frac{\frac{\hat{x}_\pi}{\hat{x}_A} e^{-2Y} - z}{(1-z)(1+\frac{\hat{x}_\pi}{\hat{x}_A} e^{-2Y})}$$

Claim: yellow terms give rise to the resummation expressions

$$\frac{C_{q\bar{q}}}{e_q^2} = \delta(1-z)\,\frac{\delta(y)+\delta(1-y)}{2}\left[1+\frac{C_F\alpha_s}{\pi}\left(\frac{3}{2}\ln\frac{M^2}{\mu_f^2}+\frac{2\pi^2}{3}-4\right)\right]$$
$$+\frac{C_F\alpha_s}{\pi}\left\{\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left[\frac{1}{1-z}\ln\frac{M^2(1-z)^2}{\mu_f^2 z}\right]_+ +1-z\right]\right.$$
$$\left.+\frac{1}{2}\left[1+\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ + \left[\frac{1}{1-y}\right]_+\right)-2(1-z)\right]\right\}$$

Claim: Red terms are power suppressed in $(1-z)$ and wouldn't contribute to the same order as the yellow terms

Generalized Threshold resummation

G. Lustermans, J. K. L. Michel, and F. J. Tackmann, arXiv:1908.00985 [hep-ph].

- Write the (z, y) coefficients in terms of (z_a, z_b), and for the red terms, you get:

$$z_a = \frac{x_\pi^0}{\hat{x}_\pi}$$

$$z_b = \frac{x_A^0}{\hat{x}_A}$$

$$\mathrm{d}z\,\mathrm{d}y\,\frac{1}{1-z}\left(\frac{1}{y}+\frac{1}{1-y}\right) = \mathrm{d}z_a\mathrm{d}z_b\,\frac{1}{(1-z_a)(1-z_b)}\big[1+\mathcal{O}(1-z_a, 1-z_b)\big]\,.$$

- This is *not* power suppressed in $(1 - z_a)$ or $(1 - z_b)$ but instead the same order as the leading power in the soft limit
- Generalized threshold resummation in the soft limit does not agree with the MF methods

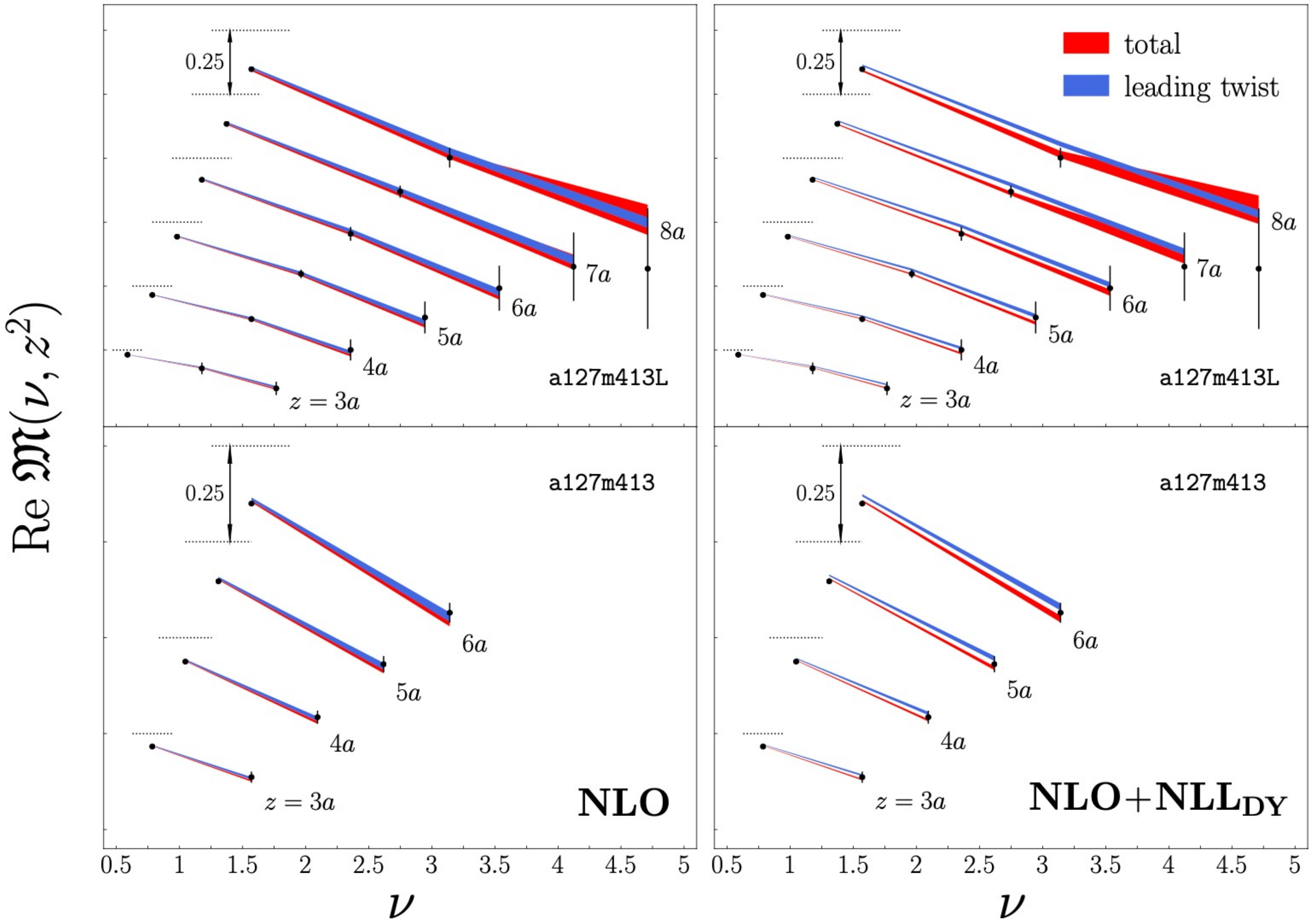
Goodness of fit

- Scenario A: experimental data alone
- Scenario B: experimental + lattice, no systematics
- Scenario C: experimental + lattice, with systematics

			Scenario A		Scenario B		**Scenario C**	
			NLO	$+\text{NLL}_{\text{DY}}$	NLO	$+\text{NLL}_{\text{DY}}$	NLO	$+\text{NLL}_{\text{DY}}$
Process	Experiment	N_{dat}	$\overline{\chi}^2$		$\overline{\chi}^2$		$\overline{\chi}^2$	
DY	E615	61	0.84	0.82	0.83	0.82	**0.84**	**0.82**
	NA10 (194 GeV)	36	0.53	0.53	0.52	0.54	**0.52**	**0.55**
	NA10 (286 GeV)	20	0.80	0.81	0.78	0.79	**0.78**	**0.87**
LN	H1	58	0.36	0.35	0.39	0.39	**0.37**	**0.37**
	ZEUS	50	1.56	1.48	1.62	1.69	**1.58**	**1.60**
Rp-ITD	`a127m413L`	18	–	–	1.04	1.06	**1.04**	**1.06**
	`a127m413`	8	–	–	1.98	2.63	**1.14**	**1.42**
Total		**251**	**0.82**	**0.80**	**0.89**	**0.92**	**0.85**	**0.87**

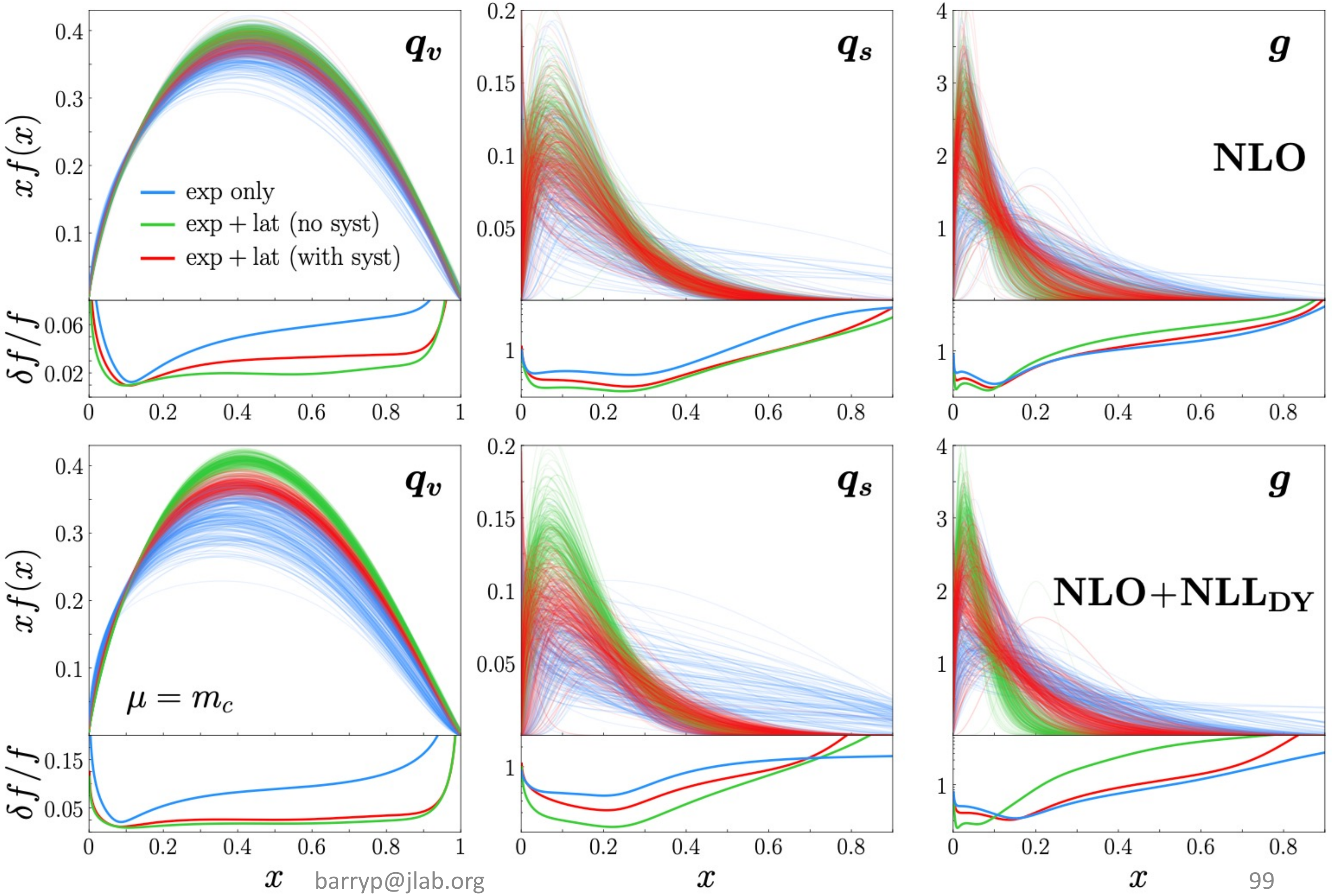
Agreement with the data

- Results from the full fit and isolating the leading twist term
- Difference between bands is the systematic correction



Resulting PDFs

- PDFs and relative uncertainties
- Including lattice reduces uncertainties
- NLO+NLL$_{DY}$ changes a lot – unstable under new data

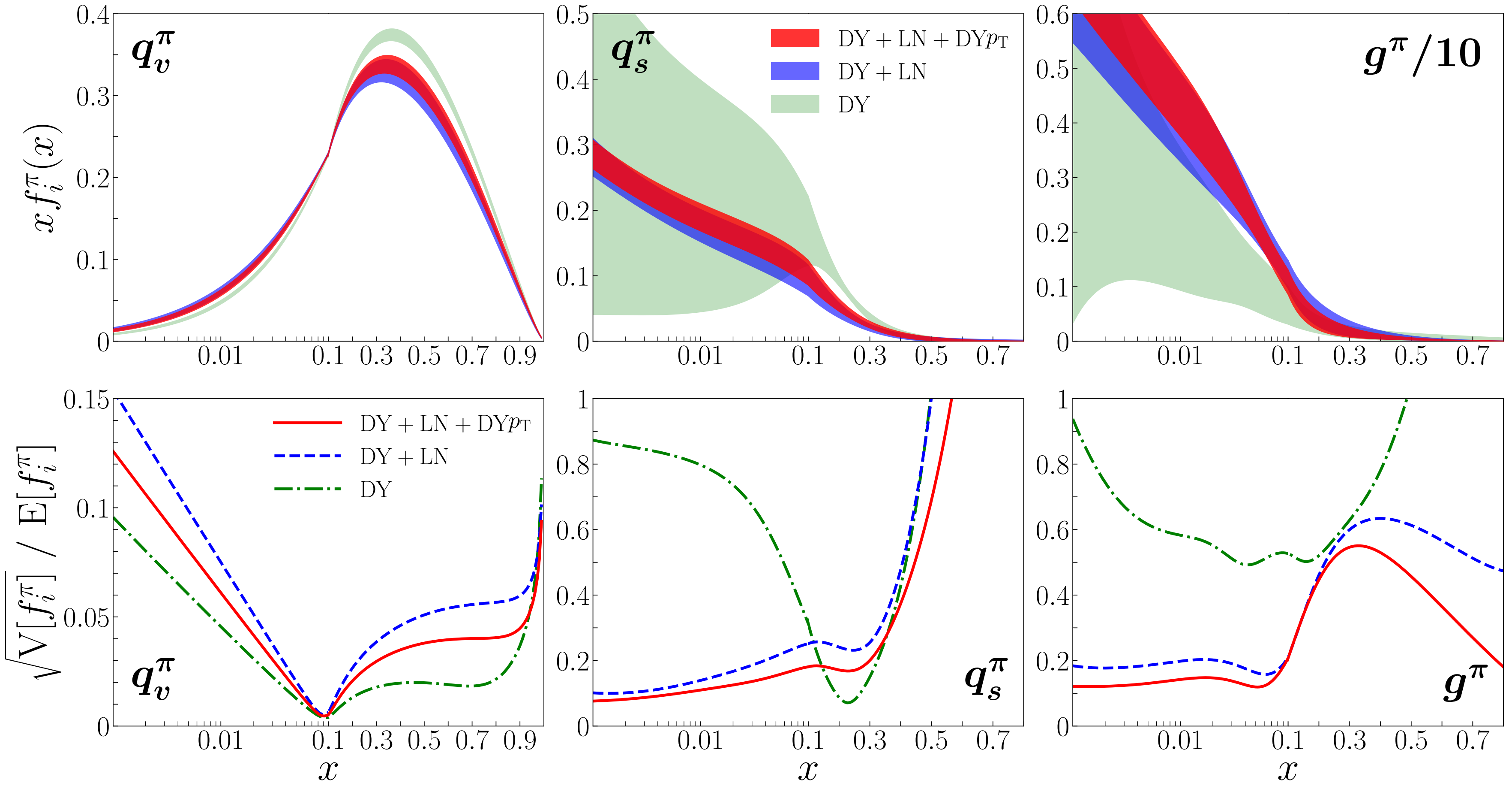


A word on Scale

- In the p_{T}-integrated Drell-Yan and Leading Neutron observables, only one hard scale appears, Q^2
  - Sensible scale for the PDFs
- However, in p_T-dependent DY, two hard momenta appear, Q as well as p_T
  - Ambiguous which scale to choose
- We run fits to scales of $\mu^2 = Q^2, \left(\frac{p_T}{2}\right)^2, p_T^2, (2p_T)^2$
- Best description of the data with $\mu^2 = \left(\frac{p_T}{2}\right)^2$

Effects of Each Dataset

- Not much impact from the transverse-momentum dependent DY data



Building of the nuclear TMD PDF

- Then taking into account the intrinsic non-perturbative, we model the flavor-dependent pieces of the TMD PDF as

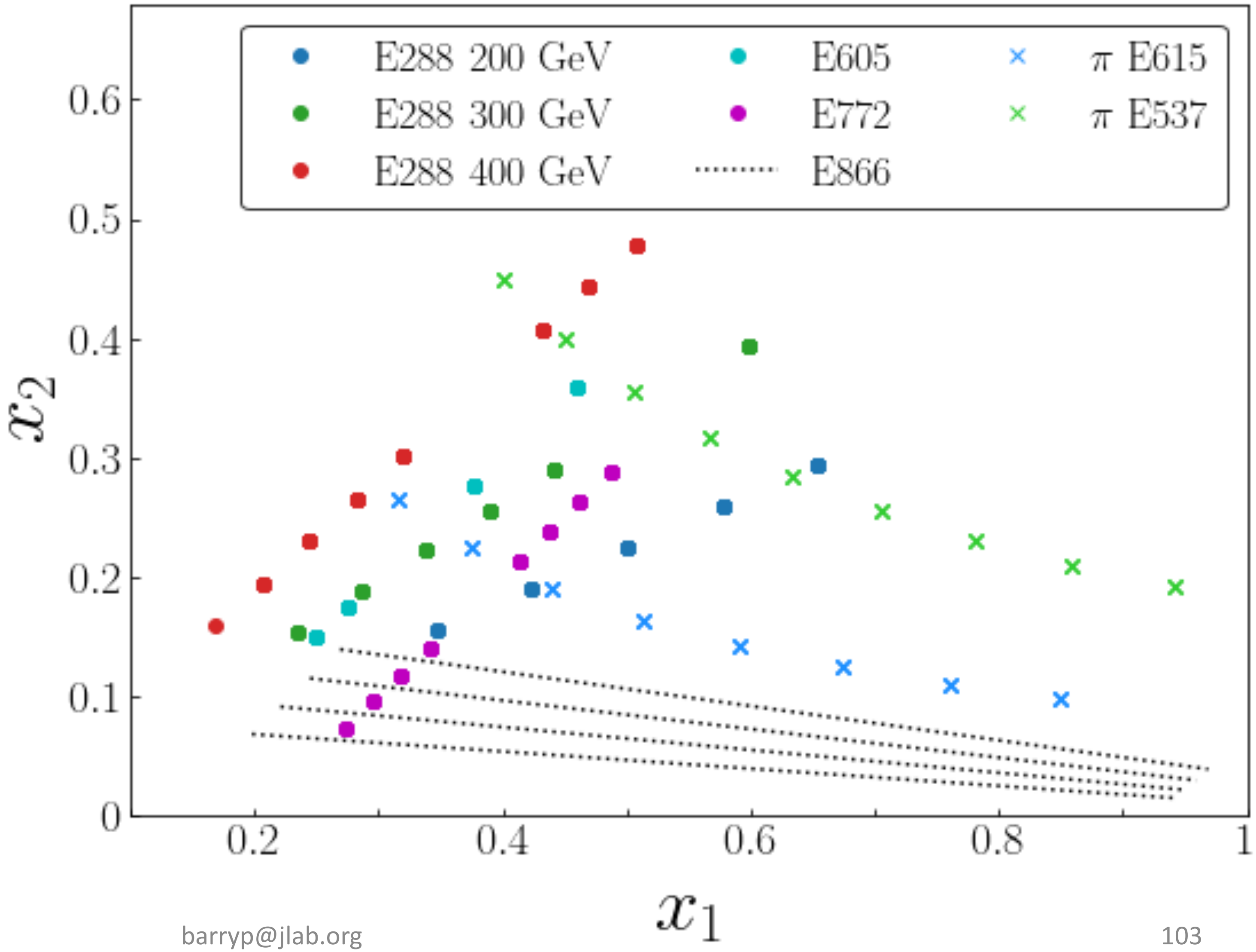
$$(C \otimes f)_{u/A}(x) e^{-g_{u/A}(x,b_T)} \to \frac{Z}{A}(C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x,b_T)} + \frac{A-Z}{A}(C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x,b_T)}$$

and

$$(C \otimes f)_{d/A}(x) e^{-g_{d/A}(x,b_T)} \to \frac{Z}{A}(C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x,b_T)} + \frac{A-Z}{A}(C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x,b_T)}.$$

Kinematics in x_1, x_2

- Using the kinematic mid-point from each of the bins, we show the range in x_1 and x_2



Parametrizations

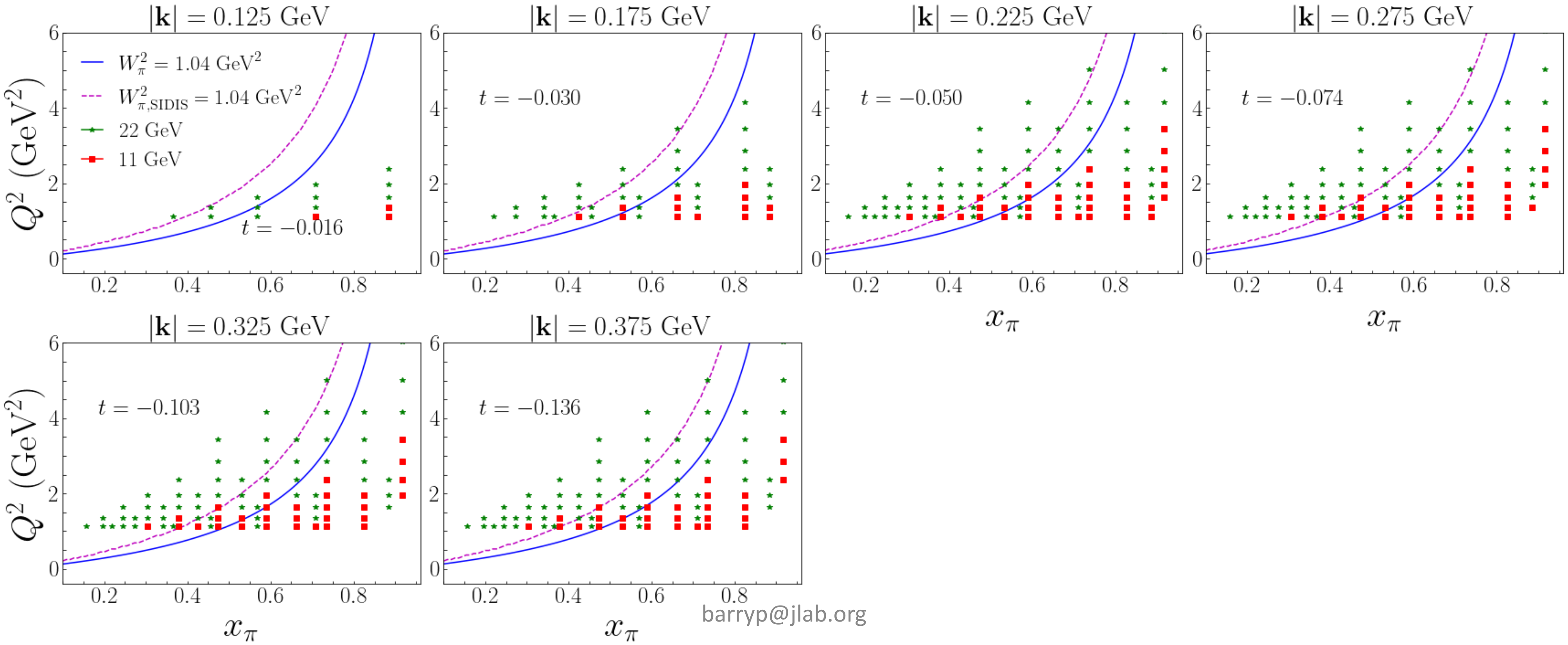
- We can test whether or not the x-dependence is important for these functions (it is!)
- For these g_q functions, we have the following

$$g_q(x, A) = |g^q + g_2^q x + g_3^q (1-x)^2| (1 + g_1 (A^{1/3} - 1)) ,$$
$$B_{NP}(x) = b_{NP} x^2 ,$$

- 4 free parameters for each scheme (5 for Gaussian-to-Exponential)
- We may also open up these for each flavor in the proton (u, d, and sea) and for the pion (val, sea)

Kinematics with 11 GeV

- Still a cut on $W_\pi^2 = 1.04 \text{ GeV}^2$, but SIDIS requires more phase space
- Hardly anything available with $z = 0.2$, $P_{h,T} = 0.2$ GeV



Trust perturbative region

- Method to keep the $\widetilde{W}$ term unaltered by b_* mechanism up to a certain $b_{\max}$
- Non-perturbative effects kick in at $b_{\max}$
- Smooth function as 1st and 2nd derivatives are continuous at $b_{\max}$

$$\widetilde{W}(b_T, x_a, x_b, Q) = \widetilde{W}_{\text{pert}}(b_T, x_a, x_b, Q) \quad \text{for} \quad b_T < b_{\max}$$
$$= \widetilde{W}_{\text{pert}}(b_{\max}, x_a, x_b, Q) f_{\text{NP}}(b_T, b_{\max}, x_a, x_b) \quad \text{for} \quad b_T > b_{\max}$$

Qiu, Zhang, PRD **63**, 114011 (2001).