Construction of a non-pertubative vertex in QED

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- Aristotle: 4 fundamental elements

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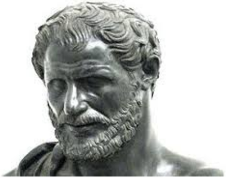
- Democritus: Introduces the term atom



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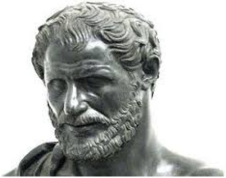

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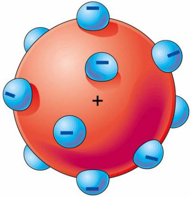

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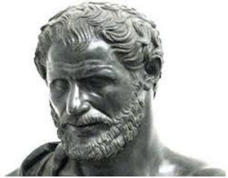

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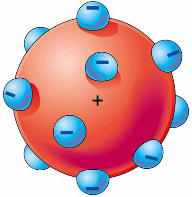

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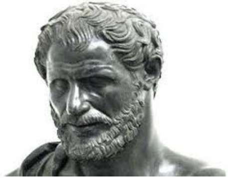

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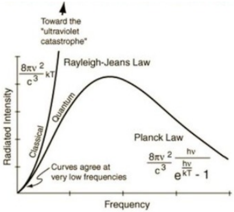

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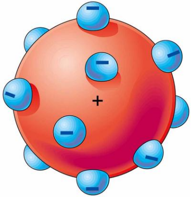

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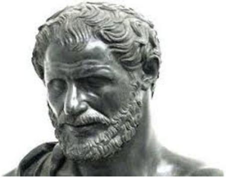
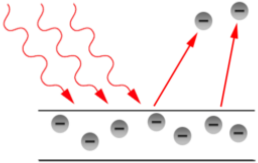


- 1900: Black body
- 1905: Photoelectric effect



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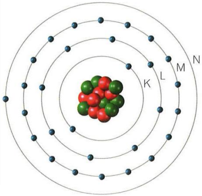

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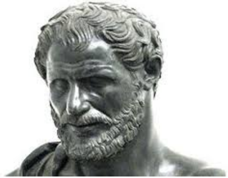

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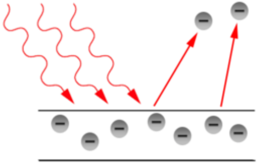

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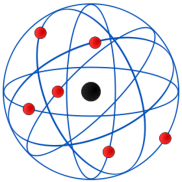

- 1803: John Dalton
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- 1913: Niels Bohr

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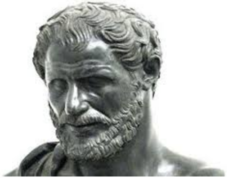

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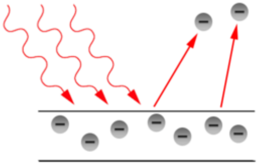


Energy quantification
Schrödinger's Equation

- Energy quantification $\Rightarrow \quad E = nh\nu$
- Schrödinger's Eq $\Rightarrow \quad i\hbar\frac{\partial}{\partial t}\Psi(r,t) = \hat{H}\Psi(r,t)$

Quantum mechanics at high speeds $\rightarrow$ Problems with the theory!

Relativistic Schrödinger equation:

- Klein-Gordon equation $\quad \left(\Box^2 + m^2\right)\phi = 0$
- Dirac equation $\quad \left(i\gamma^\mu\partial_\mu - m\right)\Psi = 0$
- Maxwell, Proca equation $\quad \partial^\alpha\partial_\alpha A^\mu(\boldsymbol{x}, t) = 0$
- Rarita-Schwinger equation $\quad (\epsilon^{\mu\kappa\rho\nu}\gamma_5\gamma_\kappa\partial_{rho} - im\sigma^{\mu\nu})\psi_\nu = 0$

Interacting forces
Quantum electrodynamics

Force	Theory	Mediator
Strong	Quantum Chromodynamics (QCD)	Gluon
Electromagnetic	Quantum electrodynamics (QED)	Photon
Weak	Weak interactions	$W^{\pm}$, Z
Gravitational	General Relativity	Graviton

Sin-Itiro Tomonaga

Julian Schwinger

Richard P. Feynman

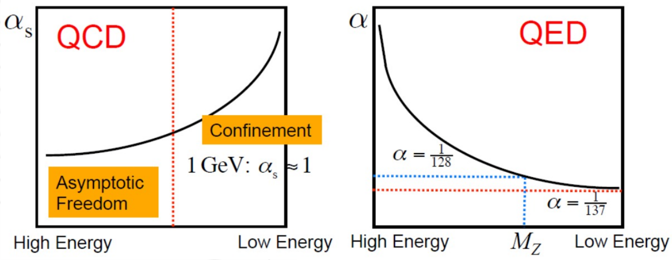
Key: Perturbative methods

Problem: Divergence

Solution: Renormalization

Non-perturbative regime

Why not pertubative?



QED Lagrangian

$$\mathcal{L}_{QED} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Free photon}} + \underbrace{\overline{\Psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\Psi} + \underbrace{e\overline{\Psi}\gamma^{\mu}\Psi A_{\mu}} + \underbrace{\frac{1}{2\xi}(\partial_{\mu}A^{\mu})^{2}}$$

QED Lagrangian

$$\mathcal{L}_{QED} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\textit{Free photon}} + \underbrace{\overline{\Psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\Psi}_{\textit{Free electron}} + \underbrace{e\overline{\Psi}\gamma^{\mu}\Psi A_{\mu}} + \underbrace{\frac{1}{2\xi}(\partial_{\mu}A^{\mu})^{2}}$$

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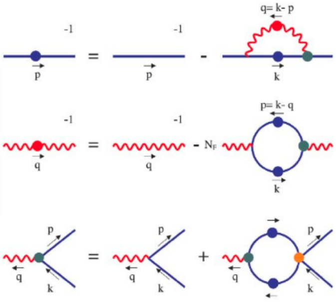
Schwinger Dyson equations
Perturbative vs. non perturbative regime

Non perturbative phenomena $\Rightarrow \begin{cases} \textit{Confinement} \\ \text{Dynamical Chiral Symmetry Breaking (DCSB)} \end{cases}$



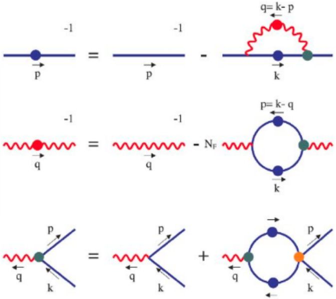
The way $\rightarrow$ Schwinger Dyson equations
They are infinite
Truncate them in the propagators

$$S_F^{-1}(k) = S_F^{0\,-1}(k) - ie^2 \int_M \frac{d^4p}{(2\pi)^4} \gamma^\nu S_F(p) \Gamma^\mu(k,p) \Delta_{\mu\nu}(q)$$

$$\Delta_{\mu\nu}^{-1}(q) = \Delta_{\mu\nu}^{0\;\;-1} - ie^2 N_f \int_M \frac{d^4p}{(2\pi)^4} \gamma_\nu S_F(p) \Gamma_\mu(k,p) S_F(k)$$

Schwinger Dyson equations
Perturbative vs. non perturbative regime

Non perturbative phenomena $\Rightarrow$ { *Confinement*; Dynamical Chiral Symmetry Breaking (DCSB) }



The way $\rightarrow$ Schwinger Dyson equations

Problem $\rightarrow$ They are infinite

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$$S_F^{-1}(k)=S_F^{0\,-1}(k)-ie^2\int_M \frac{d^4p}{(2\pi)^4}\gamma^\nu S_F(p)\Gamma^\mu(k,p)\Delta_{\mu\nu}(q)$$

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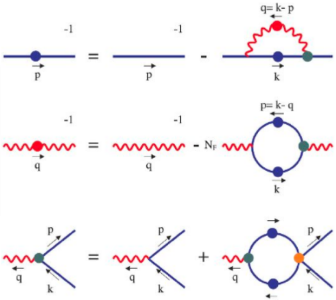

The way $\rightarrow$ Schwinger Dyson equations
Problem $\rightarrow$ They are infinite
Solution $\rightarrow$ Truncate them in the propagators

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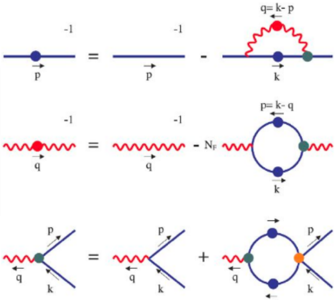
Schwinger Dyson equations
Perturbative vs. non perturbative regime

Non perturbative phenomena ⇒ { *Confinement*; Dynamical Chiral Symmetry Breaking (DCSB) }



The way → Schwinger Dyson equations
Problem → They are infinite
Solution → Truncate them in the propagators

Propagators

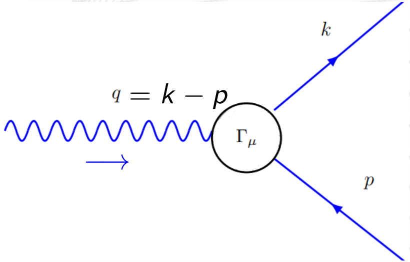
$$S_F^{-1}(k) = S_F^{0\,-1}(k) - ie^2 \int_M \frac{d^4p}{(2\pi)^4} \gamma^\nu S_F(p) \Gamma^\mu(k,p) \Delta_{\mu\nu}(q)$$

$$\Delta_{\mu\nu}^{-1}(q) = \Delta_{\mu\nu}^{0\;\;-1} - ie^2 N_f \int_M \frac{d^4p}{(2\pi)^4} \gamma_\nu S_F(p) \Gamma_\mu(k,p) S_F(k)$$

Fermion-photon vertex

It can be expressed as $\rightarrow$

$$\Gamma_\mu (k,p) = \sum_{i=1}^{12} v_i (k,p)\, V_\mu^i (k,p)$$



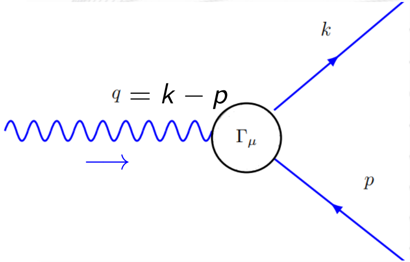
$$V_\mu^1(k,p)=\gamma_\mu \quad , \quad V_\mu^2(k,p)=k_\mu \quad , \quad V_\mu^3(k,p)=p_\mu$$
$$V_\mu^4(k,p)=\not{k}\gamma_\mu \quad , \quad V_\mu^5(k,p)=\not{k}k_\mu \quad , \quad V_\mu^6(k,p)=\not{k}p_\mu$$
$$V_\mu^7(k,p)=\not{p}\gamma_\mu \quad , \quad V_\mu^8(k,p)=\not{p}k_\mu \quad , \quad V_\mu^9(k,p)=\not{p}p_\mu$$
$$V_\mu^{10}(k,p)=\not{k}\not{p}\gamma_\mu , \quad V_\mu^{11}(k,p)=\not{k}\not{p}k_\mu , \quad V_\mu^{12}(k,p)=\not{k}\not{p}p_\mu$$

$$\Gamma_\mu (k,p) = \Gamma_\mu^L (k,p) + \Gamma_\mu^T (k,p)$$

Fermion-photon vertex

It can be expressed as $\rightarrow$

$$\Gamma_\mu\left(k,p\right)=\sum_{i=1}^{12} v_i\left(k,p\right)V_\mu^i\left(k,p\right)$$



$$V_\mu^1(k,p)=\gamma_\mu \quad , \quad V_\mu^2(k,p)=k_\mu \quad , \quad V_\mu^3(k,p)=p_\mu$$

$$V_\mu^4(k,p)=\not{k}\gamma_\mu \quad , \quad V_\mu^5(k,p)=\not{k}k_\mu \quad , \quad V_\mu^6(k,p)=\not{k}p_\mu$$

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Can be divided $\rightarrow$

$$\Gamma_\mu\left(k,p\right)=\Gamma_\mu^L\left(k,p\right)+\Gamma_\mu^T\left(k,p\right)$$

Longitudinal Vertex

Longitudinal component $\rightarrow$

$$\Gamma_L^\mu(k,p) = \sum_{i=1}^{4} a_i(k^2,p^2) L_i^\mu$$

$$\Gamma_\mu^L(k,p) = a\left(k^2,p^2\right)\gamma_\mu + \frac{1}{2} b\left(k^2,p^2\right) t_\mu \gamma \cdot t - ic\left(k^2,p^2\right) t_\mu$$

where $t = k + p$

$$a\left(k^2,p^2\right) = \frac{1}{2}\left[\frac{1}{F\left(k^2\right)} + \frac{1}{F\left(p^2\right)}\right],$$

$$b\left(k^2,p^2\right) = \frac{1}{k^2 - p^2}\left[\frac{1}{F\left(k^2\right)} - \frac{1}{F\left(p^2\right)}\right],$$

$$c\left(k^2,p^2\right) = \frac{1}{k^2 - p^2}\left[\frac{M\left(k^2\right)}{F\left(k^2\right)} - \frac{M\left(p^2\right)}{F\left(p^2\right)}\right].$$

Longitudinal Vertex

Longitudinal component →

$$\Gamma_L^{\mu}(k,p) = \sum_{i=1}^{4} a_i(k^2,p^2) L_i^{\mu}$$

Longitudinal vertex →

$$\Gamma_{\mu}^{L}(k,p) = a\left(k^2,p^2\right)\gamma_{\mu} + \frac{1}{2} b\left(k^2,p^2\right) t_{\mu}\gamma\cdot t - ic\left(k^2,p^2\right) t_{\mu}$$

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where $t = k + p$

Coefficients →

$$a\left(k^2,p^2\right) = \frac{1}{2}\left[\frac{1}{F\left(k^2\right)} + \frac{1}{F\left(p^2\right)}\right],$$

$$b\left(k^2,p^2\right) = \frac{1}{k^2-p^2}\left[\frac{1}{F\left(k^2\right)} - \frac{1}{F\left(p^2\right)}\right],$$

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Transverse Vertex

The only information $\rightarrow$ $iq_\mu \Gamma_\mu^T(k,p) = 0$

$$\Gamma_\mu^T(k,p) = \sum_{i=1}^{8} \tau_i(k,p) T_\mu^i(k,p) \rightarrow \begin{cases} T_\mu^1(k,p) = i[p_\mu(k\cdot q) - k_\mu(p\cdot q)] \\ T_\mu^2(k,p) = [p_\mu(k\cdot q) - k_\mu(p\cdot q)]\gamma\cdot t \\ T_\mu^3(k,p) = q^2\gamma_\mu - q_\mu \gamma\cdot q \\ T_\mu^4(k,p) = iq^2[\gamma_\mu \gamma\cdot t - t_\mu] + 2q_\mu p_\nu k_\rho \sigma_{\nu\rho} \\ T_\mu^5(k,p) = \sigma_{\mu\nu} q_\nu \\ T_\mu^6(k,p) = -\gamma_\mu(k^2 - p^2) + t_\mu \gamma\cdot q \\ T_\mu^7(k,p) = \frac{i}{2}(k^2 - p^2)[\gamma_\mu \gamma\cdot t - t_\mu] + t_\mu p_\nu k_\rho \sigma_{\nu\rho} \\ T_\mu^8(k,p) = -i\gamma_\mu p_\nu k_\rho \sigma_{\nu\rho} - p_\mu \gamma\cdot k + k_\mu \gamma\cdot p \end{cases}$$

where $t = k + p$ y $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$

$$\tau_i(p,k) = \tau_i(k,p), \qquad i = 1,2,3,5,7,8$$

$$\tau_i(p,k) = -\tau_i(k,p), \qquad i = 4,6$$

Transverse Vertex

The only information → $iq_\mu \Gamma_\mu^T(k,p) = 0$

Transverse part → $\Gamma_\mu^T(k,p) = \sum_{i=1}^{8} \tau_i(k,p) T_\mu^i(k,p)$ →

$$T_\mu^1(k,p) = i[p_\mu(k\cdot q) - k_\mu(p\cdot q)]$$
$$T_\mu^2(k,p) = [p_\mu(k\cdot q) - k_\mu(p\cdot q)]\gamma\cdot t$$
$$T_\mu^3(k,p) = q^2\gamma_\mu - q_\mu \gamma\cdot q$$
$$T_\mu^4(k,p) = iq^2[\gamma_\mu\gamma\cdot t - t_\mu] + 2q_\mu p_\nu k_\rho \sigma_{\nu\rho}$$
$$T_\mu^5(k,p) = \sigma_{\mu\nu} q_\nu$$
$$T_\mu^6(k,p) = -\gamma_\mu(k^2-p^2) + t_\mu \gamma\cdot q$$
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where $t = k + p$ y $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$

$$\tau_i(p,k) = \tau_i(k,p), \qquad i = 1,2,3,5,7,8$$
$$\tau_i(p,k) = -\tau_i(k,p), \qquad i = 4,6$$

Transverse Vertex

The only information → $iq_\mu \Gamma^T_\mu(k,p) = 0$

Transverse part → $\Gamma^T_\mu(k,p) = \sum_{i=1}^{8} \tau_i(k,p) T^i_\mu(k,p)$ →

$$
\begin{cases}
T^1_\mu(k,p) = i[p_\mu(k\cdot q) - k_\mu(p\cdot q)] \\
T^2_\mu(k,p) = [p_\mu(k\cdot q) - k_\mu(p\cdot q)]\gamma\cdot t \\
T^3_\mu(k,p) = q^2\gamma_\mu - q_\mu\gamma\cdot q \\
T^4_\mu(k,p) = iq^2[\gamma_\mu\gamma\cdot t - t_\mu] + 2q_\mu p_\nu k_\rho \sigma_{\nu\rho} \\
T^5_\mu(k,p) = \sigma_{\mu\nu} q_\nu \\
T^6_\mu(k,p) = -\gamma_\mu(k^2 - p^2) + t_\mu\gamma\cdot q \\
T^7_\mu(k,p) = \frac{i}{2}(k^2 - p^2)[\gamma_\mu\gamma\cdot t - t_\mu] + t_\mu p_\nu k_\rho \sigma_{\nu\rho} \\
T^8_\mu(k,p) = -i\gamma_\mu p_\nu k_\rho \sigma_{\nu\rho} - p_\mu\gamma\cdot k + k_\mu\gamma\cdot p
\end{cases}
$$

where $t = k + p$ y $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$

CP symmetry $\tau_i(k,p)$ →

$\tau_i(p,k) = \tau_i(k,p)$, $i = 1, 2, 3, 5, 7, 8$

$\tau_i(p,k) = -\tau_i(k,p)$, $i = 4, 6$

Transverse Takahashi identities (TTI)

- Vector TTI

$$q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\nu(k,p) = S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) + 2im\Gamma_{\mu\nu}(k,p) + t_\alpha \epsilon_{\alpha\mu\nu\beta}\Gamma^A_\beta(k,p) + A^V_{\mu\nu}(k,p)$$

- Axial-vector TTI

$$q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) = S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu} S^{-1}(k) + t_\alpha \epsilon_{\alpha\mu\nu\beta}\Gamma_\beta(k,p) + V^A_{\mu\nu}(k,p)$$

Axial-vector TTI is contracted with the tensors

$$T^1_{\mu\nu} = \frac{1}{2}\epsilon_{\alpha\mu\nu\beta} t_\alpha q_\beta \quad ; \quad T^2_{\mu\nu} = \frac{1}{2}\epsilon_{\alpha\mu\nu\beta}\gamma_\alpha q_\beta$$

The contraction of two Levi-Civita tensors = Kronecker delta determinant

$$\epsilon_{i_1 i_2 \dots i_n}\epsilon_{j_1 j_2 \dots j_n} = \begin{vmatrix} \delta_{i_1 j_1} & \delta_{i_1 j_2} & \cdots & \delta_{i_1 j_n} \\ \delta_{i_2 j_1} & \delta_{i_2 j_2} & \cdots & \delta_{i_2 j_n} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{i_n j_1} & \delta_{i_n j_2} & \cdots & \delta_{i_n j_n} \end{vmatrix}$$

$$\epsilon_{\rho\mu\nu\sigma}\epsilon_{\alpha\mu\nu\beta} = (D-3)(D-2)\left[g^{\alpha\rho}g^{\beta\sigma} - g^{\alpha\sigma}g^{\beta\rho}\right]$$

Transverse Takahashi identities

$$\frac{1}{2}(D-3)(D-2)q\cdot tt\cdot\Gamma(k,p) = T^1_{\mu\nu}[S^{-1}(p)\sigma^5_{\mu\nu}-\sigma^5_{\mu\nu}S^{-1}(p)]+T^1_{\mu\nu}V^A_{\mu\nu}(k,p) + \frac{1}{2}(D-3)(D-2)t^2q\cdot\Gamma(k,p)$$

$$\frac{1}{2}(D-3)(D-2)q\cdot t\gamma\cdot\Gamma(k,p) = T^2_{\mu\nu}[S^{-1}(p)\sigma^5_{\mu\nu}-\sigma^5_{\mu\nu}S^{-1}(k)]+T^2_{\mu\nu}V^A_{\mu\nu}(k,p) \quad \frac{1}{2}(D-3)(D-2)\gamma\cdot tq\cdot\Gamma(k,p)$$

$$iT^1_{\mu\nu}V^A_{\mu\nu}=I_D Y_1(k,p)+i(\gamma\cdot q)Y_2(k,p)+i(\gamma\cdot t)Y_3(k,p)+[\gamma\cdot q,\gamma\cdot t]Y_4(k,p)$$
$$iT^2_{\mu\nu}V^A_{\mu\nu}=iI_D Y_5(k,p)+(\gamma\cdot q)Y_6(k,p)+(\gamma\cdot t)Y_7(k,p)+i[\gamma\cdot q,\gamma\cdot t]Y_8(k,p)$$

The above equations represent a system of 8 linearly independent equations. They can be projected :

$$Tr\{I_D TTI_{1,2}\} \qquad Tr\{\gamma\cdot k\, TTI_{1,2}\} \qquad Tr\{\gamma\cdot p\, TTI_{1,2}\} \qquad Tr\{\gamma\cdot k\gamma\cdot p\, TTI_{1,2}\}$$

$$\tau_1 = -\frac{Y_1}{(D-3)(D-2)\left(k^2-p^2\right)\nabla(k,p)}$$

$$\tau_2 = -\frac{Y_5 - DY_3 + Y_3}{(D-3)(D-2)^2\left(k^2-p^2\right)\nabla(k,p)}$$

$$\tau_3 = \frac{1}{2}b\left(k^2,p^2\right) + \frac{(D-2)Y_2\left(k^2-p^2\right) - \left(Y_3 - Y_5\right)t^2}{2(D-3)(D-2)^2\left(k^2-p^2\right)\nabla(k,p)}$$

$Y_i(k,p)$ functions are unknown!

$$\tau_4 = -\frac{\left(k^2-p^2\right)\left[2(D-1)Y_4 + Y_6^A\right] + Y_7^S t^2}{2(D-3)(D-2)^2\left(k^2-p^2\right)\nabla(k,p)}$$

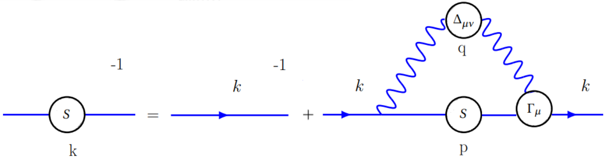
$$\tau_5 = -c\left(k^2,p^2\right) - \frac{2\left(2Y_4 + Y_6^A\right)}{(D-3)(D-2)^2\left(k^2-p^2\right)}$$

$$\tau_6 = \frac{(D-2)q^2Y_2 - \left(k^2-p^2\right)\left(Y_3 - Y_5\right)}{2(D-3)(D-2)^2\left(k^2-p^2\right)\nabla(k,p)}$$

$$\tau_7 = \frac{q^2\left[2(D-1)Y_4 + Y_6^A\right] + Y_7^S\left(k^2-p^2\right)}{(D-3)(D-2)^2\left(k^2-p^2\right)\nabla(k,p)}$$

$$\tau_8 = -\frac{8Y_8^A}{(D-3)(D-2)^2\left(k^2-p^2\right)} - \frac{1}{k^2-p^2}\left(\frac{2}{(D-2)F\left(k^2\right)} - \frac{1}{F\left(p^2\right)}\right)$$

Schwinger Dyson equation for fermion
Gap equation



The gap equation is defined as

$$S^{-1}(k) = S_0^{-1}(k) + e^2 \int_E \frac{d^d p}{(2\pi)^d} \gamma_\mu S(p) \Gamma_\nu(k,p) \Delta_{\mu\nu}(q)$$

can be decomposed into two equations by multiplying by 1 and $\gamma \cdot k$

$$\gamma \cdot k S^{-1}(k) = \gamma \cdot k S_0^{-1}(k) + e^2 \int_E \frac{d^d p}{(2\pi)^d} \gamma \cdot k \gamma_\mu S(p) \Gamma_\nu(k,p) \Delta_{\mu\nu}(q)$$

Mass function $\rightarrow$

$$\frac{M(k^2)}{F(k^2)}=m_0+e^2\xi\int_E \frac{d^dp}{(2\pi)^d}\frac{F\left(p^2\right)}{p^2+M^2\left(p^2\right)}\frac{1}{q^4}\frac{1}{F(k^2)}\left[M\left(p^2\right)k\cdot q-M\left(k^2\right)q\cdot p\right]$$
$$+e^2\int_E \frac{d^dp}{(2\pi)^d}\frac{F(p^2)}{p^2+M^2(p^2)}M(p^2)G_M(k,p)$$

Fermion wave function $\rightarrow$

$$\frac{1}{F(k^2)}=1-e^2\xi\int_E \frac{d^dp}{(2\pi)^d}\frac{1}{q^4}\frac{F(p^2)}{p^2+M^2(p^2)}\frac{1}{F(k^2)}\left\{p\cdot q+M(k^2)M(p^2)\frac{k\cdot q}{k^2}\right\}$$
$$+e^2\int_E \frac{d^dp}{(2\pi)^d}\frac{1}{k^2}\frac{F(p^2)}{p^2+M^2(p^2)}G_F(k,p)$$

Y_1, Y_6^A, Y_7^S are massive $\rightarrow$

$$\Lambda_M(p,k)=\frac{1}{(D-3)(D-2)}[-Y_1(k,p)-2(q\cdot p)Y_6^A(k,p)+2(t\cdot p)Y_7^S(k,p)]$$

Y_2, Y_3, Y_5, Y_8^A are not massive $\rightarrow$

$$\Lambda_{NM}(k,p)=\frac{1}{(D-3)(D-2)}[\left(k^2-p^2\right)Y_2(k,p)+t^2Y_3(k,p)-2(k\cdot p)Y_5(k,p)+8\nabla(k,p)Y_8^A(k,p)]$$

Ensuring Multiplicative Renormalization

In the chiral limit $\rightarrow$

$$\frac{1}{F(k^2)}=1-e^2\xi\int_E \frac{d^Dp}{(2\pi)^D}\frac{1}{q^4}\frac{F(p^2)}{F(k^2)}\frac{q\cdot p}{p^2}-\frac{e^2}{k^2}\int_E \frac{d^Dp}{(2\pi)^D}\frac{1}{p^2}\frac{F(p^2)\Delta(q^2)}{k^2-p^2}\{\Lambda_{NM}(k,p)$$

$$+(k^2-p^2)[(D-1)k^2p^2b(k^2,p^2)-u(k,p)\widetilde{b}(k^2,p^2)-(D-4)\frac{\nabla(k,p)}{k^2-p^2}\frac{1}{F(k^2)}]\}$$

$$\frac{1}{F(k^2)}=1-(4\pi)^2\int\frac{d^dp}{(2\pi)^d}\gamma_d(k^2,p^2,q^2)\frac{F(p^2)}{F(k^2)}\frac{p\cdot q}{p^2q^4}$$

$$\Lambda_{NM}(k,p)=(p^2-k^2)[(D-1)k^2p^2b(k^2,p^2)-u(k,p)\widetilde{b}(k^2,p^2)-(D-4)\frac{\nabla(k,p)}{k^2-p^2}\frac{1}{F(k^2)}]$$

[1] J. Phys. A: Math. Gen. 37 (2004) 6587–6597

Ensuring Multiplicative Renormalization

In the chiral limit →

$$\frac{1}{F(k^2)}=1-e^2\xi\int_E \frac{d^Dp}{(2\pi)^D}\frac{1}{q^4}\frac{F(p^2)}{F(k^2)}\frac{q\cdot p}{p^2}-\frac{e^2}{k^2}\int_E \frac{d^Dp}{(2\pi)^D}\frac{1}{p^2}\frac{F(p^2)\Delta(q^2)}{k^2-p^2}\{\Lambda_{NM}(k,p)$$
$$+(k^2-p^2)[(D-1)k^2p^2b(k^2,p^2)-u(k,p)\widetilde{b}(k^2,p^2)-(D-4)\frac{\nabla(k,p)}{k^2-p^2}\frac{1}{F(k^2)}]\}$$

Paper: Bashir and Delbourgo [1] →

$$\frac{1}{F(k^2)}=1-(4\pi)^2\int\frac{d^dp}{(2\pi)^d}\gamma_d(k^2,p^2,q^2)\frac{F(p^2)}{F(k^2)}\frac{p\cdot q}{p^2q^4}$$

$$\Lambda_{NM}(k,p)=(p^2-k^2)[(D-1)k^2p^2b(k^2,p^2)-u(k,p)\widetilde{b}(k^2,p^2)-(D-4)\frac{\nabla(k,p)}{k^2-p^2}\frac{1}{F(k^2)}]$$

[1] J. Phys. A: Math. Gen. 37 (2004) 6587–6597

Ensuring Multiplicative Renormalization

In the chiral limit $\rightarrow$

$$\frac{1}{F(k^2)}=1-e^2\xi\int_E\frac{d^Dp}{(2\pi)^D}\frac{1}{q^4}\frac{F(p^2)}{F(k^2)}\frac{q\cdot p}{p^2}-\frac{e^2}{k^2}\int_E\frac{d^Dp}{(2\pi)^D}\frac{1}{p^2}\frac{F(p^2)\Delta(q^2)}{k^2-p^2}\{\Lambda_{NM}(k,p)$$
$$+(k^2-p^2)[(D-1)k^2p^2b(k^2,p^2)-u(k,p)\widetilde{b}(k^2,p^2)-(D-4)\frac{\nabla(k,p)}{k^2-p^2}\frac{1}{F(k^2)}]\}$$

Paper: Bashir and Delbourgo [1] $\rightarrow$

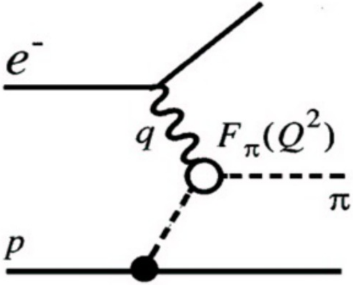
$$\frac{1}{F(k^2)}=1-(4\pi)^2\int\frac{d^dp}{(2\pi)^d}\gamma_d(k^2,p^2,q^2)\frac{F(p^2)}{F(k^2)}\frac{p\cdot q}{p^2q^4}$$

It gives us information on Y_2, Y_3, Y_5, Y_8^A $\rightarrow$

$$\Lambda_{NM}(k,p)=(p^2-k^2)[(D-1)k^2p^2b(k^2,p^2)-u(k,p)\widetilde{b}(k^2,p^2)-(D-4)\frac{\nabla(k,p)}{k^2-p^2}\frac{1}{F(k^2)}]$$

[1] J. Phys. A: Math. Gen. 37 (2004) 6587–6597

Importance



Pion Elastic Form Factor

Conclusions

[2] A nonperturbative *Ansatz* for the fermion-photon vertex $\Gamma_\mu(k,p)$ can be obtained from the WTI and TTI.

- It is free of kinematic singularities
- Ensures gauge invariance through WTIs and TTIs
- Decouples the Schwinger Dyson equations at the propagator level
- Multiplicative renormalization of the fermion SDE is ensured.
- We obtain information on the appropriate linear combinations of Y_2, Y_3, Y_5 y Y_8^A

[2] Phys. Rev. D 100, 054028 (2019)

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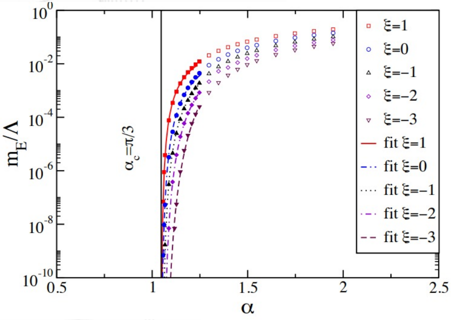
Different Gauges



To ensure MR $\rightarrow$

$$\frac{1}{F(k^2)}=1-(4\pi)^2\int_E \frac{d^Dp}{(2\pi)^D}\gamma_d\frac{F(p^2)}{F(k^2)}\frac{p\cdot q}{p^2q^4}-\frac{e^2}{k^2}\int_E\frac{d^Dp}{(2\pi)^D}\frac{1}{p^2}\frac{F(p^2)\Delta(q^2)}{k^2-p^2}\{\Lambda_{NM}(k,p)$$
$$+(k^2-p^2)[(D-1)k^2p^2b(k^2,p^2)-u(k,p)\widetilde{b}(k^2,p^2)-(D-4)\frac{\nabla(k,p)}{k^2-p^2}\frac{1}{F(k^2)}]\}$$
$$+(4\pi)^2\int_E\frac{d^Dp}{(2\pi)^D}\lambda_d\frac{F(p^2)}{F(k^2)}\frac{q\cdot p}{p^2q^4}$$

We infer $\Lambda_{NM}(k,p)$ $\rightarrow$

$$(4\pi)^2\int_E\frac{d^Dp}{(2\pi)^D}\lambda_d\frac{F(p^2)}{F(k^2)}\frac{q\cdot p}{p^2q^4}-\frac{e^2}{k^2}\int_E\frac{d^Dp}{(2\pi)^D}\frac{1}{p^2}\frac{F(p^2)}{q^2}\{\frac{\Lambda_{NM}(k,p)}{k^2-p^2}$$
$$+(D-1)k^2p^2b(k^2,p^2)-u(k,p)\widetilde{b}(k^2,p^2)-(D-4)\frac{\nabla(k,p)}{k^2-p^2}\frac{1}{F(k^2)}\}=0$$

It gives us information on Y_2, Y_3, Y_5, Y_8^A $\rightarrow$

$$\Lambda_{NM}(k,p)=(p^2-k^2)[(D-1)k^2p^2b(k^2,p^2)-u(k,p)\widetilde{b}(k^2,p^2)-(D-4)\frac{\nabla(k,p)}{k^2-p^2}\frac{1}{F(k^2)}]$$

The trace is calculated $\rightarrow$

$$S^{-1}(k)=S_0^{-1}(k)+e^2\int_E \frac{d^d p}{(2\pi)^d}\gamma_\mu S(p)\Gamma_\nu(k,p)\Delta_{\mu\nu}(q)$$

Mass function $\rightarrow$

$$\frac{M(k^2)}{F(k^2)}=m_0+e^2\xi\int_E \frac{d^d p}{(2\pi)^d}\frac{F\left(p^2\right)}{p^2+M^2\left(p^2\right)}\frac{1}{q^4}\frac{1}{F\left(k^2\right)}\left[M\left(p^2\right)k\cdot q-M\left(k^2\right)q\cdot p\right]$$
$$+e^2\int_E \frac{d^d p}{(2\pi)^d}\frac{F(p^2)}{p^2+M^2(p^2)}M(p^2)G_M(k,p)$$

It defines $\rightarrow$

$$\frac{\left(k^2-p^2\right)}{\Delta(q^2)}G_M(k,p)=\frac{2}{(D-3)(D-2)}Y_5(k,p)+\frac{\Lambda_M(p,k)}{M(p^2)}$$
$$+\left[(D-1)k^2-u(k,p)+\left[u(k,p)-(D-1)p^2\right]\frac{M(k^2)}{M(p^2)}\right]\frac{1}{F(k^2)}$$

Y_1, Y_6^A, Y_7^S are massive $\rightarrow$

$$\Lambda_M(p,k)=\frac{1}{(D-3)(D-2)}\left[-Y_1(k,p)-2(q\cdot p)Y_6^A(k,p)+2(t\cdot p)Y_7^S(k,p)\right]$$

$$u(k,p)=(D-1)(k\cdot p)-2\frac{\nabla(k,p)}{q^2}$$

Gram determinant $\rightarrow$

$$\nabla(k,p)=k^2p^2-(k\cdot p)^2$$

The trace is calculated $\rightarrow$

$$\gamma\cdot k S^{-1}(k)=\gamma\cdot k S_0^{-1}(k)+e^2\int_E \frac{d^dp}{(2\pi)^d}\gamma\cdot k\gamma_\mu S(p)\Gamma_\nu(k,p)\Delta_{\mu\nu}(q)$$

Fermion wave function $\rightarrow$

$$\frac{1}{F(k^2)}=1-e^2\xi\int_E \frac{d^dp}{(2\pi)^d}\frac{1}{q^4}\frac{F(p^2)}{p^2+M^2(p^2)}\frac{1}{F(k^2)}\left\{p\cdot q+M(k^2)M(p^2)\frac{k\cdot q}{k^2}\right\}$$
$$+e^2\int_E \frac{d^dp}{(2\pi)^d}\frac{1}{k^2}\frac{F(p^2)}{p^2+M^2(p^2)}G_F(k,p)$$

It defines $\rightarrow$

$$\frac{\left(k^2-p^2\right)}{\Delta(q^2)}G_F(k,p)=-\Lambda_{NM}(k,p)-M(p^2)\Lambda_M(k,p)+(k^2-p^2)\{-(D-1)k^2p^2b(k^2,p^2)$$
$$+u(k,p)\widetilde{b}(k^2,p^2)+M(p^2)\left[u(k,p)-(D-1)k^2\right]c(k^2,p^2)+(D-4)\frac{\nabla(k,p)}{k^2-p^2}\frac{1}{F(k^2)}\}$$

Y_2, Y_3, Y_5, Y_8^A are not massive $\rightarrow$

$$\Lambda_M(k,p)=\frac{1}{(D-3)(D-2)}[Y_1(k,p)+2(k\cdot q)Y_6^A(k,p)+2(k\cdot t)Y_7^S(k,p)]$$

$$\Lambda_{NM}(k,p)=\frac{1}{(D-3)(D-2)}[\left(k^2-p^2\right)Y_2(k,p)+t^2Y_3(k,p)-2(k\cdot p)Y_5(k,p)+8\nabla(k,p)Y_8^A(k,p)]$$

$$\widetilde{b}(k^2,p^2)=\frac{1}{k^2-p^2}\left[\frac{k^2}{F(k^2)}-\frac{p^2}{F(p^2)}\right]$$