Quarkonia as tools to study the multi-dimensional structure of the nucleon

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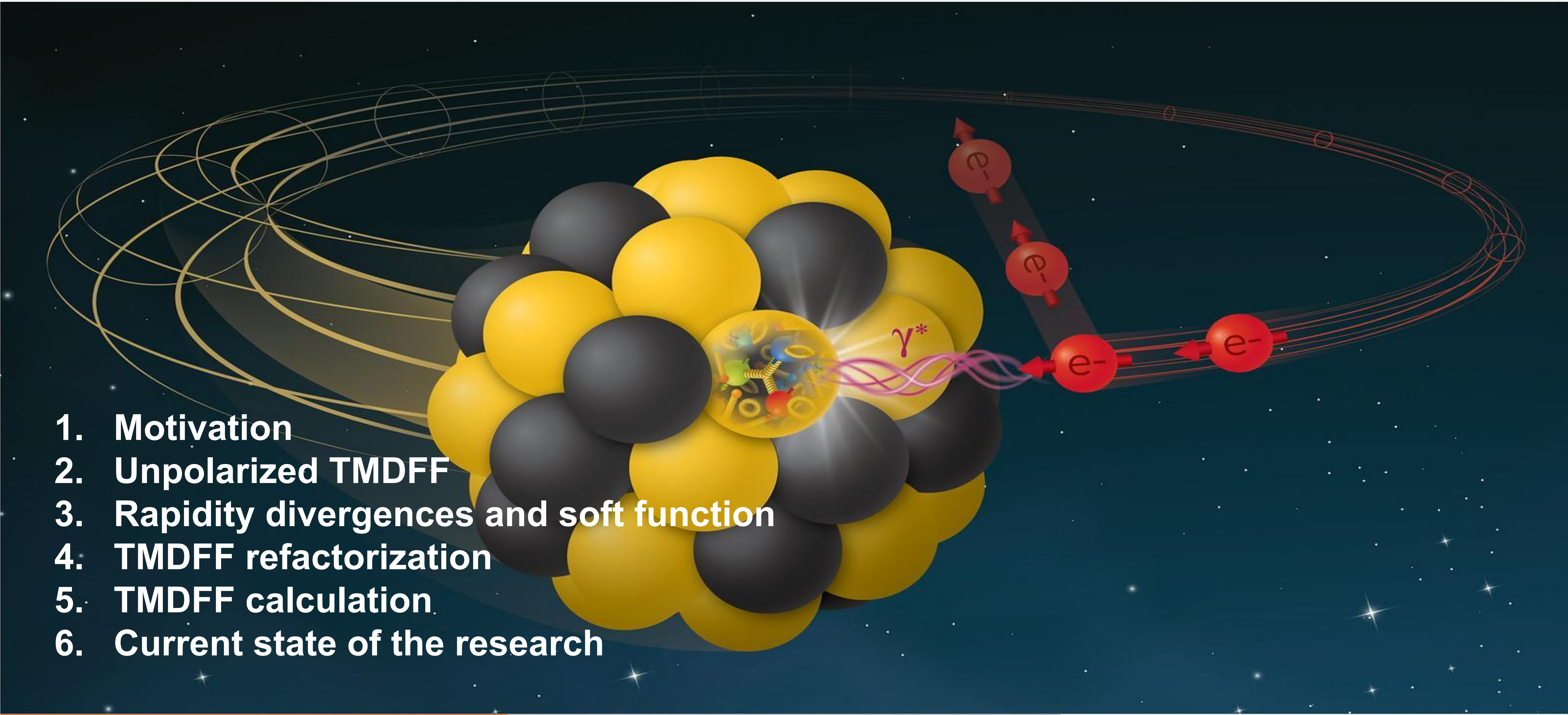
Jefferson Lab

Outline



1. Motivation
2. Unpolarized TMDFF
3. Rapidity divergences and soft function
4. TMDFF refactorization
5. TMDFF calculation
6. Current state of the research

Why quarkonia?

From an experimental point of view...

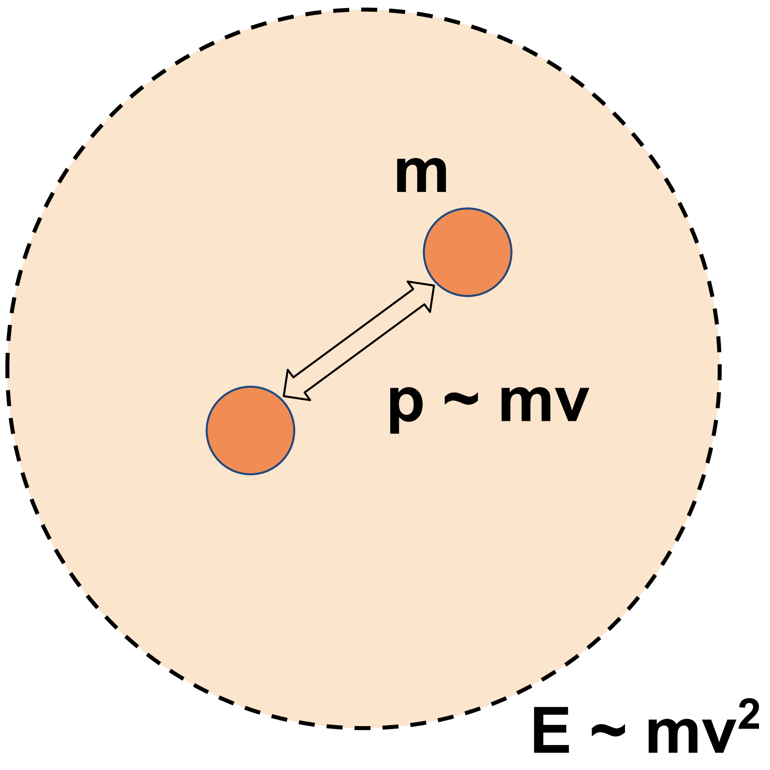
In particular quarkonium states such as J/ψ, $\psi(2S)$ and $\Upsilon(nS)$

- Easy to produce and detect ⇐
- Quarkonia production is one of the processes on which the EIC is most focused

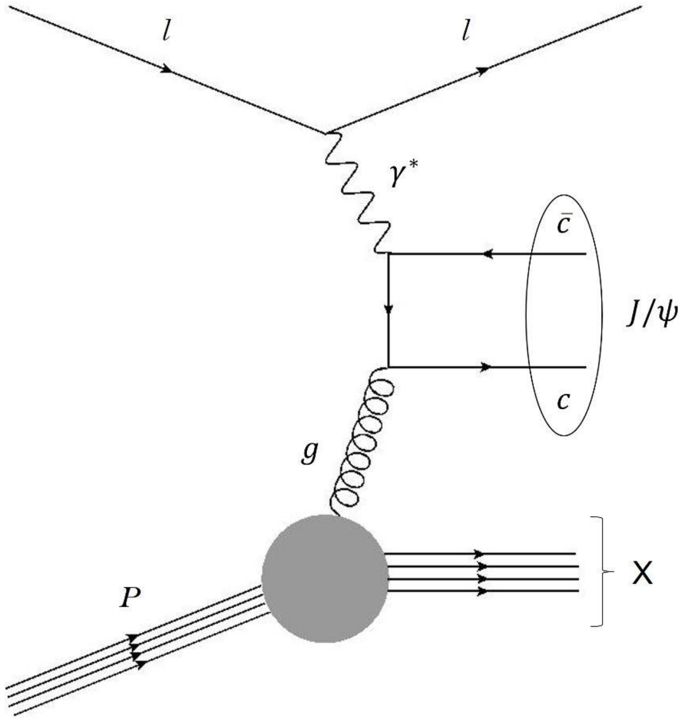
From a theoretical point of view...

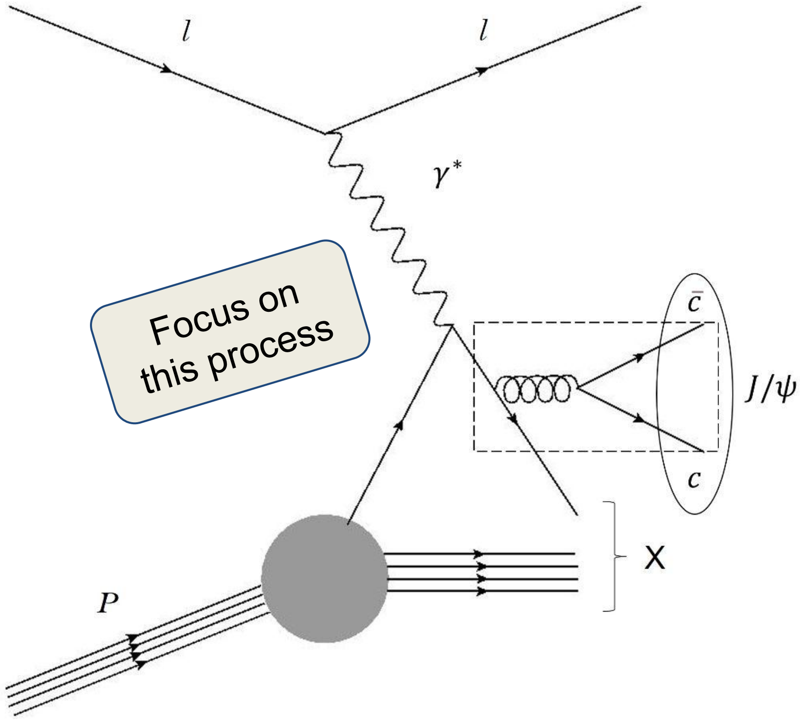
- Quarkonia are useful laboratories to study the interplay between pQCD and non-perturbative QCD — We'll see later!
- There are much more physics yet to be understood and much more work yet to be done in quarkonium production 

m >> p >> E

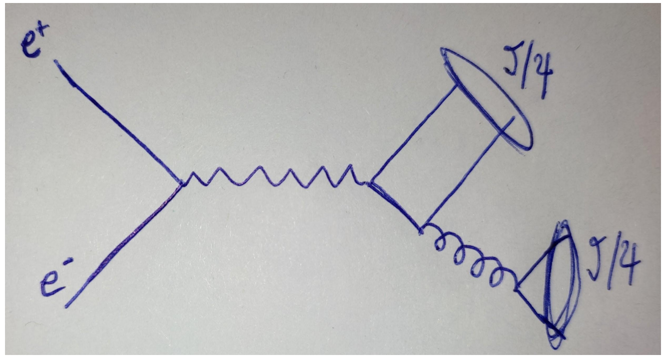


Goal





- The main motivation is complete the J/ψ production in ep scattering
- The contribution by the light-quark fragmentation have been done in arXiv:2007.05547 [hep-ph]
- Typical process carried by EIC: J/ψ production one of the most important



But we can also find the contribution of gluon TMDFF in other processes!

Unpolarized TMDFF

$$\tilde{f}_{q/\mathcal{N}}(x,b_T) = \int \frac{db^-}{4\pi} e^{-ixP^+b^-} Tr\left[< \mathcal{N}|\bar{\psi}_q(b)\gamma^+\mathcal{W}(b,0)\psi_q(0)|\mathcal{N} >\right]$$

Thank you!

Patrick Barry's lecture...

How can we describe the process when a quark is produced in a hard interaction and then fragments into a detected hadron? TMD fragmentation function

$$\Delta_{g\to J/\psi}(z,\mathbf{b}_T) \sim \sum_X \int \frac{db^-}{4\pi} e^{-iP^+b^-/z} < 0|T\left[\mathcal{B}^{\mu}_{n\perp}\right](b)\,|X, J/\psi >< X, J/\psi|\bar{T}\left[\mathcal{B}_{n\perp\mu}\right](0)\,|0 >$$

$$D_{g\to J/\psi} \sim \boxed{Z_g \times R_g} \times \Delta_{g\to J/\psi(z,\mathbf{b}_T)}$$

Renormalization factors

Gluon field

$$\mathcal{B}^{\mu}_{n\perp} = \frac{1}{g}[\mathcal{W}^{\dagger}_n(b) iD^{\mu}_{n\perp}\mathcal{W}_n(b)]$$

Overview of rapidity divergences and Soft Function (SF)

Cause

Our theory is boost invariant in the light-cone directions

Consecuence

$k^+ \to ak^+, k^- \to k^-/a, k^+k^-$

$a \to \infty \implies$ Divergence

Solution

This is well defined at operator level!

$$\frac{1}{k^-} \to \frac{1}{k^- - i\delta}$$

Now the rapidity divergences are described by the regulator. We obtain something like that… $\ln\left(\frac{\delta^2}{Q^2}\right)$

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At this point, the Soft Function enters in the game to cancel all the spurious rapidity divergences.

$$S(\mathbf{b}_\perp) = \frac{Tr}{N_c} < 0| \left[S_n^\dagger S_{\bar{n}}\right] (0^+, 0^-, \mathbf{b}_\perp) \left[S_{\bar{n}}^\dagger S_n\right] (0^+, 0^-, \mathbf{0})|0 >$$

Gluon TMDFF refactorization

Quarkonia, bound states of heavy quarks: massive quarks → perturbative QCD. The heavy quarks are not relativistics: v<<1 → QCD turns non-perturbative

→ We have two scales strongly separated describing the quarkonium production.

→ Non-relativistic QCD (NRQCD)

$$n = {}^{2S+1}L_J^{[col.]}$$

NRQCD →

$$\sigma^H = \sum_n \sigma_{Q\bar{Q}(n)} < 0|\mathcal{O}^H(n)|0 >$$

→ Can be organized in powers of v

gTMDFF in NRQCD →

$$D_{g\to J/\psi}(z, \mathbf{b}_T) = \sum_n d_{g\to Q\bar{Q}(n)}(z, \mathbf{b}_T) < 0|\mathcal{O}^{J/\psi}(n)|0 >$$

SDC

LDME → Universals

$$< \mathcal{O}_n^{J/\psi} >=< 0|\chi^\dagger \mathcal{K}_n \psi a^\dagger_{J/\psi} a_{J/\psi} \psi^\dagger \mathcal{K}'_n \chi|0 >$$

TMDFF calculation

$$D_{g\to J/\psi}(z,\mathbf{b}_T)=\sum_n d_{g\to Q\bar{Q}(n)}(z,\mathbf{b}_T) < 0|\mathcal{O}^{J/\psi}(n)|0 >$$

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- QCD calculation at next to leading order in α_s
- Expansion around the threshold (q = 0)

$$\bar{u}(p)v(\bar{p}) = -2\,\xi^\dagger(\mathbf{q}\cdot\boldsymbol{\sigma})\eta,$$

$$\bar{u}(p)\gamma^\mu v(\bar{p}) = L^\mu{}_j\left(\boxed{2E_q\,\xi^\dagger\sigma^j\eta} - \frac{2}{E_q+m_c}q^j\,\xi^\dagger(\mathbf{q}\cdot\boldsymbol{\sigma})\eta\right),$$

$$\bar{u}(p)(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)v(\bar{p}) = (P^\mu L^\nu{}_j - P^\nu L^\mu{}_j)\left(\frac{2m_c}{E_q}\,\xi^\dagger\sigma^j\eta + \frac{2}{E_q(E_q+m_c)}q^j\,\xi^\dagger(\mathbf{q}\cdot\boldsymbol{\sigma})\eta\right) + L^\mu{}_j\,L^\nu{}_k\,\xi^\dagger\{[\sigma^j,\sigma^k],\mathbf{q}\cdot\boldsymbol{\sigma}\}\eta,$$

$$\begin{aligned}\bar{u}(p)(\gamma^\mu\gamma^\nu\gamma^\lambda-\gamma^\lambda\gamma^\nu\gamma^\mu)v(\bar{p}) &= L^\mu{}_i\,L^\nu{}_j\,L^\lambda{}_k\Bigg(-E_q\,\xi^\dagger\{[\sigma^i,\sigma^j],\sigma^k\}\eta + \frac{q^i}{E_q+m_c}\,\xi^\dagger\{[\sigma^j,\sigma^k],\mathbf{q}\cdot\boldsymbol{\sigma}\}\eta \\ &\quad + \frac{q^j}{E_q+m_c}\,\xi^\dagger\{[\sigma^k,\sigma^i],\mathbf{q}\cdot\boldsymbol{\sigma}\}\eta + \frac{q^k}{E_q+m_c}\,\xi^\dagger\{[\sigma^i,\sigma^j],\mathbf{q}\cdot\boldsymbol{\sigma}\}\eta\Bigg) \\ &\quad - \frac{2}{E_q}\left(P^\mu L^\nu{}_i L^\lambda{}_j + L^\mu{}_i L^\nu{}_j P^\lambda + L^\mu{}_j P^\nu L^\lambda{}_i\right)\left(\xi^\dagger q^i\sigma^j\eta - \xi^\dagger q^j\sigma^i\eta\right).\end{aligned}$$

TMDFF calculation

$$D_{g\to J/\psi}(z,\mathbf{b}_T) = \sum_n d_{g\to Q\bar{Q}(n)}(z,\mathbf{b}_T) < 0|\mathcal{O}^{J/\psi}(n)|0 >$$

Perturbative NRQCD

$$\langle \chi^\dagger \sigma^j T^a \psi \, \mathcal{P}_{c\bar{c}',c\bar{c}} \, \psi^\dagger \sigma^i T^a \chi \rangle \;=\; 4m_c^2 \, \eta'^\dagger \sigma^j T^a \xi' \xi^\dagger \sigma^i T^a \eta$$

$$\langle \chi^\dagger \psi \, \mathcal{P}_{c\bar{c}',c\bar{c}} \, \psi^\dagger \chi \rangle \;=\; 4m_c^2 \, \eta'^\dagger \xi' \xi^\dagger \eta,$$

$$\langle \chi^\dagger T^a \psi \, \mathcal{P}_{c\bar{c}',c\bar{c}} \, \psi^\dagger T^a \chi \rangle \;=\; 4m_c^2 \, \eta'^\dagger T^a \xi' \xi^\dagger T^a \eta,$$

$$\langle \chi^\dagger (-\tfrac{i}{2}\overleftrightarrow{D}^m) T^a \psi \, \mathcal{P}_{c\bar{c}',c\bar{c}} \, \psi^\dagger (-\tfrac{i}{2}\overleftrightarrow{D}^n) T^a \chi \rangle \;=\; 4m_c^2 \, q'^m q^n \, \eta'^\dagger T^a \xi' \xi^\dagger T^a \eta,$$

$$\langle \chi^\dagger (-\tfrac{i}{2}\overleftrightarrow{D}^m) \sigma^i T^a \psi \, \mathcal{P}_{c\bar{c}',c\bar{c}} \, \psi^\dagger (-\tfrac{i}{2}\overleftrightarrow{D}^n) \sigma^j T^a \chi \rangle \;=\; 4m_c^2 \, q'^m q^n \, \eta'^\dagger \sigma^i T^a \xi' \xi^\dagger \sigma^j T^a \eta$$

TMDFF calculation

$$D_{g\to J/\psi}(z,\mathbf{b}_T) = \sum_n d_{g\to Q\bar{Q}(n)}(z,\mathbf{b}_T) < 0|\mathcal{O}^{J/\psi}(n)|0 >$$

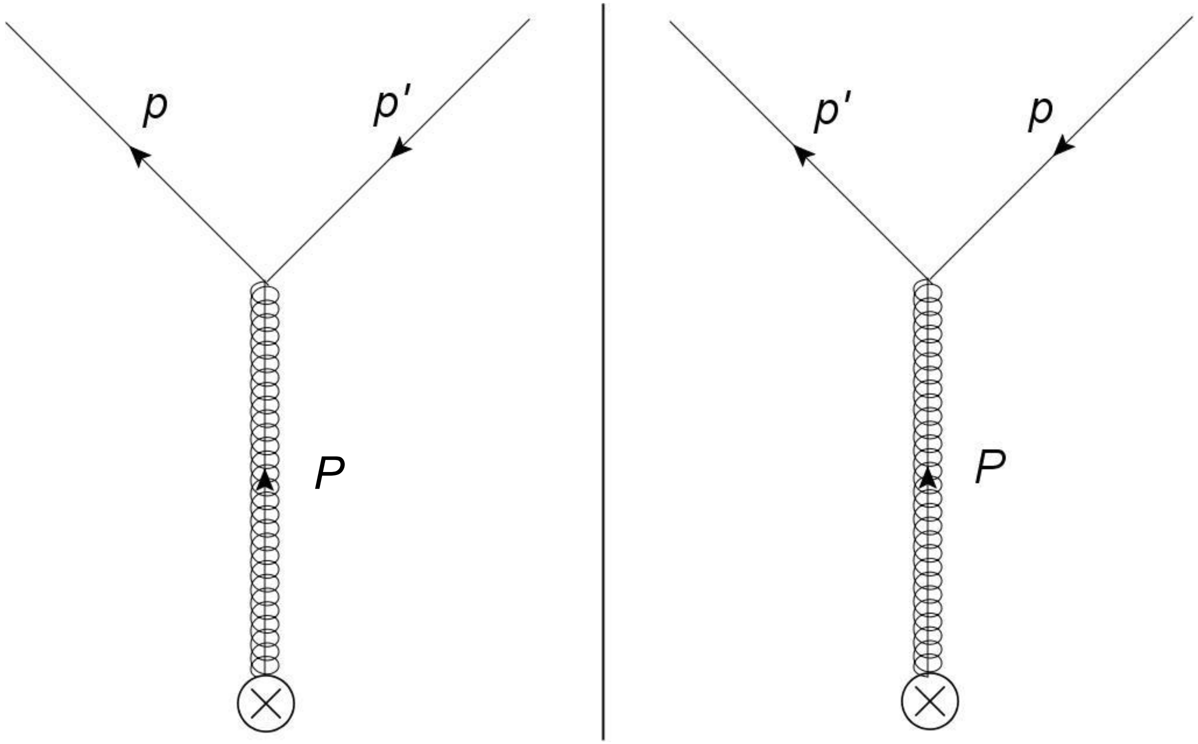
- QCD calculation at next to leading order in α_s
- Expansion around the threshold ($q = 0$)

Matching

perturbative NRQCD

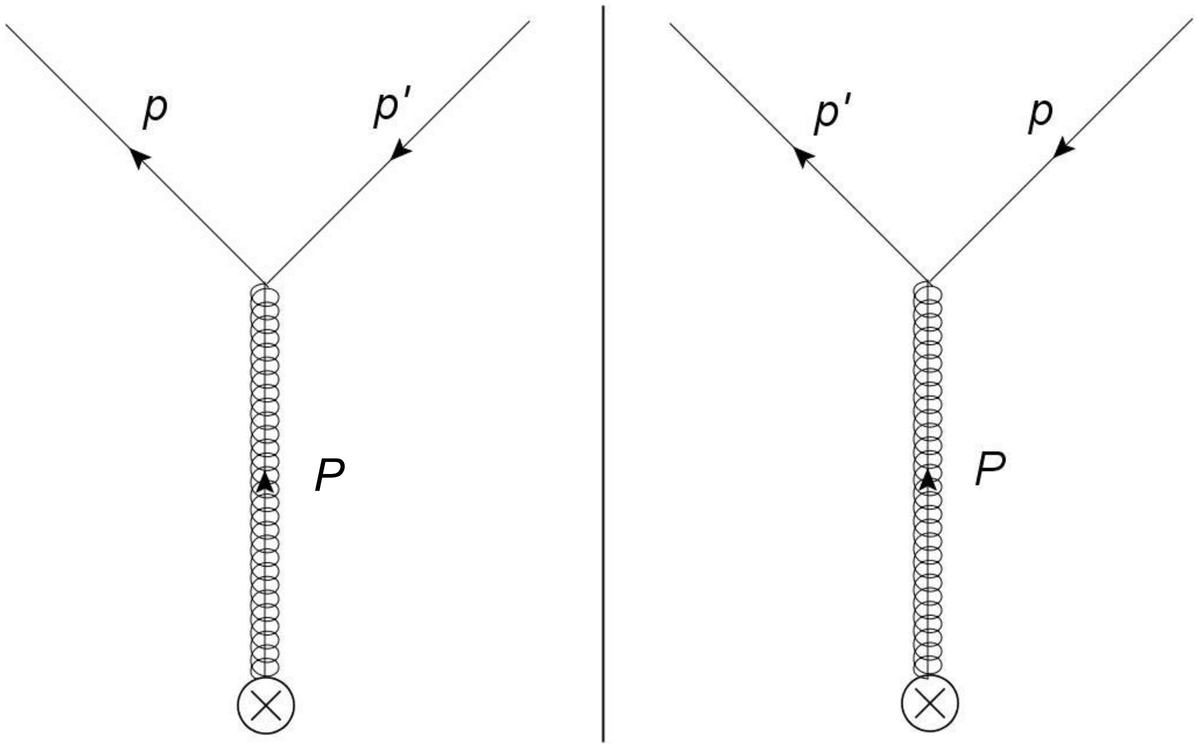
Feynman Diagrams

Leading order

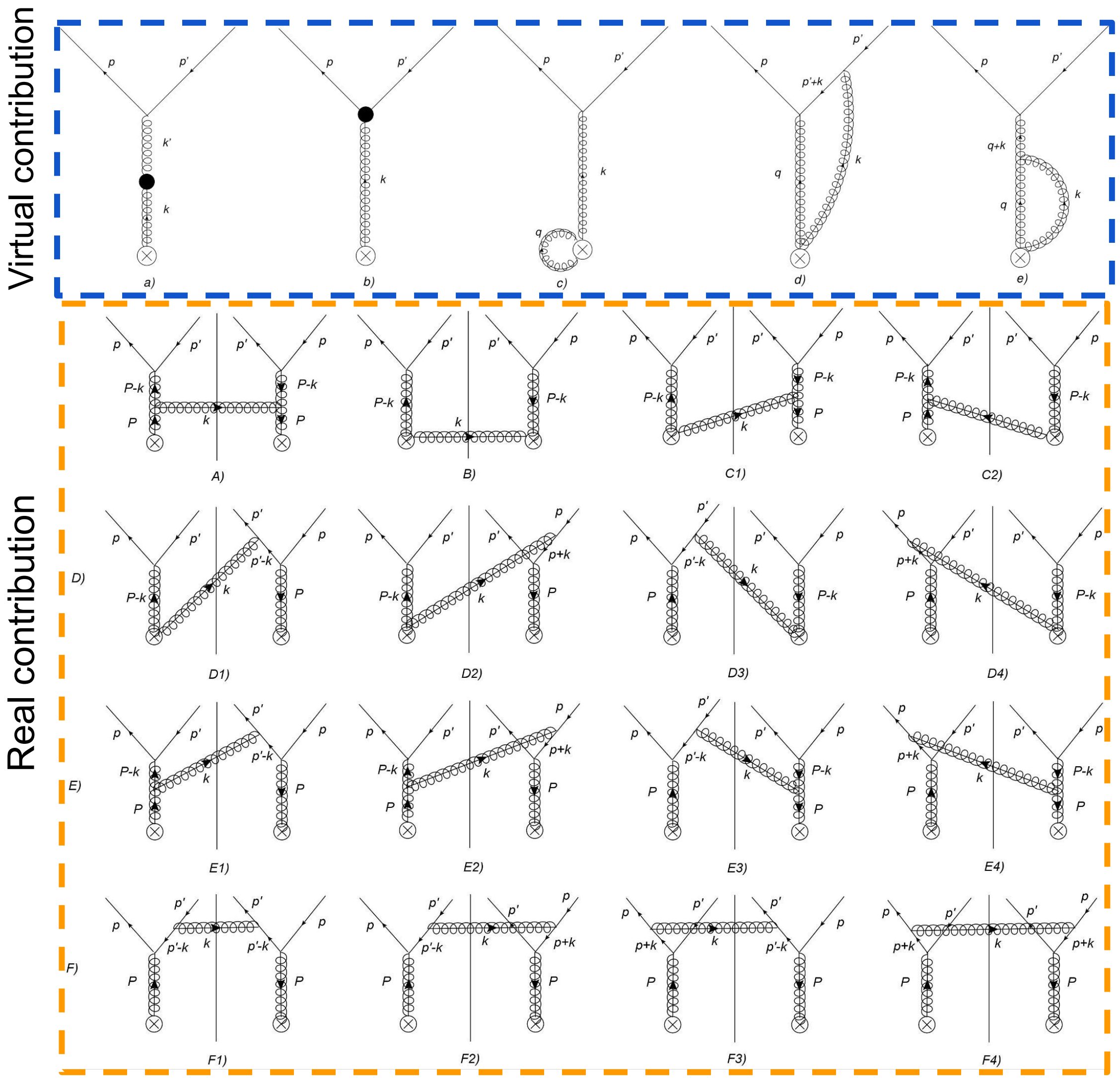


Feynman Diagrams

Leading order



Next to leading order



Some results

Virtual SDC

$$d^{a,b,c,d,e}(z,\mathbf{b}_\perp) = d^{LO}(z,\mathbf{b}_\perp)\frac{\alpha_s C_A}{2\pi}\left[\frac{1}{\epsilon_{UV}}\left(\frac{\beta_0}{2C_A} + \ln\frac{\delta^{+2}}{P^{+2}}\right) - \frac{1}{\epsilon_{IR}}\right.$$
$$+2\ln^2\frac{\delta^+}{P^+} + 2\ln\frac{\delta^+}{P^+}\ln\frac{\mu^2}{4m_c^2} + \ln\frac{\mu^2}{m_c^2}$$
$$\left. -4\ln^2 2 + \frac{10}{3}\ln 2 + 2 - \pi^2 + \frac{123-10n_f}{27}\right]$$

Virtual SF

$$S_v = \frac{\alpha_s C_A}{2\pi}\left[\frac{-2}{\epsilon_{UV}^2} + \frac{2}{\epsilon_{UV}}\ln\frac{\delta^{+2}\zeta}{(P^+)^2\mu^2} - \ln^2\frac{(\delta^+)^2}{\mu^2} - \frac{\pi^2}{2}\right]$$

arXiv:1502.05354 [hep-ph]

$$\frac{R_g d^{a,b,c,d,e}(z,\mathbf{b}_\perp)}{d^{LO}(z,\mathbf{b}_\perp)} = \frac{\alpha_s C_A}{2\pi}\left[\frac{1}{\epsilon_{UV}^2} + \frac{1}{\epsilon_{UV}}\left(\frac{\beta_0}{2C_A} + \ln\frac{\mu^2}{\zeta}\right) - \frac{1}{\epsilon_{IR}}\right.$$
$$+\frac{1}{2}\ln^2\frac{\delta^{+2}}{\mu^2} + 2\ln^2\frac{\delta^+}{P^+} + 2\ln\frac{\delta^+}{P^+}\ln\frac{\mu^2}{4m_c^2} + \ln\frac{\mu^2}{m_c^2}$$
$$\left. -\frac{3\pi^2}{4} - 4\ln^2 2 + \frac{10}{3}\ln 2 + 2 + \frac{123-10n_f}{27}\right]$$

Real contribution!

Current state of the research

Two papers in progress…

Gluon TMDFF for J/ψ production

In collaboration with Ignazio Scimemi (Complutense University of Madrid - IPARCOS) and Miguel G. Echevarría (EHU)

J/psi TMD shape function

In collaboration with Pieter Taels (University of Antwerp) and Miguel G. Echevarría (EHU)

Papers finished before August!

…or so I hope