Dynamical Chiral Symmetry Breaking and Hadrons

Institute of Physics and Mathematics
University of Michoacán, Morelia, Michoacán, Mexico

Fulbright Visiting Scientist
Jefferson Laboratory, Newport News, Virginia, USA

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Chirality

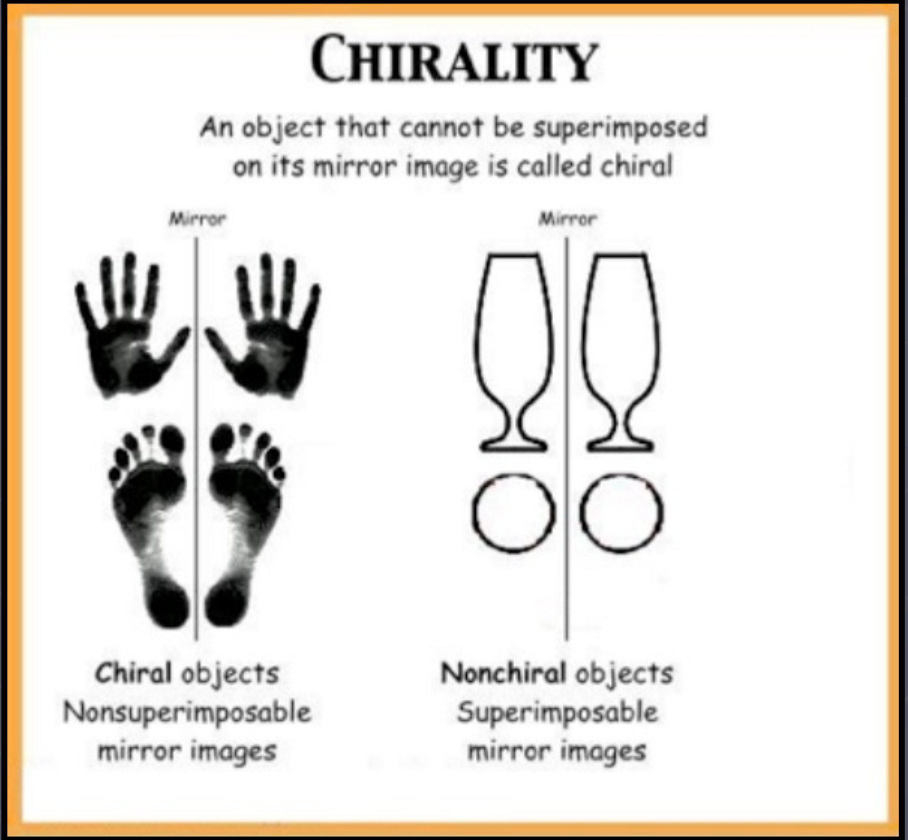
chirality [from Greek *kheir* hand + -AL[1] + -ITY]

The characteristic of a structure (usually a molecule) that makes it impossible to superimpose it on its mirror image. Also called *handedness*.

It was **Louis Pasteur**, the French scientist who had discovered **chirality** in the spin of molecules in **1848**.

"Any man who, upon looking down at his bare feet, doesn't laugh, has either no sense of symmetry or no sense of humour"

(Descartes, cf. Walker 1979)



QCD and chiral symmetry

Chiral symmetry of QCD and the manner in which it is broken has far reaching consequences for hadron spectroscopy and understanding their internal dynamics.

This is particularly important for the light quarks. So we can start from the fermionic part of QCD Lagrangian for up and down quarks alone.

$$\mathcal{L} = \bar{u}\, i \not{D} u + \bar{d}\, i \not{D} d - m_u\, \bar{u}u - m_d\, \bar{d}d$$

with the covariant derivative defined as:

$$D_\mu \psi(x) = \left(\partial_\mu + ig \frac{\tau^a A^a_\mu(x)}{2} \right) \psi(x)$$

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

Left and right spinors

We assume that m_u and m_d are so small that they can be neglected. What are the implications of this assumption?

$$\mathcal{L} = \bar{u}\, i \not{D} u + \bar{d}\, i \not{D} d$$

Let us define the left and right spinors:

$$\psi_L = \frac{1}{2}(1-\gamma_5)\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$\psi_R = \frac{1}{2}(1+\gamma_5)\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

Left and right sector of the Lagrangian is then separated.

$$\mathcal{L} = \bar{\psi}_L i \not{D} \psi_L + \bar{\psi}_R i \not{D} \psi_R \qquad \psi = \psi_L + \psi_R$$

Chiral transformations $U(1)_R \times U(1)_L$

The Lagrangian is invariant under the following $U(1)_R \times U(1)_L$ global chiral transformations:

$$\psi_L(x) \to e^{-i\theta_L}\psi_L(x) \quad \psi_R(x) \to e^{-i\theta_R}\psi_R(x)$$

Conserved currents:

$$\begin{aligned} J_L^\mu &= \bar{\psi}_L\gamma^\mu\psi_L \quad \text{with} \quad \partial_\mu J_L^\mu = 0\,, \\ J_R^\mu &= \bar{\psi}_R\gamma^\mu\psi_R \quad \text{with} \quad \partial_\mu J_R^\mu = 0\,. \end{aligned}$$

Linear combinations:

$$\begin{aligned} V^\mu &= J_R^\mu + J_L^\mu\,, \qquad V^\mu = \bar{\psi}\gamma^\mu\psi \\ A^\mu &= J_R^\mu - J_L^\mu\,, \qquad A^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi \end{aligned}$$

$$U(1)_V \times U(1)_A:\ \psi(x) \to e^{-i\theta_V}\psi(x),\ \ \psi(x) \to e^{-i\theta_A\gamma_5}\psi(x)$$

Chiral/Axial symmetry breaking

Introduce isospin invariant mass term:

$$\mathcal{L}_1 = -m\bar{\psi}\psi$$

$$m\bar{\psi}\psi = m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$$

Under $U(1)_V$ X $U(1)_A$ the addition of mass term yields:

$$\mathcal{L}_1 = -m\bar{\psi}\psi \begin{array}{l} \stackrel{V}{\longmapsto} \quad -m\bar{\psi}'\psi' = -m\bar{\psi}\psi \ , \\ \stackrel{A}{\longmapsto} \quad -m\bar{\psi}'\psi' = -m\bar{\psi}\psi - 2im\theta_A(\bar{\psi}\gamma_5\psi) \end{array}$$

Thus:

$$\begin{array}{l} \partial_\mu V^\mu = 0 \stackrel{m\neq 0}{\longmapsto} \partial_\mu V^\mu = 0 \\ \partial_\mu A^\mu = 0 \stackrel{m\neq 0}{\longmapsto} \partial_\mu A^\mu = 2m(\bar{\psi}i\gamma_5\psi) \end{array}$$

implying:

$$U(1)_V \times U(1)_A \stackrel{m\neq 0}{\longmapsto} U(1)_V$$

$SU(2)_R \times SU(2)_L$

Let us start from the massless Lagrangian again:

$$\mathcal{L} = \bar{\psi}_L i \not{D} \psi_L + \bar{\psi}_R i \not{D} \psi_R \qquad \psi = \psi_L + \psi_R$$

Consider the chiral transformations: $SU(2)_R \times SU(2)_L$

$$\begin{aligned} \psi_L &\rightarrow \psi_L' = e^{-i\frac{\vec{\tau}}{2}\cdot\vec{\theta}_L}\psi_L \\ \psi_R &\rightarrow \psi_R' = e^{-i\frac{\vec{\tau}}{2}\cdot\vec{\theta}_R}\psi_R \end{aligned}$$

τ are the Pauli matrices.

Lagrangian remains invariant. Thus the Lagrangian is chirally symmetric with conserved chiral currents:

$$J^{i\mu}_{L(R)} = \bar{\psi}_{L(R)}\gamma^\mu \frac{\tau_i}{2}\psi_{L(R)}$$

$SU(2)_V \times SU(2)_A$

We can make linear combinations again:

Vector and axial vector currents:

$$J_V^{i\mu} = J_R^{i\mu} + J_L^{i\mu} \qquad (i = 1, 2, 3)$$
$$J_A^{i\mu} = J_R^{i\mu} - J_L^{i\mu} ,$$

Corresponding vector and axial vector transformations are:

$$\Lambda_V : \ \psi \longrightarrow e^{-i\frac{\vec{\tau}}{2}\cdot\vec{\Theta}}\psi \simeq (1 - i\frac{\vec{\tau}}{2}\cdot\vec{\Theta})\psi$$

$$\Lambda_A : \ \psi \longrightarrow e^{-i\gamma_5\frac{\vec{\tau}}{2}\cdot\vec{\Theta}}\psi \simeq (1 - i\gamma_5\frac{\vec{\tau}}{2}\cdot\vec{\Theta})\psi$$

The mass term again breaks chiral or axial symmetry.

Explicit chiral symmetry breaking

Introduce isospin invariant mass term:

$$\delta\mathcal{L} = -m\left(\bar{\psi}\psi\right)$$

Lagrangian is still invariant under vector transformations. Under axial transformations:

$$\Lambda_A : \; m\left(\bar{\psi}\psi\right) \longrightarrow m\bar{\psi}\psi - 2im\vec{\Theta}\cdot\left(\bar{\psi}\frac{\vec{\tau}}{2}\gamma_5\psi\right)$$

It is because the mass term mixes the chiral partners:

$$m\bar{\psi}\psi = m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$$

As long as masses are small as compared to a relevant mass scale, the symmetry is almost (partially) conserved. u and d masses are 5-10 MeV which is much smaller than Λ_{QCD}.

Dynamical chiral symmetry breaking

Nambu and Jona-Lasinio proposed in 1960 that chiral symmetry of massless QCD is broken dynamically.

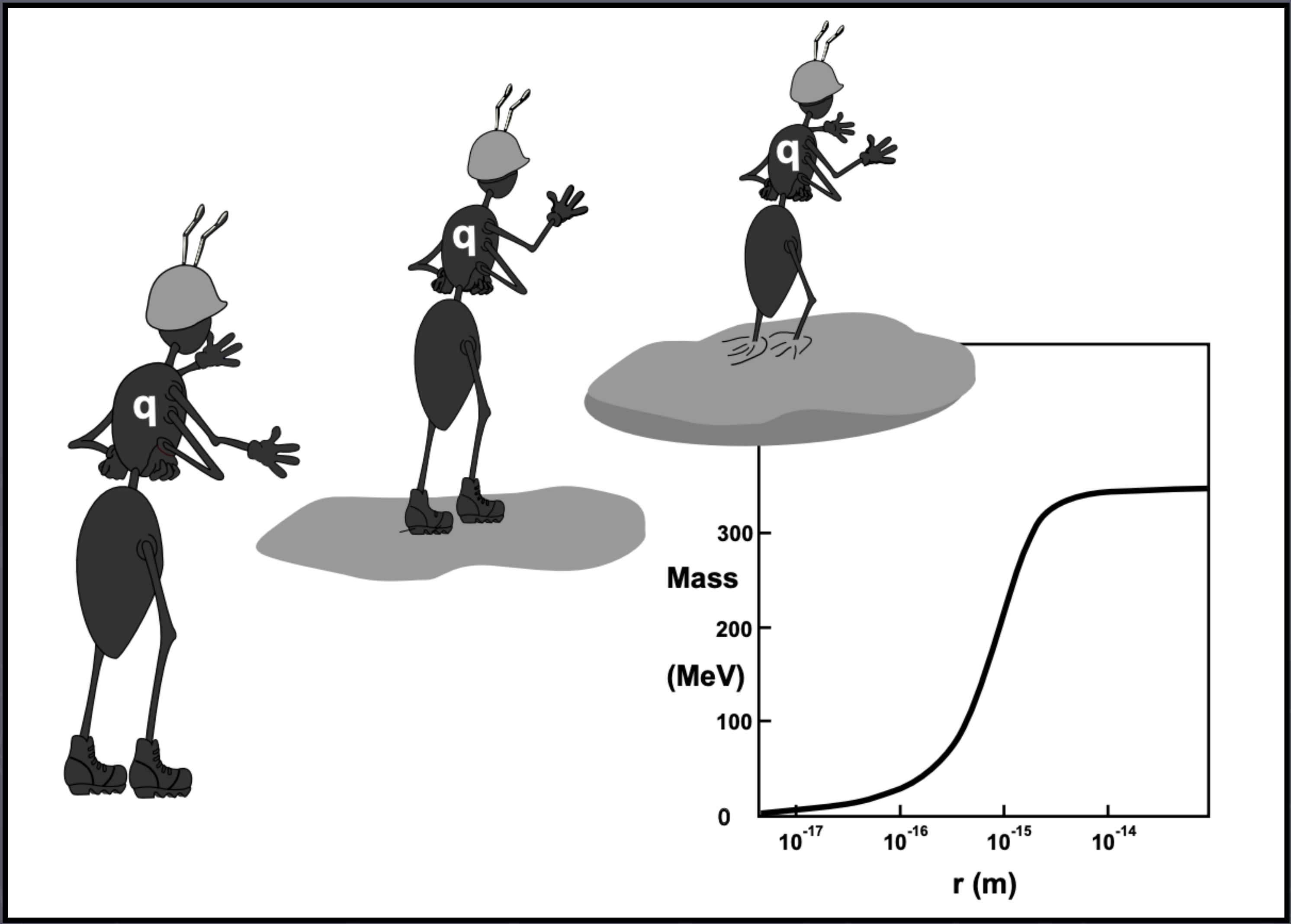
Though the massless Lagrangian remains invariant, vacuum is not due to strong QCD interactions.

$$\langle 0|\bar{\psi}\psi|0\rangle = \langle 0|\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L|0\rangle \neq 0$$

Hence the vacuum mixes the light quark chiralities. This allows "u" and "d" quarks to acquire masses when they travel through the QCD vacuum, i.e., inside quark-anti- quark bound states.

Thus inside the bound states, the "u" and "d" quarks would acquire a large dressed mass even when they have zero mass in the Lagrangian.

Dynamical chiral symmetry breaking



Dynamical chiral symmetry breaking

A typical meson like a ρ has a mass of 770 MeV while the nucleon has a mass of 940 MeV. This is consistent with a constituent u,d, mass of around 300 MeV.

However, pions only weigh about 140 MeV, which is about 1/5th of the mass of the ρ.

Dynamical breakdown of chiral symmetry generated by

$$j^{\mu 5a} = \bar{Q}\gamma^{\mu}\gamma^{5}\tau^{a}Q$$

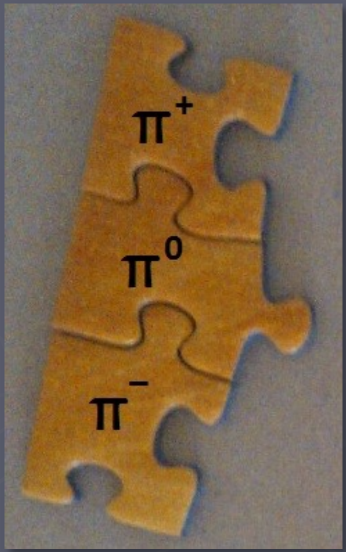
in fact gives rise to three massless Goldstone bosons, pions.

Pions are the lightest of hadrons. They do not have zero mass due to explicit chiral symmetry breaking.

Pion – Nambu-Goldstone boson of DCSB

The three broken generators give rise to three pions observed in nature.

$$
\begin{aligned}
|\, T=1, T_3=1\rangle &= -\,|\, u\bar{d}\rangle \\
|\, T=1, T_3=0\rangle &= \frac{1}{\sqrt{2}}\,|\, u\bar{u}-d\bar{d}\rangle \\
|\, T=1, T_3=-1\rangle &= |\, d\bar{u}\rangle .
\end{aligned}
$$



They are created by axial isospin current whose relevant matrix element can then be parameterized as:

$$\langle 0|j_5^{\mu a}(x)|\pi^b(p)\rangle = i f_\pi p^\mu \delta^{ab}\, \mathrm{e}^{-ip\cdot x}$$

where f_π=93 MeV is the pion leptonic decay constant.

Pion – Nambu-Goldstone boson of DCSB

For finite current quark masses, we have the Gell-Mann-Oakes-Renner formula (1968):

$$m_\pi^2 = (m_u + m_d)\,\frac{\langle 0|\overline{u}u + \overline{d}d|0\rangle}{2 f_\pi^2}$$

Thus there are two sources of chiral symmetry breaking, explicit and dynamical. In the absence of explicit chiral symmetry breaking, the pion mass is strictly zero as it should be, for it is a Goldstone boson.

The relation $m_\pi^2 \propto m$ is impossible in quantum mechanics for which one always finds:

$$m_{bound-state} \propto m_{constituent}$$

Chiral transformations and parity partners

How π, ϱ, and several other mesons transform under chiral transformations?

Let us consider combinations of quark fields, which carry the quantum numbers of the mesons under consideration:

$$\text{pion-like state} \quad (0^-) : \vec{\pi} \equiv i\bar{\psi}\vec{\tau}\gamma_5\psi$$

$$\text{rho-like state} \quad (1^-) : \vec{\rho}_\mu \equiv \bar{\psi}\vec{\tau}\gamma_\mu\psi$$

The vector sign indicates the iso-vector nature of the particle. The μ index is the Lorentz index (vector particle).

$$\text{sigma-like state } (0^+): \quad \sigma \equiv \bar{\psi}\psi$$

$$a_1\text{-like state } (1^+) \quad : \quad \vec{a_1}_\mu \equiv \bar{\psi}\vec{\tau}\gamma_\mu\gamma_5\psi$$

SU(2)$_V$ transformations

See how a pion transforms under SU(2)$_V$ transformations.

$$\begin{aligned}
\pi_i = i\bar\psi\tau_i\gamma_5\psi \quad &\Rightarrow \quad i\bar\psi\left(1 + i\frac{\vec\tau}{2}\cdot\vec\theta\right)\tau_i\gamma_5\left(1 - i\frac{\vec\tau}{2}\cdot\vec\theta\right)\psi \\
&\Rightarrow \quad i\bar\psi\tau_i\gamma_5\psi + \theta_j\left[\bar\psi\tau_i\gamma_5\frac{\tau_j}{2} - \bar\psi\frac{\tau_j}{2}\tau_i\gamma_5\right]\psi \\
&\Rightarrow \quad i\bar\psi\tau_i\gamma_5\psi + 2\theta_j\bar\psi\gamma_5\left[\frac{\tau_i}{2}\frac{\tau_j}{2} - \frac{\tau_j}{2}\frac{\tau_i}{2}\right]\psi \\
&\Rightarrow \quad i\bar\psi\tau_i\gamma_5\psi + 2\theta_j\bar\psi\gamma_5\left[i\epsilon_{ijk}\frac{\tau_k}{2}\right]\psi \\
&\Rightarrow \quad i\bar\psi\tau_i\gamma_5\psi + \epsilon_{ijk}\theta_j\left[i\bar\psi\tau\gamma_5\psi\right]_k \\
\pi_i \quad &\Rightarrow \quad \pi_i + \epsilon_{ijk}\theta_j\left[i\bar\psi\tau\gamma_5\psi\right]_k \\
\vec\pi \quad &\Rightarrow \quad \vec\pi + \vec\theta\times\vec\pi
\end{aligned}$$

$SU(2)_V$ transformations

The pion:

$$\vec{\pi} \longrightarrow \vec{\pi} + \vec{\Theta} \times \vec{\pi}$$

This simply means that the isospin direction of the pion is rotated by an angle θ.

The rho:

$$\vec{\rho_\mu} \longrightarrow \vec{\rho_\mu} + \vec{\Theta} \times \vec{\rho_\mu}$$

Again it means that the isospin direction of the rho meson is rotated by an angle θ.

That is why these rotations are identified with isospin rotations and the conserved vector current with the isospin current.

Axial vector $SU(2)_A$ transformations

The fermions:

$$\psi \longrightarrow e^{-i\gamma_5 \frac{\vec{\tau}}{2}\cdot\vec{\Theta}}\psi \simeq \left(1 - i\gamma_5 \frac{\vec{\tau}}{2}\cdot\vec{\Theta}\right)\psi$$

$$\bar{\psi} \longrightarrow \bar{\psi}e^{-i\gamma_5 \frac{\vec{\tau}}{2}\cdot\vec{\Theta}} \simeq \bar{\psi}\left(1 - i\gamma_5 \frac{\vec{\tau}}{2}\cdot\vec{\Theta}\right)$$

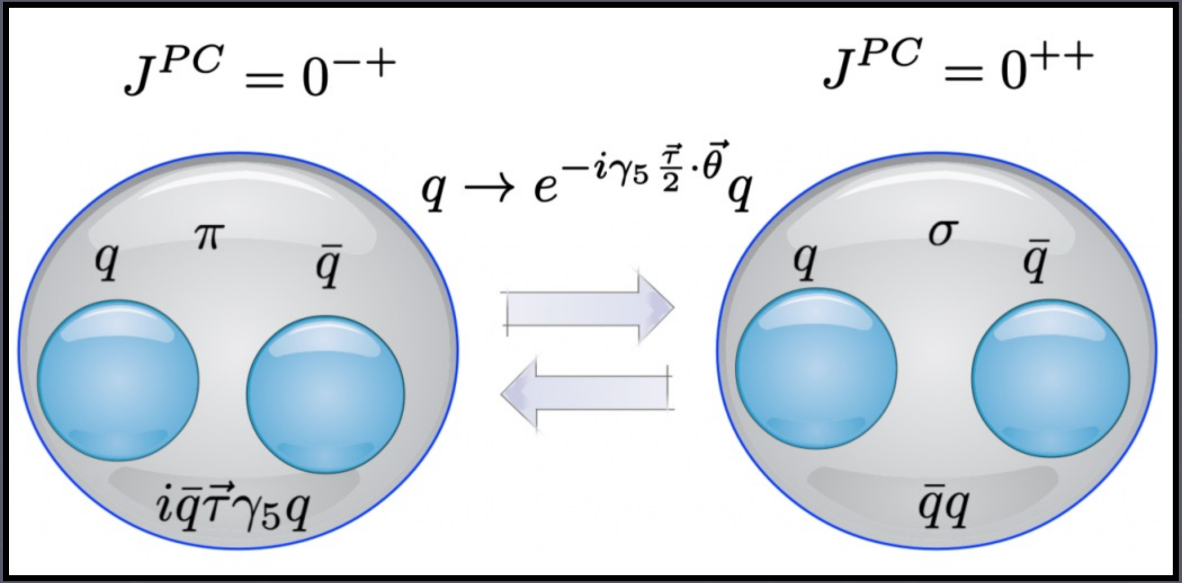
$$\pi_i\colon\ i\bar{\psi}\tau_i\gamma_5\psi \rightarrow i\bar{\psi}\tau_i\gamma_5\psi + \Theta_j\left(\bar{\psi}\tau_i\gamma_5\gamma_5\frac{\tau_j}{2}\psi + \bar{\psi}\gamma_5\frac{\tau_j}{2}\tau_i\gamma_5\psi\right)$$
$$= \ i\bar{\psi}\tau_i\gamma_5\psi + \Theta_i\bar{\psi}\psi$$

The mesons:

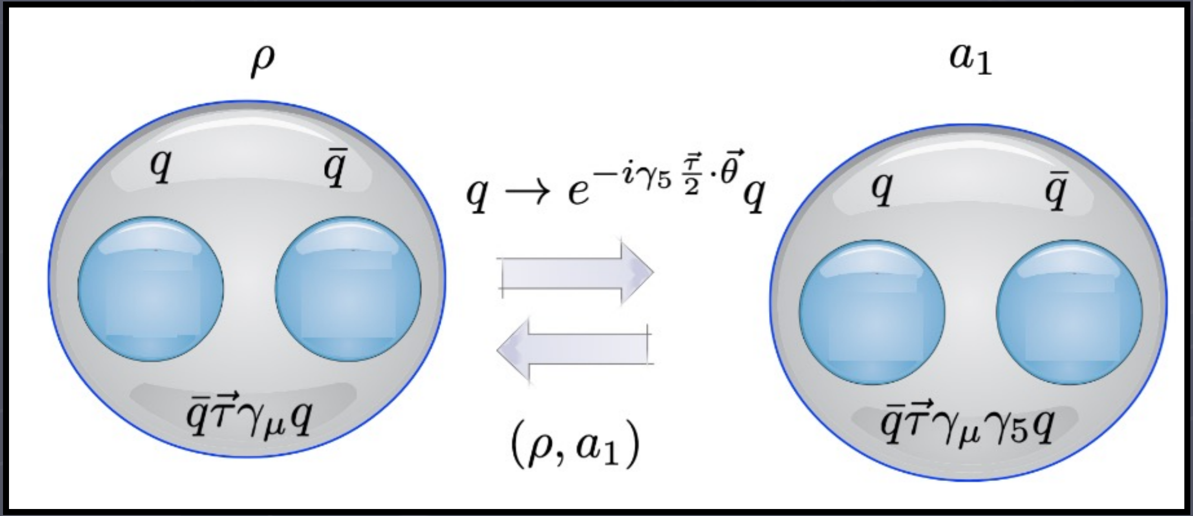
$$\vec{\pi} \longrightarrow \vec{\pi} + \vec{\Theta}\sigma \qquad \sigma \longrightarrow \sigma - \vec{\Theta}.\vec{\pi}$$

$$\vec{\rho}_\mu \longrightarrow \vec{\rho}_\mu + \vec{\Theta}\times\vec{a_1}_\mu$$

Axial vector $SU(2)_A$ transformations



The parity partners!

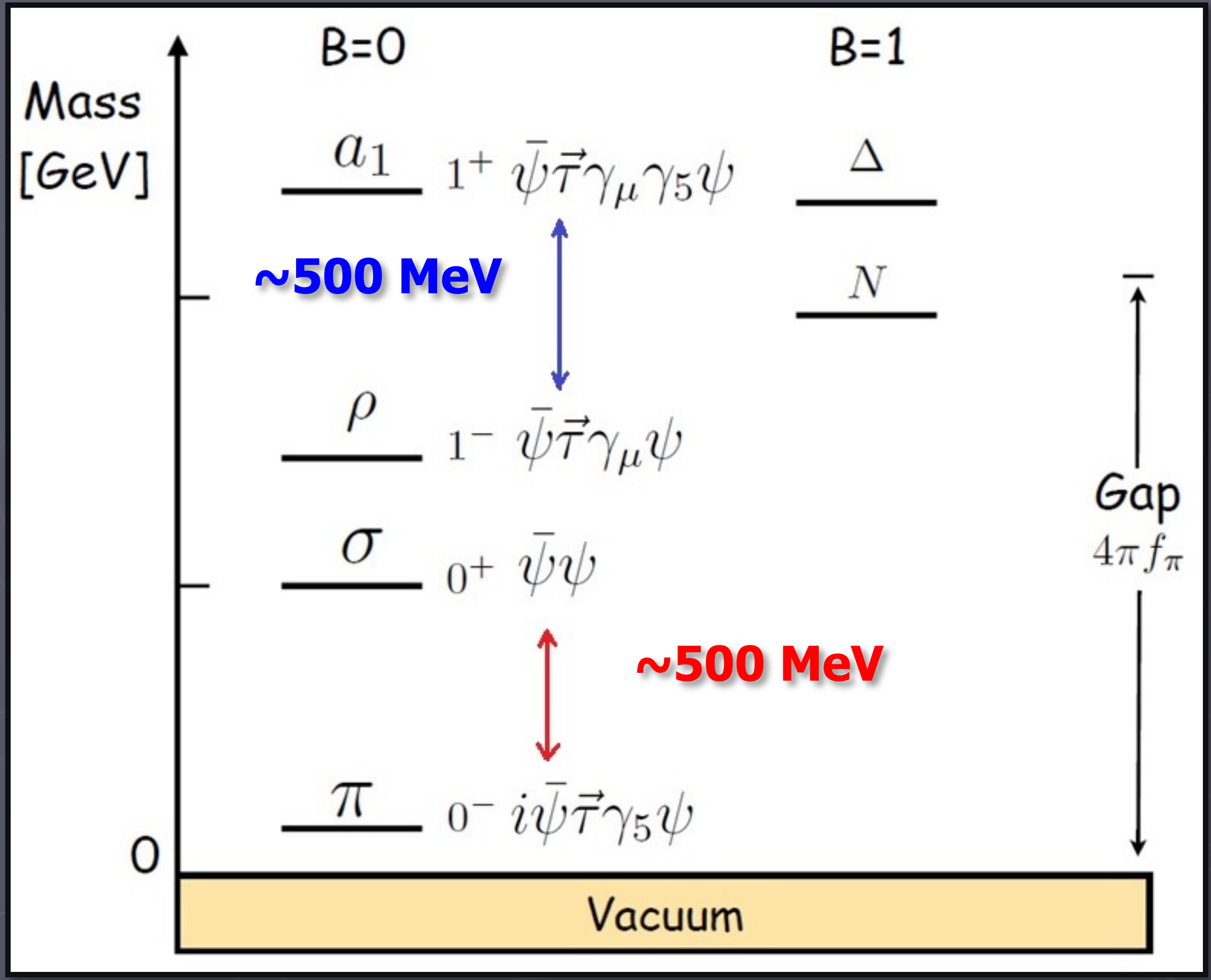


DCSB and meson spectrum

Parity Partners

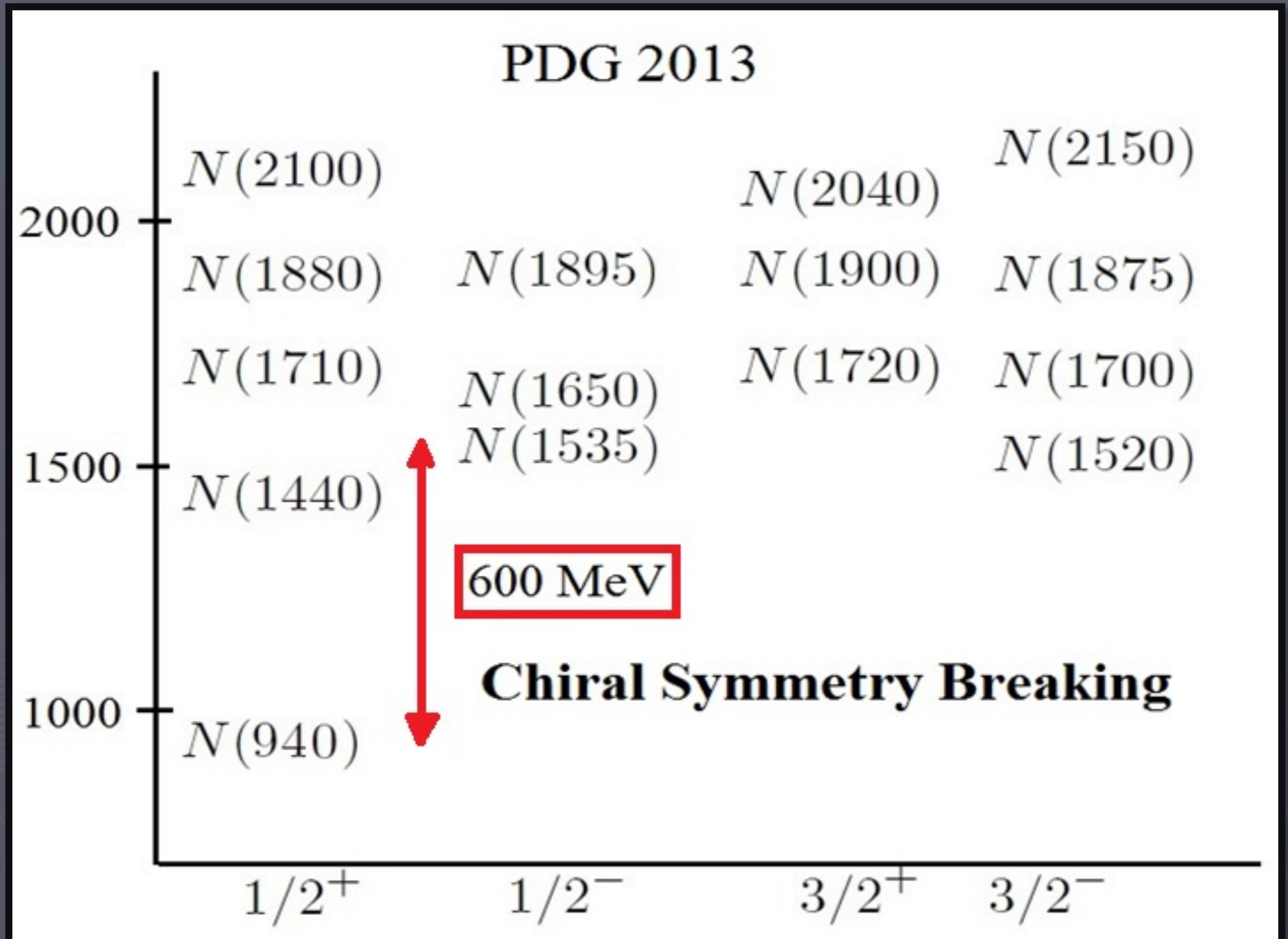
&

Dynamical Chiral Symmetry Breaking



DCSB and baryon spectrum

Nucleon
and its
Parity
Partner



DCSB and baryon spectrum

Particle J^P	overall	$N\gamma$	$N\pi$
$\Delta(1232)\ 3/2^+$	****	****	****
$\Delta(1600)\ 3/2^+$	***	***	***
$\Delta(1620)\ 1/2^-$	****	***	****
$\Delta(1700)\ 3/2^-$	****	****	****

500 MeV

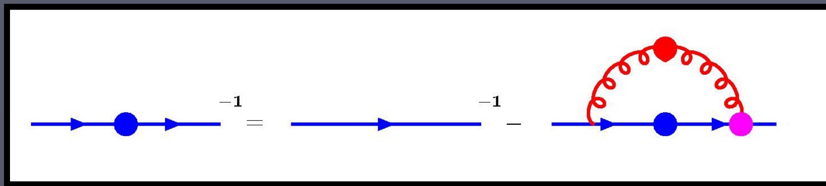
Chiral Symmetry Breaking

Dynamical Chiral symmetry breaking is the single most important phenomenon to dictate hadrons properties.

This can be studied through systematically improvable formalism of continuum QCD: Schwinger-Dyson equations.

A contact interaction model

The SDE for the quark propagator of flavor f:

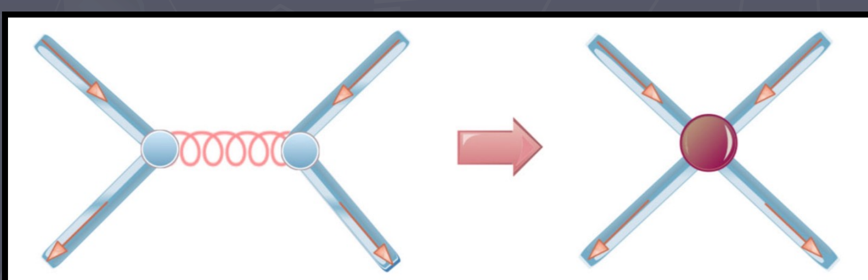


$$S(p)^{-1} = i\gamma \cdot p + m_f + \Sigma(p)$$

$$\Sigma(p) = \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q)\gamma_\mu S(q)\Gamma_\nu(q,p)$$

Landau gauge gluon propagator saturates in the infrared and a large effective mass scale is generated for the gluon.

A. Ayala, AB, D. Binosi, M. Cristoforetti, J. Rodriguez, Phys. Rev. D 86, 074512 (2012).



$$g^2 D_{\mu\nu}(k) = 4\pi\hat{\alpha}_{\rm IR}\delta_{\mu\nu}, \;\; \hat{\alpha}_{\rm IR} = \alpha_{\rm IR}/m_g^2,$$
$$m_g = 500 \text{ MeV}$$

$$\Gamma_\nu(q,p) = \gamma_\nu$$

L.X. Gutierrez, AB, I.C. Cloet, C.D. Roberts, Phys. Rev. C 81 (2010) 065202.

The gap equation

The general solution of the gap equation:

$$\Sigma(p) = \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q)\gamma_\mu S(q)\Gamma_\nu(q,p)$$

is:

$$S(q,M_f) \equiv -i\gamma \cdot q\sigma_V(q,M_f) + \sigma_S(q,M_f)$$

$$\sigma_V(q,M_f) = \frac{1}{q^2+M_f^2}, \qquad \sigma_S(q,M_f) = M_f\sigma_V(q,M_f)$$

where M_f is the dynamically generated dressed quark mass obtained by solving the equation:

$$M_f = m_f + M_f \frac{4\hat{\alpha}_{\rm IR}}{3\pi}\int_0^\infty ds\, s\, \frac{1}{s+M_f^2}$$

The interaction needs to be regularized for its divergence.

The solutions to the gap equation

In the chiral limit the gap equation becomes:

$$M_f = \frac{4 M_f \alpha_{\rm IR}^2}{3\pi m_g^2} \int_0^{\Lambda^2} ds \frac{s}{s + M_f^2}$$

The trivial solution is the Wigner mode solution M_f=0 which is realized in perturbation theory. We set α_{IR}=1.

We look for a non-perturbative solution and set α_{IR}=1.

$$1 = \frac{4}{3\pi m_g^2}\left[\Lambda^2 - M_f^2 \log\left(1 + \frac{\Lambda^2}{M_f^2}\right)\right]$$

Let us also set Λ=1 GeV, typical hadronic scale.

$$1 = \frac{4}{3\pi m_g^2}\left[1 - M_f^2 \log\left(1 + \frac{1}{M_f^2}\right)\right] \equiv \frac{4}{3\pi m_g^2}\mathcal{D}(M_f, 1)$$

The gap equation

The D[M,1] function has maximum value 1 at M=0.

It is a monotonically decreasing function of M.

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In[26]:= Df[x_] := 1 - x Log[1 + 1/x]

In[27]:=
      Plot[Df[x], {x, 0, 5}]
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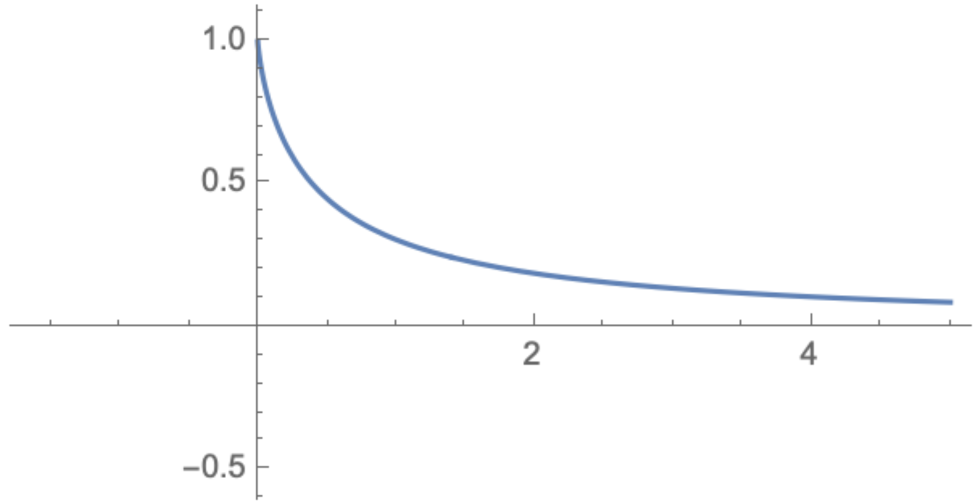
Out[27]=



The gap equation

Let's focus on:

$$1 = \frac{4}{3\pi m_g^2}\mathcal{D}(M_f, 1)$$

As D[M,1] has maximum at 1 a and is monotonically decreasing,

$$\frac{3\pi m_g^2}{4} \leq 1$$

We could reinstate Λ:

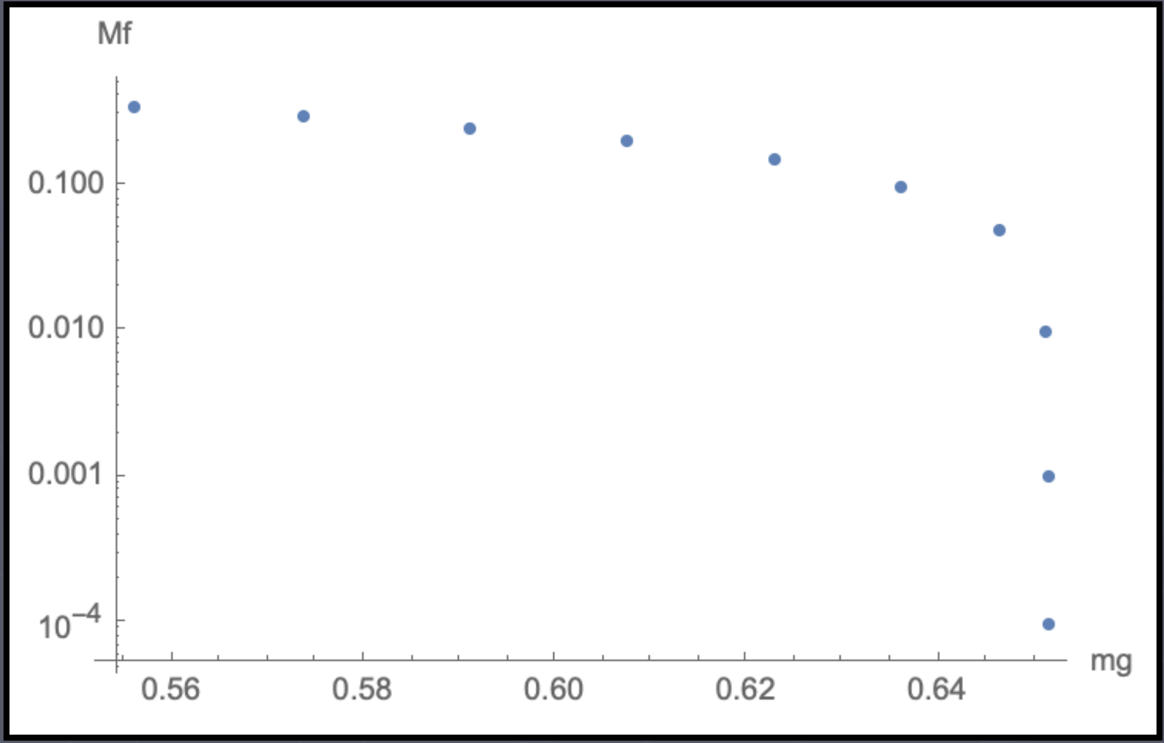
$$m_g^2 \leq \frac{4\Lambda^2}{3\pi}$$

There is a critical value of the effective coupling α_{IR}/m_g^2 or for m_g for fixed $\alpha_{IR}=1$.

$$m_g \leq \sqrt{\frac{4}{3\pi}} \simeq 650\text{MeV}$$

The non-perturbative solution of gap equation

The non-perturbative solution arises only above a critical value of the coupling like in QED:



Perturbation theory solution:

$$M(p^2) \quad = \quad m\left(1 - \frac{\alpha}{\pi}\,\mathrm{Log}\left[\frac{p^2}{m^2}\right] + \cdots\right)$$

The regularization

We adopt the proper time regularization scheme:

$$\frac{1}{s+M_f^2} = \int_0^\infty d\tau \mathrm{e}^{-\tau(s+M_f^2)} \to \int_{\tau_{\mathrm{UV}}^2}^{\tau_{\mathrm{IR}}^2} d\tau \mathrm{e}^{-\tau(s+M_f^2)}$$

$$= \frac{\mathrm{e}^{-(s+M_f^2)\tau_{\mathrm{UV}}^2} - e^{-(s+M_f^2)\tau_{\mathrm{IR}}^2}}{s+M_f^2}$$

τ_{IR}, τ_{UV} are infrared and ultraviolet regulators.

$\tau_{IR}=1/\Lambda_{IR}$ implements confinement by ensuring the absence of quark production thresholds.

As the model is not renormalizable, $\tau_{UV}=1/\Lambda_{UV}$ cannot be removed but instead takes a dynamical role.

The gap equation can now be easily solved.

Dynamical chiral symmetry breaking

The solution of this gap equation is:

$$M_f = m_f + M_f \frac{4\hat{\alpha}_{\rm IR}}{3\pi} \mathcal{C}(M_f^2)$$

$$\frac{\mathcal{C}(M^2)}{M^2} = \Gamma(-1, M^2\tau_{\rm UV}^2) - \Gamma(-1, M^2\tau_{\rm IR}^2)$$

$\Gamma(\alpha,x)$ is the incomplete gamma function.

We choose the parameters of the model:

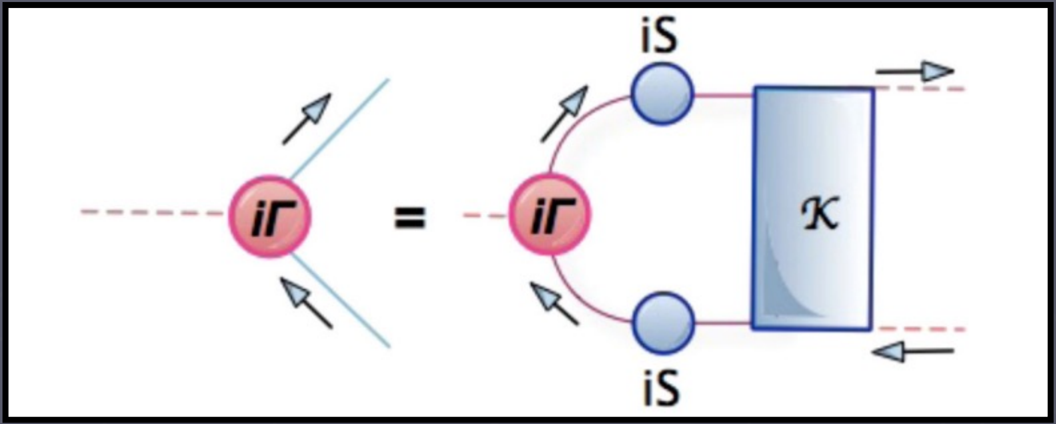
m_u [GeV]	$\Lambda_{\rm IR}$ [GeV]	$\Lambda_{\rm UV}$ [GeV]	$\alpha_{\rm IR}$
0.007	0.24	0.905	0.36π

which yield:

$$M_u = 0.367 \text{ GeV}$$

The Bethe-Salpeter equation

Bethe-Salpeter (BS) equation for a meson is:



$$[\Gamma(k;P)]_{tu} = \int \frac{d^4q}{(2\pi)^4}[\chi(q;P)]_{sr}\mathcal{K}^{rs}_{tu}(q,k;P)$$

$$\chi(q;P) = S(q+P)\Gamma S(q)$$

BS amplitude

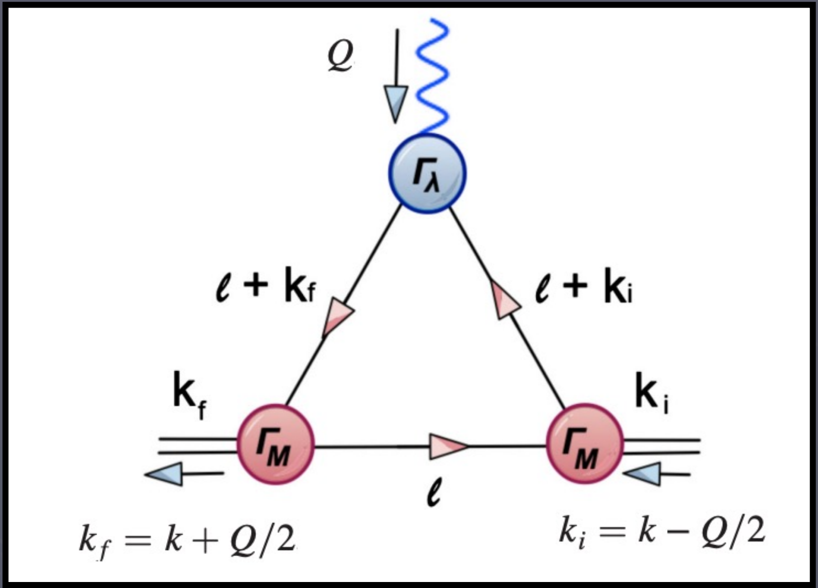
BS wave function

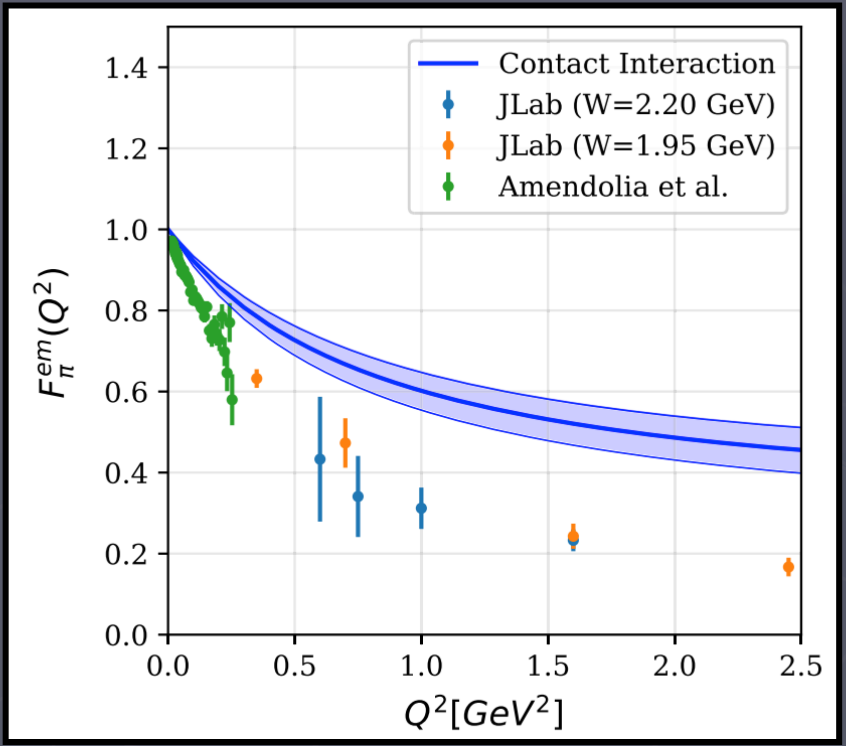
quark-antiquark scattering kernel

The indices r, s, t, u to color, flavor and spinor indices. P is the total meson momentum. For a PS meson in the CI:

$$\Gamma_{PS}(P) = i\gamma_5 E_{PS}(P) + \frac{1}{2M_R}\gamma_5\gamma\cdot P\, F_{PS}(P)$$

Electromagnetic PS Meson Form Factor





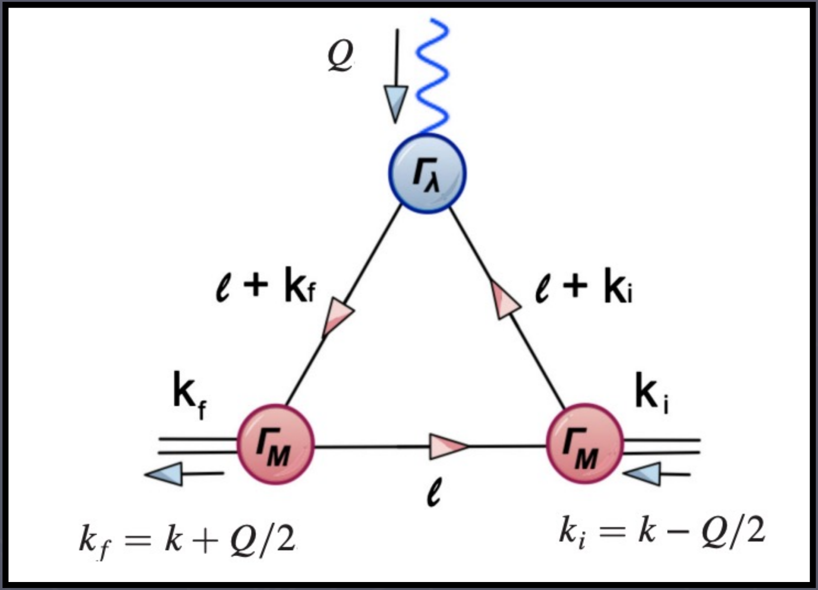
$$F^M(Q^2) = e_{f_1} F^{M,f_1}(Q^2) + e_{\overline{f_2}} F^{M,\overline{f_2}}(Q^2)$$

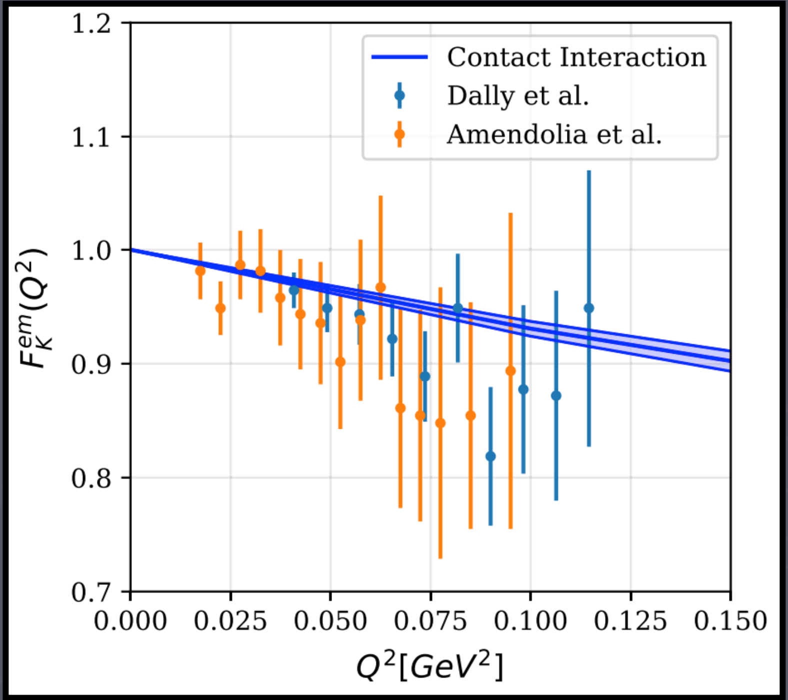
$$r_M^2 = -6 \frac{\mathrm{d}F_M(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q^2=0}$$

The triangle diagram for the impulse approximation to the $M\gamma M$ vertex

The band allows a 5% variation in the charge radius.

Electromagnetic PS Mesons Form Factor





$$F^M(Q^2) = e_{f_1} F^{M,f_1}(Q^2) + e_{\overline{f_2}} F^{M,\overline{f_2}}(Q^2)$$

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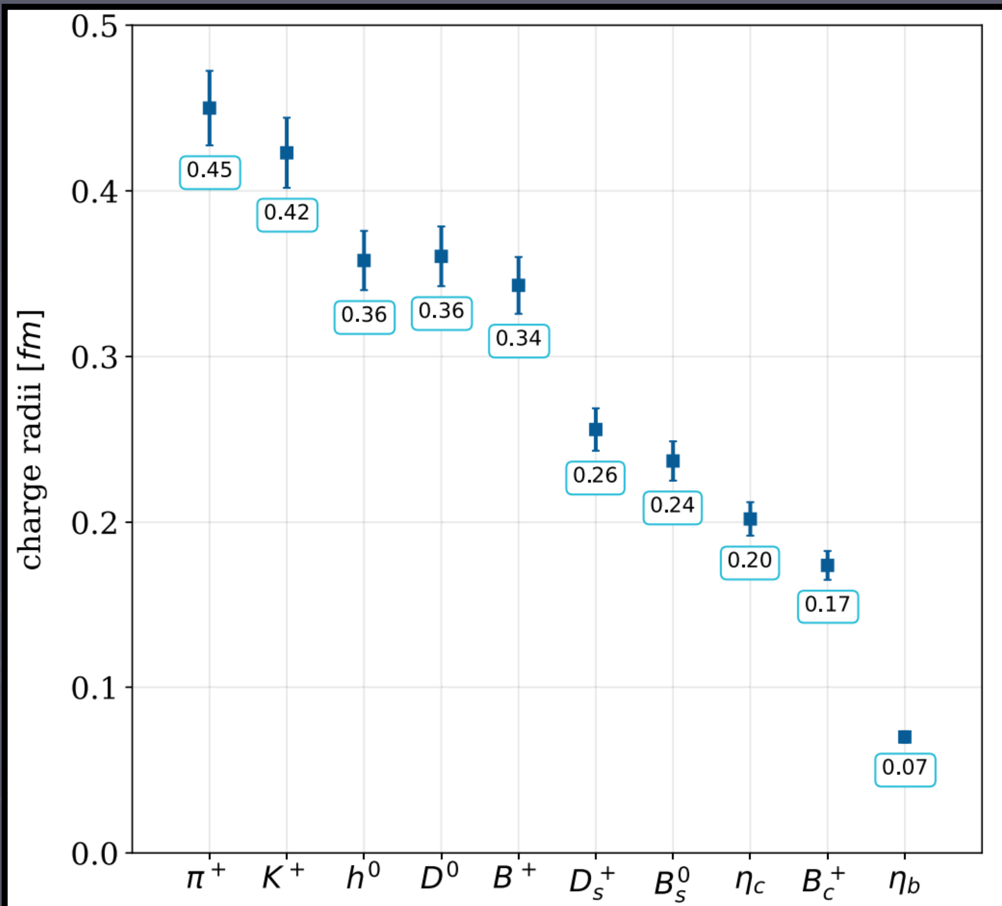
Electromagnetic PS Mesons Form Factor



	$u\bar{d}$	$u\bar{s}$
Our Result	0.45	0.42
SDE [4,23]	0.676 ± 0.002	0.593 ± 0.002
Lattice [7,8,62]	0.648 ± 0.141	0.566 (extracted)
Exp. [63]	0.659 ± 0.004	0.560 ± 0.031
r_M^t [61]	0.658	0.568
r_M^t(CI)	0.45	0.38
HM [16]	0.66	0.65
LFF [17]	0.66	0.58
PM [18]	...	...

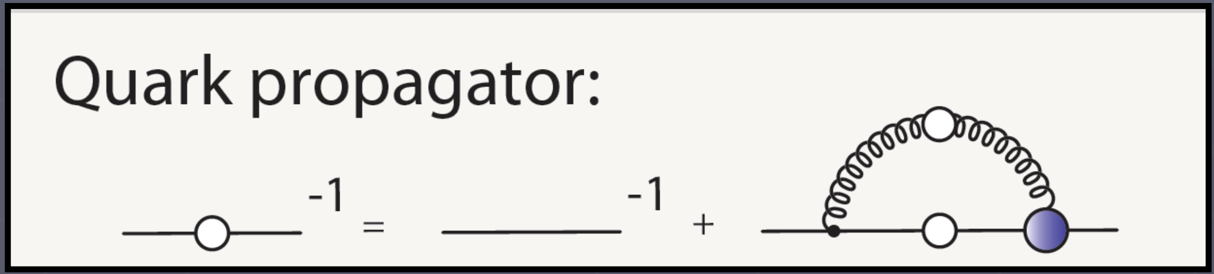
$$r_{u\bar{d}} > r_{u\bar{s}} > r_{c\bar{u}} > r_{u\bar{b}},$$
$$r_{u\bar{s}} > r_{s\bar{s}} > r_{c\bar{s}} > r_{s\bar{b}},$$
$$r_{c\bar{u}} > r_{c\bar{s}} > r_{c\bar{c}} > r_{c\bar{b}},$$
$$r_{u\bar{u}} > r_{s\bar{s}} > r_{c\bar{c}} > r_{b\bar{b}}.$$

The model is so simple that we can evaluate the **form factors** and **charge radii** of all **PS mesons**.

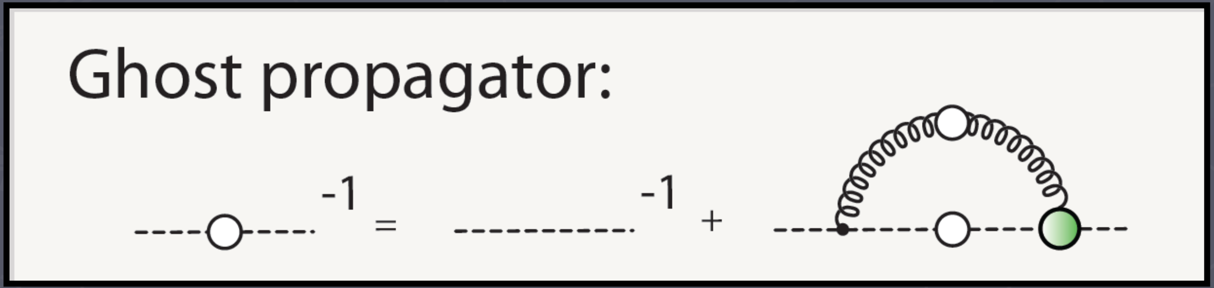
R. Hernández, L.X. Gutierrez, AB, M. Bedolla, H. Melany, Phys. Rev. D 107 (2023) 054002.

Schwinger-Dyson equations (QCD)

Schwinger-Dyson equations in covariant gauges:

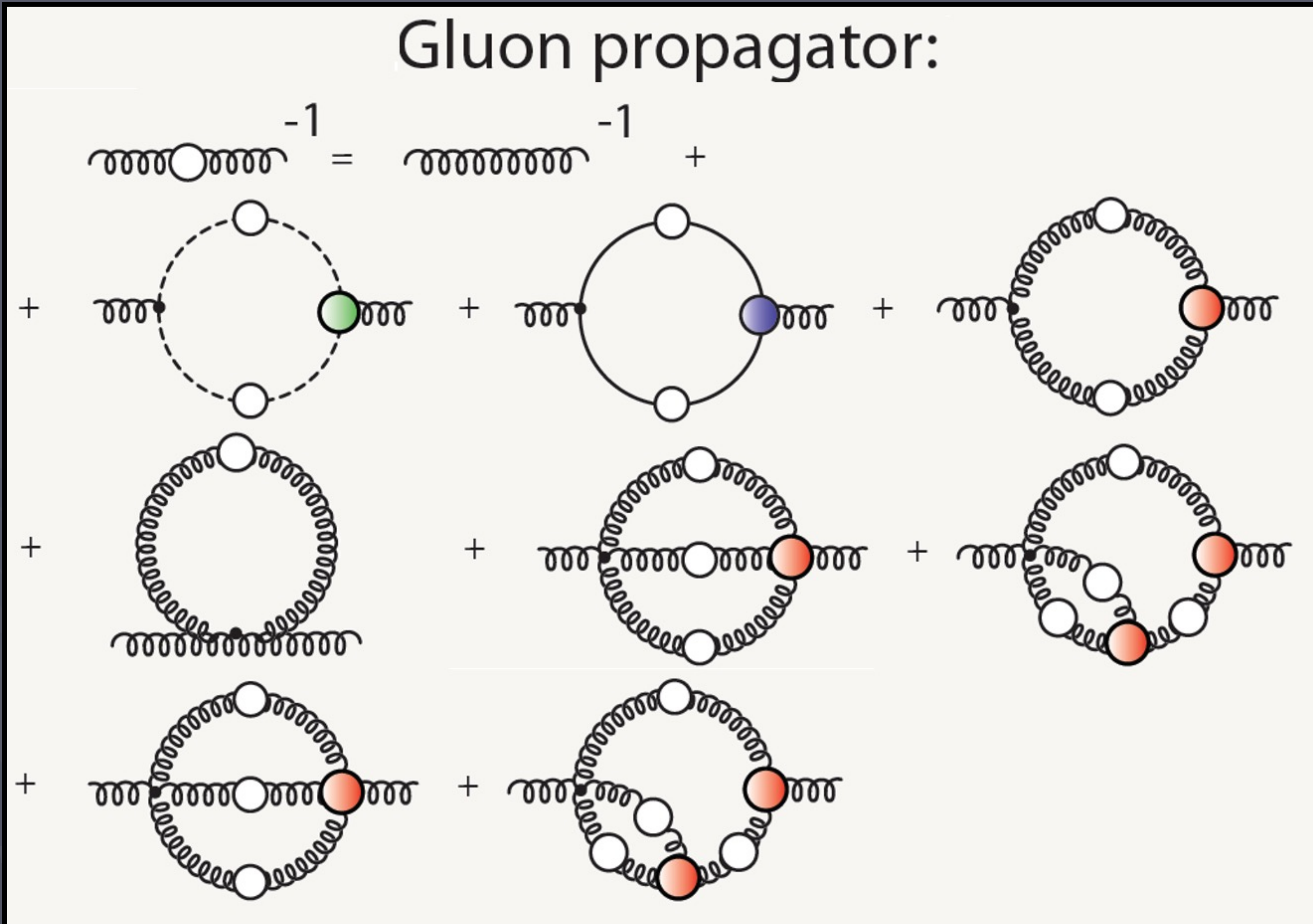


Solution of the quark propagator requires the knowledge of the gluon propagator and the quark-gluon vertex.



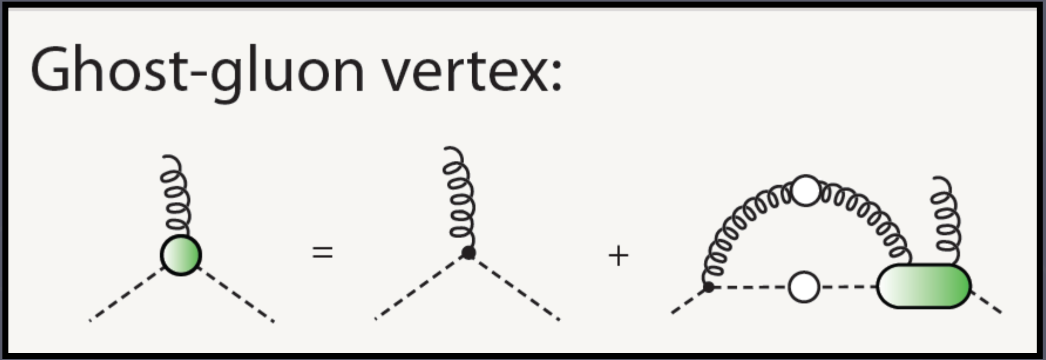
The ghost propagator is intertwined with the gluon propagator and the ghost-gluon vertex.

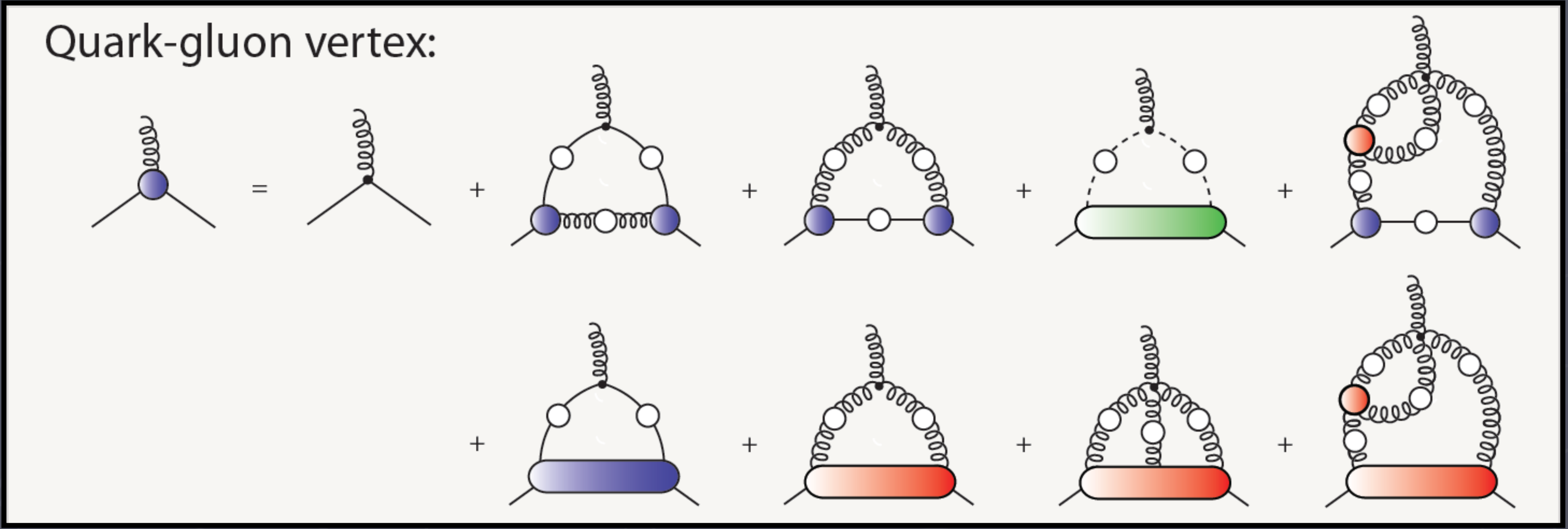
Schwinger-Dyson equations (QCD)



Schwinger-Dyson equations (QCD)

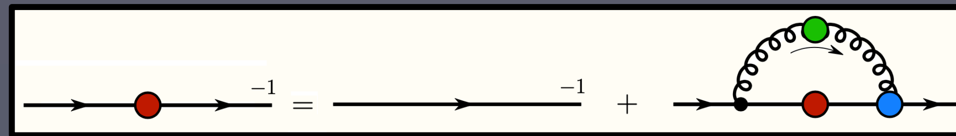
The 3-point vertices are connected to the 2-point and 3-point and 4-point vertices:





The Quark Propagator

The quark propagator:



$$S_B^{-1}(p,\Lambda) = S_0^{-1}(p) + \int d^4q\; g_B^2(\Lambda) D^B_{\mu\nu}(p-q,\Lambda)\frac{\lambda^a}{2}\gamma_\mu S_B(q;\Lambda)\Gamma^a_{B\nu}(q,p;\Lambda)$$

$$\begin{aligned} g_B(\Lambda) &= \mathcal{Z}_g g(p-q,\mu) \\ D^B_{\mu\nu}(p-q,\Lambda) &= \mathcal{Z}_3 D_{\mu\nu}(p-q,\mu) \\ S_B(q;\Lambda) &= \mathcal{Z}_{2F} S(p,\mu) \\ \Gamma^a_{B\nu}(q,p;\Lambda) &= \mathcal{Z}_{1F}^{-1}\Gamma(p,q,\mu) \end{aligned}$$

$$\frac{\mathcal{Z}_1}{\mathcal{Z}_3} = \frac{\tilde{\mathcal{Z}}_1}{\tilde{\mathcal{Z}}_3} = \frac{\mathcal{Z}_5}{\mathcal{Z}_1} = \frac{\mathcal{Z}_{1Fj}}{\mathcal{Z}_{2Fj}}$$

$$S^{-1}(p,\mu) = \mathcal{Z}_{2F} i\gamma\cdot p + \mathcal{Z}_4 m(\mu) + \mathcal{Z}_{1F}\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q,\mu)\frac{\lambda^a}{2}\gamma_\mu S(p,\mu)\Gamma(p,q,\mu)$$

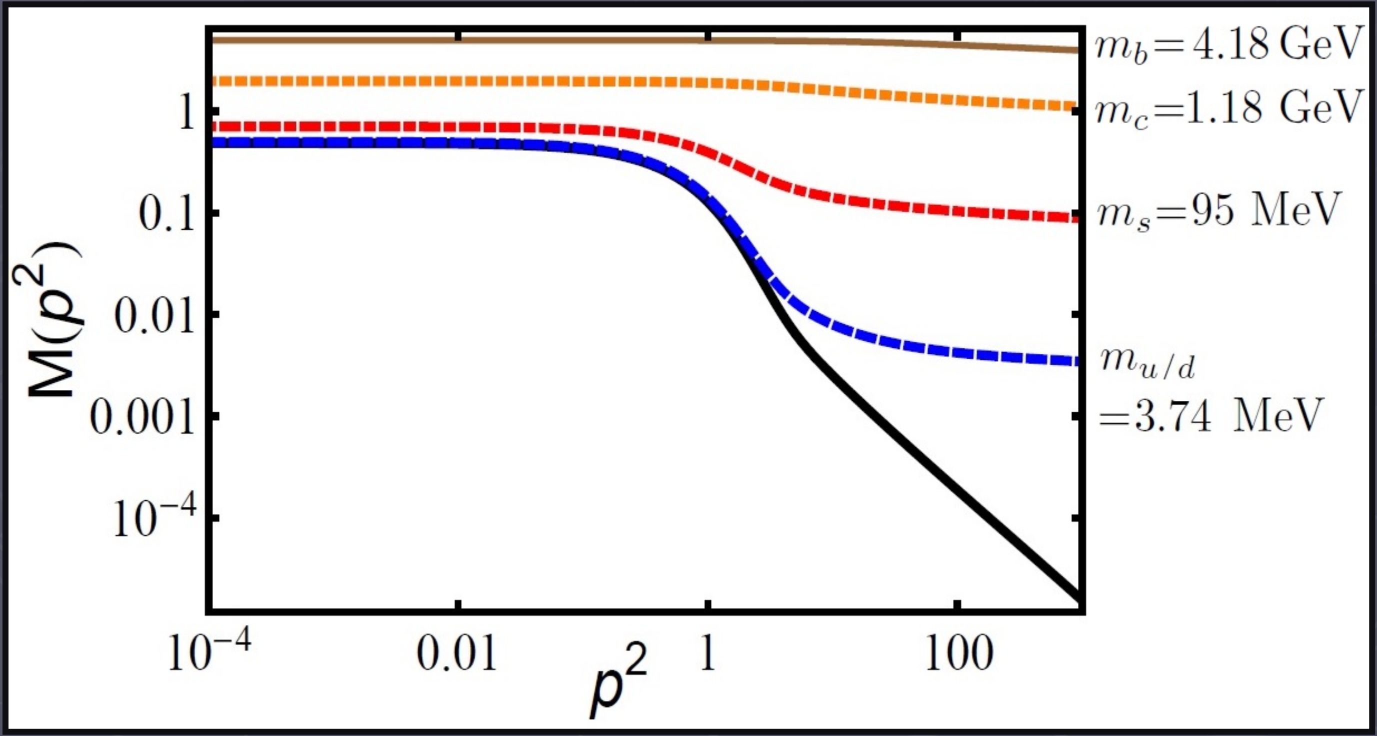
$$S^{-1}(p,\mu) = \mathcal{Z}_{2F} S_0^{-1}(p) + \frac{\tilde{\mathcal{Z}}_1 \mathcal{Z}_{2F}}{\tilde{\mathcal{Z}}_3}\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q,\mu)\frac{\lambda^a}{2}\gamma_\mu S(p,\mu)\Gamma(p,q,\mu)$$

The Quark Propagator

The quark propagator:

$$S(p^2,\mu^2) = i\,\gamma\cdot p\,A(p^2,\mu^2) + B(p^2,\mu^2) = \frac{Z(p^2,\mu^2)}{i\,\gamma\cdot p + M(p^2)}$$

Within Maris-Tandy truncation of the QCD SDEs:



What Next?

- What is the status of modern results for the hadron observables, form factors, GPDs, PDFs, PDAs, etc.?
- Can we strike a balance between the complexity of the hadron physics study through SDEs and aims of the theoretical and experimental program?