Schwinger-Dyson Equations
Symmetries of a QFT

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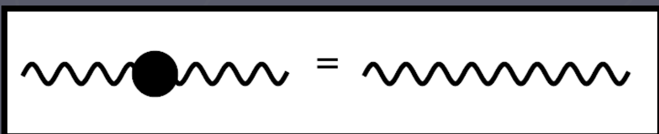
Dynamical mass generation in QED

Rainbow approximation in the Landau gauge:

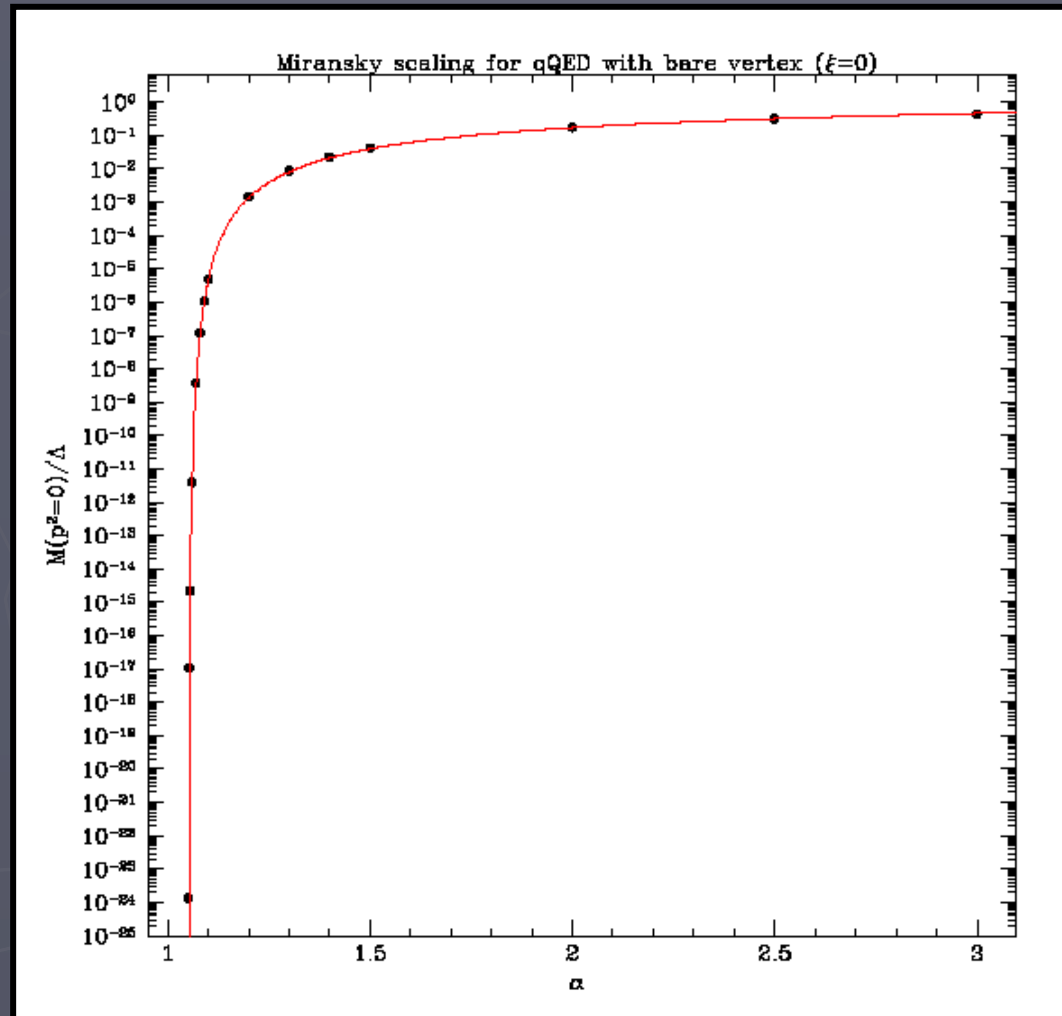


$$\Gamma^\mu(k,p) = \gamma^\mu$$

Miransky scaling

$$M(0) = \Lambda \exp\left[-\frac{A}{\sqrt{\alpha/\alpha_c - 1}} + B\right]$$

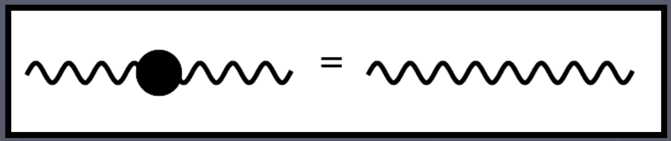
$$\alpha_c = \pi/3$$



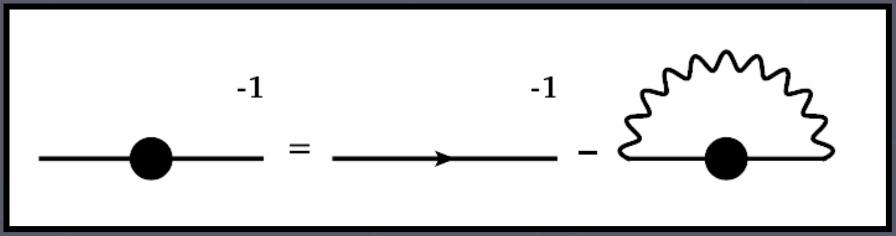
α_c is a physical observable

Dynamical mass generation in QED

Rainbow approximation beyond the Landau gauge:



$$\Gamma^\mu(k,p) = \gamma^\mu$$



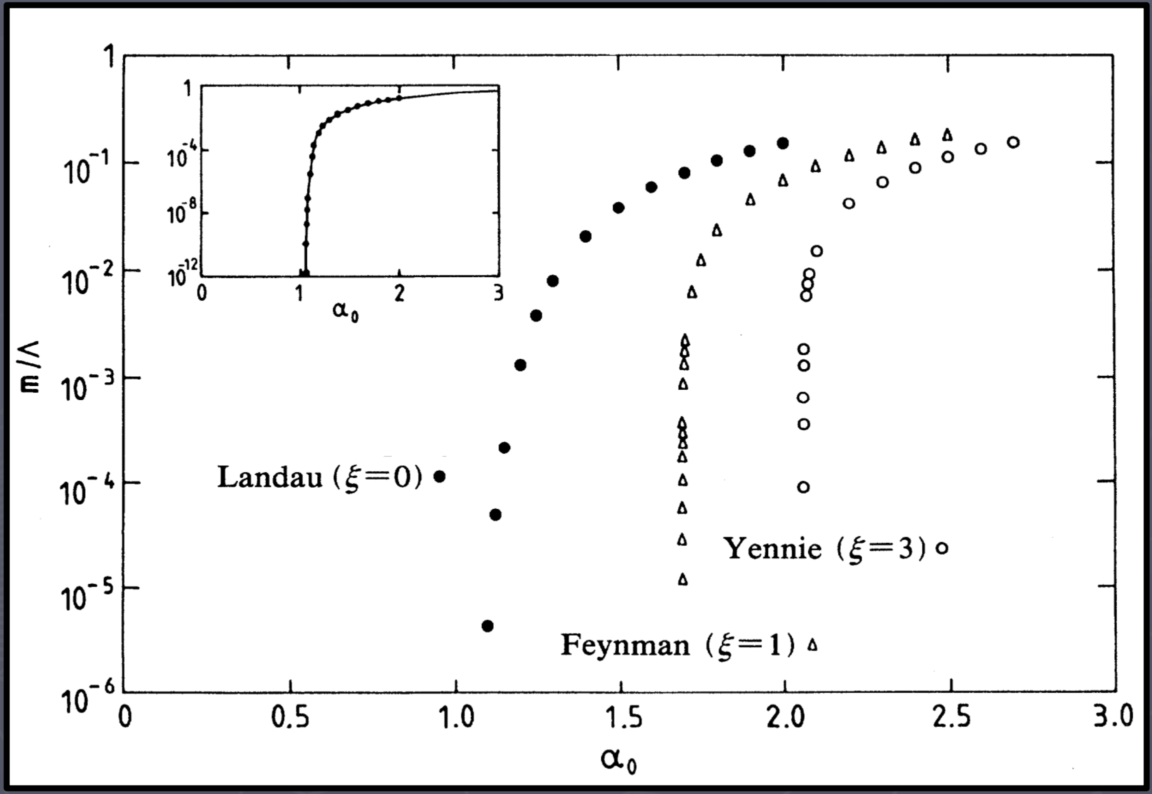
$$\frac{\mathcal{M}(p)}{F(p)} = m_0 + \frac{\alpha}{4\pi}(3+\xi)\int_0^{\Lambda^2} dk^2 \frac{F(k)\mathcal{M}(k)}{k^2+\mathcal{M}^2(k)}\left[\frac{k^2}{p^2}\theta(p^2-k^2)+\theta(k^2-p^2)\right]$$

$$\frac{1}{F(p)} = 1 + \frac{\alpha\xi}{4\pi}\int_0^{\Lambda^2} dk^2 \frac{F(k)}{k^2+\mathcal{M}^2(k)}\left[\frac{k^4}{p^4}\theta(p^2-k^2)+\theta(k^2-p^2)\right]$$

This system of equations decouples in the Landau gauge but in practice we can solve it in any gauge numerically.

Dynamical mass generation in QED

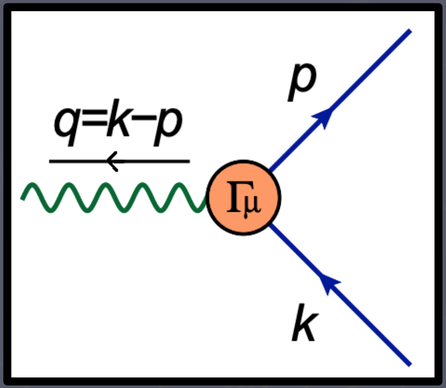
The bifurcation of dynamical mass takes place at a different value of the gauge parameter:



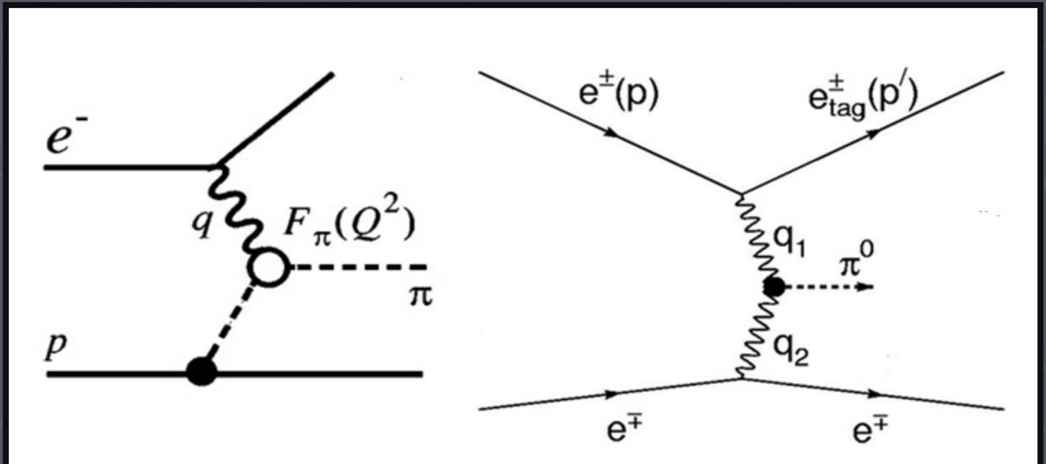
The photon dressing function does not depend upon the gauge parameter. Need to focus on the fermion-photon interaction.

The fermion-photon interaction

The electron-photon interaction plays a key role in unraveling the internal structure of hadrons in laboratories like the JLab.

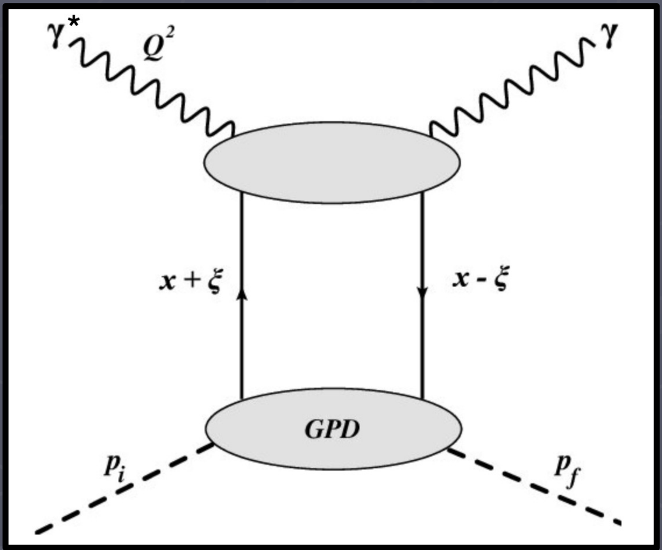


The photons interact with electrically charged quarks inside hadrons to study their internal structure which stems primarily from QCD.

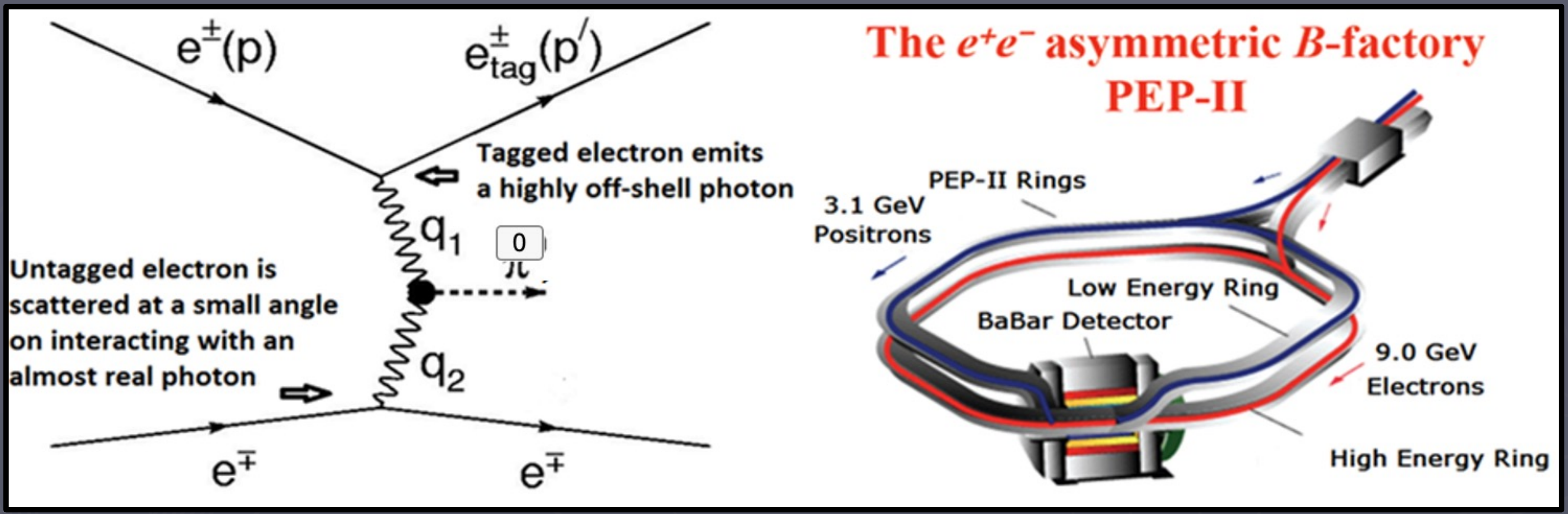


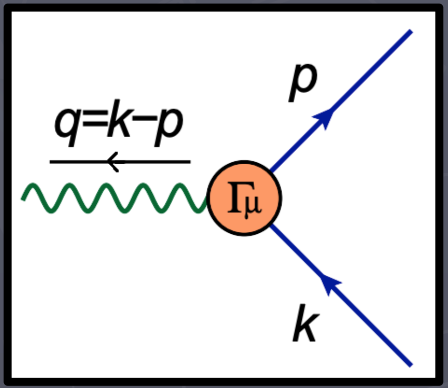
Pion Elastic Form Factor

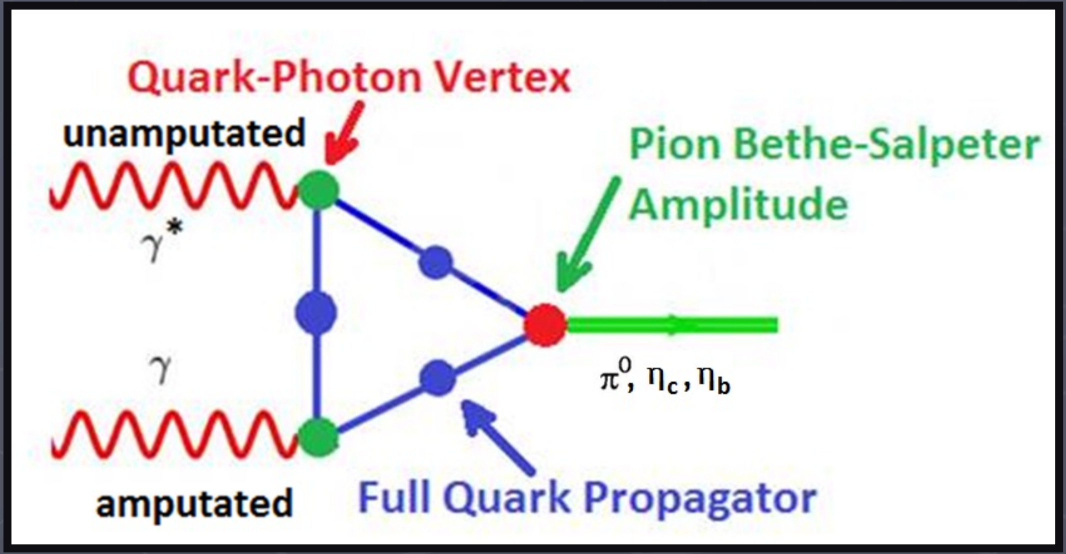
Pion Transition Form Factor



Meson to γγ* transition form factor







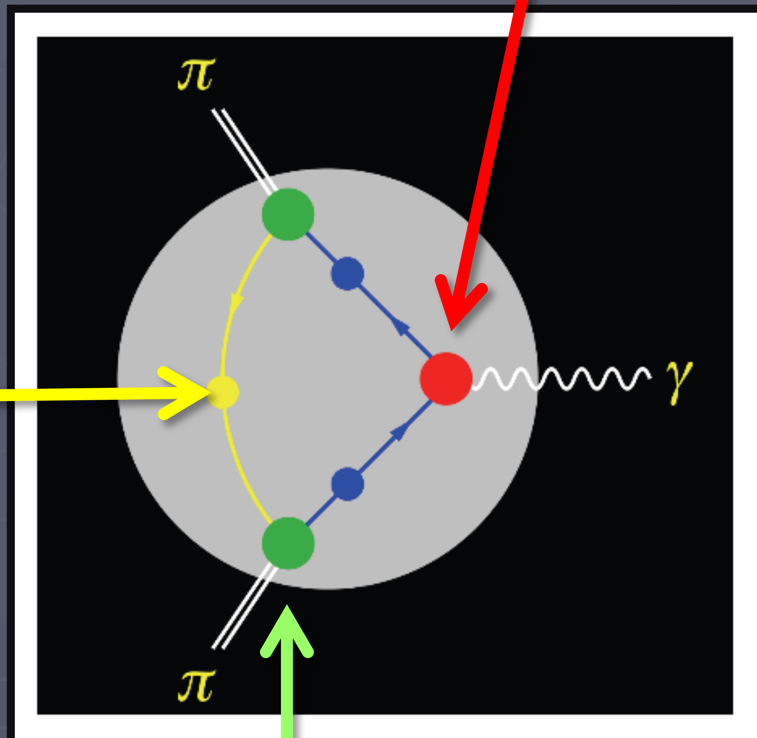
Pion electromagnetic form factor

Quark-Photon Vertex

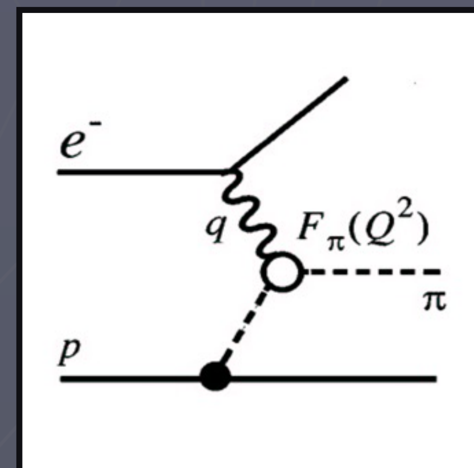
Gluon Propagator

Quark Propagator

Quark-Gluon Vertex

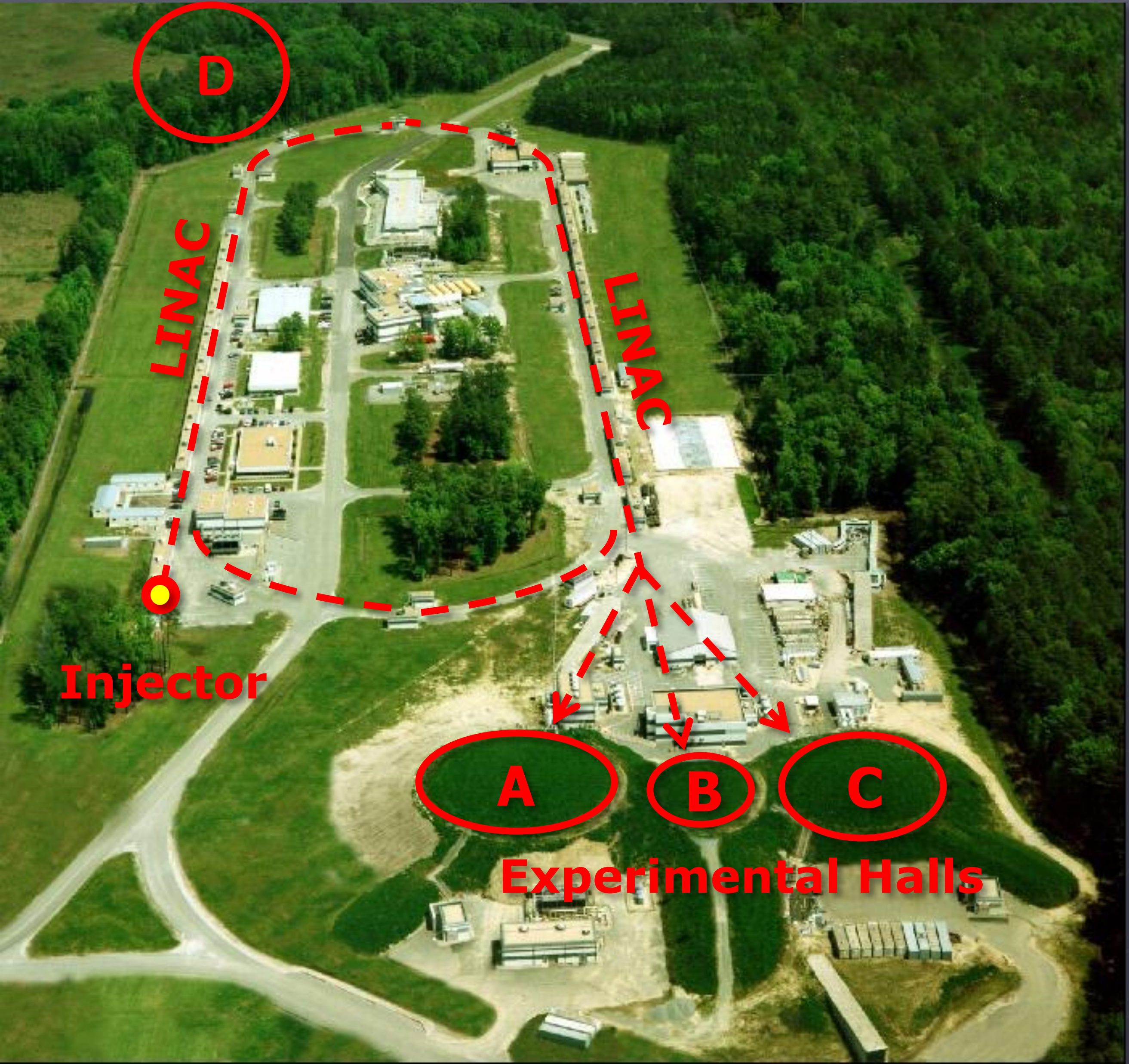


Bethe-Salpeter Amplitude



Pion Elastic Form Factor

Photon-quark interaction & hadron structure



Continuous
Electron
Beam
Accelerator
Facility

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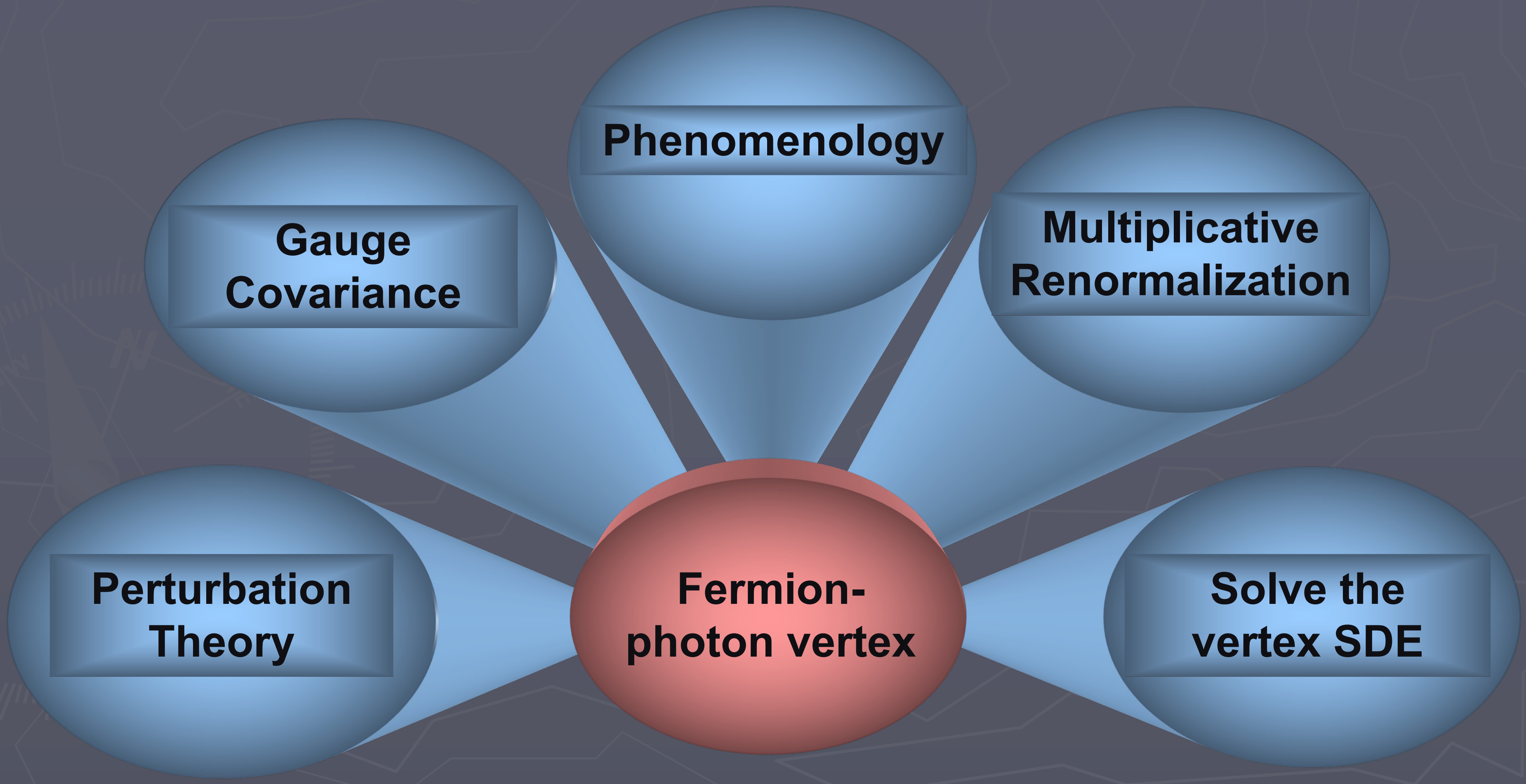
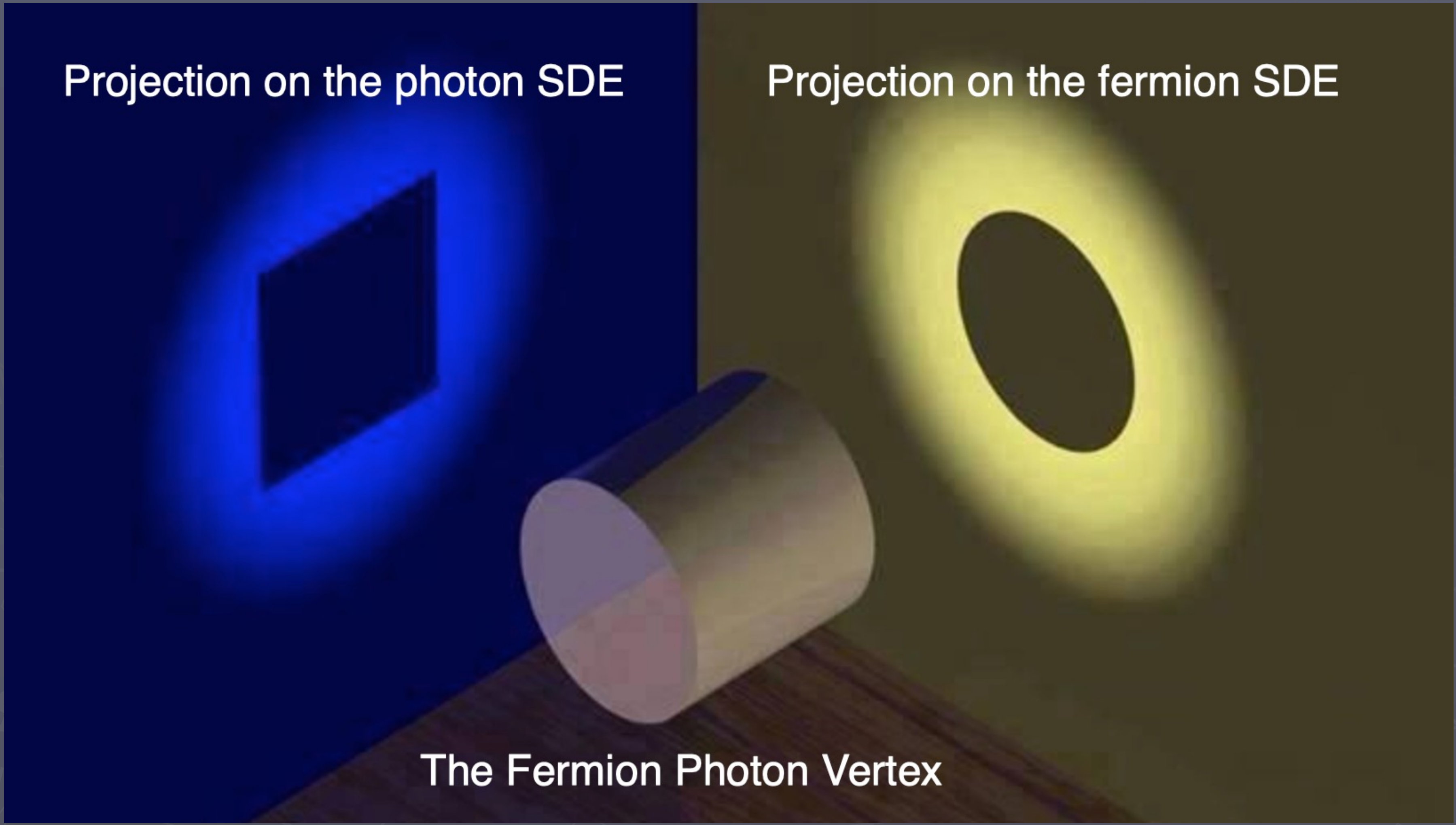
USA

Truncating the SDEs

The fermion-photon vertex must satisfy the key features of the associated quantum field theory, i.e., QED.

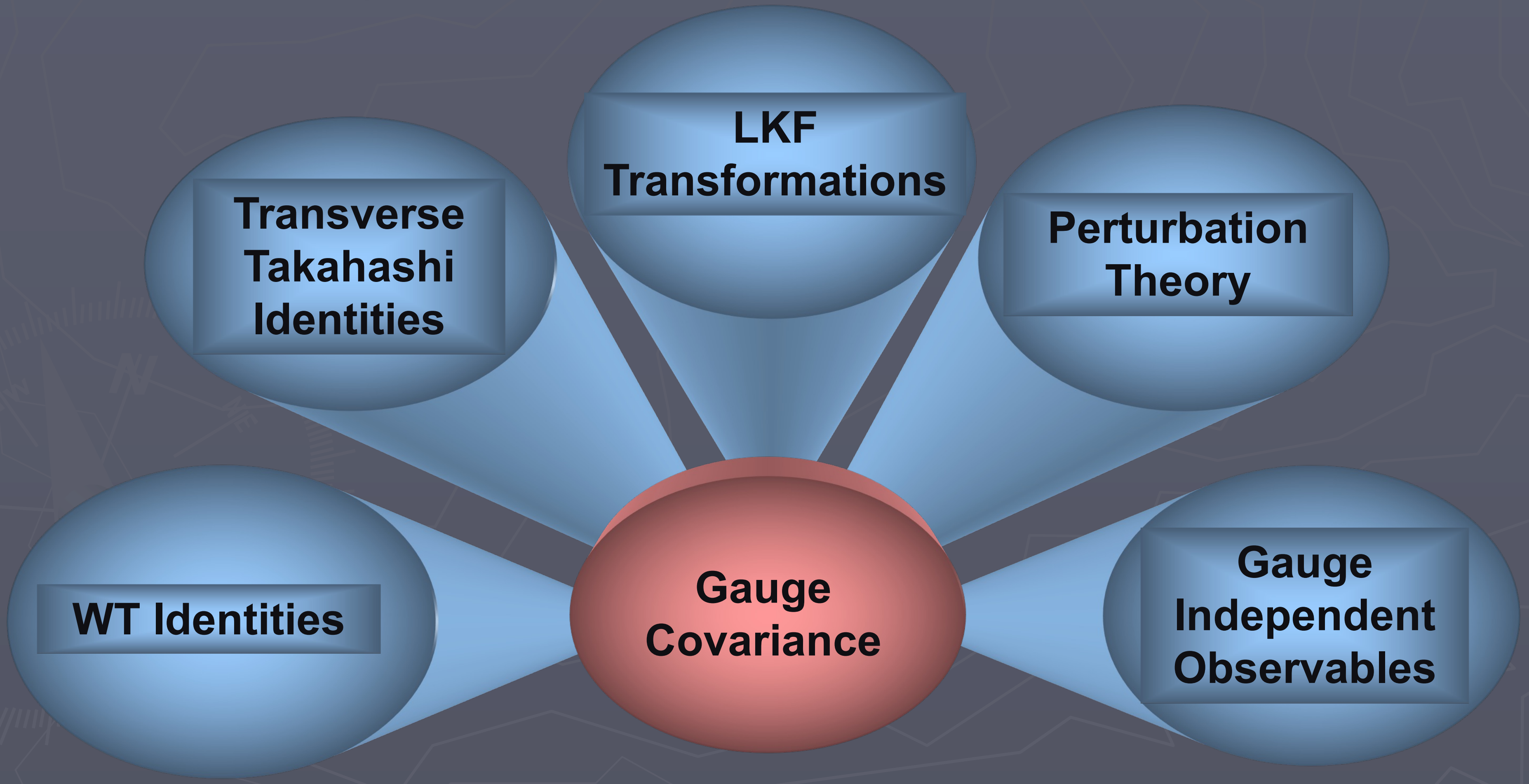


Be mindful of the projections



The transverse vertex

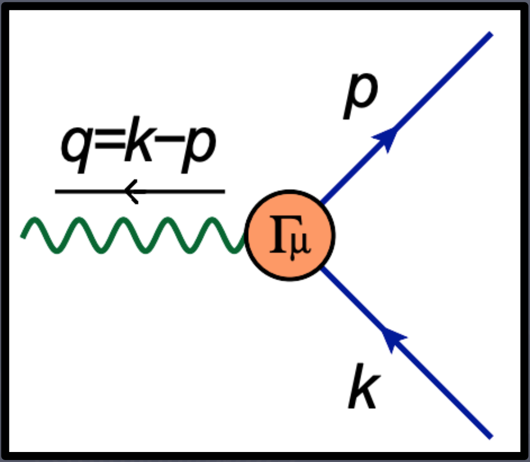
There are numerous implications of gauge covariance which need to be respected.



Ward Takahashi identities

Ward-Takahashi identity

The Ward and Ward-Takahashi identities:



$$\Gamma^{\mu}(p,p) = \frac{\partial}{\partial p_{\mu}} S_F^{-1}(p)$$

$$q_{\mu}\Gamma^{\mu}(k,p) = S_F^{-1}(k) - S_F^{-1}(p)\ , \qquad q = k - p$$

$$\Gamma^{\mu}(k,p) = \gamma^{\mu}$$

$$S_F(p) = \frac{F(p)}{\not{p} - \mathcal{M}(p)}$$

$$\not{q} = \frac{\not{k}}{F(k)} - \frac{\not{p}}{F(p)} - \frac{\mathcal{M}(k)}{F(k)} + \frac{\mathcal{M}(p)}{F(p)}$$

In the Landau gauge F(k)=F(p) and M(k)~M(p) at and below the pole mass.

The WTI is almost satisfied in the Landau gauge for momenta of interest for non-perturbative mass generation.

Ward-Takahashi identity

The structure of the Ward-Takahashi identity:

$$q_\mu \Gamma^\mu(k,p) = S_F^{-1}(k) - S_F^{-1}(p)$$

is such that we can divide it into two components, longitudinal and transverse to the photon momentum.

$$\Gamma^\mu(k,p) = \Gamma_L^\mu(k,p) + \Gamma_T^\mu(k,p) \; , \qquad q_\mu \Gamma_T^\mu = 0$$

The WTI only provides information on the longitudinal part of the vertex. We can think of the following ansatz:

$$\Gamma_L^\mu(k,p) = \frac{q^\mu}{q^2}[S_F^{-1}(k) - S_F^{-1}(p)]$$

but it has a kinematic singularity at $q^2 \to 0$.

Constructing the longitudinal vertex

Let's start from the Ward identity and the general form of the fermion propagator:

$$\Gamma^\mu(p,p) = \frac{\partial}{\partial p_\mu} S_F^{-1}(p)$$

$$S_F(p) = \frac{F(p)}{\not p - \mathcal{M}(p)}$$

$$\Gamma^\mu(p,p) = \frac{\gamma^\mu}{F(p)} + 2p^\mu \not p \frac{\partial}{\partial p^2}\frac{1}{F(p)} - 2p^\mu \frac{\partial}{\partial p^2}\frac{\mathcal{M}(p)}{F(p)}$$

To generalize it to k ≠ p, we can symmetrize it:

$$\frac{1}{F(p)} \to \frac{1}{2}\left[\frac{1}{F(k)} + \frac{1}{F(p)}\right], \quad p^\mu \to \frac{1}{2}(k^\mu + p^\mu), \quad \not p \to \frac{1}{2}(\not k + \not p)$$

$$\frac{\partial}{\partial p^2}\frac{1}{F(p)} \to \frac{1}{k^2 - p^2}\left[\frac{1}{F(k)} - \frac{1}{F(p)}\right], \quad \frac{\partial}{\partial p^2}\frac{\mathcal{M}(p)}{F(p)} \to \frac{1}{k^2 - p^2}\left[\frac{\mathcal{M}(k)}{F(k)} - \frac{\mathcal{M}(p)}{F(p)}\right]$$

The longitudinal vertex

We can thus write the longitudinal vertex as follows which is attributed to the work of Ball and Chiu:

$$\Gamma^{\mu}_{BC} = \frac{\gamma^{\mu}}{2}\left[\frac{1}{F(k)} + \frac{1}{F(p)}\right] + \frac{1}{2}\,\frac{(\not k + \not p)(k+p)^{\mu}}{(k^2 - p^2)}\left[\frac{1}{F(k)} - \frac{1}{F(p)}\right]$$
$$+ \frac{(k+p)^{\mu}}{(k^2 - p^2)}\left[\frac{\mathcal{M}(k)}{F(k)} - \frac{\mathcal{M}(p)}{F(p)}\right]$$

It has no kinematic singularities at $k^2 \to p^2$.

It satisfies the Ward-Takahashi identity.

It obeys the CP symmetry of the fermion-photon vertex.

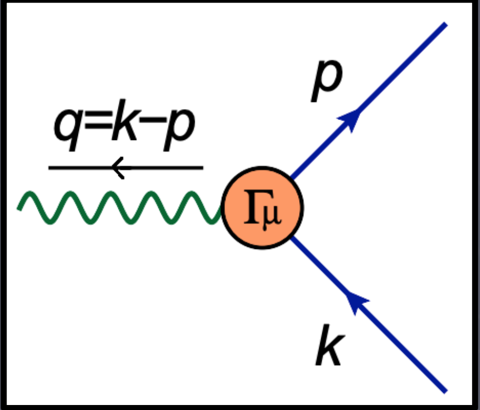
Is it the complete vertex? Is there a transverse part?

The bare vertex does not satisfy WTI beyond Landau gauge.

The vertex – its complexities

What is the complete structure of the fermion-photon vertex we can construct with 3 vectors and 4 independent scalars.

γ^μ		k^μ	p^μ
1	$\not{k}$	$\not{p}$	$\not{k}\not{p}$



$$\Gamma^\mu(k,p) = \sum_{i=1}^{12} v_i(k,p) V_i^\mu \ ,$$

$$\begin{aligned}
V_1^\mu &= \gamma^\mu, & V_2^\mu &= k^\mu, & V_3^\mu &= p^\mu, \\
V_4^\mu &= \not{k}\gamma^\mu, & V_5^\mu &= \not{k}k^\mu, & V_6^\mu &= \not{k}p^\mu, \\
V_7^\mu &= \not{p}\gamma^\mu, & V_8^\mu &= \not{p}k^\mu, & V_9^\mu &= \not{p}p^\mu, \\
V_{10}^\mu &= \not{k}\not{p}\gamma^\mu, & V_{11}^\mu &= \not{k}\not{p}k^\mu, & V_{12}^\mu &= \not{k}\not{p}p^\mu.
\end{aligned}$$

$$\Gamma^\mu(k,p) = \Gamma_L^\mu(k,p) + \Gamma_T^\mu(k,p)$$

$$q_\mu \, \Gamma_T^\mu(k,p) = 0$$

The transverse vertex

Longitudinal vertex consumes 4 basis vectors. The other 8 allow us to expand out the transverse vertex systematically:

$$\Gamma_T^\mu(k,p) = \sum_{i=1}^{8} \tau_i(k^2,p^2,q^2) T_i^\mu(k,p) \ ,$$

$$\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$$

$$\begin{aligned}
T_1^\mu &= p^\mu (k\cdot q) - k^\mu (p\cdot q) \\
T_2^\mu &= \left[p^\mu (k\cdot q) - k^\mu (p\cdot q)\right] (\not{k} + \not{p}) \\
T_3^\mu &= q^2 \gamma^\mu - q^\mu \not{q} \\
T_4^\mu &= \left[p^\mu (k\cdot q) - k^\mu (p\cdot q)\right] k^\lambda p^\nu \sigma_{\lambda\nu} \\
T_5^\mu &= q_\nu \sigma^{\nu\mu} \\
T_6^\mu &= \gamma^\mu (p^2 - k^2) + (p+k)^\mu \not{q} \\
T_7^\mu &= \frac{1}{2}(p^2 - k^2)\left[\gamma^\mu(\not{p} + \not{k}) - p^\mu - k^\mu\right] + (k+p)^\mu \, k^\lambda p^\nu \sigma_{\lambda\nu} \\
T_8^\mu &= -\gamma^\mu k^\nu p^\lambda \sigma_{\nu\lambda} + k^\mu \not{p} - p^\mu \not{k}
\end{aligned}$$

Transverse Takahashi identities

Transverse Takahashi identities

To know a vector completely, we need to know its divergence as well its curl. Note that for the bare vertex:

$$\begin{aligned} q_\mu \gamma^\mu &= \not{q} = (\not{k} - m) - (\not{p} - m) \\ iq^\mu \gamma^\nu - iq^\nu \gamma^\mu &= (\not{k} - m)\sigma^{\mu\nu} - \sigma^{\mu\nu}(\not{p} - m) \\ &\quad + (k+p)_\lambda \epsilon^{\lambda\mu\nu\rho}\gamma_\rho\gamma_5 \end{aligned}$$

Transverse Takahashi identities

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) &= S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu}S^{-1}(k) \\ &\quad + 2im_0\Gamma_{\mu\nu}(k,p) + t_\alpha \epsilon_{\alpha\mu\nu\beta}\Gamma^A_\beta(k,p) \\ &\quad + A^V_{\mu\nu}(k,p) \\ q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) &= S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k) \\ &\quad + t_\alpha \epsilon_{\alpha\mu\nu\beta}\Gamma_\beta(k,p) + V^A_{\mu\nu}(k,p) \end{aligned}$$

$$t = k + p, \qquad \sigma^5_{\mu\nu} = \gamma_5 \sigma_{\mu\nu}$$

Transverse Takahashi identities

In order to project out transverse form factors from the TTIs, it is convenient to introduce the following projectors:

$$T^1_{\mu\nu} = \frac{1}{2}\epsilon_{\alpha\mu\nu\beta}\, t_\alpha\, q_\beta$$

$$T^2_{\mu\nu} = \frac{1}{2}\epsilon_{\alpha\mu\nu\beta}\, \gamma_\alpha\, q_\beta$$

Contract the first TTI with both these projection operators and simplify the results:

$$\begin{aligned} q_\mu\Gamma_\nu(k,p) - q_\nu\Gamma_\mu(k,p) \;&=\; S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu}S^{-1}(k) \\ &\quad + 2im_0\Gamma_{\mu\nu}(k,p) + t_\alpha\epsilon_{\alpha\mu\nu\beta}\Gamma^A_\beta(k,p) \\ &\quad + A^V_{\mu\nu}(k,p) \end{aligned}$$

Transverse Takahashi identities

The result can be written in the following form:

$$
\begin{aligned}
q \cdot t\, t \cdot \Gamma(k,p) &= T^1_{\mu\nu}\left[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)\right] \\
&\quad + t^2 q \cdot \Gamma(k,p) + T^1_{\mu\nu}V^A_{\mu\nu}\,, \\
q \cdot t\gamma \cdot \Gamma(k,p) &= T^2_{\mu\nu}\left[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)\right] \\
&\quad + \gamma \cdot tq \cdot \Gamma(k,p) + T^2_{\mu\nu}V^A_{\mu\nu}\,.
\end{aligned}
$$

These Lorentz scalar objects can be expressed:

$$
\begin{aligned}
iT^1_{\mu\nu}V^A_{\mu\nu} &= \mathbf{I}_D Y_1(k,p) + i(\gamma \cdot q)Y_2(k,p) \\
&\quad + i(\gamma \cdot t)Y_3(k,p) + [\gamma \cdot q, \gamma \cdot t]\, Y_4(k,p) \\
iT^2_{\mu\nu}V^A_{\mu\nu} &= i\mathbf{I}_D Y_5(k,p) + (\gamma \cdot q)Y_6(k,p) \\
&\quad + (\gamma \cdot t)Y_7(k,p) + i\,[\gamma \cdot q, \gamma \cdot t]\, Y_8(k,p)
\end{aligned}
$$

and implemented into the gap equation.

Landau-Khalatnikov-Fradkin Transformations

Landau-Khalatnikov-Fradkin transformations

Ward-Takahashi identities relate different Green functions to each other.

Landau-Khalatnikov-Fradkin transformations tell us how a Green function will change under a variation of gauge.

These transformations are non-perturbative and most compactly written in the coordinate space.

In QED, for the fermion propagator:

$$S_F(x;\xi) = S_F(x;0)\,\mathrm{e}^{-i[\Delta_d(0)-\Delta_d(x)]}$$

$$\Delta_d(x) = -i\xi e^2 \mu^{4-d} \int_0^\infty \frac{d^d p}{(2\pi)^d} \frac{e^{-ip\cdot x}}{p^4}$$

LKFT for the fermion propagator

In 4-dimensions, we can calculate:

$$\Delta_4(x_{min}) - \Delta_4(x) = -i \ln\left(\frac{x^2}{x_{min}^2}\right)^{\nu}, \qquad \nu = \alpha\xi/4\pi$$

$$S_F(x;\xi) = S_F(x;0)\left(\frac{x^2}{x_{min}^2}\right)^{-\nu}$$

We work with momentum space fermion propagator generally:

$$S_F(p;\xi) = \int d^d x e^{ip\cdot x} S_F(x;\xi)$$

$$S_F(x;\xi) = \int \frac{d^d p}{(2\pi)^d} e^{-ip\cdot x} S_F(p;\xi)$$

LKFT for the fermion propagator

The general form of massless fermion propagator is

$$S_F(p;\xi) = \frac{F(p;\xi)}{i\not{p}}$$

$$S_F(x;\xi) = \not{x}X(x;\xi)$$

Based upon our one-loop perturbation theory calculation

$$F(p;0) = 1$$

Fourier transform

$$X(x;0) = -\frac{1}{4\pi x^3}$$

$$X(x;0) = -\frac{1}{4\pi x^3}$$

LKFT

$$X(x;\xi) = X(x;0)\left(\frac{x^2}{x_{min}^2}\right)^{-\nu}$$

LKFT for the fermion propagator

Inverse Fourier transform yields:

$$F(p/\Lambda;\xi)=\left(\frac{p^2}{\Lambda^2}\right)^{\nu}, \qquad \nu=\frac{\alpha\xi}{4\pi}$$

The renormalized wavefunction renormalization is:

$$F_R(p/\mu;\xi)=\mathcal{Z}_2^{-1}(\mu/\Lambda;\xi)\ F(p/\Lambda;\xi)$$

$$F(p/\Lambda;\xi)=\left(\frac{p^2}{\Lambda^2}\right)^{\nu}$$

$$\mathcal{Z}_2(\mu/\Lambda;\xi)=\left(\frac{\mu^2}{\Lambda^2}\right)^{\nu}$$

$$F_R(p/\mu;\xi)=\left(\frac{p^2}{\mu^2}\right)^{\nu}$$

These are not at a given order in perturbation theory.

These are an all order re-summation of leading logarithms.

LKFT for the fermion propagator

$$F(p^2,\Lambda^2) = 1+\sum_{n=1}^{\infty}\alpha^n A_n \ln^n\left(\frac{p^2}{\Lambda^2}\right)$$

$$\mathcal{Z}_2^{-1}(\mu^2,\Lambda^2) = 1+\sum_{n=1}^{\infty}\alpha^n B_n \ln^n\left(\frac{\mu^2}{\Lambda^2}\right)$$

$$F_R(p^2,\mu^2) = 1+\sum_{n=1}^{\infty}\alpha^n C_n \ln^n\left(\frac{p^2}{\mu^2}\right)$$

$$A_n = C_n = (-1)^n B_n = \frac{A_1^n}{n!}$$

$$A_1 = \frac{\xi}{4\pi}$$

The bare vertex truncation or even the Ball-Chiu longitudinal vertex do not respect it in all gauges.

This requirement puts tight constraint on the choice of the transverse vertex.

"The momentum space Landau-Khalatnikov-Fradkin transformation of interaction vertices in quantum electrodynamics", AB, J. P. Edwards, U.D. Jentschura, J. Nicasio, in progress.

Perturbation Theory

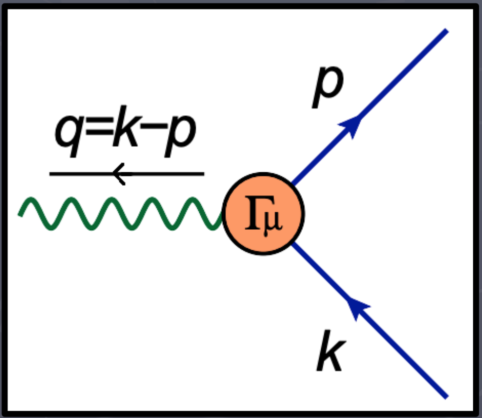
Perturbation Theory

In perturbation theory, all the key properties of a gauge field theory are maintained order by order.

So it is natural to demand all Green functions to reproduce perturbation theory in the weak coupling regime.

At tree level, the fermion-photon vertex is merely γ^μ.

At one-loop, it already becomes very complicated. However, for the asymptotic values of fermion momenta: k >> p:

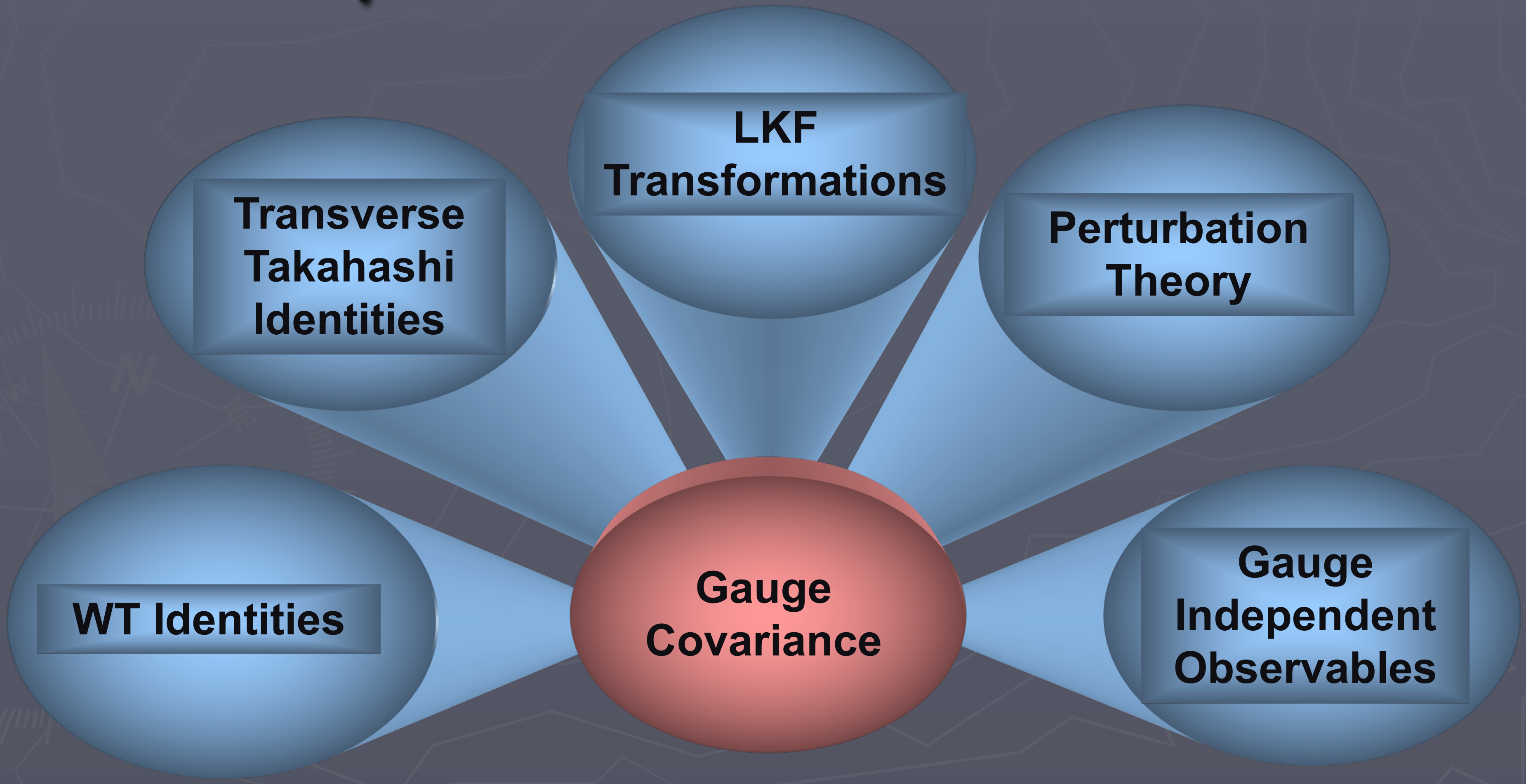


$$\Gamma^T_\mu(k,p) \stackrel{k^2 >> p^2}{=} \frac{\alpha\xi}{8\pi k^2} \log\left(\frac{p^2}{k^2}\right) T^{asy}_\mu$$

$$T^{asy}_\mu \equiv T^{3\,asy}_\mu = T^{6\,asy}_\mu = k^2\gamma_\mu - k_\mu \gamma\cdot k$$

The transverse vertex

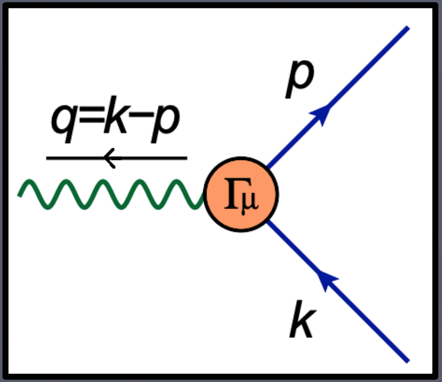
Recall the numerous implications of gauge covariance which need to be respected.



The transverse vertex

$$\Gamma^T_\mu(k,p) = \sum_{i=1}^{8} \tau_i(k,p) T^i_\mu(k,p)$$

$$\begin{aligned}
T^1_\mu(k,p) &= i\left[p_\mu(k\cdot q) - k_\mu(p\cdot q)\right] \\
T^2_\mu(k,p) &= \left[p_\mu(k\cdot q) - k_\mu(p\cdot q)\right]\gamma\cdot t \\
T^3_\mu(k,p) &= q^2\gamma_\mu - q_\mu\gamma\cdot q\,, \\
T^4_\mu(k,p) &= iq^2\left[\gamma_\mu\gamma\cdot t - t_\mu\right] + 2q_\mu p_\nu k_\rho \sigma_{\nu\rho} \\
T^5_\mu(k,p) &= \sigma_{\mu\nu}q_\nu\,, \\
T^6_\mu(k,p) &= -\gamma_\mu\left(k^2 - p^2\right) + t_\mu\gamma\cdot q \\
T^7_\mu(k,p) &= \frac{i}{2}(k^2 - p^2)\left[\gamma_\mu\gamma\cdot t - t_\mu\right] + t_\mu p_\nu k_\rho \sigma_{\nu\rho} \\
T^8_\mu(k,p) &= -i\gamma_\mu p_\nu k_\rho\sigma_{\nu\rho} - p_\mu\gamma\cdot k + k_\mu\gamma\cdot p
\end{aligned}$$



$$\sigma_{\nu\rho} = \frac{i}{2}\left[\gamma_\nu,\gamma_\rho\right]$$

Coefficients $\tau_i(k,p)$ are constrained by symmetries of QED.

The transverse vertex

$$\tau_1(k^2,p^2) = \frac{a_1}{(k^2+p^2)}\; c(k^2,p^2)$$

$$\tau_2(k^2,p^2) = \frac{a_2}{(k^2+p^2)}\; b(k^2,p^2)$$

$$\tau_3(k^2,p^2) = a_3\; b(k^2,p^2)$$

$$\tau_4(k^2,p^2) = \frac{a_4(k^2-p^2)}{4k^2p^2}\; c(k^2,p^2)$$

$$\tau_5(k^2,p^2) = -a_5\; c(k^2,p^2)\,,$$

$$\tau_6(k^2,p^2) = -\frac{a_6(k^2+p^2)}{(k^2-p^2)}\; b(k^2,p^2)$$

$$\tau_7(k^2,p^2) = -\left[\frac{a_4 q^2}{2k^2p^2} + \frac{a_7}{k^2+p^2}\right] c(k^2,p^2)$$

$$\tau_8(k^2,p^2) = a_8\; b(k^2,p^2)$$

Transverse vertex consists of the same structures as the longitudinal vertex.

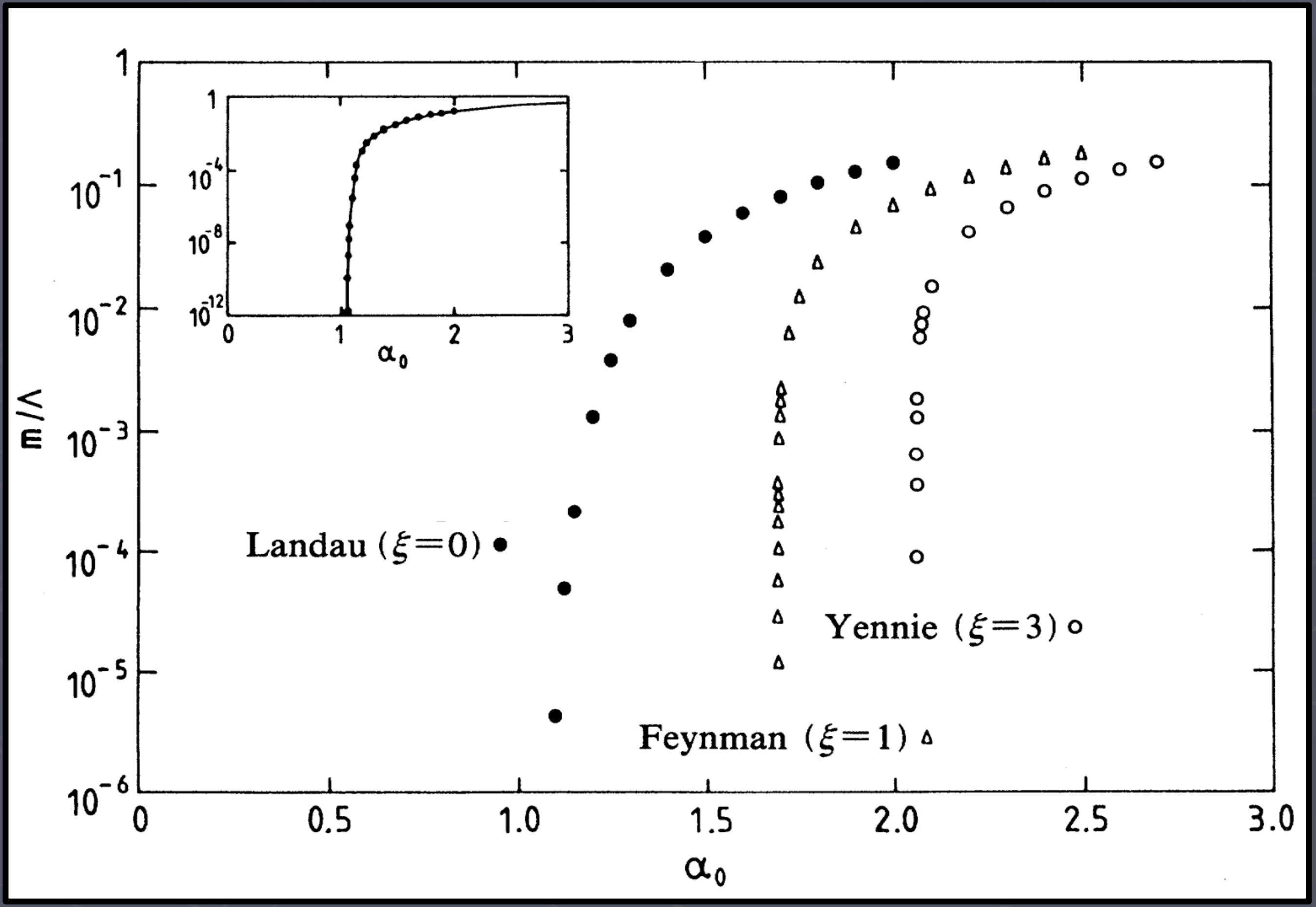
$$b = \left[\frac{1}{F(k^2)} - \frac{1}{F(p^2)}\right]\frac{1}{k^2-p^2}$$

$$c = \left[\frac{\mathcal{M}(k^2)}{F(k^2)} - \frac{\mathcal{M}(p^2)}{F(p^2)}\right]\frac{1}{k^2-p^2}$$

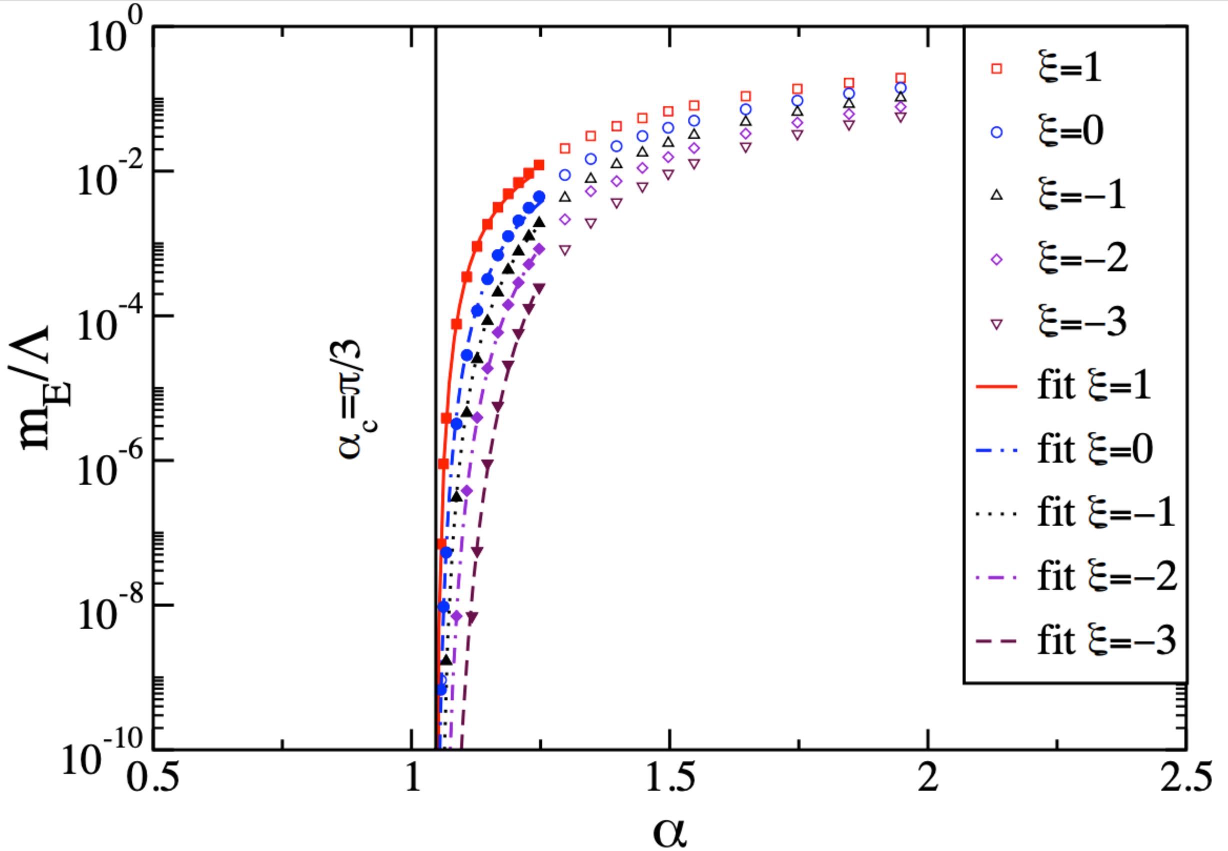
Coefficients a_i are constrained by the symmetries of QED.

The bare vertex in different gauges

Recall the bifurcation of dynamical mass takes place at a different critical value of the coupling in gauge parameter:



Constrained vertex in different gauges

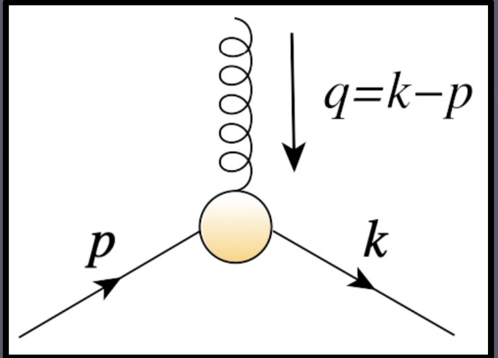


Ratio of the Euclidean mass and ultraviolet cut-off in different gauges. $\alpha_c = \pi/3$ within the numerical accuracy of computation.

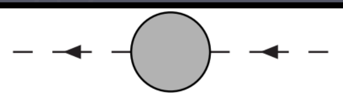
L. Albino, AB, L.X. Gutiérrez-Guerrero, B. El Bennich, E. Rojas, Phys. Rev. D100 054028 (2019)

Salvnov-Taylor identities

For the quark-gluon vertex, Slavnov-Taylor identity replaces the Ward-Takahashi identity.



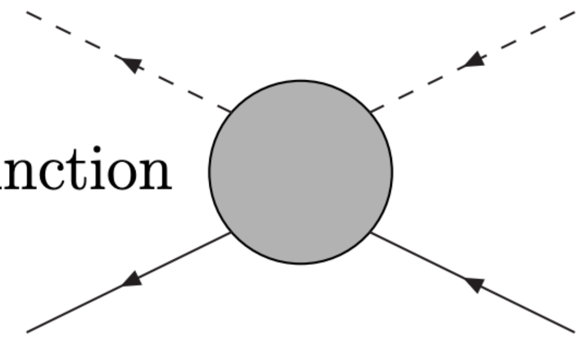
$$i\, q \cdot \Gamma^a(k,p) = G(q^2)\left[S^{-1}(k)\, H^a(k,p) - \bar{H}^a(p,k) S^{-1}(p)\right]$$



Ghost propagator

$$D^{ab}(q^2) = -\,\delta^{ab}\,\frac{G(q^2)}{q^2}$$

H function



quark-quark-ghost-ghost amplitude

$$H(k,p) = X_0(k,p)\mathbb{1}_D + X_1(k,p)\,\gamma\cdot k + X_2(k,p)\,\gamma\cdot p + X_3(k,p)\,\left[\gamma\cdot k, \gamma\cdot p\right]$$

Transverse Slavnov-Taylor identities

In analogy to QED, the transverse part of the quark-gluon vertex can be constrained by the TSTI relate to its curl.

The Dirac structure of these identities is identical to that in QED but they also involve the ghost dressing function and quark-ghost scattering kernel in QCD.

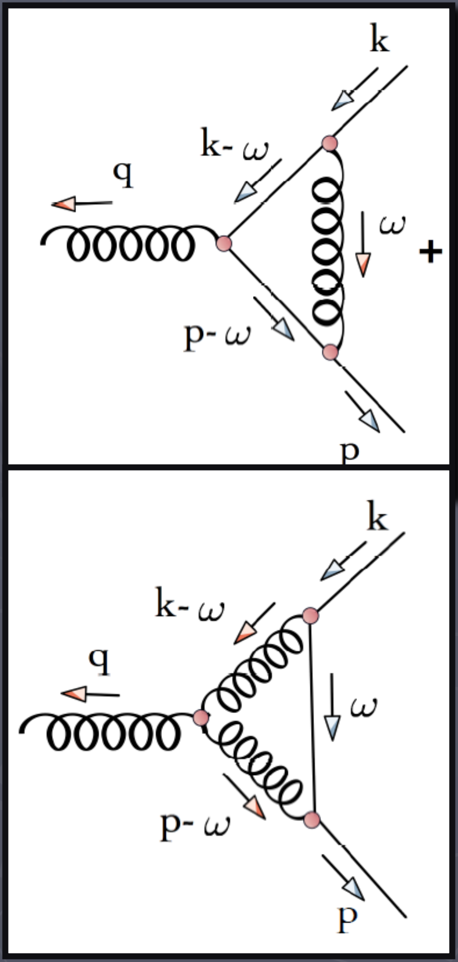
$$
\begin{aligned}
& q_\mu \Gamma^a_\nu(k,p) - q_\nu \Gamma^a_\mu(k,p) \\
& = G(q^2)\left[S^{-1}(p)\sigma_{\mu\nu}\, H^a(k,p) + \bar{H}^a(p,k)\,\sigma_{\mu\nu} S^{-1}(k)\right] \\
& \quad +2im\,\Gamma^a_{\mu\nu}(k,p) + t_\alpha \epsilon_{\alpha\mu\nu\beta}\,\Gamma^{5a}_\beta(k,p) + A^a_{\mu\nu}(k,p)\,, \\
& q_\mu \Gamma^{5a}_\nu(k,p) - q_\nu \Gamma^{5a}_\mu(k,p) \\
& = G(q^2)\left[S^{-1}(p)\sigma^5_{\mu\nu}\, H^a(k,p) - \bar{H}^a(p,k)\,\sigma^5_{\mu\nu} S^{-1}(k)\right] \\
& \quad +t_\alpha \epsilon_{\alpha\mu\nu\beta}\,\Gamma^a_\beta(k,p) + V^a_{\mu\nu}(k,p)\,,
\end{aligned}
$$

$$
t = p + k
$$

$$
\sigma^5_{\mu\nu} = \gamma_5 \sigma_{\mu\nu}
$$

Perturbation theory

At 1-loop, in the asymptotic limit of incoming and outgoing fermion momenta: k>>p, QED result can be extended to QCD:



$$\Gamma_T^\mu \overset{k^2>>p^2}{=} -\alpha \frac{C_A(2-\xi) - 8C_F(1-\xi)}{64k^2\pi} \ln\frac{p^2}{k^2} T^\mu$$

$$T_\mu = k^2\gamma_\mu - \not{k} k_\mu$$

R. Bermudez, L. Albino, L.X. Gutiérrez, M. Tejeda, AB
Phys. Rev. D95 034041 (2017)

The QED result can be readily inferred .

$$\text{QED:} \quad \Gamma_T^\mu \overset{k^2>>p^2}{=} \frac{\alpha(1-\xi)}{8k^2\pi} \ln\frac{p^2}{k^2} T^\mu$$

Local gauge transformation

The expression for these transformations is not closed in QCD due to its non-abelian nature.

$$
\begin{aligned}
iS_{ij}^{F}(x,x') \;=\; & iS_{ij}^{0F}(x,x')\Big[\mathrm{e}^{g_s^2\,C_F\,[i\Delta_F(x-x')-i\Delta_F(0)]} \\
& -\frac{g_s^4\,C_A\,C_F}{(2!)(3!2!1!)}\{[i\Delta_F(x-x')-i\Delta_F(0)]\,[3i\Delta_F(x-x')-i\Delta_F(0)]\} \\
& \times\left[1+g_s^2C_F\left(i\Delta_F(x-x')-i\Delta_F(0)\right)\right] \\
& +\frac{g_s^6\,C_FC_A^2}{(1!)(4!3!2!1!)}\,[i\Delta_F(x-x')-i\Delta_F(0)] \\
& \times\left[8(i\Delta_F(x-x'))^2-7(i\Delta_F(x-x'))(i\Delta_F(0))+(i\Delta_F(0))^2\right]+\mathcal{O}(g_s^8)\Big]
\end{aligned}
$$

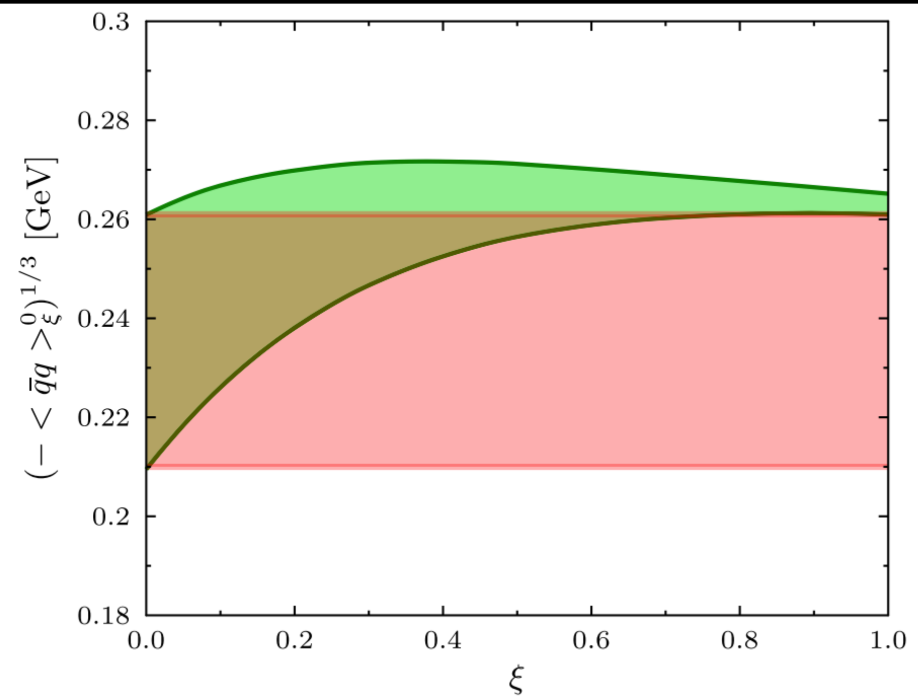
M.J. Aslam, A. Bashir, L.X. Gutierrez-Guerrero,
Phys. Rev.D 93 (2016) 7, 076001.

The QED result is recovered for C_A=0, C_F=1.

Condensate – exploratory study

The chiral quark condensate is explicitly gauge invariant.

$$\langle\bar{\psi}\psi\rangle_\xi = -\mathrm{Tr}\left[S^F(x,x')\right]_{x'=x}$$
$$= -\mathrm{Tr}\left[S^{0F}(x,x')\right]_{x'=x}$$
$$= \langle\bar{\psi}\psi\rangle_0$$



Gauge dependence of the quark condensate. The horizontal pink-shaded band indicates the admissible region of a gauge-independent chiral quark condensate as implied by LKFT in QCD.

H.R. Lessa, F.E. Serna, B. El Bennech, AB, O. Oliveira
Phys. Rev. D107 074017 (2023)

What next?

- What role does chiral symmetry and its dynamical breaking play in QCD and hadron physics?
- Can we start from a simple illustrative example of how we can start from SDEs of QCD and from their extract physical observables of hadron physics laboratories?
- How do we study bound states, mesons, baryons?
- How do we improve upon our studies?