Introduction to HEP simulations on Quantum Computers

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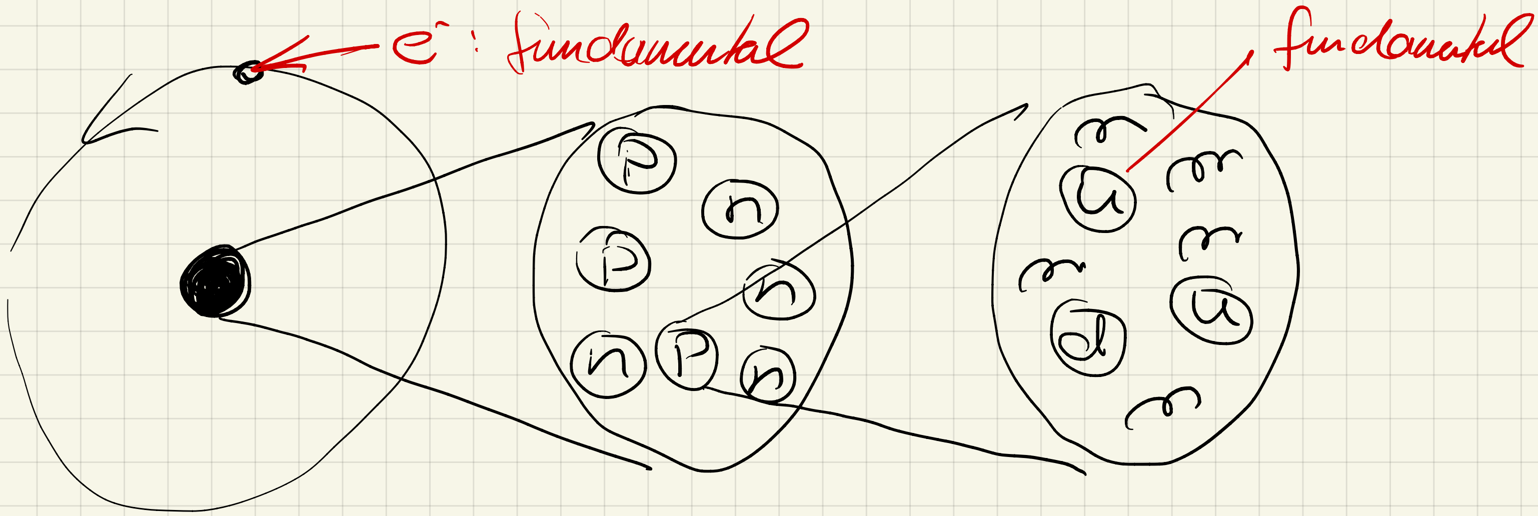
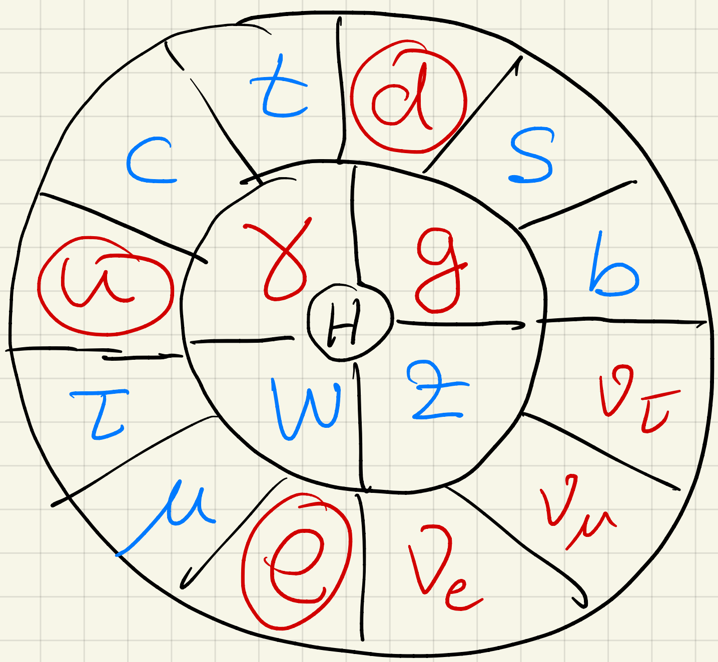
Intro to HEP

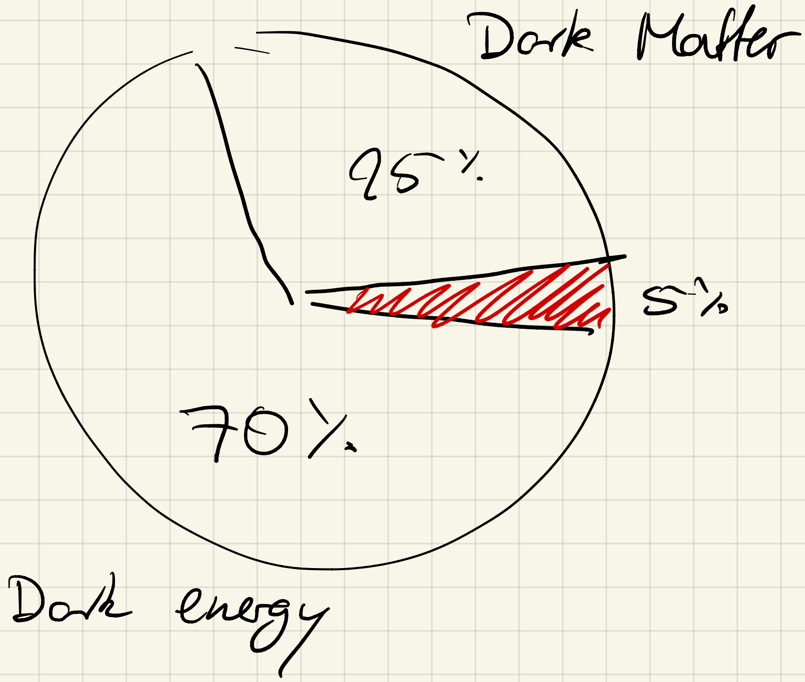
- What is HEP



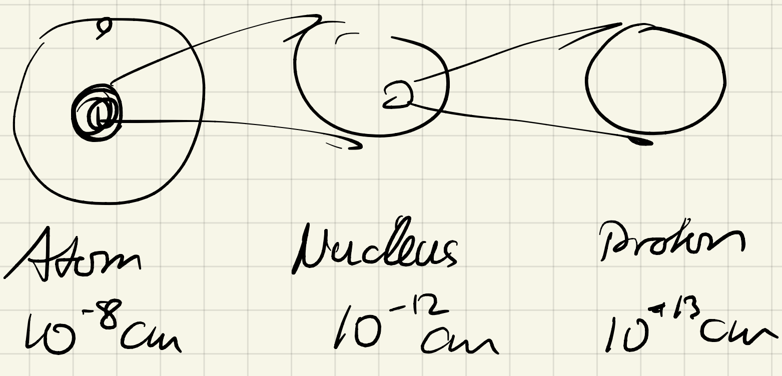
- What are interactions of fundamental particles



- Quarks, leptons



- We know that the SM is <u>incomplete</u>
- How to look for BSM ?
- Trying to develop good microscopes so we can see smaller



- Wavelength of visible light is $\sim 10^{-5}$ cm
- x-rays $\lambda \sim 10^{-8}$ cm

- To see smaller than atoms, need objects with smaller wavelengths.

- Particle-matter duality

$$\boxed{\lambda \sim \frac{hc}{E}}$$

- Can look at smaller distances by colliding objects with large energy with object.

- Particle colliders

- $\underline{E} \sim \underline{m}c^2$

- Best microscope: Large Hadron Collider

$E \sim 14\,\text{TeV}$

$\lambda \sim 10^{-17}\,\text{cm}$

$d_p \sim 10^{-13}\,\text{cm}$

- What we measure:

$$\boxed{pp \xrightarrow{\text{time}} \text{lots of particles}}$$

- $$\left| \langle X(T) | \, U(-T, T) \, | pp(-T) \rangle \right|^2$$

$$= \left| \langle X(T) | \, e^{-iH_{SM}(-T-T)} \, | pp(-T) \rangle \right|^2$$

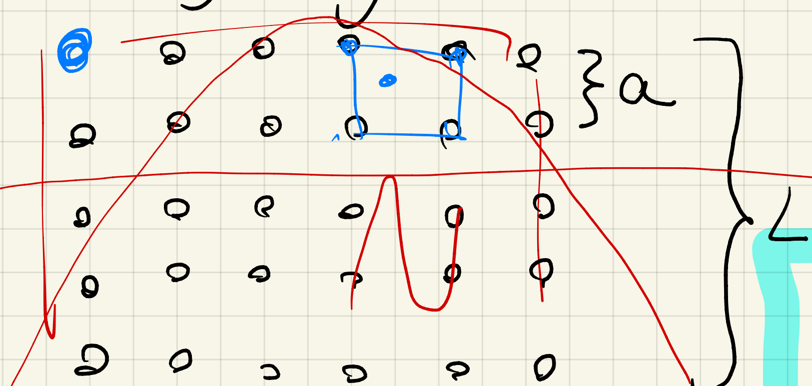
$$= \left| \langle X(T) | \, e^{2iH_{SM}T} \, | pp(-T) \rangle \right|^2$$

Simulation: Calculate expectation values of time evolution operator.

<u>Dealing with infinite dimensional Hilbert space</u>

- What is a QFT
- A field theory is described in terms of fields $E(x)$, $B(x)$

- Infinitely many points in space
- Infinitely many possible field values



$$n_L = \left(\frac{L}{a}\right)^d$$

$$\boxed{\frac{L}{a} \sim \frac{E_{max}}{E_{min}}}$$

- $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \cancel{\frac{\lambda}{4!}\phi^4}$ 0

$= \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\vec{\nabla}\phi)^2 - \frac{m^2}{2}\phi^2$

$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}$

$\partial_\mu : \mu = 0, 1, \dots, 3$

$\partial_0 = \partial/\partial t$

$\partial_i = \partial/\partial x_i$

$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

- $H = \dot{\phi}\pi - \mathcal{L}$

$= \dot{\phi}^2 - \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{m^2}{2}\phi^2$

$$\boxed{\begin{aligned} H &= \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{m^2}{2}\phi^2 \\ &= \frac{1}{2}\pi^2 + \frac{m^2}{2}\phi^2 + \frac{1}{2}(\nabla\phi)^2 \end{aligned}}$$

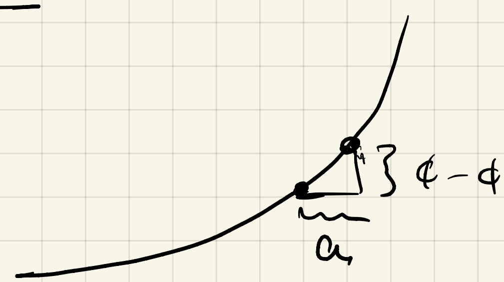
$[\hat{\phi}(x), \hat{\pi}(y)] = i\delta(x-y)$

$[\hat{\phi}(x), \hat{\phi}(y)] = [\hat{\pi}(x), \hat{\pi}(y)] = 0$

$$H = \int d^dx \left[\frac{1}{2}\hat{\pi}^2(x) + \frac{m^2}{2}\hat{\phi}^2(x) + \frac{1}{2}(\nabla\phi)^2(x) \right.$$

$$\boxed{H = a^d \sum_{x_i} \left[\frac{1}{2}\hat{\pi}^2(x_i) + \frac{m^2}{2}\hat{\phi}^2(x_i) + \frac{1}{2}[\phi\, \nabla_i^2 \phi](x) \right)}$$

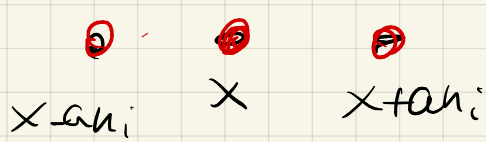
$$\nabla_i \phi(x) = \frac{\phi(x+an_i) - \phi(x)}{a}$$



$$\nabla_i^2 \phi(x) = \frac{\phi(x+an_i) + \phi(x-an_i) - 2\phi(x)}{a^2}$$

$$[\phi \nabla_i^2 \phi](x) = \phi(x) \left[\frac{\phi(x+an_i) + \phi(x-an_i) - 2\phi(x)}{a^2} \right]$$

$$= \frac{1}{a^2} \left[\phi(x)\phi(x+an_i) + \phi(x)\phi(x-an_i) - 2\phi(x)^2 \right]$$



- $[\phi(x_i), \Pi(x_j)] = i\,\delta_{ij}\,\mathbb{1}$

- $\hat{H} = \frac{1}{2}\hat{\Pi}^2 + \frac{m_0^2}{2}\hat{\phi}^2$

$$\hat{\phi} \to \frac{1}{\sqrt{m_0}}\hat{\phi} \qquad \hat{\Pi} \to \sqrt{m_0}\,\hat{\Pi}, \quad \hat{H} \to m_0 \hat{H}$$

$$\Rightarrow \boxed{\hat{H} = \frac{\hat{\Pi}^2}{2} + \frac{\hat{\phi}^2}{2}} \qquad [\phi, \Pi] = i$$

- $\hat{H}_{HO} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2$

$\hat{x} \to \frac{1}{\sqrt{m}\sqrt{\omega}} \hat{x}$

$\hat{p} \to \sqrt{m}\sqrt{\omega}\, \hat{p}$

$$\boxed{\hat{H}_{HO} = \frac{\hat{p}^2}{2} + \frac{\hat{x}^2}{2}}$$

$[x, p] = i$

$|\psi_n\rangle$

$\hat{H}|\psi_n\rangle = (n + \frac{1}{2})\hbar\omega\, |\psi_n\rangle$

$n = 0, 1, \ldots, \infty$

$H_{HO}^{(D)} = (n \times n)$ matrix

$\underline{1/2},\ \underline{3/2},\ \underline{5/2},\ \underline{7/2},\ \ldots$

$$\hat{H} = \begin{pmatrix} 1/2 & & & \\ & 3/2 & & \\ & & 5/2 & \\ & & & 7/2 \end{pmatrix} \hbar\omega$$

$$H_{HO} = \begin{pmatrix} 1/2 & & & & \\ & 3/2 & & & \\ & & 5/2 & & \\ & & & \ddots & \\ & & & & \infty \end{pmatrix} \underline{\hbar\omega}$$

$$\omega = \bar{\omega} + \delta x$$

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2$$

$$= \boxed{\frac{p^2}{2m} + \frac{1}{2} m\bar{\omega}^2 x^2} + \frac{1}{2} m(\bar{\omega}^2 - \omega^2)x^2$$

$$= \frac{p^2}{2m} - \frac{1}{2} m\bar{\omega}^2 x^2 - \frac{1}{2} m(\bar{\omega}\delta\omega - \delta\omega^2)x^2$$

relative error $\boxed{\frac{\delta\omega}{\omega}}$

$$\frac{L}{a} = \frac{E_{max}}{E_{min}} = \underline{n_L}$$

Tot number of lattice sites is $\left(\frac{E_{max}}{E_{min}}\right)^d$

$E_{max} = 7\,\text{TeV}$ $\qquad \frac{E_{max}}{E_{min}} \sim 70\,000$

$E_{min} = 1\,\text{GeV}$

$$(70000)^3 \sim 10^{14}$$

$$d_H \sim 4^{10^{14}}$$

$$H = \frac{1}{2}\hat{\pi}^2 + \frac{1}{2}\hat{\phi}^2 = H_\pi + H_\phi$$

Choice 2 ϕ-basis

$$\underline{\{|\phi\rangle\}} \qquad -\infty < \phi < \infty$$

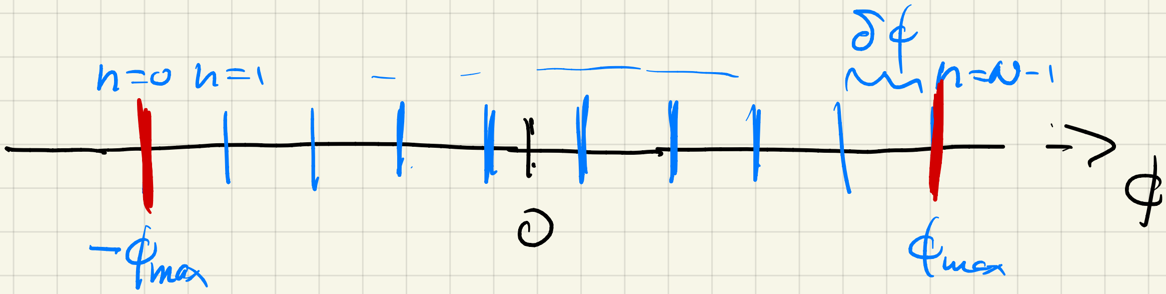
$$\hat{\phi}|\phi\rangle = \phi|\phi\rangle$$

$$\langle\phi'|\hat{\phi}^2|\phi\rangle = \phi^2 \underbrace{\langle\phi'|\phi\rangle}_{\delta_{\phi'\phi}}$$

$$\langle\phi'|H_\phi|\phi\rangle = \frac{\phi^3}{2}\,\delta_{\phi\phi'}$$

$$\langle\phi'|H_\pi|\phi\rangle = ?$$

$$[\hat{\phi}, \hat{\pi}] = i \implies \hat{\pi} = -i\frac{\partial}{\partial\phi}$$



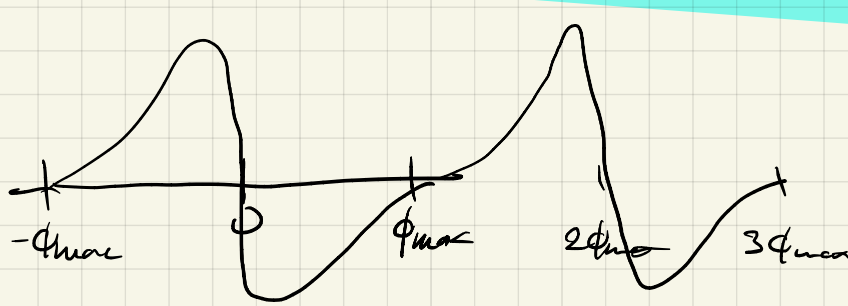
$$\phi_n = -\phi_{max} + n\,\delta\phi \qquad \delta\phi = \frac{2\phi_{max}}{\underline{\underline{N-1}}}$$

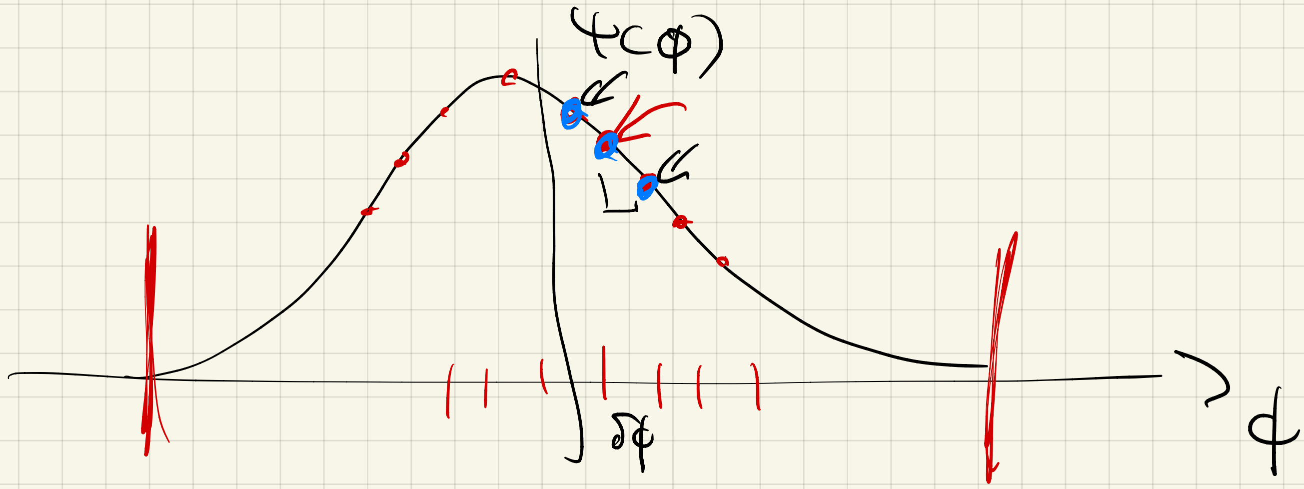
$n = 0, \cdots, N-1$

$$\{|\phi\rangle\} \longrightarrow \{|\phi_n\rangle\} \longrightarrow \{|n\rangle\}$$

$$\begin{aligned}
\langle n' | H_\phi | n \rangle &= \tfrac{1}{2}\phi_n^2 \langle n' | n \rangle \\
&= \tfrac{1}{2}\left(-\phi_{max} + n\,\delta\phi\right)^2 \delta_{n'n} \\
&= \frac{\phi_{max}^2}{2}\left(1 + 2n - 2N\right)^2 \delta_{nn'}
\end{aligned}$$

$$= \frac{\phi_{max}^2}{2} \begin{pmatrix} (1-2N)^2 & & & & \\ & (3-2N)^2 & & & \\ & & (5-2N)^2 & & \\ & & & \ddots & \\ & & & & (2N-1)^2 \end{pmatrix}$$





$$\pi = -i\frac{\partial}{\partial\phi} \Rightarrow \pi^2 = -\frac{\partial^2}{\partial\phi^2}$$

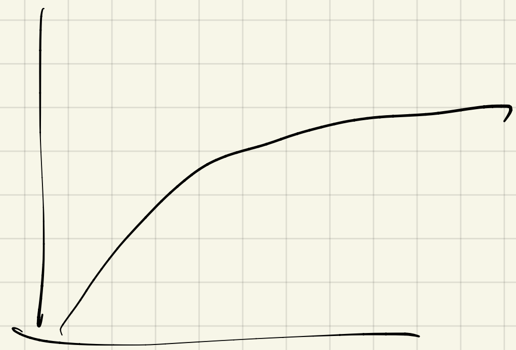
$$\hat{\pi}^2 = \frac{-1}{\delta\phi^2}\left[\phi_{n+1} + \phi_{n-1} - \underline{\underline{2\phi_n}}\right]$$

$$\xrightarrow{\delta\phi\to 0} -\frac{\partial^2}{\partial\phi^2}$$

$$\langle n'|\hat{\pi}^2|n\rangle = \boxed{\frac{1}{\delta\phi^2}\begin{bmatrix} 2 & -1 & & & -1 \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & & \ddots & \\ -1 & & & -1 & 2 \end{bmatrix}}$$

$$\langle n'|H|n\rangle = \langle n'|H_\phi|n\rangle + \langle n'|H_\pi|n\rangle$$

N	E_0
8	0.474
16	0.487
32	0.494
64	0.497
128	0.498



Can we do better?

$$i\frac{\partial}{\partial x} \xrightarrow{FT} p$$

$$i\frac{\partial}{\partial x} \xrightarrow{DFT} ?$$

<u>DFT</u> :

$$FT_{kn} = \frac{1}{\sqrt{N}} \exp\left[-\frac{2\pi i}{N} kn\right]$$

$$\begin{pmatrix} \frac{1}{\sqrt{N}} & \frac{1}{\sqrt{N}} & \frac{1}{\ } & - & - \\ \frac{1}{\sqrt{N}} & \frac{1}{\sqrt{N}} e^{-\frac{2\pi i}{N}} & & - & \\ & & - & & \end{pmatrix}$$

$$[FT]^{-1}_{kn} = \frac{1}{\sqrt{N}} \exp\left[\frac{2\pi i}{N} kn\right]$$

• Let's work out $\qquad x \leftrightarrow \phi$

$$FT\,\hat{\Pi}^2\,FT^{-1}$$

$$|\Pi\rangle = \underbrace{FT\,|\phi\rangle}$$

$$\langle\phi'|\,\hat{\Pi}^2\,|\phi\rangle$$

$$\langle\phi'|\Pi^2|\phi\rangle = \underbrace{\langle\phi'|FT^{-1}}_{\langle\Pi'|}\,FT\,\hat{\Pi}^2\,FT^{-1}\,\underbrace{FT|\phi\rangle}_{|\Pi\rangle}$$

$$= \langle\Pi'|\,FT\,\hat{\Pi}\,FT^{-1}\,|\Pi\rangle$$

$$\left[FT\,\hat{\Pi}^2\,FT^{-1}\right]_{nn'} = \left[FT\right]_{n\ell}\left[\hat{\Pi}^2\right]_{\ell\ell'}\left[FT^{-1}\right]_{\ell' n'}$$

$$= \frac{1}{\sqrt{N}}\exp\left[-\frac{2\pi i}{N}n\ell\right]\frac{1}{\delta\phi^2}\left[2\delta_{\ell\ell'} - \delta_{\ell'(\ell-1)} - \delta_{\ell'(\ell+1)}\right]$$

$$\frac{1}{\sqrt{N}}\exp\left[\frac{2\pi i}{N}n'\ell'\right]$$

$$= \frac{1}{N}\,\frac{1}{\delta\phi^2}\,\exp\left[-\frac{2\pi i}{N}n\ell\right]$$

$$\times\left[2\exp\left[\frac{2\pi i}{N}n'\ell\right] - \exp\left[\frac{2\pi i}{N}n'(\ell-1)\right]\right.$$

$$\left. - \exp\left[\frac{2\pi i}{N}n'(\ell+1)\right]\right]$$

$$= \frac{1}{N}\frac{1}{\delta\phi^2}\overbrace{\sum_l \exp\left[-\frac{2\pi i}{N}\,\underline{nl}\right]\exp\left[\frac{2\pi i}{N}\,\underline{n'l}\right]}$$

$$\times\left[2-\exp\left[-\frac{2\pi i}{N}n'\right]-\exp\left[+\frac{2\pi i}{N}n'\right]\right]$$

$$= \frac{1}{N}\frac{1}{\delta\phi^2}\underbrace{\sum_l \exp\left[\frac{2\pi i}{N}\,\underline{l(n-n')}\right]}_{N\delta_{nn'}}$$

$$\times\left[2-\exp\left[-\frac{2\pi i}{N}n'\right]-\exp\left[+\frac{2\pi i}{N}n'\right]\right]$$

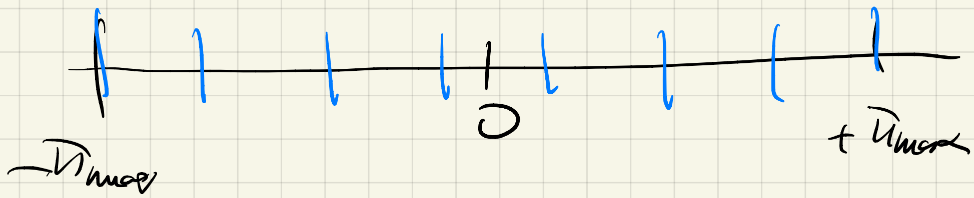
$$= \frac{1}{\delta\phi^2}\delta_{nn'}\left[2-\exp\left[-\frac{2\pi i}{N}n\right]-\exp\left[\frac{2\pi i}{N}n\right]\right]$$

$$= \frac{1}{\delta\phi^2}\delta_{nn'}\left[2-2\cos\left(\frac{2\pi n}{N}\right)\right]$$

$$= \boxed{\frac{4}{\delta\phi^2}\,\underline{\delta_{nn'}}\,\underline{\sin^2\left(\frac{\pi n}{N}\right)}}$$

- $\underline{|\pi_k\rangle} = \delta\phi\sum_n [FT]_{kn}\,|\phi_n\rangle$

$$\pi_k = -\pi_{max} + k\,\delta\pi \Rightarrow \underline{\delta\pi = \frac{2\pi}{N\delta\phi}}$$



$$\Pi_{max} = \frac{N-1}{2}\,\delta\pi$$

$\hat{\Pi}^2$ operator had $\frac{4}{\delta\phi^2}\sin^2\left(\frac{\pi n}{N}\right)$

$$\Pi_n = -\Pi_{max} + n\,\delta\pi = \left(\frac{N-1}{2} + n\right)\delta\pi = \left(\frac{N-1}{2} + n\right)\frac{\pi}{\phi_{max}}\frac{N-1}{N}$$

$$\frac{\pi}{2} + \left(\Pi_n - \frac{\delta\pi}{2}\right)\frac{\delta\phi}{2} = \Pi_n + \frac{\pi}{2} - \frac{\delta\pi\,\delta\phi}{4}$$

$$= \frac{\pi}{2} + \left(-\Pi_{max} + n\,\delta\pi - \frac{\delta\pi}{2}\right)\frac{\delta\phi}{2}$$

$$= \frac{\pi}{2} + \left(-\frac{N-1}{2} + n - \frac{1}{2}\right)\frac{\delta\pi\,\delta\phi}{2}$$

$$= \frac{\pi}{2} + \left(n - \frac{N}{2}\right)\frac{\pi}{N}$$

$$= \frac{\pi n}{N}$$

$$\frac{\pi n}{N} = \left(\Pi_n\,\delta\phi + \frac{\pi}{2} - \frac{\delta\pi\,\delta\phi}{2}\right)$$

$$\langle \pi_{k'} | \hat{\Pi}^2 | \pi_k \rangle = \delta_{kk'} \frac{4}{\delta\phi^2} \sin^2\left[\pi_n \frac{\delta\phi}{2} - \frac{\delta\phi\,\delta\pi}{2} + \frac{\pi}{2} \right]$$

$$= \delta_{kk'} \left(\pi_k - \frac{\delta\pi}{2} + \frac{\pi}{\delta\phi} \right)^2 + \dots$$

FT of $\hat{\Pi}^2$ linearized is equal to square of (shifted) momentum values.

$$[FT]_{kn} = \frac{1}{\sqrt{N}} \exp\left[\frac{2\pi i}{N} \left(k - \frac{N-1}{2}\right)\left(n - \frac{N}{2}\right) \right]$$

$$= \frac{1}{\sqrt{N}} \exp\left[i\,\delta\phi\,\delta\pi \left(k - \frac{N-1}{2}\right)\left(n - \frac{N}{2}\right) \right]$$

$$= \frac{1}{\sqrt{N}} \exp\left[i \phi_k \left(\pi_n - \frac{\delta\pi}{2} \right) \right]$$

$$\langle \pi_k | \hat{\Pi}^2 | \pi_{k'} \rangle = \delta_{kk'} \frac{4}{\delta\phi^2} \sin^2\left(\frac{\delta\phi}{2} \left(\pi_n - \frac{\pi}{2} \right) \right)$$

$$\rightarrow \delta_{kk'} \frac{4}{\delta\phi^2} \sin^2\left(\frac{\delta\phi\,\pi_k}{2} \right)$$

$$\langle \pi_k | \hat{\pi}^2 | \pi_{k'} \rangle = \delta_{kk'} \frac{\cancel{4}}{\cancel{\delta\phi^2}} \left(\frac{\cancel{\delta\phi}\, \pi_k}{\cancel{2}} \right)^2 + O(\pi_k^4)$$

$$= \delta_{kk'} \pi_k^2 + \underline{O(\pi_k^4)}$$

$$\langle \pi_k | \hat{\pi}^2 | \pi_{k'} \rangle \overset{!}{=} \delta_{kk'} \pi_k^2$$

$$\pi_k = -\pi_{max} + k\delta\pi$$

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1) Pick discrete ϕ basis

$\Rightarrow$ choose N, $\underline{\delta\phi}$, symmetric

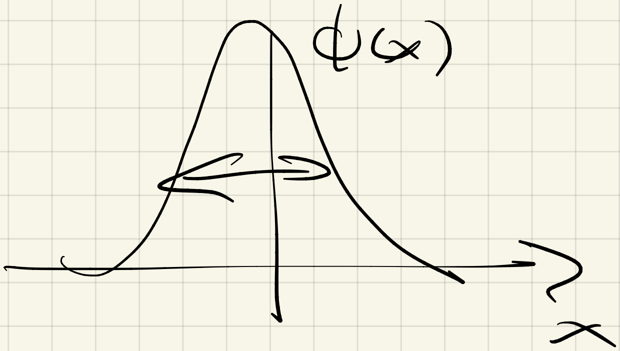
2) Choosen π values making symmetric around 0

3) $\langle \phi' | \hat{\phi}^2 | \phi \rangle = \underline{\delta_{\phi\phi'} \phi^2}$

$$\langle \phi' | \hat{\pi}^2 | \phi \rangle = FT\left[\delta_{\pi\pi'}\, \pi^2\right] FT^{-1}$$

$$H = \frac{1}{2}\phi^2 + \frac{1}{2}\pi^2 \qquad |\pi\rangle = FT\,|\phi\rangle$$

$$H = \frac{1}{2}x^2 + \frac{1}{2}p^2$$



$$\delta\phi\,\delta\pi = \frac{2\pi}{N} \qquad \delta\phi = \delta\pi$$

$$\delta\phi^2 = \frac{2\pi}{N} \Rightarrow \delta\phi = \sqrt{\frac{2\pi}{N}}$$

$$\boxed{H = \frac{1}{2} m_0^2 \phi^2 + \frac{1}{2}\pi^2}$$

rescaling

$$\Rightarrow \frac{1}{2}\phi^2 + \frac{1}{2}\pi^2$$

$$\hat{\phi} = \begin{pmatrix} \phi_0 & & & \\ & \phi_1 & & \\ & & \ddots & \\ & & & \ddots \end{pmatrix}$$

$$\hat{\Pi} = FT \begin{pmatrix} \ddots & & \\ & \ddots & \\ & & \ddots \end{pmatrix} FT^{-1}$$

$$\boxed{[\hat{\phi}, \hat{\Pi}] = i\mathbb{1}\,?}$$

$$\underline{N=8} = 2^3 = 2^{n_q}$$

$$\phi_k = \pm\underline{\phi_{max}}, \pm\underline{\tfrac{5}{7}}\phi_{max}, \pm\underline{\tfrac{3}{7}}\phi_{max}, \pm\underline{\tfrac{1}{7}\phi_{max}}$$

$$\hat{\phi} = \frac{\phi_{max}}{7}\sum_{j=0}^{2} 2^j \,\underline{\hat{\sigma}_z^{(j)}} \qquad \hat{\sigma}_z = \begin{pmatrix} 1 & \\ & \underline{-1} \end{pmatrix}$$

$$\hat{\sigma}^{(0)} \quad \hat{\sigma}^{1}$$

$$\hat{\phi}|000\rangle = \frac{\phi_{max}}{7}\left[2^0 + 2^1 + 2^2\right] = \phi_{max}\frac{7}{7}$$

$$\hat{\phi}|00\underline{1}\rangle = \frac{\phi_{max}}{7}\left[-2^0 + 2^1 + 2^2\right] = \phi_{max}\frac{5}{7}$$

$$\hat{\phi}\,|111\rangle = \frac{\phi_{max}}{7}\left[-2^0 - 2^1 - 2^2\right] = -\phi_{max}$$

$$\hat{\phi}^2 = \frac{\phi_{max}^2}{49}\left[2^0\sigma_z^{(0)} + 2^1\sigma_z^{(1)} + 2^2\sigma_z^{(2)}\right]^2$$

$$= \frac{\phi_{max}^3}{49}\left[21\,\mathbb{1} + \sum_{i<j} 2^{i+j+1}\,\sigma_z^{(i)}\sigma_z^{(j)}\right]$$

$$= \frac{\phi_{max}^3}{49}\left[21\,\mathbb{1} + 4\,\sigma_z\otimes\sigma_z\otimes\mathbb{1} + 8\,\sigma_z\otimes\mathbb{1}\otimes\sigma_z + 16\,\mathbb{1}\otimes\sigma_z\otimes\sigma_z\right]$$

$$e^{iHt} = e^{i\frac{1}{2}\phi^2 t} = e^{i\frac{\phi_{max}^2}{2\times 49}[\,---\,]}$$

$$= e^{i\frac{\phi_{max}^3}{2\times 49}}\exp\left[4i\,\sigma_z\otimes\sigma_z\otimes 1\right]\exp[\,\_\,]$$

$$\times\exp[\,\_\,]$$

$$e^{i\tilde{\alpha}(\sigma_z\otimes\sigma_z)}\,|00\rangle = e^{i\tilde{\alpha}}\,|00\rangle$$

$$e^{i\tilde{\alpha}\,\sigma_z\otimes\sigma_z}\,|11\rangle = e^{i\tilde{\alpha}}\,|11\rangle$$

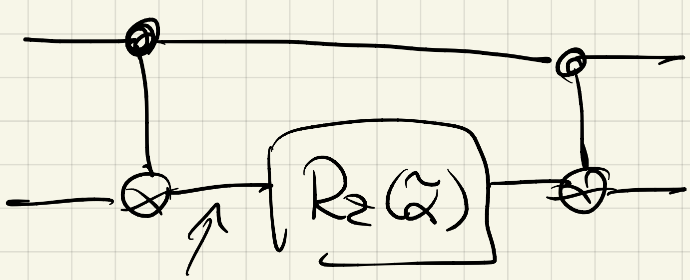
$$e^{i\tilde{\alpha}\, \sigma_z \otimes \sigma_z} \Big| \; |01\rangle = e^{-i\tilde{\alpha}}\, |01\rangle$$

$$|10\rangle = e^{-i\tilde{\alpha}}\, |10\rangle$$

$$R_z(\alpha) = \begin{pmatrix} e^{i\alpha} & \\ & e^{-i\alpha} \end{pmatrix}$$

$$R_z|0\rangle = e^{i\alpha}\, |0\rangle$$

$$R_z|1\rangle = e^{-i\alpha}\, |1\rangle$$



$0 \quad e^{i\tilde{\alpha}}$

$1 \quad e^{-i\tilde{\alpha}}$