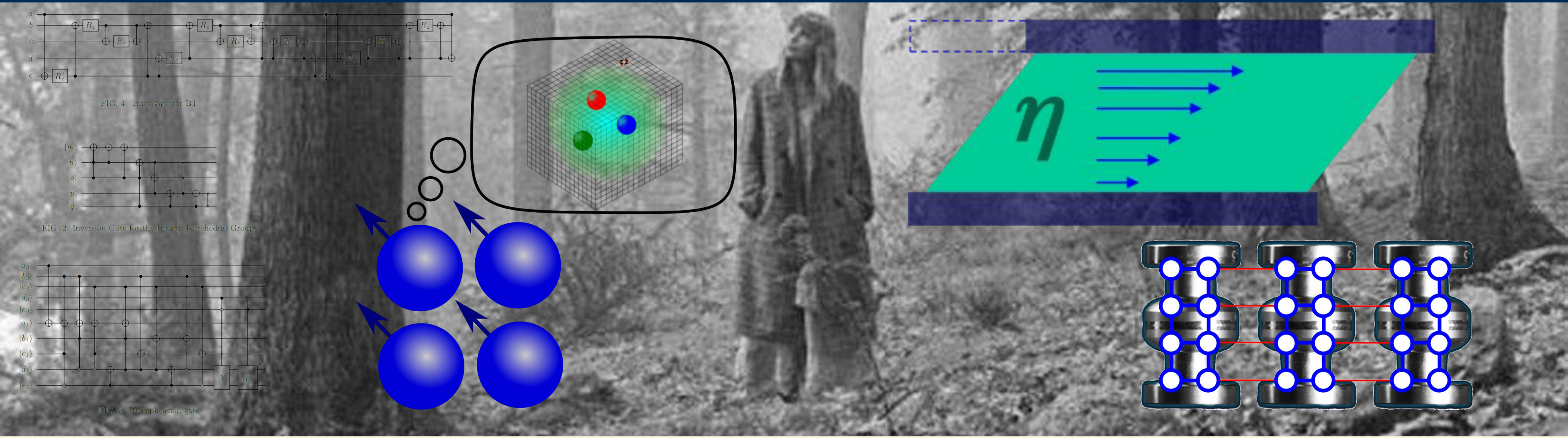


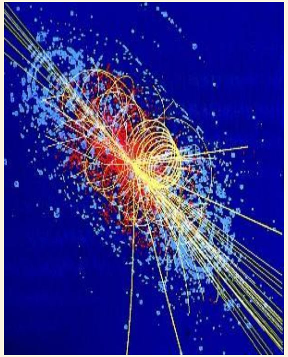
I’m the problem, it’s me: Viscosity in 3+1d QCD

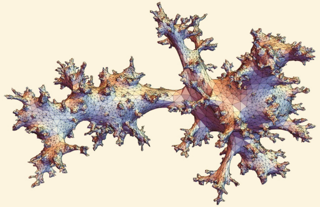
Hank Lamm

June 27, 2023

Quantum Computing for Particle Physics, it's a need

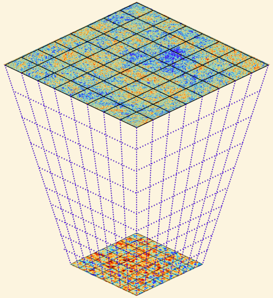
- **The world is quantum, and we are lucky anything is amenable to classical computers**
  - Large-scale quantum computers can tackle computations in HEP otherwise inaccessible
  - This opens up new frontiers & extends the reach of LHC, LIGO, EIC & DUNE

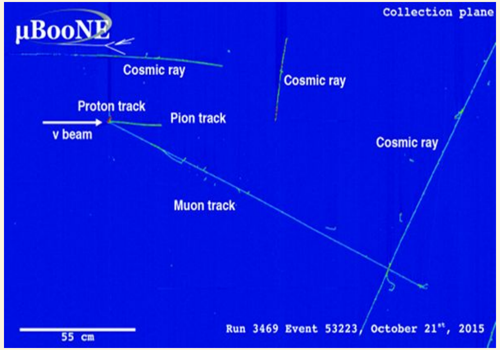




- ***Ab initio* cross sections for colliders and neutrino experiments**
- **Cosmic inflation and the evolution of matter asymmetry in the early universe**
- **Explorations of BSM, supersymmetry, and quantum gravity**
- **Hadronization and Hydrodynamics in Heavy-Ion collisions**

While broad, these topics often are formulated as **lattice field theories**





Quantum Simulation for High-Energy Physics
Bauer, Davoudi *et al.* - *PRX Quantum 4 (2023) 2, 027001*
Wonderful survey of physics questions, methods, and outstanding problems in field

Don’t let anyone fool you…

There is **so much** to be done

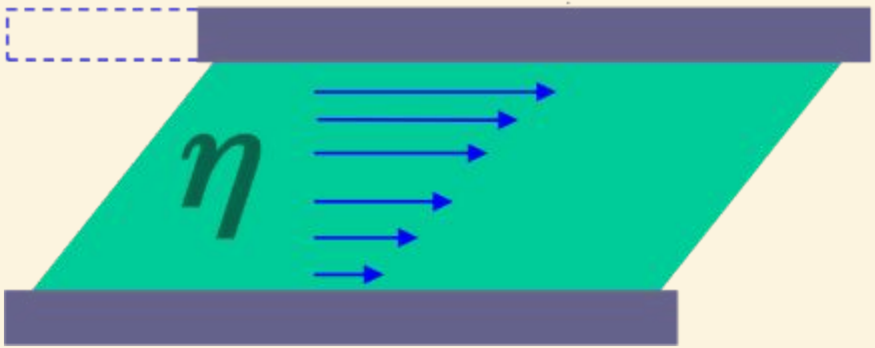
As a target, today we are going to consider the viscosity of QCD

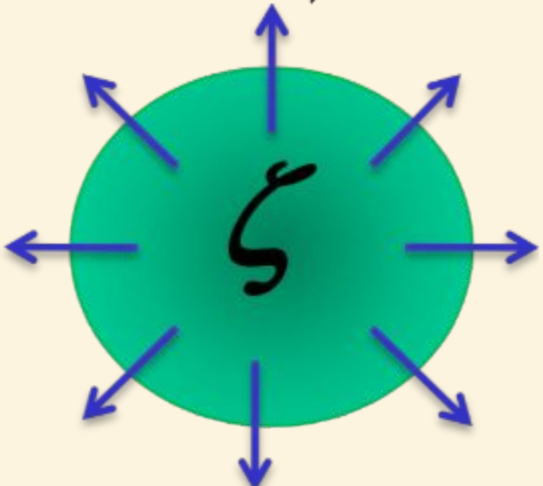
- $\eta = \frac{V}{T}\int_0^\infty \langle T_{12}(t) T_{12}(0)\rangle$
- I believe its a “near-term” goal and allows for focus…
- …while introducing all the necessary pieces

Quantum algorithms for transport coefficients in gauge theories
NuQS Collaboration - *Phys.Rev.D 104 (2021) 9, 094514*
Formulates lattice operators and propose correlators

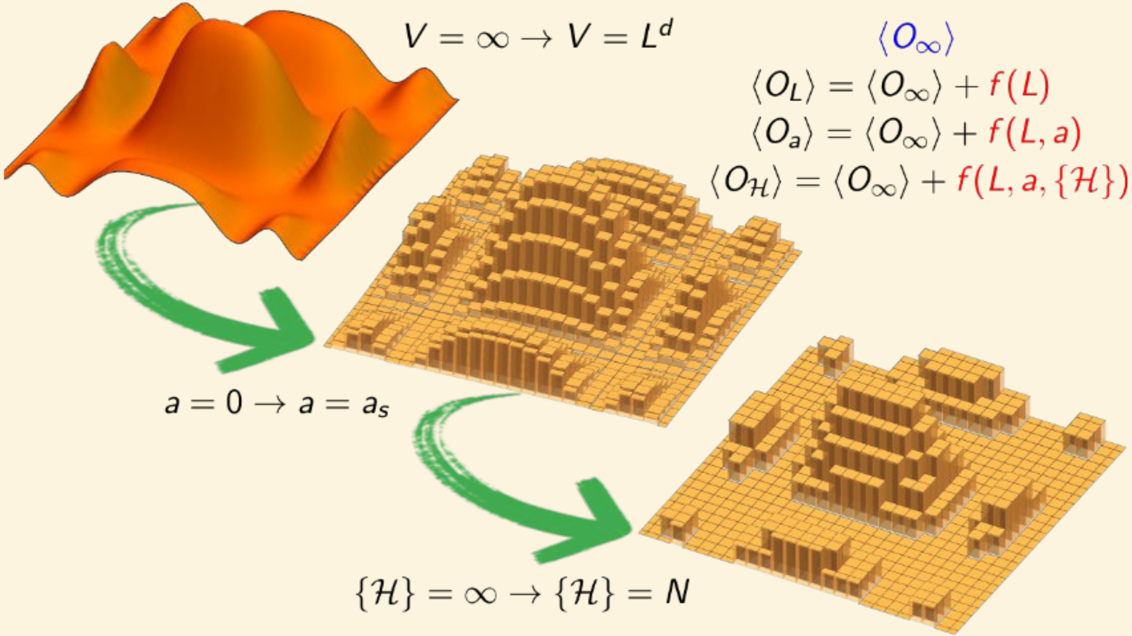
Viscosity of pure-glue QCD from the lattice
Altenkort *et al.* - *2211.08230 [hep-lat]*
State of the art lattice results, but massive uncertainties persist

$$\eta/s = 0.15 - 0.48, T = 1.5T_c$$
$$\zeta/s = 0.017 - 0.059, T = 1.5T_c$$





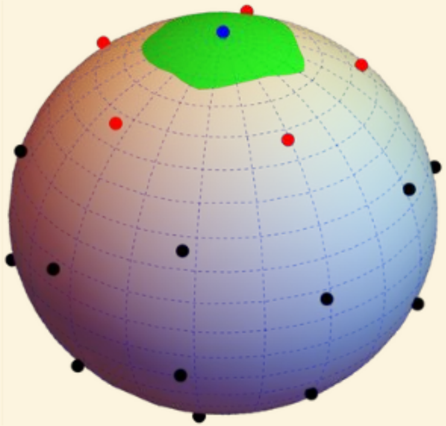
Take it to the limit

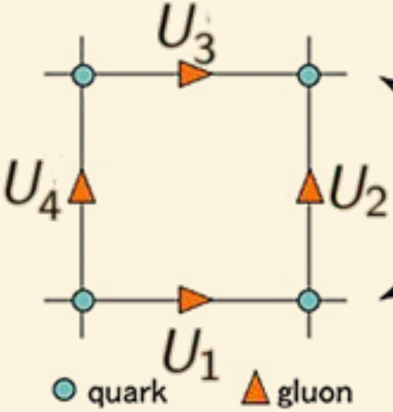


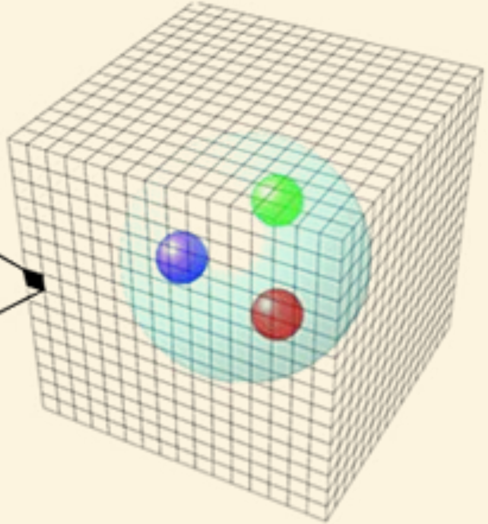
- O(L,a,$\mathcal{H}$) is an **approximation** for HEP
- Truncations leads to **systematic errors**
- **Extrapolating** is done on results, reducing computational resources…
- …but **obscures** precise resource estimates

Qubit Costs for Lattice Field Theory

- Lattice field theory discretizes spacetime into a lattice of size $(La)^d$
  - $L \to \infty$ and $a \to 0$ must be taken
- Matter fields are placed on sites, gauge fields on links
  - Fermionic matter need **Ƒ=Spin x Color x Flavor qubits per site** e.g. 12 for staggered QCD
  - Gauge links are bosonic and need efficient truncation **Λ qubits per link** e.g. SU(3) ~ **???**q
  - Scalar (bosonic) matter is infinite-dimensional, so must be truncated as well
- So qubit cost is: $(d\Lambda + \digamma)(L/a)^d$

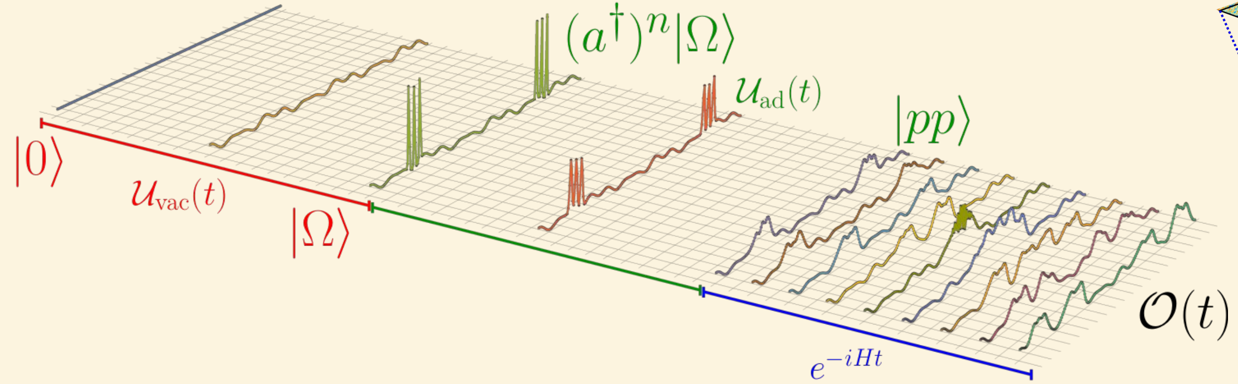


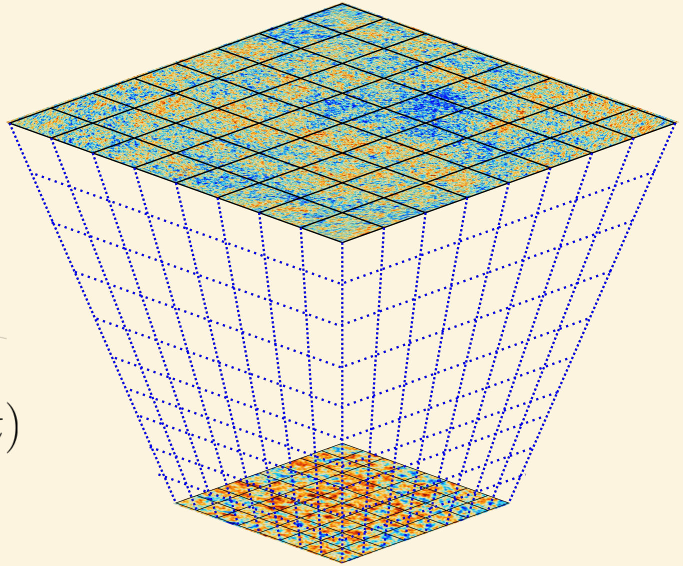




Gate Costs for Lattice Field Theory

- Lattice field theory approximates $U(T) = e^{-iHT}$ which *can* corresponds to into a lattice of size Ta_t
  - $a_t \rightarrow 0$ or equivalent limit must be taken
  - Trottertization has this property, others less clear i.e. potentially variable temporal spacing
- Gate cost is heuristically: $\frac{T}{a_t} \times [\mathcal{O}(1)(d\Lambda + F)(L/a)^d]^{\mathcal{O}(1)}$





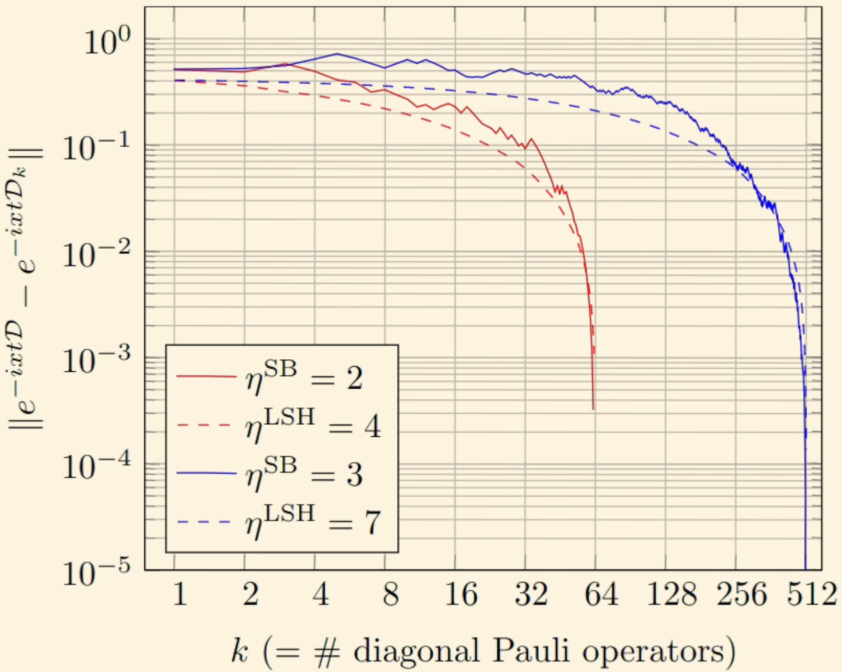
Exercise 1: What will viscosity take?

Qubits: $(d\Lambda + \digamma)(L/a)^d$ Gates: $\frac{T}{a_t} \times [\mathcal{O}(1)(d\Lambda + \digamma)(L/a)^d]^{\mathcal{O}(1)}$

- d=3
- What is Ƒ? (Staggered=12 Wilson=24)
  - Note: **a_t scaling of errors**
- How will you truncate Λ? (9 64-bit ℂ floats = 1152)
  - Note: **truncation errors**
- How small will you take a? (1fm^{-1}~ 200 MeV)
  - Note: **discretization errors**
- How large will you take L?
  - Note: **finite volume errors**
- Gate cost prefactor ~ 10 and exponent~2
- How small will you take a_t?
  - Note: **Trotter errors**
- How long do you need to run for (T)?
  - Note: **Signal resolution errors**

What didja get?

- Qubit costs: 10^3-10^9
  - 10q for SU(3) might be reasonable
  - a~0.5 fm, L~3 fm
  - Perhaps we drop fermions
  - Perhaps lower dimensions
- Gate costs: 10^7-10^{40}
  - a_t~0.1 fm, T~1 fm
  - Quantum arithmetic can hurt
  - Perhaps sloppy synthesis
  - Perhaps improved algorithms



General quantum algorithms for Hamiltonian simulation with applications to a non-Abelian lattice gauge theory
Davoudi, Shaw, Stryker - 2212.14030 [hep-lat]
Understanding the synthesis and Trotter errors, along with algorithmic choices in 1+1 SU(2)

But we don't today have a good sense of **theoretical** errors…

Exercise 2: What gate fidelities do you need?

- Consider your gate cost N_g
- Assume that every gate has a infidelity of p
- "Simulation fidelity" is $(1-p)^{N_g}$ i.e the probability your result is without error.

What must p be such that the simulation fidelity is 50%

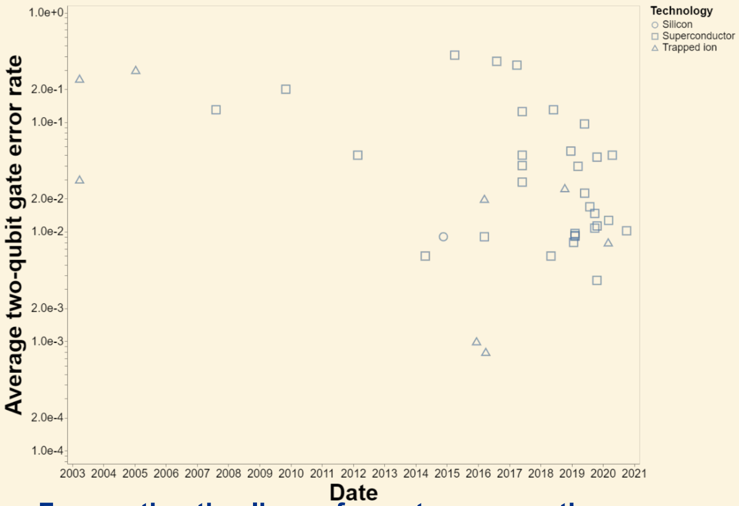
Exercise 2: What gate fidelities do you need?

- Consider your gate cost N_g
- Assume that every gate has a infidelity of p
- "Simulation fidelity" is $(1-p)^{N_g}$ i.e the probability your result is without error.

What must p be such that the simulation fidelity is 50%

10^{-8}-10^{-40}

Today, we talk about p ~10^{-3}



Forecasting timelines of quantum computing
Sevilla and Riedel - 2009.05045 [quant-ph]
Current hardware properties and projections for future

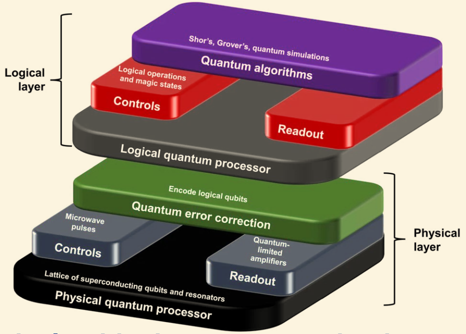
Noisy Intermediate-Scale Quantum vs Fault-Tolerance

NISQ

- Exists today!
- Limited number of qubits
  - Probably $<10^4$
- Basic gate set is native one
  - Often included arbitrary rotations
- Speed limited by 2q gate
- Errors tolerated or mitigated
  - Probably $>10^{-7}$
  - Measurement slow
  - Count CNOTs

FT

- Scalable, networked qubits
  - No limits on number of logical qubits
- Requires error correction
  - Potentially huge overhead
  - Threshold error rates
  - Measurement + Classical compute
- Gate set limited
  - Must synthesize
  - Count nontransverse T-gates



Building logical qubits in a superconducting quantum computing system
Gambetta, Chow, Steffen - npj Quantum Information 3, 2 (2017)
Discusses possible architectures for FT devices

Your paradigm will greatly affect your research projects

Hamiltonians for Nonabelian Gauge Theories in the Continuum

H in terms of CEM fields $$H = \int d^d x \, \mathrm{Tr}(\mathbf{E}^2 + \mathbf{B}^2)$$

Fields & Field-strength tensor $$E_i = \frac{1}{2} F_{ii} \qquad B_i = \frac{1}{2}\epsilon_{ijk} F_{jk}$$

FS tensor & gluon field $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu, A_\nu]$$

Chromo-field components $$E_i = \lambda^a E_i^a \qquad B_i = \lambda^a B_i^a$$

Gell-Mann matrices $$\mathrm{Tr}(\lambda^a \lambda^b) = \frac{1}{2}\delta_{ab} \qquad [\lambda^a, \lambda^b] = i f^{abc} \lambda^c$$

Improvement and analytic techniques in Hamiltonian lattice gauge theory
Carlsson - PhD thesis, 0309138 [hep-lat]
Derivation of KS and Improved Hamiltonians and variational techniques

Approximating gauge fields

- For *reasons* of gauge symmetry, discretizing A_μ is fraught with danger
- Instead, define an *average* A_μ along a link in direction μ as

$$\mathcal{A}_\mu = \frac{1}{a}\int_\mu d\mathbf{x}\cdot\mathbf{A}$$

- On the lattice, this definition leads to a discretization error since the field at all points between $\mathbf{x}$ & $\mathbf{x}+\hat{\mu}$
- Since we are considering lattice Hamiltonians, for now we restrict ourselves to the spatial lattice with latin indices i,j,k…
- $$\mathcal{A}_i = \frac{1}{a}\int_{-a/2}^{a/2} dx_i[A_i(\mathbf{x}) + x_i\partial_i A_i(\mathbf{x}) + \frac{1}{2}x_i^2 A_i(\mathbf{x}) + \cdots]$$
$$= A_i(\mathbf{x}) + \frac{a^2}{24}\partial_i^2 A_i(\mathbf{x}) + \frac{a^4}{1920}\partial_i^4 A_i(\mathbf{x}) + \cdots$$

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Wilson lines and gauge links

- To avoid the dangers of using A_μ we use the average to define a gauge link which is a *Wilson line*:

$$U_l = e^{iea\mathcal{A}_l}$$

- Taylor expanding and using relations between $\mathcal{A}_l$ & $A_l(x)$, we see

$$U_l(\mathbf{x}) = 1 + ieaA_l(\mathbf{x}) - \frac{e^2a^2}{2!}A_l(\mathbf{x})A_l(\mathbf{x}) + \cdots$$

- As we will see, while this can reproduce the continuum theory when a=0, at finite lattice spacing, there will be new interactions in the Hamiltonian

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Commutation relations

- We would like to recover in the continuum

$$[E_i^a(\mathbf{x}), A_j^b(\mathbf{x})] = i\delta_{ij}\delta_{ab}\delta(\mathbf{x}-\mathbf{y})$$
$$[A_i^a(\mathbf{x}), A_j^b(\mathbf{x})] = [E_i^a(\mathbf{x}), E_j^b(\mathbf{x})] = 0$$

- To see how to define our lattice kinetic term, we should investigate $\mathcal{E}_l^a$ one way this can be done is through the lattice commutator

$$[\mathcal{E}_l^a, U_m] = [\mathcal{E}_l^a, e^{iea\mathcal{A}_m}]$$

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Lattice commutation relations

Using a BCH relation:

$$e^{-A}Be^{A} = B + [B, A] + \frac{1}{2!}[[B, A], A] + \frac{1}{3!}[[[B, A], A], A] + \cdots$$

It is possible to show:

$$\begin{aligned}[\mathcal{E}_l^a, U_m] =& [\mathcal{E}_l^a, e^{iea\mathcal{A}_m}] \\ =& iea[\mathcal{E}_l^a, \mathcal{A}_m]U_m\end{aligned}$$

Check this for yourself tonight!

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Lattice electric field

Now, using the definition,

$$\mathcal{A}_i = A_i(\mathbf{x}) + \frac{a^2}{24}\partial_i^2 A_i(\mathbf{x}) + \frac{a^4}{1920}\partial_i^4 A_i(\mathbf{x}) + \cdots$$

We find that the the commutator with the continuum field is:

$$[\mathcal{E}_l^a, U_m] = iea[\mathcal{E}_l^a, A_m^b + \frac{a^2}{24}\partial_m^2 A_m^b + \cdots]\lambda^b U_m$$

Which implies that to ensure the continuum relations, we should associate:

$$\mathcal{E}_l^a = -\frac{a^{d-1}}{e}[E_l^a - \frac{a^2}{24}\partial_i^2 E_i^a + \cdots]$$

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Derivation of KS and Improved Hamiltonians and variational techniques

Lattice Kinetic Energy

- With this definition and imposing gauge invariance, we find:

$$\mathrm{Tr}[\mathbf{E}^2(\mathbf{x})] \approx$$
$$\frac{g^2}{2a}\mathrm{Tr}[X\mathcal{E}_i(\mathbf{x})\mathcal{E}_i(\mathbf{x}) + Y\mathcal{E}_i(\mathbf{x})U_i(\mathbf{x})\mathcal{E}_i(\mathbf{x}+a\hat{i})U_i^\dagger(\mathbf{x})]$$

- Expanding E and U in terms of their continuum fields, we find

$$K = \frac{X+Y}{2}E_i^2 + \frac{5Y-X}{12}E_i\partial_i^2 E + \mathcal{O}(ea^2, a^4)$$

- Setting X=1, Y=0 we obtain the KS kinetic term with errors scaling with a^2

Improvement and analytic techniques in Hamiltonian lattice gauge theory
Carlsson - PhD thesis, 0309138 [hep-lat]
Derivation of KS and Improved Hamiltonians and variational techniques

Exercise 3: Improved Lattice Kinetic Energy

- What values of X,Y would cancel of all classical a^2 errors?

$$K = \frac{X+Y}{2}E_i^2 + \frac{5Y-X}{12}E_i\partial_i^2 E + \mathcal{O}(ea^2, a^4)$$

Improvement and analytic techniques in Hamiltonian lattice gauge theory
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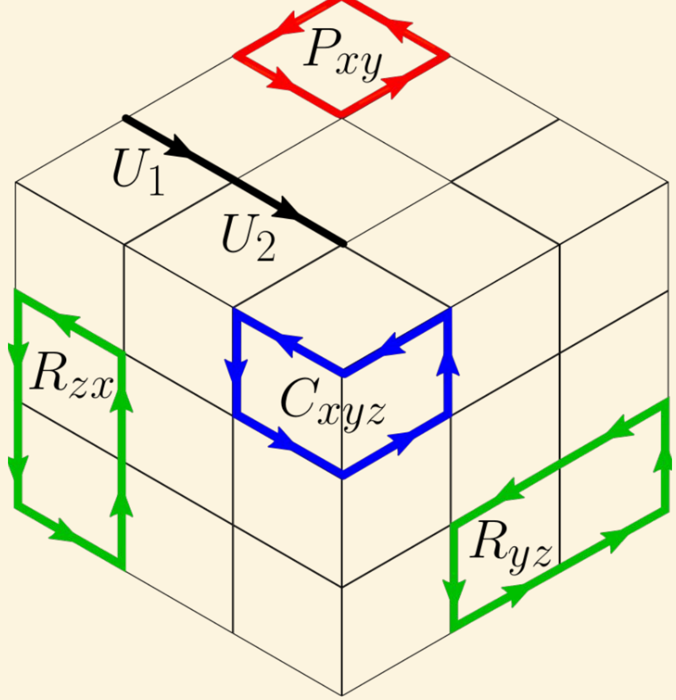
Lattice Potential Energy

- Constructed form closed loops of Wilson lines



- The simplest nontrivial *Wilson loop* is the plaquette:

$$P_{xy} = 1 - \frac{1}{N}\mathrm{ReTr}[U_x(\mathbf{x})U_y(\mathbf{x}+a\hat{\mathbf{x}})U_x^\dagger(\mathbf{x}+a\hat{\mathbf{y}})U_y^\dagger(\mathbf{x})]$$

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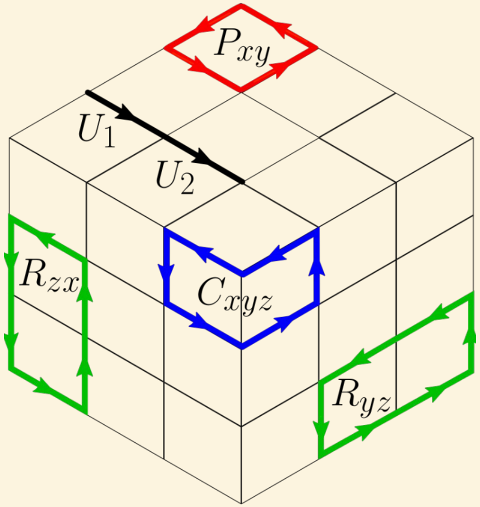
Lattice Potential Energy

- Including R_{ij} and R_{ji} yields:

$$V = \frac{2N}{ag^2}\left[XP_{ij}(\mathbf{x}) + \frac{Y}{2}(R_{ij}(\mathbf{x}) + R_{ji}(\mathbf{x})\right]$$

- Which can be related to the continuum, obtaining:

$$V \approx a^d\left[(X+4Y)\mathrm{Tr}(F_{ij}^2) + \frac{a^2}{12}(X+10Y)\mathrm{Tr}(F_{ij}\{D_i^2 + D_j^2\}F_{ij}) + \mathcal{O}(e^2a^2, a^4)\right]$$



- So if you are satisfied with a^2 errors, X=1,Y=0 yields the KS Hamiltonian

Improvement and analytic techniques in Hamiltonian lattice gauge theory
Carlsson - PhD thesis, 0309138 [hep-lat]
Derivation of KS and Improved Hamiltonians and variational techniques

Exercise 4:

- What values of X and Y will yield an a^2 improved Hamiltonian?

$$V \approx a^d[(X + 4Y)\mathrm{Tr}(F_{ij}^2) + \frac{a^2}{12}(X + 10Y)\mathrm{Tr}(F_{ij}\{D_i^2 + D_j^2\}F_{ij}) + \mathcal{O}(e^2a^2, a^4)]$$

Improvement and analytic techniques in Hamiltonian lattice gauge theory
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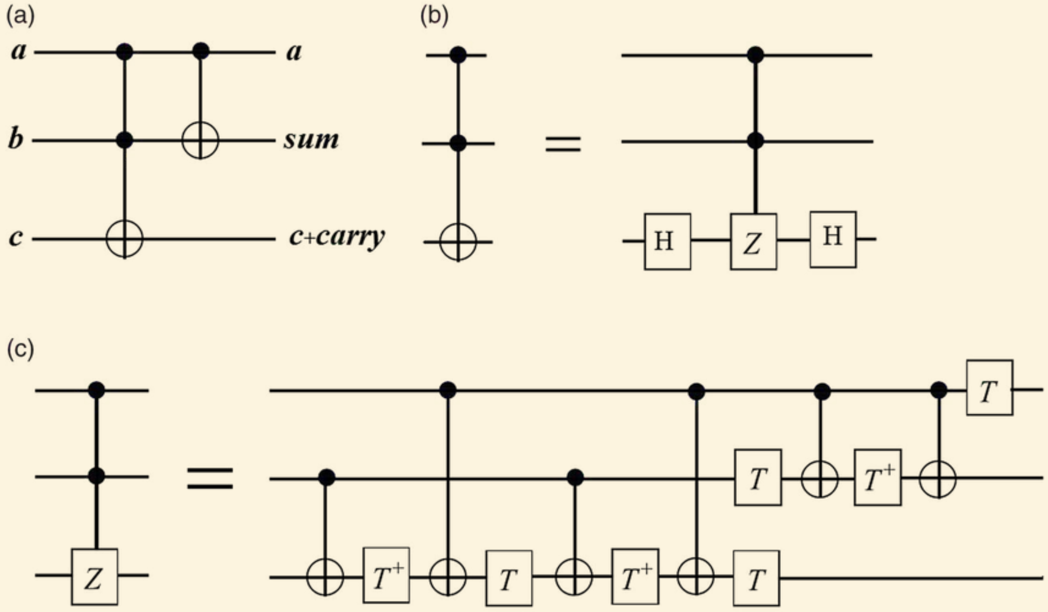
Regardless of your choice, you will need to do some math

- e.g. V=Tr(g)
- Floating point or fixed point arithmetic is expensive in qubits and gates
- Consider the half-adder



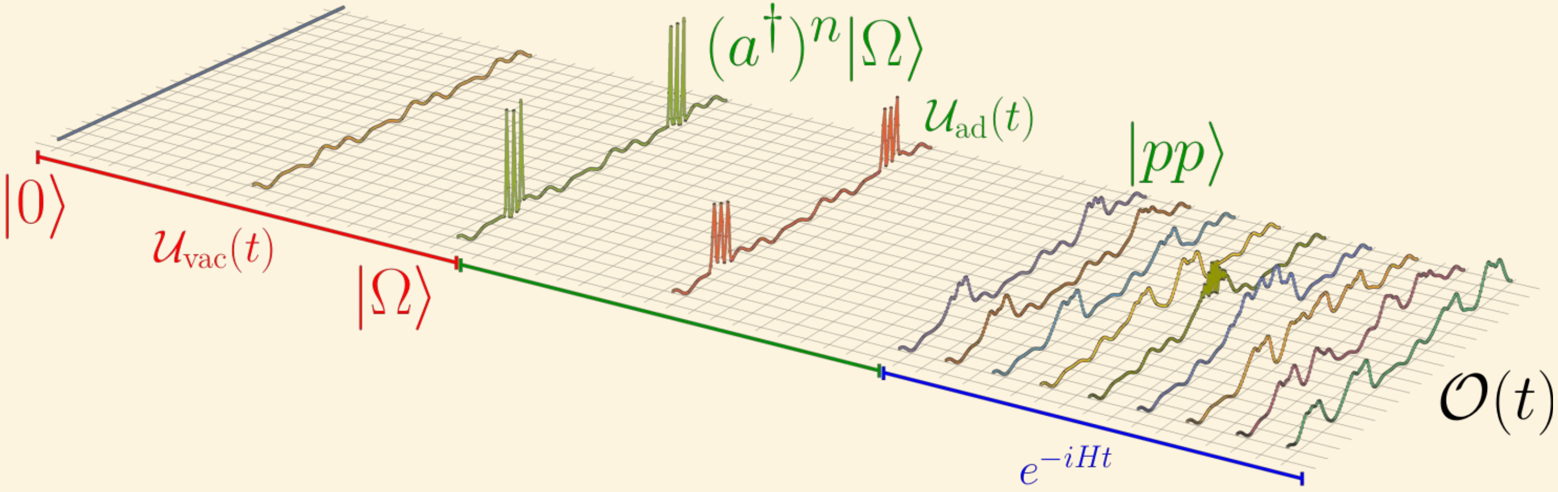
A transmon-based quantum half-adder scheme
Chatterjee and Roy- PTEP 2015 9, September 2015, 093A02
Described a specific hardware implementation of the general half-adder algorithm

Questions?

Exercise 5: Implement the Adders

- Take a look at ***lab_quantum_adder.ipynb***

…but I really want physics



99.998% cost is **QFOPs** for **< 3 yrs** on an **exascale** quantum computer.

Lattice Quantum Chromodynamics and Electrodynamics on a Universal Quantum Computer
Kan and Nam - 2107.12769 [quant-ph]
Rough, conservative, model- and algorithm-dependent estimates for viscosity and heavy-ion collisions

Primitives as a construction method

- These are lattice ***gauge*** theories, so we need to ability to perform group operations on the local registers
  - Think **native gates** for gauge theories

- U_i and E_i are conjugates related* by **group Fourier transform** (gFT)
  - *Depending on your digitization, the exact conjugate relations can be broken, in which cases there is an approximate gFT

- Further, group theory and gauge invariance requires:
  - **Inversion**: g -> g^{-1}
  - **Multiplication**: g,h -> gh
  - **Trace**: Tr(g)

General Methods for Digital Quantum Simulations of Gauge Theories
Lamm, Lawrence, Yamauchi - *Phys.Rev.D* 100 (2019) 3, 034518
Constructed this general formalism for group independent implementation

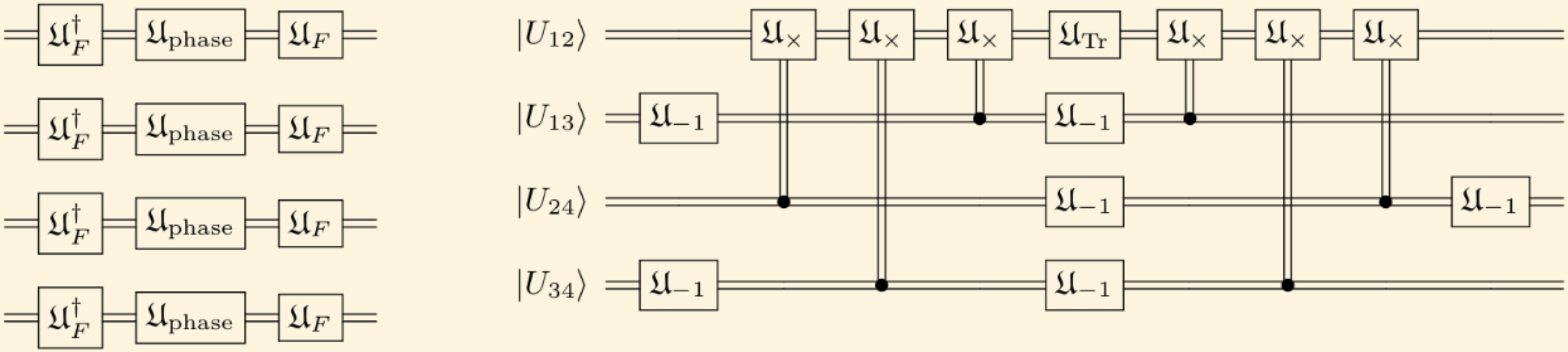
Primitives as gates

- Inversion gate: $\mathfrak{U}_{-1}\left|g\right\rangle = \left|g^{-1}\right\rangle$
- Multiplication gate: $\mathfrak{U}_{\times}\left|g\right\rangle\left|h\right\rangle = \left|g\right\rangle\left|gh\right\rangle$
- Trace gate $\mathfrak{U}_{\mathrm{Tr}}(\theta)\left|g\right\rangle = e^{i\theta\,\mathrm{Re}\,\mathrm{Tr}\,g}\left|g\right\rangle$
- Fourier Transform gate: $\mathfrak{U}_F\sum_{g\in G} f(g)\left|g\right\rangle = \sum_{\rho\in\hat{G}}\hat{f}(\rho)_{ij}\left|\rho, i, j\right\rangle$

General Methods for Digital Quantum Simulations of Gauge Theories
Lamm, Lawrence, Yamauchi - *Phys.Rev.D* 100 (2019) 3, 034518
Constructed this general formalism for group independent implementation

Circuits for Kogut-Susskind without regard for connectivity

- With these gates, the time evolution operators are given for Kogut-Susskind by:

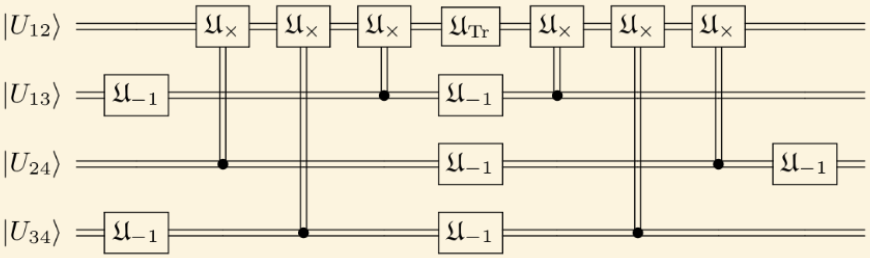


- Need **Λ A2A in-register** and **1:(2d) register** connectivity

General Methods for Digital Quantum Simulations of Gauge Theories
Lamm, Lawrence, Yamauchi - *Phys.Rev.D* 100 (2019) 3, 034518
Constructed this general formalism for group independent implementation

Exercise 6: $U_{V,KS}$ with only linear register connectivity

- Real hardware commonly has limited connectivity.
- The $U_{V,KS}$ assumed 1 register per plaquette could be coupled to the other 3

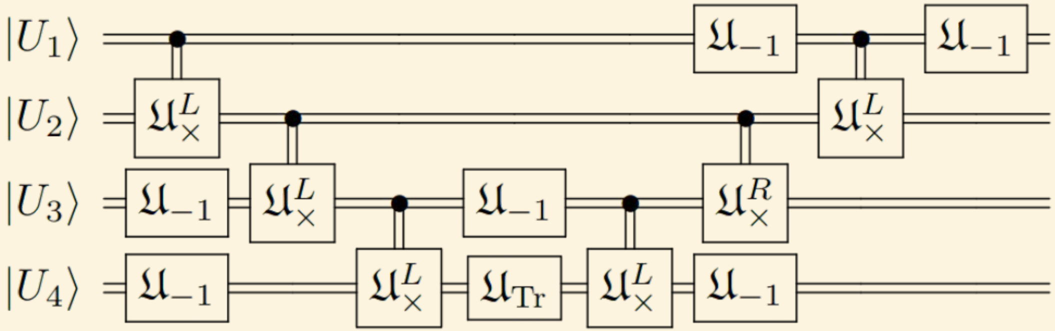


- Inversion gate: $\mathfrak{U}_{-1}|g\rangle = |g^{-1}\rangle$
- Multiplication gate: $\mathfrak{U}_\times |g\rangle|h\rangle = |g\rangle|gh\rangle$
- Trace gate $\mathfrak{U}_{\mathrm{Tr}}(\theta)|g\rangle = e^{i\theta\,\mathrm{Re\,Tr}\,g}|g\rangle$

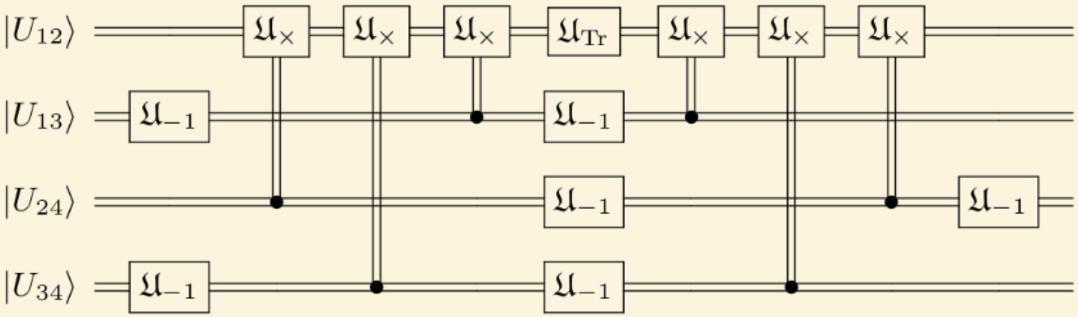
- **Can you construct a $U_{V,KS}$ where only linear (nearest-neighbor) register interactions?**
  - **It might prove useful to consider** $\mathcal{U}_\times^R|g\rangle|h\rangle = |gh\rangle|h\rangle$

Exercise 6: $U_{V,KS}$ with only linear register connectivity

- One possible solution:



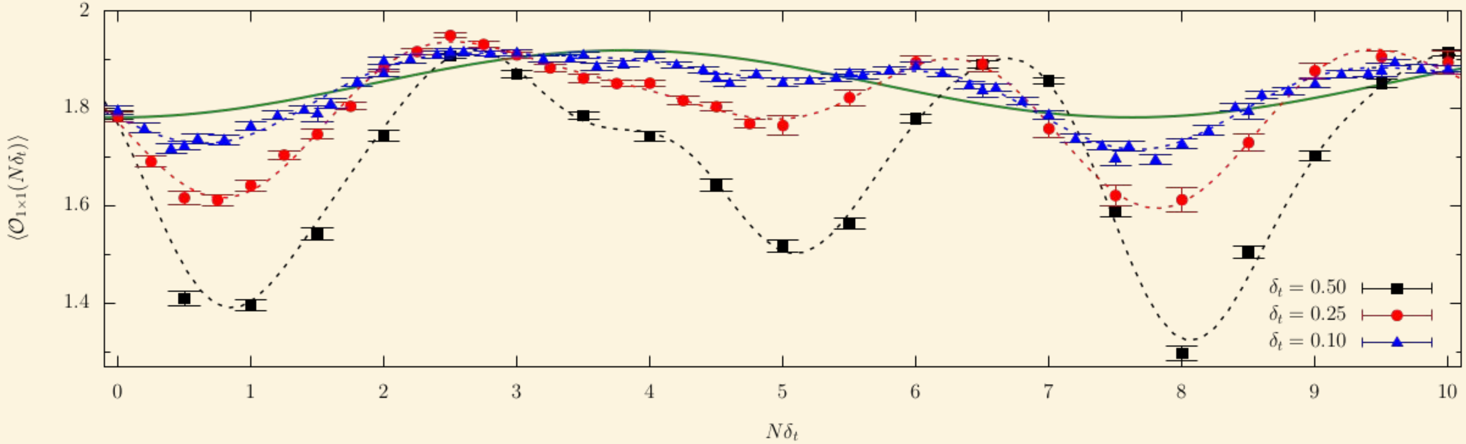
- Notice difference to previous, including total clock cycles



What is trotterization?

$$\mathcal{U}(t) = e^{-iHt} \approx \left(e^{-i\delta t\frac{H_V}{2}} e^{-i\delta t H_K} e^{-i\delta t\frac{H_V}{2}}\right)^{\frac{t}{\delta t}}$$

$$\approx \exp\left\{-it\left(H_K + H_V + \frac{\delta t^2}{24}(2[H_K,[H_K,H_V]] - [H_V,[H_V,H_K]])\right)\right\}$$



- δt is bare $c(a, a_t)$ **not** physical a_t
- Introduces **higher dimension operators**

Lattice renormalization of quantum simulations
Carena, Lamm, Li, Liu - *Phys.Rev.D 104 (2021) 9, 094519*
Investigated trotterization, renormalization, and Euclidean calculations

How to estimate Trotter errors

General quantum algorithms for Hamiltonian simulation with applications to a non-Abelian lattice gauge theory
Davoudi, Shaw, Stryker - 2212.14030 [hep-lat]
Understanding the synthesis and Trotter errors, along with algorithmic choices in 1+1 SU(2)

- Loose error bounds obtained from

$$||U(t) - U_{trott}(t))|| \leq (\delta t)^n \sum_{i,j,\cdots} [[H_i, H_j], \cdots]$$

- **Overly** conservative: cutoff states are largest eigenvalues
  - Empirically, we find MUCH smaller

State-dependent error bound for digital quantum simulation of driven systems
Hatomura - PRA 105, L050601 (2022)
Compares trotter errors for given initial state to norm-based estimates

$$\begin{aligned}
\|\mathcal{O}_3\| &= \left\| \left[H_I^{(j)}(r), [H_I^{(j)}(r), H_I^{(k)}(r)] \right] \right\| && \leq 4x^3 \quad (k > j), \\
\|\mathcal{O}_5\| &= \left\| \left[H_I^{(j)}(r), [H_I^{(j)}(r), H_I^{(k)}(r+1)] \right] \right\| && \leq 4x^3, \\
\|\mathcal{O}_{13}\| &= \left\| \left[H_I^{(l)}(r), [H_I^{(k)}(r), H_I^{(j)}(r)] \right] \right\| && \leq 4x^3 \quad (k > j, l > j), \\
\|\mathcal{O}_{14}\| &= \left\| \left[H_M(r+1), [H_I^{(k)}(r), H_I^{(j)}(r)] \right] \right\| && \leq 4x^2\mu \quad (k > j), \\
\|\mathcal{O}_{15}\| &= \left\| \left[H_I^{(l)}(r+1), [H_I^{(k)}(r), H_I^{(j)}(r)] \right] \right\| && \leq 4x^3 \quad (k > j), \\
\|\mathcal{O}_{19}\| &= \left\| \left[H_I^{(l)}(r), [H_I^{(k)}(r+1), H_I^{(j)}(r)] \right] \right\| && \leq 4x^3 \quad (l > j), \\
\|\mathcal{O}_{20}\| &= \left\| \left[H_M(r+1), [H_I^{(k)}(r+1), H_I^{(j)}(r)] \right] \right\| && \leq 4x^2\mu, \\
\|\mathcal{O}_{22}\| &= \left\| \left[H_I^{(l)}(r+1), [H_I^{(k)}(r+1), H_I^{(j)}(r)] \right] \right\| && \leq 4x^3, \\
\|\mathcal{O}_{24}\| &= \left\| \left[H_I^{(l)}(r+2), [H_I^{(k)}(r+1), H_I^{(j)}(r)] \right] \right\| && \leq 4x^3.
\end{aligned}$$

- Can we use Euclidean calculations to compute $\langle\psi|O_i|\psi\rangle$
  - Another interesting research topic

Reduce excited state contamination with smearing

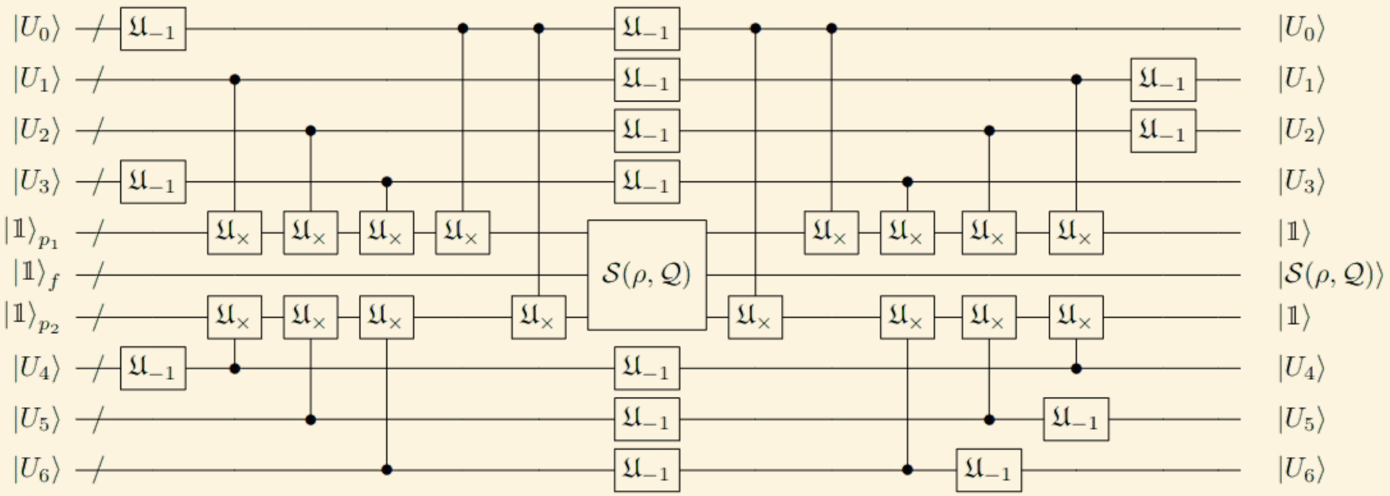
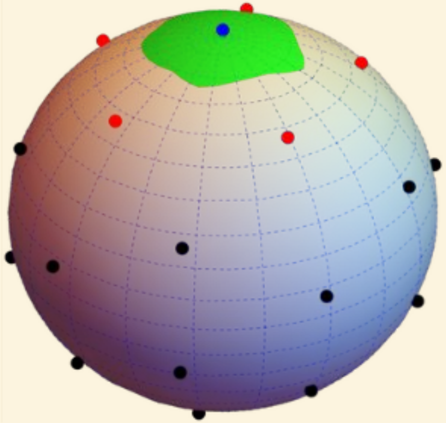


FIG. 3. Quantum circuit for constructing the projected element $\mathcal{S}(\rho, \mathcal{Q})$ on a 2d lattice. The forward slash indicates that the register maybe composed of multiple qubits. The registers $|U_m\rangle$ correspond to the links in Fig. 1. The three register gate $\mathcal{S}(\rho, \mathcal{Q})$ takes the $|\rangle_{p_1}$ and $|\rangle_{p_2}$ scratch plaquette registers as inputs and outputs the closest group element to $e^{i\rho\mathcal{Q}}$ onto the scratch register $|\mathbb{1}\rangle_f$.

Stout Smearing on a Quantum Computer
Gustafson - *2211.05607 [hep-lat]*
Explores how otherwise nonunitary smearing can be implemented

Digitization of nonabelian gauge theories

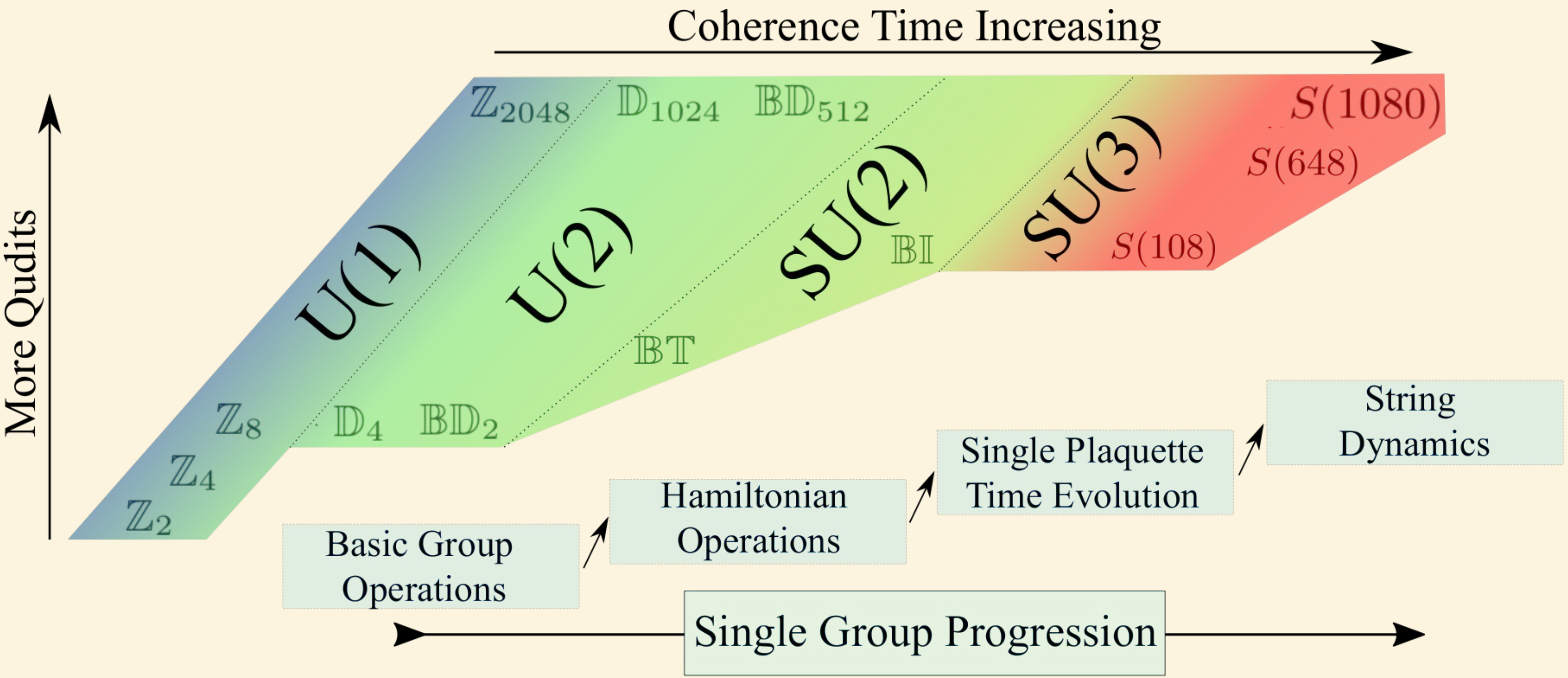
- Need to find a way to map infinite-dimensional Hilbert space of gauge field to finite quantum register built from qubits



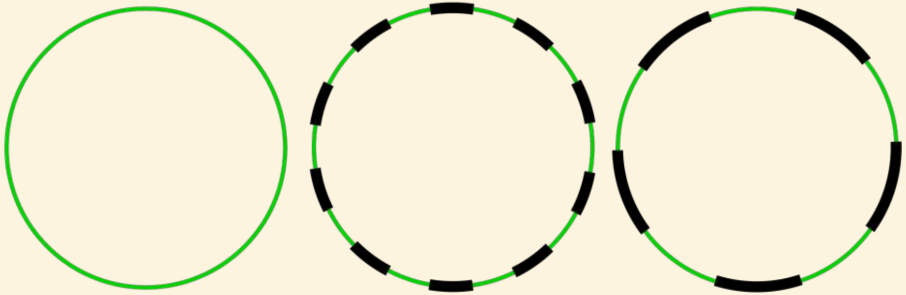
- This is not a trivial decision, it breaks some symmetries and are simulating

$$H + \hat{O}_{trunc}$$

The ladder of discrete gauge theories in HEP calculations



What does this digitization cost?



$$\beta_{f,U(1)} = \frac{\log(1+\sqrt{2})}{1-\cos\left(\frac{2\pi}{N}\right)} \approx \kappa_2 N^2, \text{ which extends to } \beta_{f,SU(N_c)} \approx \kappa N^{\frac{N_c^2-1}{2}}$$

But whereas $\mathbb{Z}_N$ can be **taken to ∞**, **limited** number for $SU(N_c)$

$$\beta \propto \frac{1}{\log(a)} \implies a_f \propto e^{-\beta_f}$$

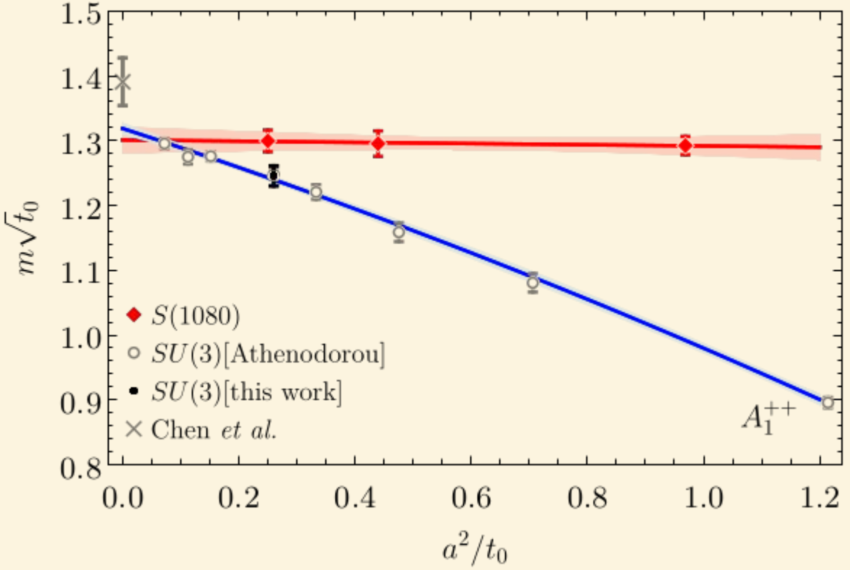
So the important question is $a_s > a_f$ for $SU(3)$?

Nope!

Digitising SU(2) gauge fields and the freezing transition
Hartung *et al.* - *Eur.Phys.J.C 82 (2022) 3, 237*
Understanding the scaling of freezing transitions with approximations

…but why use Wilson action or Kogut-Susskind Hamiltonian?

$$S_M = \beta \,\mathrm{Re}\,\mathrm{Tr}[1 - U_p] + \beta_a \,\mathrm{Re}\,\mathrm{Tr}[U_p]\,\mathrm{Tr}[U_p^\dagger]$$



Glueballs at $a = 0.08$ fm $\rightarrow 10^3$ **lattices** $\sim 10^5$ **lq**

Spectrum of digitized QCD: Glueballs in a S(1080) gauge theory
Alexandru *et al.* - *Phys.Rev.D 105 (2022) 11, 114508*
Can the low-lying spectrum of an S(1080) approximate SU(3)

How do we represent discrete groups?

- Ordered product of generators

$$h_{\{o_k\}} = \prod_k \lambda_k^{o_k} = h_d$$

$$\mathbb{D}_4 : \qquad h_d = \boldsymbol{s}^a \boldsymbol{r}^b$$

$$\mathbb{Q}_8 : \qquad h_d = (-1)^a \mathbf{i}^b \mathbf{j}^c$$

$$\mathbb{BT} : \qquad h_d = (-1)^a \mathbf{i}^b \mathbf{j}^c \mathbf{l}^d$$

$$\Sigma(36 \times 3) : \qquad h_d = \omega_3^a \mathbf{C}^b \mathbf{E}^c \mathbf{V}^d$$

$$\rightarrow |abcd \cdots\rangle$$

Robustness of Gauge Digitization to Quantum Noise
Gustafson, Lamm - 2301.10207 [hep-lat]
Discusses quantum registers with qubits, qudits for U(1), SU(2), SU(3)

Exercise 7: Inverse operation for D_4

Consider the group $\mathbb{D}_4$: $h_d = s^a r^b$

which have the relations: $srs = r^{-1} = r^3,\ sr = r^3 s,\ s = s^{-1}$

What is $h_d^{-1} = (s^a r^b)^{-1}$ in the standard presentation?

General Methods for Digital Quantum Simulations of Gauge Theories
Lamm, Lawrence, Yamauchi - *Phys.Rev.D* 100 (2019) 3, 034518
Constructed this general formalism for group independent implementation

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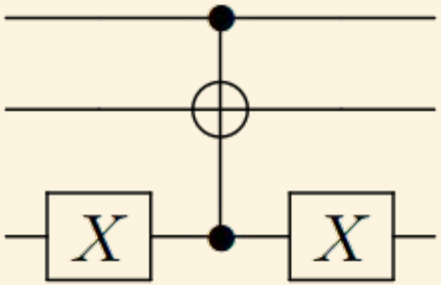
$$(s^a r^b)^{-1} = s^a r^{(3-b)(1-a)+ab}$$

General Methods for Digital Quantum Simulations of Gauge Theories
Lamm, Lawrence, Yamauchi - *Phys.Rev.D* 100 (2019) 3, 034518
Constructed this general formalism for group independent implementation

Take Home Exercise 8 : Inverse gate for D_4

$$(s^a r^b)^{-1} = s^a r^{(3-b)(1-a)+ab}$$

- Can you construct a $\mathcal{U}_{-1}|ab_0b_1\rangle \to |a'b_0'b_1'\rangle$



General Methods for Digital Quantum Simulations of Gauge Theories
Lamm, Lawrence, Yamauchi - *Phys.Rev.D* 100 (2019) 3, 034518
Constructed this general formalism for group independent implementation

Group Primitives for BT

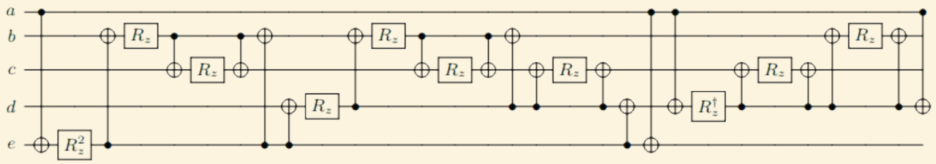


FIG. 4. Trace gate for BT

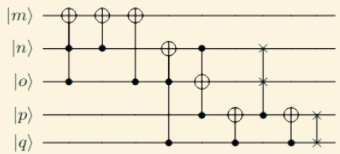


FIG. 2. Inversion Gate for the Binary Tetrahedral Group.

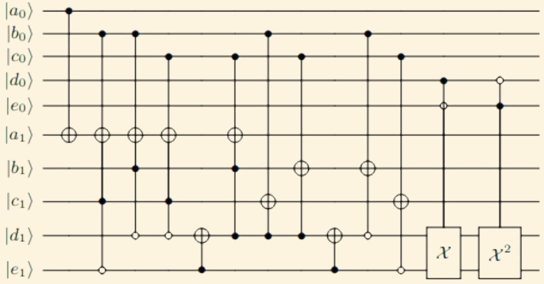
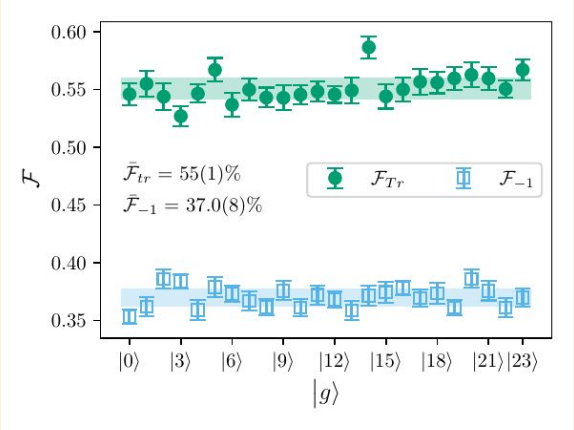
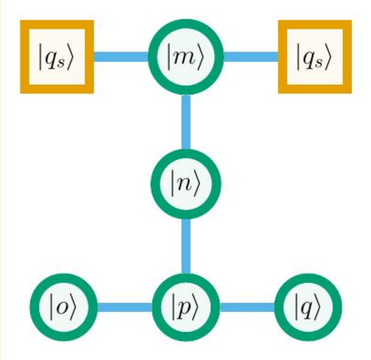


FIG. 3. Multiplication gate





Primitive Quantum Gates for an SU(2) Discrete Subgroup: BT

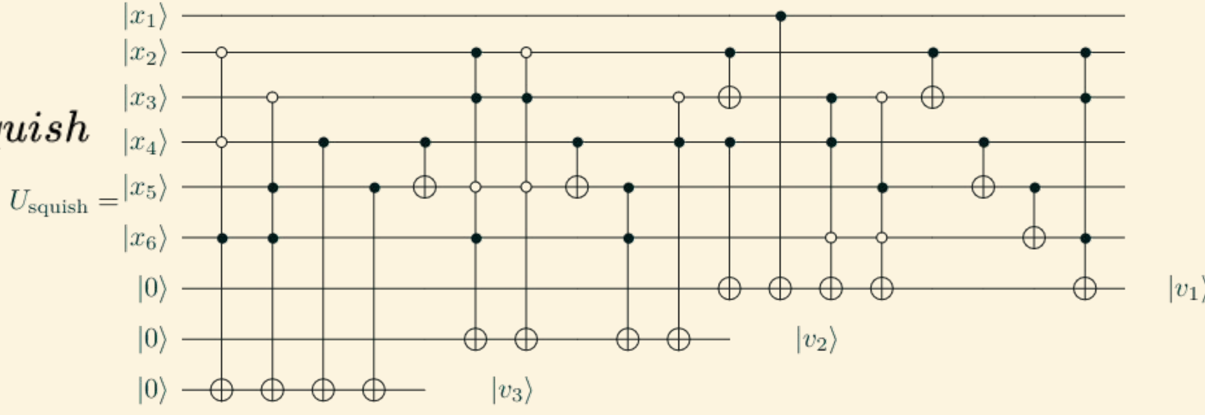
Gustafson, Lamm, Lovelace, Musk - *Phys.Rev.D* 106 (2022) 11, 114501

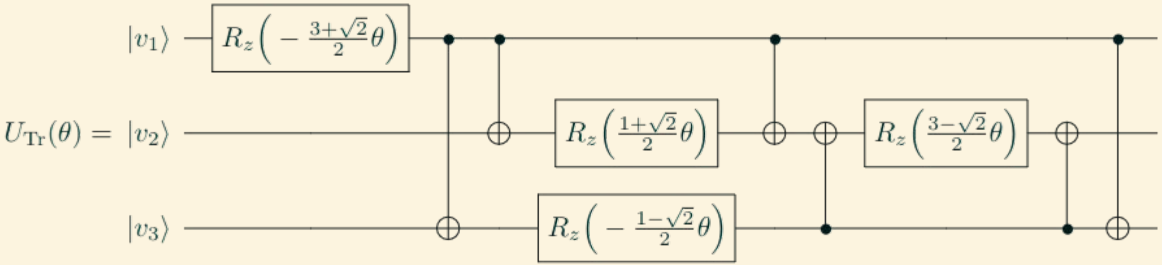
Derived and implemented using custom QEM necessary primitives for HEP simulations

Group Primitives for other groups active area of research

- For example, BO needs

$$\mathcal{U}_{tr}(\theta) = U_{squish} U_{Tr}(\theta) U^{\dagger}_{squish}$$





Resource Estimation for Lattice Simulations of Z_2, BT, S(1080)

TABLE I. C^nNOT gates required for $\mathbb{BT}$ (top) primitive gates (bottom) H_I simulations per link per δt.

Gate	CNOT	C^2NOT	C^3NOT
$\mathcal{U}_{-1}$	6	4	0
$\mathcal{U}_{\times}$	5	8	4
$\mathcal{U}_{Tr}$	20 QFT→O(100)	0	0
$\mathcal{U}_{FT}$	1025	0	0
$e^{-iH_I\delta t}$	$226d+3906$	$252d-212$	$104d-88$

Gate depth rather than memory limits options

Proof of concept — **2x1 Lattice**
- 7 links → {7, 35, 77}q
- N_t=1→ {28, 70k, ~200m} CNOTs*

Nontrivial Physics — **4x4 Lattice**
- 24 links → {24, 120, 264}q
- N_t=10→{96, 220k, ~700m} CNOTs*

HEP Applications — **10^3 Lattice**
- 3k links → {3k, 15k, 33k}q
- N_t=10→{4m, 450m, ~10t} CNOTs*

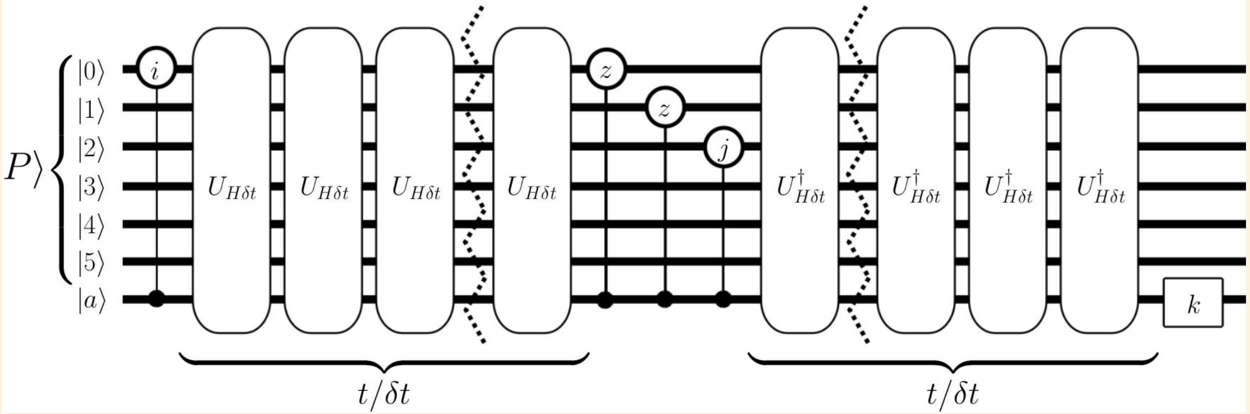
N-point correlators and Quantum Advantage

- **Nearly all** HEP QA is time-evolution + *n* Hermitian insertions

$$\langle \prod_i \mathcal{O}_i(t_i) \rangle = \int_{\psi(0)}^{\psi(T)} \mathcal{D}\psi \prod_i \mathcal{O}_i(t_i) e^{-iS} = \langle \psi(T) | \prod_i \mathcal{O}_i(t_i) | \psi(0) \rangle$$

- Example: Hadronic Tensor ***which requires Hadamard test***

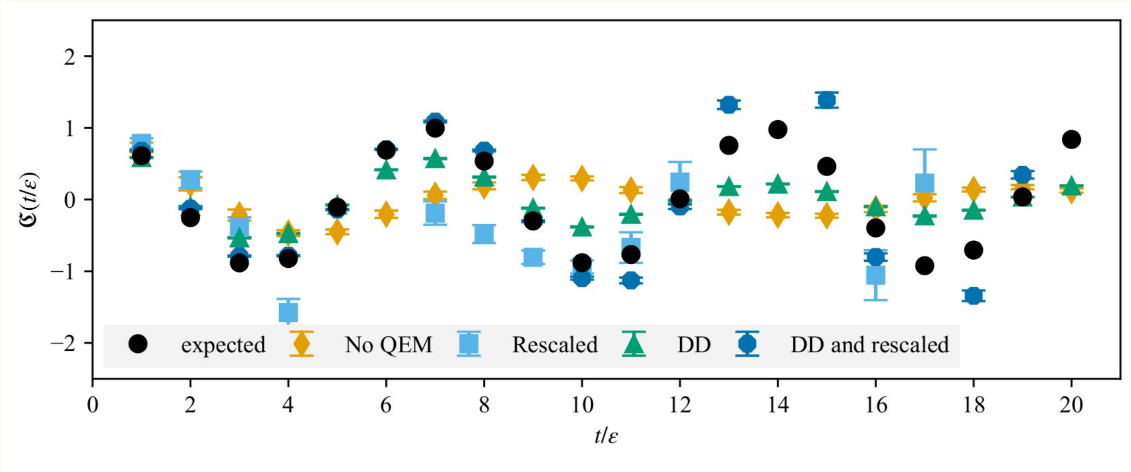
$$\langle P | \chi^\dagger(tn^\mu)\chi(0)|P\rangle = \sum_{i,j,k=\{x,y\}} \frac{c_{ij}}{4} \langle P, a | U_{i,j,k} | P, a\rangle$$

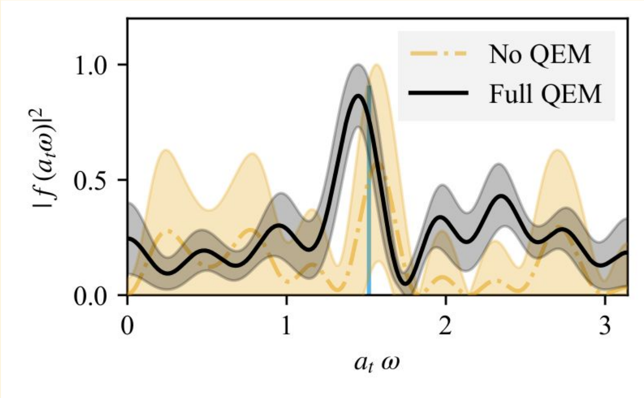


Parton Physics on Quantum Computers
Lamm, Lawrence, Yamauchi - *Phys.Rev.Res* 2 (2020) 1, 013272
Formulation of Practical HEP Quantum Advantage Problem

At some point, you need to determine a and a_t

- Pick M_{phys}
- Prep state, measure ωa_t
- $\omega = M_{phys}$ to get a_t





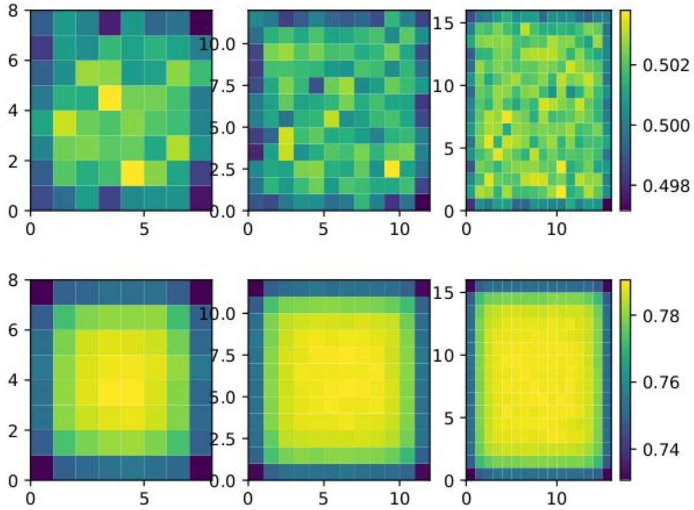
Simulating Z_2 lattice gauge theory on a quantum computer
Clement *et al.* - *2305.02361 [hep-lat]*
Extending the time-evolution using multiple error mitigation strategies

Theoretical errors

- Working at finite coupling means a>0
  - usable discretization errors are a^n
- Working at finite volume mean states get squished
  - Errors often scale exp(-mL) for QCD
  - … but L^{-n} for QED
  - … although Minkowski can lead to difference
- Boundary Conditions also affect things

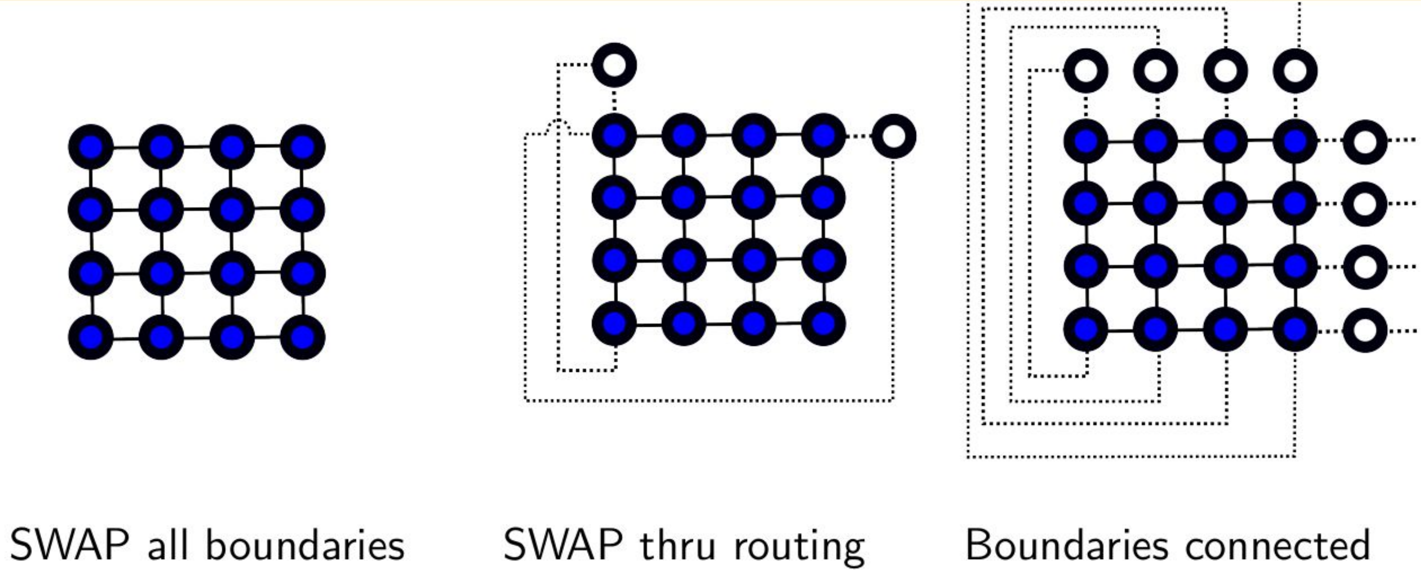
Periodic Boundary Conditions are *HIGHLY* desirable

$$\langle O(t)\rangle_{OBC} \approx \langle O(t)\rangle_{PBC} + Ae^{-mT/2}\cosh m\left(\frac{T}{2}-t\right)$$



To obtain same results as L^d_{PBC} requires $[x(a)L]^d_{OBC}$ where $x(a) > 1$ grows with a

SWAPs, Routes, and *Circuit Cutting*



Going to right you are **infuriating** experimentalists more

For gauge registers, should determine fidelity thresholds

Circuit Cutting

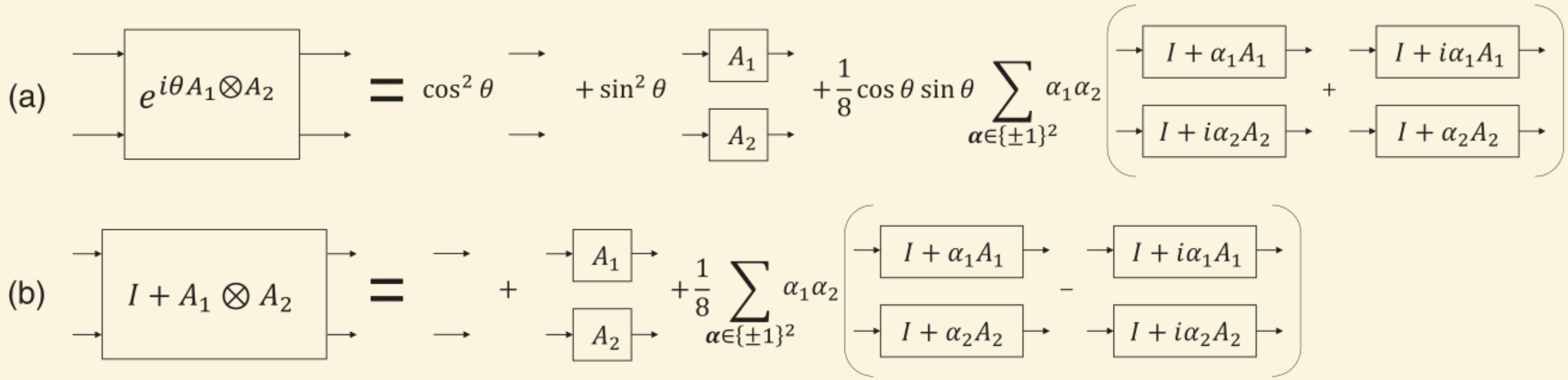
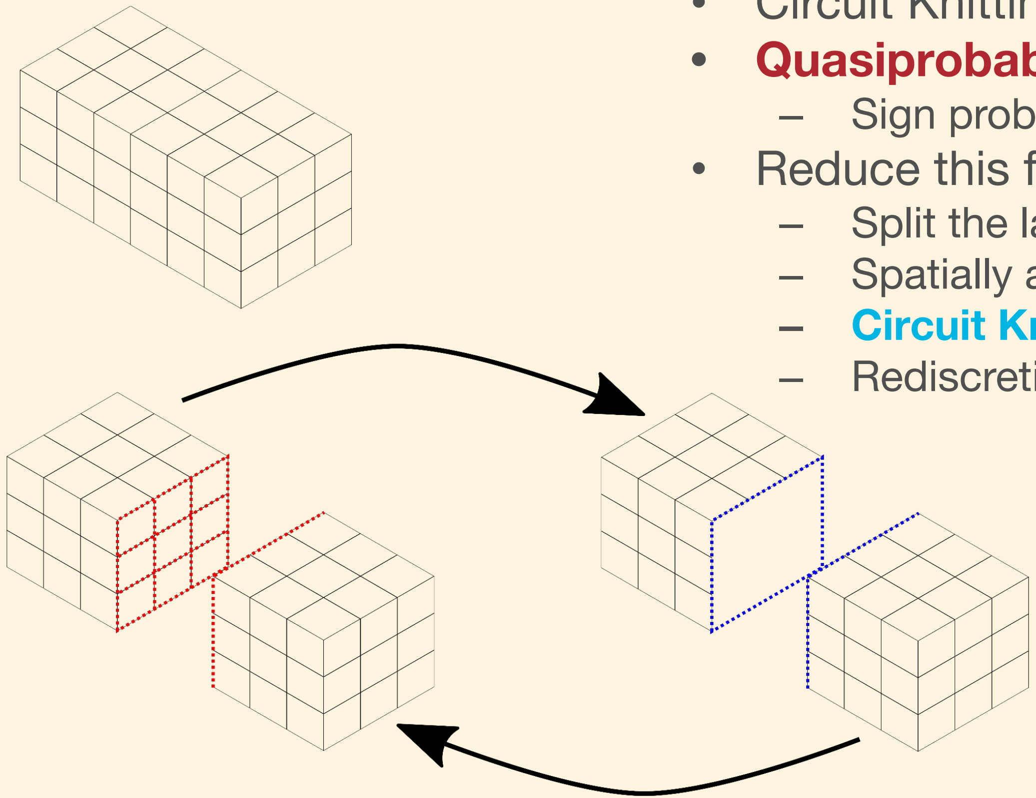


Figure 1. Decomposition of (a) a non-local gate and (b) a non-local non-destructive measurement into a sequence of local operations. A_1 and A_2 are operators such that $A_1^2 = I$ and $A_2^2 = I$.

Constructing a virtual two-qubit gate by sampling single-qubit operations
Mitarai, Fujii - New J. Phys. 23 023021 2021
A particularly good explanation and lit review of topic

Multigrid and Circuit Knitting



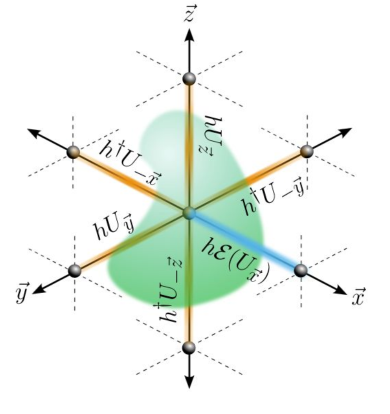
- Circuit Knitting has **<O(9^N) scaling**
- **Quasiprobabilities** will also increase costs
  - Sign problem!
- Reduce this for LFT through **multigrid** techniques?
  - Split the larger **lattice** → **sublattices**, 1 per QPU
  - Spatially average ***a spacing*** → **larger *a'*** for fixed L
  - **Circuit Knitting** time evolution on *a'* lattice
  - Rediscretize ***a'*** → ***a*** with pseudorandom sampling

Partial Error Correction, Probabilistic Error Mitigation for LFT

- Given a register, **prioritize error channels** for mitigation and correction
- Reduction of **large theoretical error** at **lower cost**

TABLE I. $\mathcal{N}_i$ vs. $\mathbb{G}$ for $U(1)$ subgroups: $\mathbb{Z}_N$ where $N = 2^n$.

Binary	Gray	Qudits	$\mathbb{G}$
$\hat{Y}_0$	$\hat{Y}_0$	$\hat{B}^{(i,j)}$, $\hat{Z}^{(i)}$	—
—	$\hat{B}_{a\neg 0}$, $\hat{Z}_a$	$\hat{\mathcal{V}}^m$	$\mathbb{Z}_2$
$\hat{X}_{a\neg 0}$	—	—	$\mathbb{Z}_{2^{n-a}}$
$\hat{Z}_a$, $\hat{Y}_{a\neg 0}$	$\hat{X}_0$	—	$\mathbb{Z}_{2^{n-1}}$
$\hat{X}_0$	—	$\hat{\chi}^m$	$\mathbb{Z}_N$



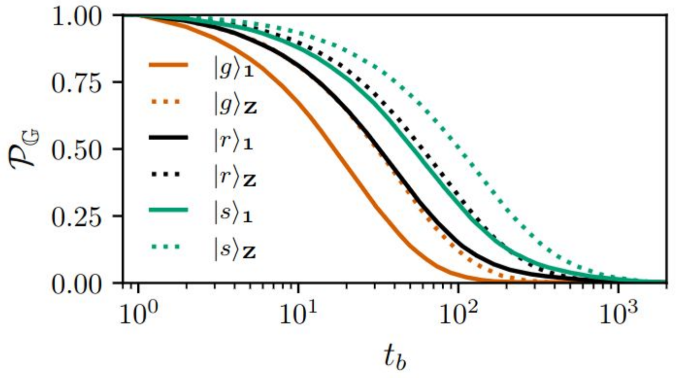


FIG. 2. $\mathcal{P}_{\mathbb{G}}(t_b)$ for $\mathbb{Z}_8$ versus t_b using $|g\rangle$, $|r\rangle$, and $|s\rangle$ for depolarizing and dephasing channels.

Robustness of Gauge Digitization to Quantum Noise
Gustafson, Lamm - 2301.10207 [hep-lat]
Classification of Gauge Violating noise for qubits, qudits for U(1), SU(2), SU(3)

Endgame

- The road to practical quantum advantage in HEP will be long and winding
- We **do not have** anything close to optimal resource estimates
- **Hardware limitations**, **quantum software stack**, and **classical overhead** are just now being investigated
- Some exciting stepping stones for you to work on.