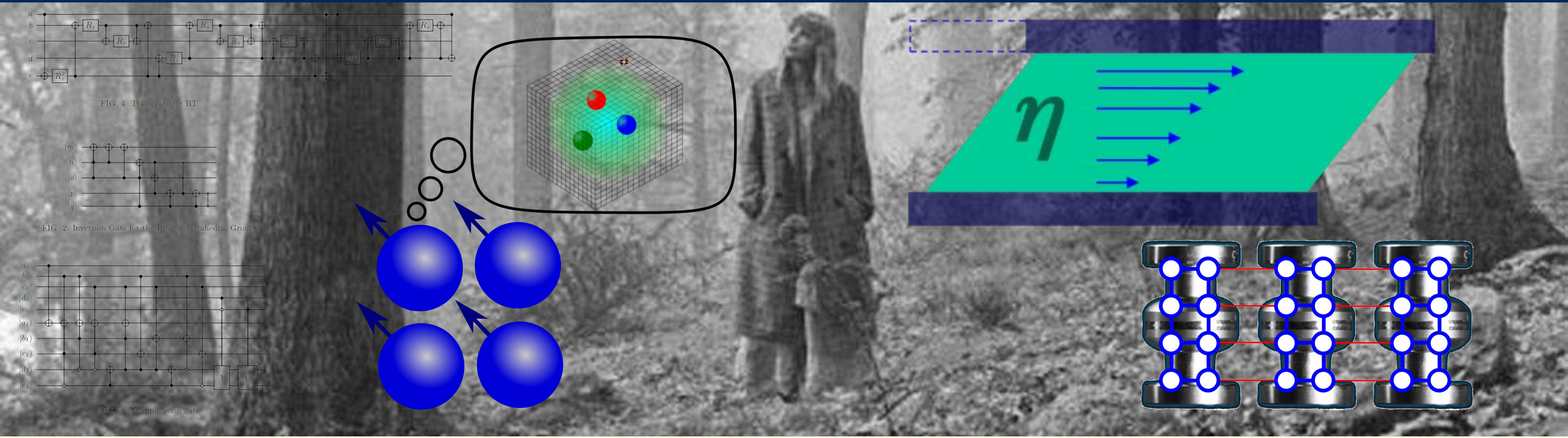


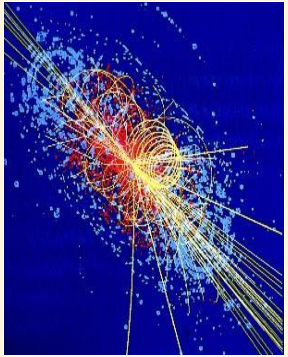
I’m the problem, it’s me: Viscosity in 3+1d QCD

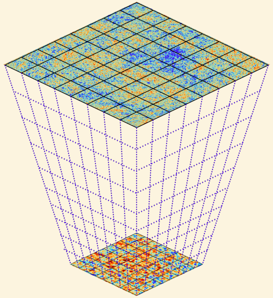
Hank Lamm

June 27, 2023

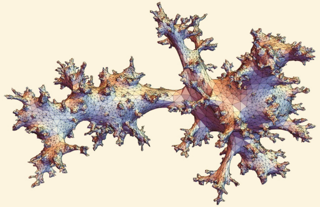
Quantum Computing for Particle Physics, it's a need

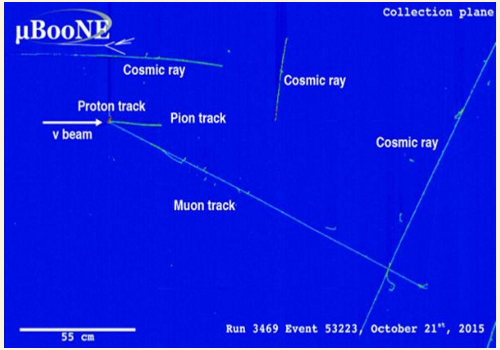
- **The world is quantum, and we are lucky anything is amenable to classical computers**
  - Large-scale quantum computers can tackle computations in HEP otherwise inaccessible
  - This opens up new frontiers & extends the reach of LHC, LIGO, EIC & DUNE





- ***Ab initio* cross sections for colliders and neutrino experiments**
- **Cosmic inflation and the evolution of matter asymmetry in the early universe**
- **Explorations of BSM, supersymmetry, and quantum gravity**
- **Hadronization and Hydrodynamics in Heavy-Ion collisions**





While broad, these topics often are formulated as **lattice field theories**

Quantum Simulation for High-Energy Physics
Bauer, Davoudi *et al.* - *PRX Quantum 4 (2023) 2, 027001*
Wonderful survey of physics questions, methods, and outstanding problems in field

Don’t let anyone fool you…

There is so much to be done

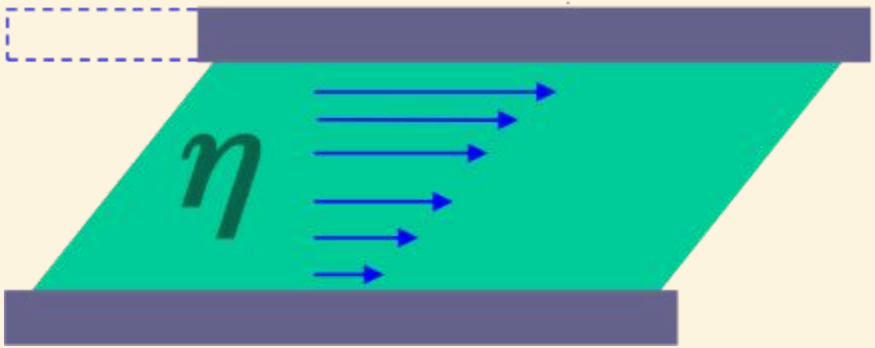
As a target, today we are going to consider the viscosity of QCD

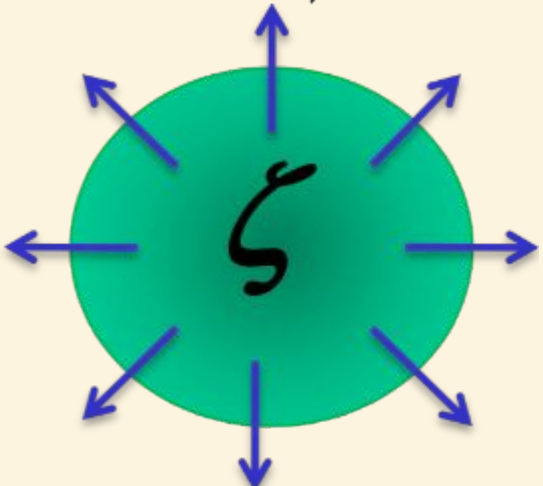
- $\eta = \frac{V}{T}\int_0^\infty \langle T_{12}(t) T_{12}(0)\rangle$
- I believe its a “near-term” goal and allows for focus…
- …while introducing all the necessary pieces

Quantum algorithms for transport coefficients in gauge theories
NuQS Collaboration - *Phys.Rev.D 104 (2021) 9, 094514*
Formulates lattice operators and propose correlators

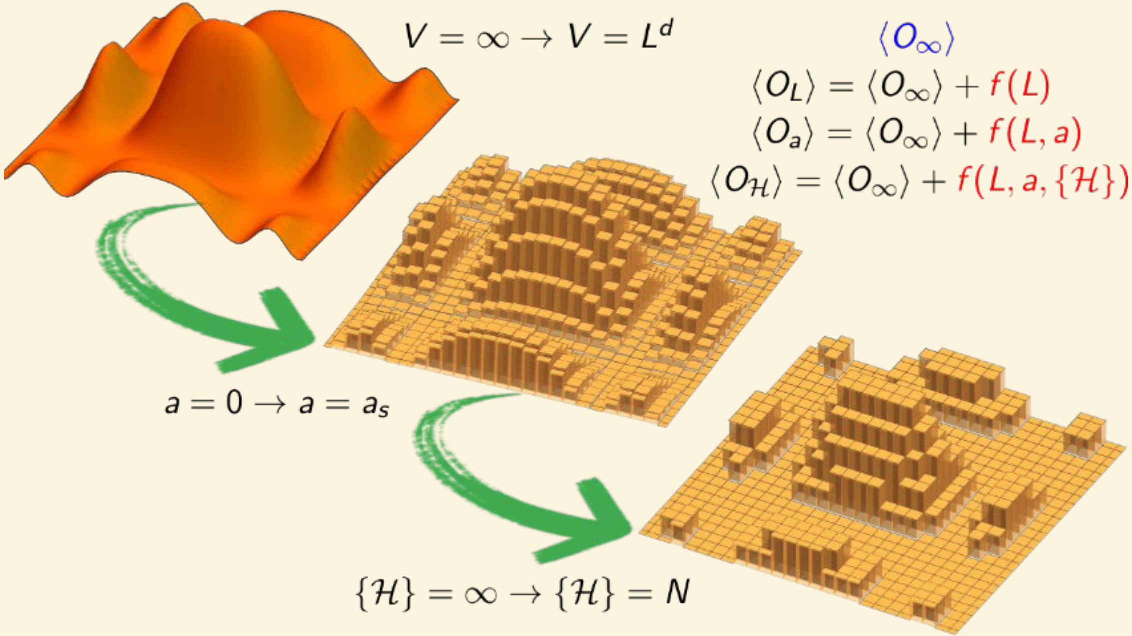
Viscosity of pure-glue QCD from the lattice
Altenkort *et al.* - *2211.08230 [hep-lat]*
State of the art lattice results, but massive uncertainties persist

$$\eta/s = 0.15 - 0.48, T = 1.5T_c$$
$$\zeta/s = 0.017 - 0.059, T = 1.5T_c$$





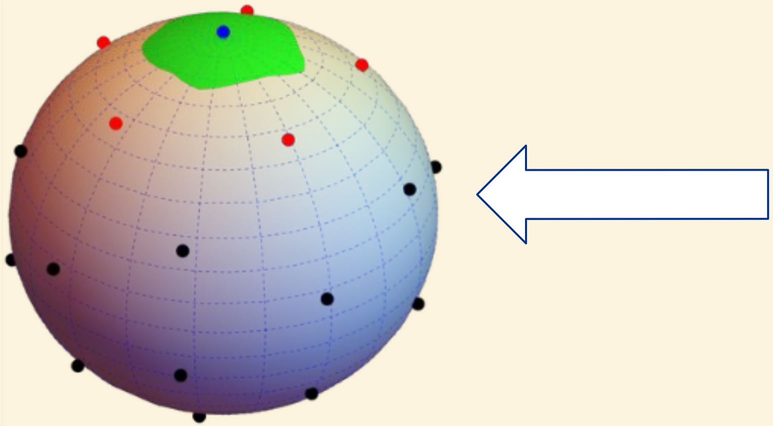
Take it to the limit

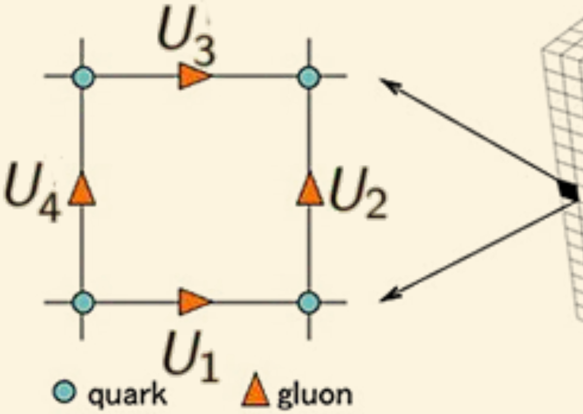


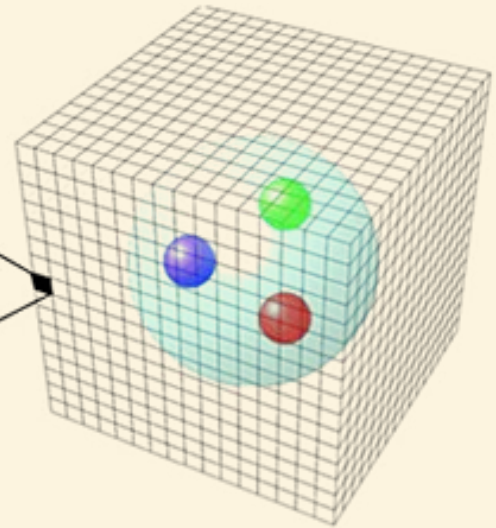
- O(L,a,$\mathcal{H}$) is an **approximation** for HEP
- Truncations leads to **systematic errors**
- **Extrapolating** is done on results, reducing computational resources…
- …but **obscures** precise resource estimates

Qubit Costs for Lattice Field Theory

- Lattice field theory discretizes spacetime into a lattice of size $(La)^d$
  - $L\rightarrow\infty$ and $a\rightarrow 0$ must be taken
- Matter fields are placed on sites, gauge fields on links
  - Fermionic matter need **Ƒ=Spin x Color x Flavor qubits per site** e.g. 12 for staggered QCD
  - Gauge links are bosonic and need efficient truncation **Λ qubits per link** e.g. SU(3) ~ **???**q
  - Scalar (bosonic) matter is infinite-dimensional, so must be truncated as well
- So qubit cost is: $(d\Lambda + \mathcal{F})(L/a)^d$

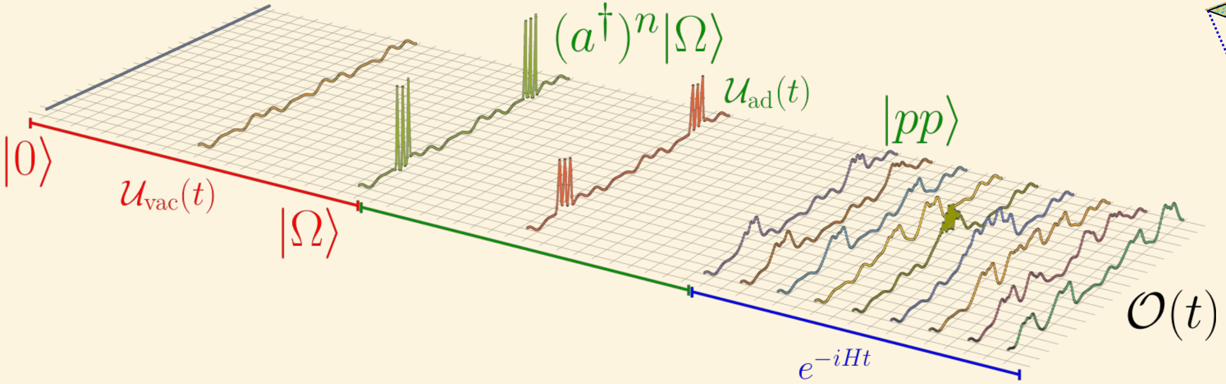


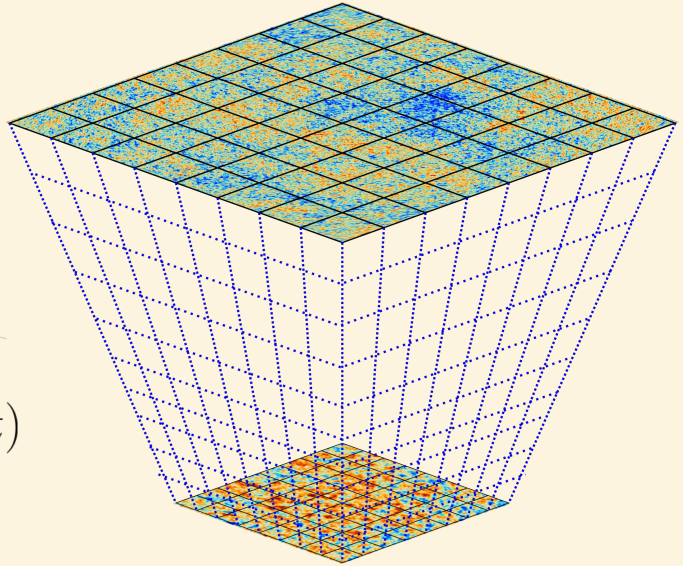




Gate Costs for Lattice Field Theory

- Lattice field theory approximates $U(T) = e^{-iHT}$ which *can* corresponds to into a lattice of size Ta_t
  - $a_t \to 0$ or equivalent limit must be taken
  - Trottertization has this property, others less clear i.e. potentially variable temporal spacing
- Gate cost is heuristically: $\frac{T}{a_t} \times [\mathcal{O}(1)(d\Lambda + F)(L/a)^d]^{\mathcal{O}(1)}$





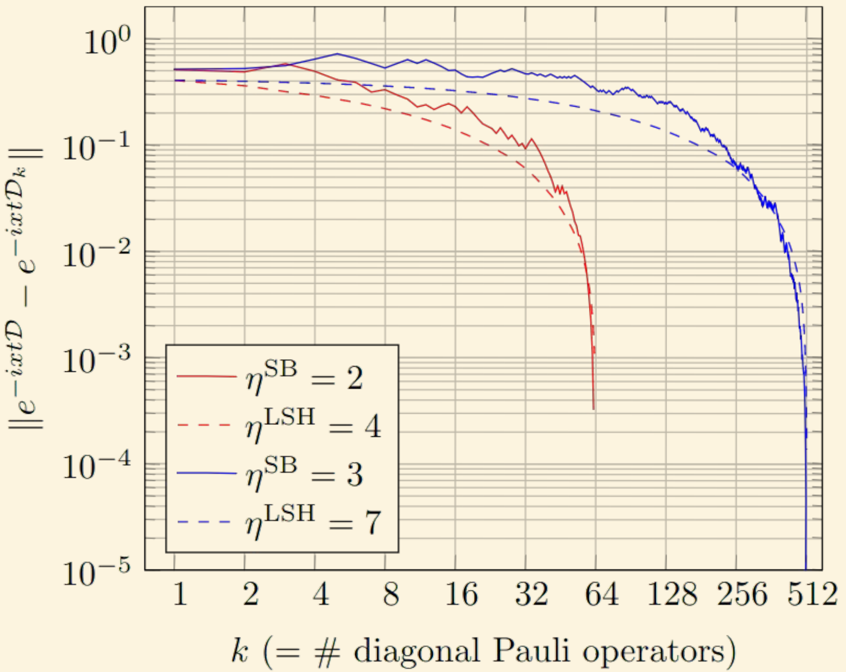
Exercise 1: What will viscosity take?

Qubits: $(d\Lambda + \mathcal{F})(L/a)^d$ Gates: $\frac{T}{a_t} \times [\mathcal{O}(1)(d\Lambda + \mathcal{F})(L/a)^d]^{\mathcal{O}(1)}$

- d=3
- What is $\mathcal{F}$? (Staggered=12 Wilson=24)
  - Note: **a_t scaling of errors**
- How will you truncate Λ? (9 64-bit $\mathbb{C}$ floats = 1152)
  - Note: **truncation errors**
- How small will you take a? ($1\text{fm}^{-1} \sim 200$ MeV)
  - Note: **discretization errors**
- How large will you take L?
  - Note: **finite volume errors**
- Gate cost prefactor ~ 10 and exponent~2
- How small will you take a_t?
  - Note: **Trotter errors**
- How long do you need to run for (T)?
  - Note: **Signal resolution errors**

What didja get?

- Qubit costs: 10^3-10^9
  - 10q for SU(3) might be reasonable
  - a~0.5 fm, L~3 fm
  - Perhaps we drop fermions
  - Perhaps lower dimensions
- Gate costs: 10^7-10^{40}
  - a_t~0.1 fm, T~1 fm
  - Quantum arithmetic can hurt
  - Perhaps sloppy synthesis
  - Perhaps improved algorithms



General quantum algorithms for Hamiltonian simulation with applications to a non-Abelian lattice gauge theory
Davoudi, Shaw, Stryker - 2212.14030 [hep-lat]
Understanding the synthesis and Trotter errors, along with algorithmic choices in 1+1 SU(2)

But we don't today have a good sense of **theoretical** errors…

Exercise 2: What gate fidelities do you need?

- Consider your gate cost N_g
- Assume that every gate has a infidelity of p
- "Simulation fidelity" is $(1-p)^{N_g}$ i.e the probability your result is without error.

What must p be such that the simulation fidelity is 50%

Exercise 2: What gate fidelities do you need?

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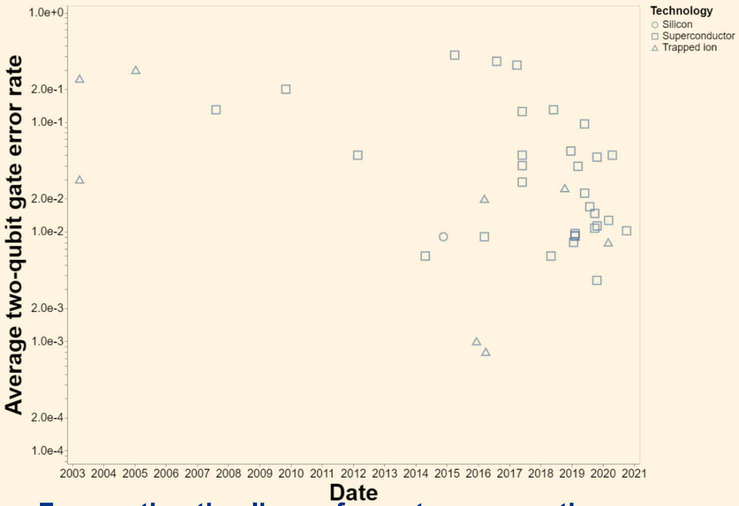
What must p be such that the simulation fidelity is 50%

10^{-8}-10^{-40}

Today, we talk about p ~10^{-3}



Forecasting timelines of quantum computing
Sevilla and Riedel - 2009.05045 [quant-ph]
Current hardware properties and projections for future

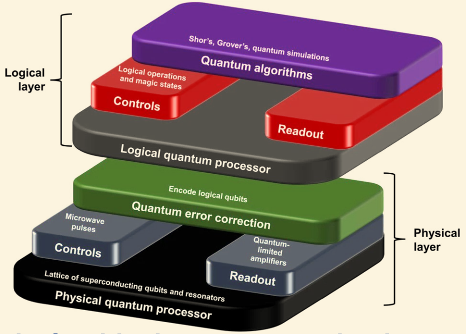
Noisy Intermediate-Scale Quantum vs Fault-Tolerance

NISQ

- Exists today!
- Limited number of qubits
  - Probably $<10^4$
- Basic gate set is native one
  - Often included arbitrary rotations
- Speed limited by 2q gate
- Errors tolerated or mitigated
  - Probably $>10^{-7}$
  - Measurement slow
  - Count CNOTs

FT

- Scalable, networked qubits
  - No limits on number of logical qubits
- Requires error correction
  - Potentially huge overhead
  - Threshold error rates
  - Measurement + Classical compute
- Gate set limited
  - Must synthesize
  - Count nontransverse T-gates



Building logical qubits in a superconducting quantum computing system
Gambetta, Chow, Steffen - npj Quantum Information 3, 2 (2017)
Discusses possible architectures for FT devices

Your paradigm will greatly affect your research projects

Hamiltonians for Nonabelian Gauge Theories in the Continuum

H in terms of CEM fields

$$H = \int d^d x \, \mathrm{Tr}(\mathbf{E}^2 + \mathbf{B}^2)$$

Fields & Field-strength tensor

$$E_i = \frac{1}{2} F_{ii} \qquad B_i = \frac{1}{2} \epsilon_{ijk} F_{jk}$$

FS tensor & gluon field

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu, A_\nu]$$

Chromo-field components

$$E_i = \lambda^a E_i^a \qquad B_i = \lambda^a B_i^a$$

Gell-Mann matrices

$$\mathrm{Tr}(\lambda^a \lambda^b) = \frac{1}{2} \delta_{ab} \qquad [\lambda^a, \lambda^b] = i f^{abc} \lambda^c$$

Improvement and analytic techniques in Hamiltonian lattice gauge theory
Carlsson - PhD thesis, 0309138 [hep-lat]
Derivation of KS and Improved Hamiltonians and variational techniques

Approximating gauge fields

- For *reasons* of gauge symmetry, discretizing A_μ is fraught with danger
- Instead, define an *average* A_μ along a link in direction μ as

$$\mathcal{A}_\mu = \frac{1}{a}\int_\mu d\mathbf{x}\cdot\mathbf{A}$$

- On the lattice, this definition leads to a discretization error since the field at all points between $\mathbf{x}$ & $\mathbf{x}+\hat{\mu}$
- Since we are considering lattice Hamiltonians, for now we restrict ourselves to the spatial lattice with latin indices i,j,k…
-

$$\begin{aligned}\mathcal{A}_i =&\frac{1}{a}\int_{-a/2}^{a/2} dx_i[A_i(\mathbf{x}) + x_i\partial_i A_i(\mathbf{x}) + \frac{1}{2}x_i^2 A_i(\mathbf{x}) + \cdots]\\ =&A_i(\mathbf{x}) + \frac{a^2}{24}\partial_i^2 A_i(\mathbf{x}) + \frac{a^4}{1920}\partial_i^4 A_i(\mathbf{x}) + \cdots\end{aligned}$$

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Wilson lines and gauge links

- To avoid the dangers of using A_μ we use the average to define a gauge link which is a *Wilson line*:

$$U_l = e^{iea\mathcal{A}_l}$$

- Taylor expanding and using relations between $\mathcal{A}_l$ & $A_l(x)$, we see

$$U_l(\mathbf{x}) = 1 + ieaA_l(\mathbf{x}) - \frac{e^2a^2}{2!}A_l(\mathbf{x})A_l(\mathbf{x}) + \cdots$$

- As we will see, while this can reproduce the continuum theory when a=0, at finite lattice spacing, there will be new interactions in the Hamiltonian

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Commutation relations

- We would like to recover in the continuum

$$[E_i^a(\mathbf{x}), A_j^b(\mathbf{x})] = i\delta_{ij}\delta_{ab}\delta(\mathbf{x}-\mathbf{y})$$
$$[A_i^a(\mathbf{x}), A_j^b(\mathbf{x})] = [E_i^a(\mathbf{x}), E_j^b(\mathbf{x})] = 0$$

- To see how to define our lattice kinetic term, we should investigate $\mathcal{E}_l^a$ one way this can be done is through the lattice commutator

$$[\mathcal{E}_l^a, U_m] = [\mathcal{E}_l^a, e^{ieaA_m}]$$

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Lattice commutation relations

Using a BCH relation:

$$e^{-A}Be^{A} = B + [B, A] + \frac{1}{2!}[[B, A], A] + \frac{1}{3!}[[[B, A], A], A] + \cdots$$

It is possible to show:

$$\begin{aligned}[\mathcal{E}_l^a, U_m] =& [\mathcal{E}_l^a, e^{iea\mathcal{A}_m}] \\ =& iea[\mathcal{E}_l^a, \mathcal{A}_m]U_m\end{aligned}$$

Check this for yourself tonight!

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Lattice electric field

Now, using the definition,

$$\mathcal{A}_i = A_i(\mathbf{x}) + \frac{a^2}{24}\partial_i^2 A_i(\mathbf{x}) + \frac{a^4}{1920}\partial_i^4 A_i(\mathbf{x}) + \cdots$$

We find that the the commutator with the continuum field is:

$$[\mathcal{E}_l^a, U_m] = iea[\mathcal{E}_l^a, A_m^b + \frac{a^2}{24}\partial_m^2 A_m^b + \cdots]\lambda^b U_m$$

Which implies that to ensure the continuum relations, we should associate:

$$\mathcal{E}_l^a = -\frac{a^{d-1}}{e}[E_l^a - \frac{a^2}{24}\partial_i^2 E_i^a + \cdots]$$

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Lattice Kinetic Energy

- With this definition and imposing gauge invariance, we find:

$$\mathrm{Tr}[\mathbf{E}^2(\mathbf{x})] \approx$$
$$\frac{g^2}{2a}\mathrm{Tr}[X\mathcal{E}_i(\mathbf{x})\mathcal{E}_i(\mathbf{x}) + Y\mathcal{E}_i(\mathbf{x})U_i(\mathbf{x})\mathcal{E}_i(\mathbf{x}+a\hat{i})U_i^\dagger(\mathbf{x})]$$

- Expanding E and U in terms of their continuum fields, we find

$$K = \frac{X+Y}{2}E_i^2 + \frac{5Y-X}{12}E_i\partial_i^2 E + \mathcal{O}(ea^2, a^4)$$

- Setting X=1, Y=0 we obtain the KS kinetic term with errors scaling with a^2

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Exercise 3: Improved Lattice Kinetic Energy

- What values of X,Y would cancel of all classical a^2 errors?

$$K = \frac{X+Y}{2} E_i^2 + \frac{5Y-X}{12} E_i \partial_i^2 E + \mathcal{O}(ea^2, a^4)$$

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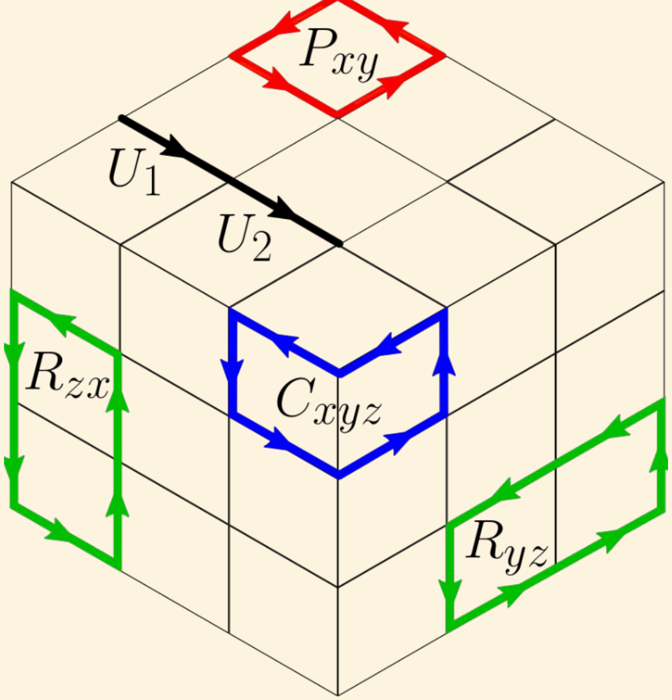
Lattice Potential Energy



- Constructed form closed loops of Wilson lines

- The simplest nontrivial *Wilson loop* is the plaquette:

$$P_{xy} = 1 - \frac{1}{N}\mathrm{ReTr}[U_x(\mathbf{x})U_y(\mathbf{x}+a\hat{\mathbf{x}})U_x^\dagger(\mathbf{x}+a\hat{\mathbf{y}})U_y^\dagger(\mathbf{x})]$$

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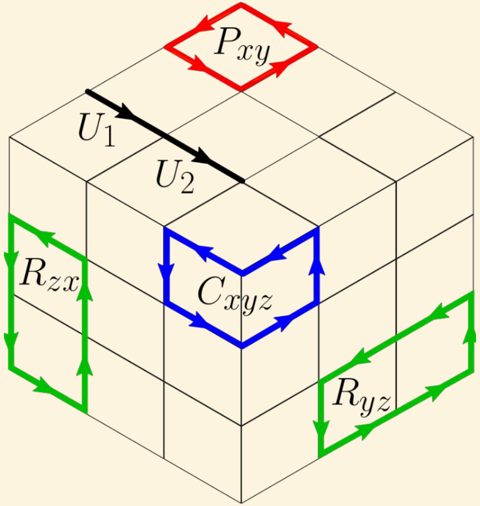
Lattice Potential Energy



- Including R_{ij} and R_{ji} yields:

$$V = \frac{2N}{ag^2}\left[XP_{ij}(\mathbf{x}) + \frac{Y}{2}\left(R_{ij}(\mathbf{x}) + R_{ji}(\mathbf{x})\right]\right.$$

- Which can be related to the continuum, obtaining:

$$V \approx a^d\Big[(X+4Y)\mathrm{Tr}(F_{ij}^2)$$
$$+\frac{a^2}{12}(X+10Y)\mathrm{Tr}(F_{ij}\{D_i^2+D_j^2\}F_{ij}) + \mathcal{O}(e^2a^2, a^4)\Big]$$

- So if you are satisfied with a^2 errors, X=1,Y=0 yields the KS Hamiltonian

Improvement and analytic techniques in Hamiltonian lattice gauge theory
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Exercise 4:

- **What values of X and Y will yield an a^2 improved Hamiltonian?**

$$V \approx a^d\Big[(X+4Y)\mathrm{Tr}(F_{ij}^2) + \frac{a^2}{12}(X+10Y)\mathrm{Tr}(F_{ij}\{D_i^2 + D_j^2\}F_{ij}) + \mathcal{O}(e^2a^2, a^4)\Big]$$

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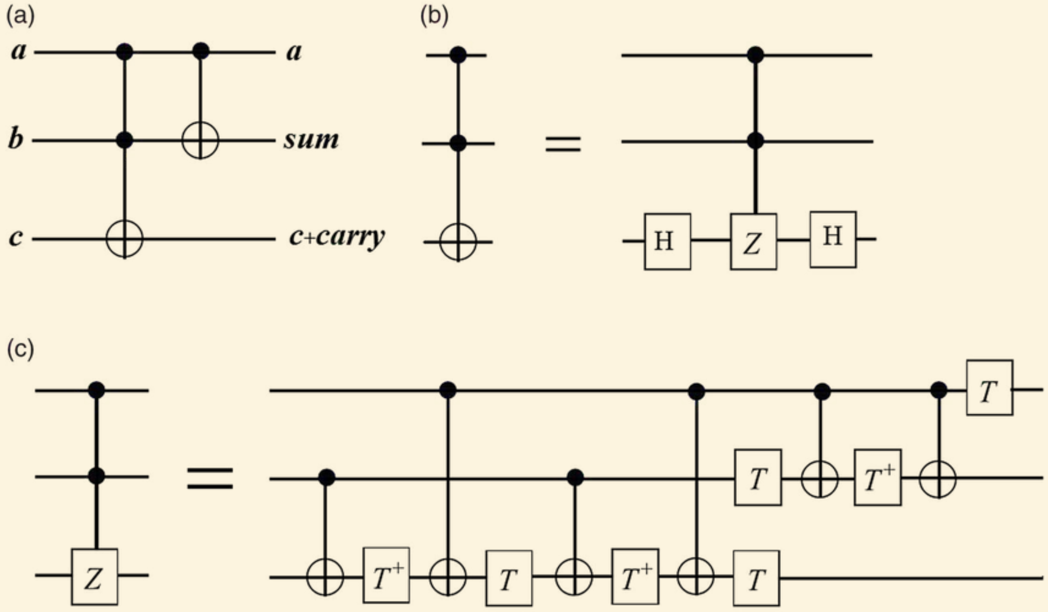
Regardless of your choice, you will need to do some math

- e.g. V=Tr(g)
- Floating point or fixed point arithmetic is expensive in qubits and gates
- Consider the half-adder



A transmon-based quantum half-adder scheme
Chatterjee and Roy- PTEP 2015 9, September 2015, 093A02
Described a specific hardware implementation of the general half-adder algorithm

Questions?

Exercise 5: Implement the Adders

- Take a look at ***lab_quantum_adder.ipynb***