Quantum Computing Bootcamp for High-Energy and Nuclear Physics

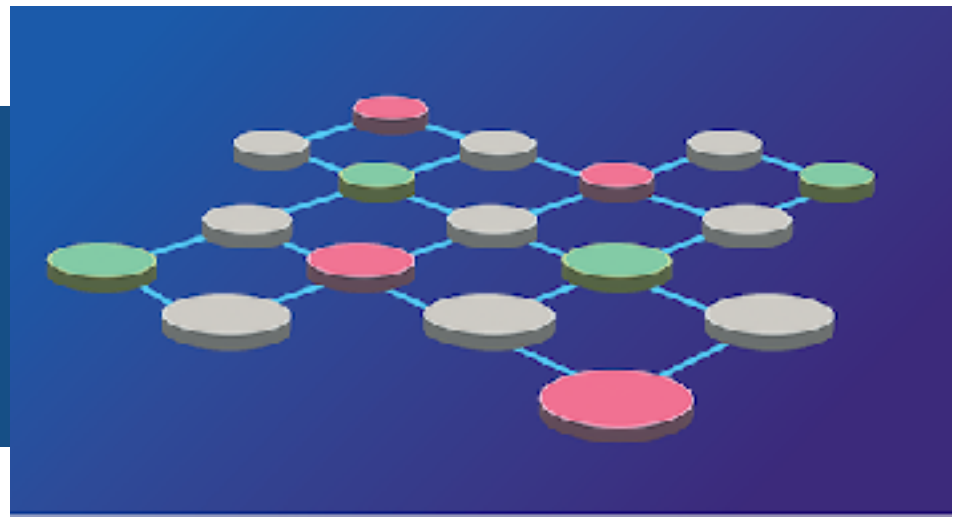
Jefferson Lab

June 21, 2023

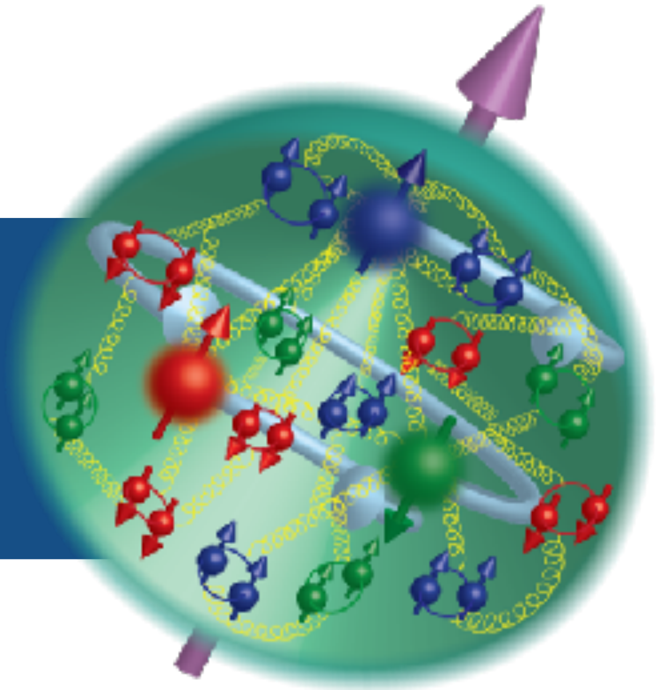
James Mulligan
University of California, Berkeley

Outline

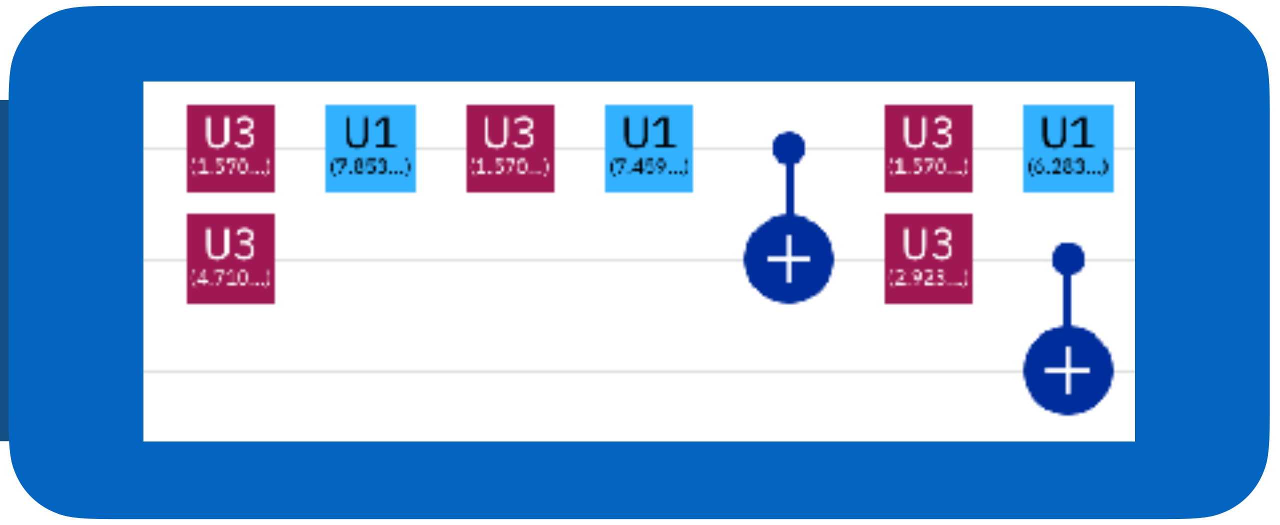
1. Quantum advantage



2. QC for HEP/NP



3. Hands-on: Circuit synthesis



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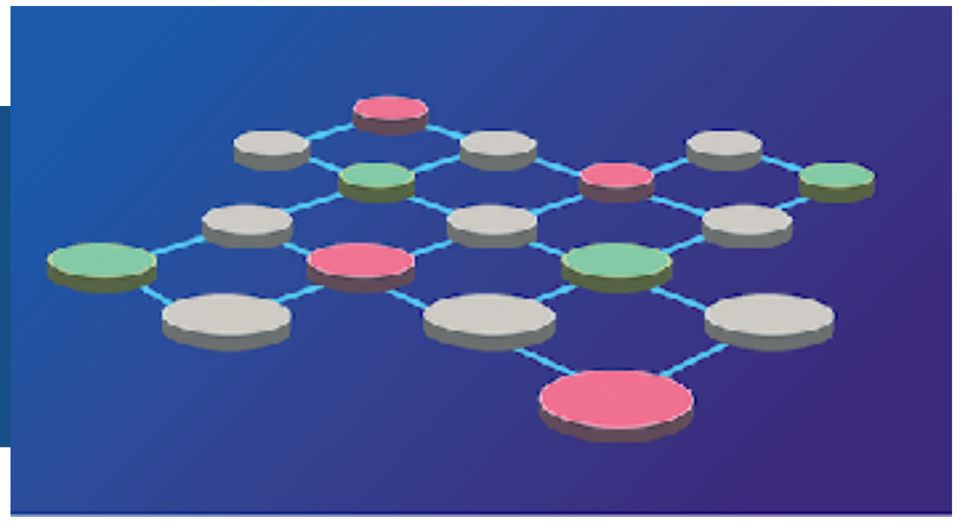

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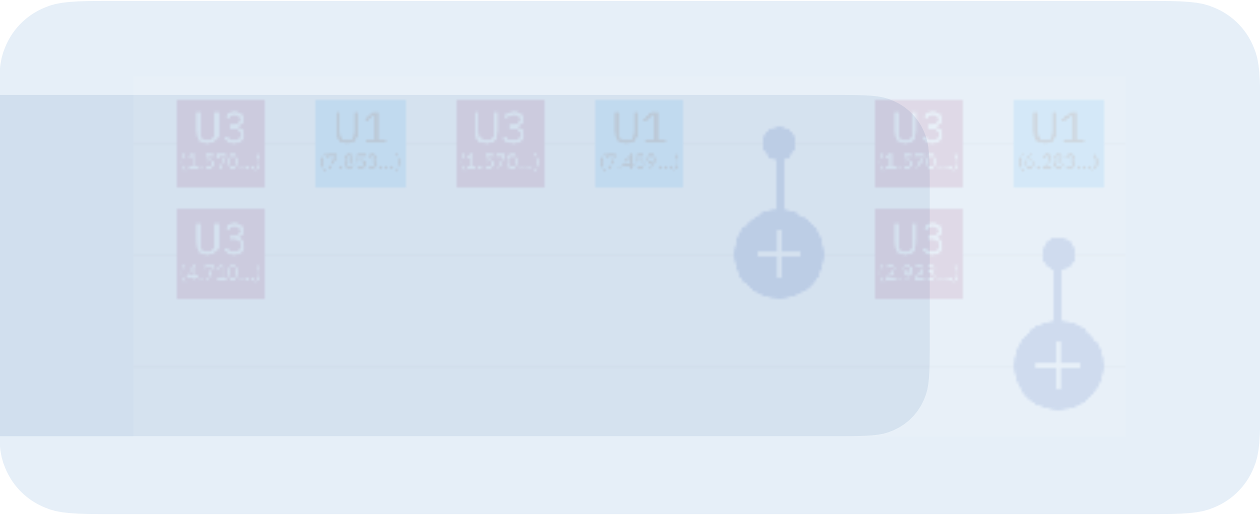
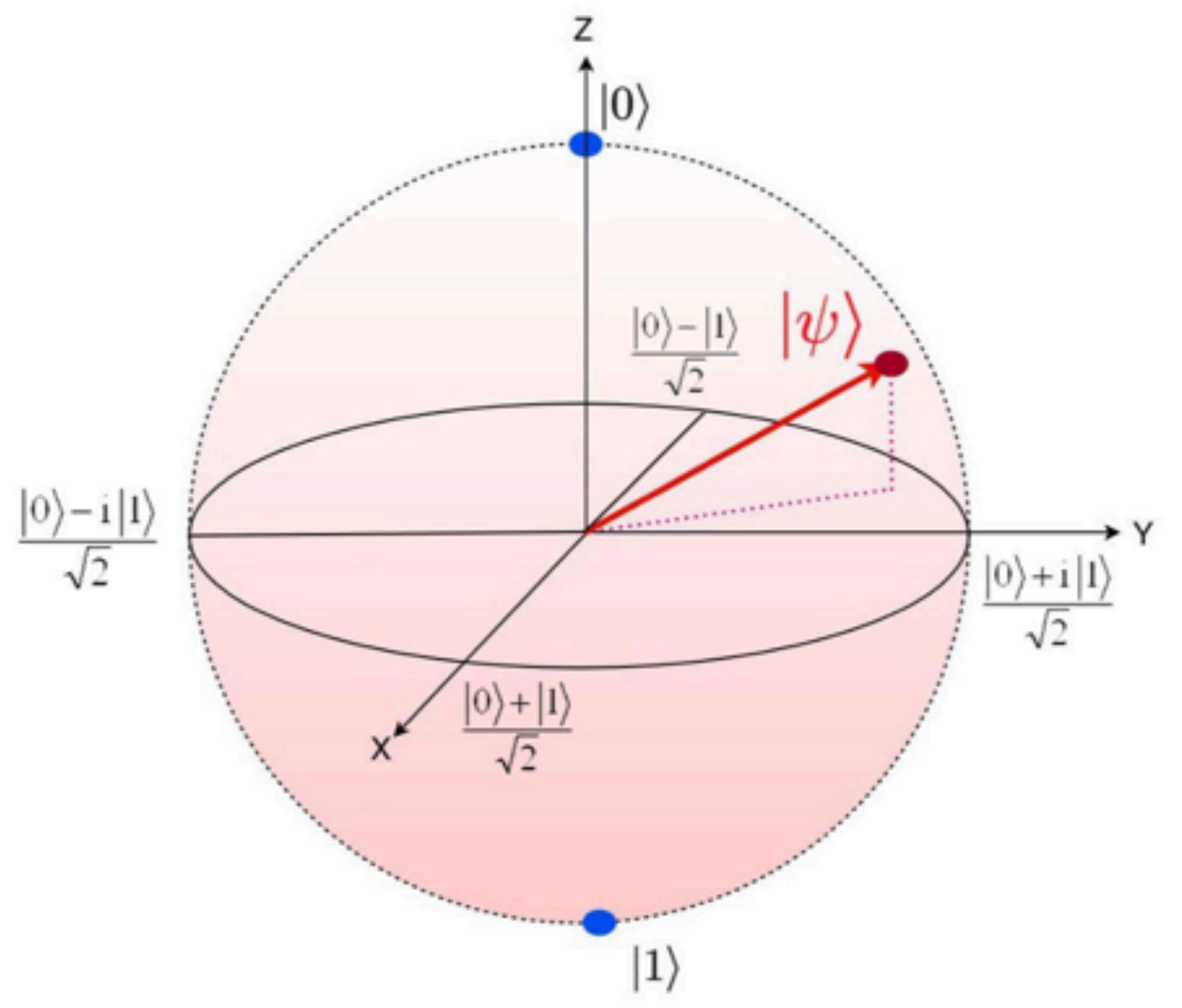
3. Hands-on: Circuit synthesis



Recap

Quantum bit (qubit): $|\psi\rangle = a_0|0\rangle + a_1|1\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$



When we measure the state $|\psi\rangle$, we obtain either:

- State $|0\rangle$, with a probability $|a_0|^2$
- State $|1\rangle$, with a probability $|a_1|^2$

For N qubits, there are 2^N amplitudes

e.g. $|\psi\rangle = a_1|000\rangle + a_2|001\rangle + a_3|010\rangle + a_4|011\rangle + a_5|100\rangle + a_6|101\rangle + a_7|110\rangle + a_8|111\rangle$

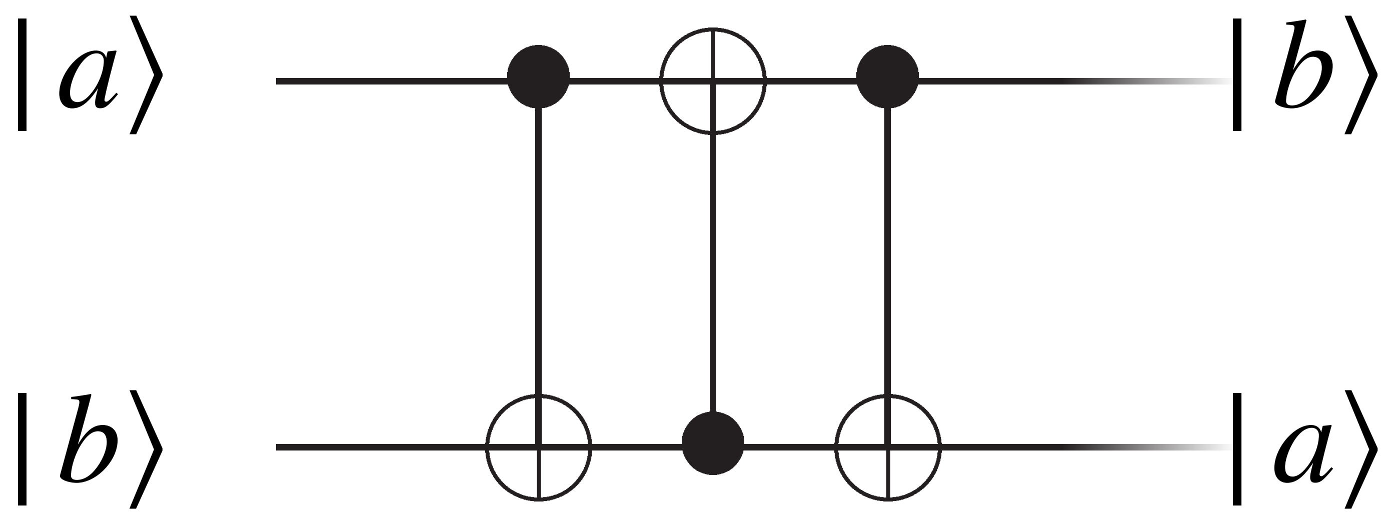
A quantum operation modifies all of these 2^N amplitudes simultaneously!

$$|a\rangle = \sum_{i=1}^{2^N} a_i|\psi_i\rangle \rightarrow |b\rangle = \sum_{i=1}^{2^N} b_i|\psi_i\rangle$$

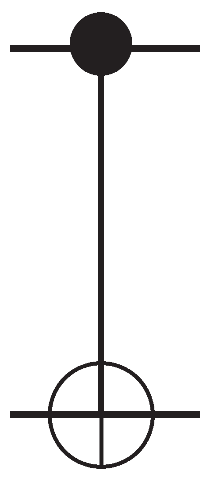
Quantum circuits

Nothing more than (clever) unitary matrix multiplications!

Example: SWAP circuit



where



$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

CNOT gate

$$SWAP\left(|a\rangle \otimes |b\rangle\right) = CNOT_{0,1} \times CNOT_{1,0} \times CNOT_{0,1} \times \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} \otimes \begin{pmatrix} b_0 \\ b_1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a_0 b_0 \\ a_0 b_1 \\ a_1 b_0 \\ a_1 b_1 \end{pmatrix} = \begin{pmatrix} b_0 a_0 \\ b_0 a_1 \\ b_1 a_0 \\ b_1 a_1 \end{pmatrix} = |b\rangle \otimes |a\rangle$$

Where does quantum advantage come from?

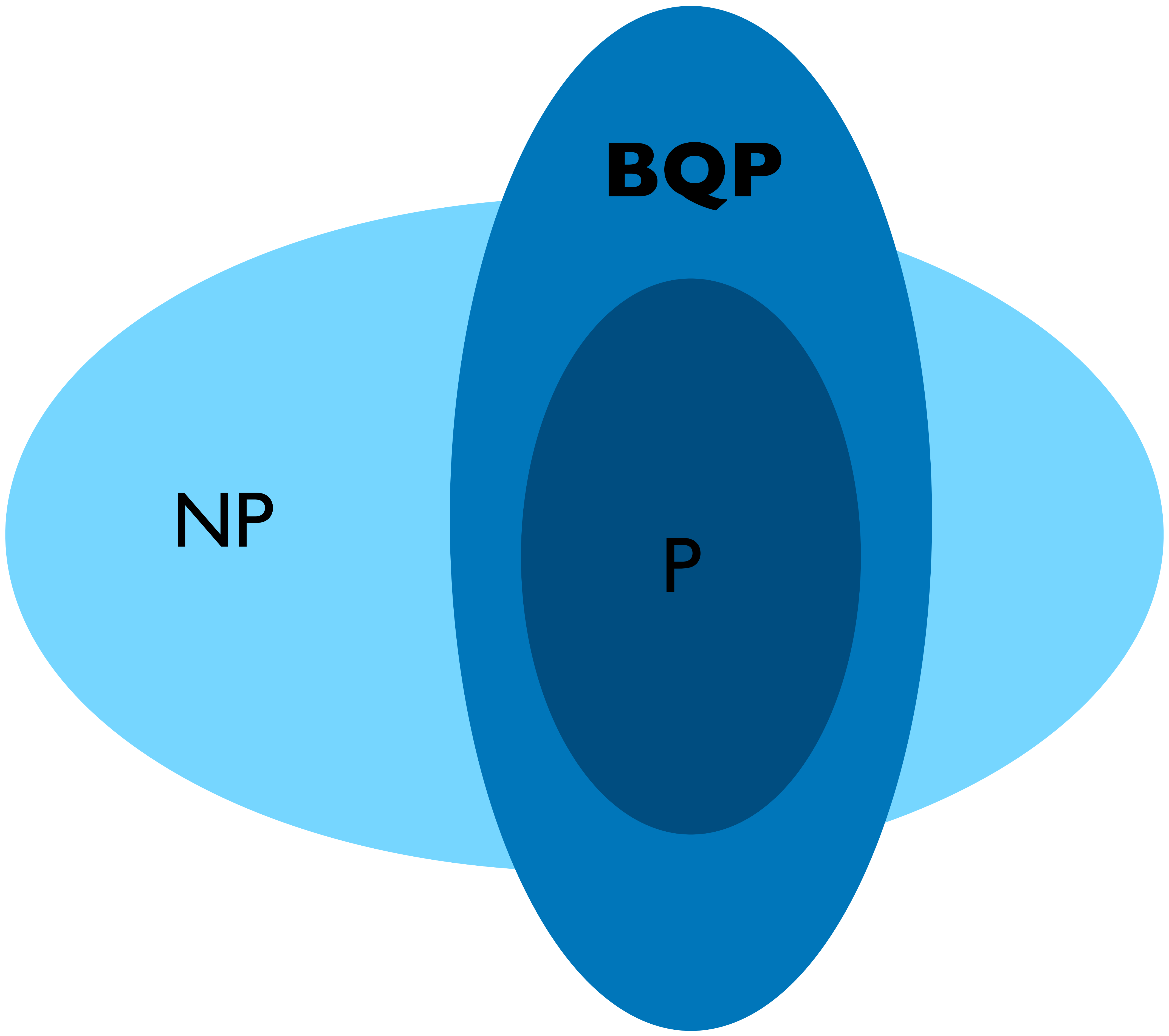
A quantum operation modifies 2^N amplitudes simultaneously

$$|a\rangle = \sum_{i=1}^{2^N} a_i |\psi_i\rangle \rightarrow |b\rangle = \sum_{i=1}^{2^N} b_i |\psi_i\rangle$$

However: we cannot access the quantum amplitudes $\{a_i\}$ directly!

This is the major challenge: How can we take advantage of the exponential efficiency of quantum operations when we only access one randomly sampled state at a time?

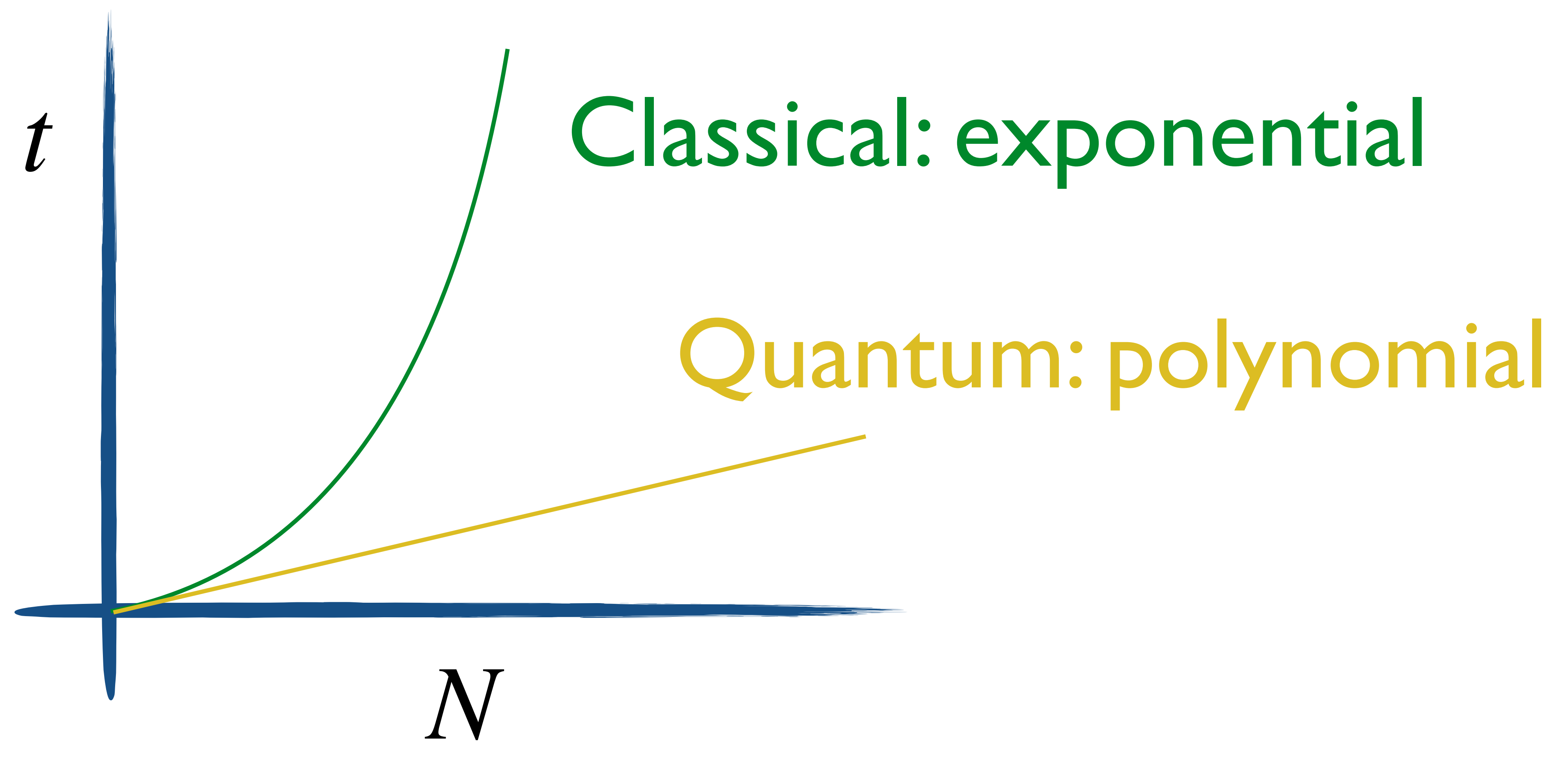
QC can solve *some* classically hard problems



P: Polynomial-time solution on classical computer

NP: Polynomial-time verification on classical computer

BQP: Polynomial-time solution on quantum computer



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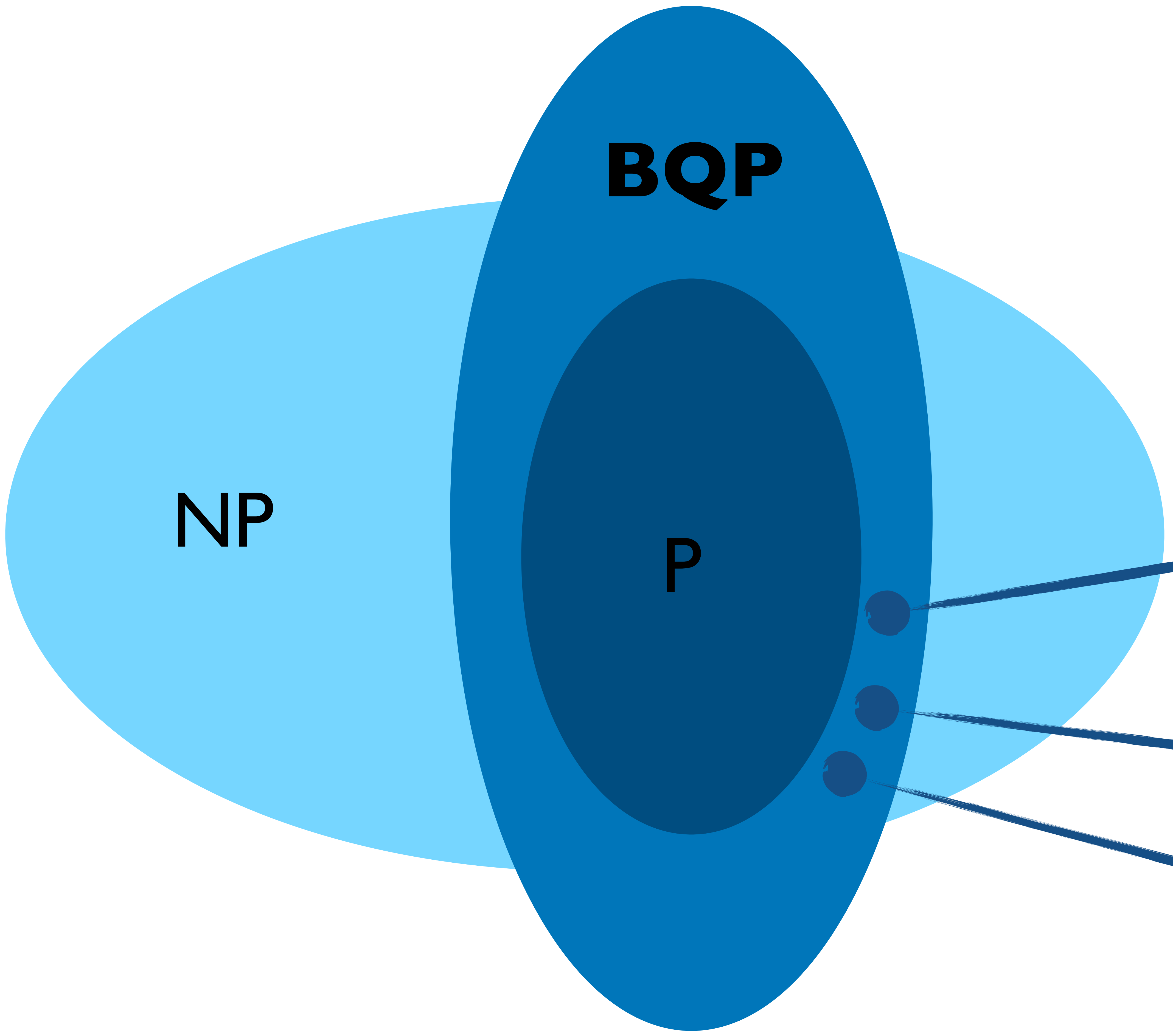


Dynamics of many-body non-relativistic quantum system

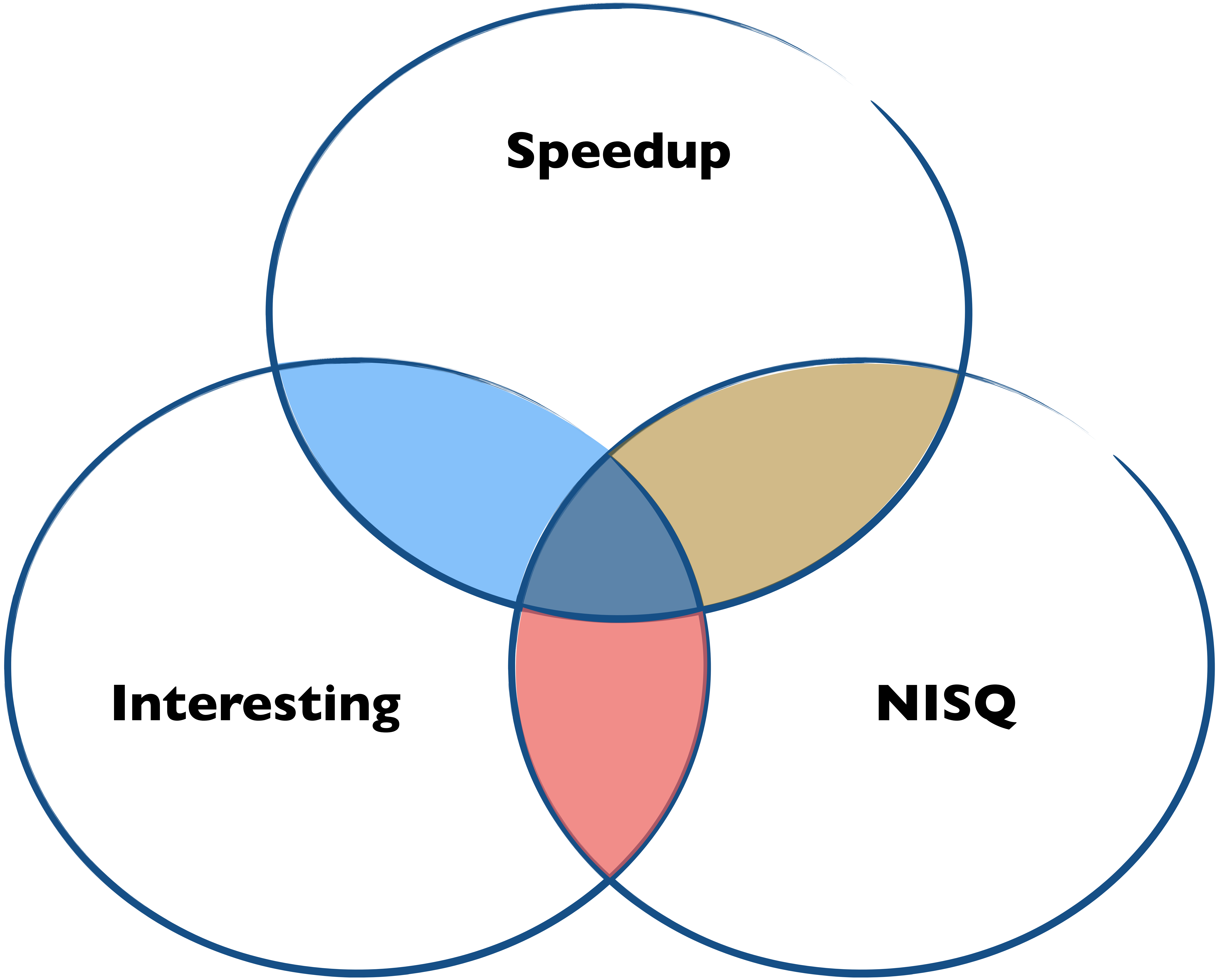
Feynman (1982), Lloyd (1996)

Scattering in scalar field theory

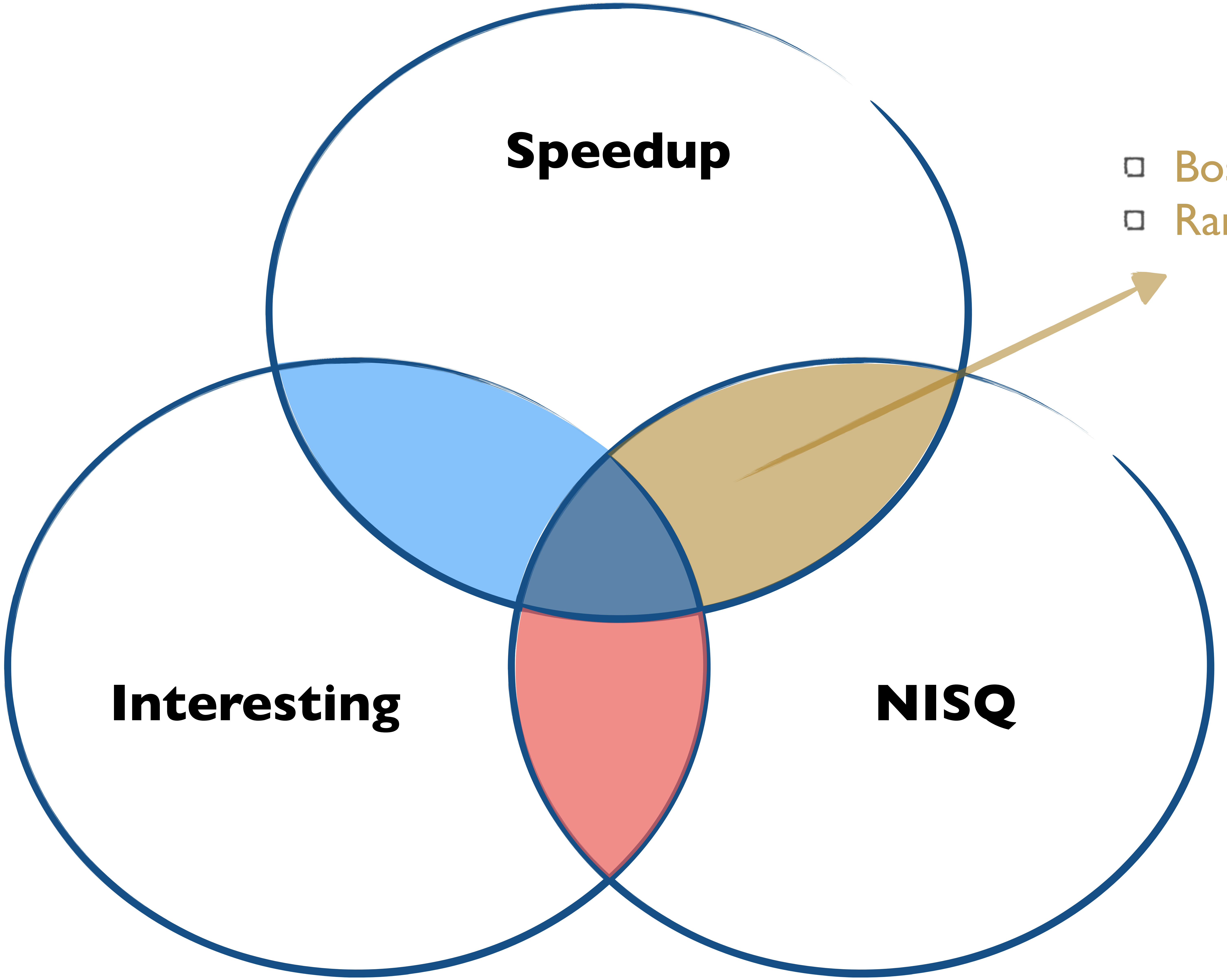
Jordan, Lee, Preskill (2012-2018)

QCD?

Preskill (2018), Klco, Savage (2018), Muschik et al. (2016), Davoudi et al. (2019), …



Based on Scott Aaronson



- Boson sampling
- Random circuit sampling

Based on Scott Aaronson

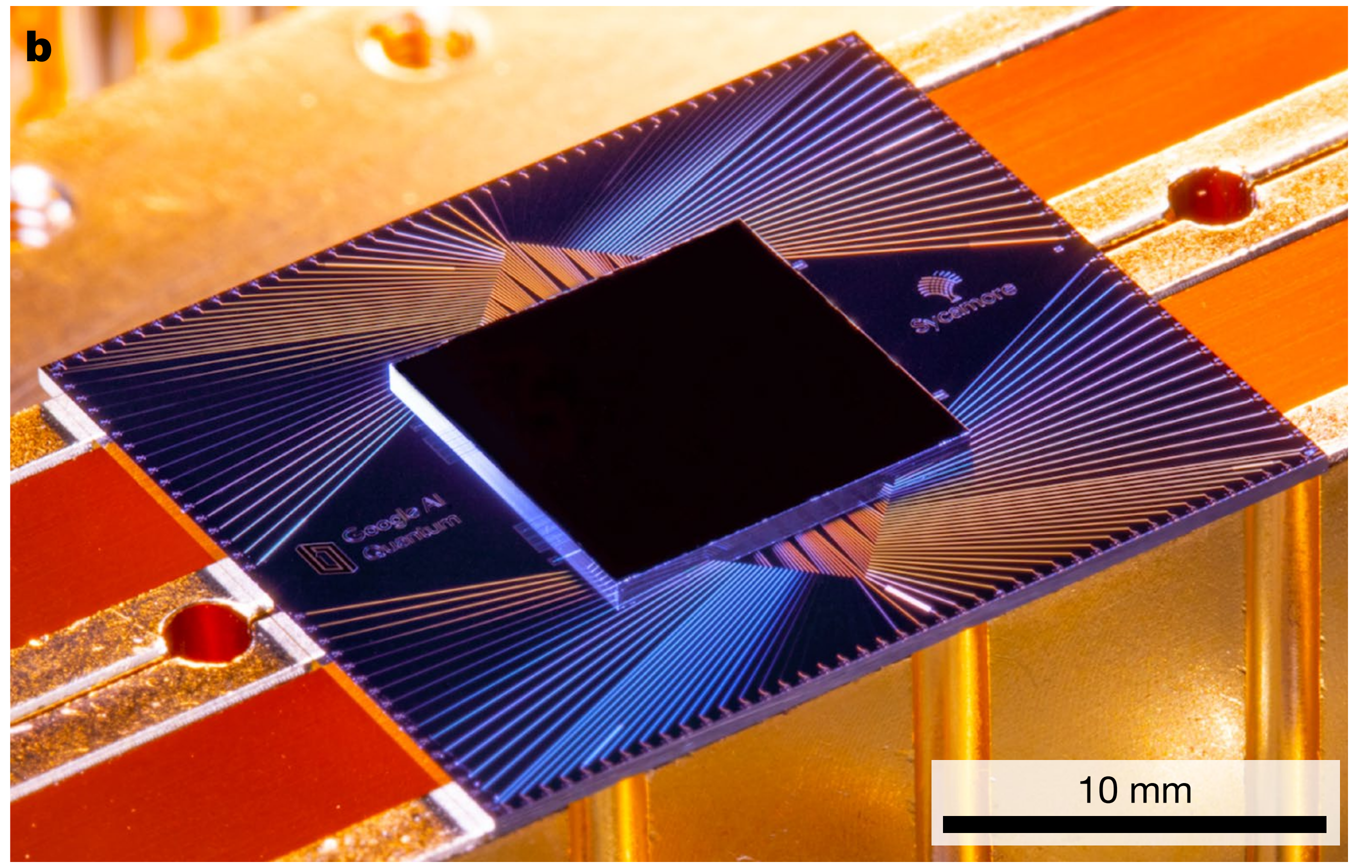
Quantum advantage

2019

Article

Quantum supremacy using a programmable superconducting processor

Martinis et al., Nature (2019)



53-qubit superconducting circuit device

Algorithm: sampling of random circuits

$\mathcal{O}\left(10^3\right)$ times faster than best classical supercomputers

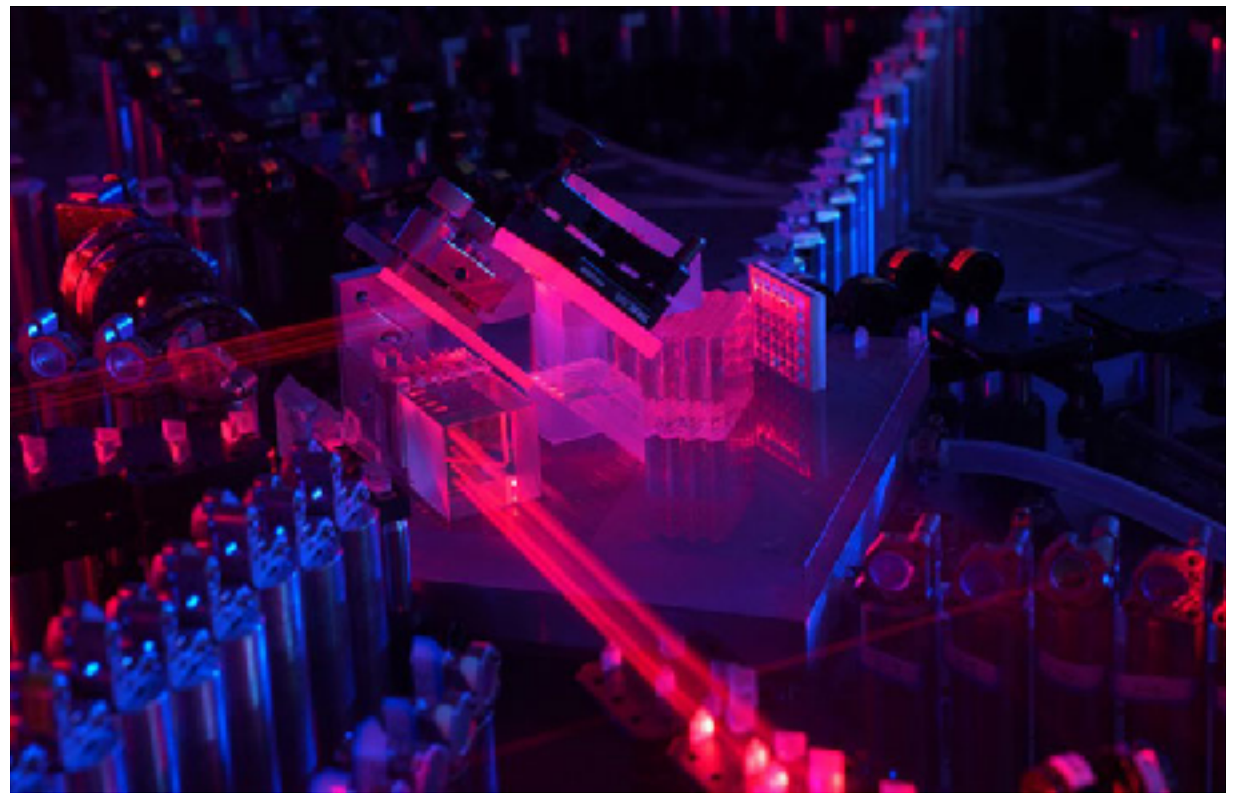
See also: Pan et al., PRL (2021)

2020-2021

Quantum computational advantage using photons

Han-Sen Zhong[1,2*], Hui Wang[1,2*], Yu-Hao Deng[1,2*], Ming-Cheng Chen[1,2*], Li-Chao Peng[1,2], Yi-Han Luo[1,2], Jian Qin[1,2], Dian Wu[1,2], Xing Ding[1,2], Yi Hu[1,2], Peng Hu[3], Xiao-Yan Yang[3], Wei-Jun Zhang[3], Hao Li[3], Yuxuan Li[4], Xiao Jiang[1,2], Lin Gan[4], Guangwen Yang[4], Lixing You[3], Zhen Wang[3], Li Li[1,2], Nai-Le Liu[1,2], Chao-Yang Lu[1,2], Jian-Wei Pan[1,2†]

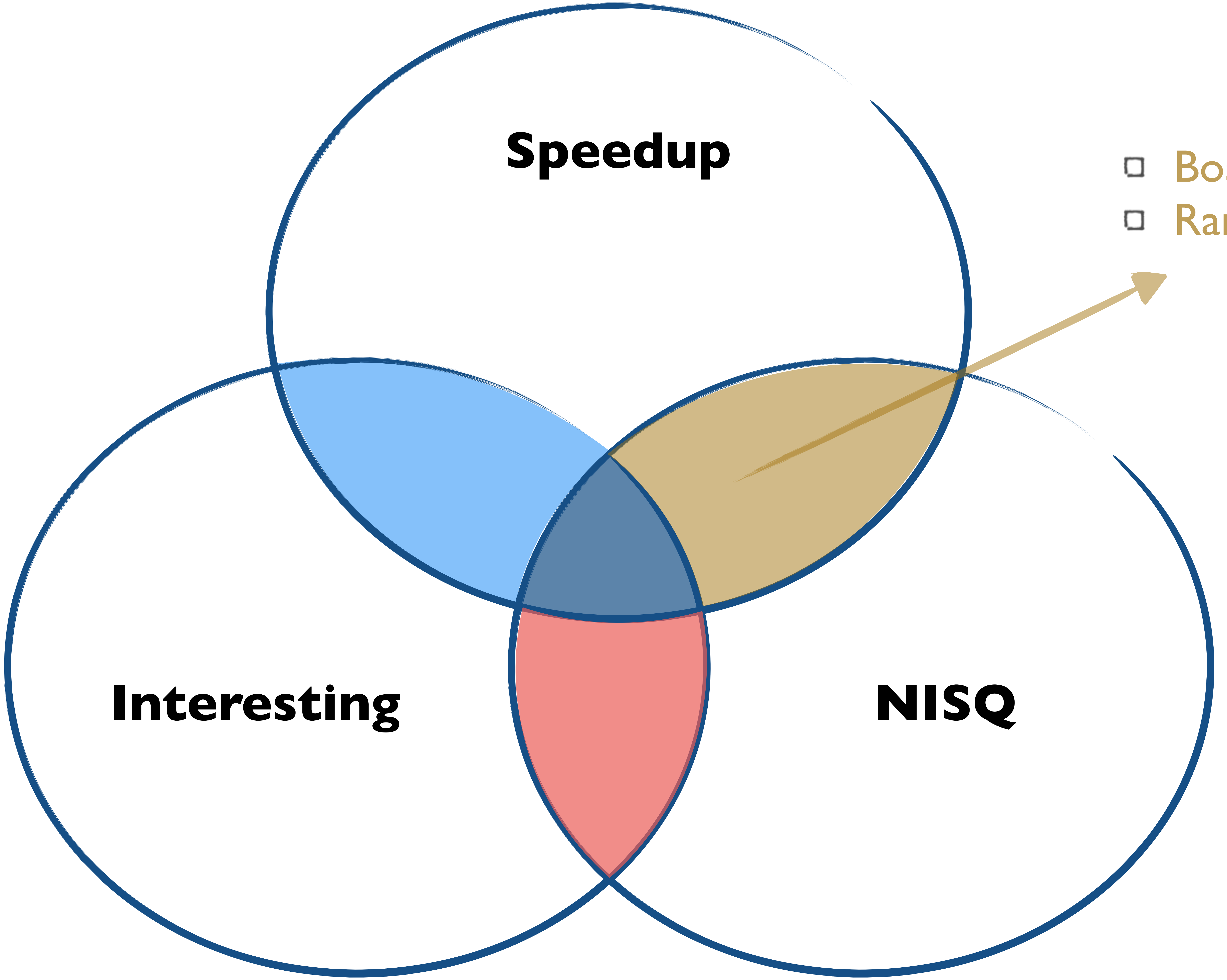
Pan et al., Science (2020)



Photonic device — special-purpose

Algorithm: boson sampling

Claim: $\mathcal{O}\left(10^{14}\right)$ times faster than best classical supercomputers



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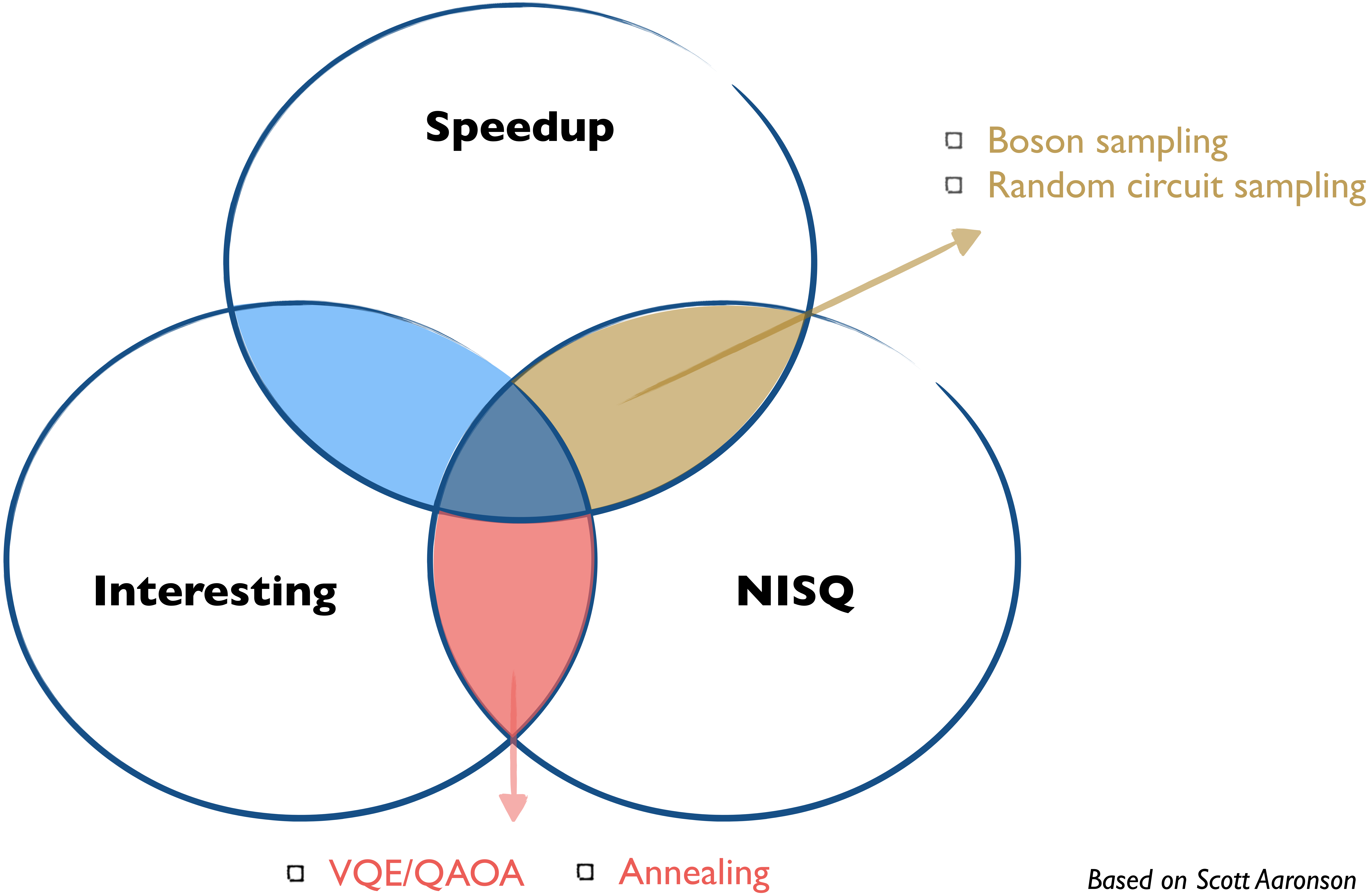
Based on Scott Aaronson

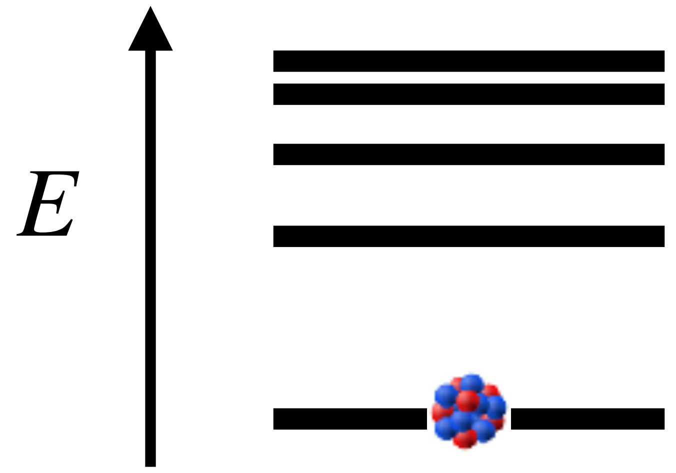


Based on Scott Aaronson

Variational Quantum Eigensolver

Use the variational principle to estimate ground state energy:

$$E_{\mathrm{trial}} = \langle \psi_{\mathrm{trial}} | H | \psi_{\mathrm{trial}} \rangle \geq E_0$$



Hybrid quantum-classical algorithm

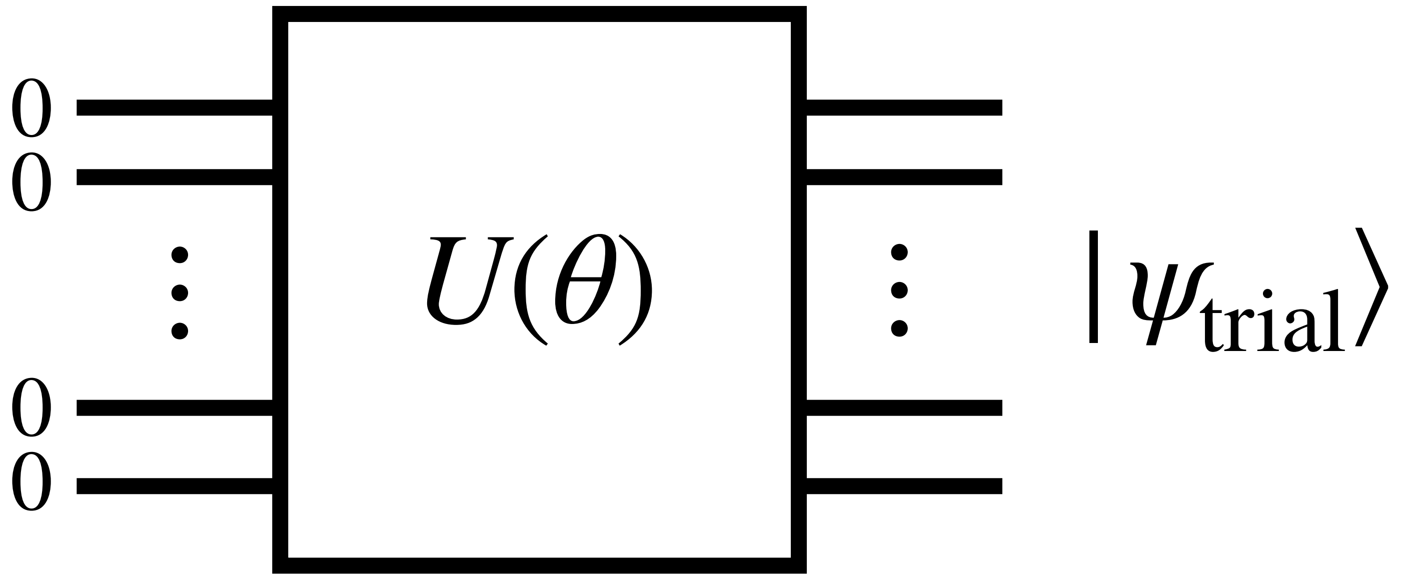
Quantum computer

Classical computer

Choose parameters θ_{trial} in a quantum circuit $U(\theta)$

Initialize the trial wavefunction: $|\psi_{\mathrm{trial}}\rangle = U(\theta)|0 \cdots 0\rangle$

Measure the energy: $E_{\mathrm{trial}} = \langle \psi_{\mathrm{trial}} | H | \psi_{\mathrm{trial}} \rangle$



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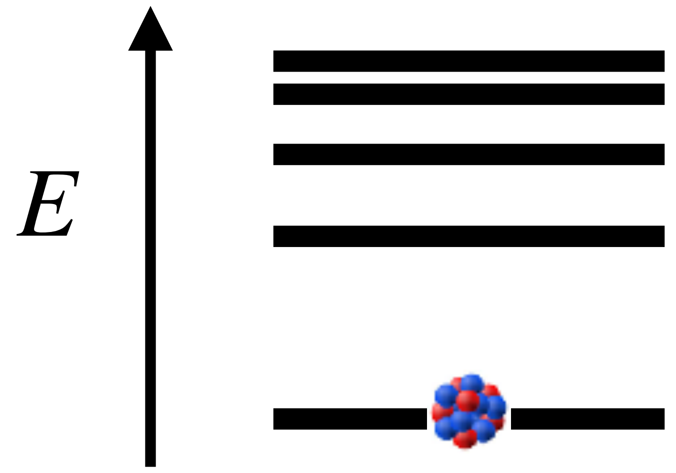

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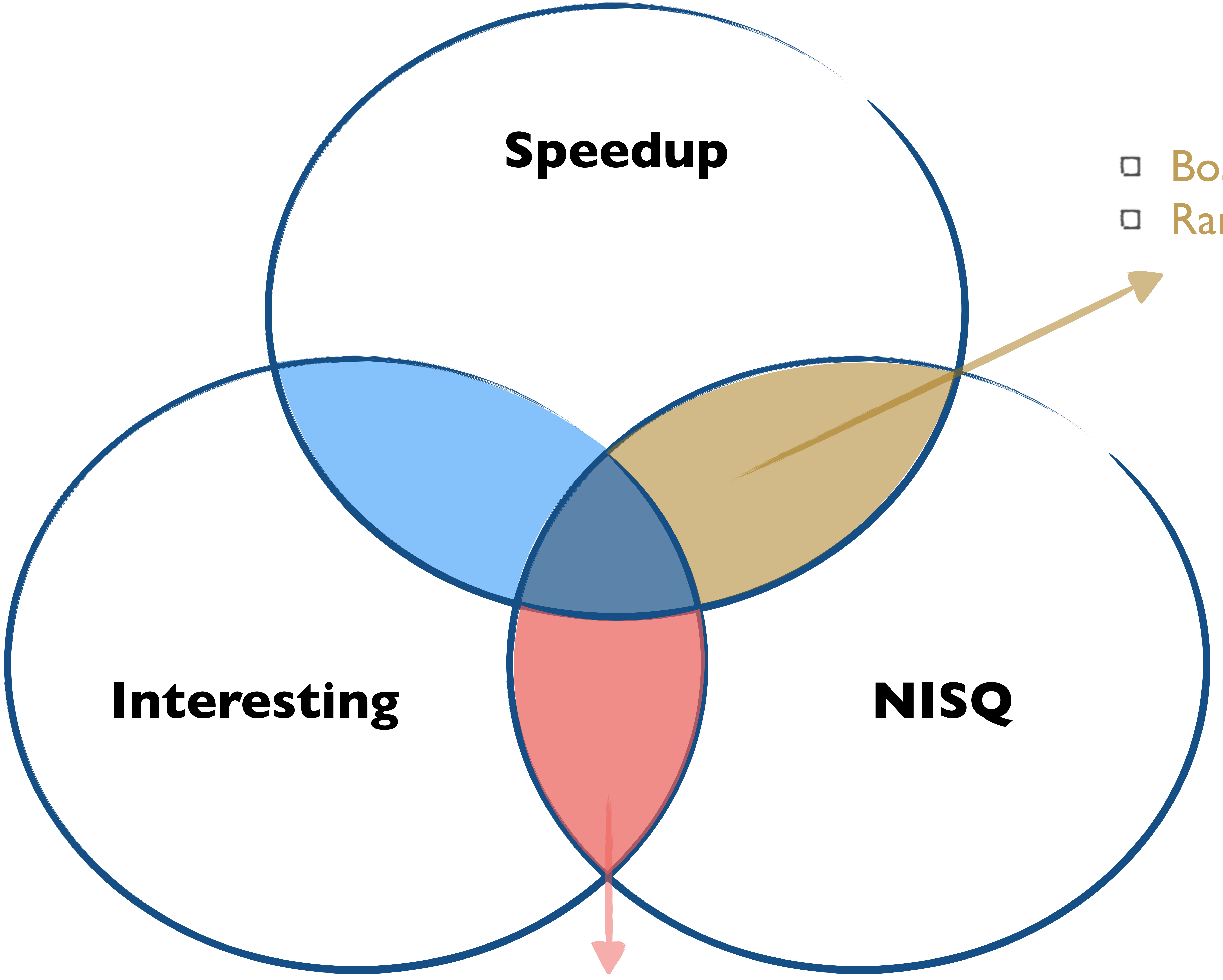
Measure the energy: $E_{\text{trial}} = \langle \psi_{\text{trial}} | H | \psi_{\text{trial}} \rangle$

Compare E_{trial} to E_{min}

Classical optimizer (e.g. SPSA)



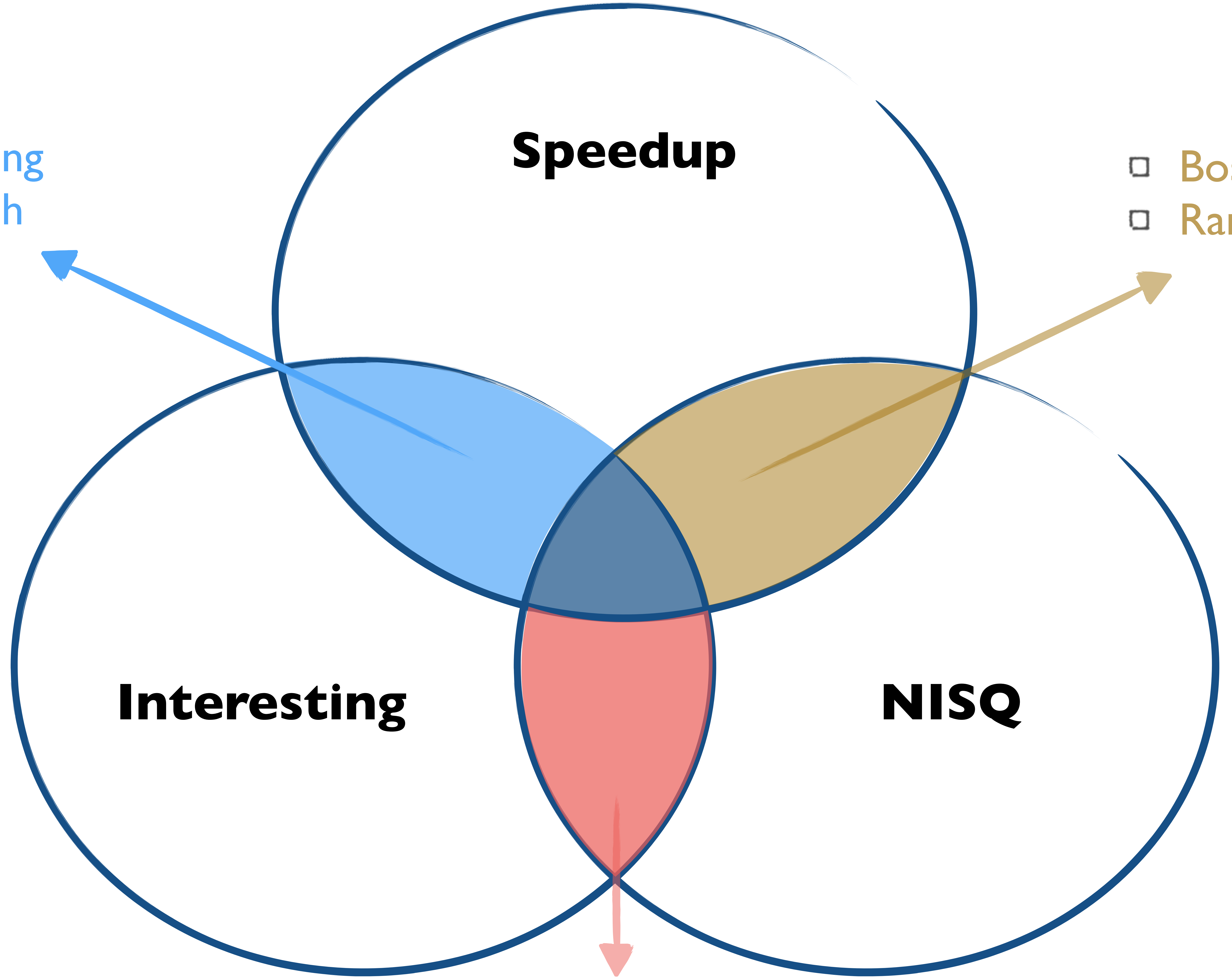
- Boson sampling
- Random circuit sampling

- VQE/QAOA
- Annealing

Based on Scott Aaronson

- Shor's factoring
- Grover search
- Simulation

- Boson sampling
- Random circuit sampling



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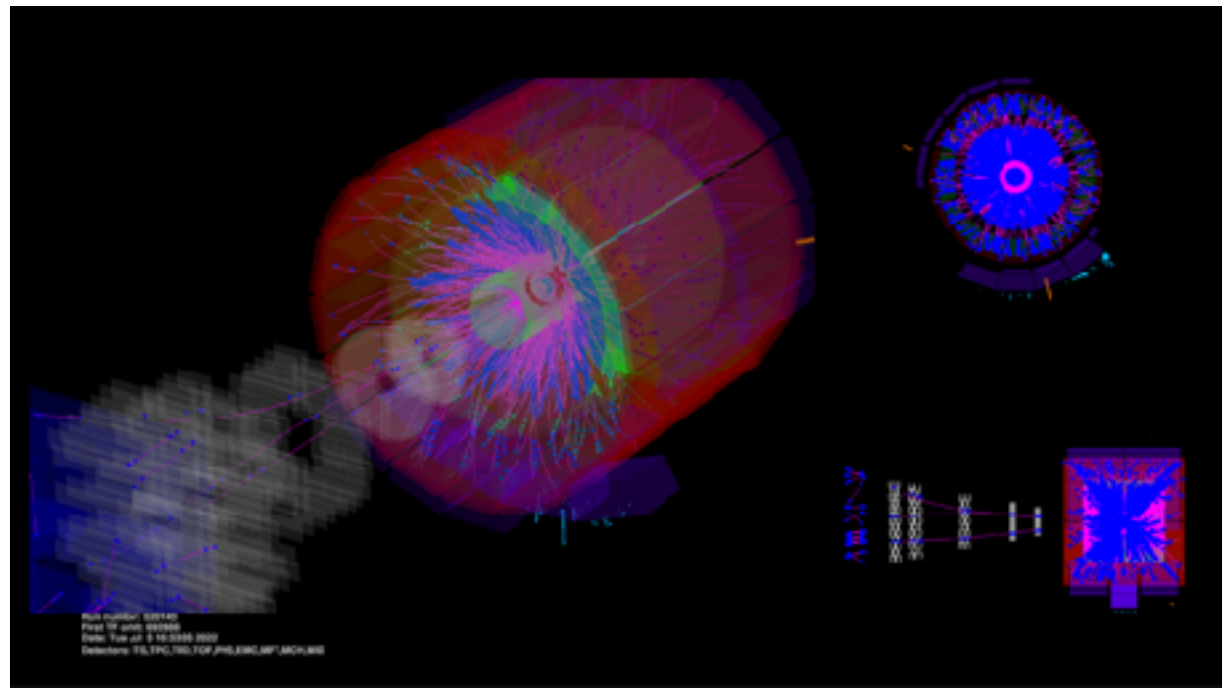
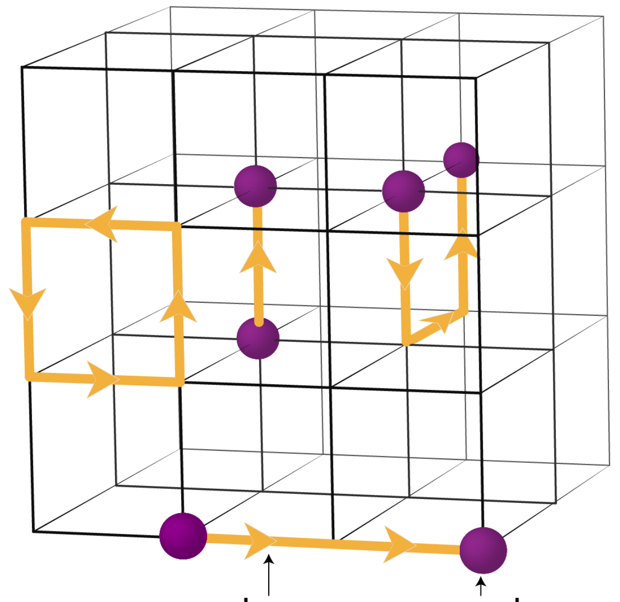
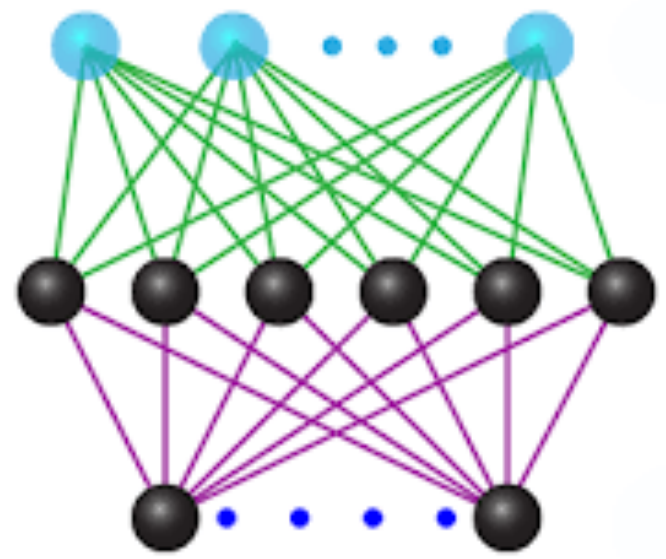
Future applications of quantum computers

Simulation of quantum field theory

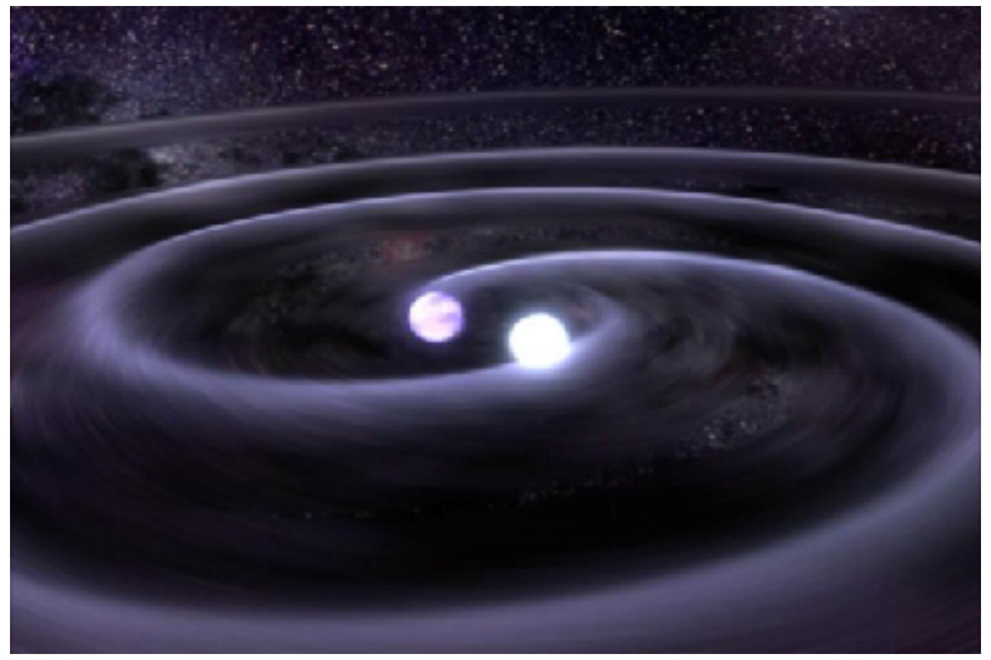




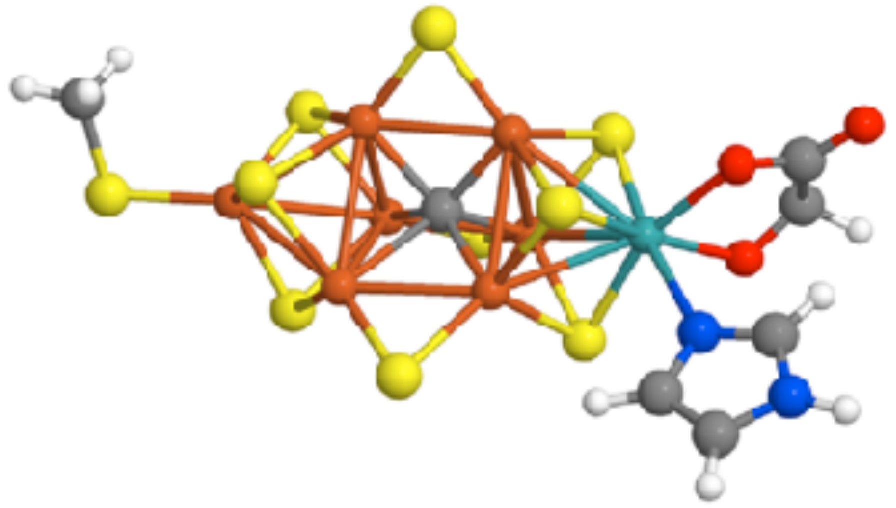
Quantum machine learning



Dense nuclear matter



Molecular dynamics



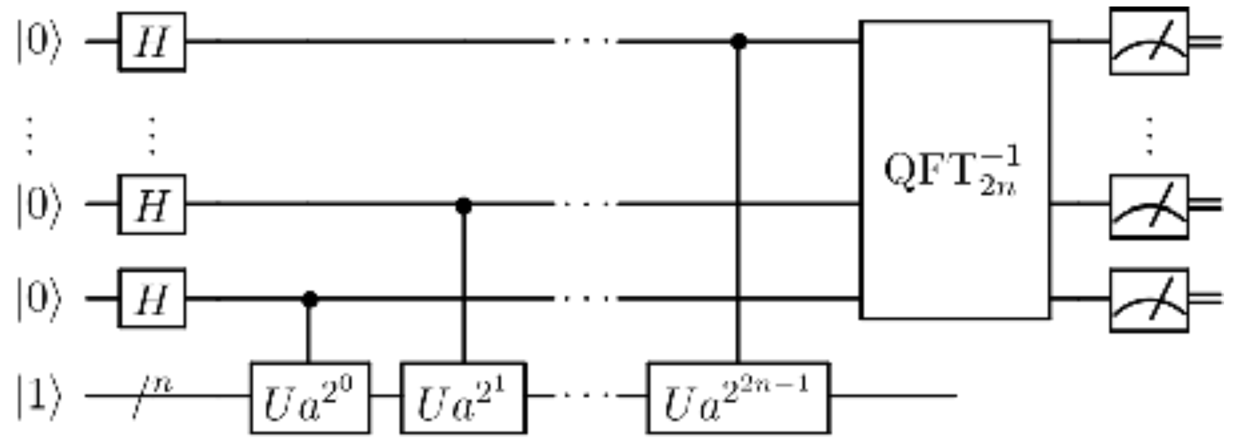
Cryptography

…

Quantum algorithms

Shor's factoring algorithm

Task: Find prime factors of an integer

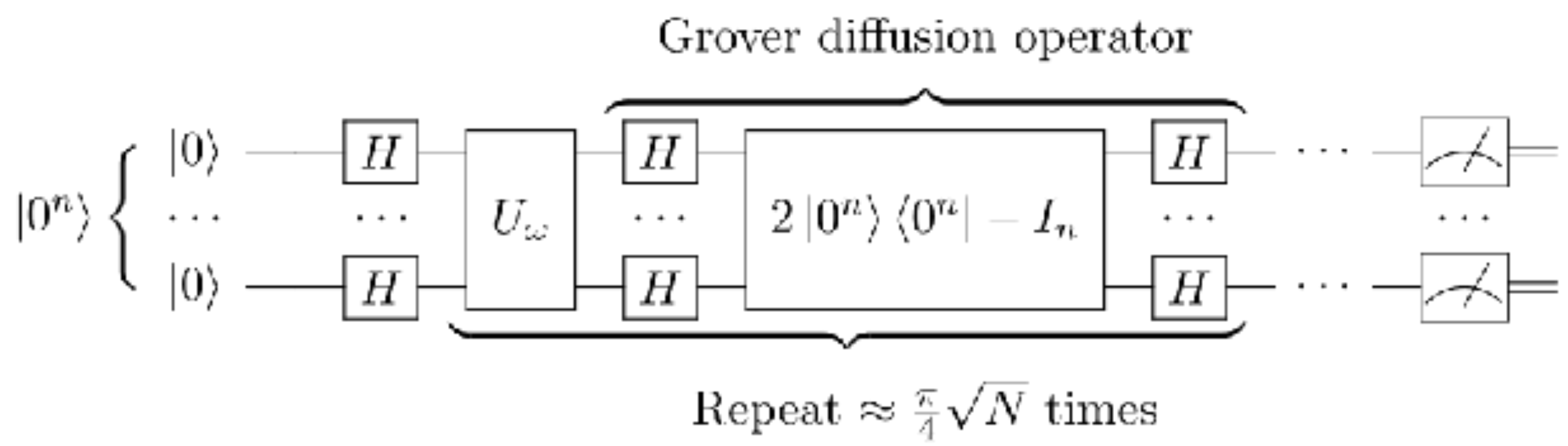


Exponential speedup compared to classical algorithms

$\mathcal{O}\left((\log N)^2 \ldots\right)$ vs. $\mathcal{O}\left(e^{1.9(\log N)^{1/3}\ldots}\right)$

Grover's search algorithm

Task: Find marked entry in an unordered list



Polynomial speedup compared to classical algorithms

$\mathcal{O}\left(\sqrt{(N)}\right)$ vs. $\mathcal{O}\left(N\right)$

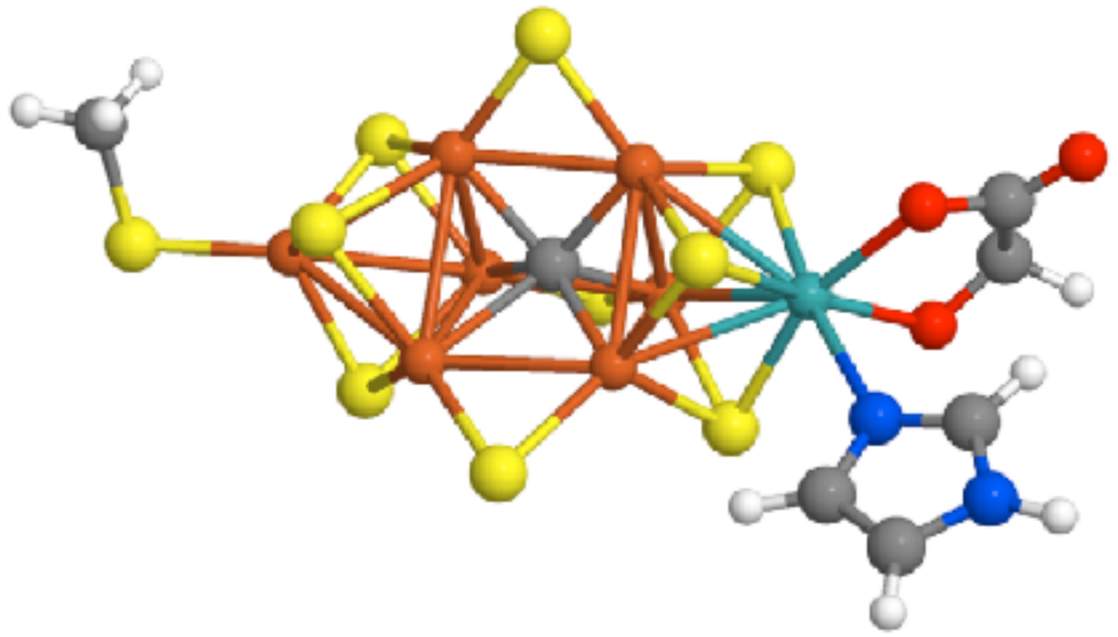
And more…

Quantum simulation

Feynman `81
Lloyd `96

Task: Given the Hamiltonian of a quantum mechanical system, simulate its dynamical evolution

- Quantum chemistry, material design, nuclear dynamics, …



That is, solve the time-dependent Schrödinger equation:

$$H|\psi(t)\rangle = i\hbar\frac{d}{dt}|\psi(t)\rangle$$

The solution is just a unitary evolution!

$$|\psi(t)\rangle = U_H|\psi(0)\rangle \qquad \text{where} \qquad U_H = e^{-iHt/\hbar}$$

It is exponentially expensive to simulate an N-body quantum system on a classical computer: 2^N amplitudes!

- Cannot simulate more than $\mathcal{O}(10-100)$ particles

Quantum simulation

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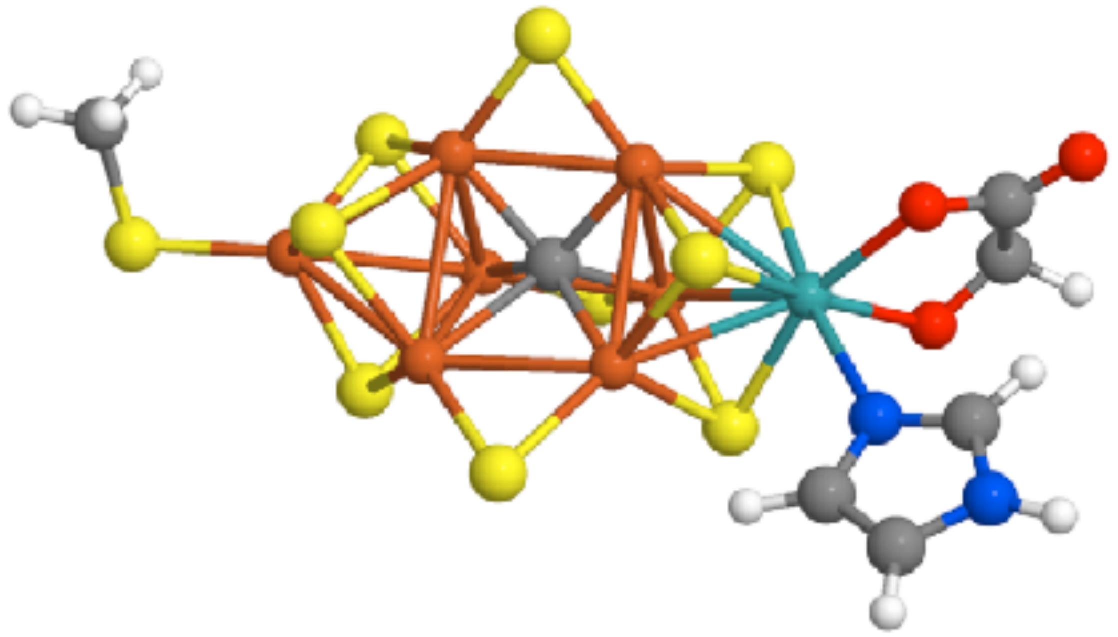
A quantum computer can naturally simulate a quantum system



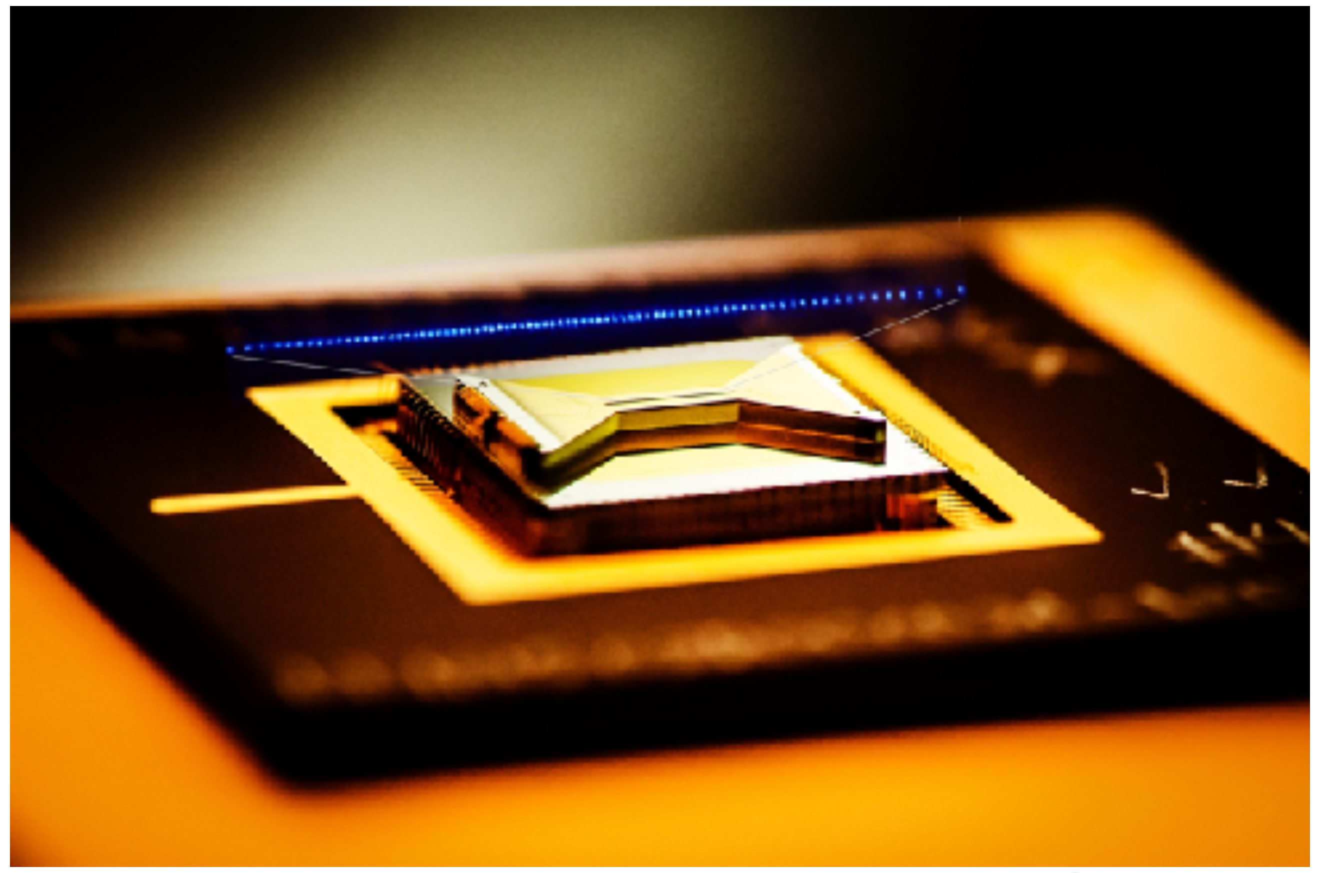
(1) Initial state preparation

$$|0\cdots0\rangle \rightarrow |\psi(0)\rangle$$

(2) Time evolution

$$|\psi(0)\rangle \; \text{—} \boxed{U_H(t)} \text{—} \; |\psi(t)\rangle$$

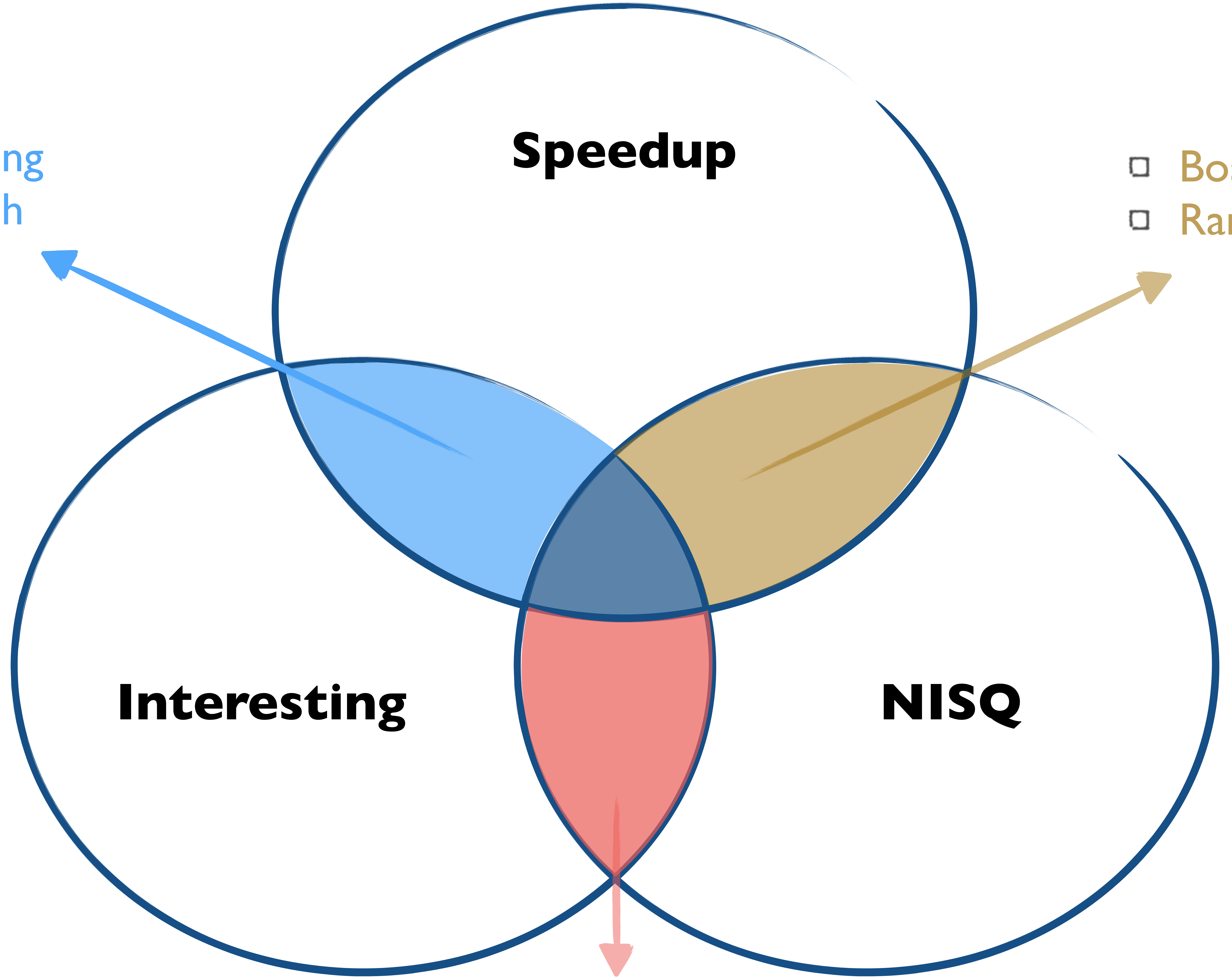
Need efficient encoding of U_H into quantum gates, e.g. local interactions



IONQ

(3) Measurement

- Shor’s factoring
- Grover search
- Simulation



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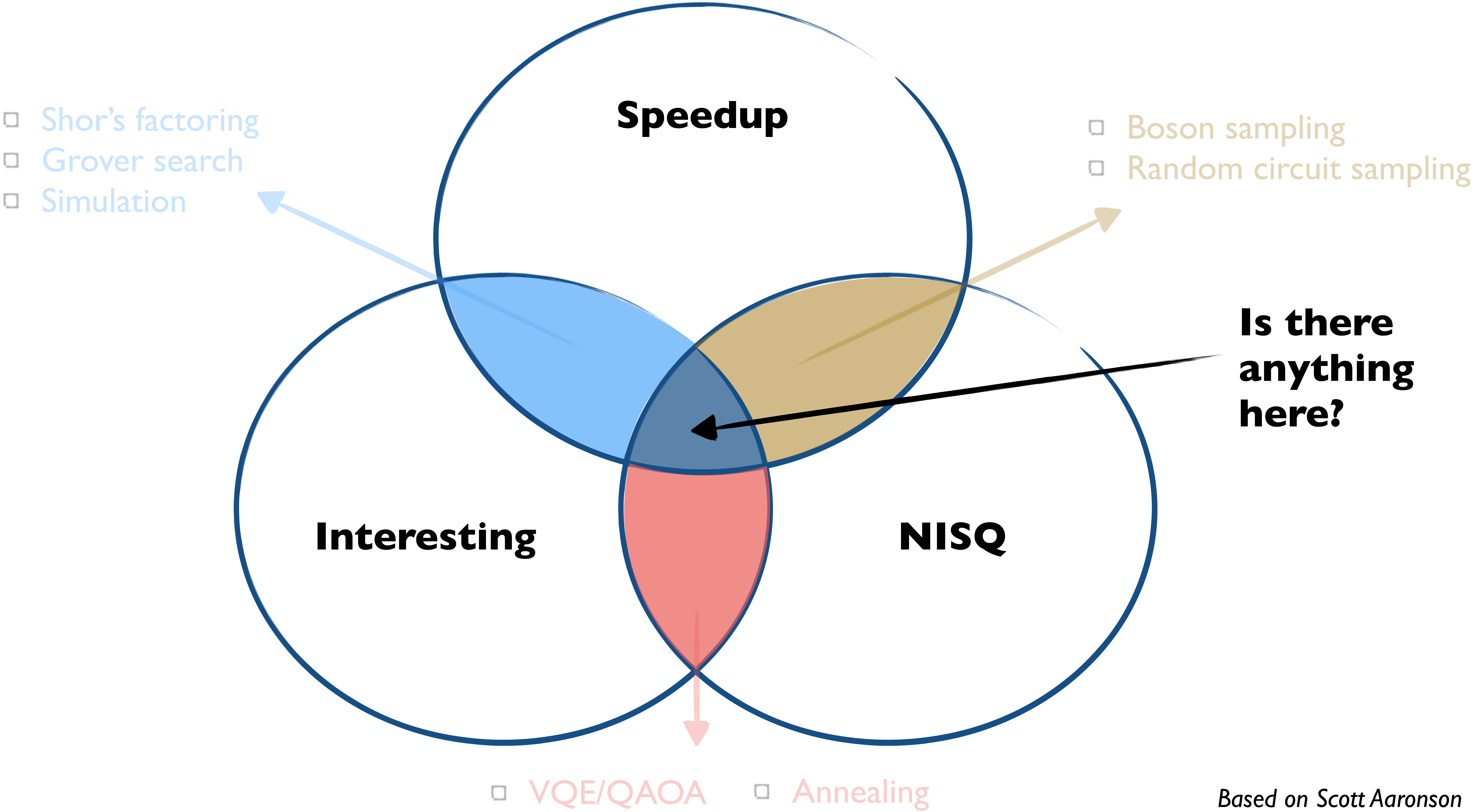
Based on Scott Aaronson



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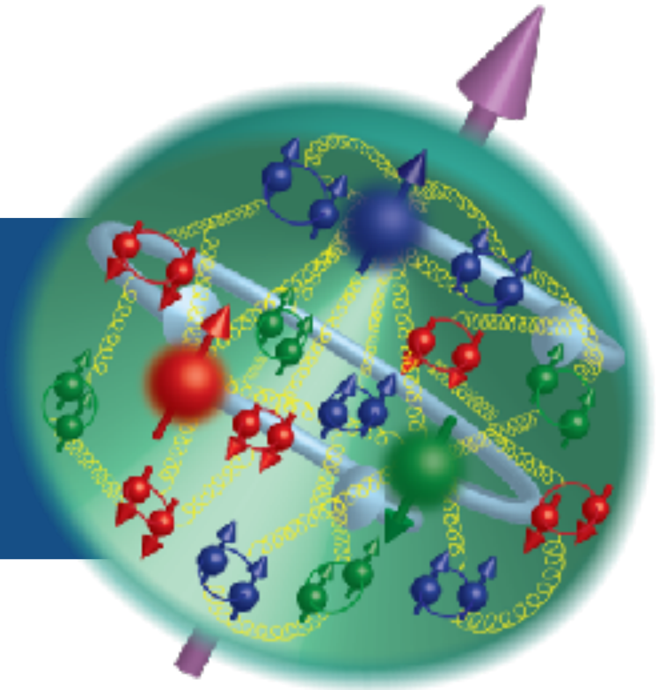

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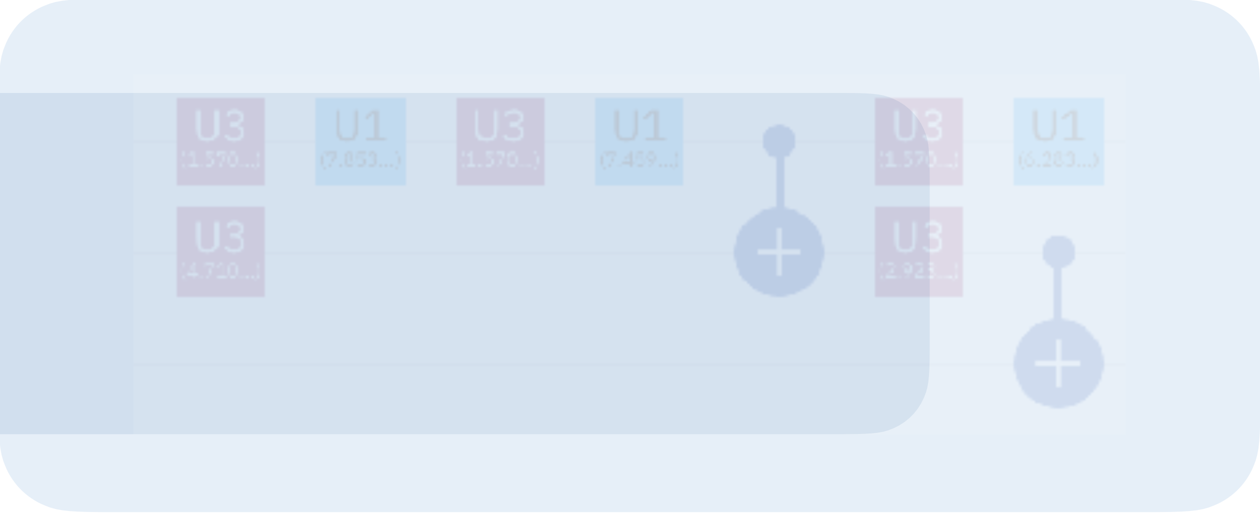
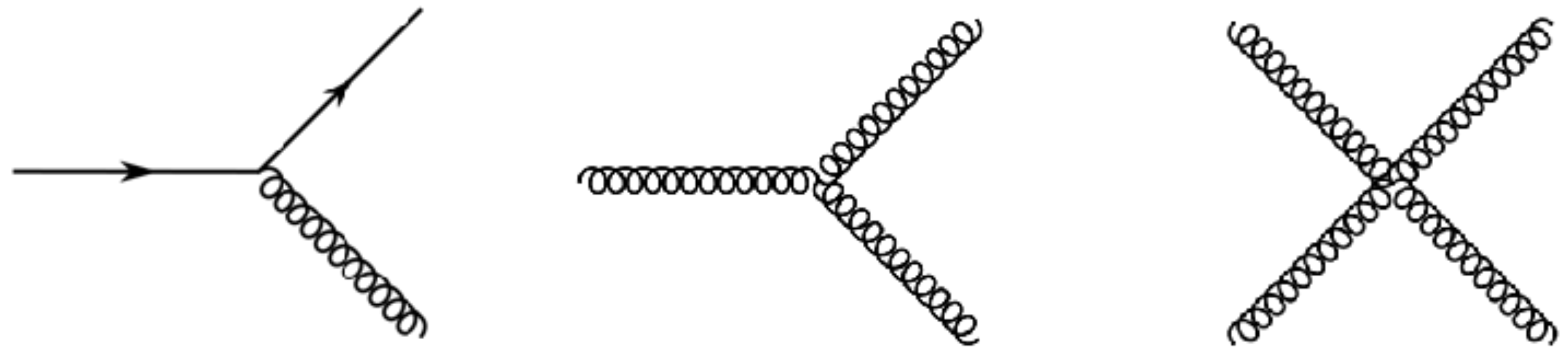


Solving the equations of QCD

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_{j=1}^{6} \bar{q}_j\left(i\gamma^\mu D_\mu - m_j\right)q_j$$

Perturbative QCD

For $\alpha_s \ll 1$, compute scattering amplitudes with Feynman diagrams

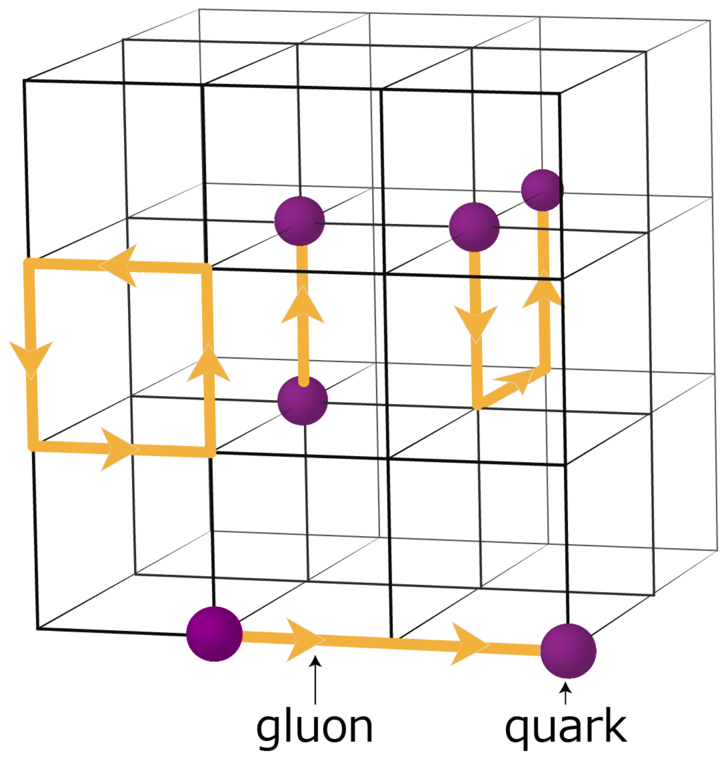


$$\sigma = \sigma^{(0)} + \alpha_s\sigma^{(1)} + \alpha_s^2\sigma^{(2)} + \ldots$$

...but no strong coupling!

Lattice QCD

For low-density systems, compute static quantities with lattice regularization

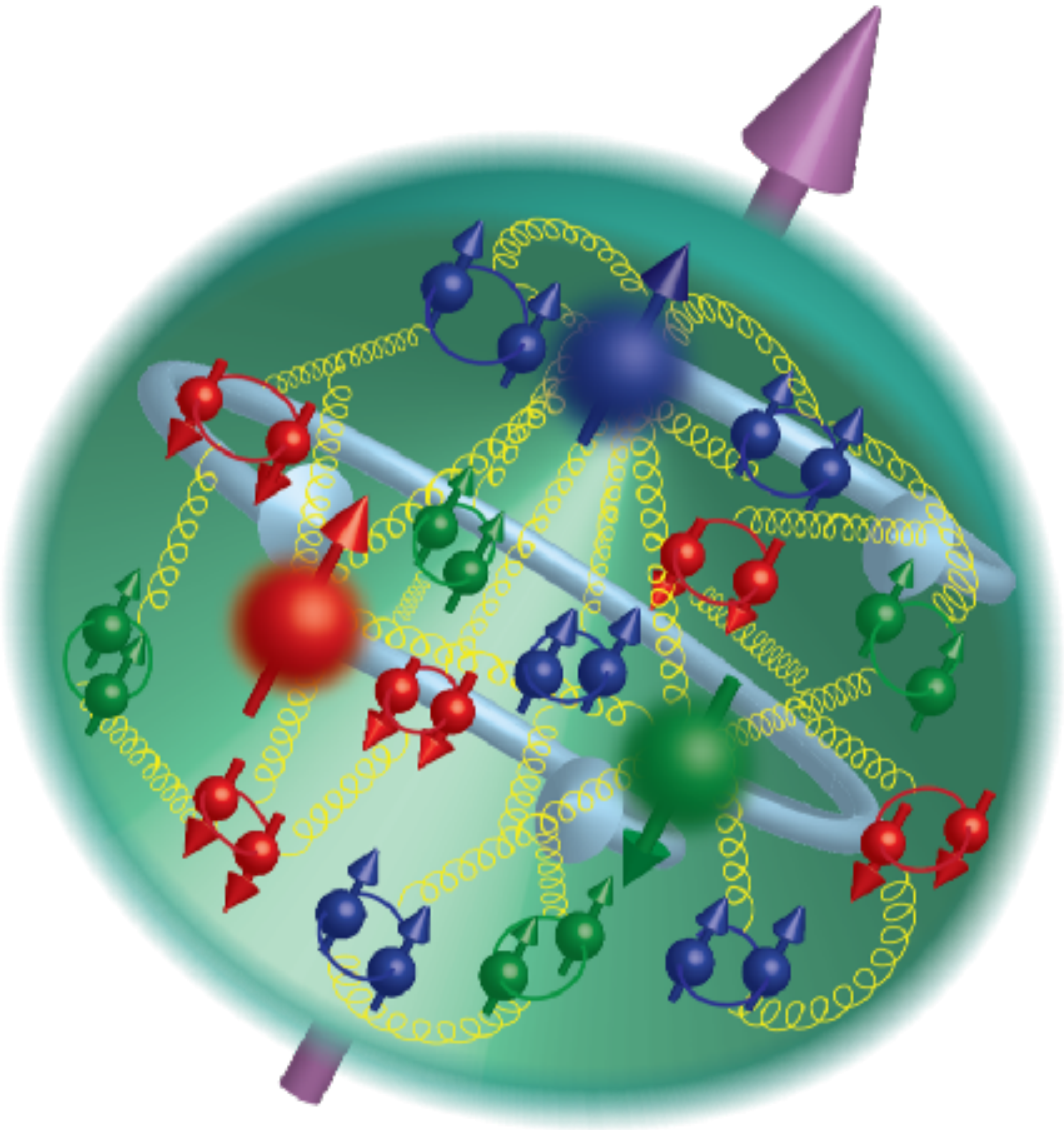


- Hadron spectra
- Deconfinement transition
- Chiral symmetry restoration

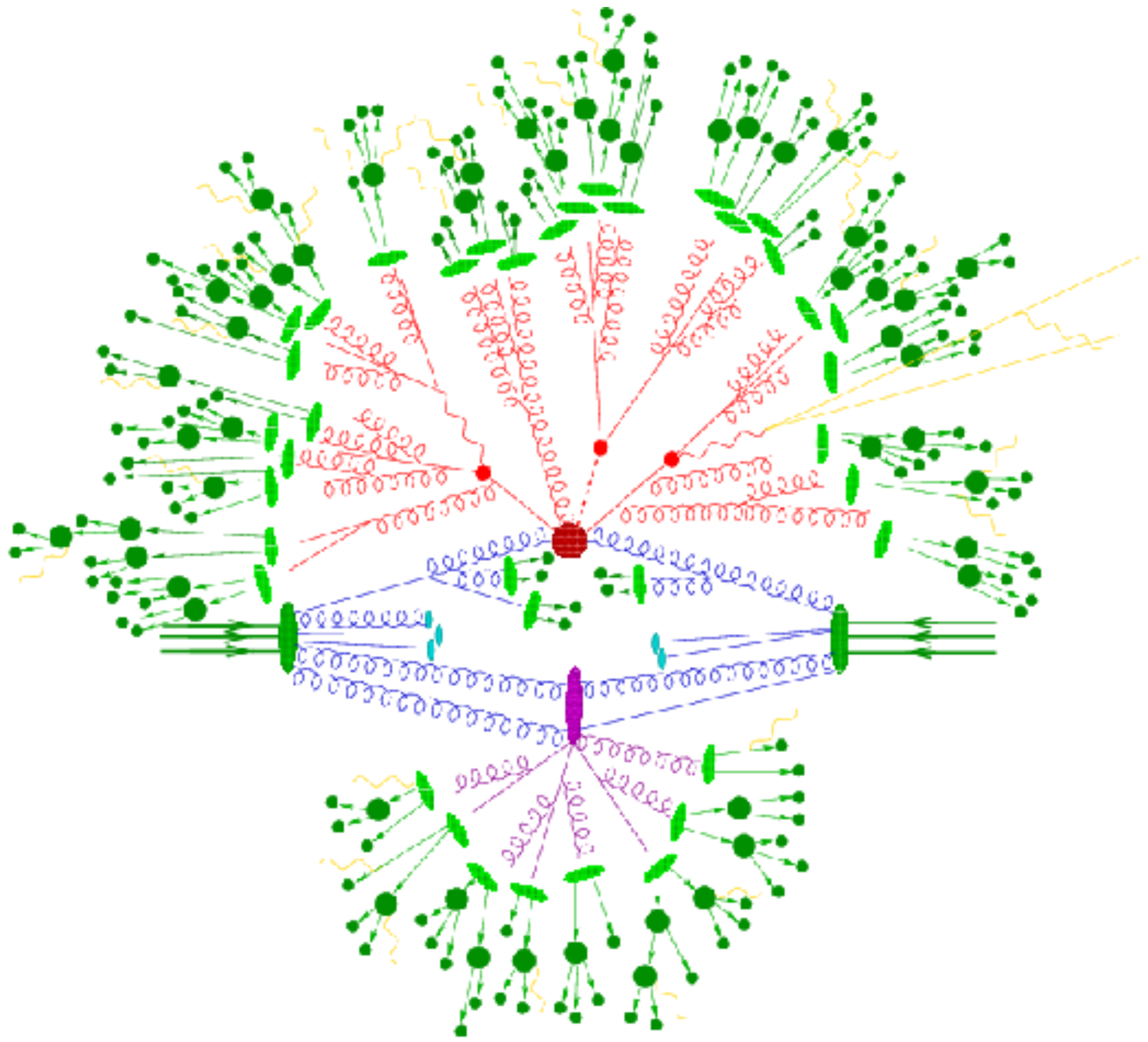
...but no dynamics!

Real-time dynamics

What are the *dynamics* that confine quarks and gluons into hadrons?



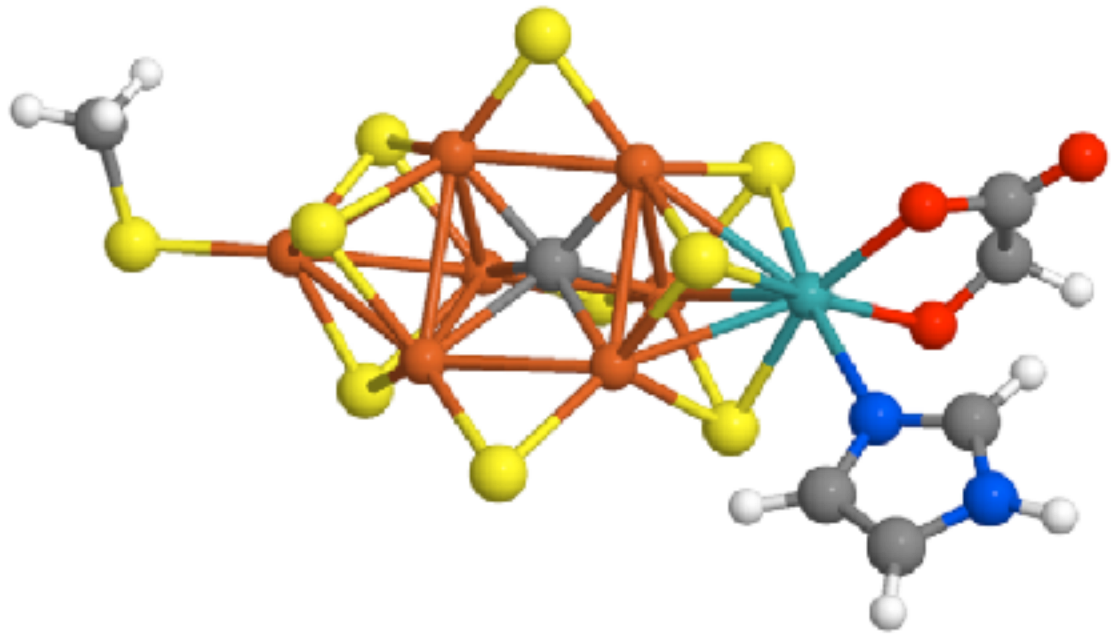
How does a high-energy quark or gluon fragment into a jet?



Quantum simulation

Feynman `81
Lloyd `96

A quantum computer can naturally simulate a quantum system described by a Hamiltonian H



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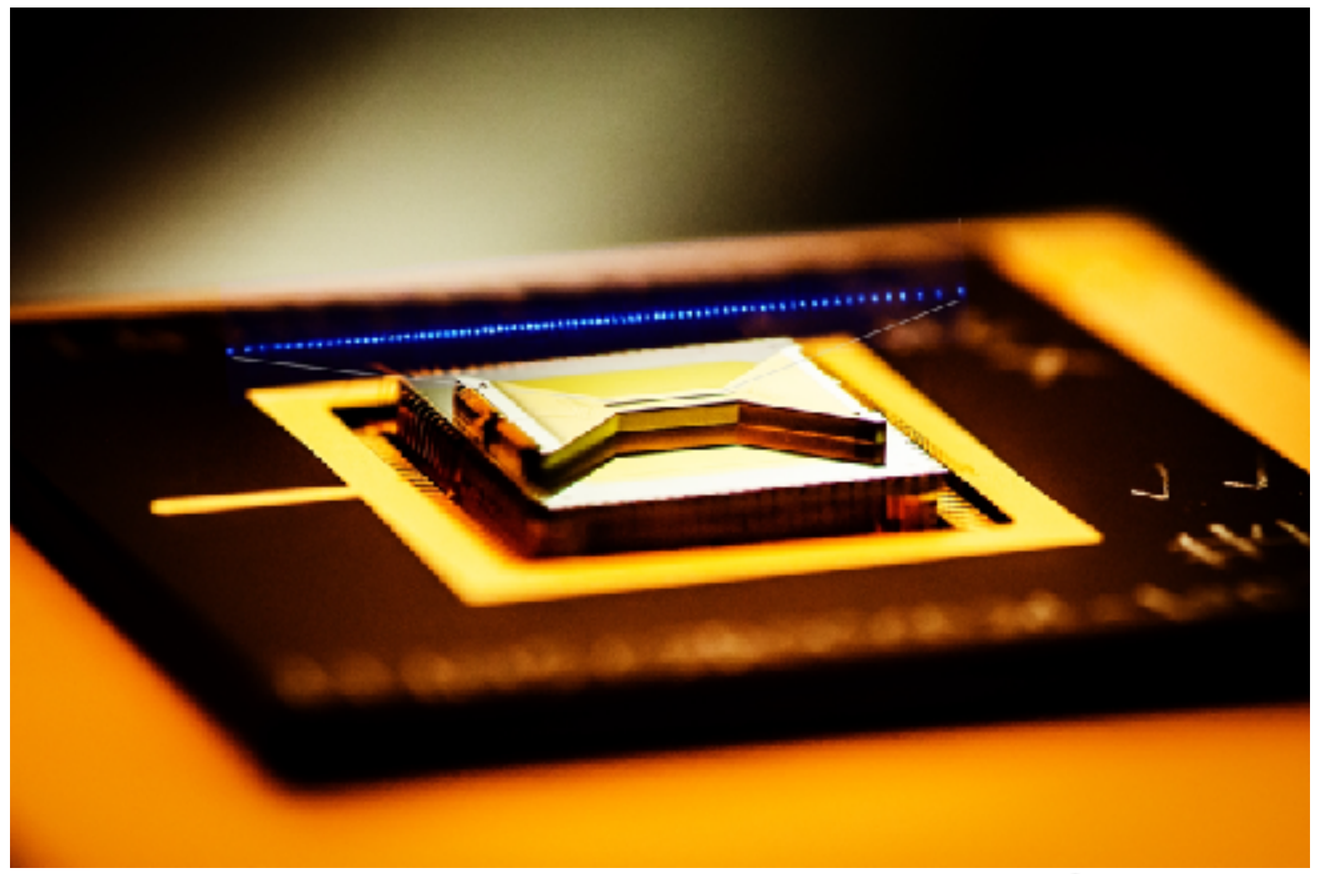
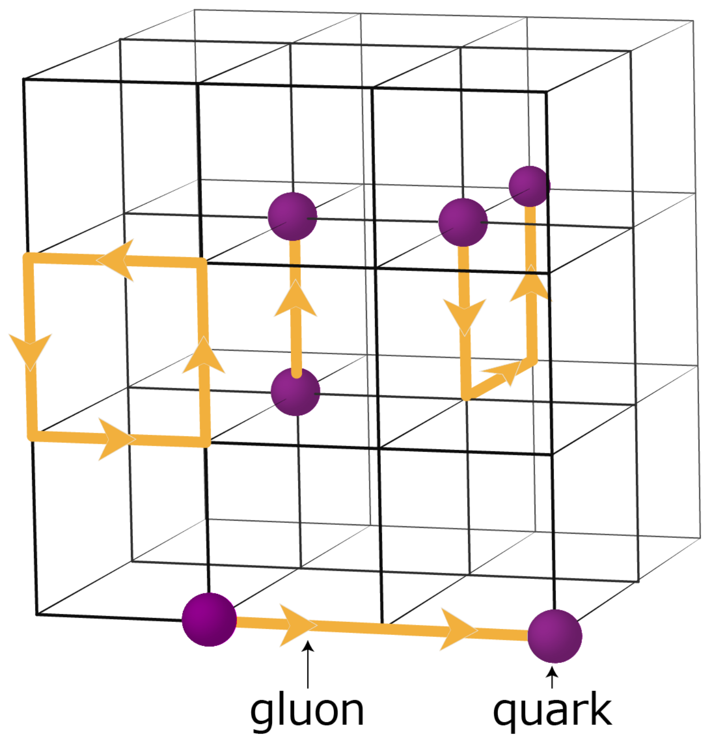


IONQ

(3) Measurement

Simulating quantum field theories

There is an extra complication if we want to simulate QCD: it is a *quantum field theory* — the particle number is not fixed



→ This requires us to simulate fields at all points in spacetime: lattice QCD

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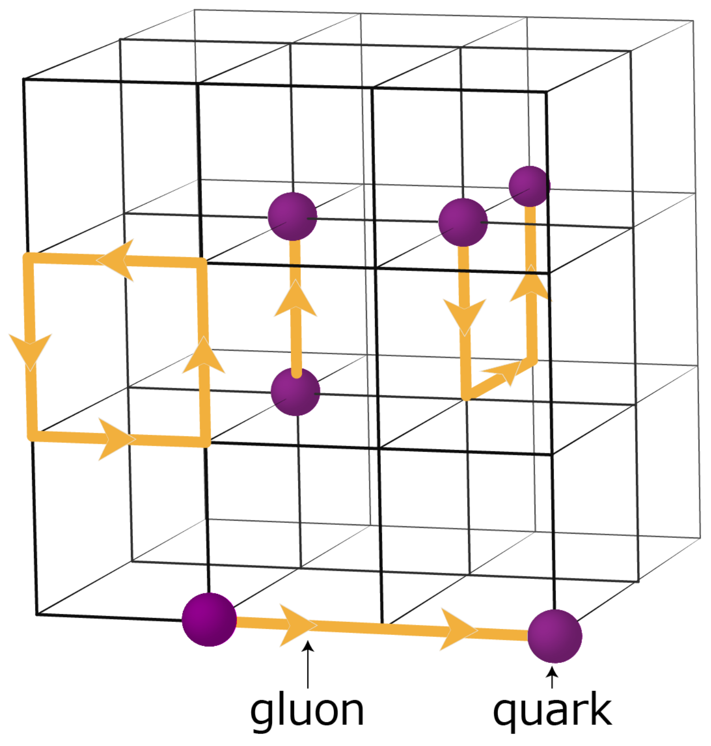
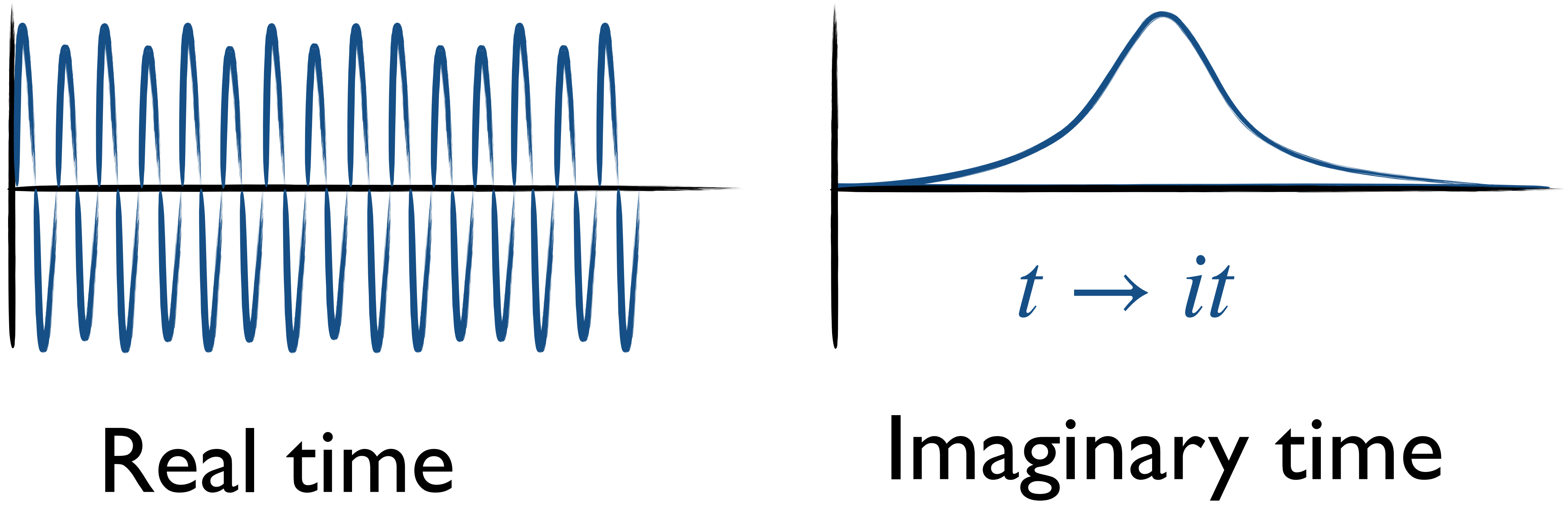


→ This requires us to simulate fields at all points in spacetime: lattice QCD

However, traditional Lattice QCD cannot simulate dynamics due to infamous sign problem

Integrals of form: $\int e^{i\mathscr{L}t}$



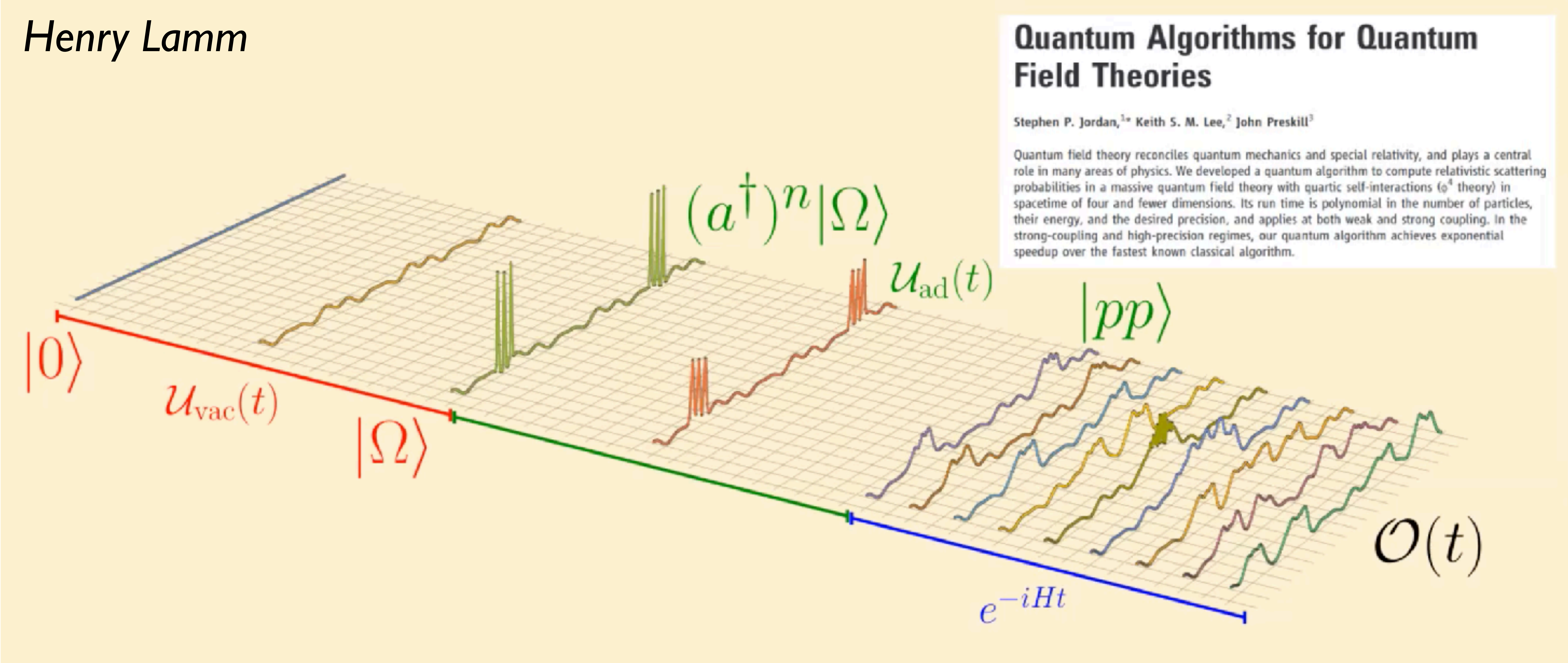
Quantum computers: directly simulate the Hamiltonian formulation of QCD

Kogut, Susskind `75

Example 1: Scattering in scalar field theories

Can be simulated efficiently using quantum computers!

Jordan, Lee, Preskill (2014)

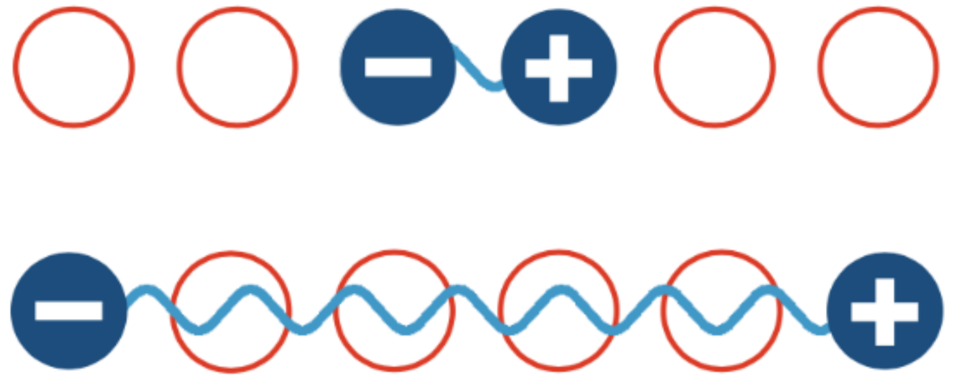


Example 2: Hadronization

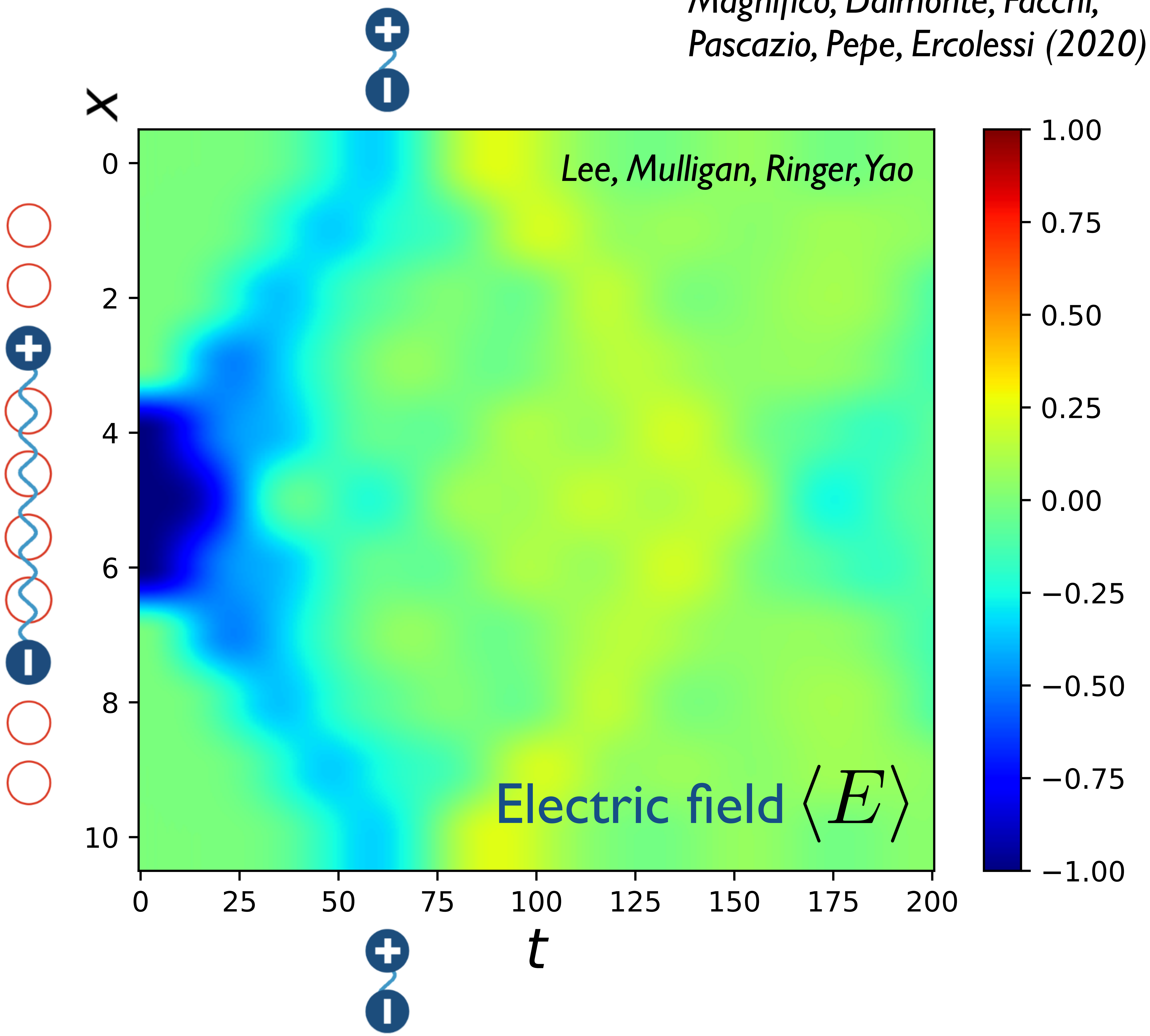
Magnifico, Dalmonte, Facchi, Pascazio, Pepe, Ercolessi (2020)

Schwinger model: QED in 1+1D

- Confinement
- Chiral symmetry breaking



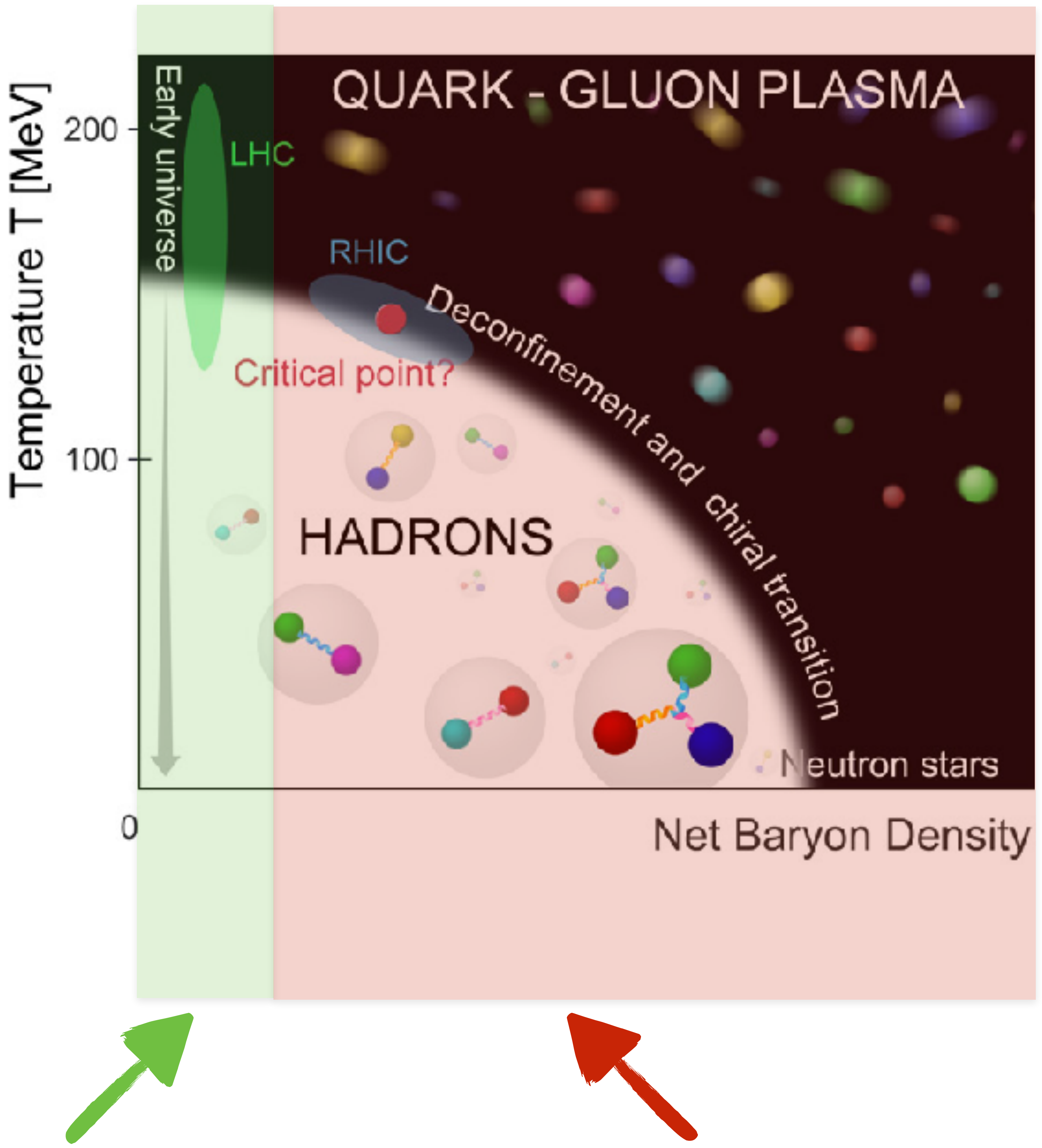
Real-time picture of string breaking mechanism



Long-term goal: QCD hadronization

Example 3: QC for hot/dense QCD

High density QCD: Lattice QCD can only calculate static quantities at **low density**



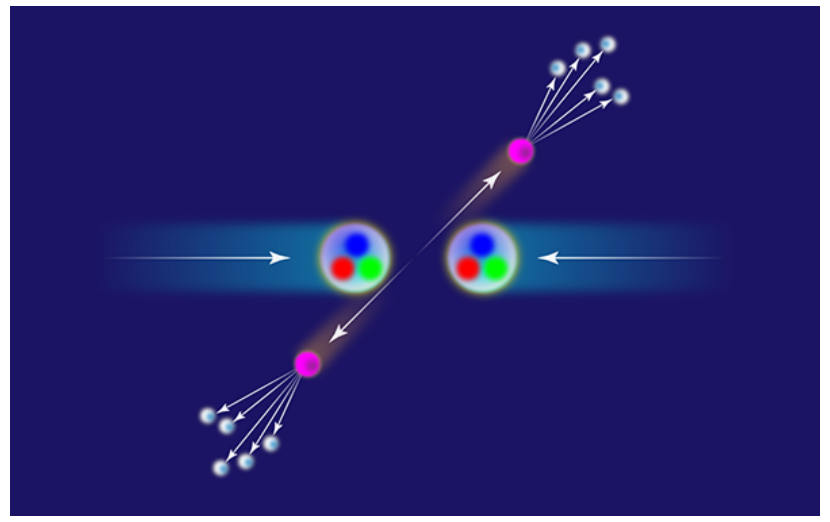
calculable

not calculable: sign problem

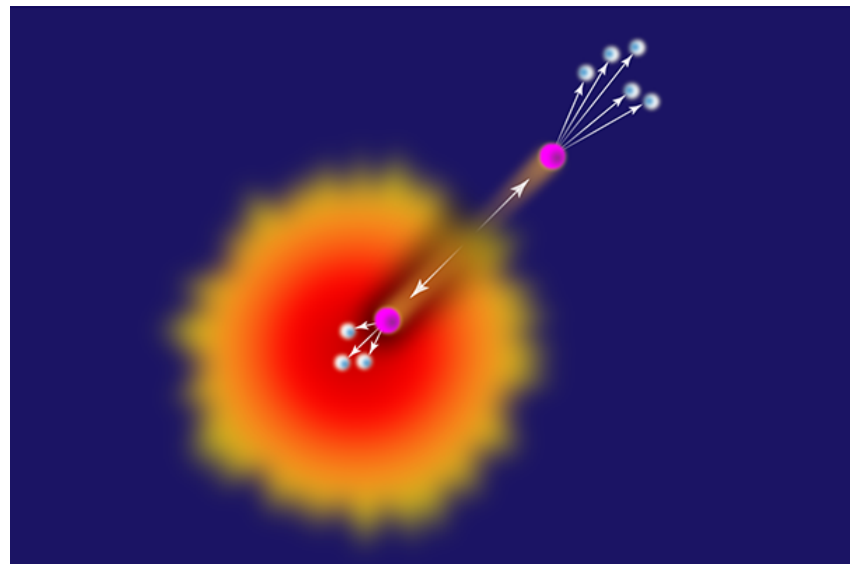
Real-time dynamics of probes evolving through the quark-gluon plasma

In vacuum: perturbative QCD

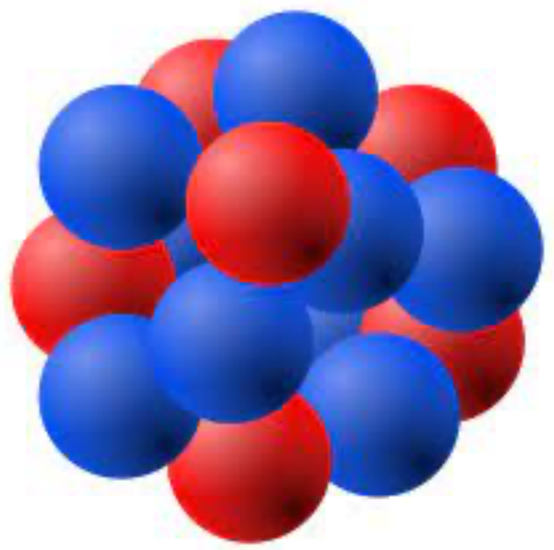
- No sense of "time evolution"



In medium: must combine probe evolution with hydrodynamic evolution of the QGP

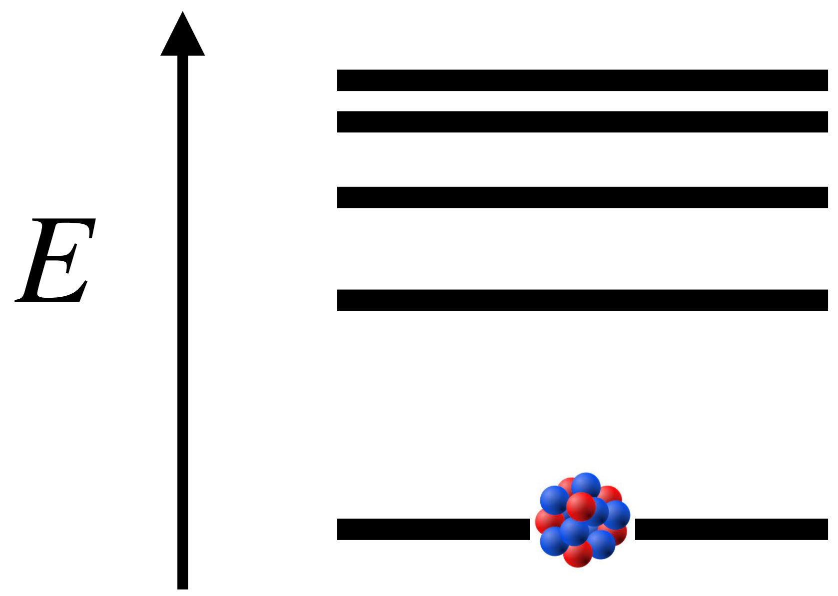


Example 4: Many-body nuclear physics

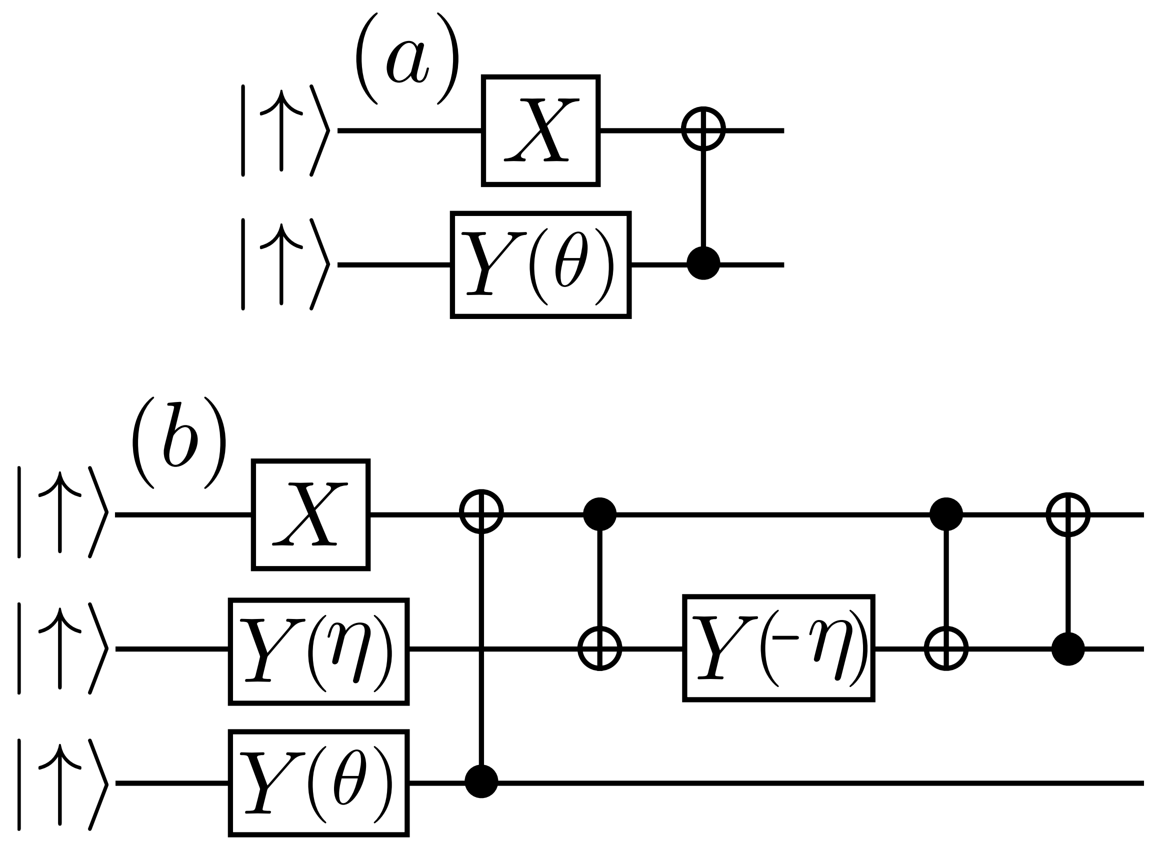


Hamiltonian obtained from **effective field theory**

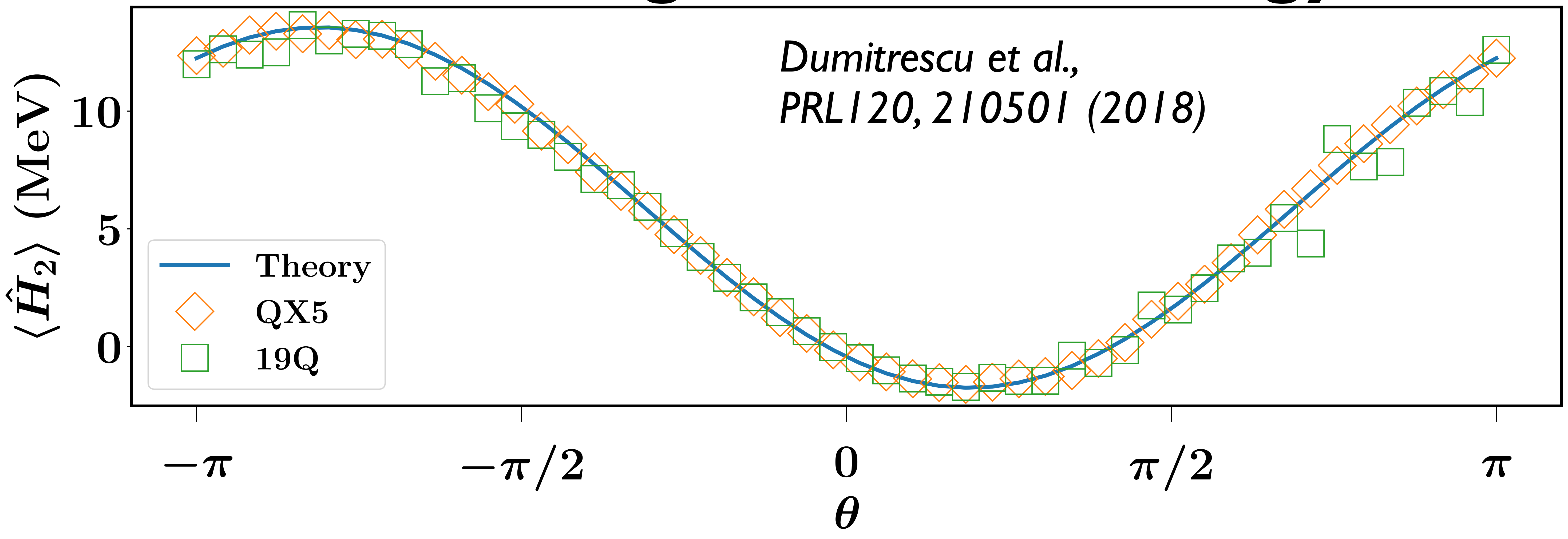
$$H_N = \sum_{n,n'=0}^{N-1} \langle n'|(T+V)|n\rangle a^\dagger_{n'} a_n$$



VQE circuit

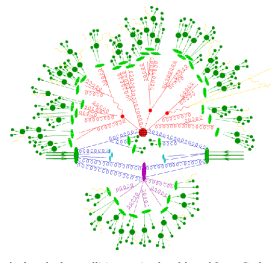
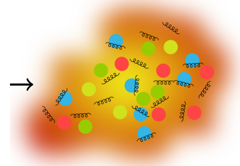
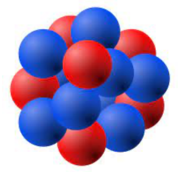


Deuteron ground state energy



Summary

Quantum computing offers potential opportunities to vastly expand our understanding of QCD

- Real-time dynamics of scattering and hadronization 
- High-temperature/density QCD 
- Many-body nuclear structure 
- …

Short-term: Current quantum hardware is too small and noisy to achieve quantum advantage, but it is an important time to explore potential applications

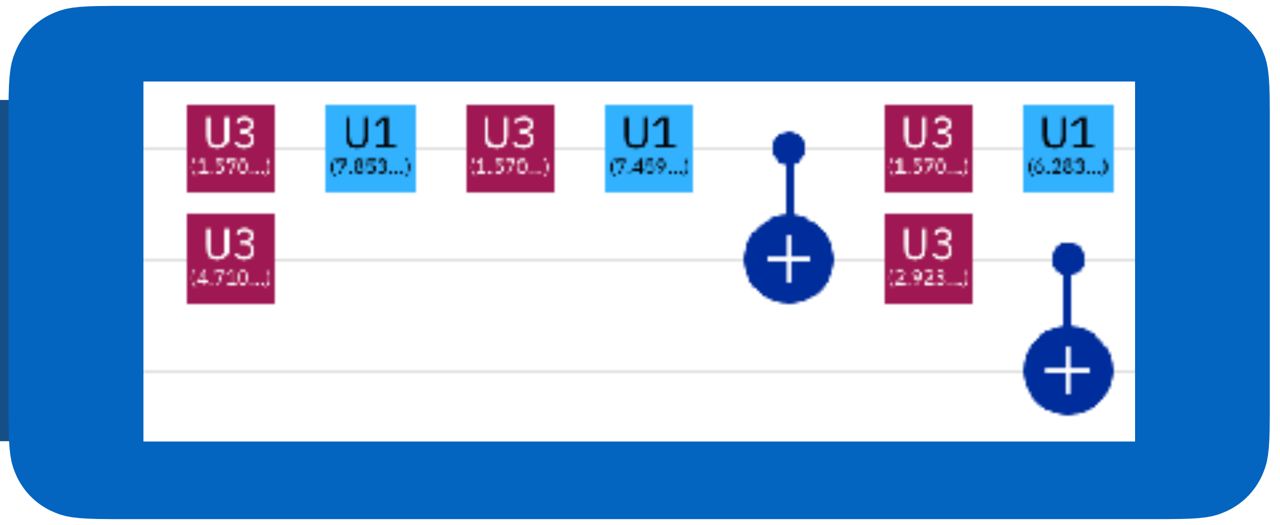
Long-term: Determining whether QCD can be simulated efficiently by quantum computers will give us profound insights about nature

Outline

1. Quantum advantage

2. QC for HEP/NP

3. Hands-on: Circuit synthesis



Hands-on: Circuit synthesis

https://colab.research.google.com/drive/1fSWR0q8y7vDxotqaVGvT_Or3uw5GaBTG?usp=share_link

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