

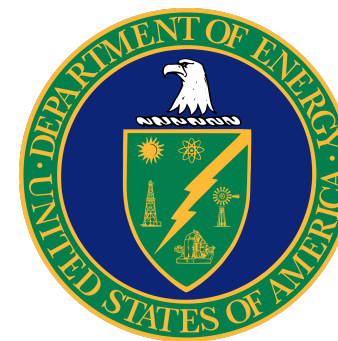


Toward Digital Quantum Simulations of Standard Model Physics

- a look from my path

Martin Savage
 InQubator for Quantum Simulation
 University of Washington

June 30, 2023

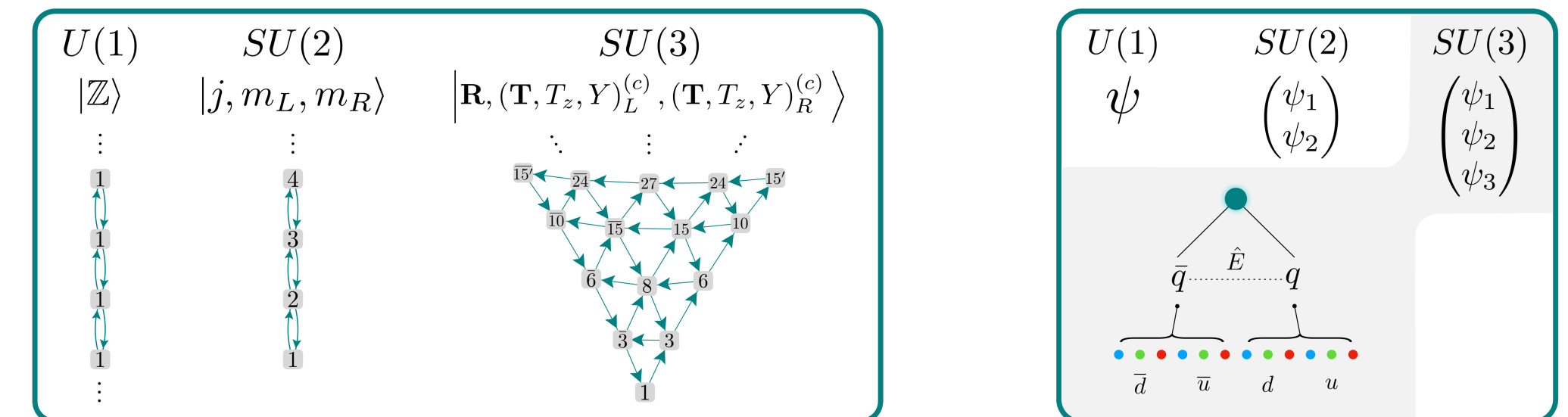


Office of Science
 U.S. Department of Energy



Quantum Computing Boot Camp

June 20-30, 2023 • Jefferson Lab • Newport News, VA



$$\hat{\mathcal{H}} \sim g^2 |\hat{E}(\mathbf{x})|^2 - \frac{1}{g^2} \text{Tr} [\hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4^\dagger + \text{h.c.}] + \hat{\mathcal{H}}_\psi$$

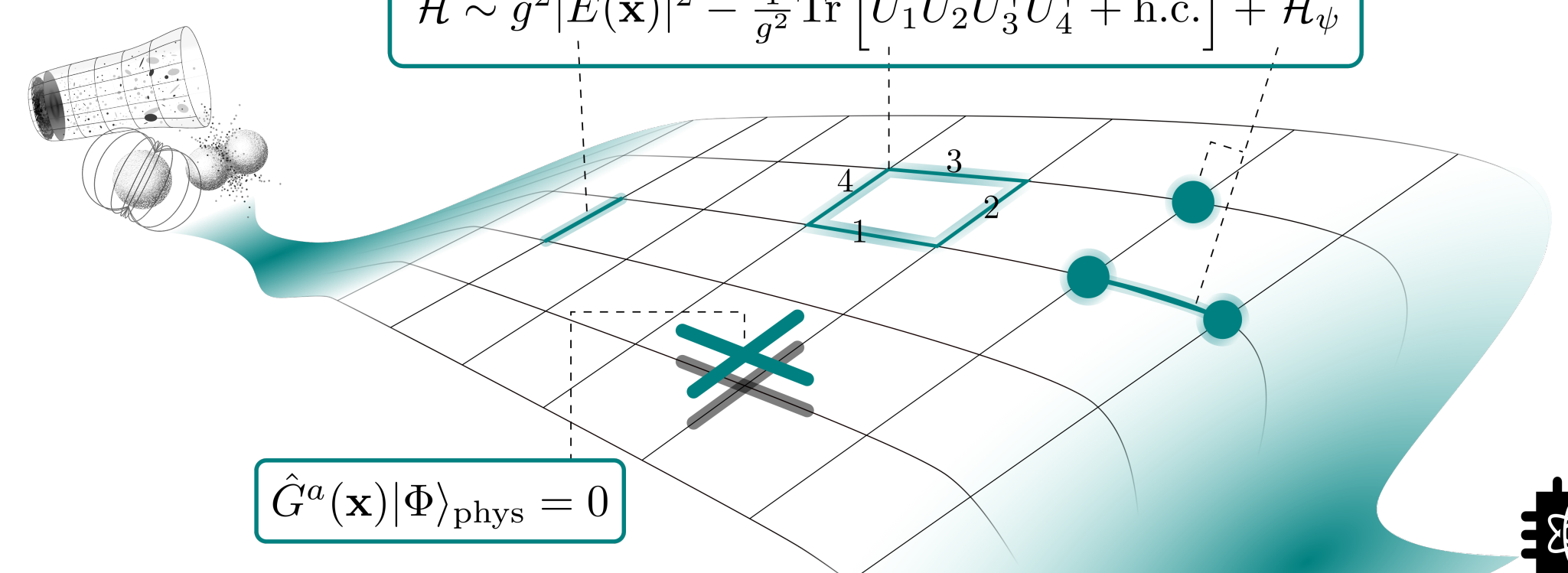
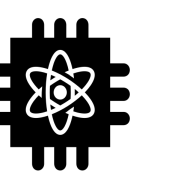
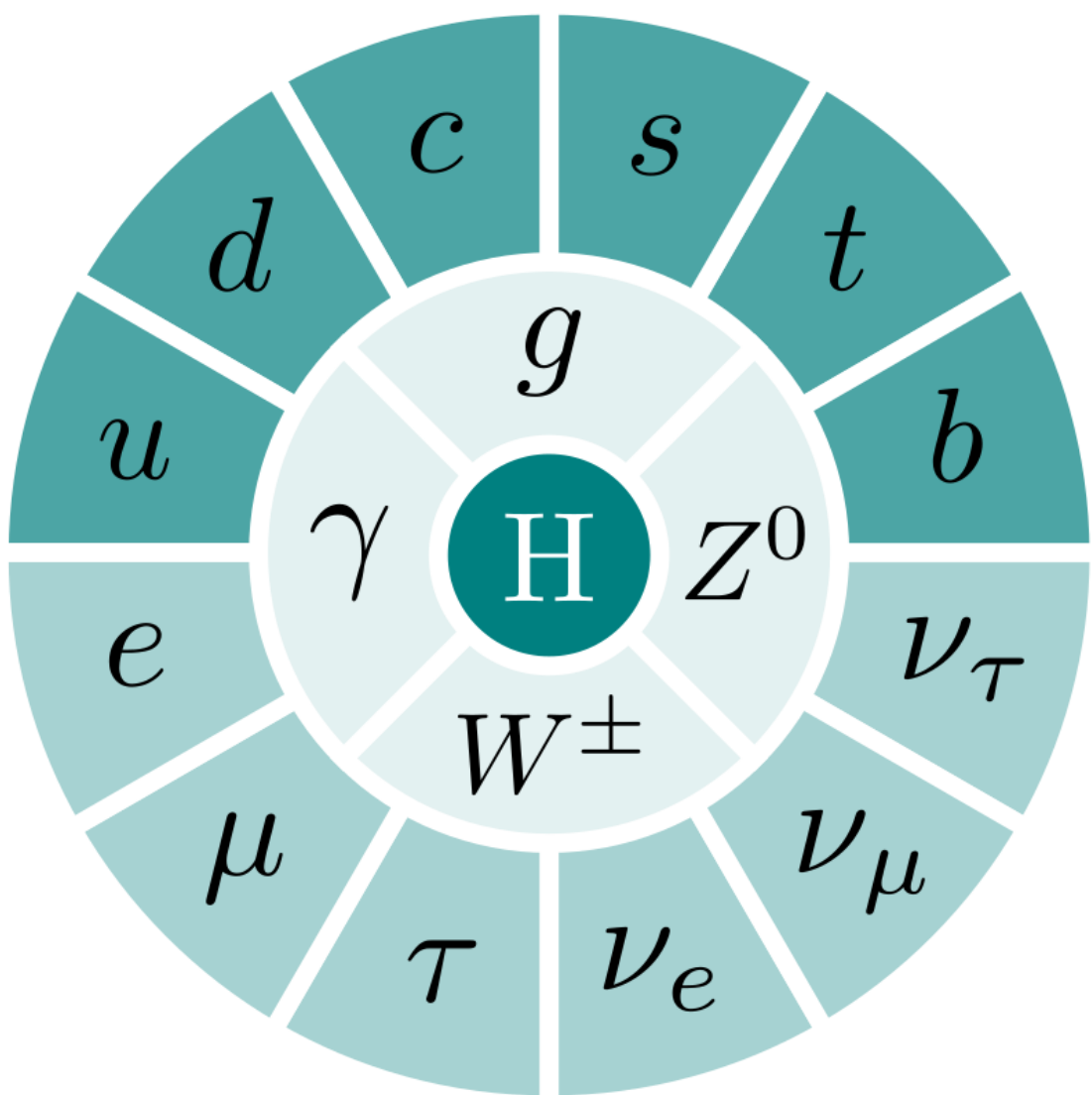


Image: Bauer, Davoudi, Klco, Savage: Nature Review



Particles & Interactions

- Quarks
- Leptons
- Gauge Bosons
- Higgs Boson

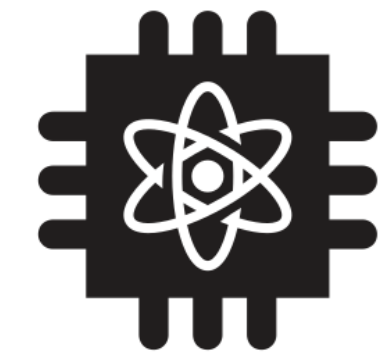


Standard Model

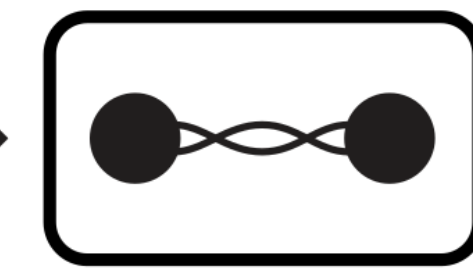
Simulation

0100
0011

Classic Computing

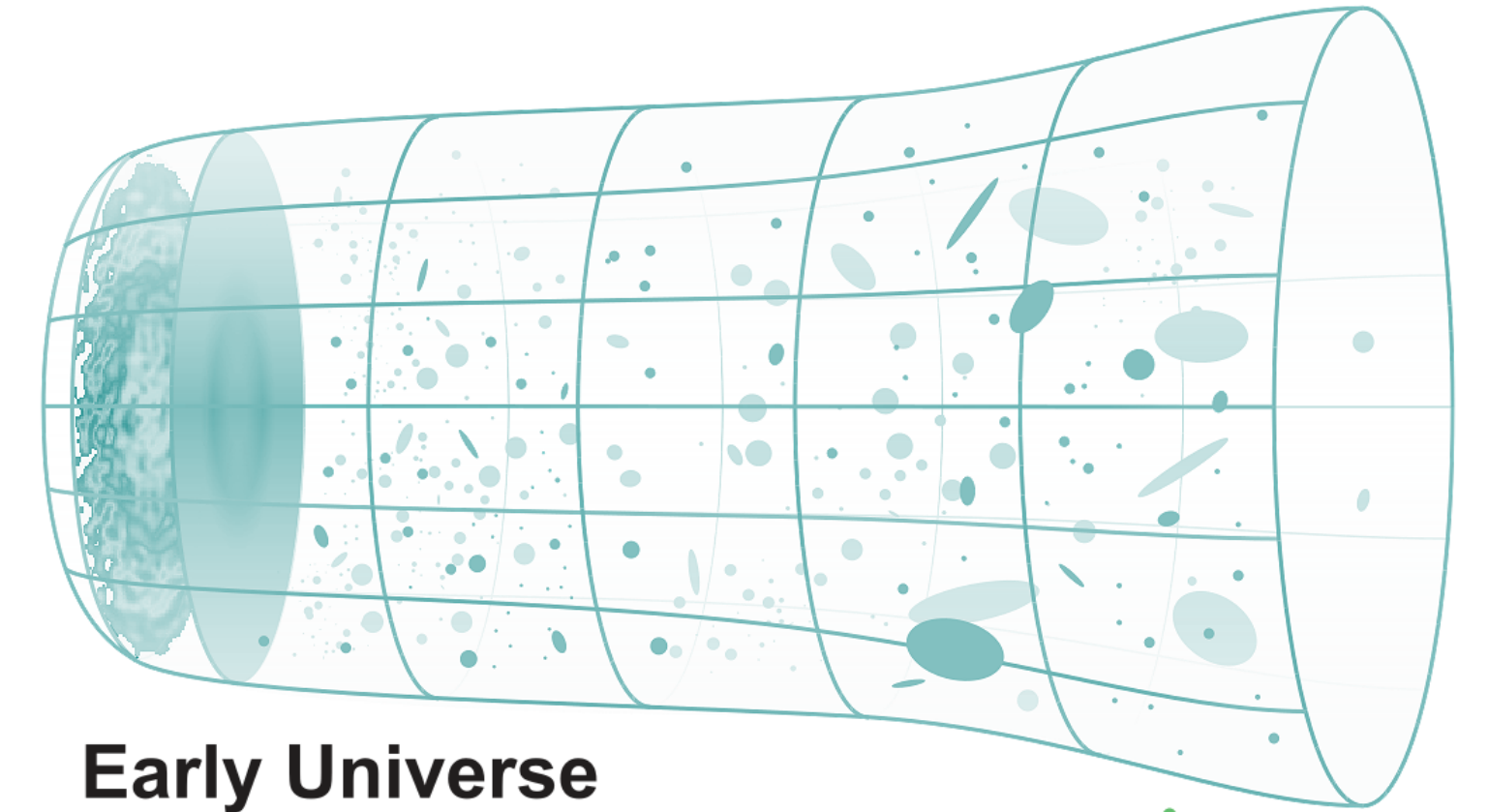


Quantum Computing



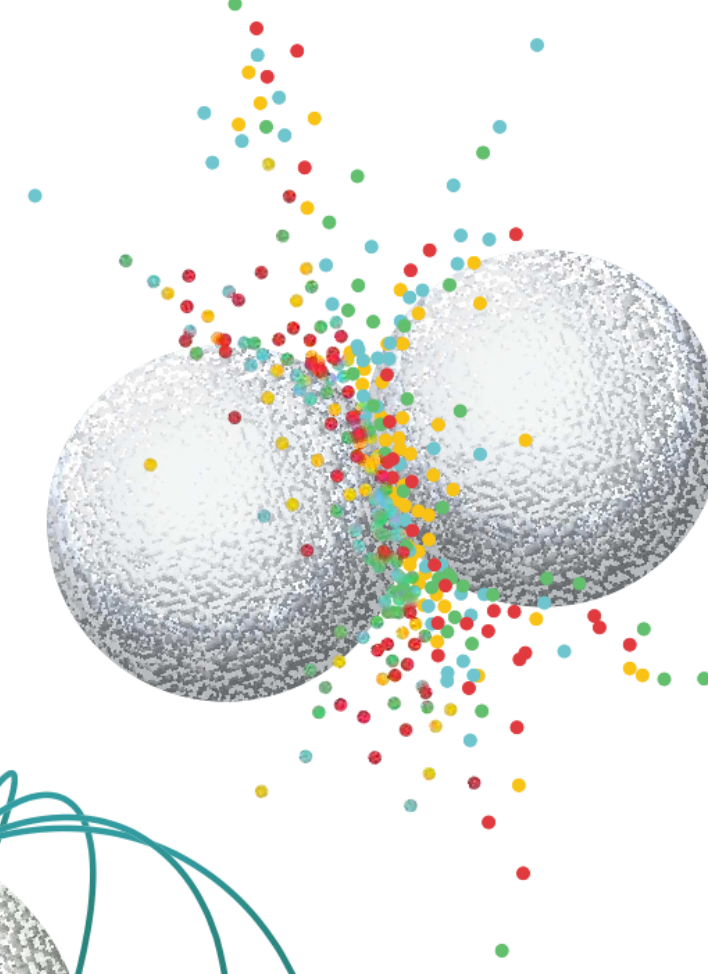
Quantum Entanglement

Phases & Dynamics of Matter

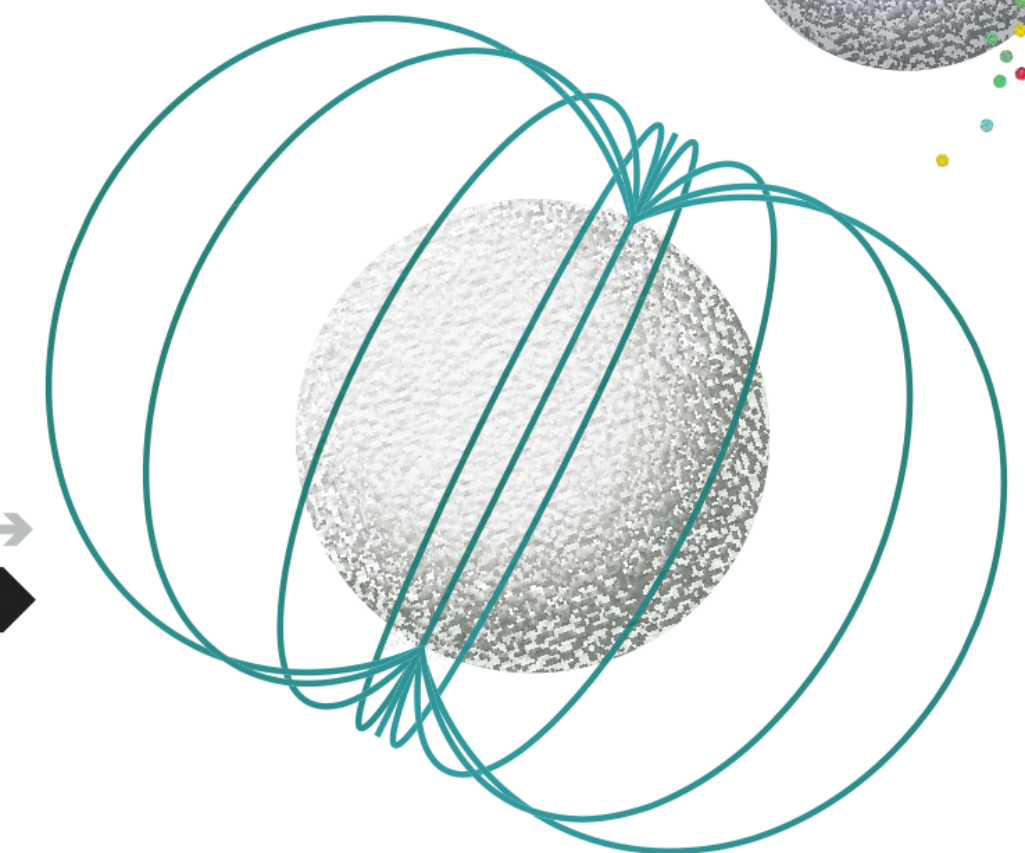


Early Universe

High-energy Particle Collisions

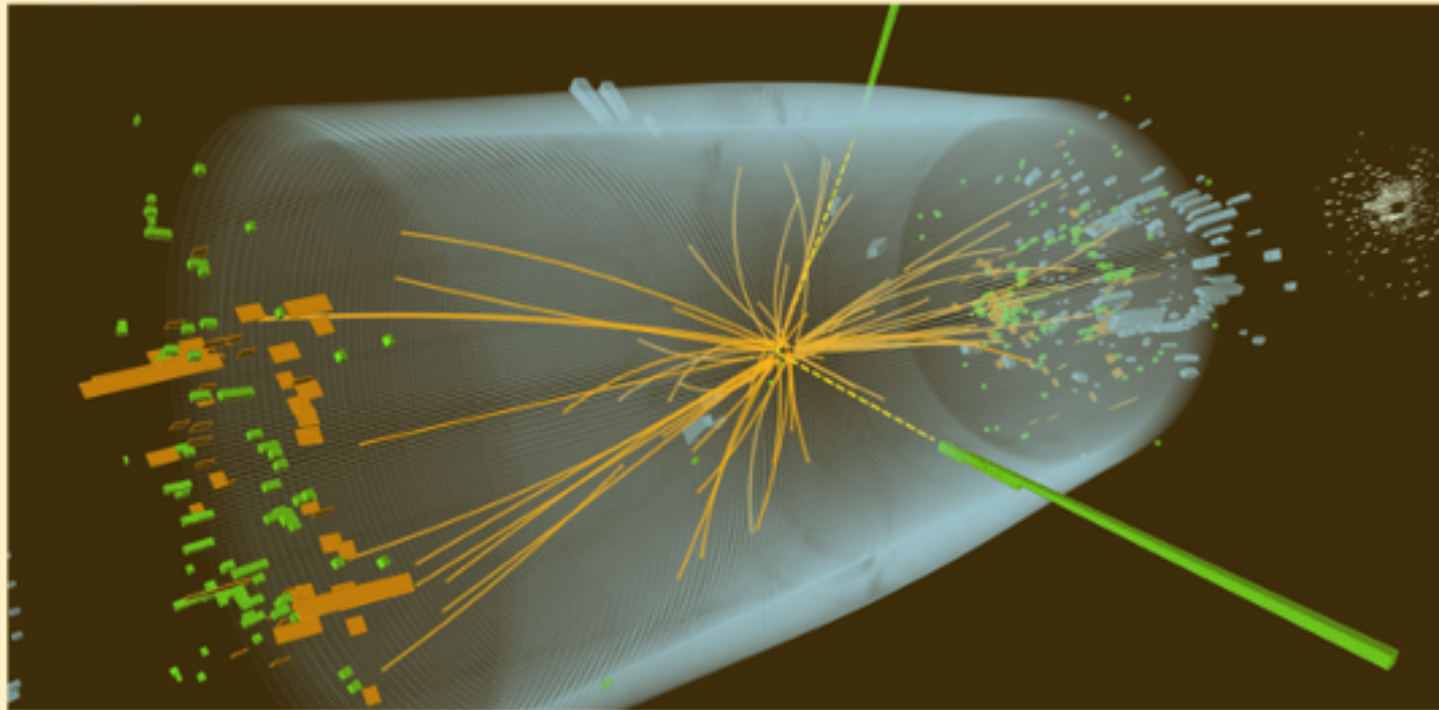


Neutron Star Core



Simulation Objectives for the Standard Model and Beyond

Gauge Theories and Descendent Effective Field Theories and Models



Real-time dynamics
particle production, fragmentation
vacuum and in medium

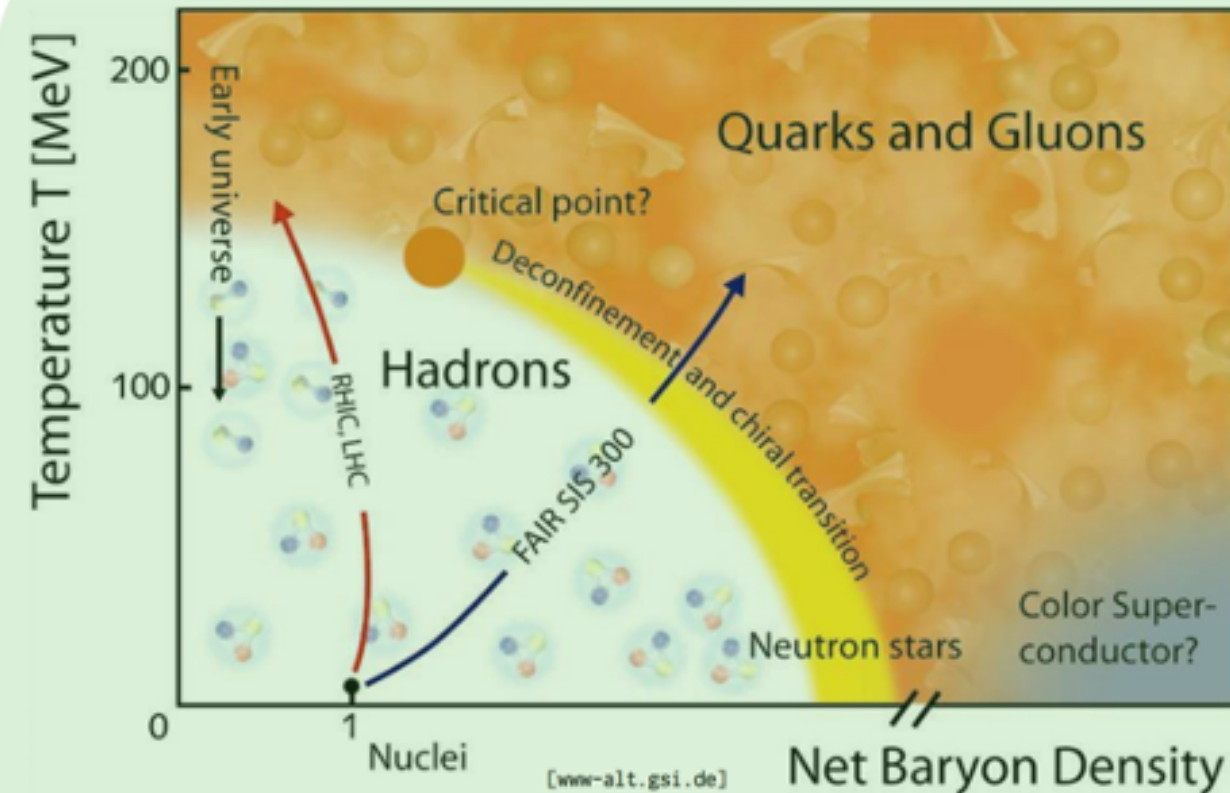
Low-energy reactions

Electroweak processes (e.g., ν -A)

Neutrino dynamics

Matter-antimatter asymmetry

BQP

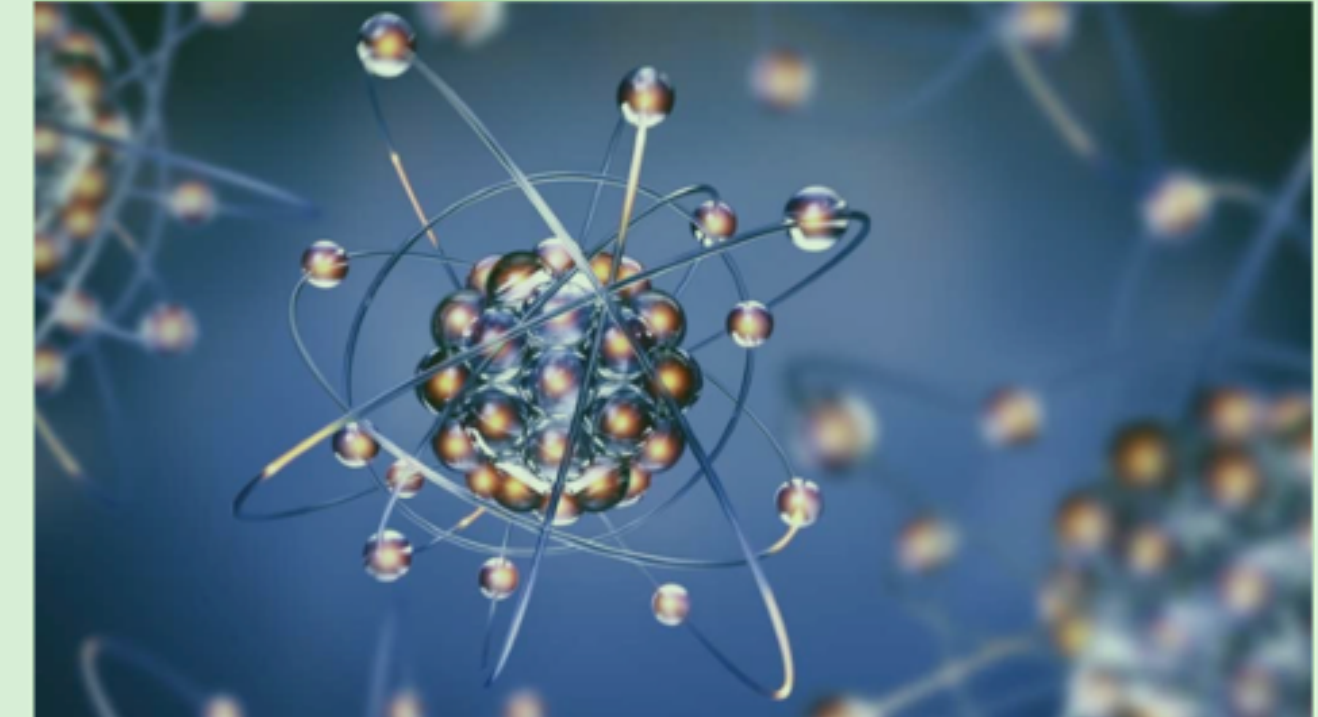


Equation of state of dense
hot matter and dynamics
viscosity, etc

Conquering some "sign problems"

The early universe

Supernova/Neutron stars



Precision structure and interactions
of nuclei

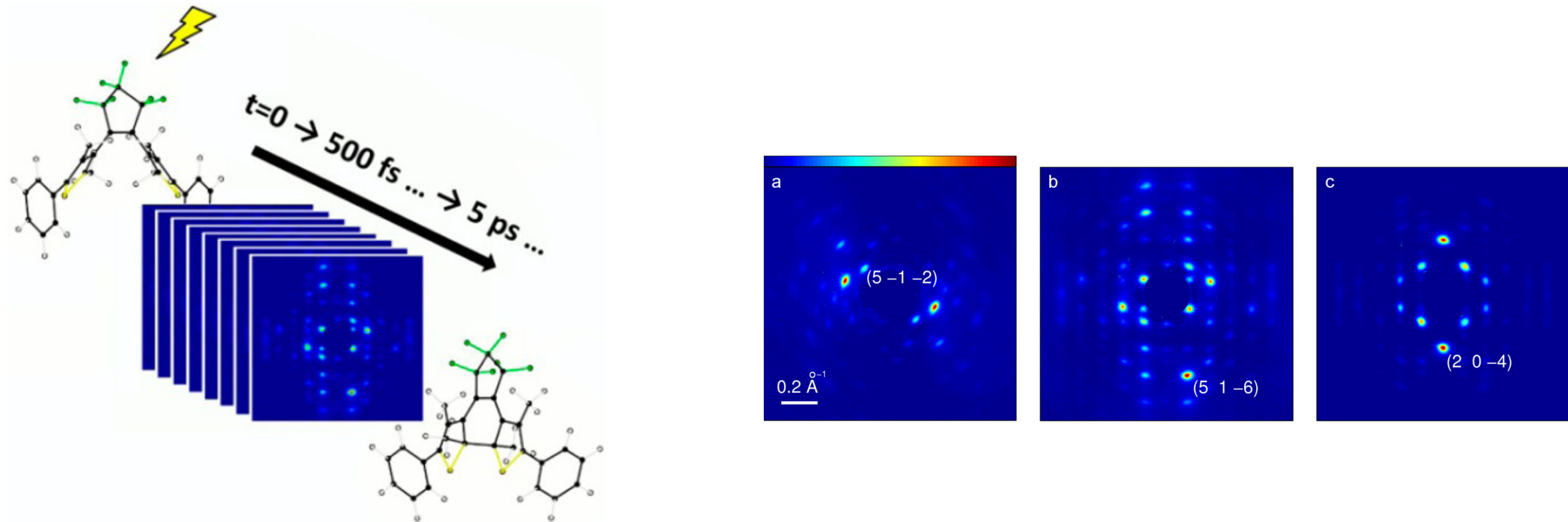
Many-body systems

Rare processes, double-beta decay

QMA

— symmetries_x

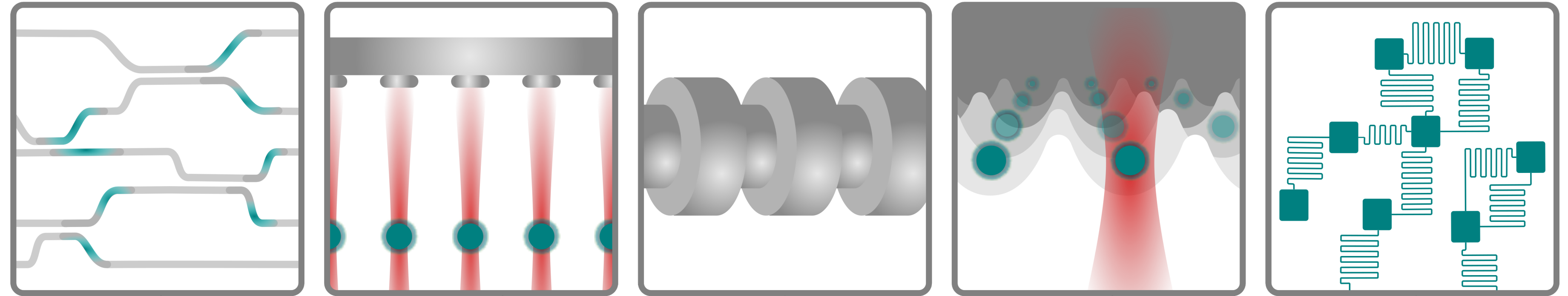
Real-Time Dynamics and Improved Modeling of Reaction Pathways



J. Phys. Chem. B 2013, 117, 49, 15894-15902

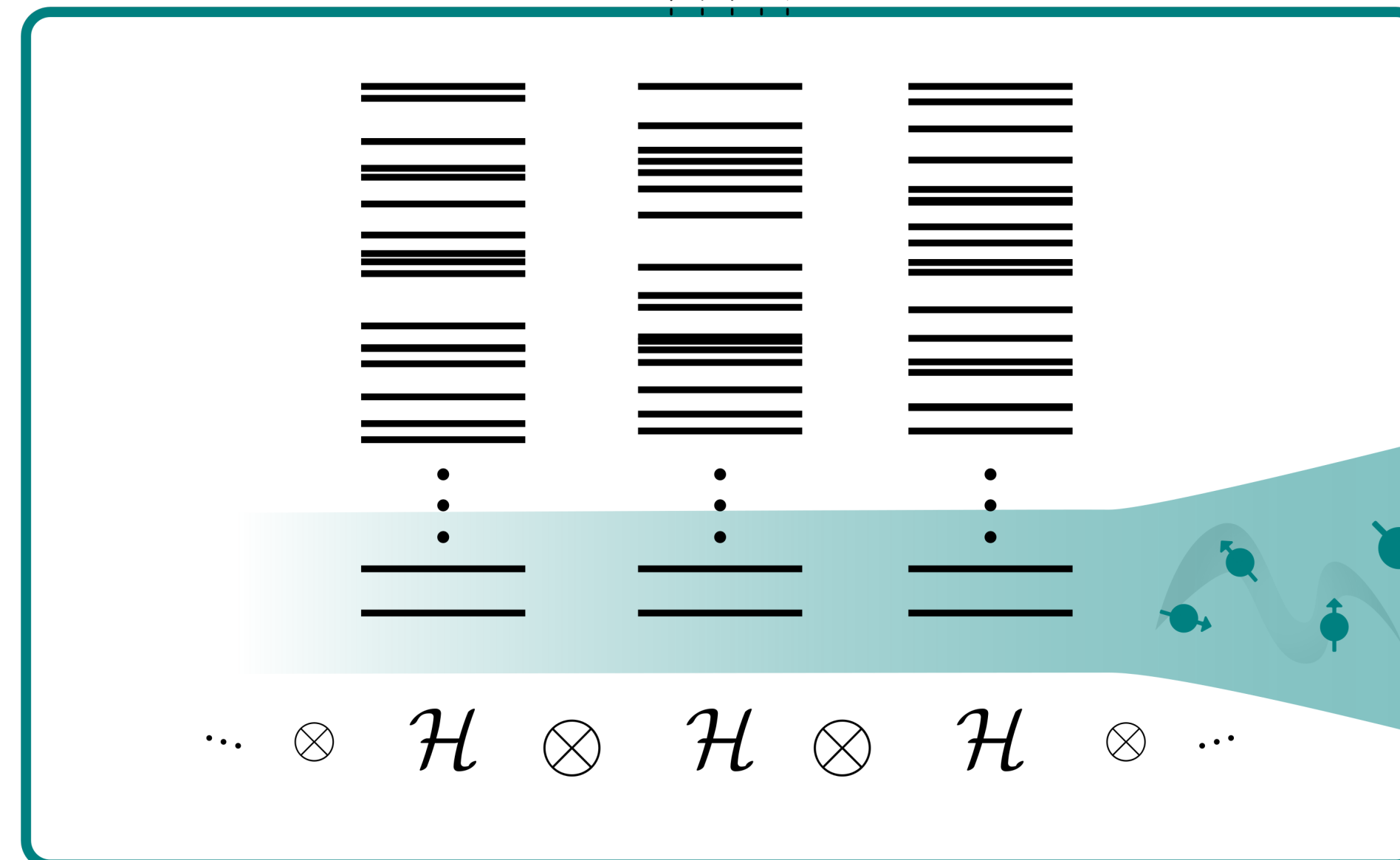
Femto-second chemistry reveals reaction mechanisms
Quantum simulations will reveal the reactions pathways of QCD

Physical Systems in Multi-Hilbert Space, Hybrid Devices



Map scalar, fermion
and vector systems

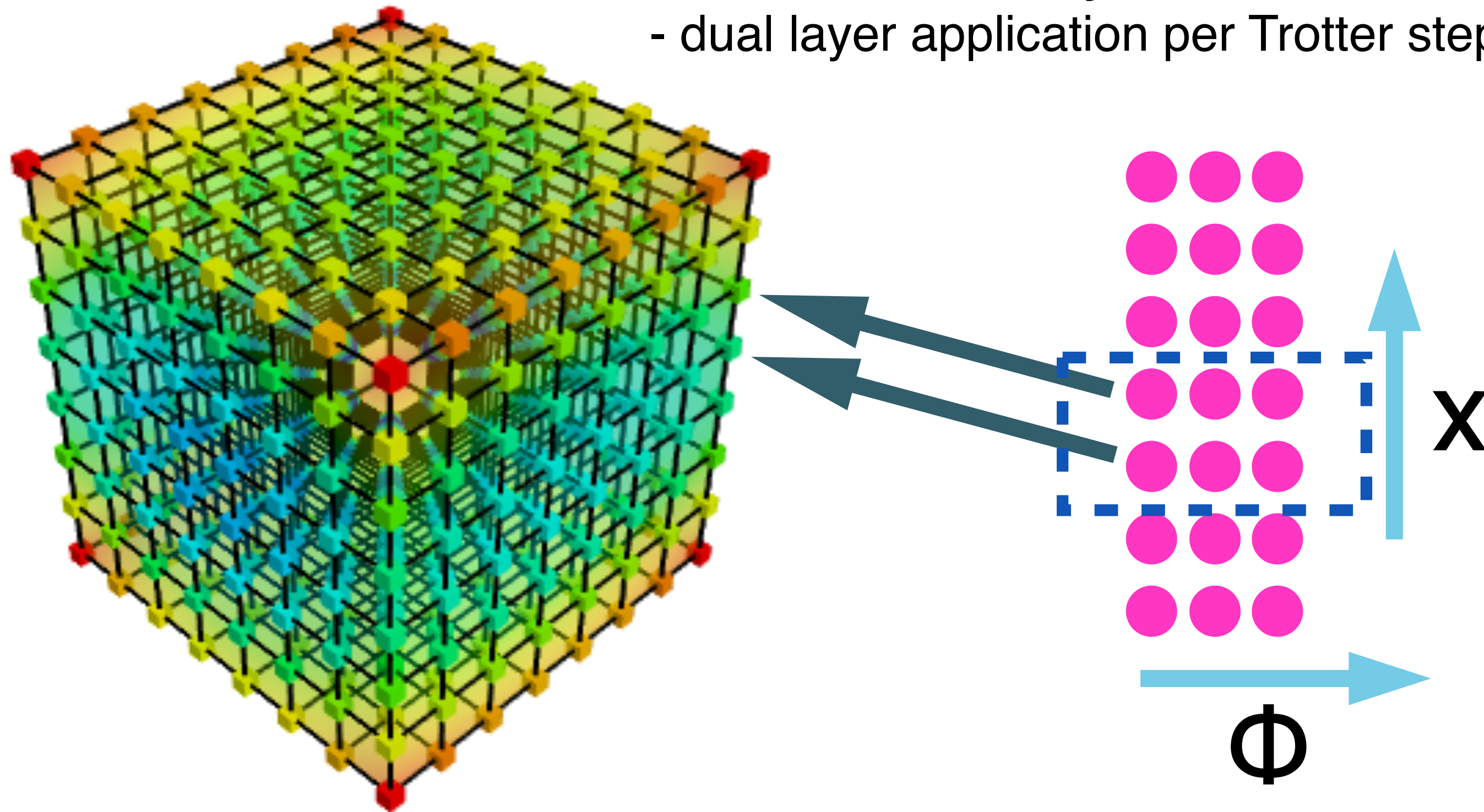
Optimize for target
observables



Gold-Standard for QFT - Lattice Scalar Field Theory

Jordan, Lee, Preskill

Parallelizes easily at the circuit level
- dual layer application per Trotter step



Could it be done better ?
Can entanglement be used more strategically?

Double exponential convergence of field digitization

- Nyquist-Shannon - JLP, FNAL, UW
- QFT and exact conjugate-momentum space operator

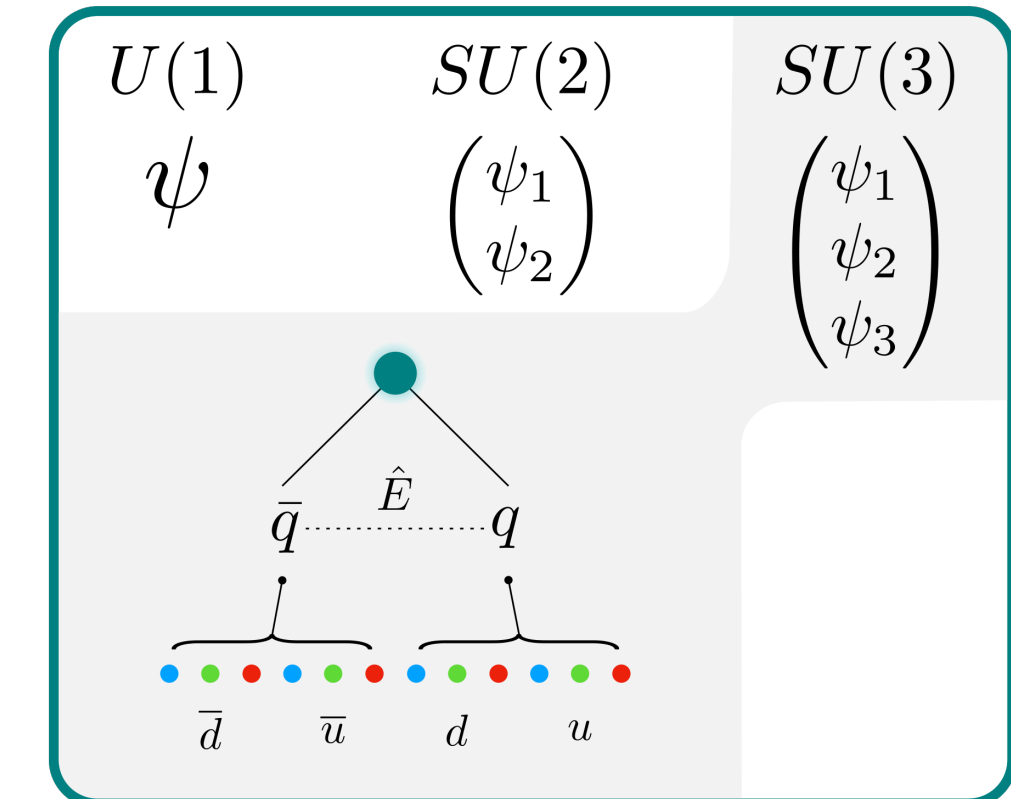
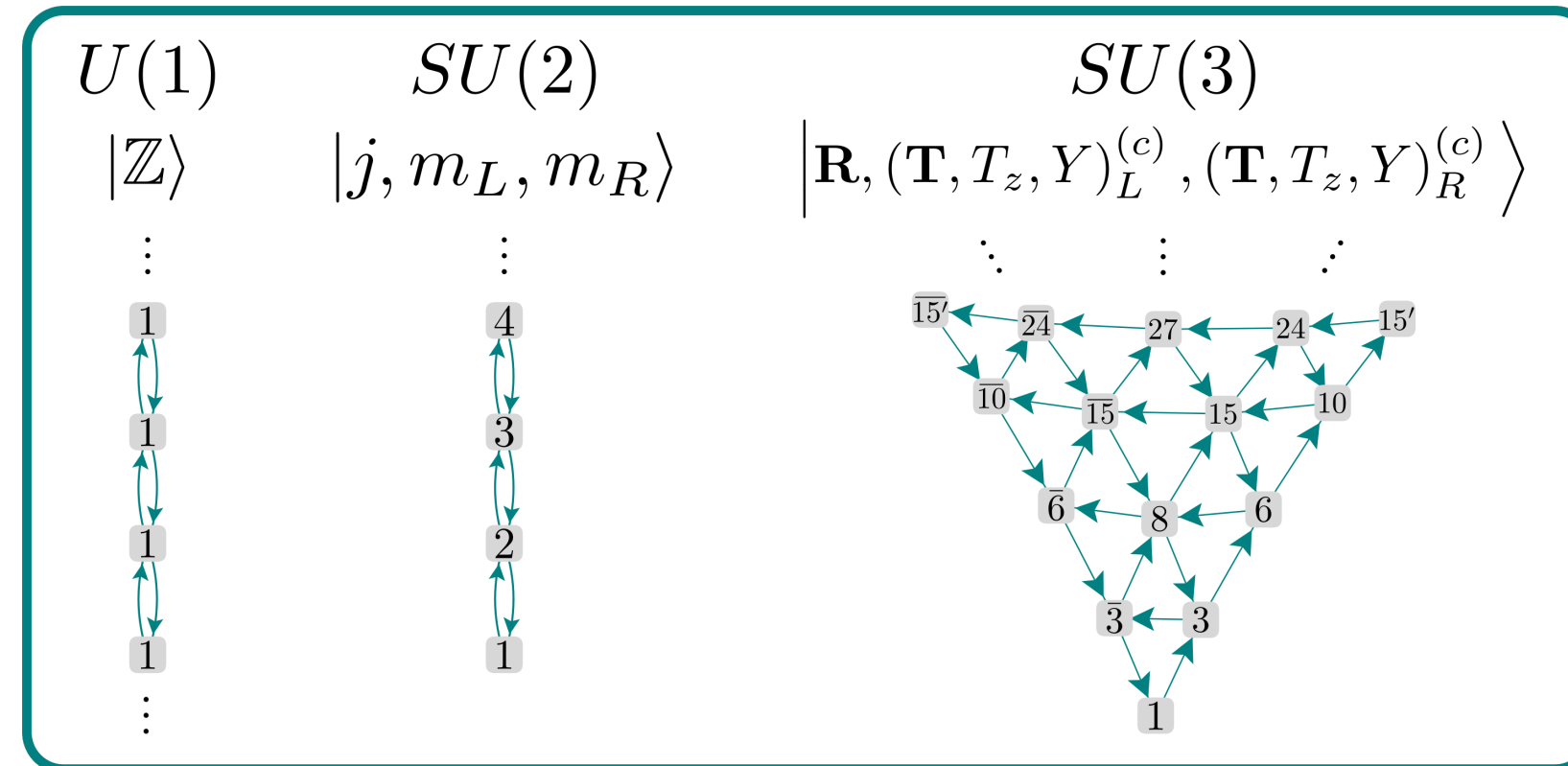
Lattice Gauge Field Theories and the Standard Model

Hamiltonian
Kogut-Susskind
1970's

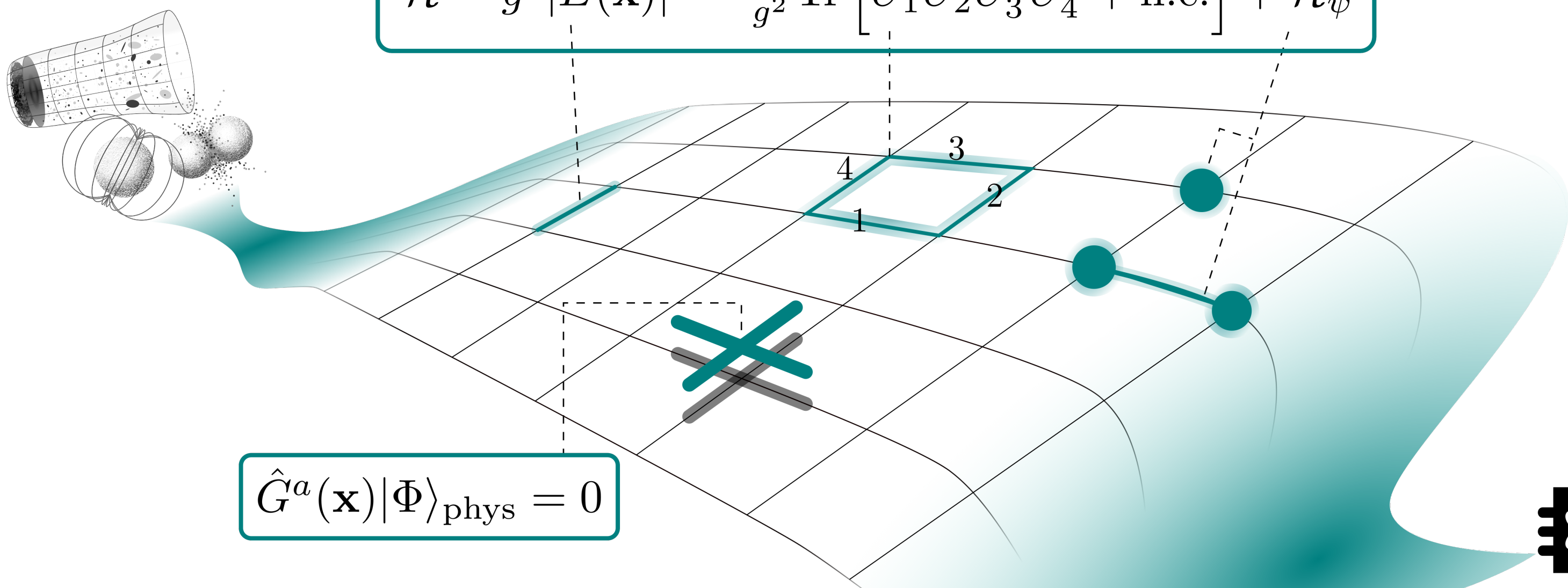
Yang-Mills:
Byrnes-Yamamoto
2005

SU(N):
Zohar et al
(2013)

QLM
Banerjee et al
Tagliacozzo et al
(2013)

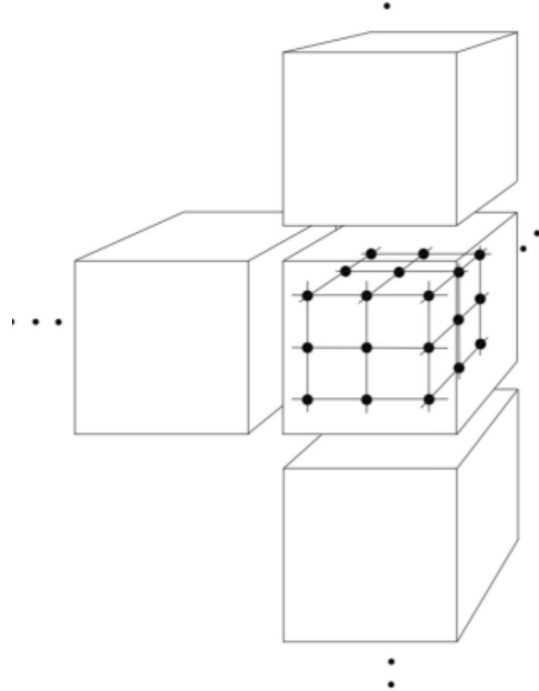


$$\hat{\mathcal{H}} \sim g^2 |\hat{E}(\mathbf{x})|^2 - \frac{1}{g^2} \text{Tr} [\hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 + \text{h.c.}] + \hat{\mathcal{H}}_\psi$$



Yang-Mills

Byrnes-Yamamoto – Kogut-Susskind



Many ways to map/distribute the field(s) in the UV (lattice spacing)
 Consider the Kogut-Susskind basis = electric basis

$$\hat{H} = \frac{g^2}{2} \sum_{\text{links}} \hat{E}^2 - \frac{1}{2g^2} \sum_{\square} \left(\hat{\square} + \hat{\square}^\dagger \right)$$

Electric Field Casimir operator

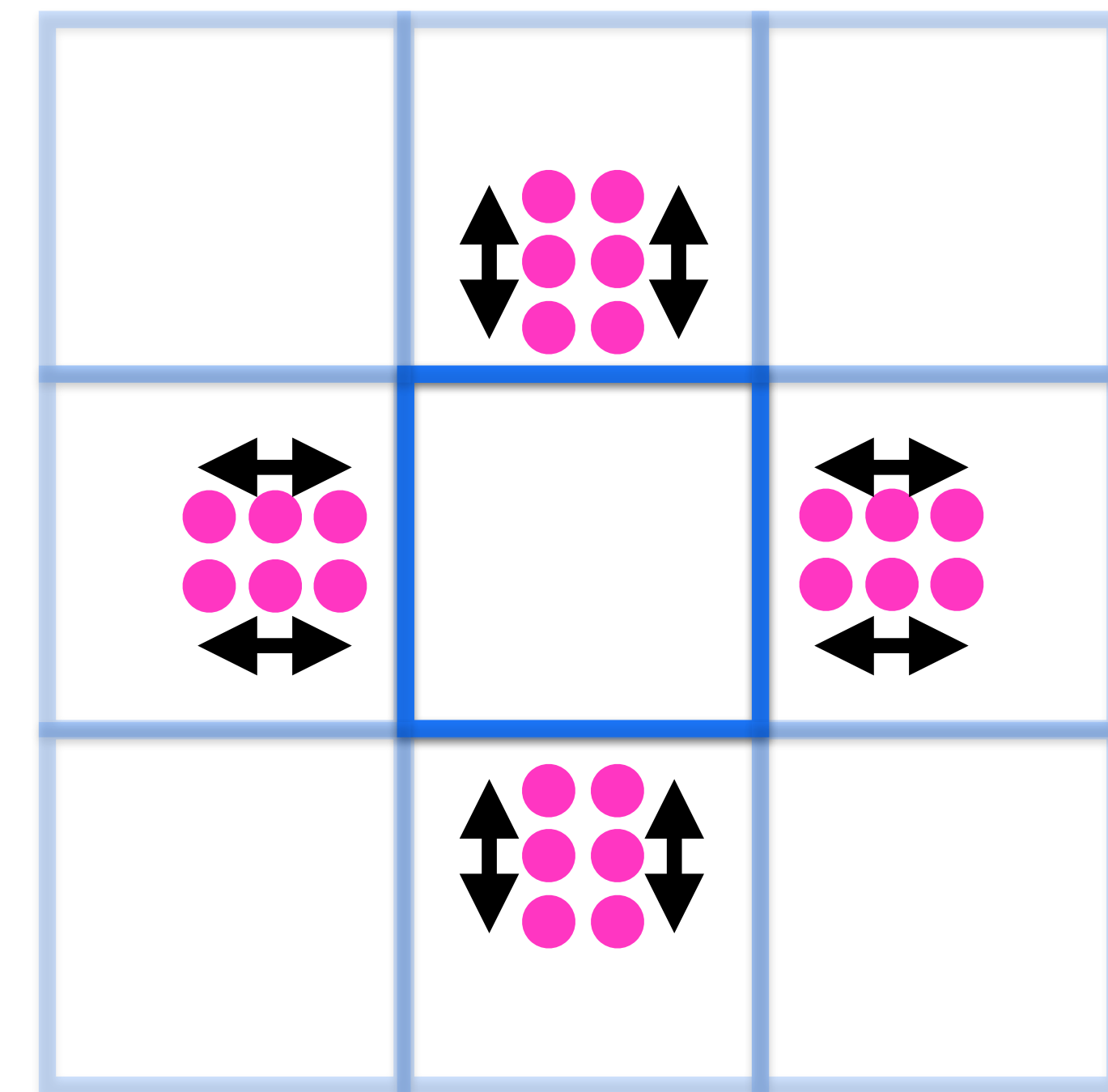
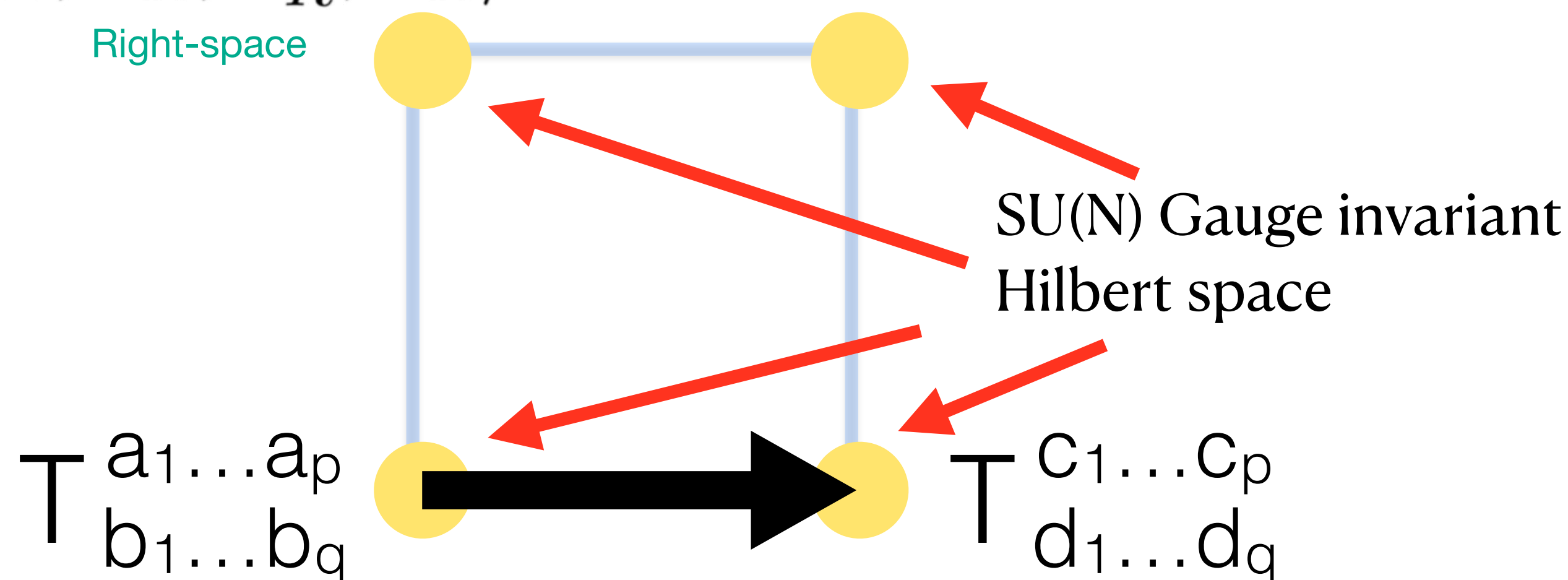
$$|p, q, T_L, T_L^z, Y_L, T_R, T_R^z, Y_R\rangle$$

Left-space

Right-space

Magnetic Field operator

Off-diagonal on electric basis

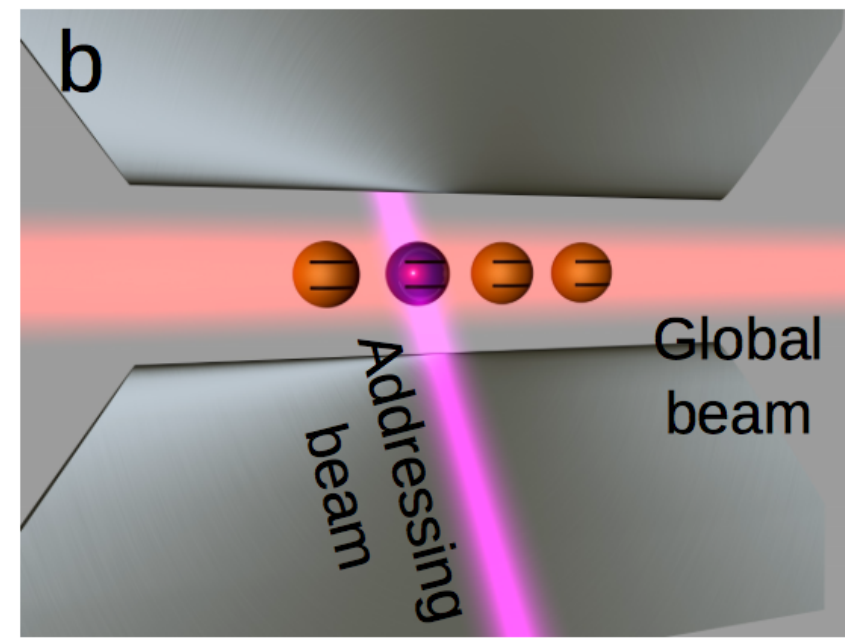
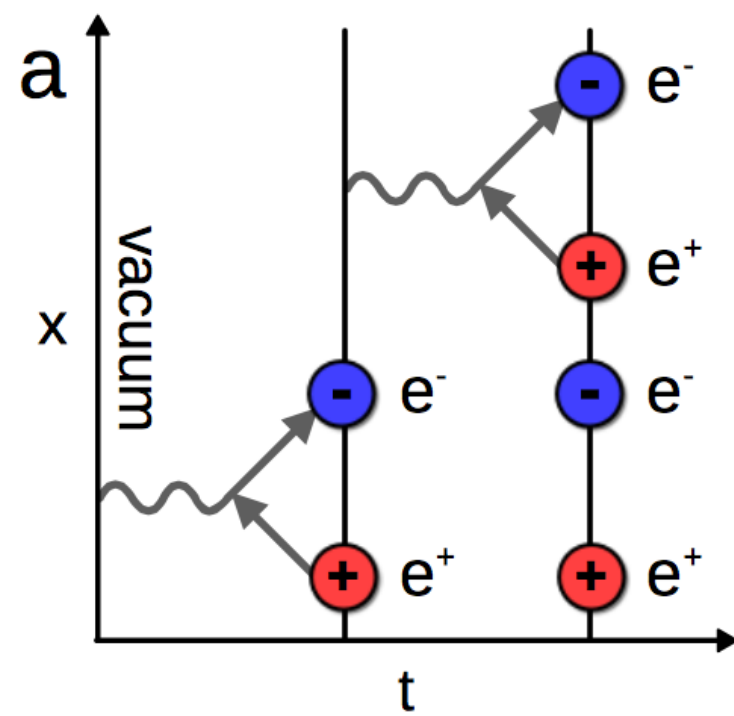


Truncate in Casimir
 = dimensionality of irrep

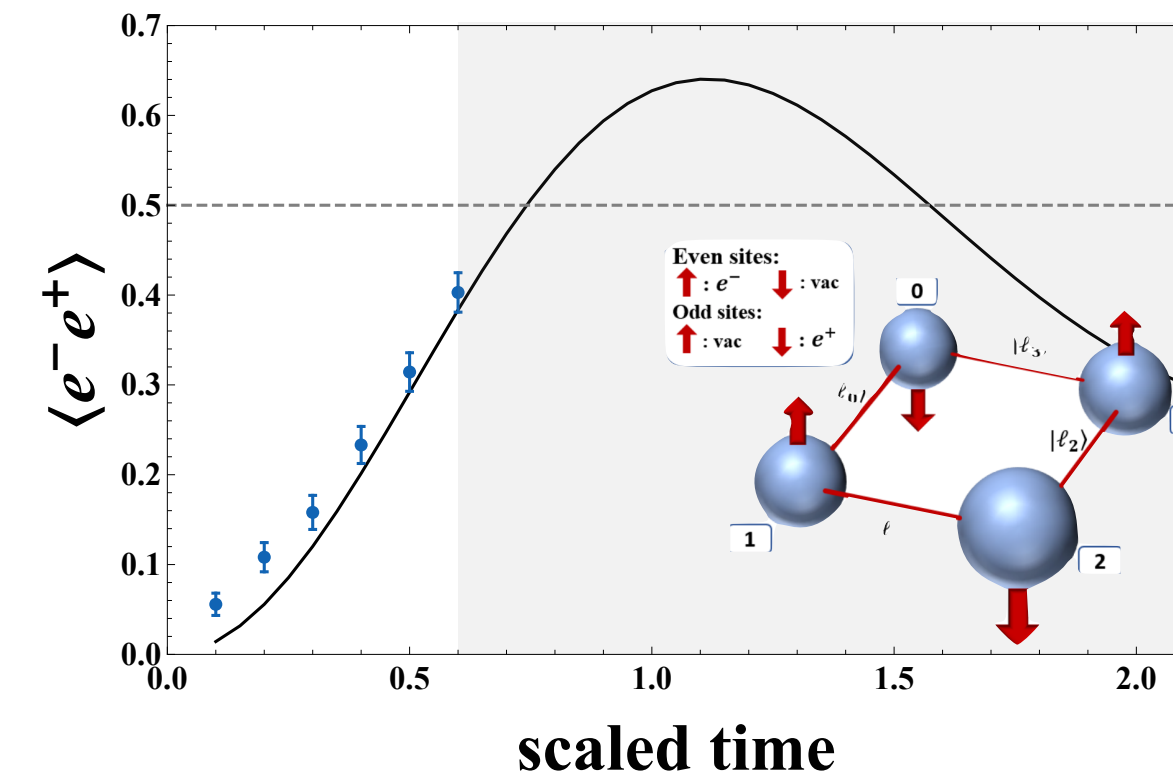
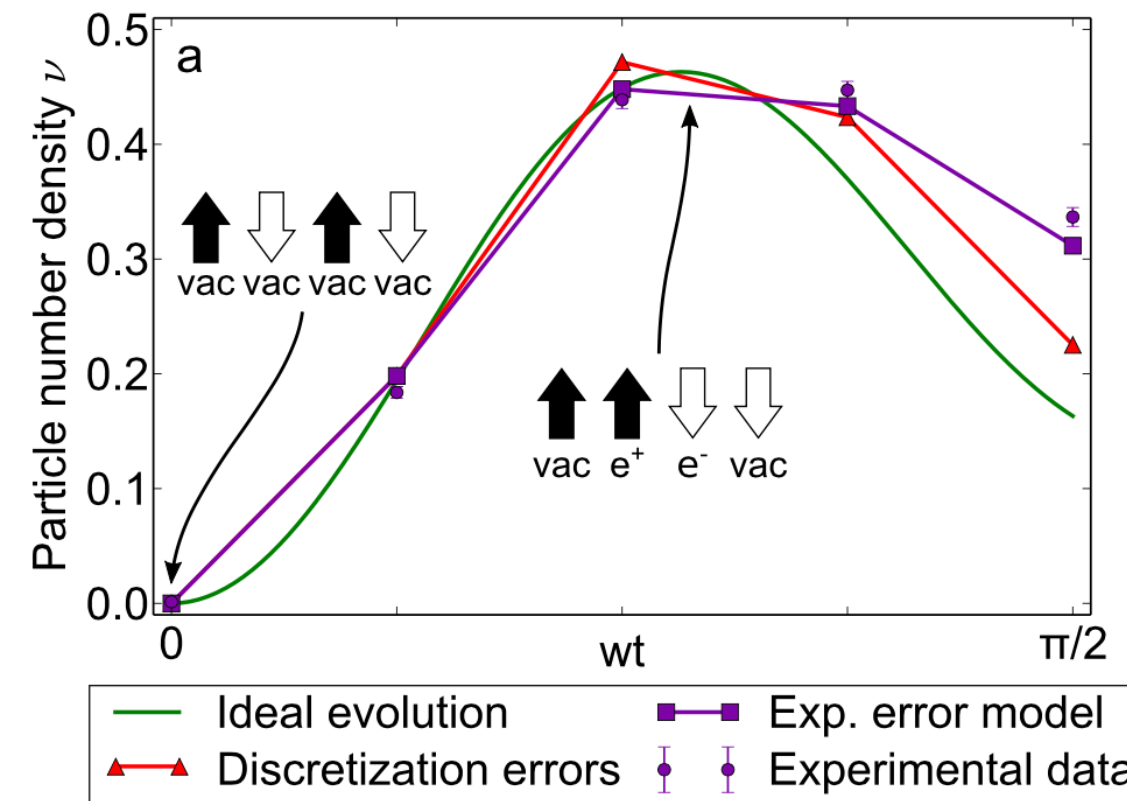
Continuum limit

Dynamics in the Schwinger Model - Abelian Gauge Theory

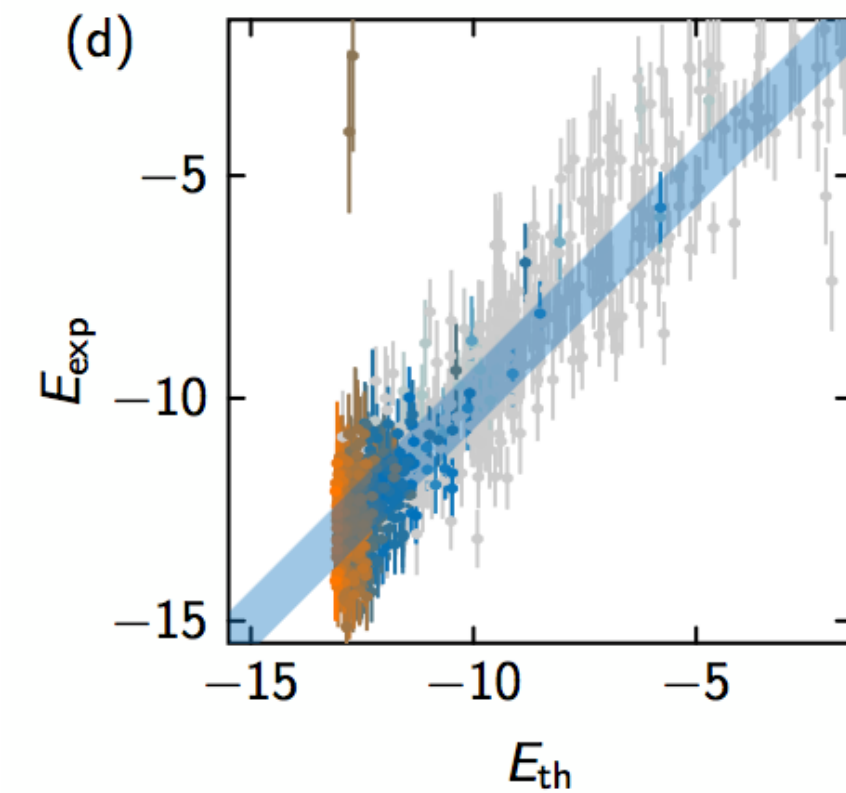
1+1D QED



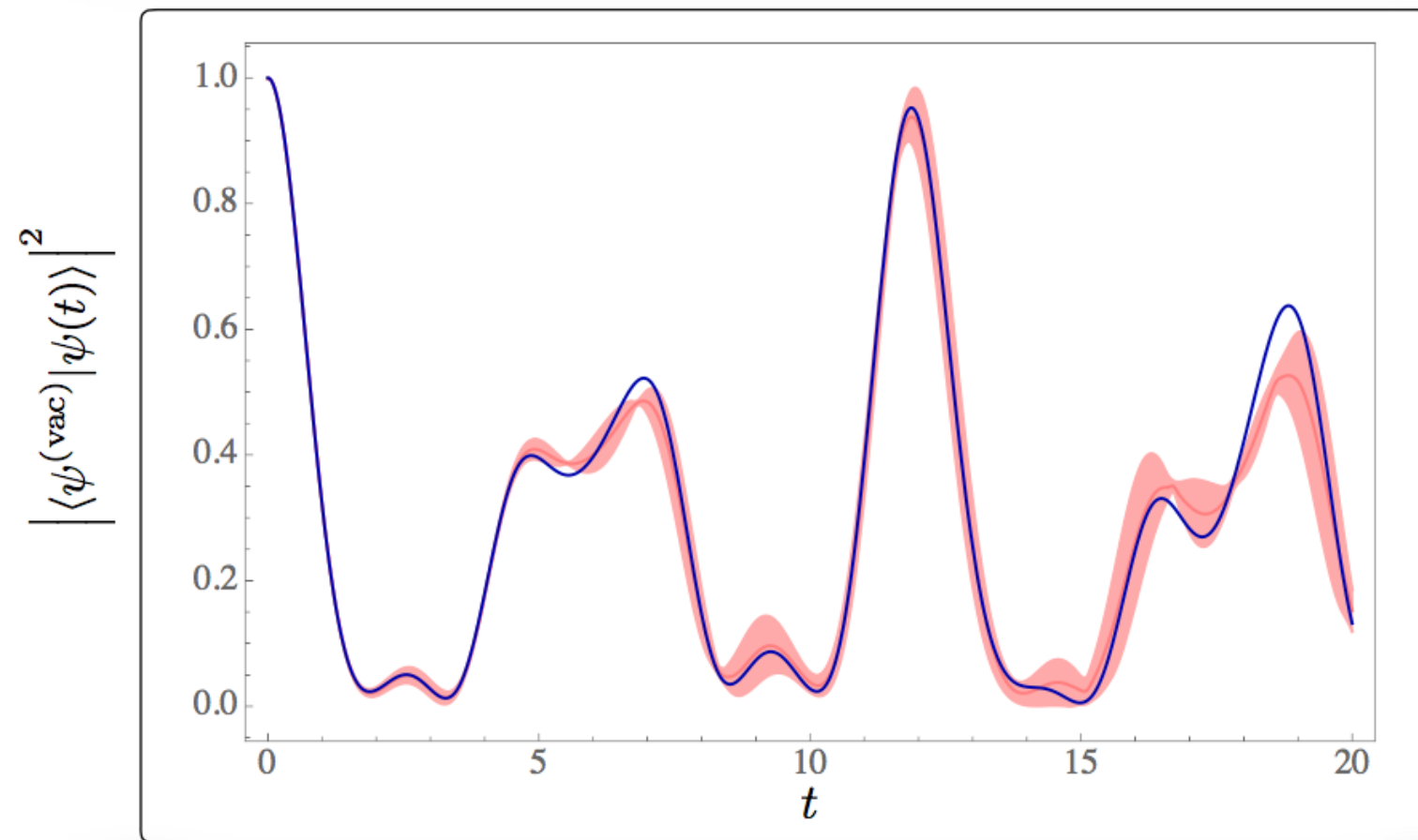
Innesbruck



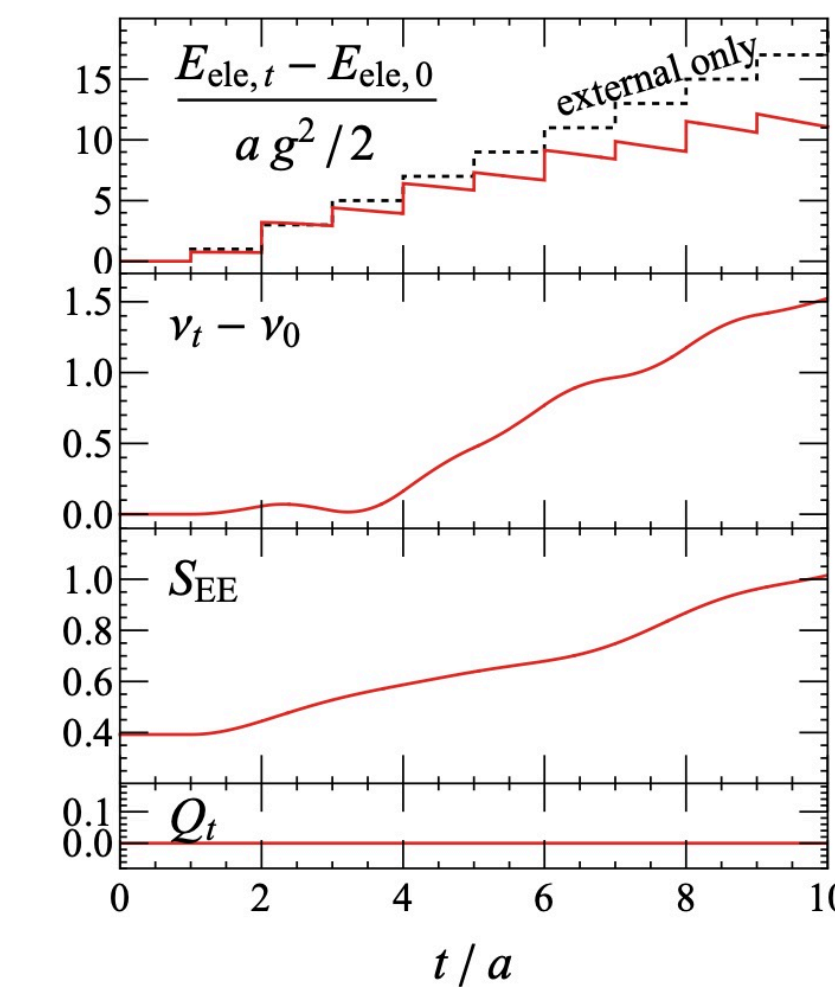
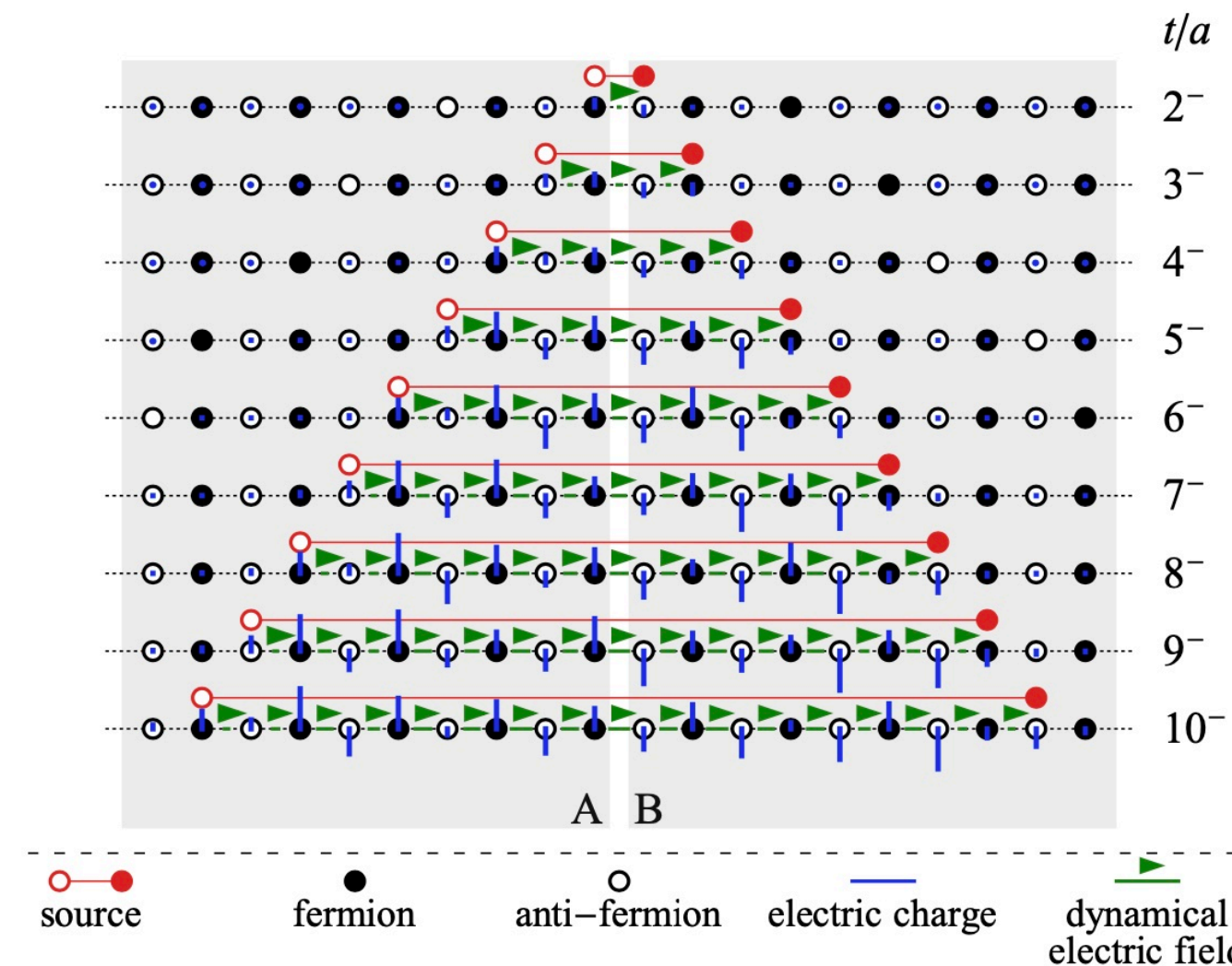
ORNL-Washington-Basque



Innesbruck



Maryland

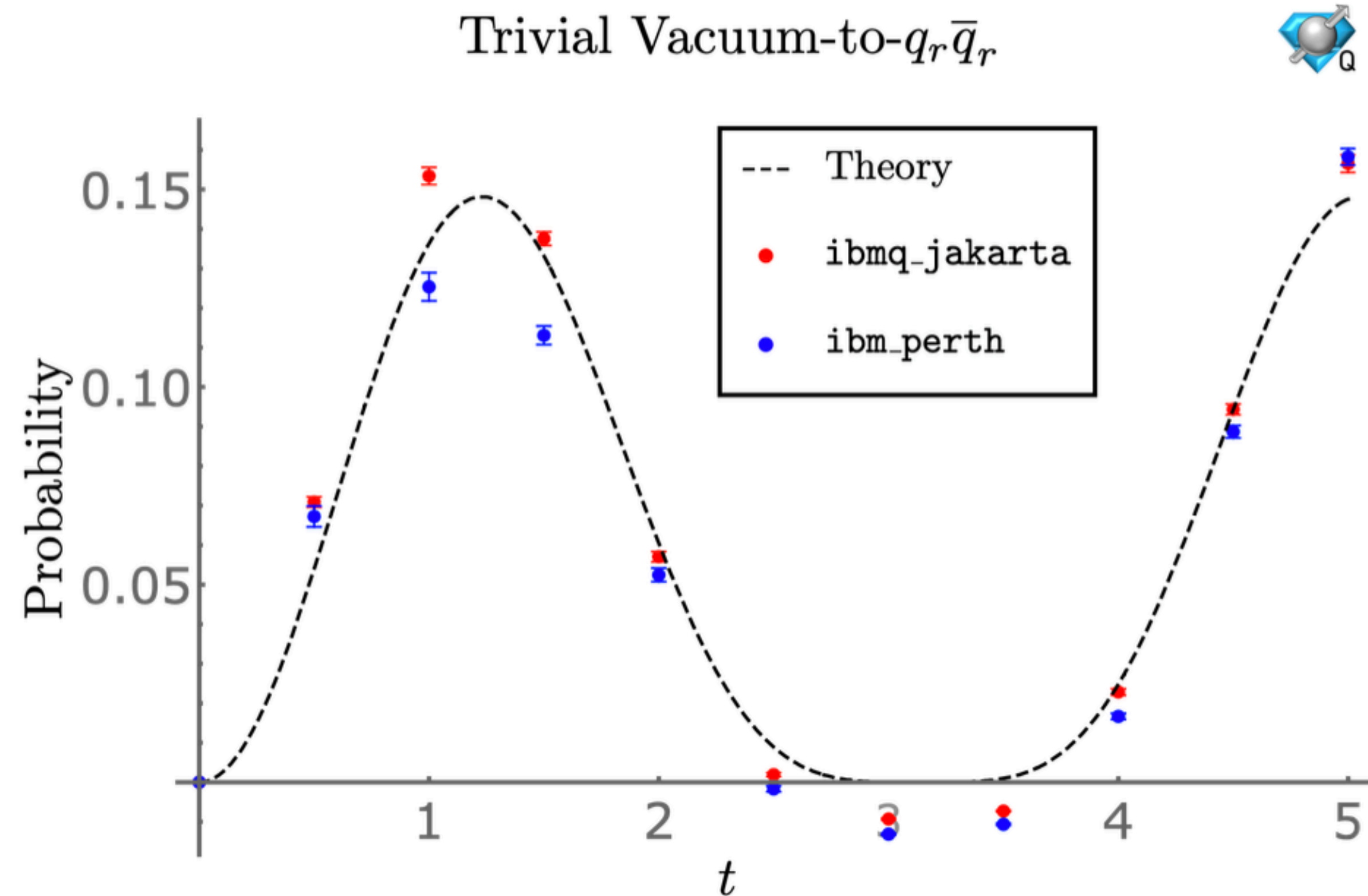
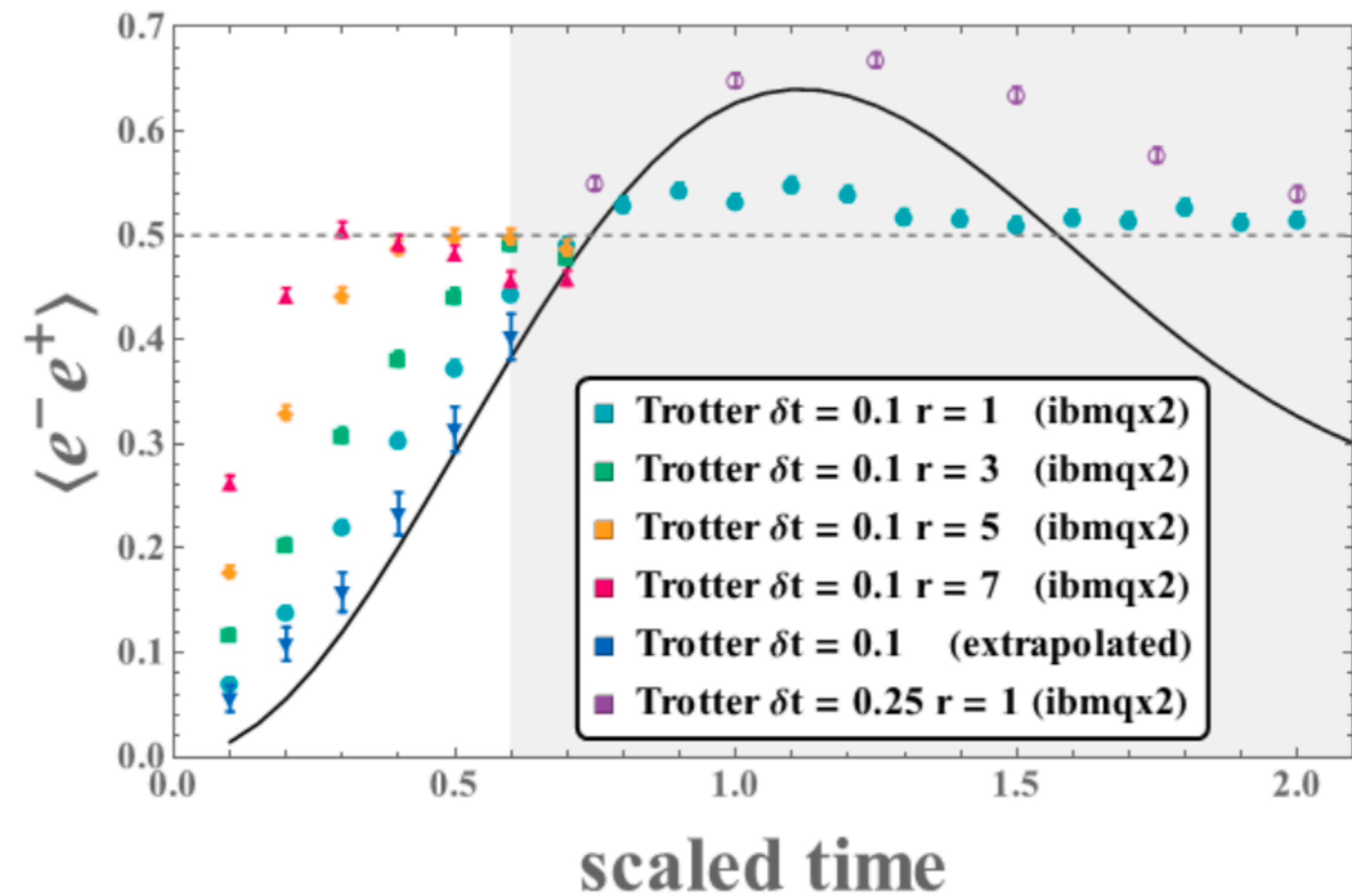


Florio, Kharzeev et al 2023

The Difference 5 Years Makes

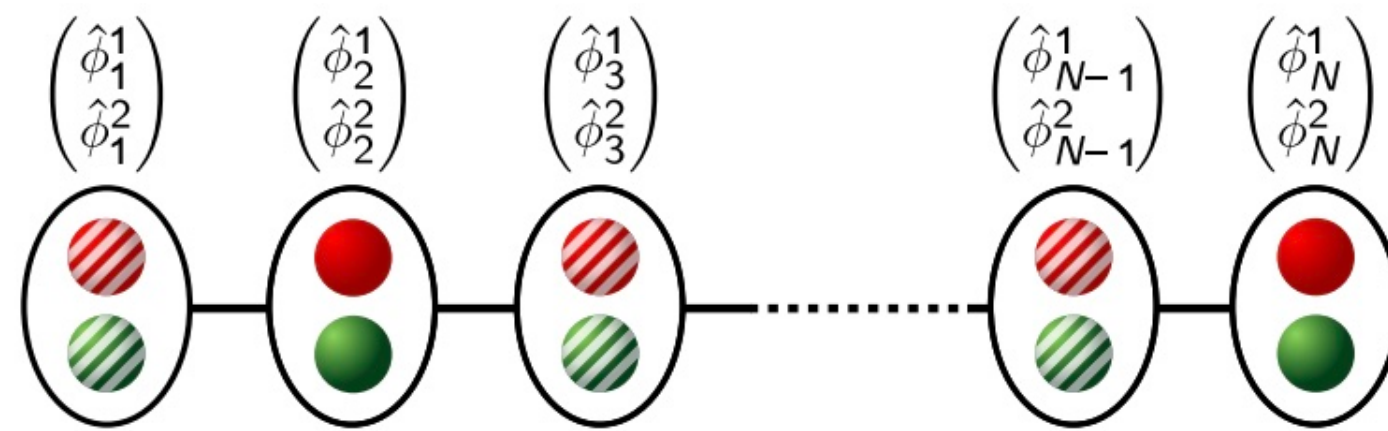
2017-8

2022

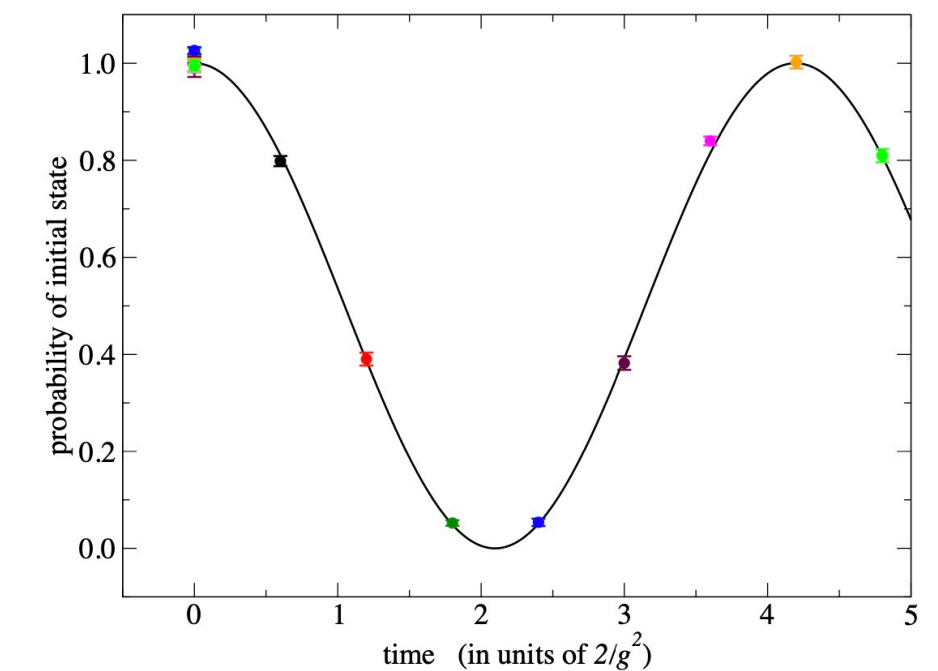
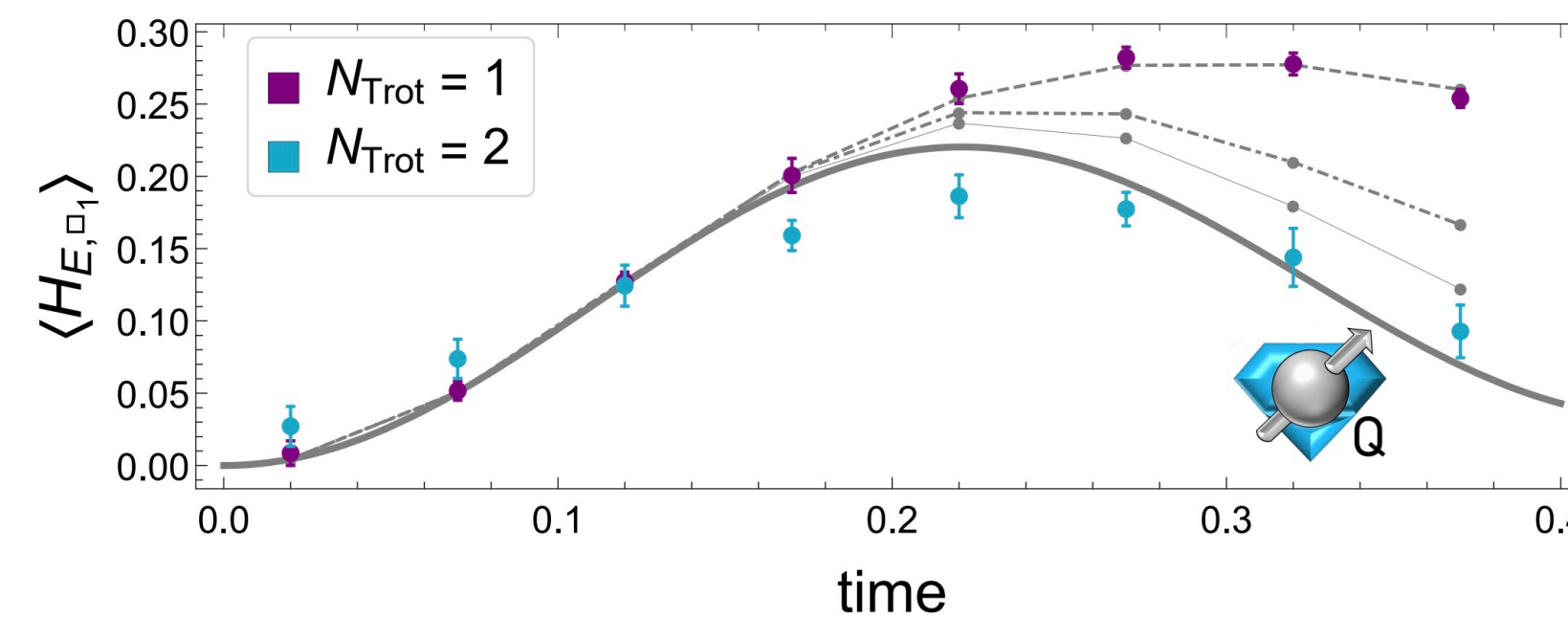
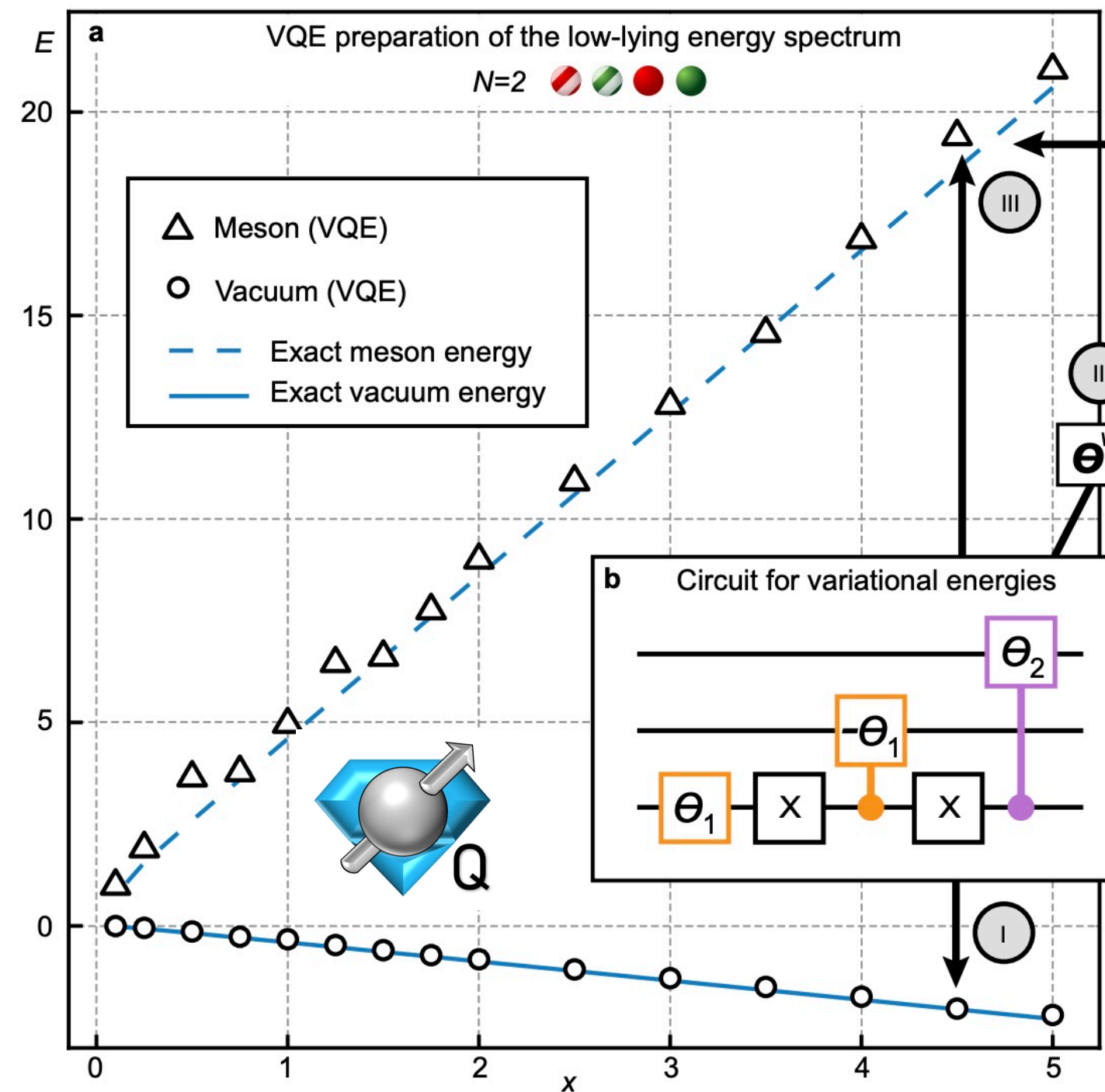
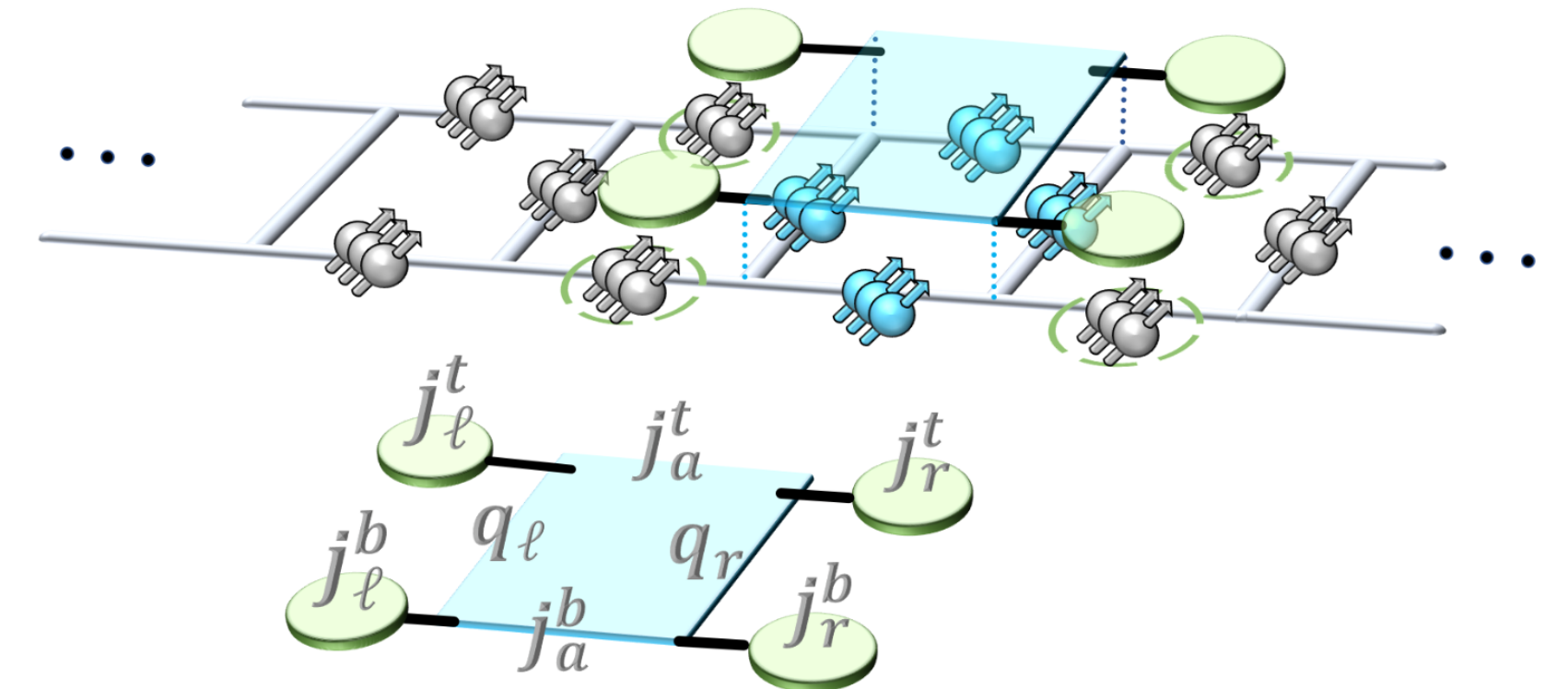


Non-Abelian GFT

SU(2) LGT - 1+1, 2+1 D



Also see Mari Carmen Banuls, Karl Jansen et al



- Only dynamical gauge fields
- Gauge Variant Completions (GVC)
- Severely truncated in field space
- 2D, but really 1D

Muschik, Lewis, et al (2021)

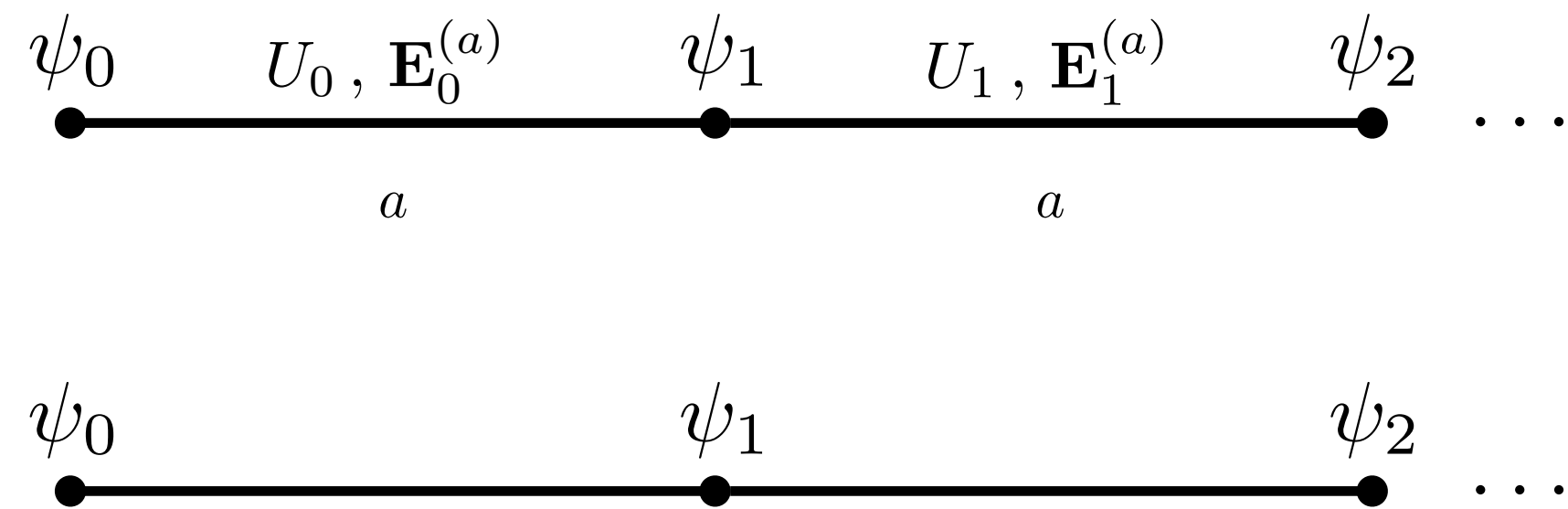
Klco, Stryker, MJS (2019), A Rahman et al (2021)

1+1 Dimensional SU(3) [QCD]



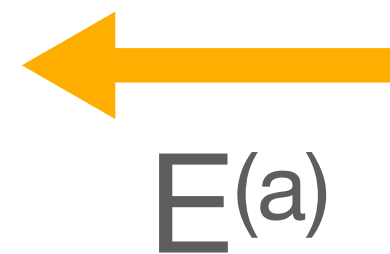
Building on the works of others, Banuls, Dirac, Jansen, Muschik, Lewis,

Gauge Choice : Axial Gauge Vs Weyl Gauge

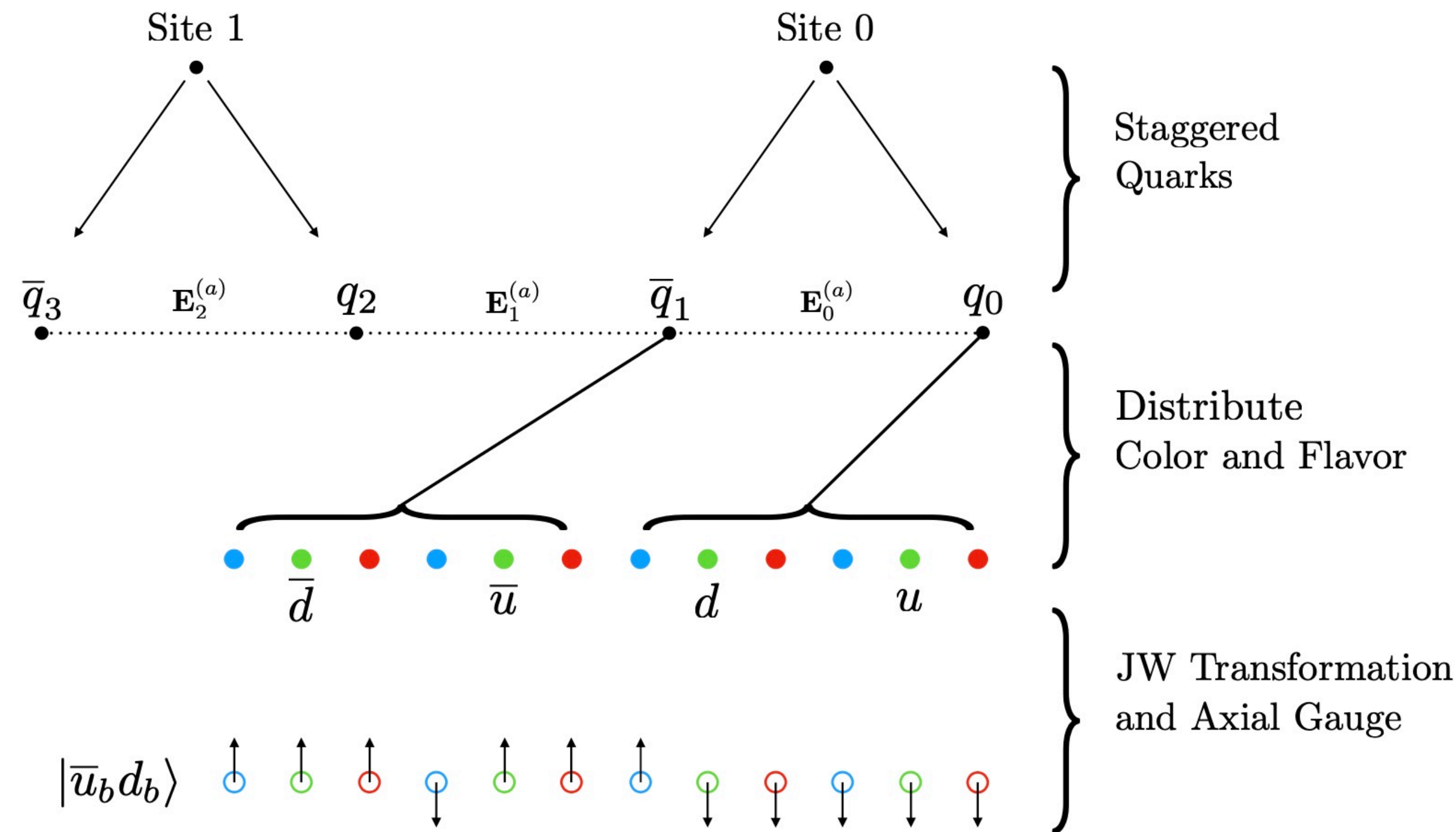


Found time-evolution requirements to be approx independent of gauge choice

Color edge states



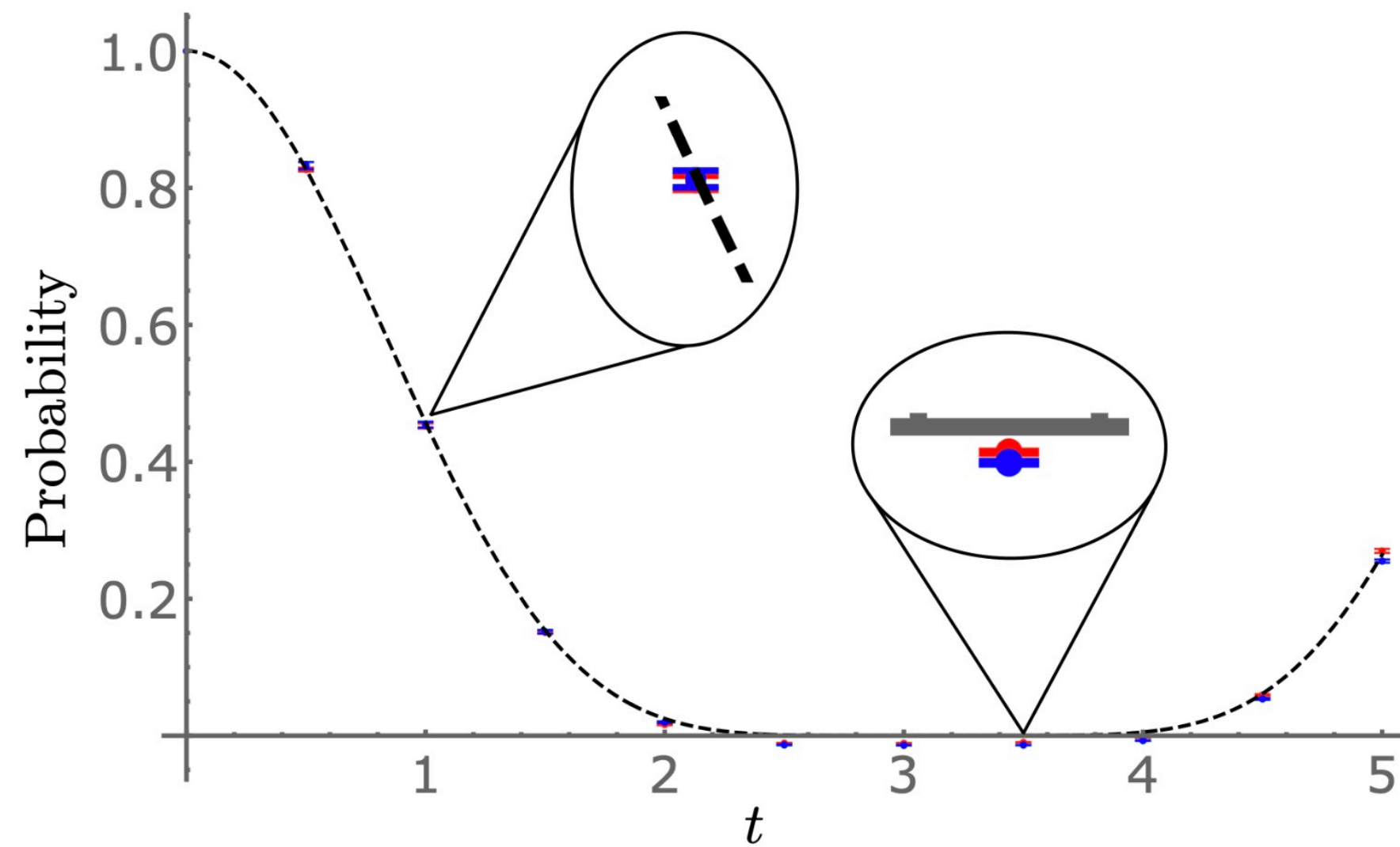
$$\mathbf{E}^{(a)} = 0$$



Simulations using IBM's Quantum Computers

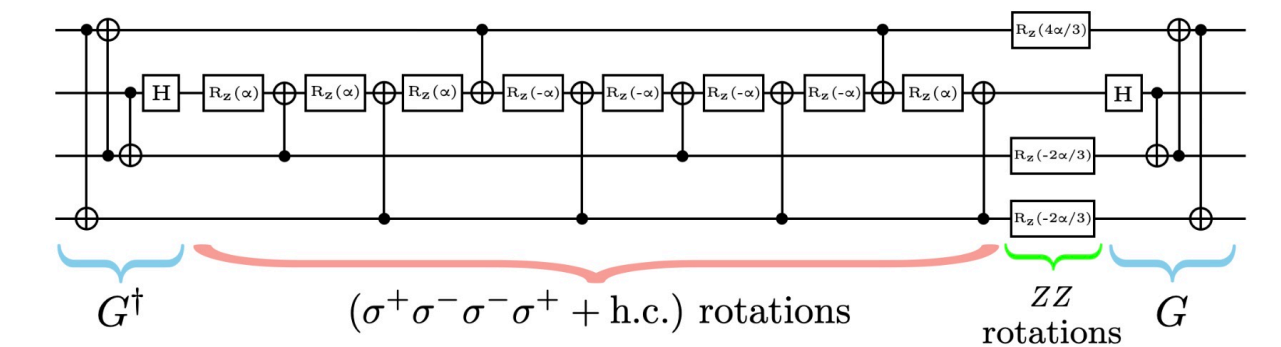
1-site, 3 colors, 1 flavor

Trivial Vacuum-to-Vacuum

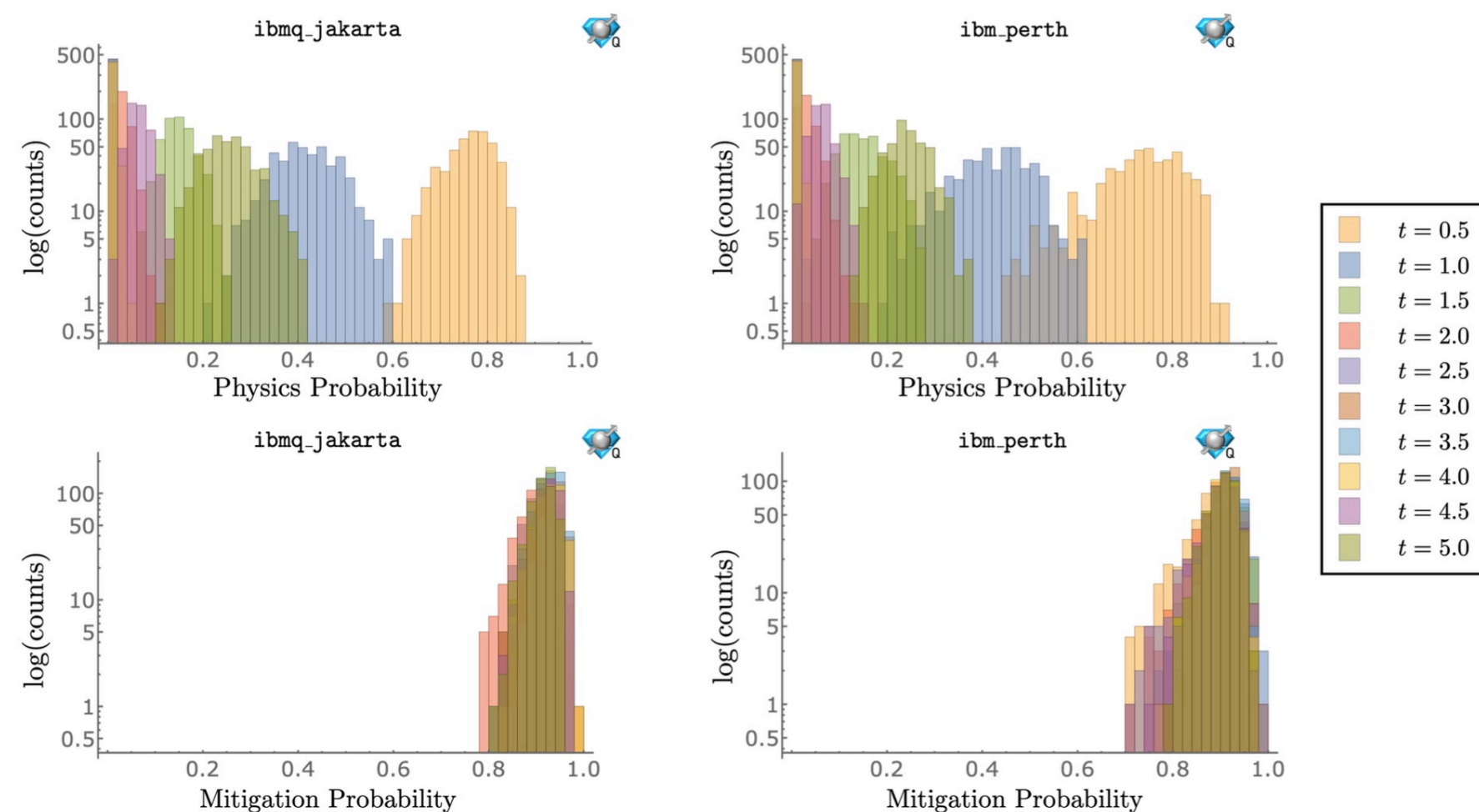
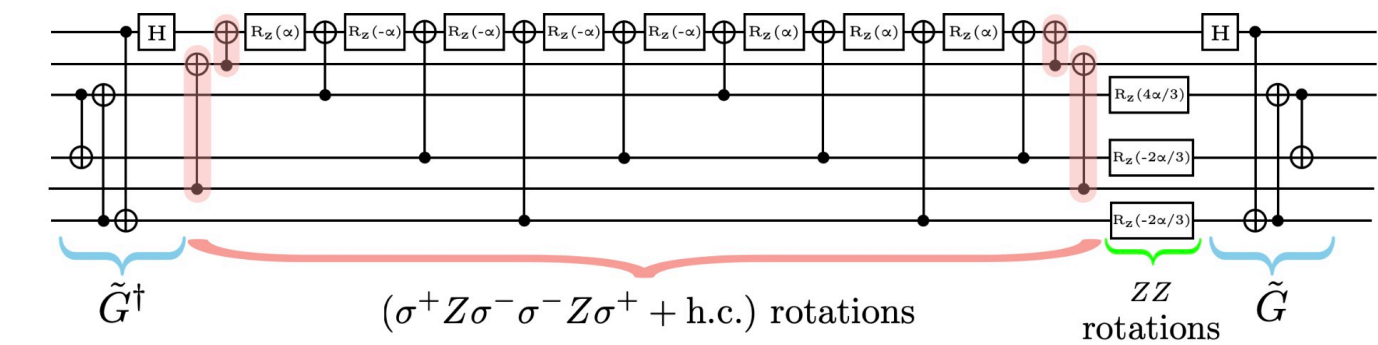


IBM 7 qubit Perth and Jakarta

34 CNOTs per step
447 Pauli-Twirled circuits
1000 shots per circuits



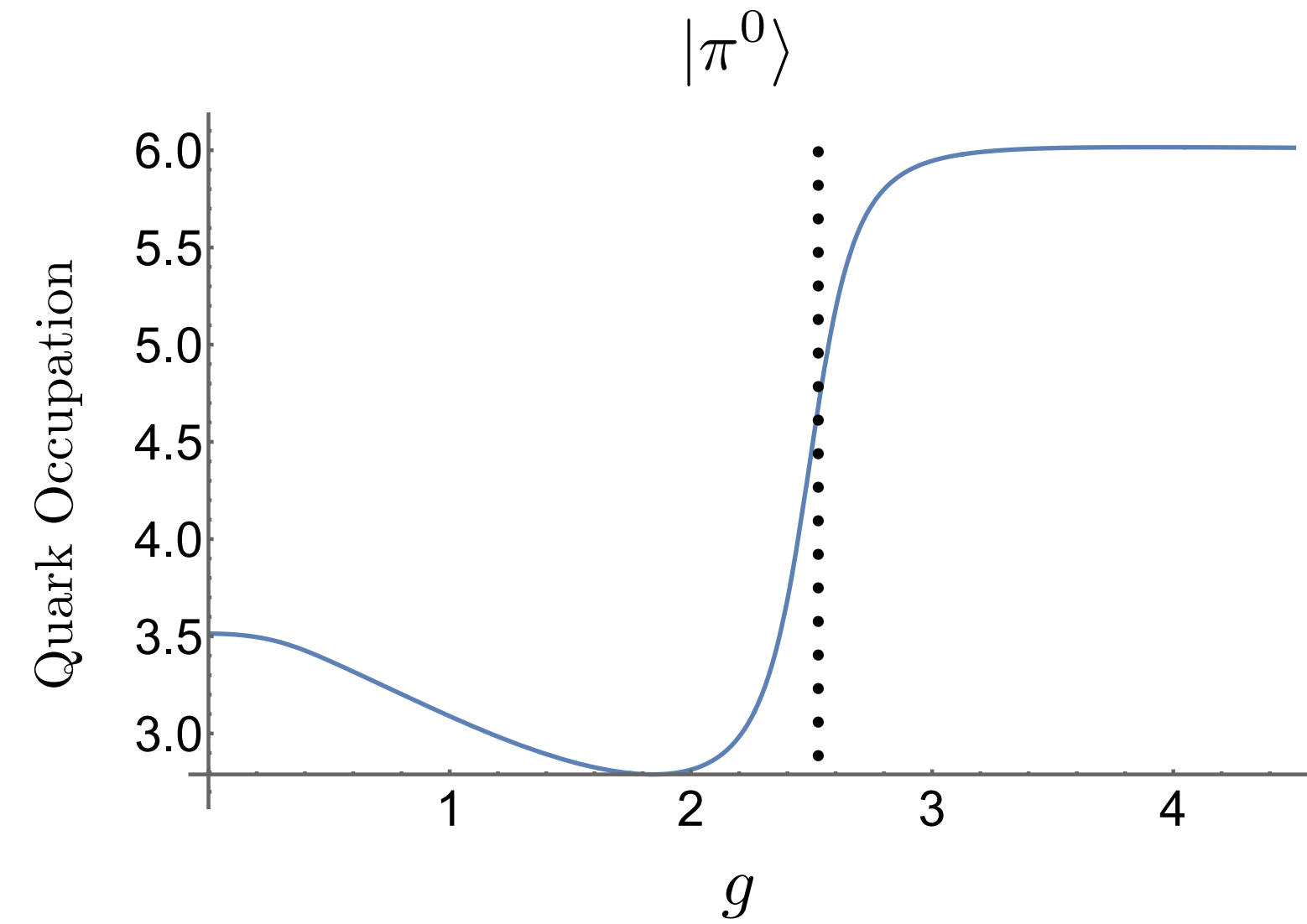
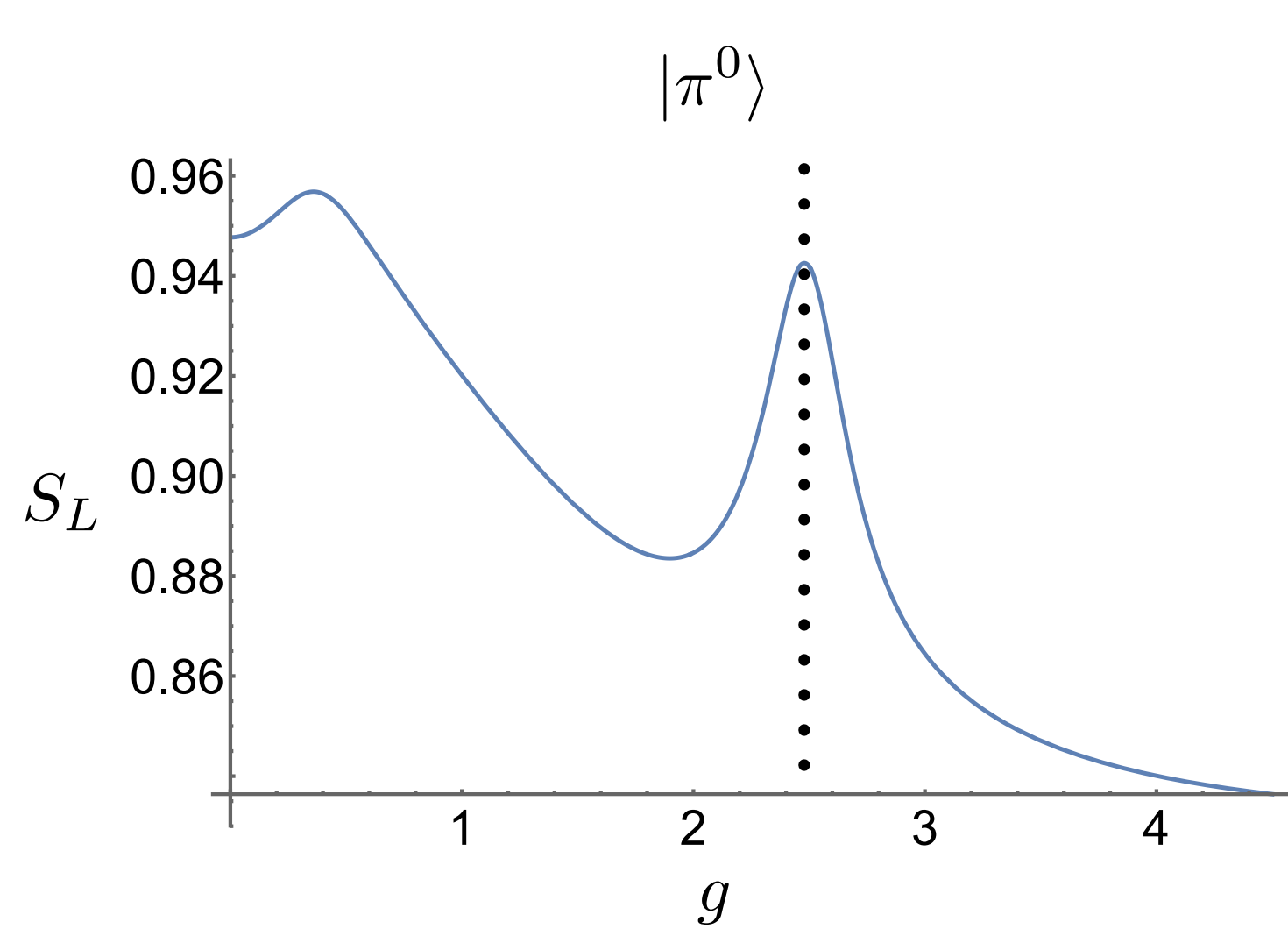
Dynamic Decoupling
Pauli-Twirling
Post selection
De-coherence renormalization (Bauer et al, Lewis et al)



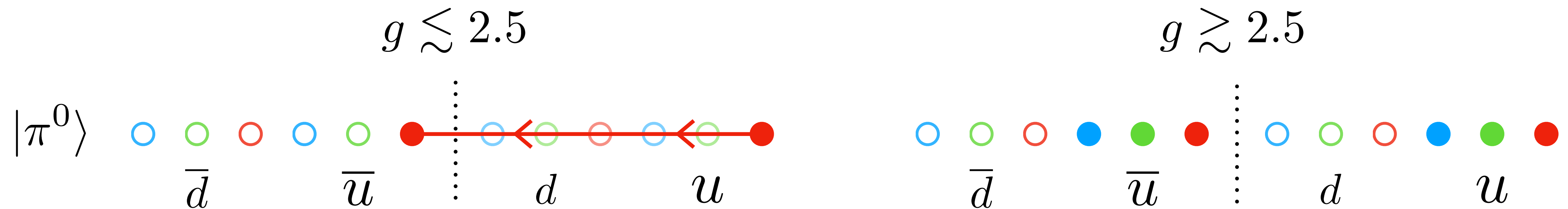
| Number of CNOT gates for one Trotter step of $SU(3)$ | | | |
|------------------------------------------------------|-----------|-----------|-----------|
| L | $N_f = 1$ | $N_f = 2$ | $N_f = 3$ |
| 1 | 30 | 114 | 242 |
| 2 | 228 | 878 | 1,940 |
| 5 | 1,926 | 7,586 | 16,970 |
| 10 | 8,436 | 33,486 | 75,140 |
| 100 | 912,216 | 3,646,086 | 8,201,600 |

Entanglement structures

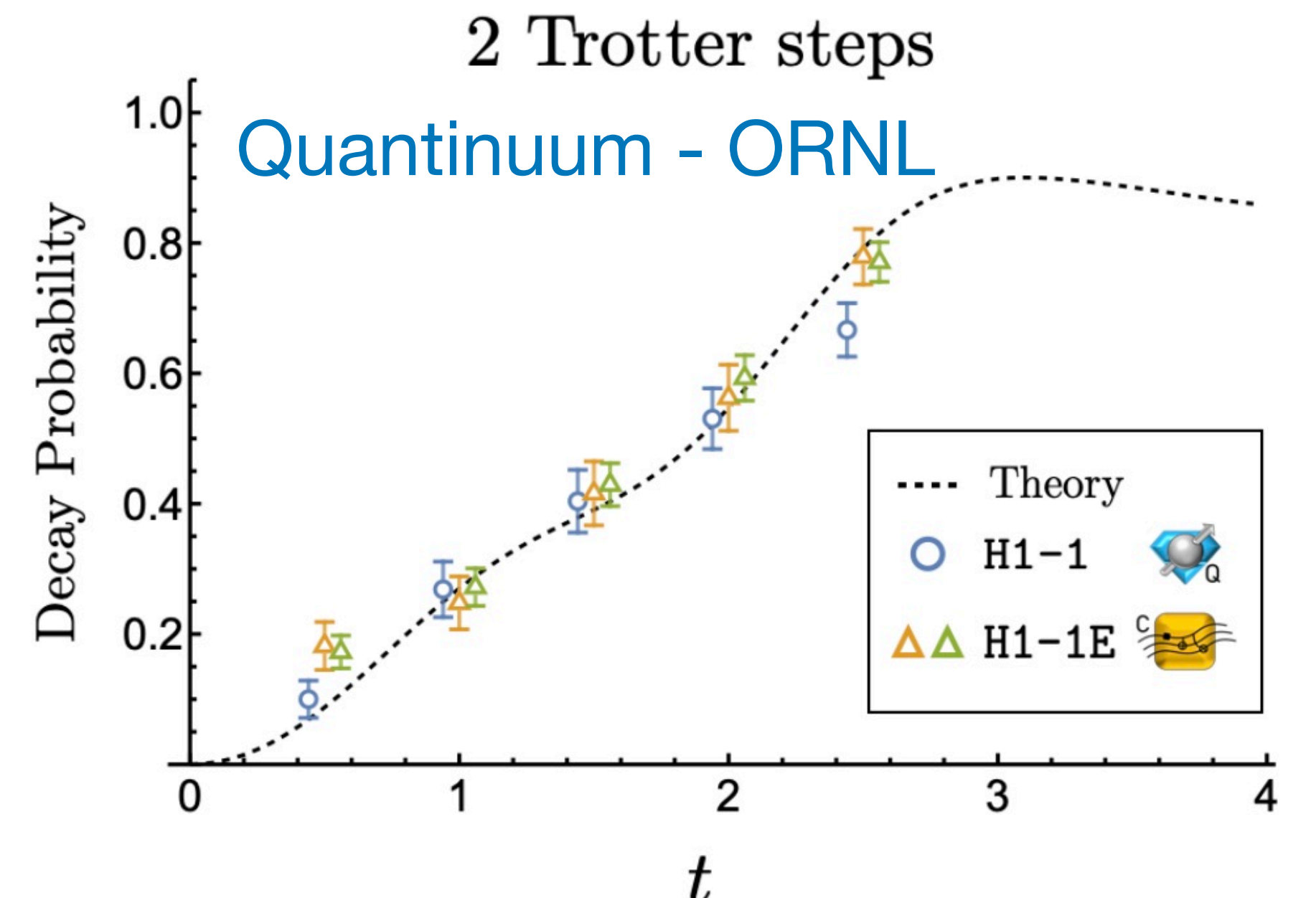
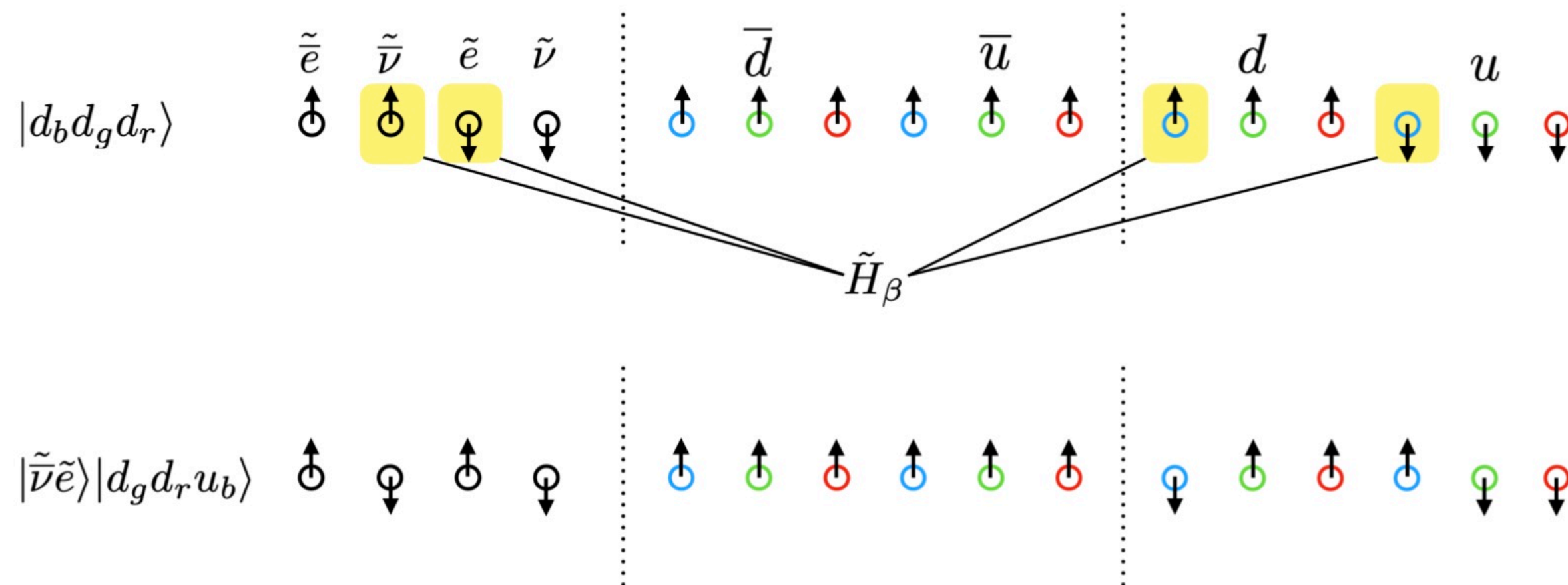
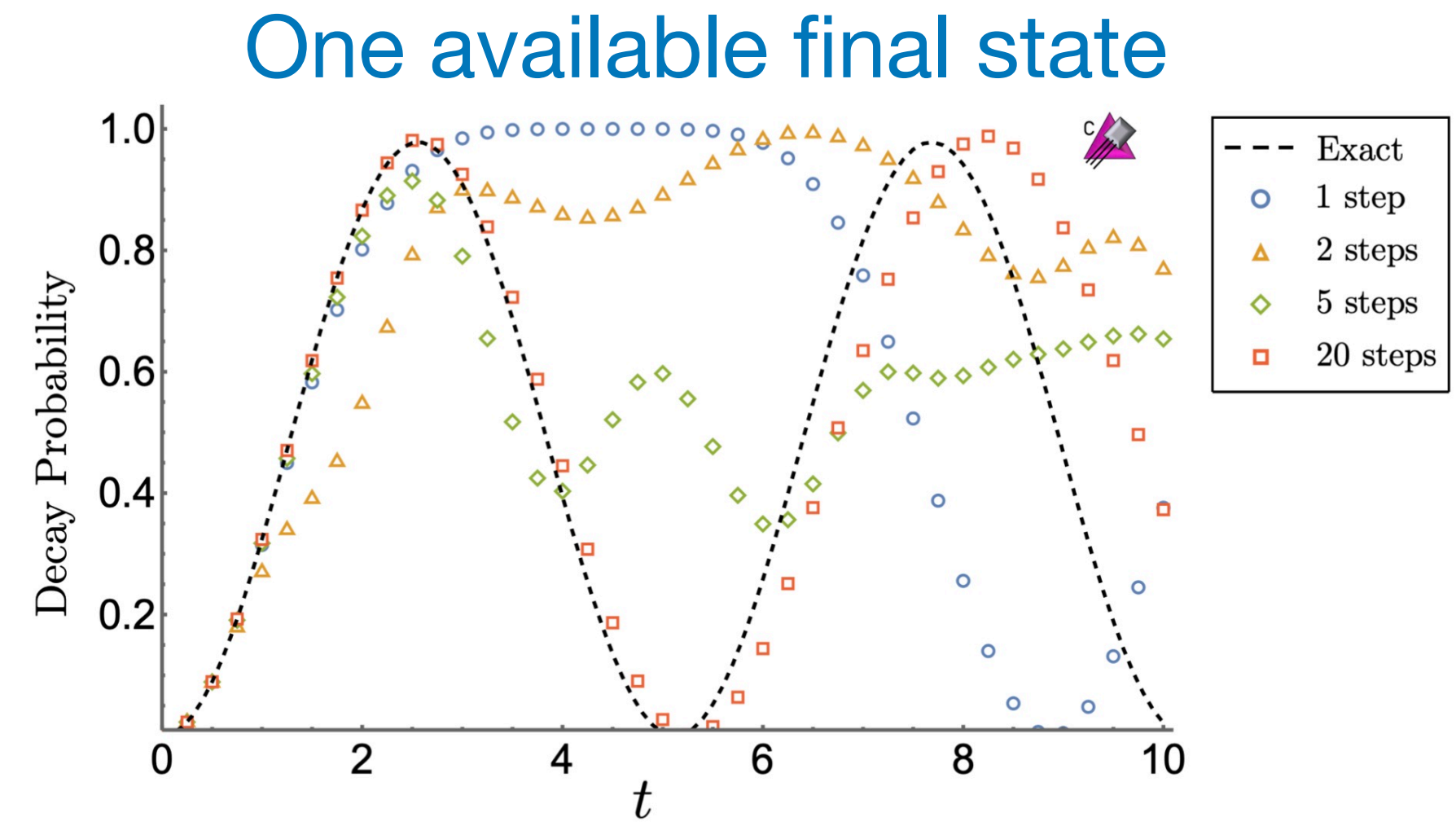
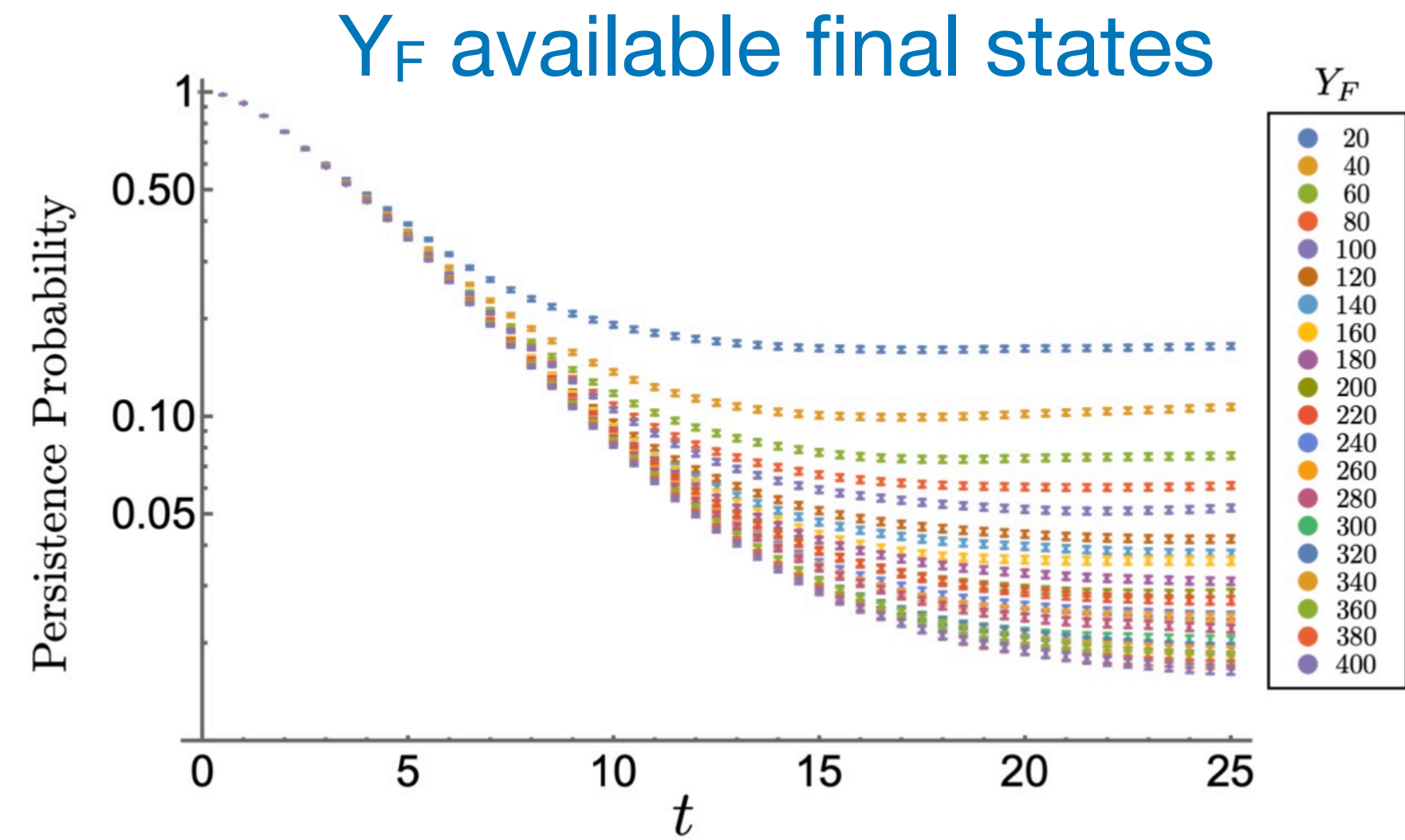
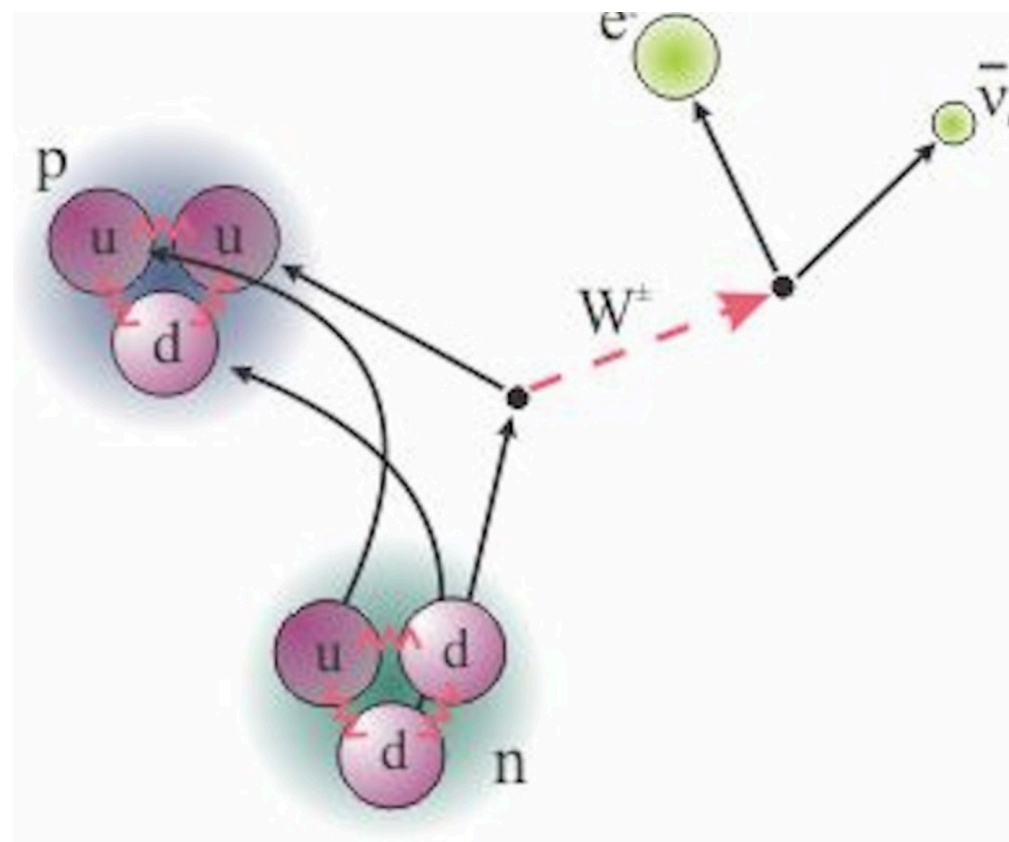
Entanglement in SM physics: Extensive literature that is rapidly growing.



Peak in entanglement coincides with transition from quark-antiquark to baryon-anti-baryon structure



Recovering Real-time Exponential-Decay Weak Interactions



Decoherence Renormalization The Difference 1 Year Can Make!

Self-mitigating Trotter circuits for SU(2) lattice gauge theory on a quantum computer

Sarmed A Rahman, Randy Lewis, Emanuele Mendicelli, and Sarah Powell
Department of Physics and Astronomy, York University,
Toronto, Ontario, Canada, M3J 1P3

(Dated: May 2022. Updated: October 2022.)

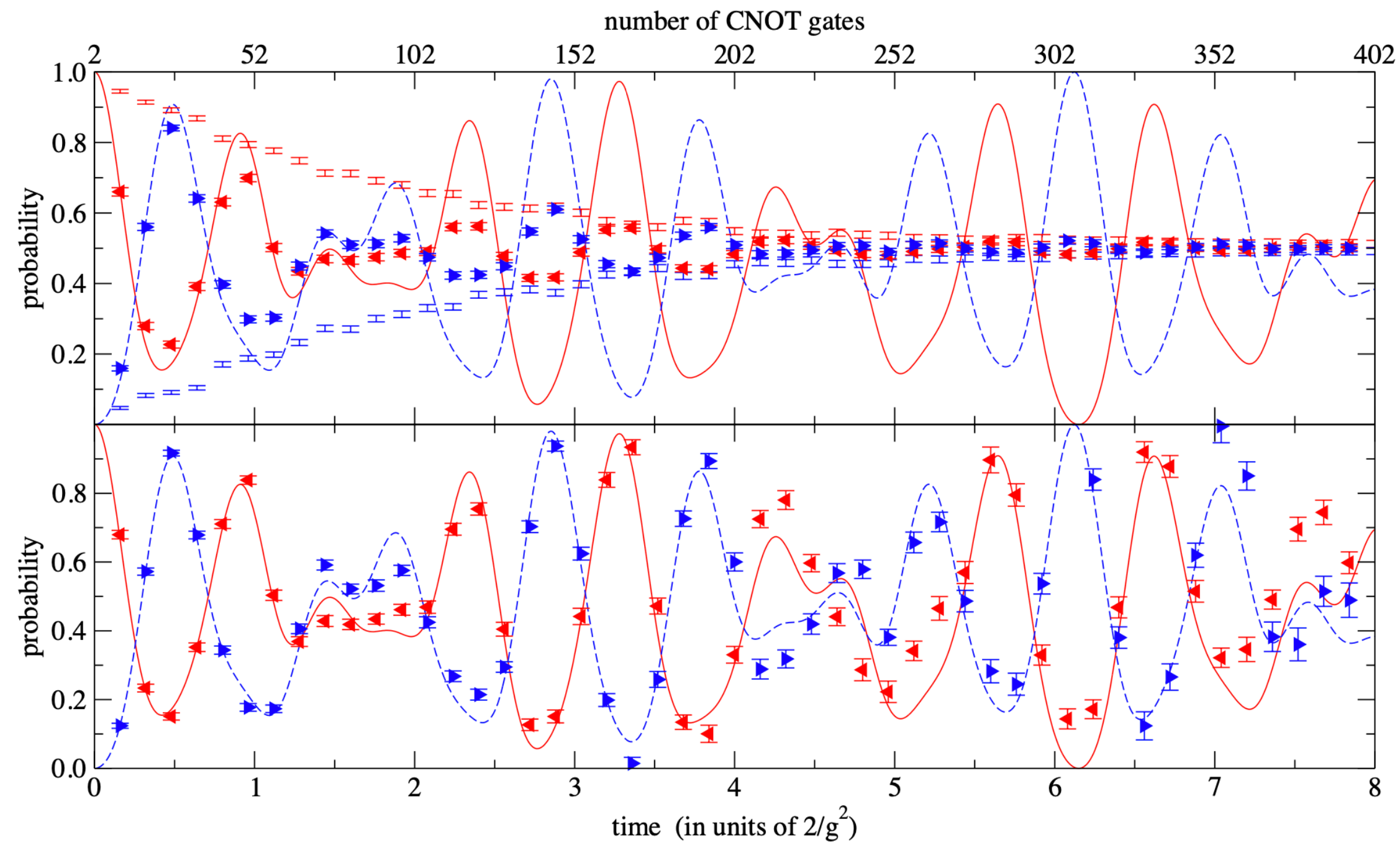
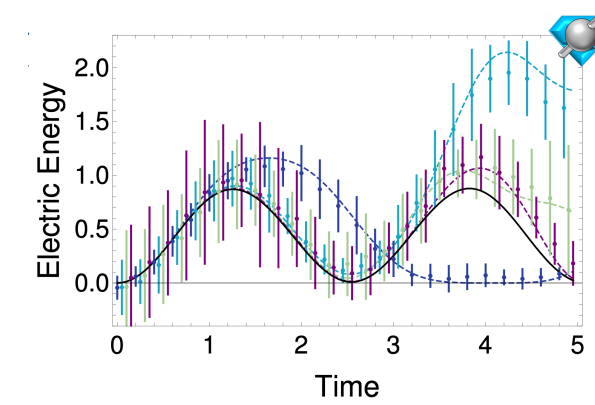
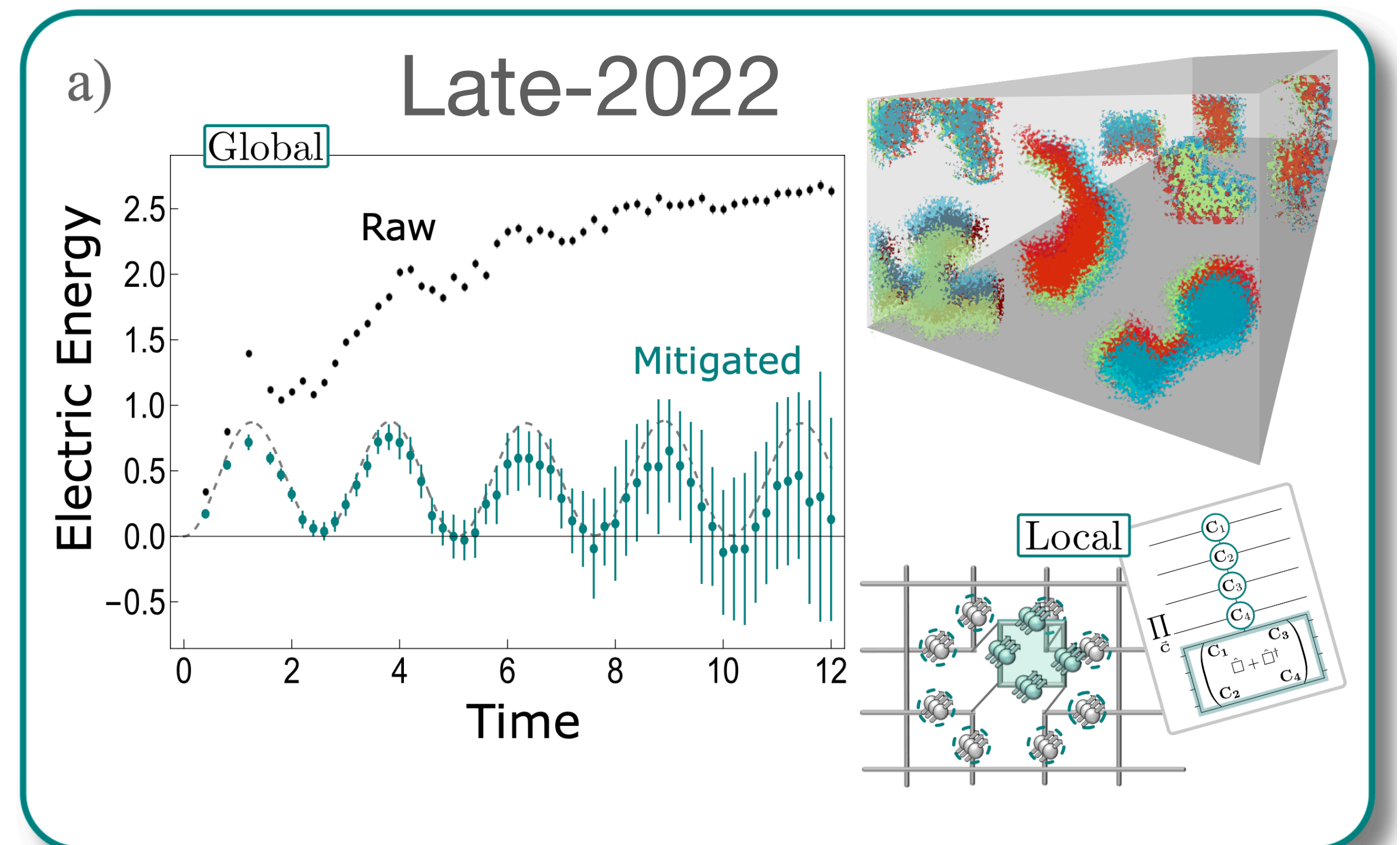


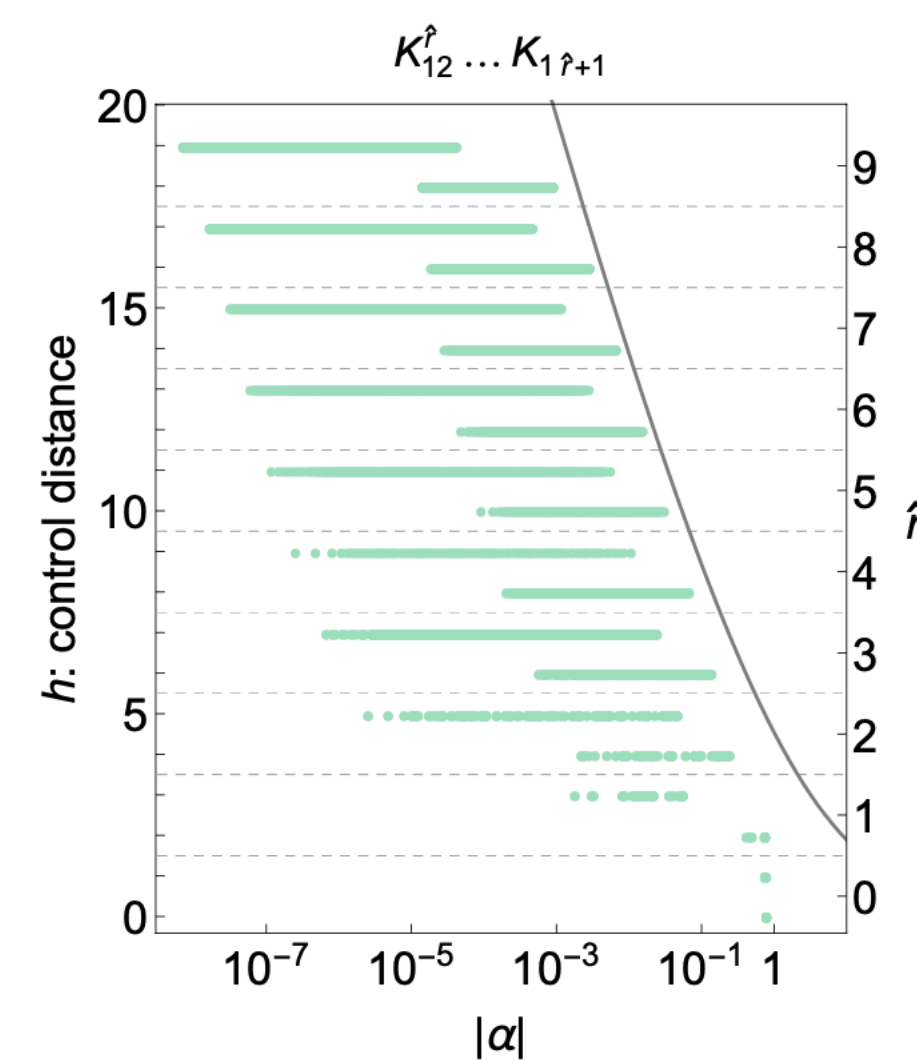
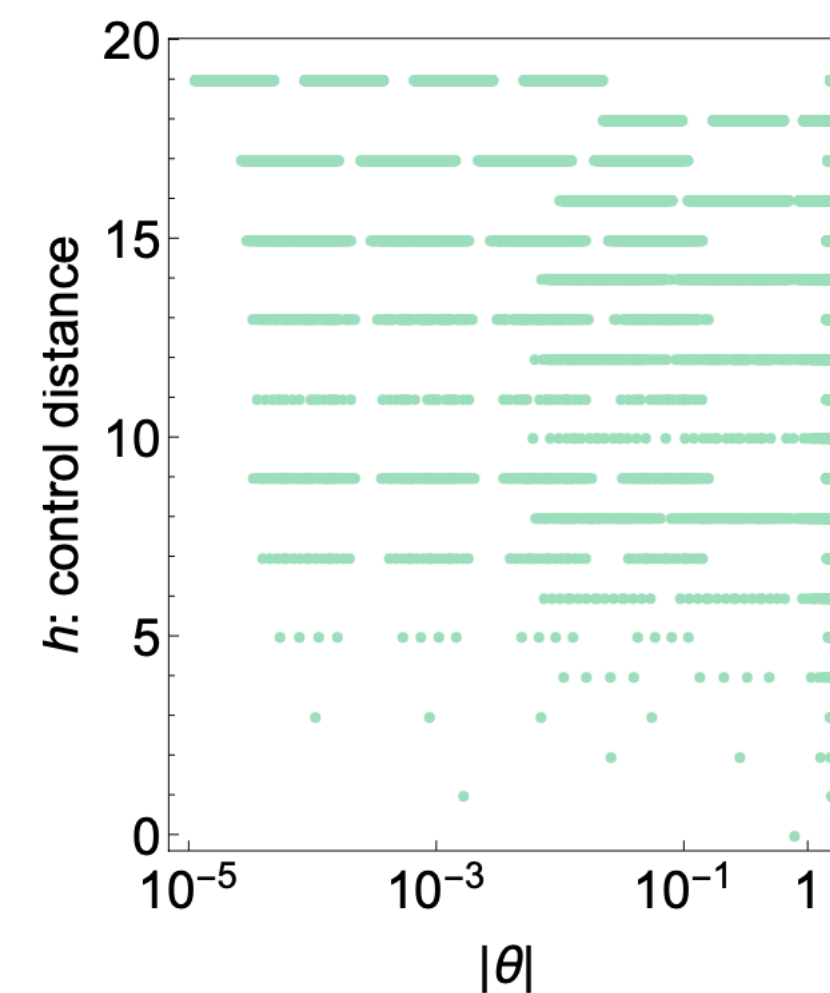
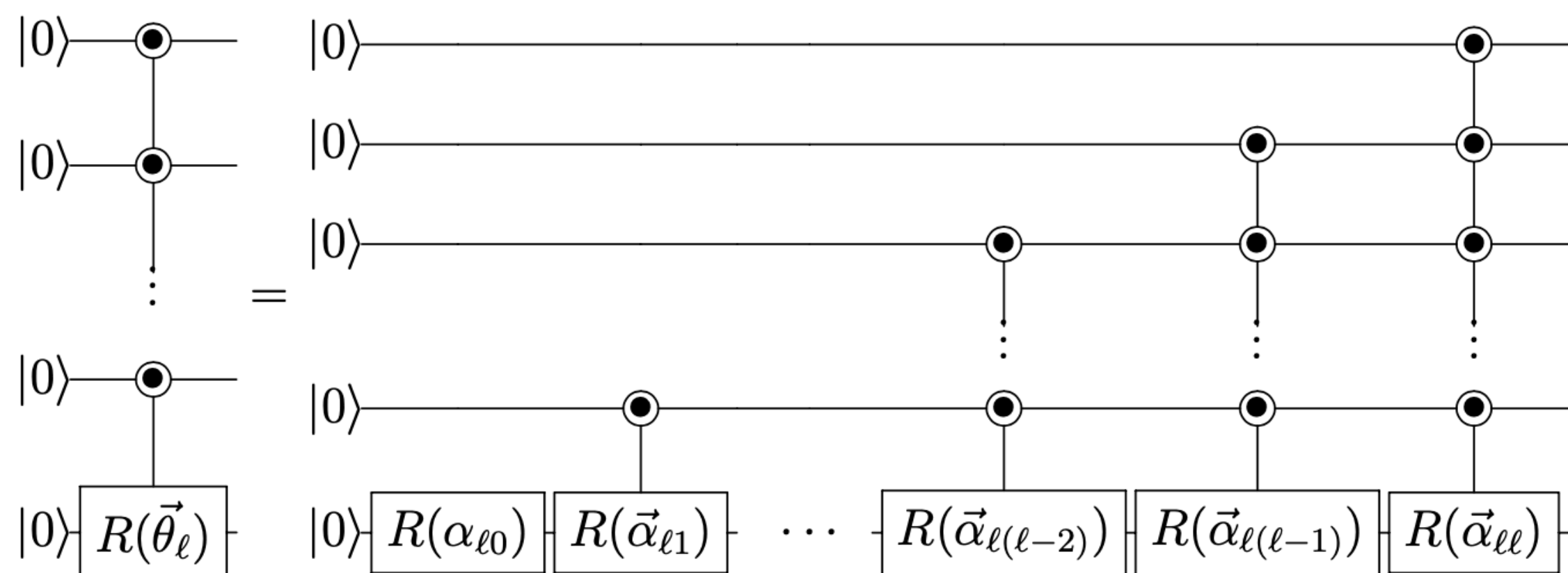
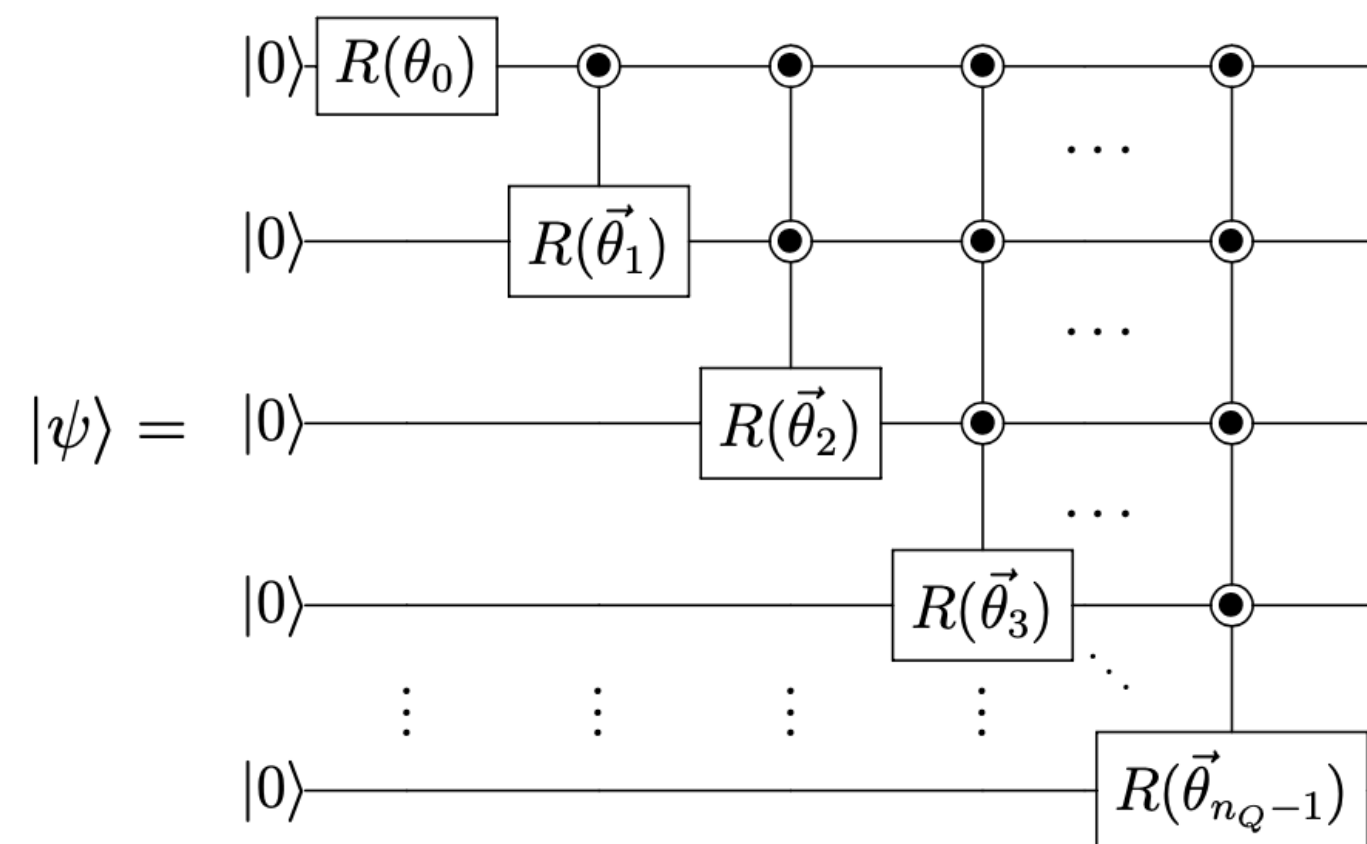
FIG. 3. Time evolution by self-mitigation on a two-plaquette lattice from the initial state of Fig. 1 with gauge coupling $x = 2.0$ and time step $dt = 0.08$. In both panels, the red solid (blue dashed) curve is the exact probability of the left (right) plaquette being measured to have $j = \frac{1}{2}$. **Upper panel:** The red left-pointing (blue right-pointing) triangles are the physics data computed from the `ibm_lagos` quantum processor. The red (blue) error bars without symbols are the mitigation data computed on `ibm_lagos` from the same circuit but with half the steps forward in time and then half backward in time. **Lower panel:** The triangles are the physics results obtained by applying Eq. (8) to the data from the upper panel.



Early-2022

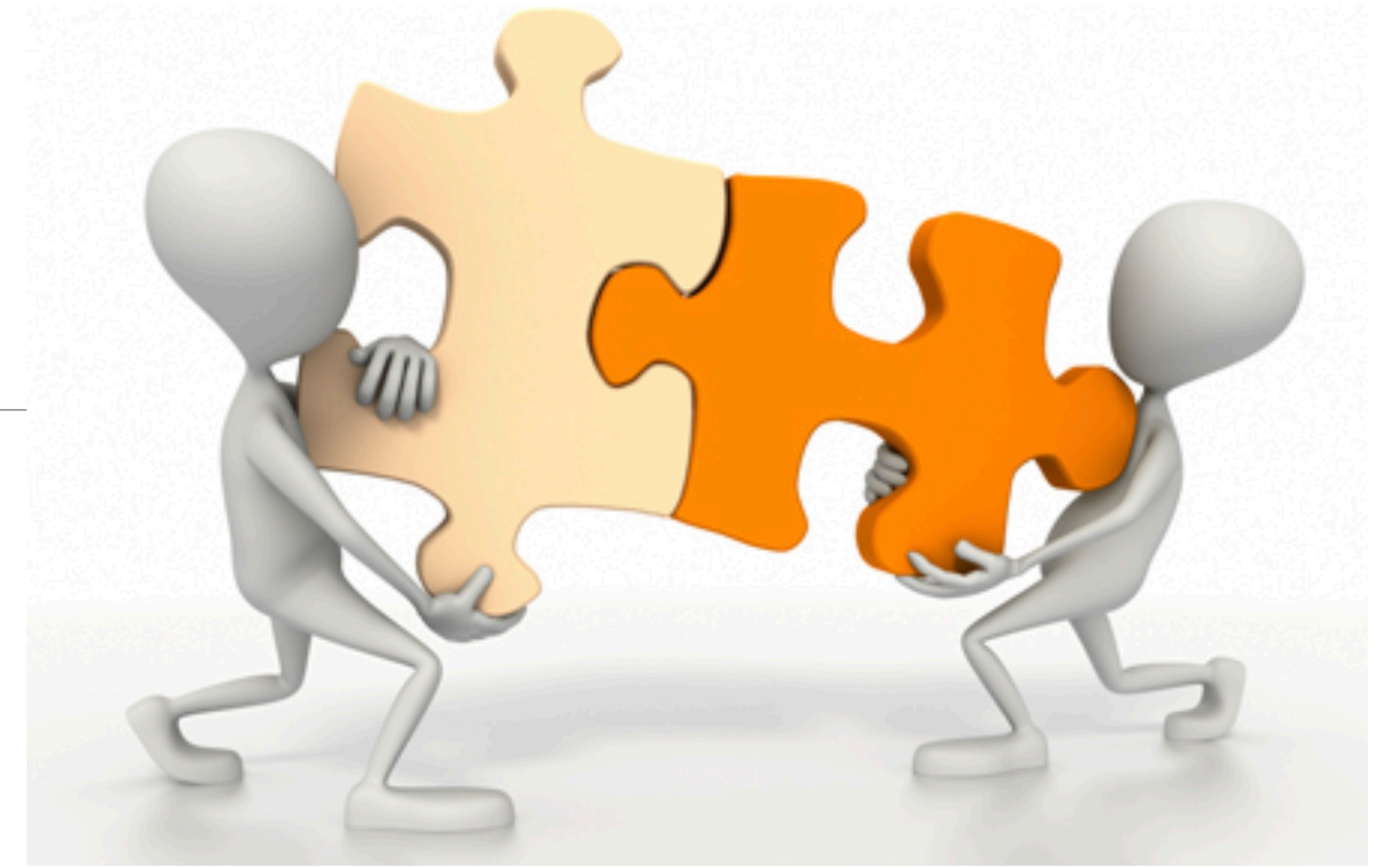


State Preparation with Localizable or **Physics-Aware** Quantum Circuits



Correlation length allows for fixed-point angles to be determined exponentially well with small-scale simulations

A Conjecture



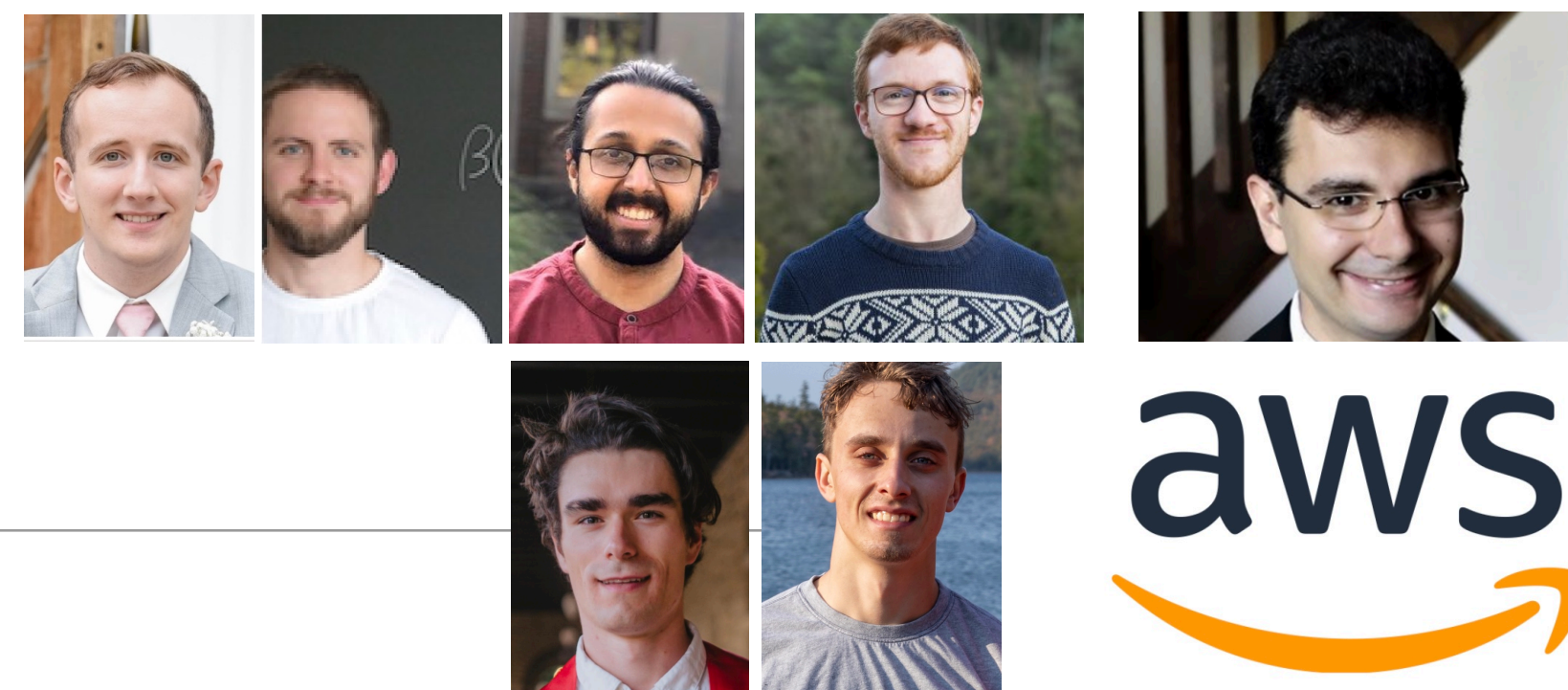
We are likely missing an important ingredient so far:

- all of the “power” of computation - the gates - are being applied at the scale of the (unphysical) lattice spacing

***Conjecture: efficient digital quantum circuits exist for Standard Model simulations where the gate-structure, or power, is dominantly focused at the scale of the physics/observable(s).
i.e., EFTs can manifest at the quantum circuit level.***

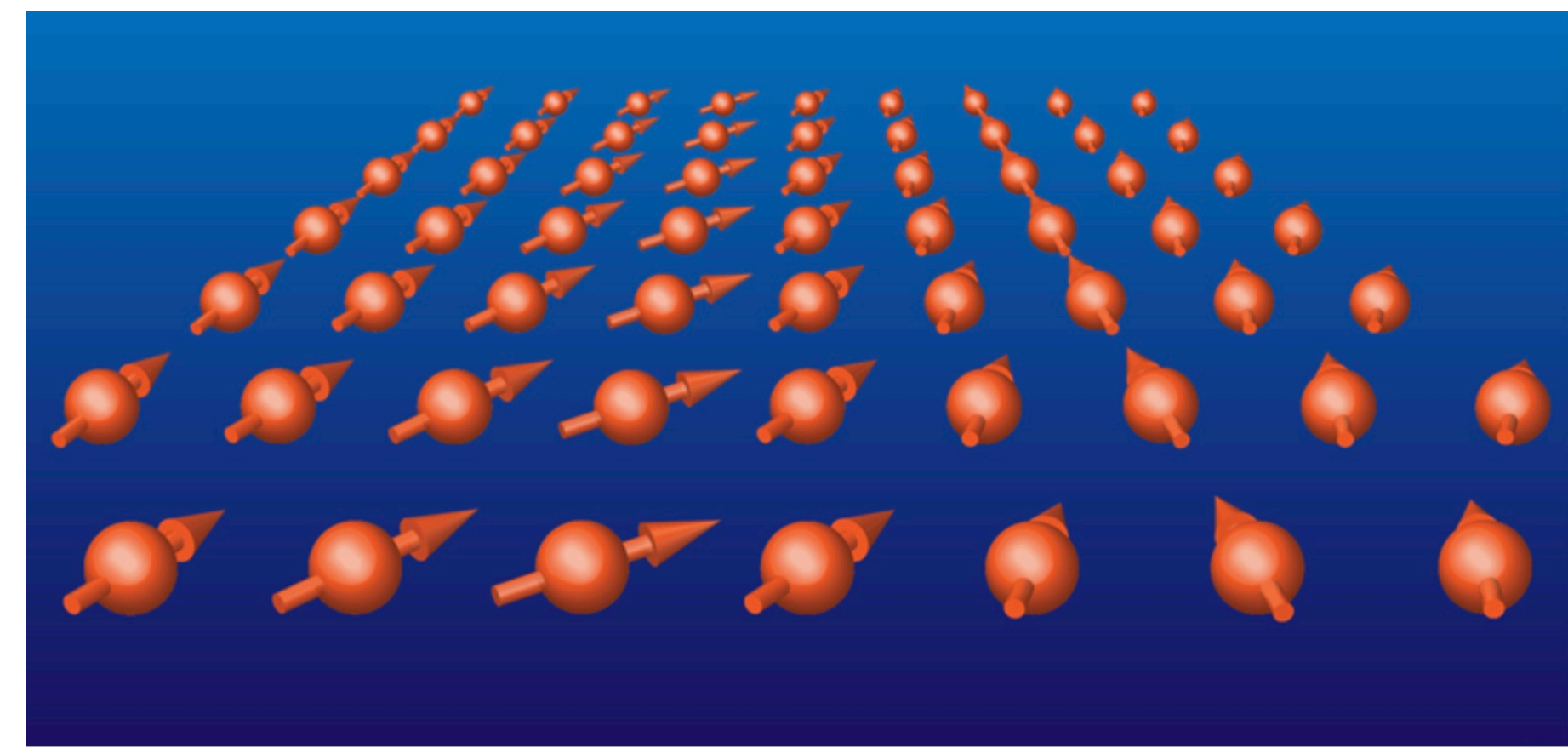
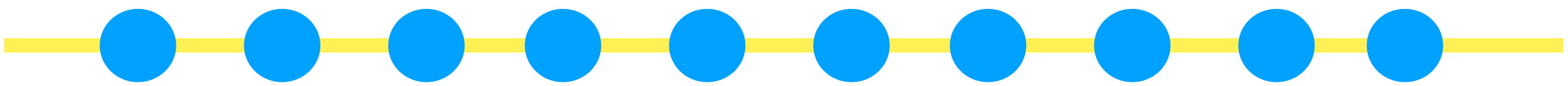
Asymptotically-Free Quantum Field Theory - Lattice Control

For Cold-Atom Systems Dimensional Reduction



[Submitted on 14 Nov 2022 (v1), last revised 22 Nov 2022 (this version, v2)]
Preparation for Quantum Simulation of the 1+1D O(3) Non-linear σ -Model using Cold Atoms
 Anthony N. Ciavarella, Stephan Caspar, Hersh Singh, Martin J. Savage

with Amazon



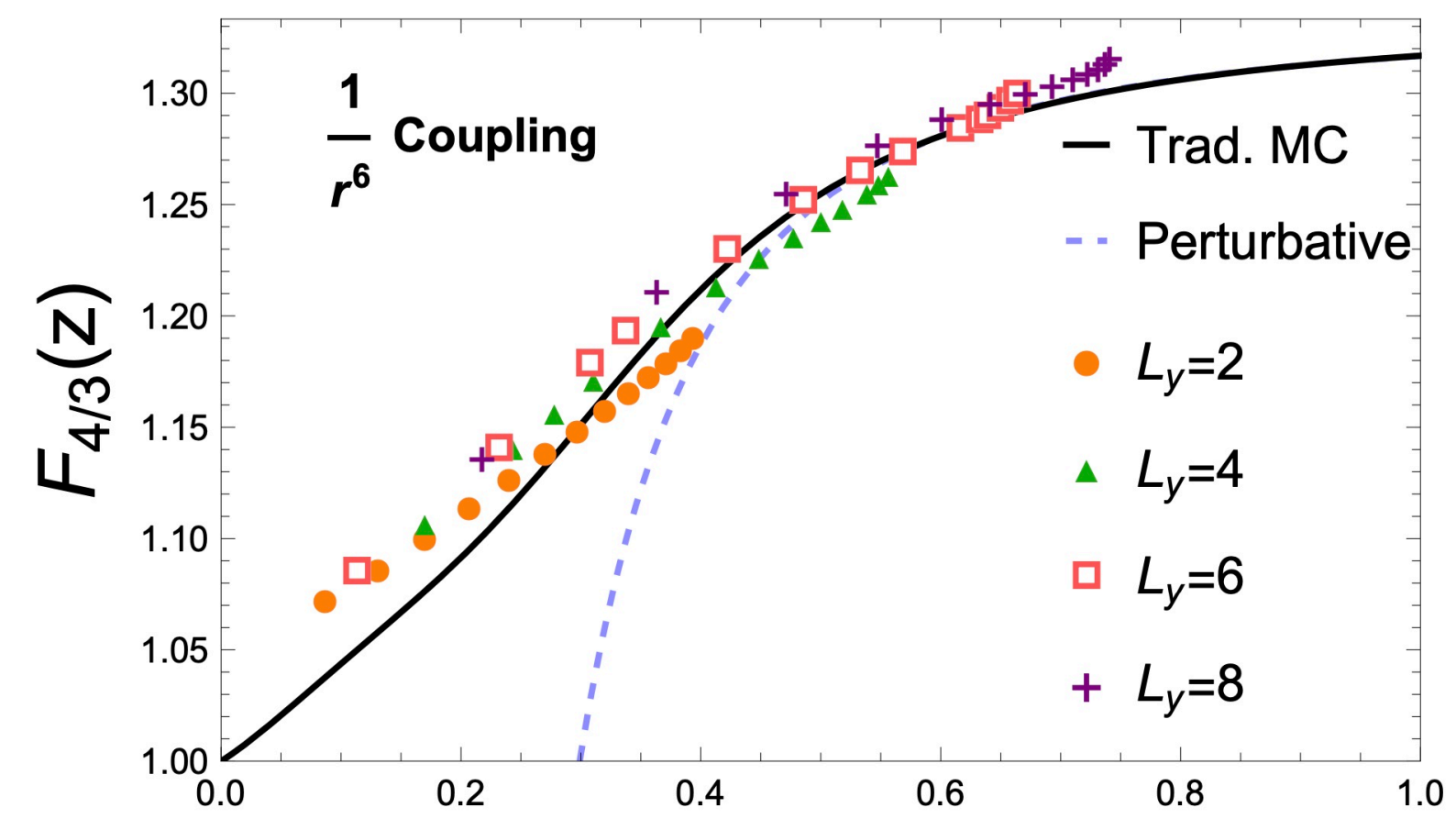
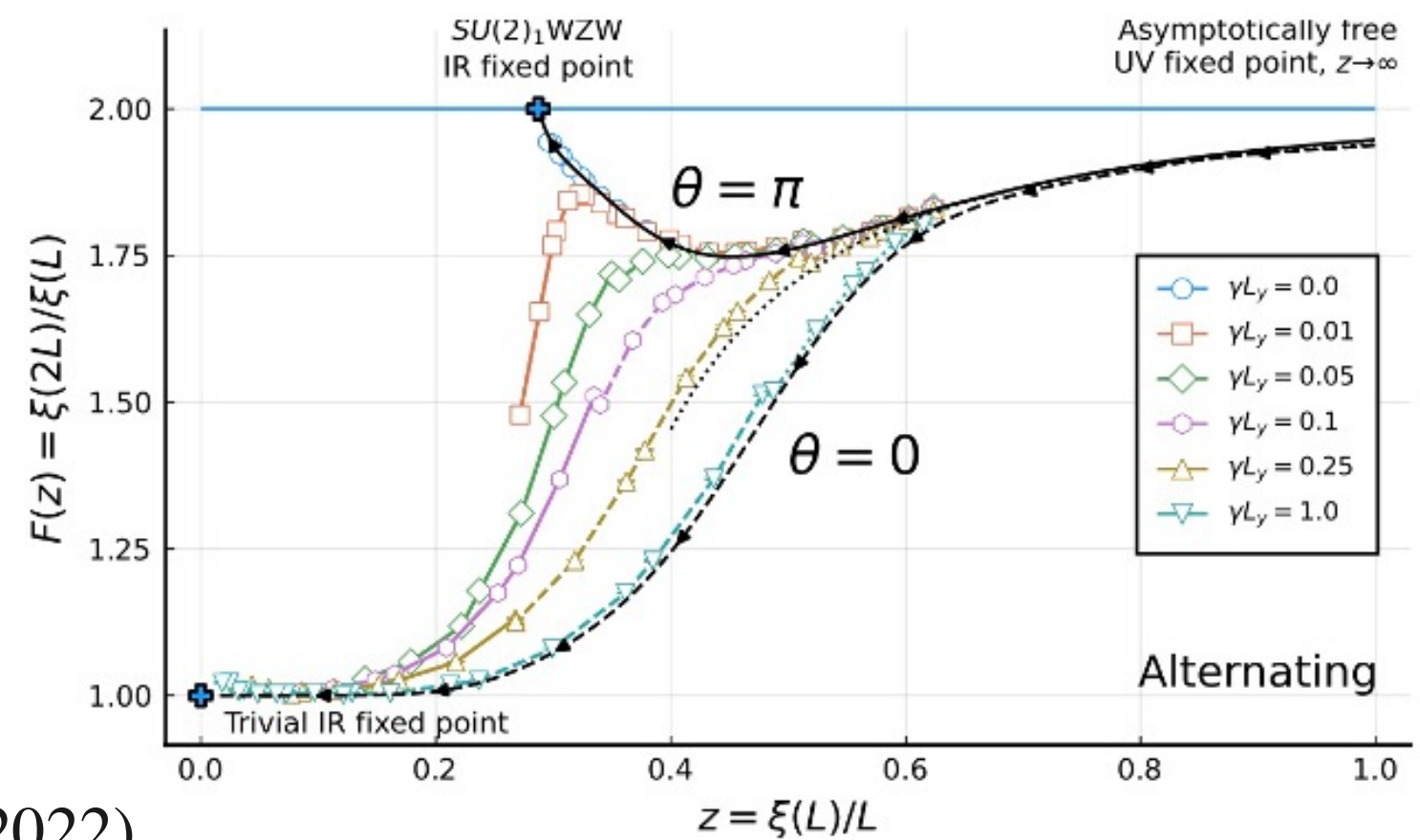
APS/Alan Stonebraker

$$S = \frac{1}{2g} \int dt dx \partial_\mu \vec{\phi}(x, t) \cdot \partial^\mu \vec{\phi}(x, t)$$

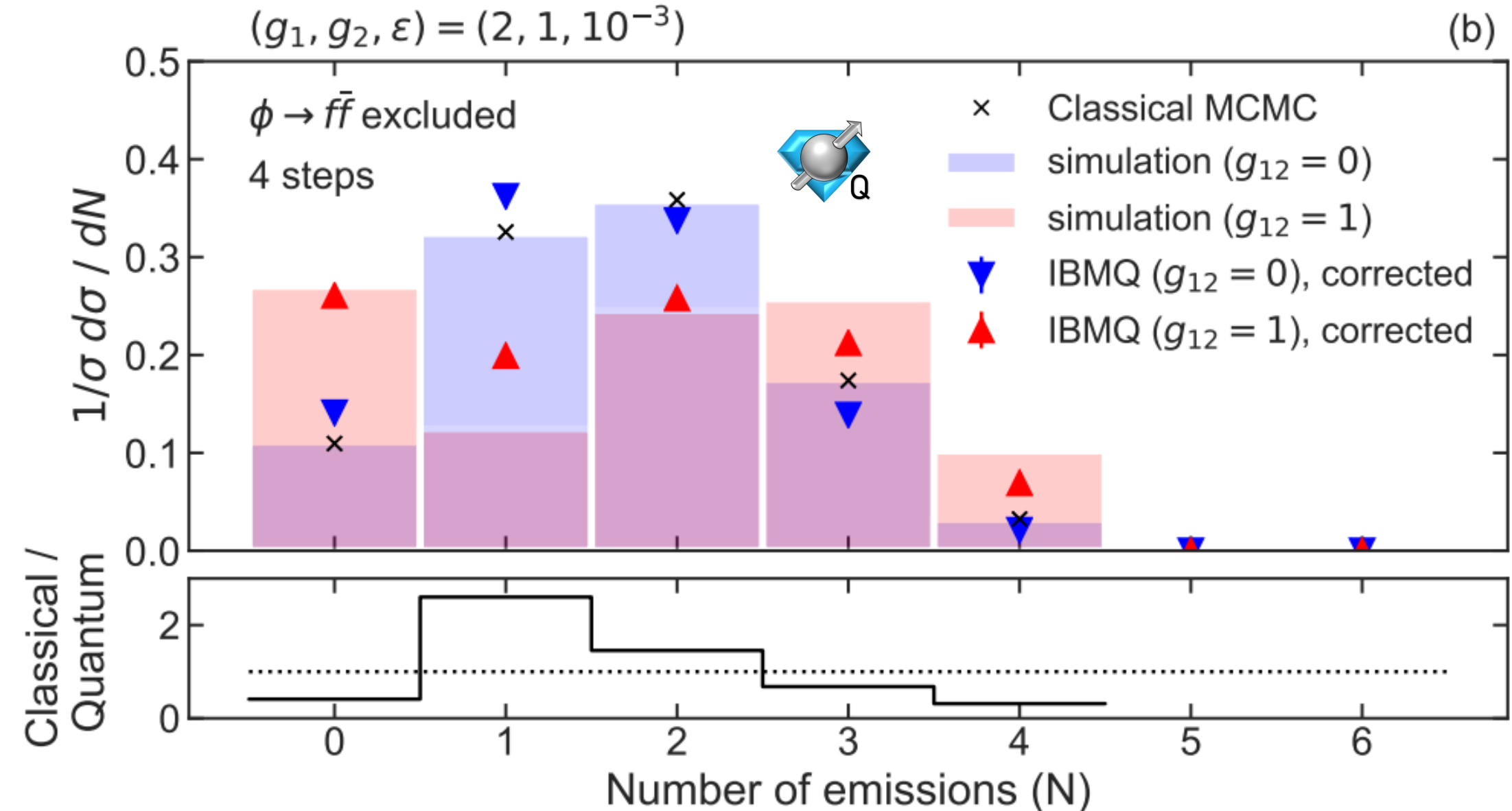
$$\hat{H}^D = J_x \sum_{x,y} \vec{S}_{x,y} \cdot \vec{S}_{x+1,y} + J_y \sum_{x,y} \vec{S}_{x,y} \cdot \vec{S}_{x,y+1}$$

2+1D

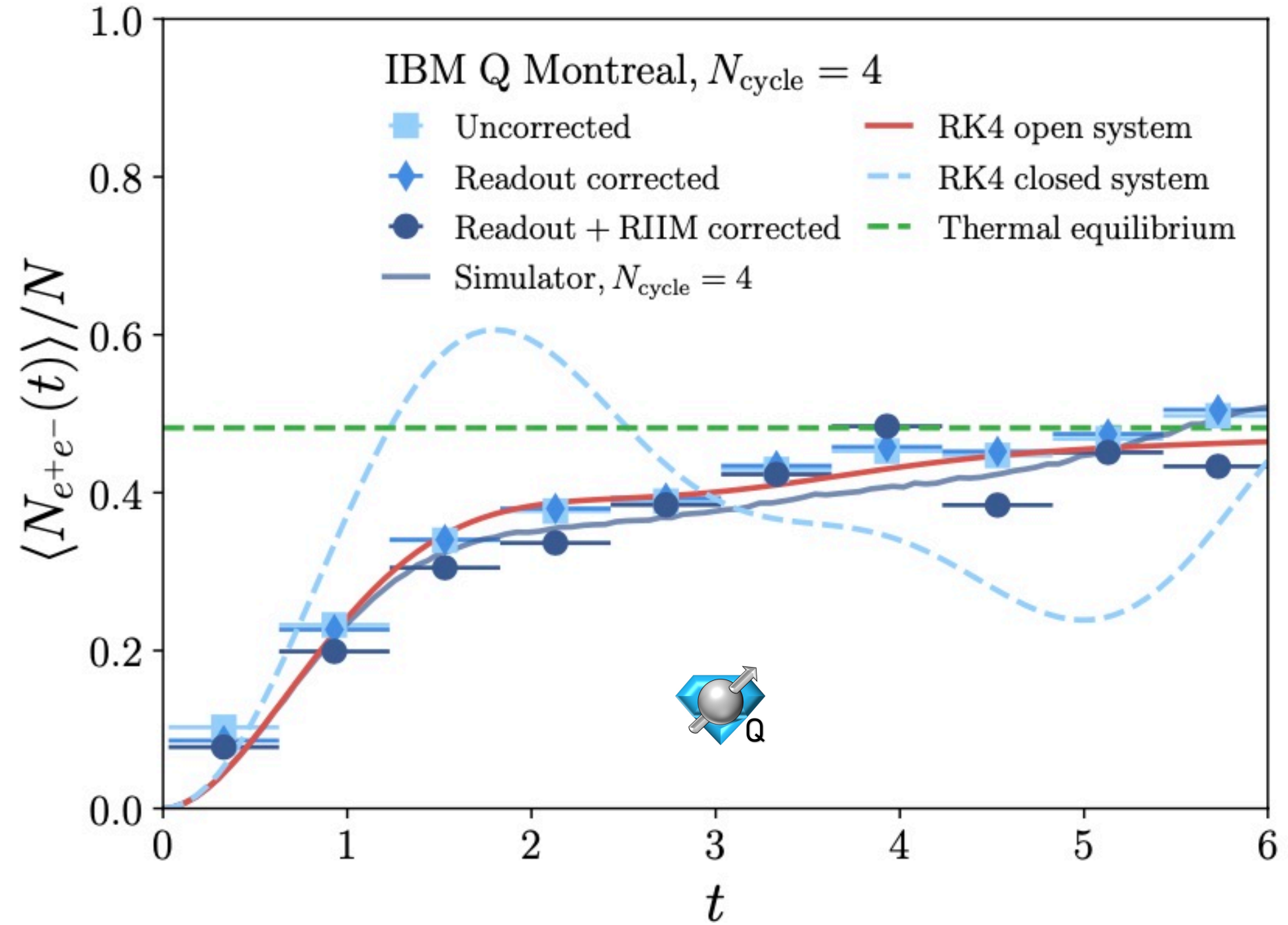
1+1D



Fragmentation and Collisions Vacuum and In-Medium



$$\mathcal{L} = \bar{f}_1(i\partial + m_1)f_1 + \bar{f}_2(i\partial + m_2)f_2 + (\partial_\mu\phi)^2 + g_1\bar{f}_1f_1\phi + g_2\bar{f}_2f_2\phi + g_{12}[\bar{f}_1f_2 + \bar{f}_2f_1]\phi.$$



Fragmentation

A quantum algorithm for high energy physics simulations

Christian W. Bauer, Wibe A. de Jong, Benjamin Nachman, Davide Provasoli, arXiv:1904.03196 [hep-ph]

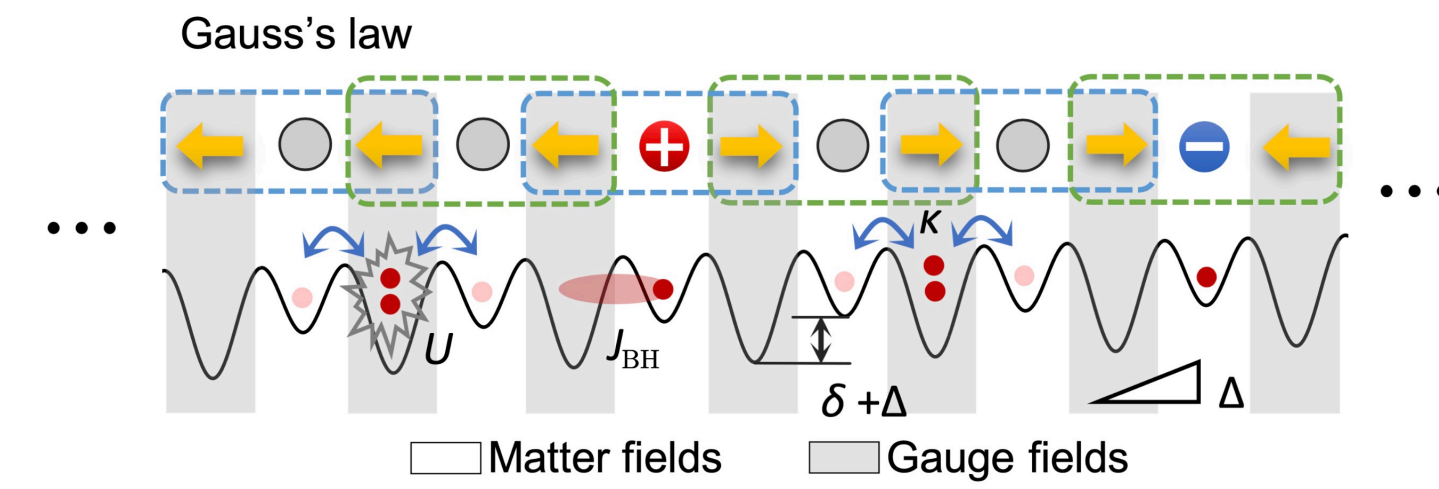
Simulating Collider Physics on Quantum Computers using Effective Field Theories

Christian W. Bauer, Benjamin Nachman, Marat Freytsis, arXiv:2102.05044 [hep-ph]

QQbar moving in medium

Quantum simulation of non-equilibrium dynamics and thermalization in the Schwinger model, Wibe A. de Jong, Kyle Lee, James Mulligan, Mateusz Płoskoń, Felix Ringer et al. e-Print: [2106.08394](https://arxiv.org/abs/2106.08394) [quant-ph]

Preserving Gauge Invariance



Stabilizing Gauge Theories in Quantum Simulators: A Brief Review

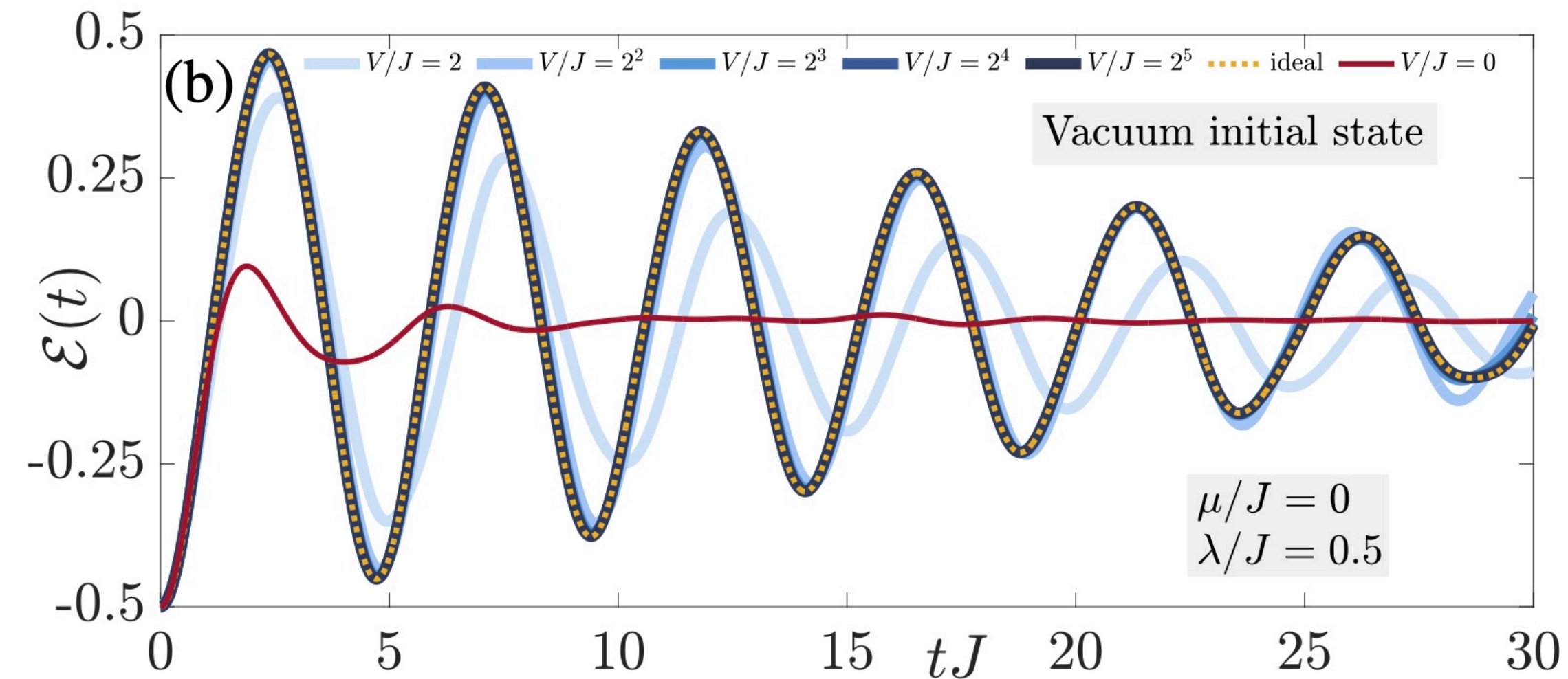
Invited Contribution to Proceedings of the Quantum Simulation for Strong Interactions

(QuaSi) Workshops 2021 [1] at the InQubator for Quantum Simulation (IQUS)

Jad C. Halimeh^{1,2,*} and Philipp Hauke^{3,4,†}

$$\hat{H} = \hat{H}_0 + \lambda \hat{H}_1 + V \hat{H}_{\text{quad}}$$

$$V \hat{H}_{\text{quad}} = V \sum_{j=1}^L (\hat{G}_j - g_j^{\text{tar}})^2,$$



Scar States in Gauge Theories and Delayed Thermalization

March 2022

Scar States in Deconfined \mathbb{Z}_2 Lattice Gauge Theories

Adith Sai Aramthottil,¹ Utso Bhattacharya,² Daniel González-Cuadra,^{2,3,4}
Maciej Lewenstein,^{2,5} Luca Barbiero,^{6,2} and Jakub Zakrzewski^{1,7}

- Anomalously-low bi-partite entanglement
- Distributed throughout spectrum
- Weakly connected to evolution Hamiltonian (cold sub-space)
- Delay thermalization

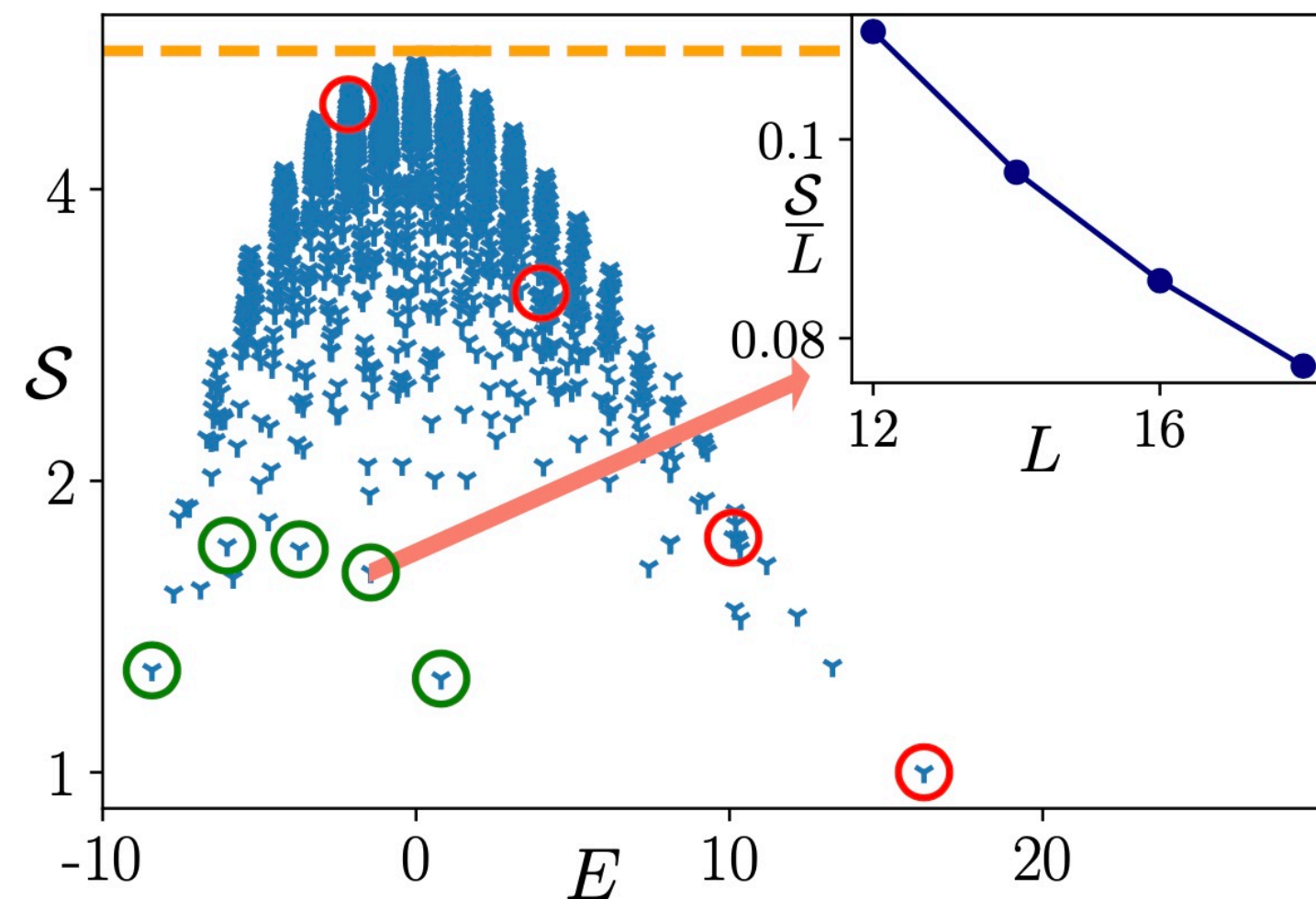


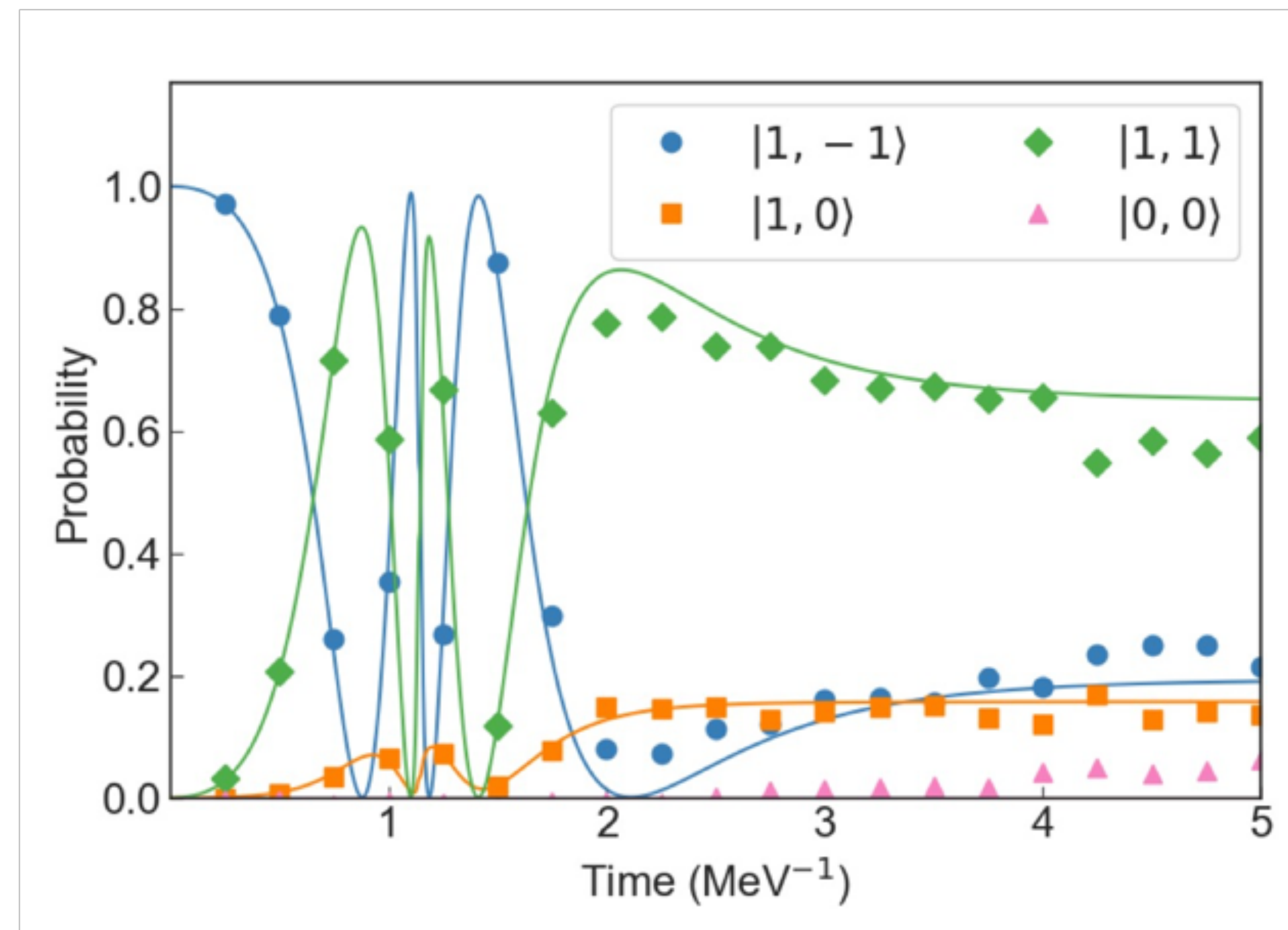
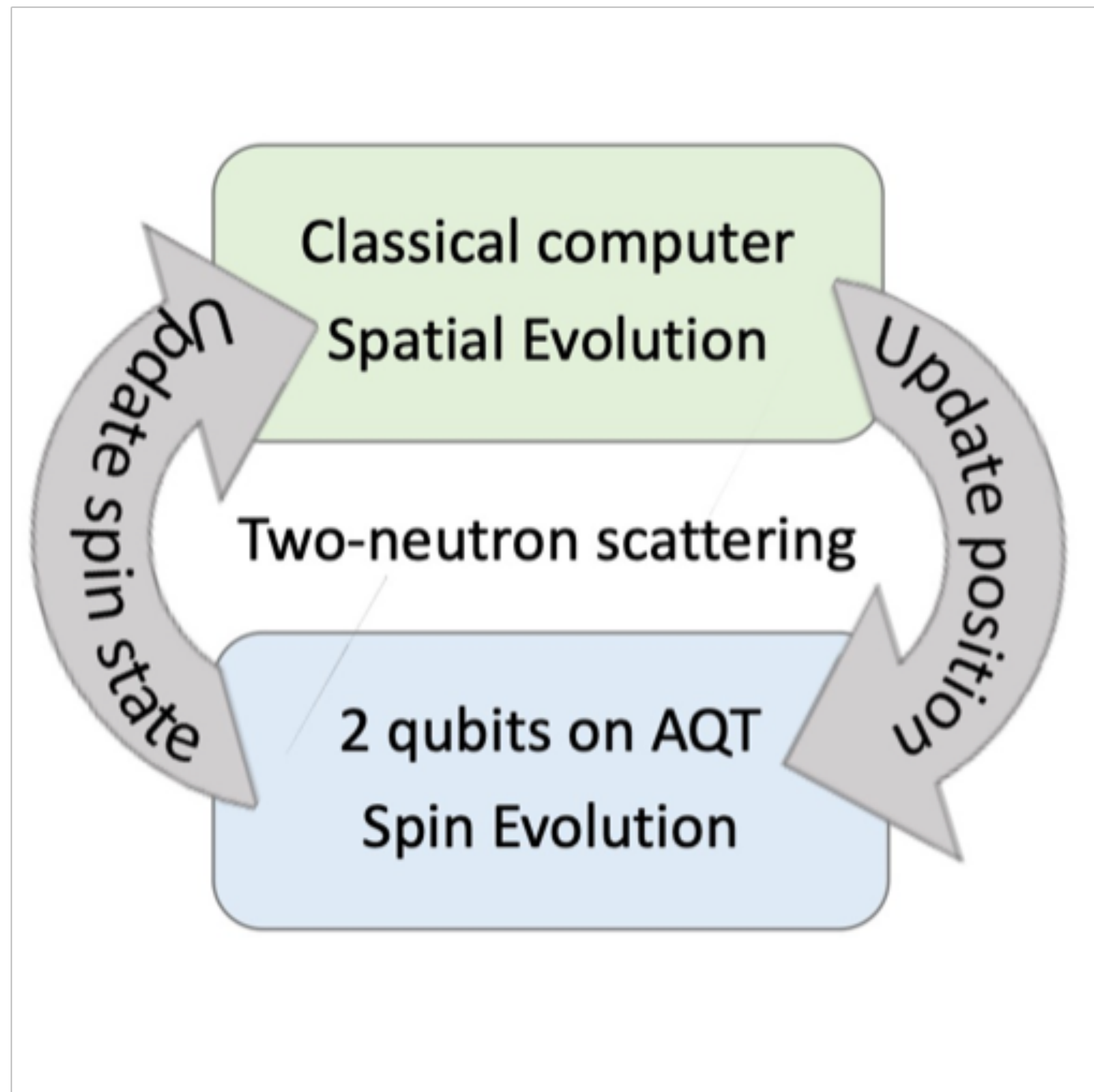
FIG. 4. The half-chain entanglement entropy(\mathcal{S}) of all the eigenstates at $t = 0.2$, $h = 0.5$ for $L = 16$. The orange dashed line gives the \mathcal{S}_{RMT} value. Circles denote different QMBS obtained via our tracking procedure. Green circles denote antimagnon-like family \mathcal{S}_n^2 for $n = 0, 2, 4, 6, 8$ while red circles magnon-like states, \mathcal{S}_n^1 with $n = 0, \dots, 6$ counting from the right hand side. *Inset:* The half-chain Entanglement Entropy divided by system size ($\frac{\mathcal{S}}{L}$) for \mathcal{S}_2^2 state showing its sub-volume property as expected for QMBS.

$$H = -t \sum_j \left(c_j^\dagger - c_j \right) \sigma_{j+1/2}^z \left(c_{j+1}^\dagger + c_{j+1} \right) - \mu \sum_j \left(c_j^\dagger c_j - \frac{1}{2} \right) - h \sum_j \sigma_{j+1/2}^x.$$

- Previously: only confining systems exhibited scars
- Shown to exist in de-confined regime
- Shown not to exist in confining regime

Neutron Scattering with Hybrid Quantum Simulation

LLNL+Trento



Hybrid Analogue-Digital using Trapped Ions

Toward simulating quantum field theories with controlled phonon-ion dynamics:
A hybrid analog-digital approach

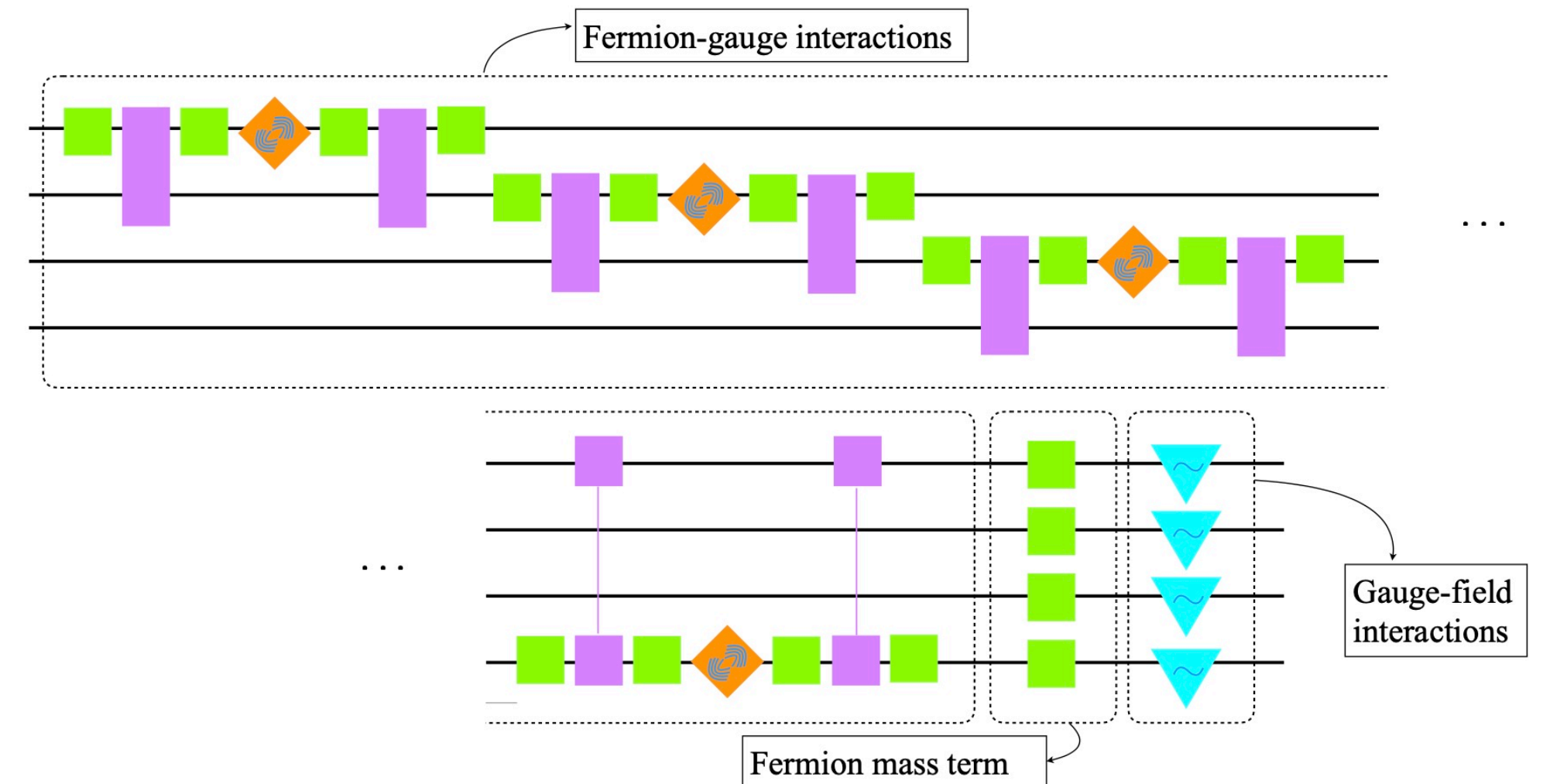
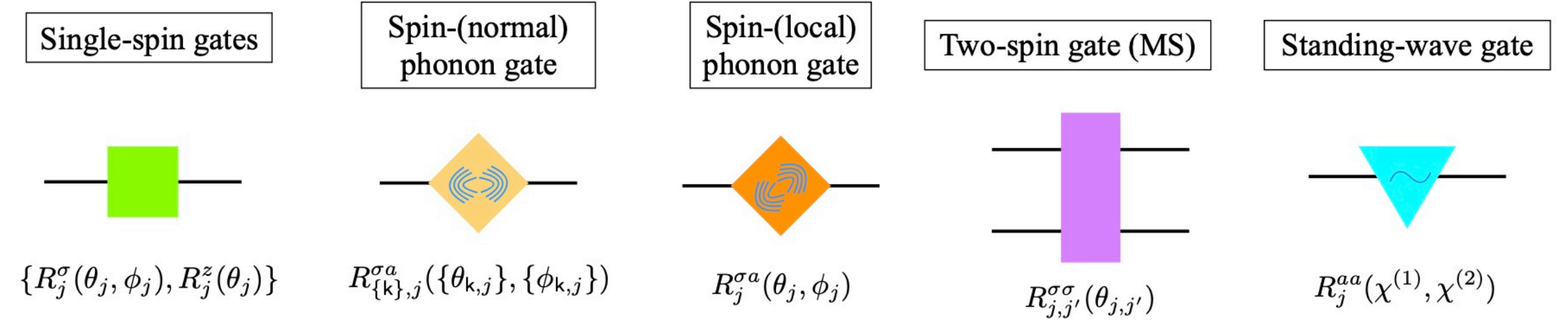
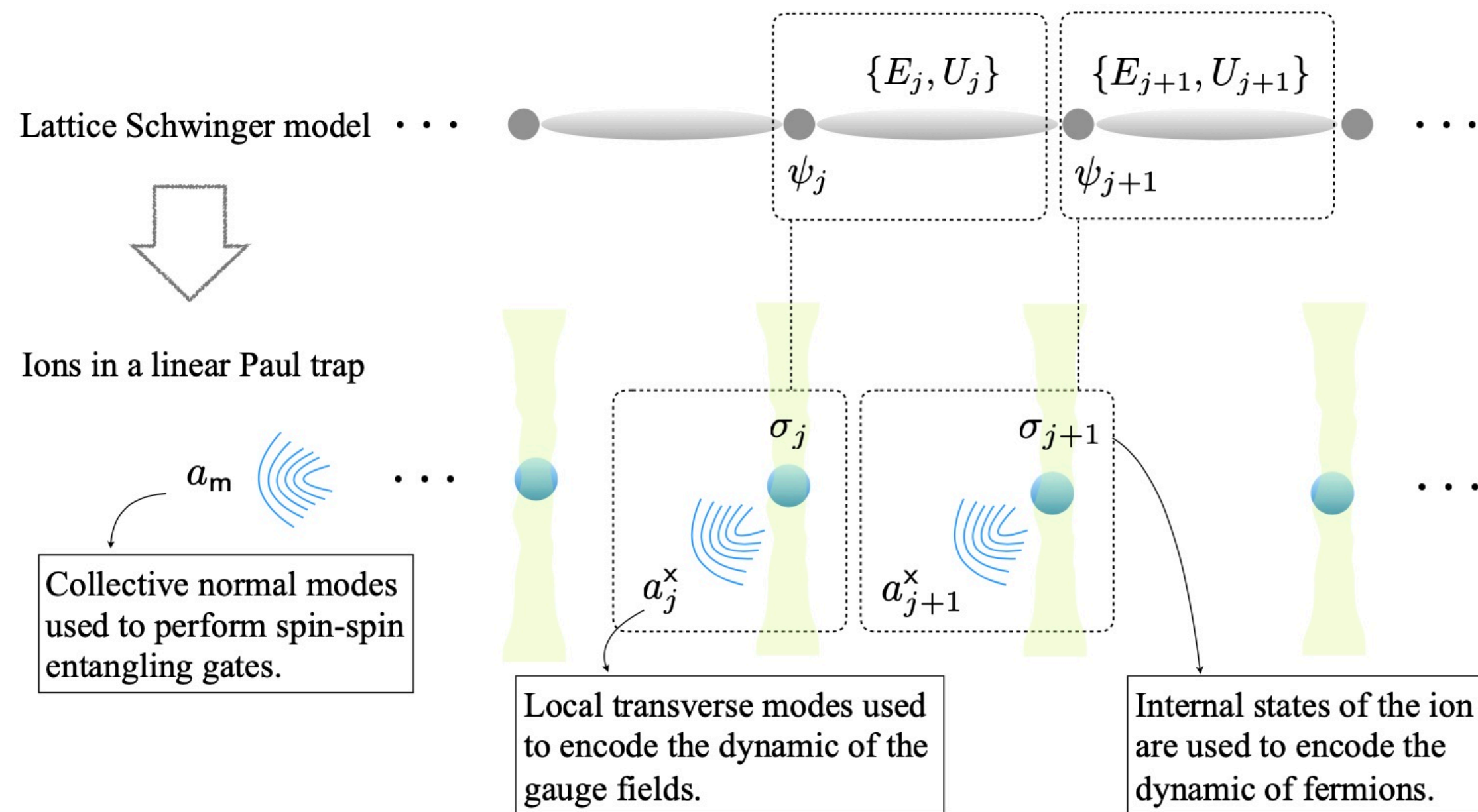
Zohreh Davoudi,^{1,*} Norbert M. Linke,² and Guido Pagano³

¹Maryland Center for Fundamental Physics and Department of Physics,
University of Maryland, College Park, MD 20742, USA.

²Joint Quantum Institute and Department of Physics,
University of Maryland, College Park, MD 20742

³Department of Physics and Astronomy, Rice University, 6100 Main Street, Houston, TX 77005, USA.

(Dated: April 20, 2021)



Examples of Co-Designing for Standard Model Physics

N-body Gates in Trapped Ion Systems

Engineering an Effective Three-spin Hamiltonian in Trapped-ion Systems for Applications in Quantum Simulation

N-body interactions between trapped ion qubits via spin-dependent squeezing

Or Katz,^{1,2,3,*} Marko Cetina,^{1,3} and Christopher Monroe^{1,2,3,4}

Bárbara Andrade,¹ Zohreh Davoudi,² Tobias Graß,¹ Mohammad Hafezi,^{3,4} Guido Pagano,⁵ and Alireza Seif^{6,*}

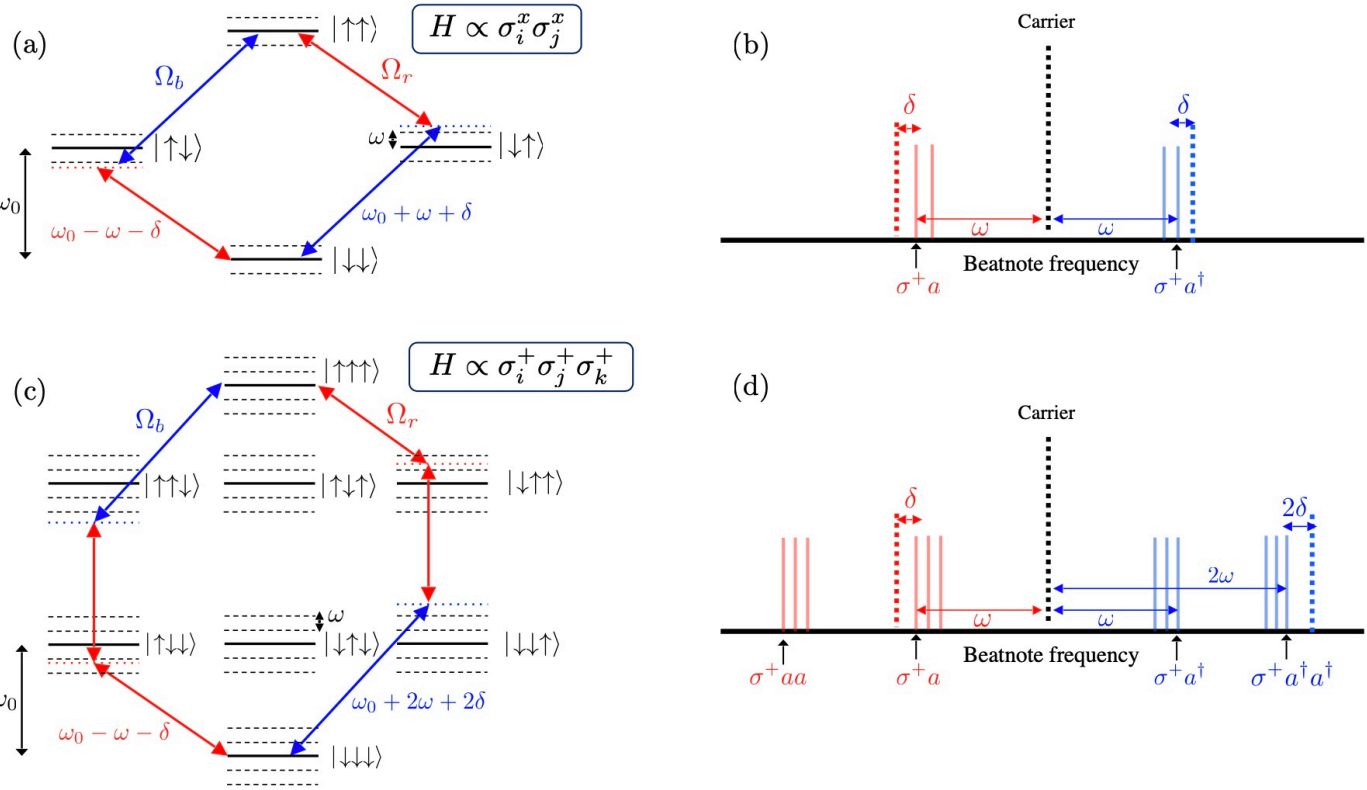
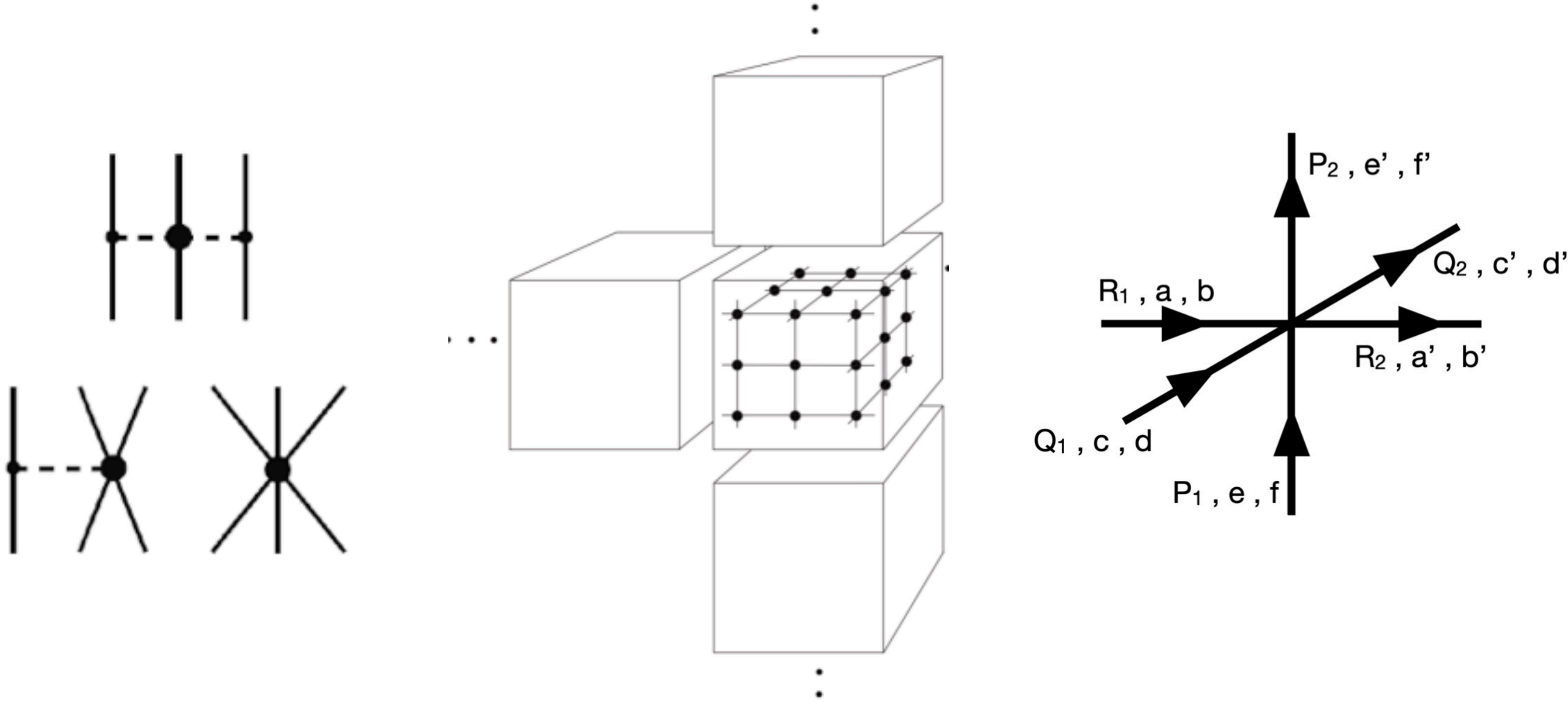
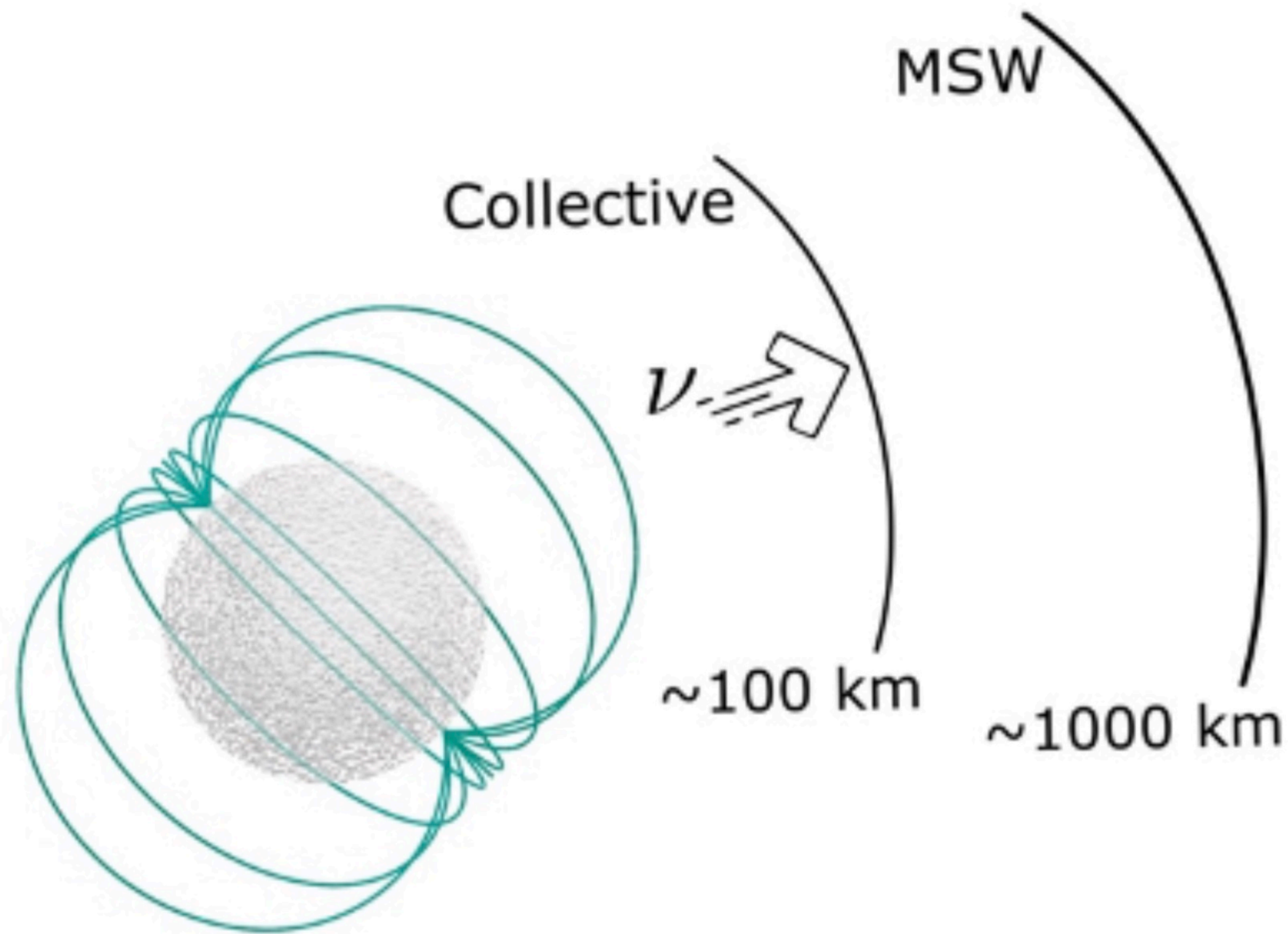


FIG. 1. (a,b) Traditional Mølmer-Sørensen scheme based on a pair of bichromatic laser beatnotes off-resonantly driving first-order spin-phonon couplings with symmetric detuning ($\pm\delta$), giving rise to an effective spin-spin interaction. The two-ion case is shown for simplicity. (c,d) Generalized Mølmer-Sørensen scheme to generate an effective three-spin coupling. A second-order blue sideband is driven with twice the detuning (2δ) as the first-order red ($-\delta$) sideband. As shown in (c), this process creates two virtual phonons with a second-order process and annihilates the same number of phonons through two first-order processes. Note that only two out of several possibilities are depicted. In all subfigures, Ω_r and Ω_b are the Rabi frequencies of the red and blue beatnotes, respectively. ω_0 is the qubit frequency, and $\omega [\equiv \omega_{\text{com}}]$ is the transverse center-of-mass frequency.

Neutrino Flavor Dynamics in Supernova



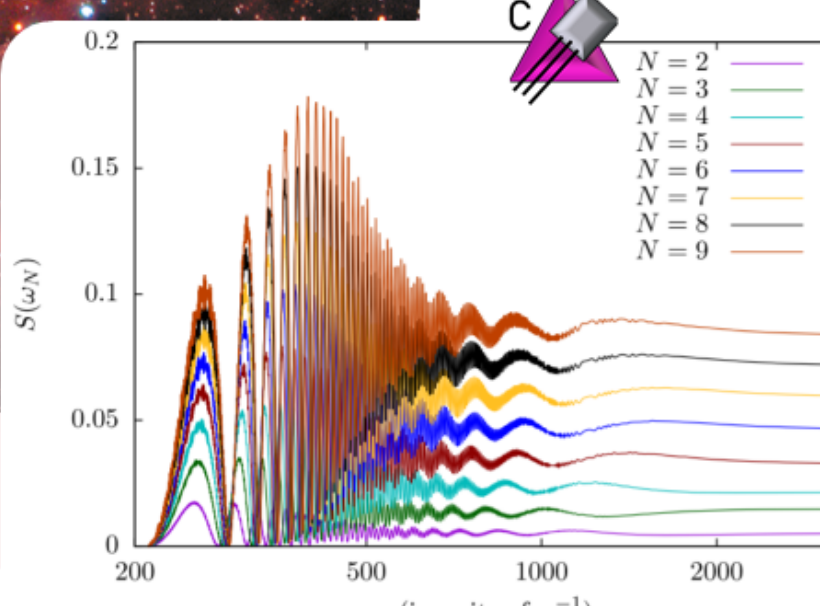
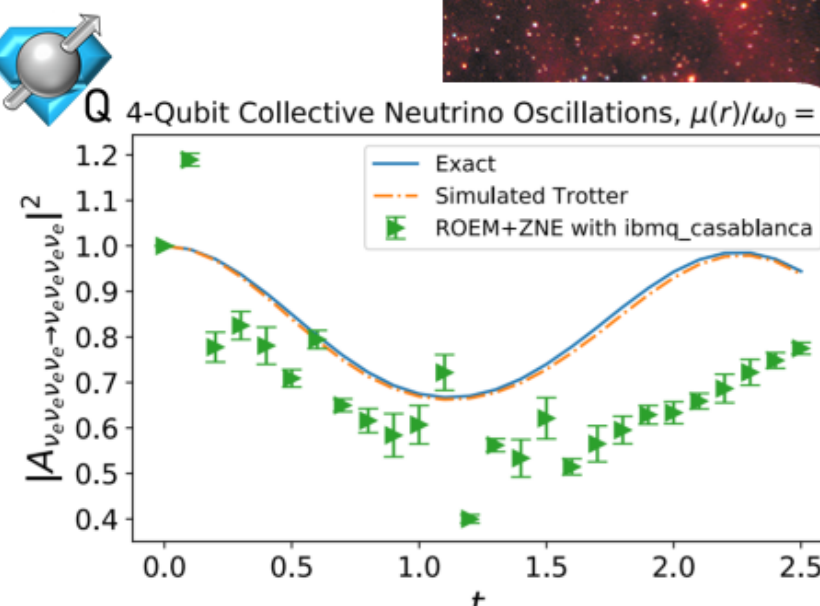
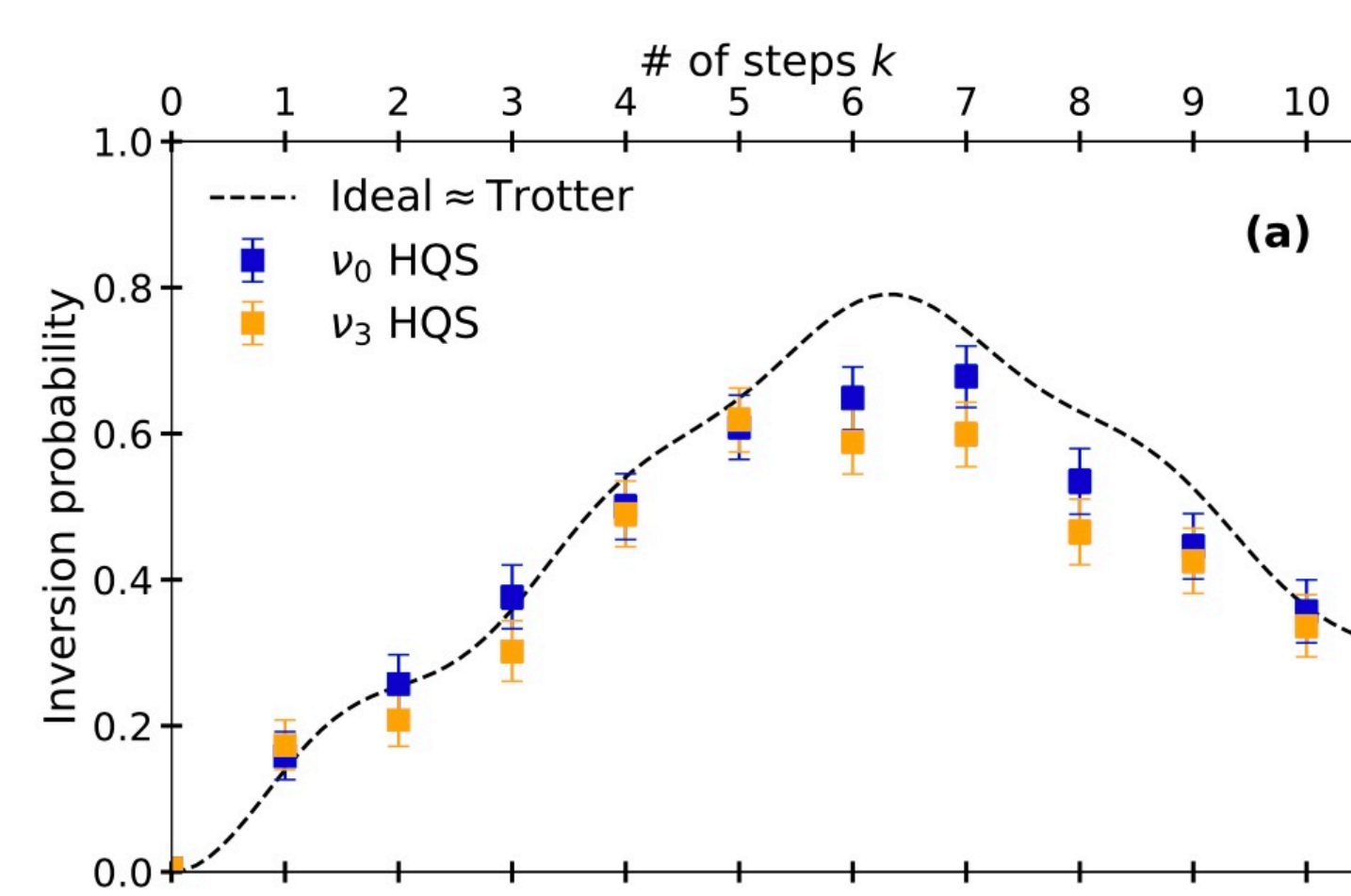
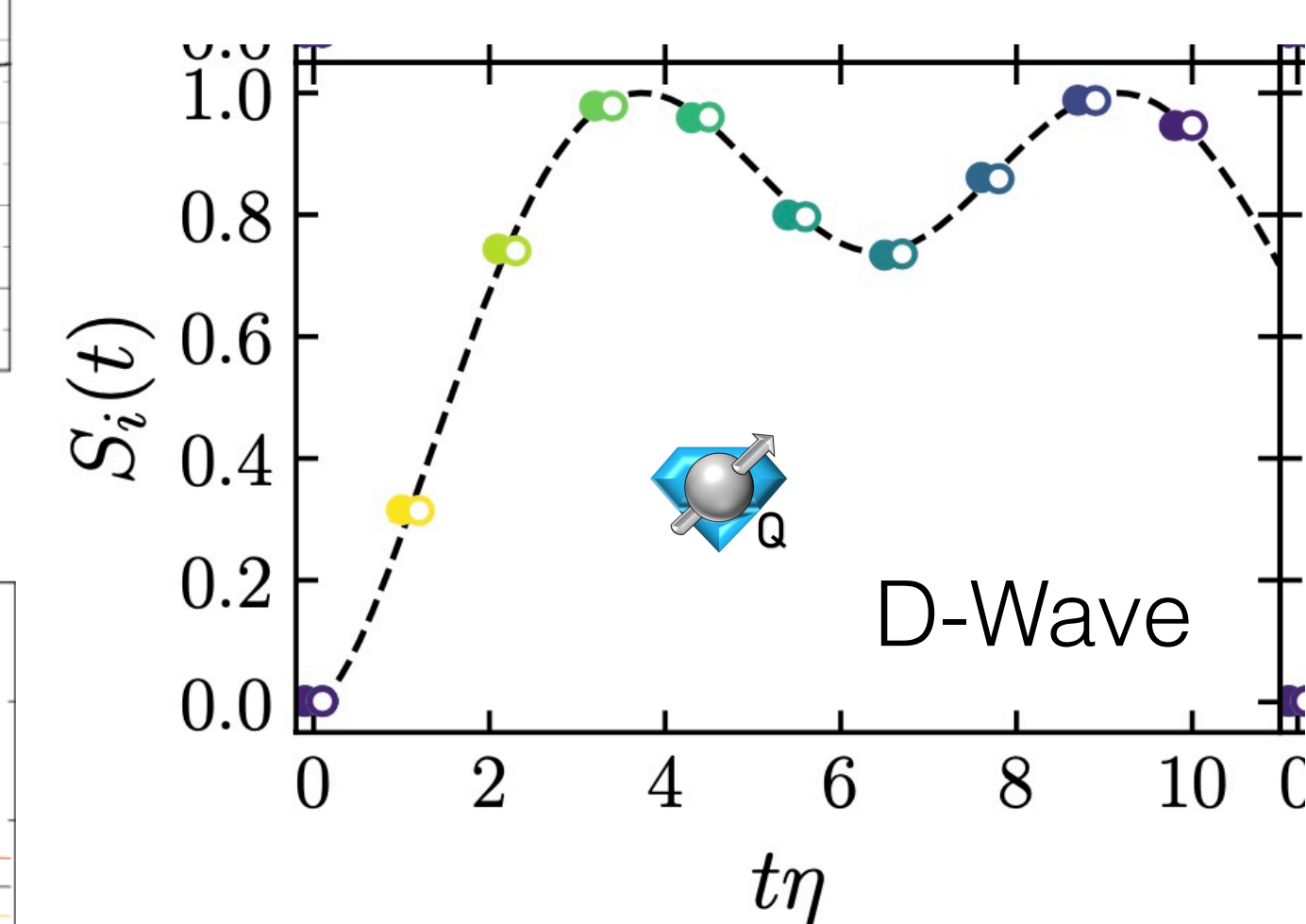
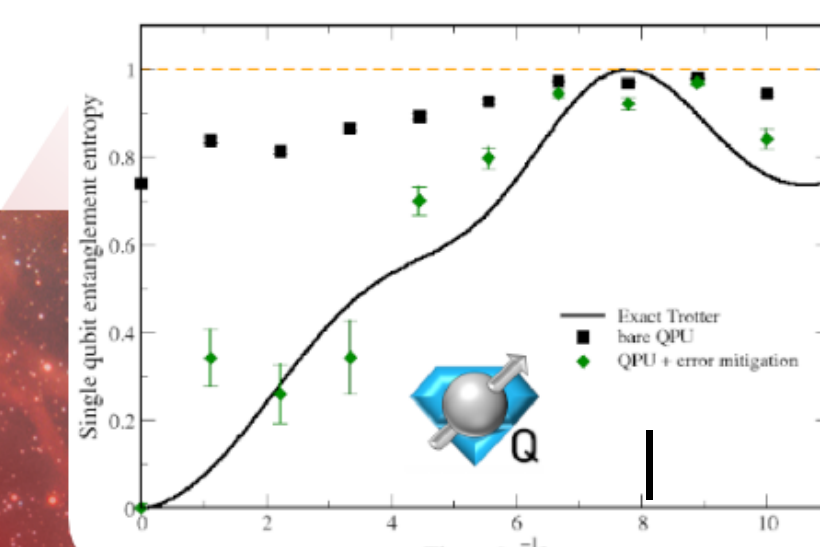
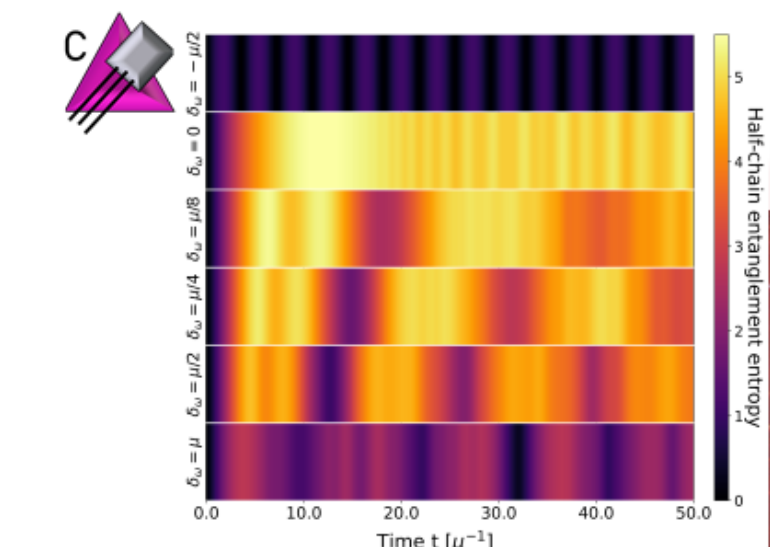
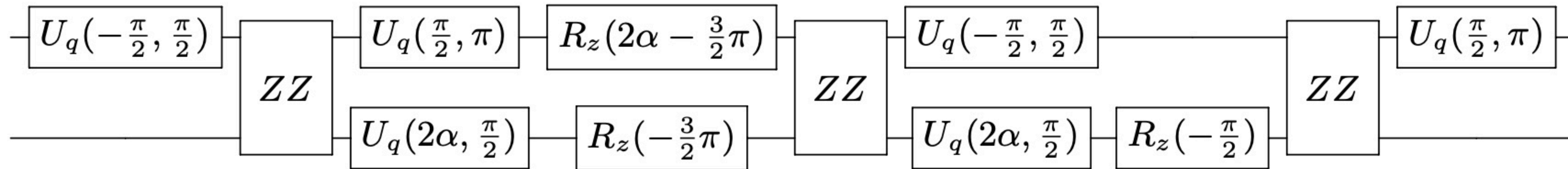
$$|\Psi\rangle = \alpha |\nu_e\rangle + \beta |\nu_x\rangle$$

$$\begin{aligned} H &= H^{(1)} + H^{(2)} = \sum_{i=0}^{N-1} h_i + \sum_{i<j}^{N-1} h_{ij} \\ &= \sum_{i=0}^{N-1} \mathbf{b} \cdot \boldsymbol{\sigma}_i + \sum_{i<j}^{N-1} J_{ij} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, \end{aligned}$$

Coherent Neutrino Systems



Marc Illa



18 CNOTS per step
Quantinuum H1-2

Trapped-Ion Quantum Simulation of Collective Neutrino Oscillations

Valentina Amitrano,^{1,2} Alessandro Roggero,^{1,2} Piero Luchi,^{1,2} Francesco Turro,^{1,2} Luca Vespucchi,^{1,3} and Francesco Pederiva^{1,2}

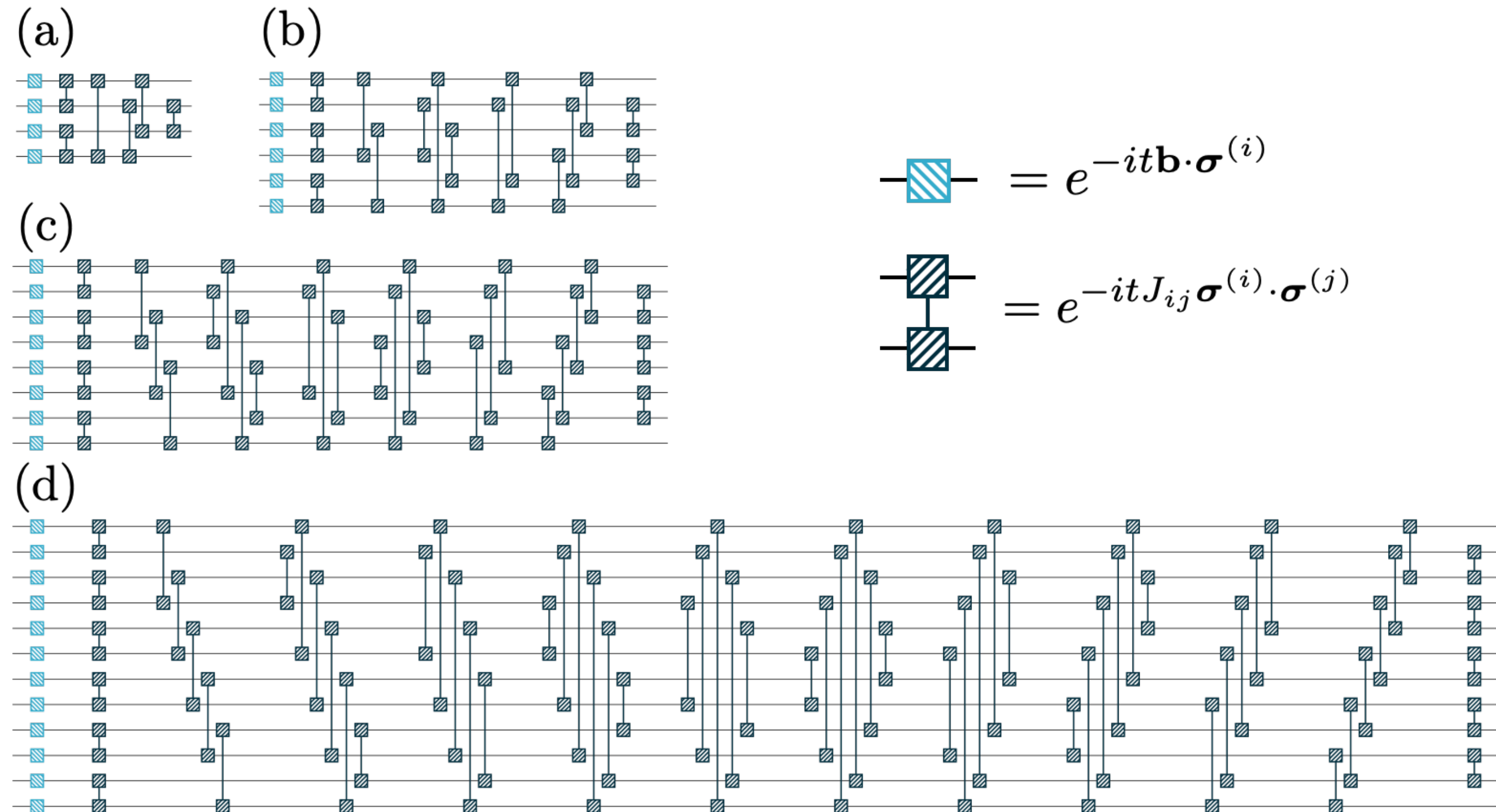
A number of independent teams are pursuing these systems

Coherent Neutrino Systems

$$H = H^\nu + H^{\nu\nu} = \sum_i \mathbf{b} \cdot \boldsymbol{\sigma}^{(i)} + \frac{1}{N} \sum_{i < j} J_{ij} \boldsymbol{\sigma}^{(i)} \cdot \boldsymbol{\sigma}^{(j)}$$

All-to-all connectivity ideal

$$|\Psi_0\rangle = |\nu_e\rangle^{\otimes N/2} \otimes |\nu_x\rangle^{\otimes N/2}$$

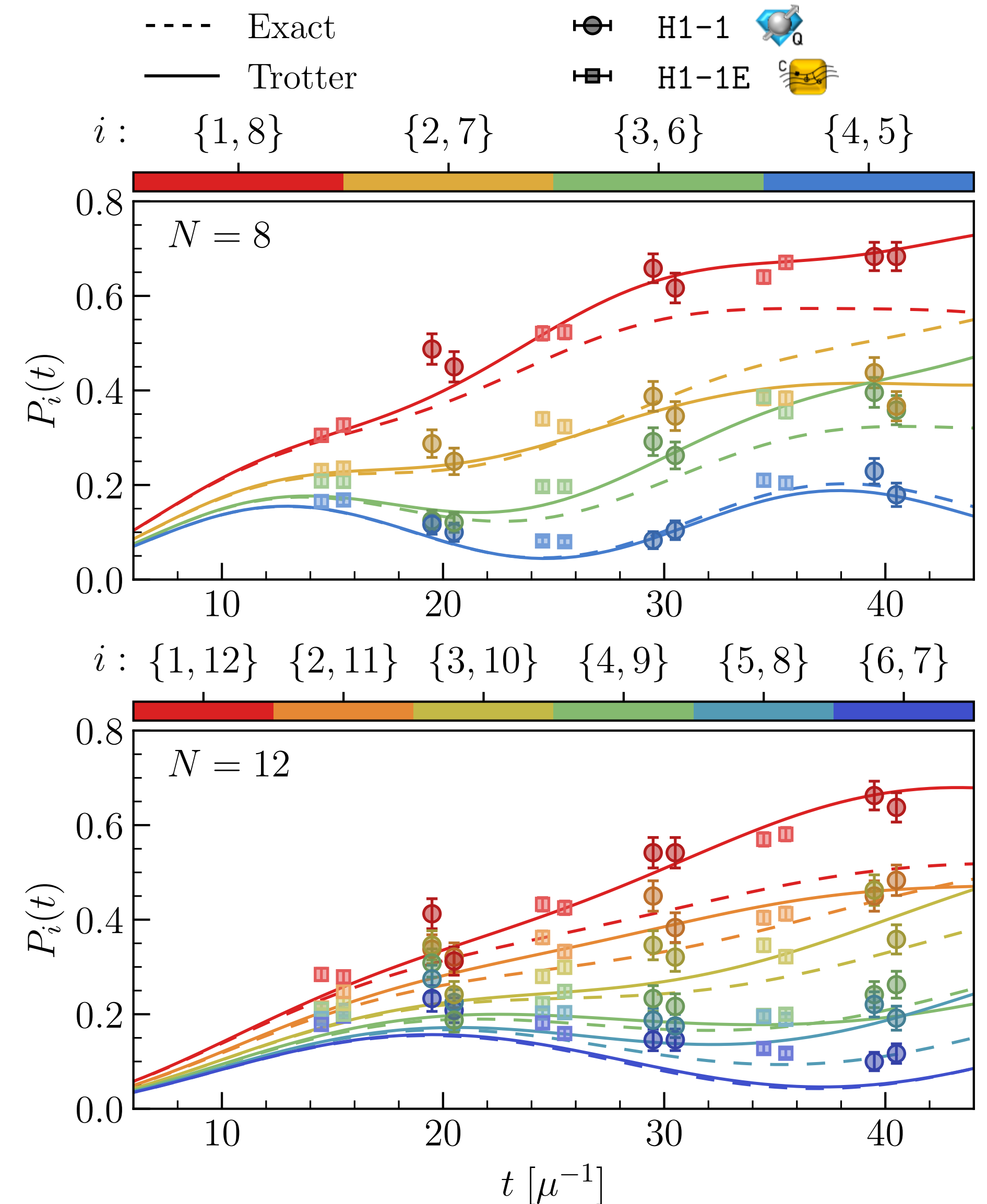


Coherent Neutrino Systems

Our simulations:
 Quantinuum's H1-1, H1-1E 20 qubit trapped ion
 quantum computer and emulator

N=4,6,8,12 neutrinos

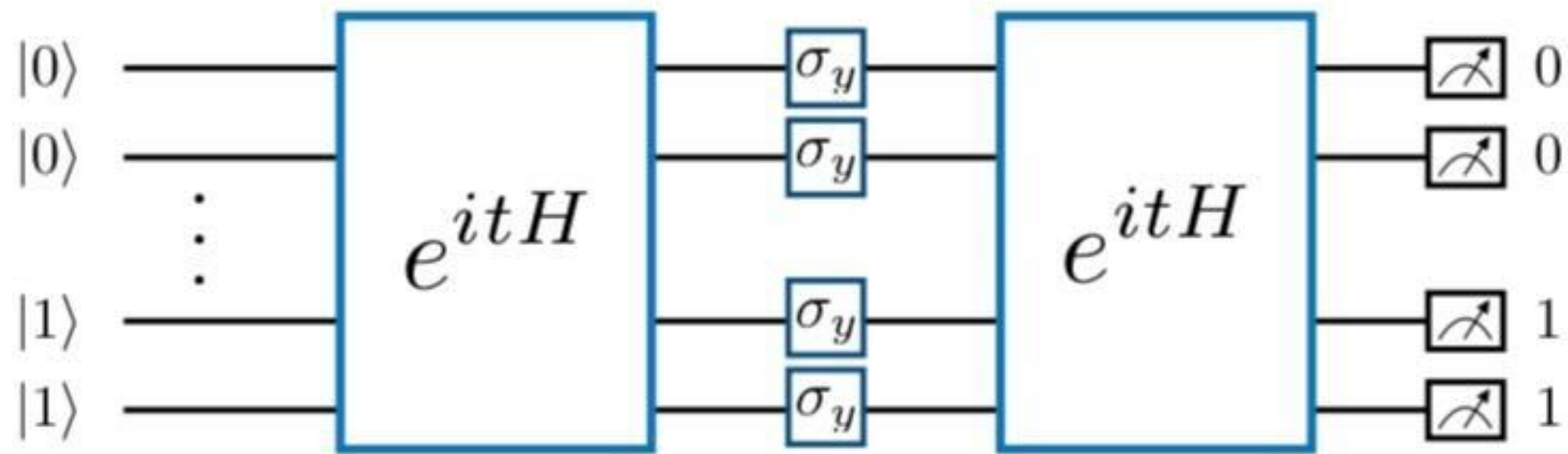
Time evolution
 compute state probabilities
 correlations
 n-tangles



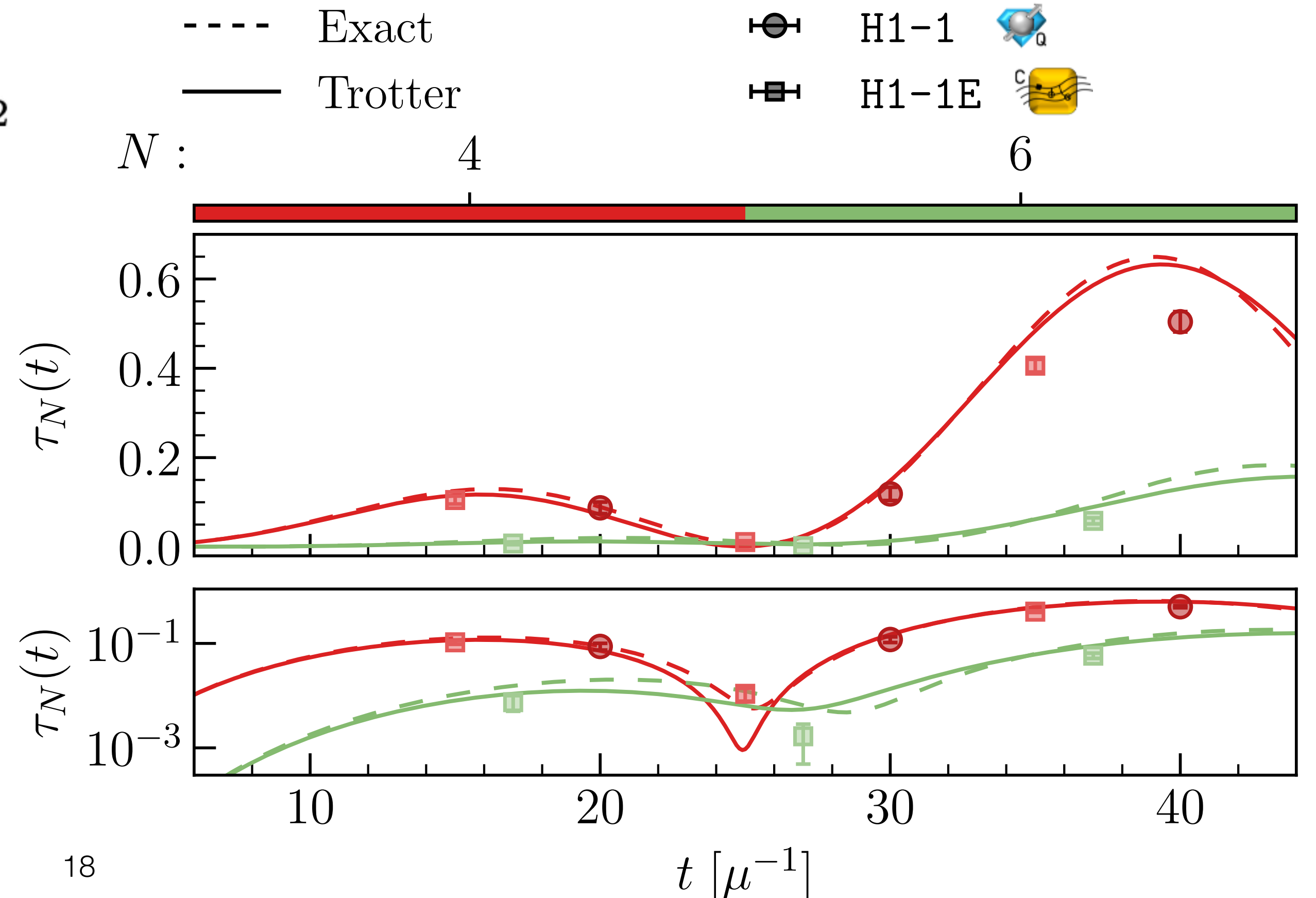
Multi-Neutrino Entanglement

e.g.,
$$|\Psi\rangle = \frac{1}{\sqrt{3}} [|11110\rangle + |00000\rangle + |10101\rangle]$$

$$\tau_N(t) = |\langle \Psi_t | \sigma_y^{\otimes N} | \Psi_t^* \rangle|^2 = |\langle \Psi_0 | e^{itH} \sigma_y^{\otimes N} e^{itH} | \Psi_0 \rangle|^2$$



Tractable by tensor-product initial state

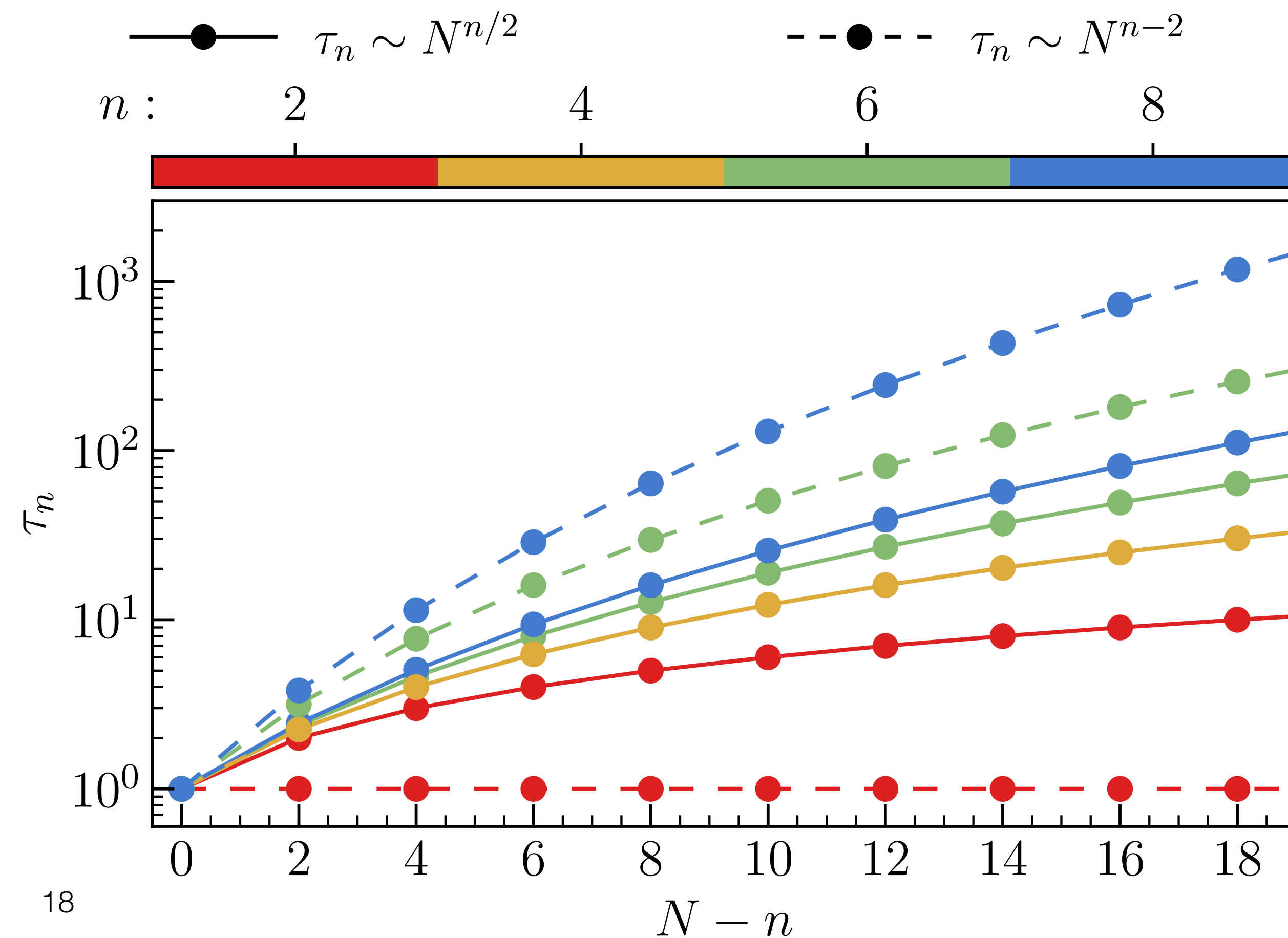
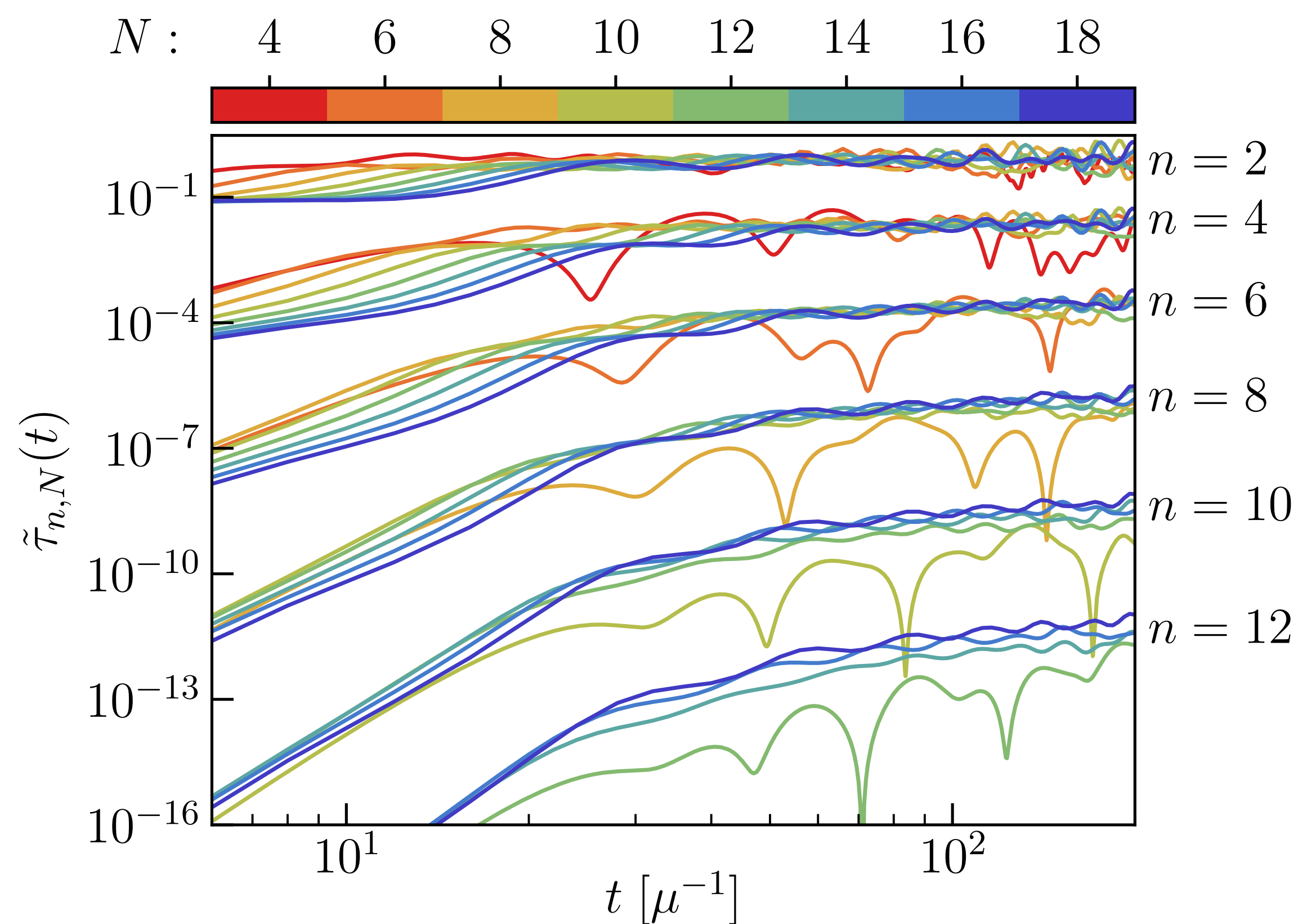


Multi-Neutrino Entanglement

$$\tilde{\tau}_{n,N} = \frac{1}{N^{n-2}} \sum_i \tau_n^{(i)}$$

$$\tau_n = \binom{N/2}{N/2-n/2} \sim N^{n/2}$$

Bell pairs only

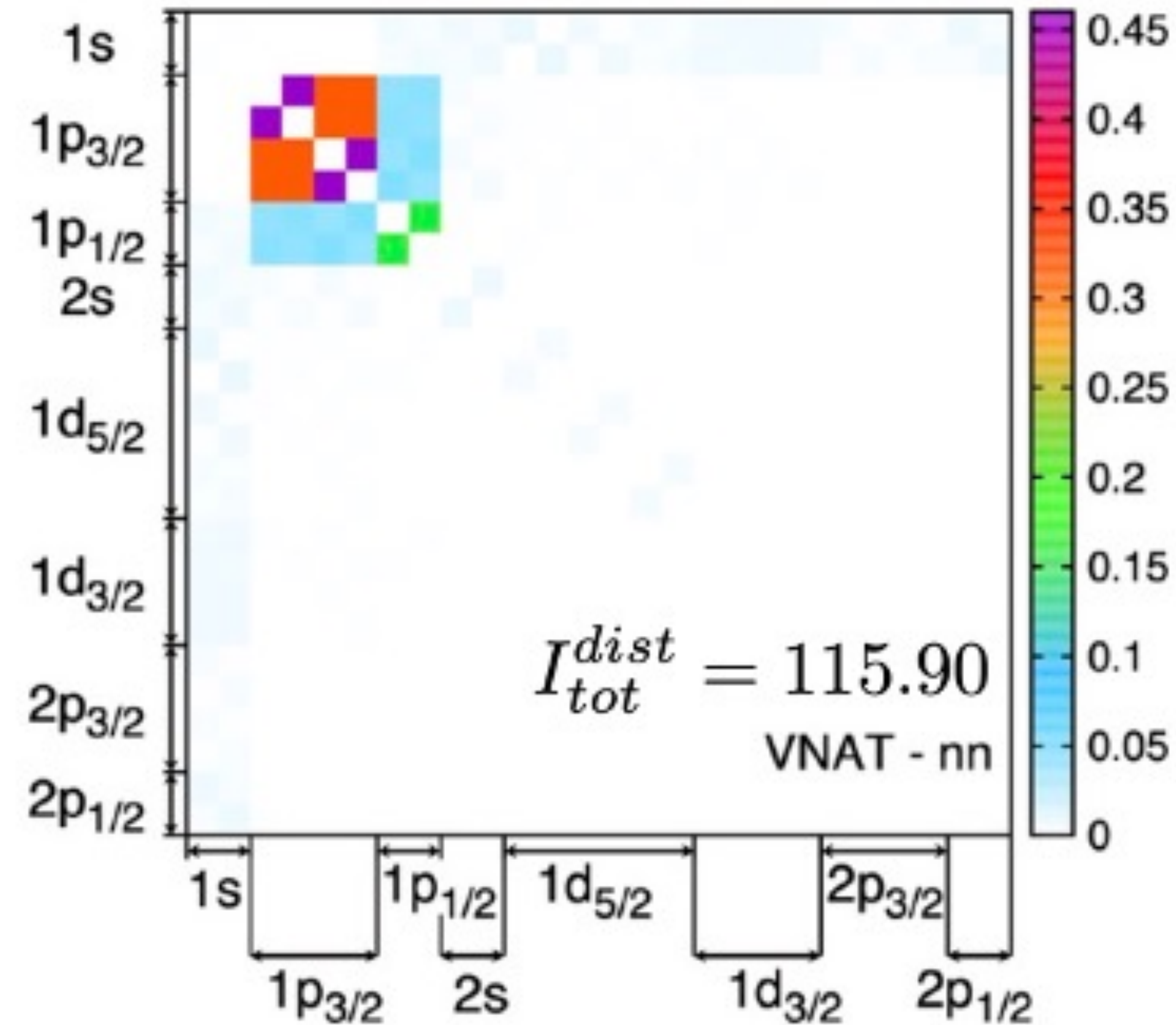
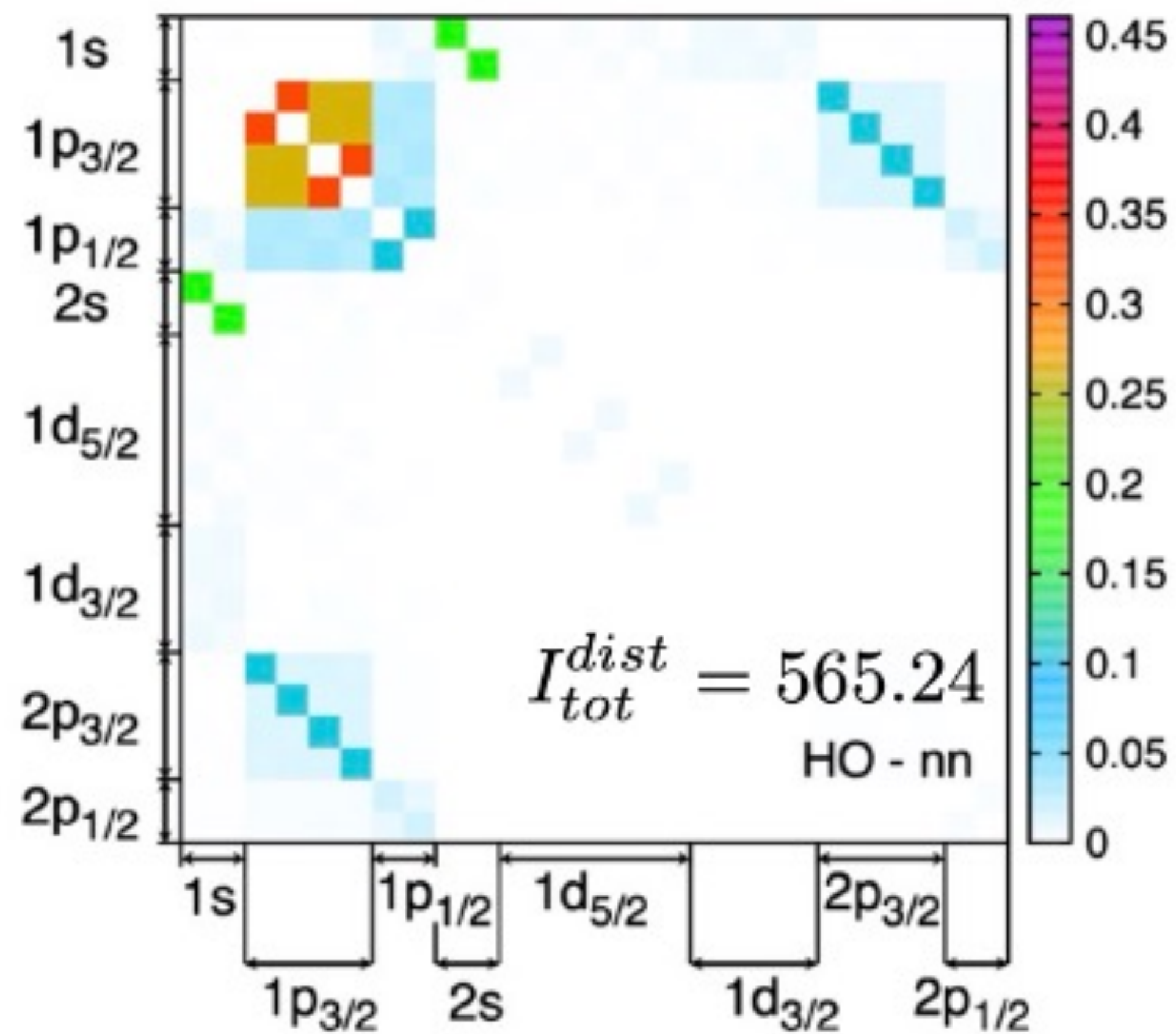


Entanglement Rearrangement and Hamiltonian Learning in Nuclei and Spin Systems

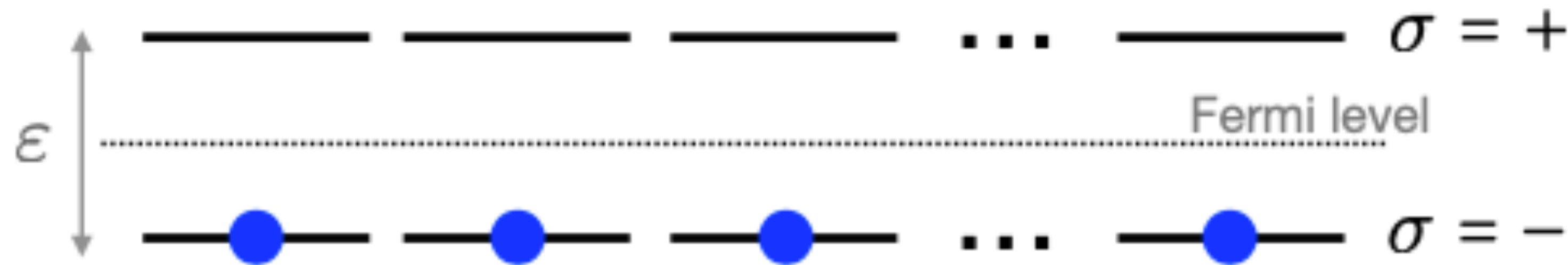


Caroline Robin

Entanglement re-arrangement
Variational natural orbitals



Lipkin-Meshkov-Glick Model and Effective Model Spaces



N particles distributed on two N-fold degenerate levels

$$\begin{aligned}
 H &= \frac{\varepsilon}{2} \sum_{\sigma p} \sigma c_{p\sigma}^\dagger c_{p\sigma} - \frac{V}{2} \sum_{pq\sigma} c_{p\sigma}^\dagger c_{q\sigma}^\dagger c_{q-\sigma} c_{p-\sigma} \\
 &= \varepsilon J_z - \frac{V}{2} (J_+^2 + J_-^2)
 \end{aligned}$$

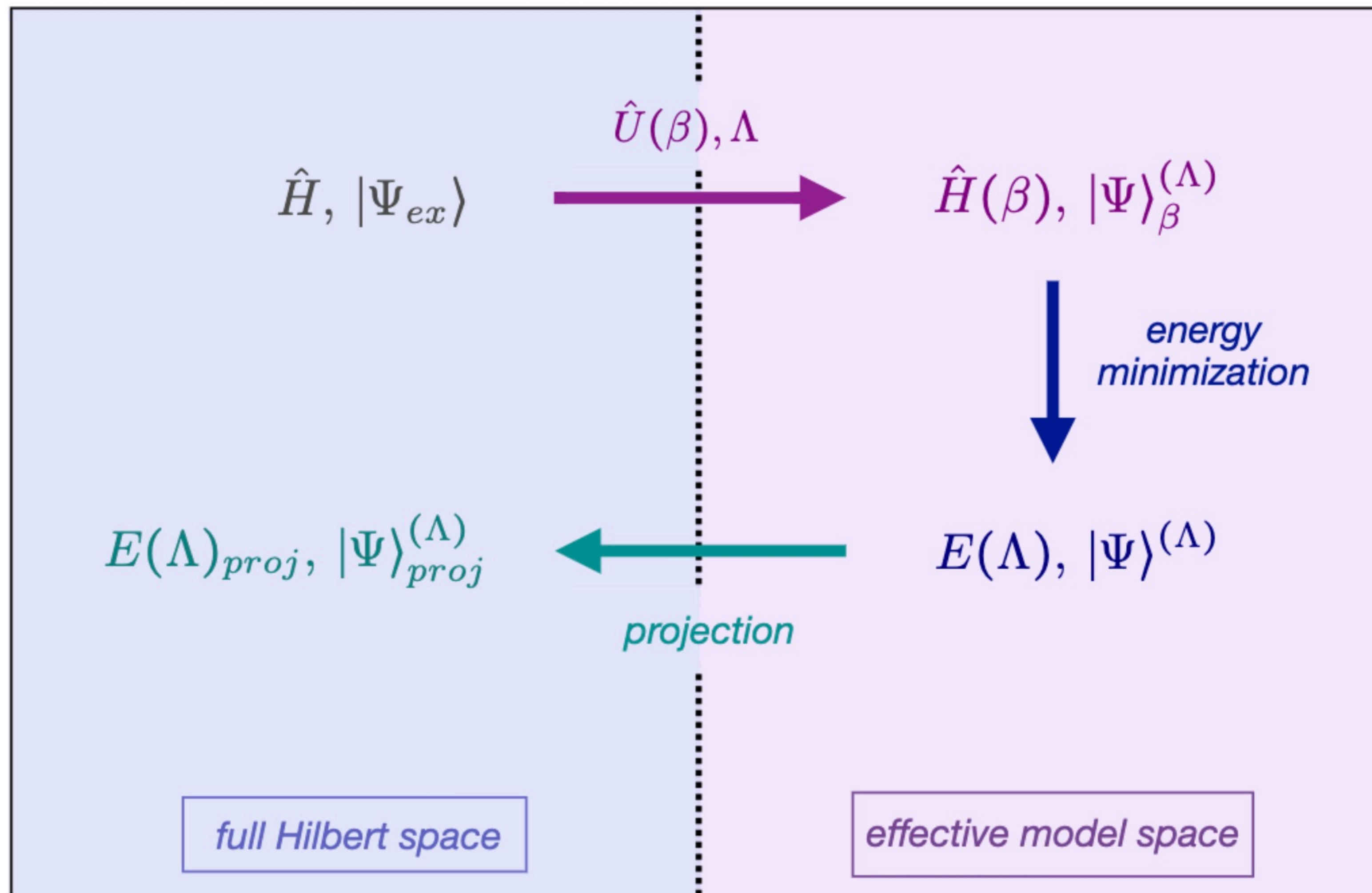
$$J_z = \frac{1}{2} \sum_{p\sigma} \sigma c_{p\sigma}^\dagger c_{p\sigma}$$

$$J_+ = \sum_p \sigma c_{p+}^\dagger c_{p-}, \quad J_- = (J_+)^{\dagger}$$

exact solutions: $|\Psi_{ex}^{(J)}\rangle = \sum_{M=-J}^J A_{J,M} |J, M\rangle \equiv \sum_{n=0}^{2J} A_n |n\rangle$

\swarrow
np-nh excitation

Effective Model Spaces



Effective Model Spaces

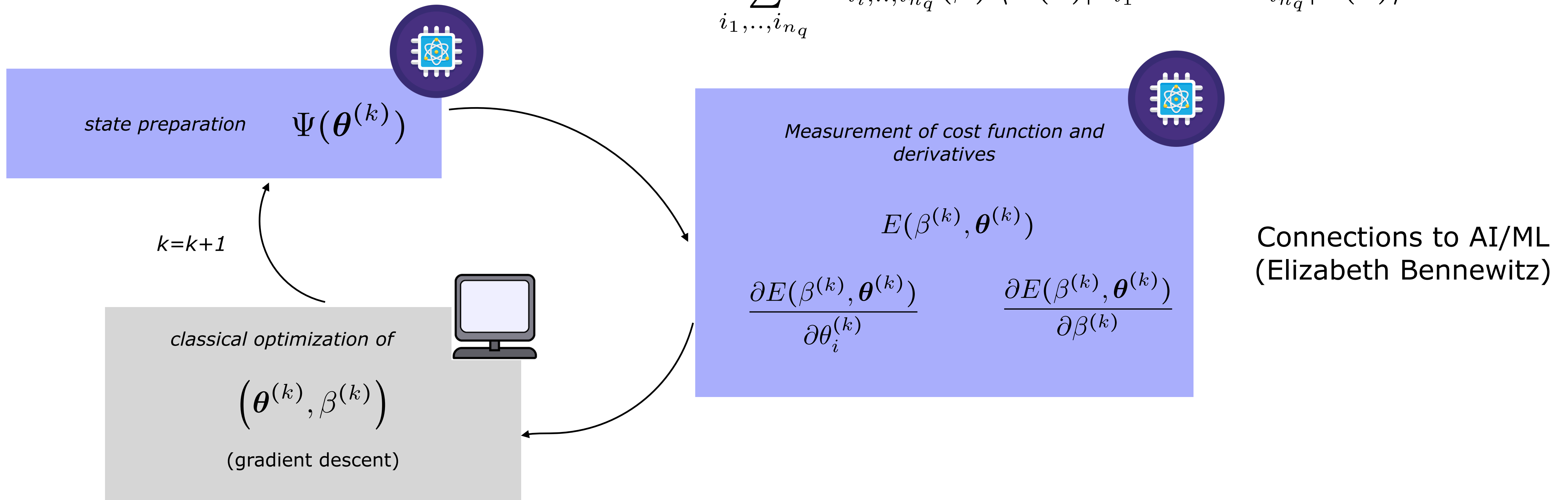
HL-VQE

★Hamiltonian-Learning-VQE:

$$\bar{\sigma} = \{\hat{I}, \hat{X}, \hat{Y}, \hat{Z}\}$$

Cost function to minimize: $E(\beta, \theta) = \langle \Psi(\theta) | \hat{H}(\beta) | \Psi(\theta) \rangle$

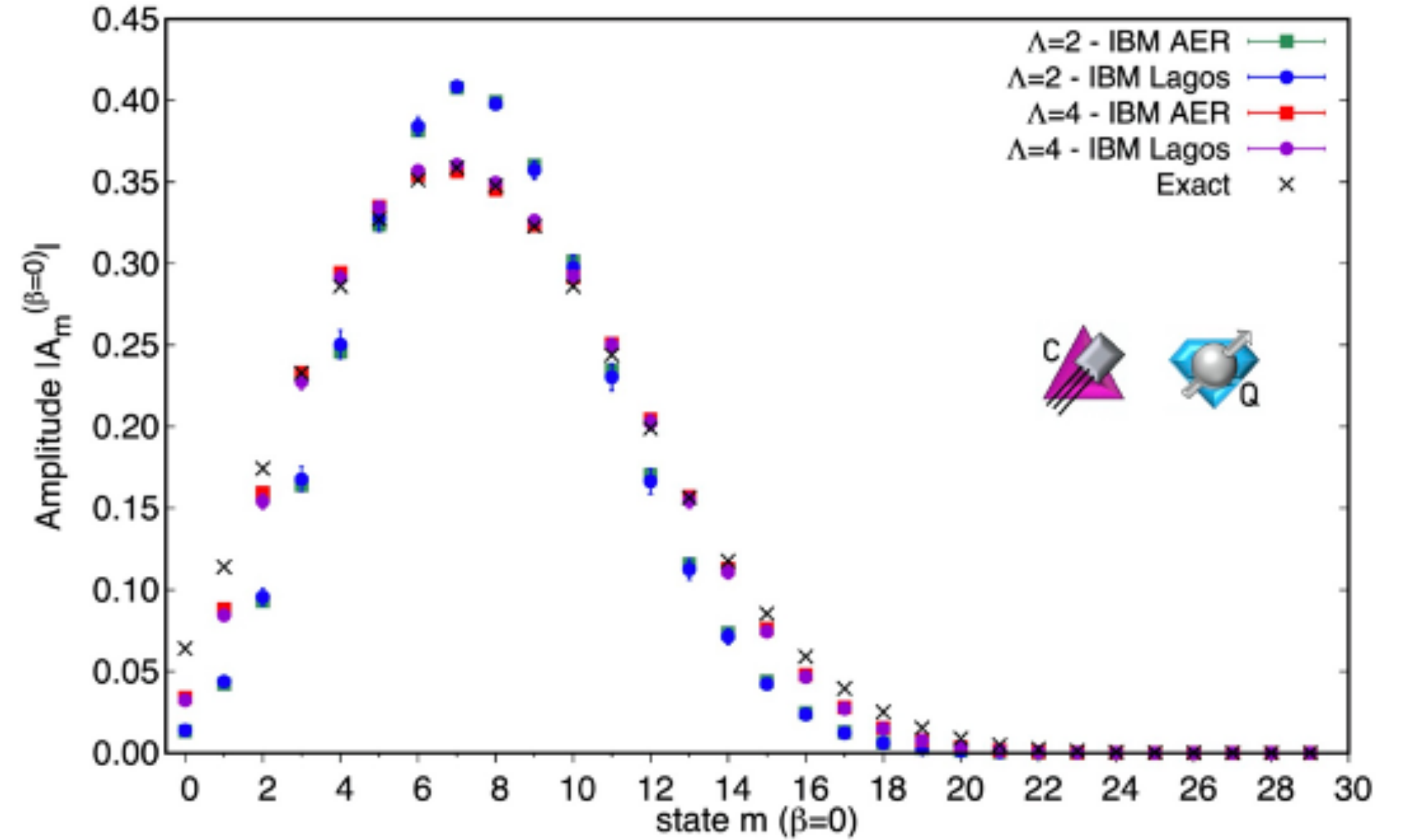
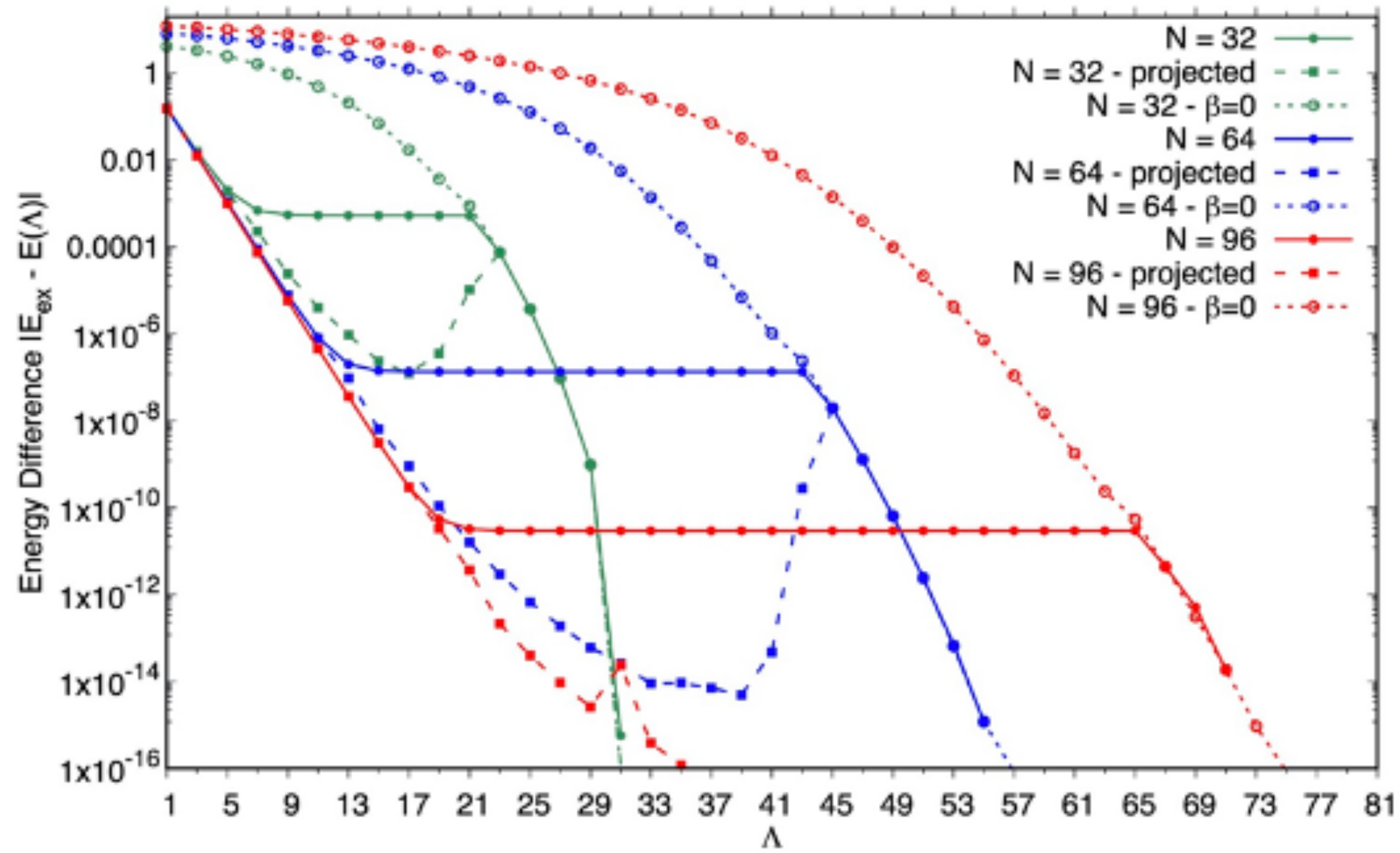
$$= \sum_{i_1, \dots, i_{n_q}} h_{i_1, \dots, i_{n_q}}(\beta) \langle \Psi(\theta) | \bar{\sigma}_{i_1} \otimes \dots \otimes \bar{\sigma}_{i_{n_q}} | \Psi(\theta) \rangle$$



⇒ learns the effective Hamiltonian and identifies the associated ground state simultaneously

Effective Model Spaces

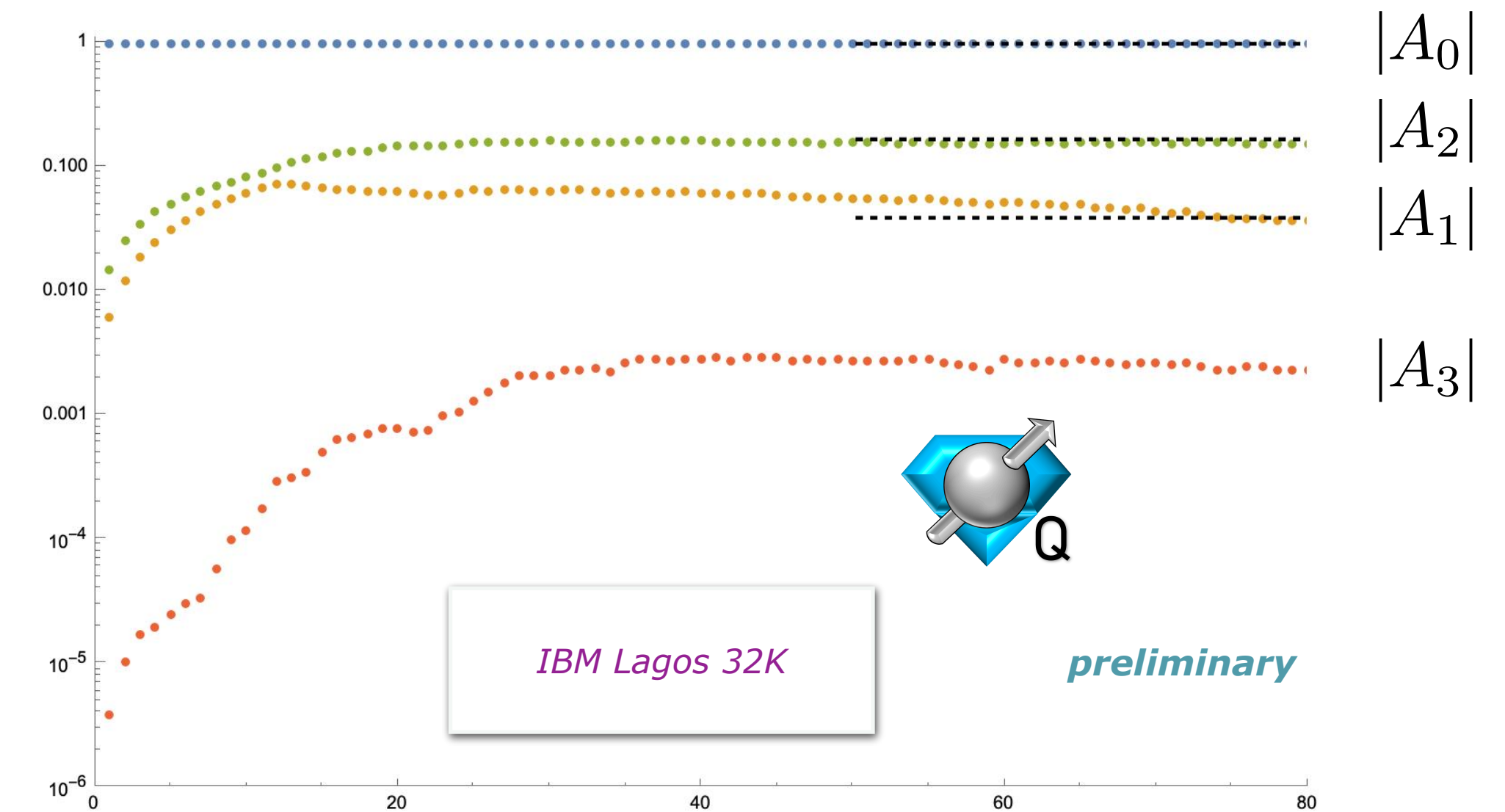
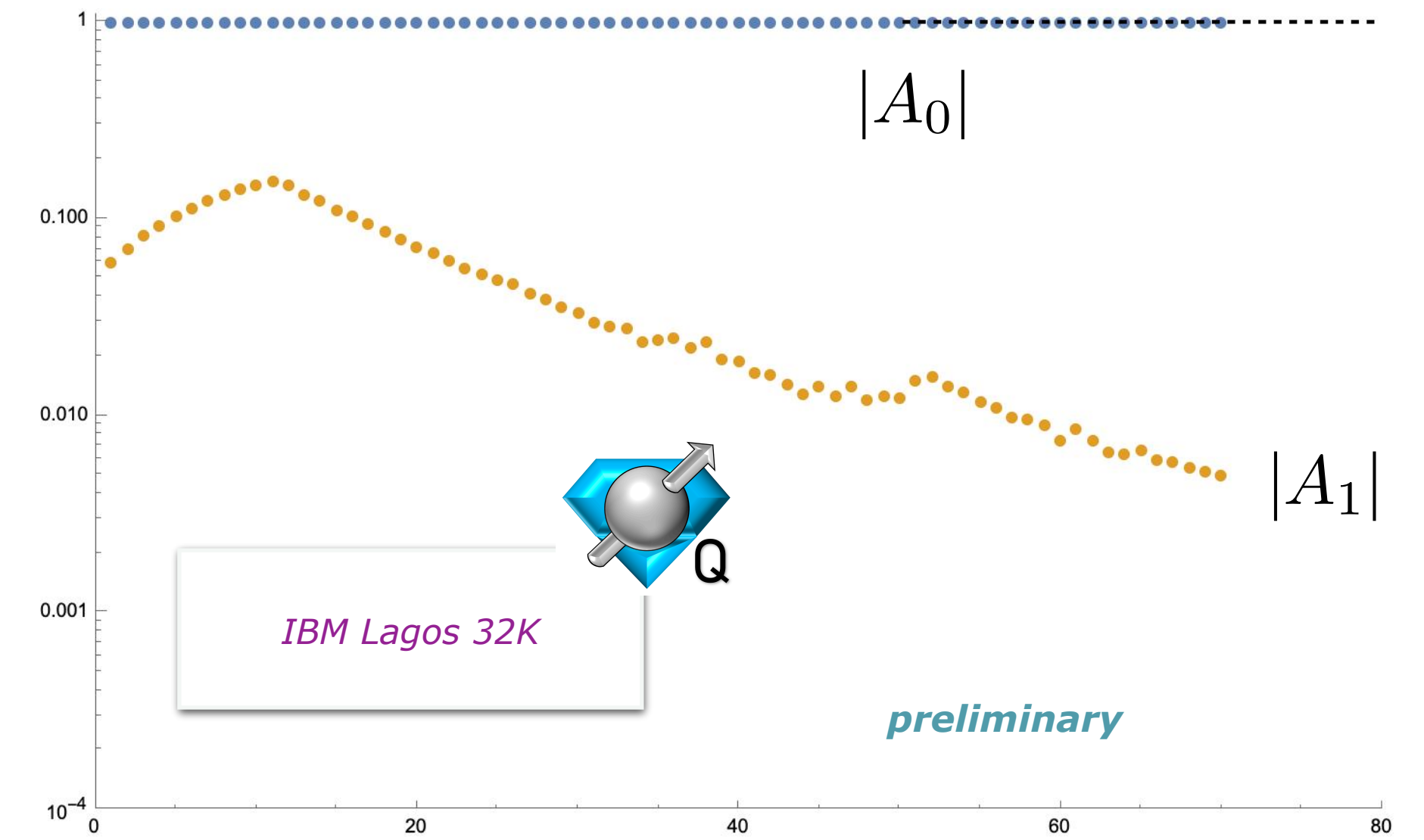
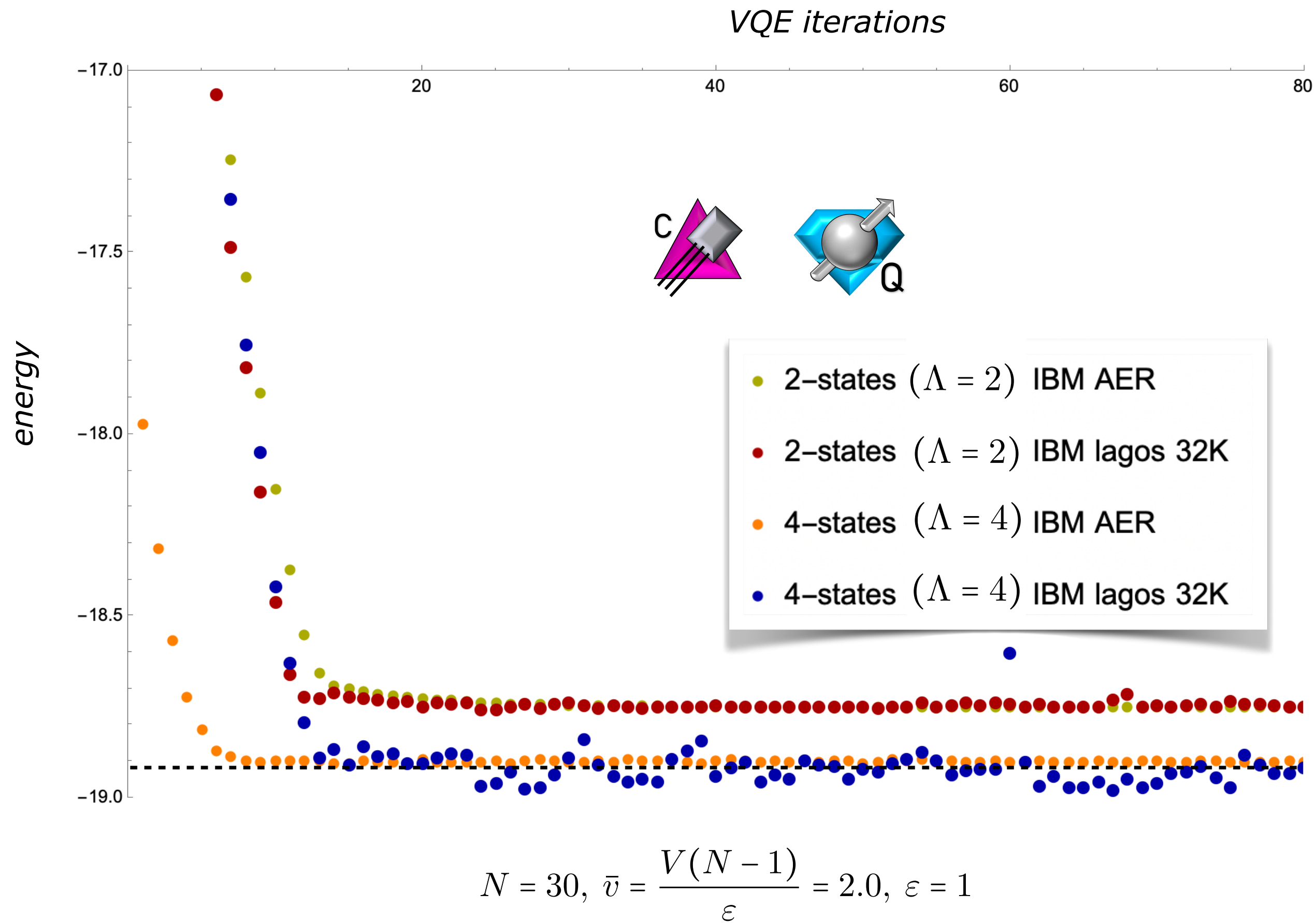
Convergence in Truncation



Effective Model Spaces

Results from IBM's Simulators and Quantum Computers

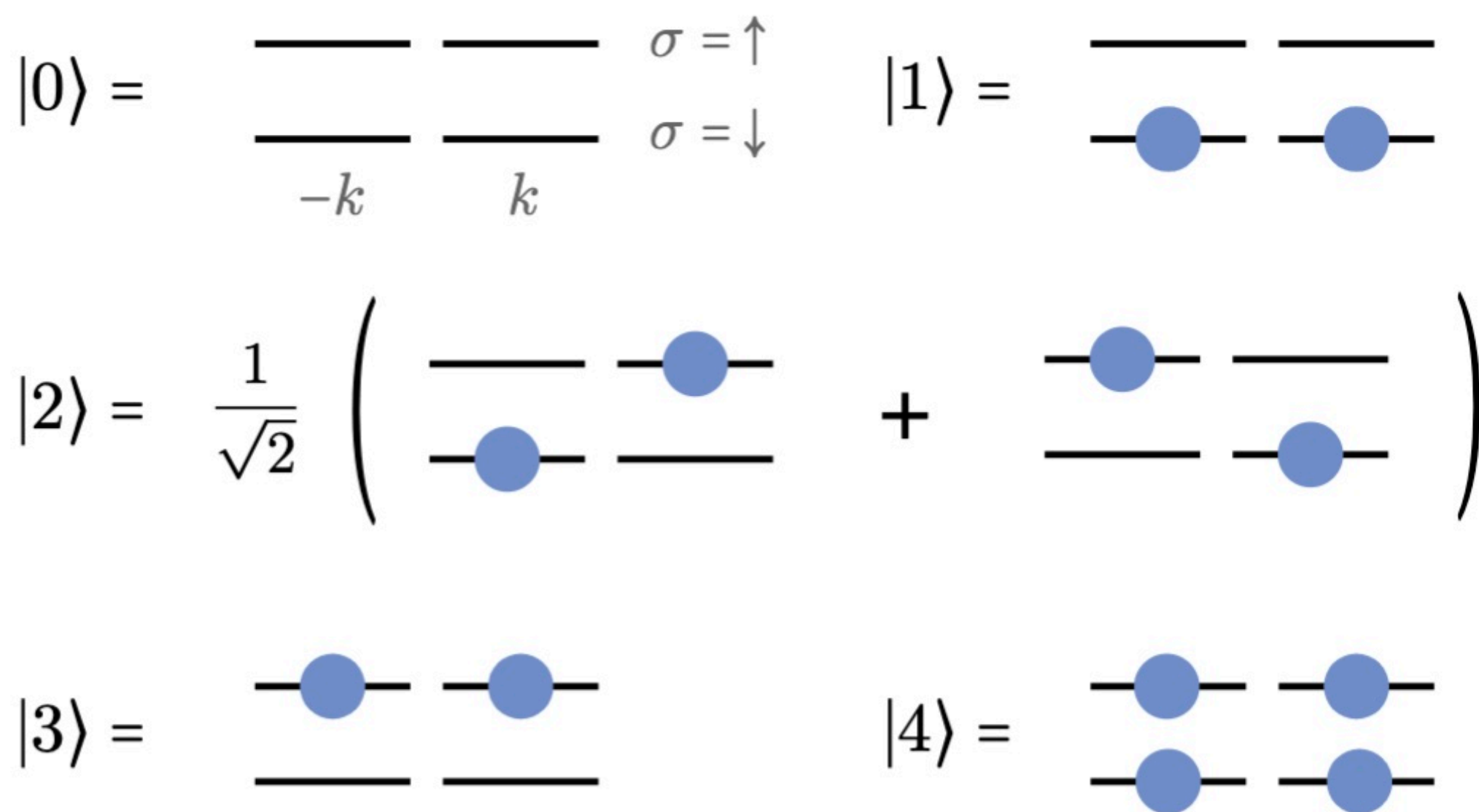
- *1 qubit ($\Lambda = 2$):
- *2 qubits ($\Lambda = 4$):



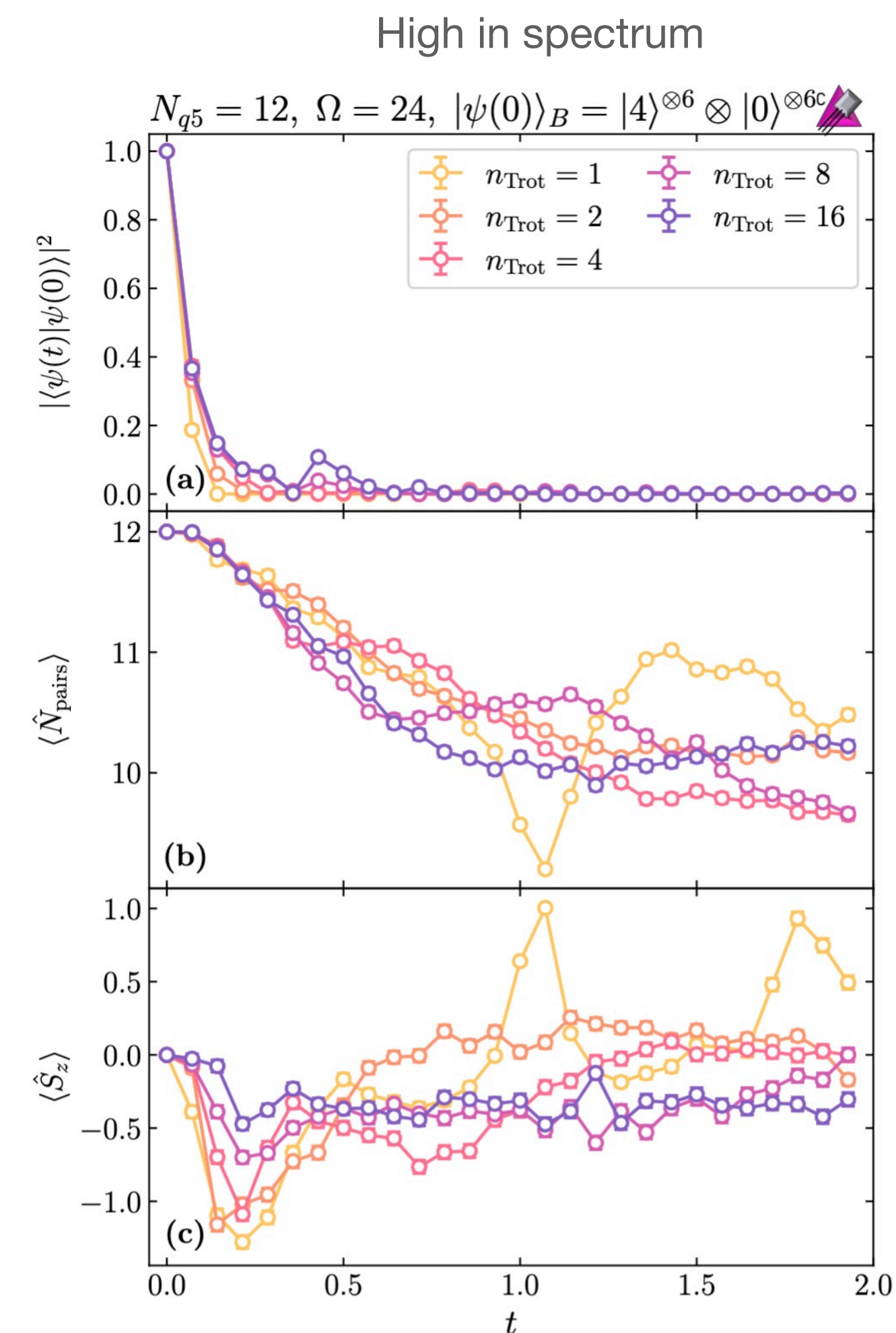
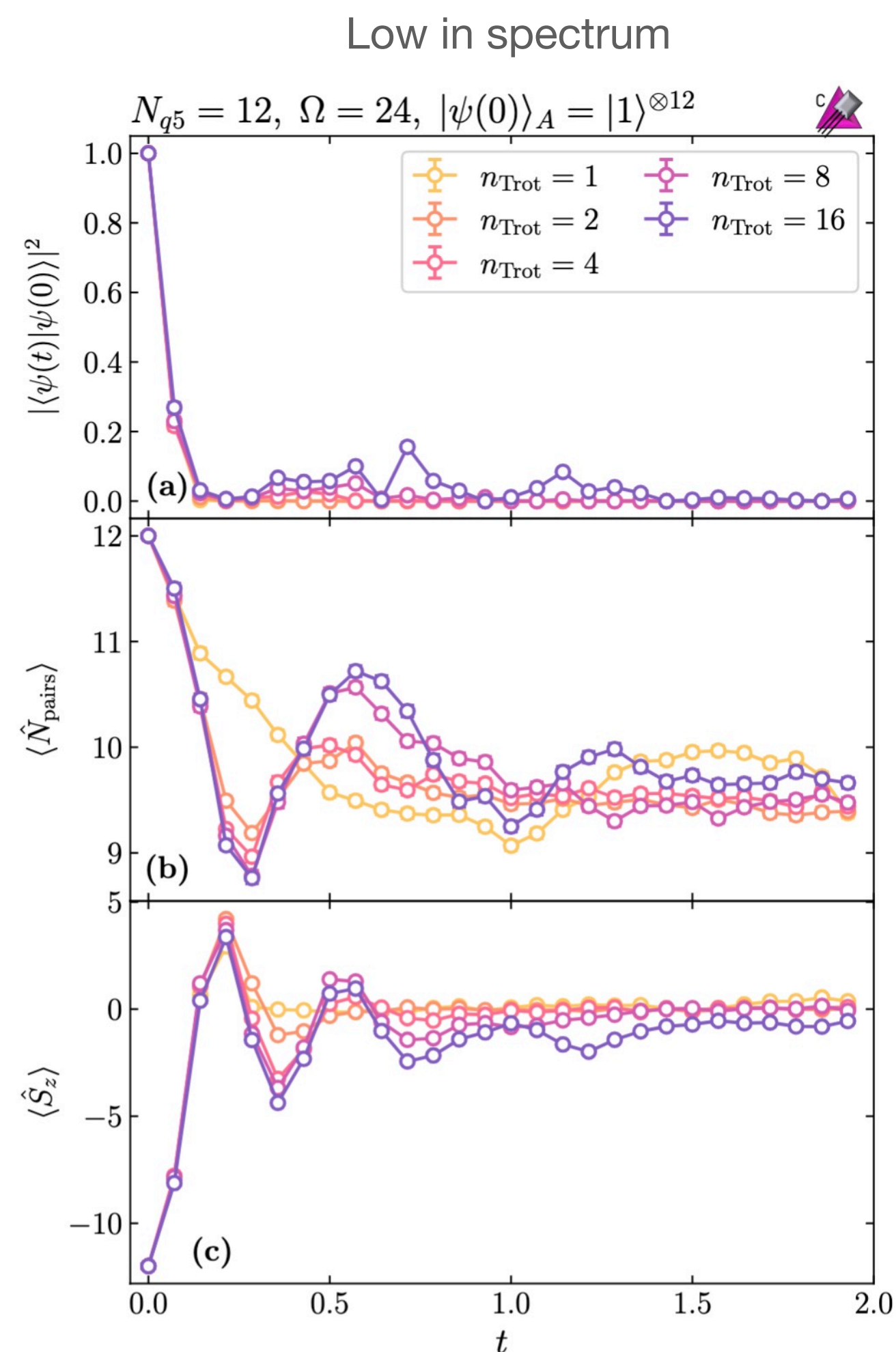
Lipkin-Meshkov-Glick Model with Pairing

Quantum Simulations of SO(5) Many-Fermion Systems using Qudits

Marc Illa ^{1,*} Caroline E. P. Robin ^{2,3,†} and Martin J. Savage ^{1,‡}



Basis that naturally embeds in a qu5it but also a physics-aware JW mapping

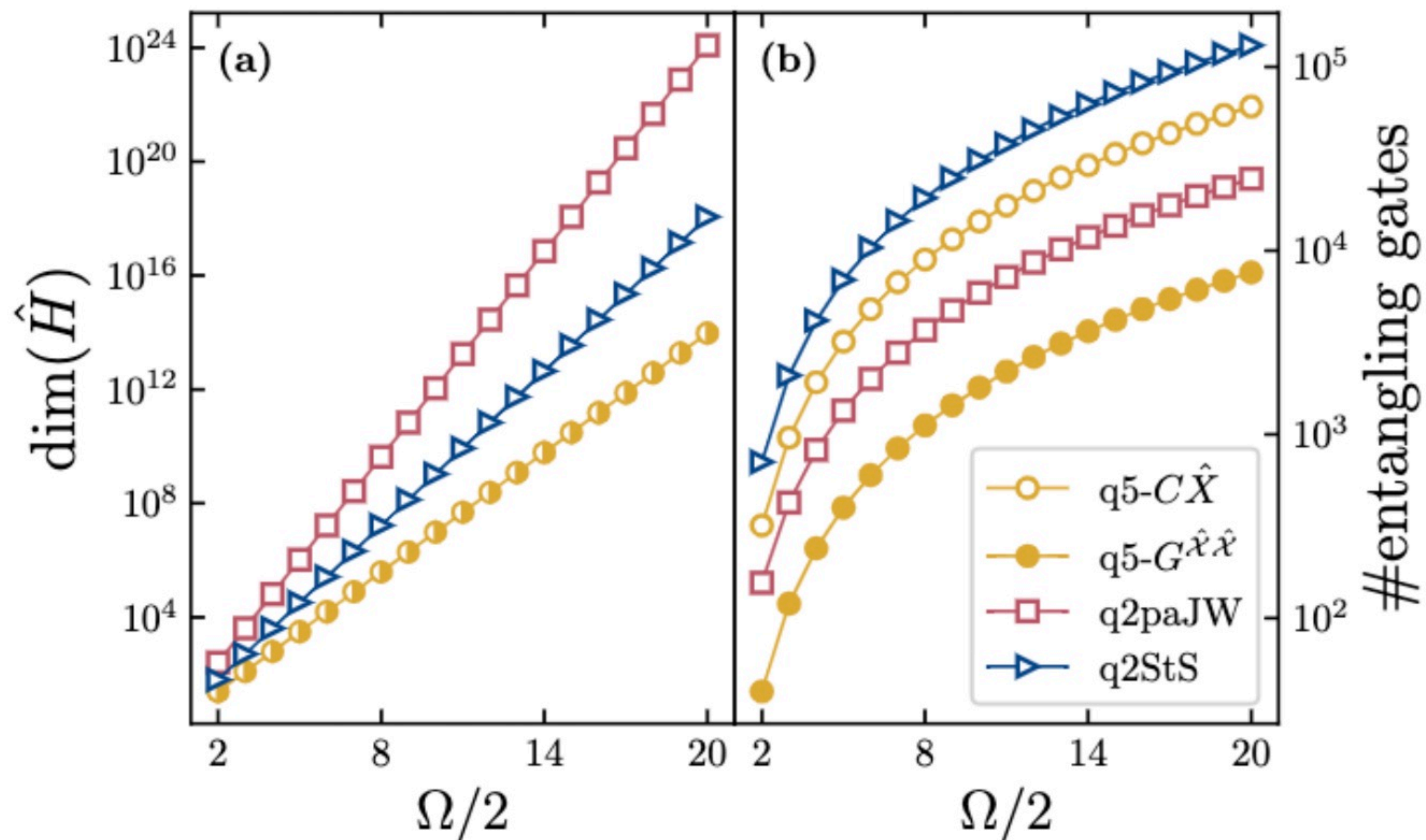


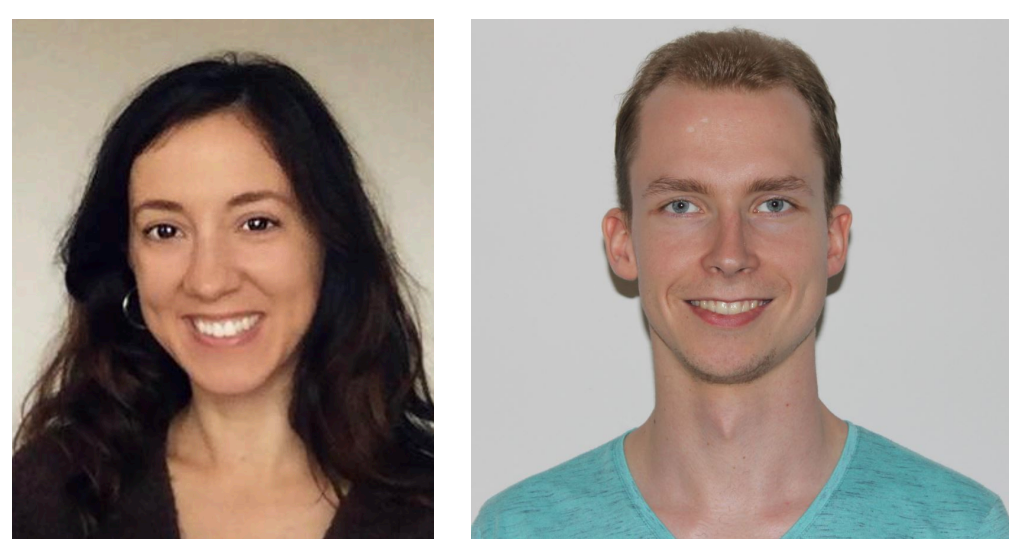
Trotter errors magnified in coherently summing over many states with small amplitudes

Lipkin-Meshkov-Glick Model with Pairing

Quantum Simulations of $SO(5)$ Many-Fermion Systems using Qudits

Marc Illa ^{1,*} Caroline E. P. Robin ^{2,3,†} and Martin J. Savage ^{1,‡}



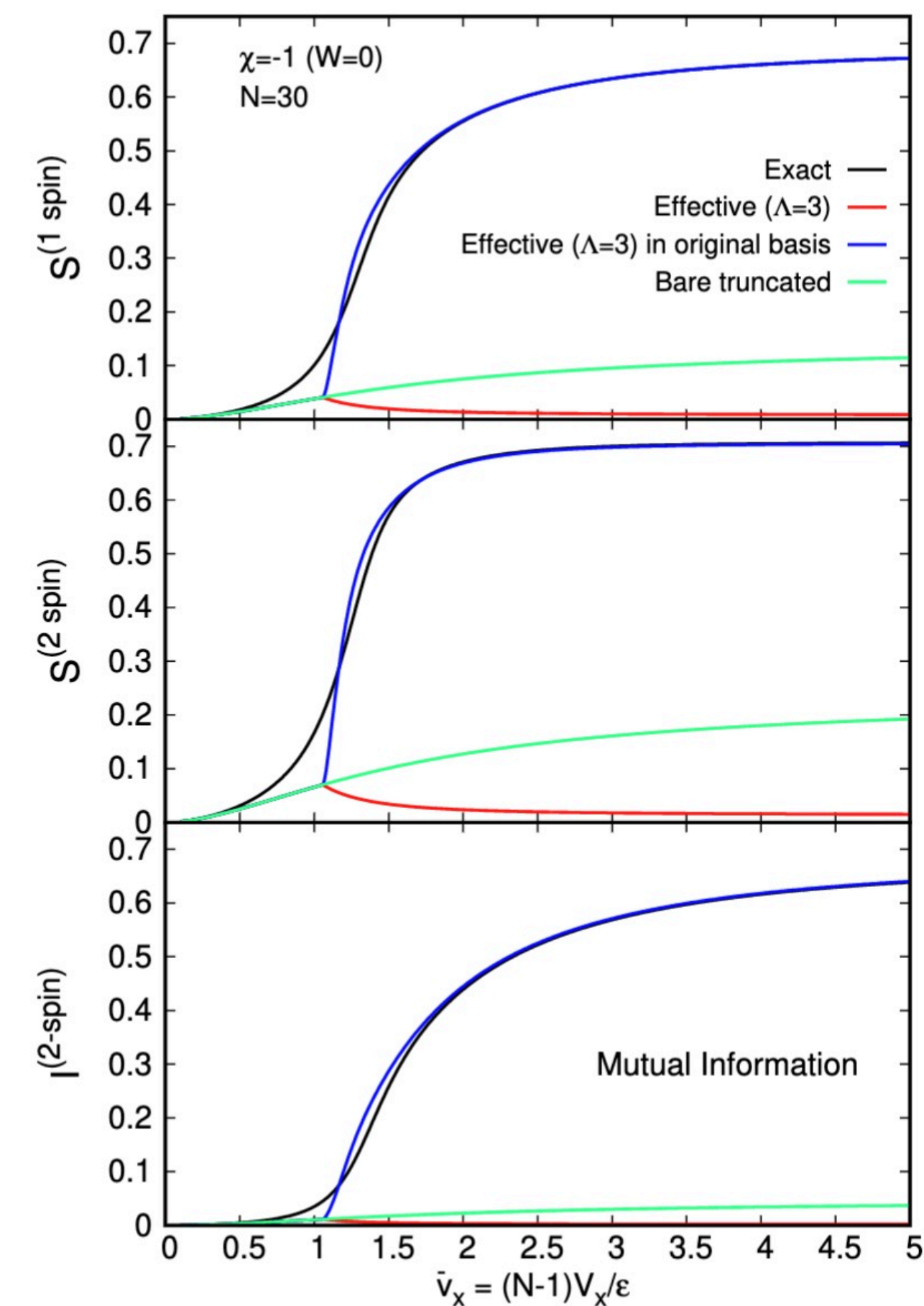
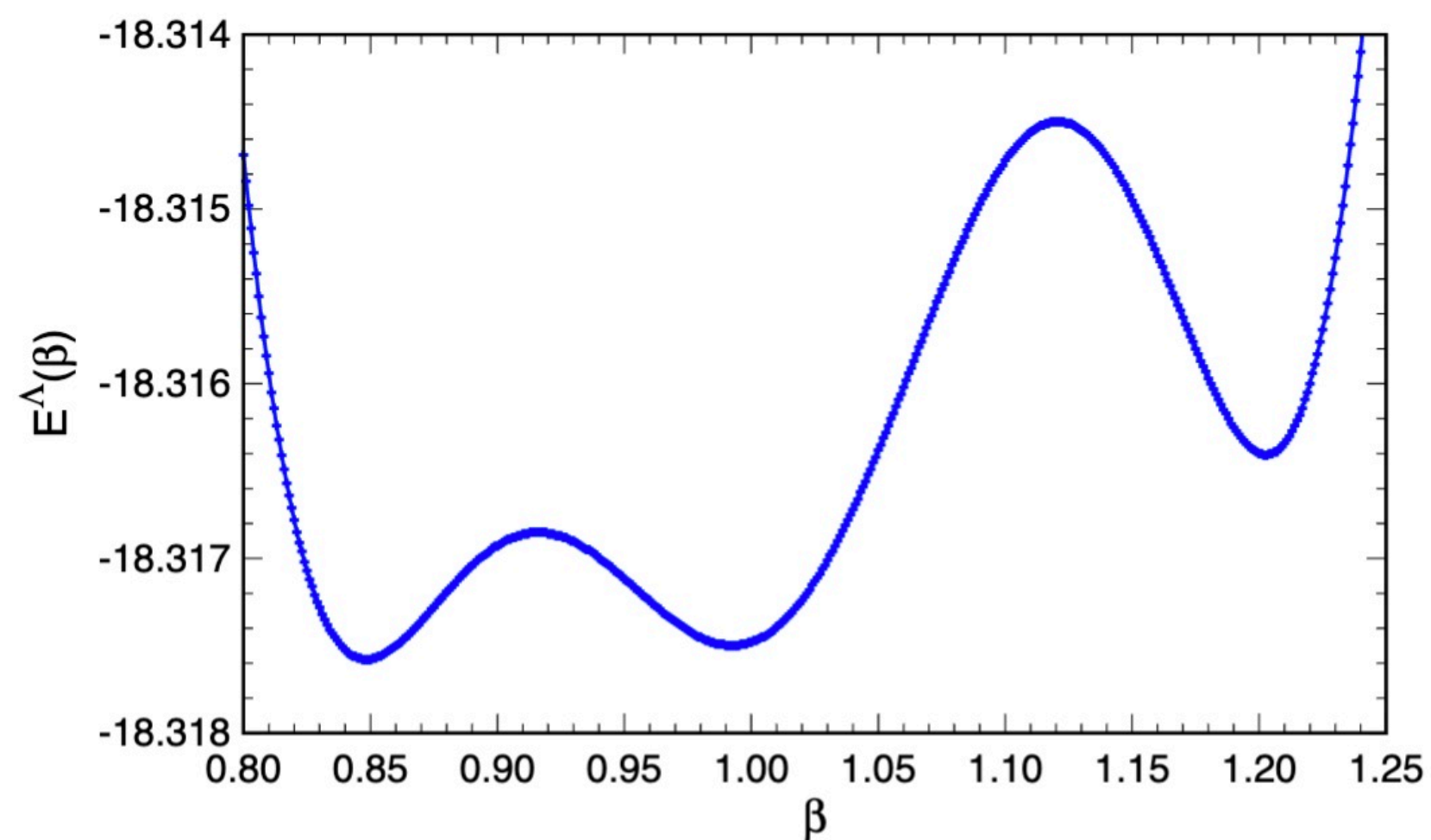
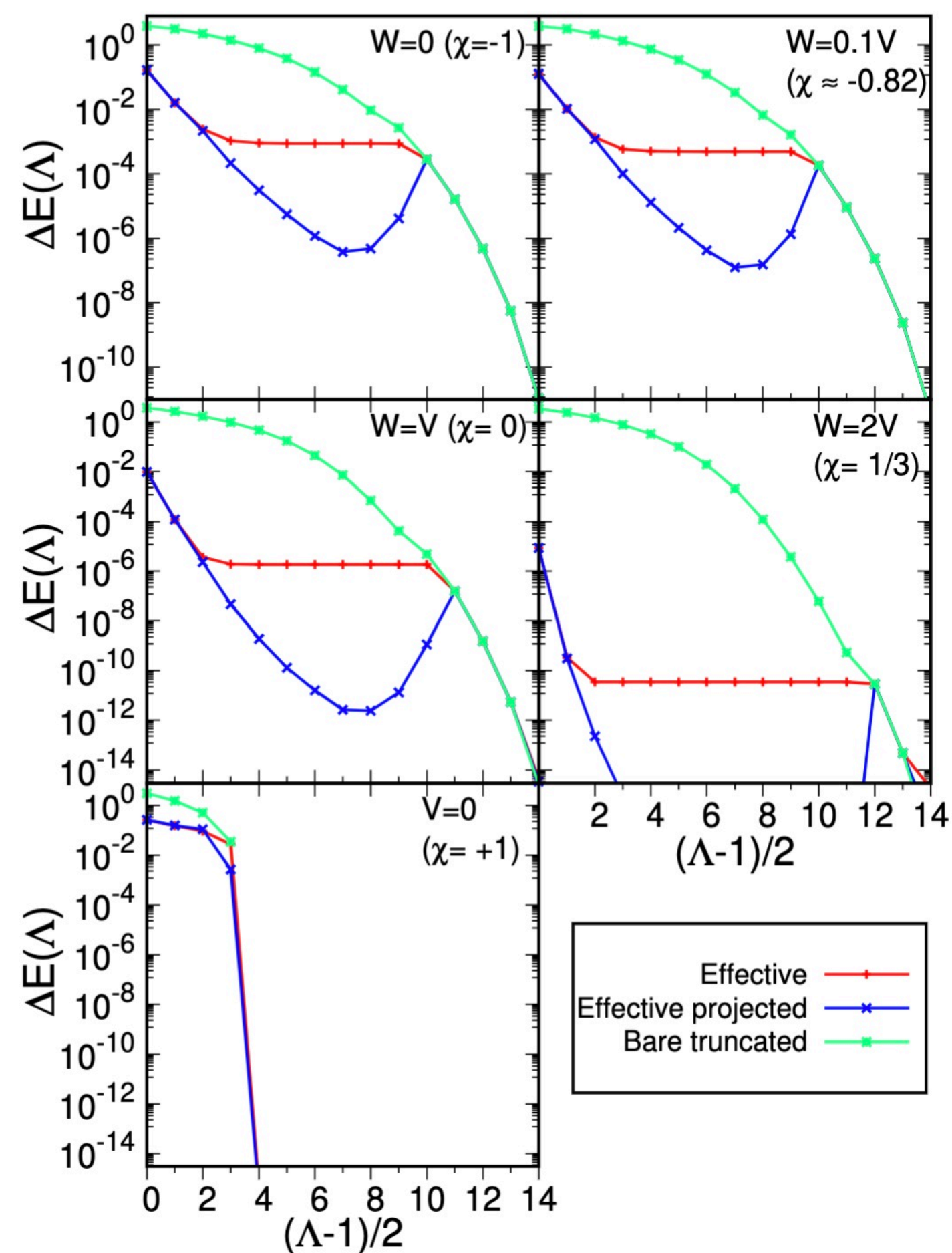


Lipkin-Meshkov-Glick Model Generalized

Multi-Body Entanglement and Information Rearrangement in Nuclear Many-Body Systems

$$\hat{H} = \varepsilon \hat{J}_z - V_x (\hat{J}_x^2 + \chi \hat{J}_y^2) + V_x \frac{1 + \chi}{4} \hat{N}$$

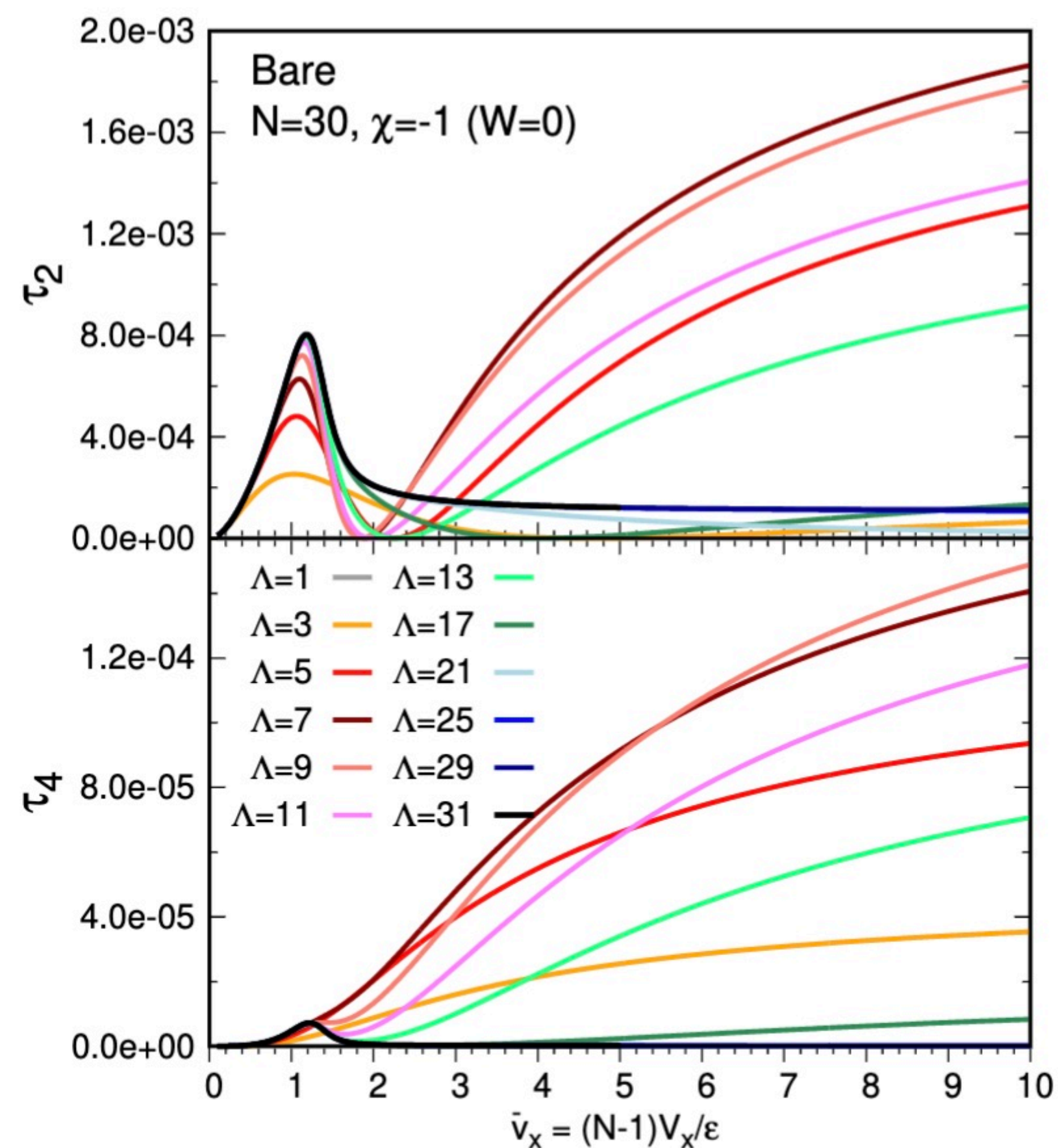
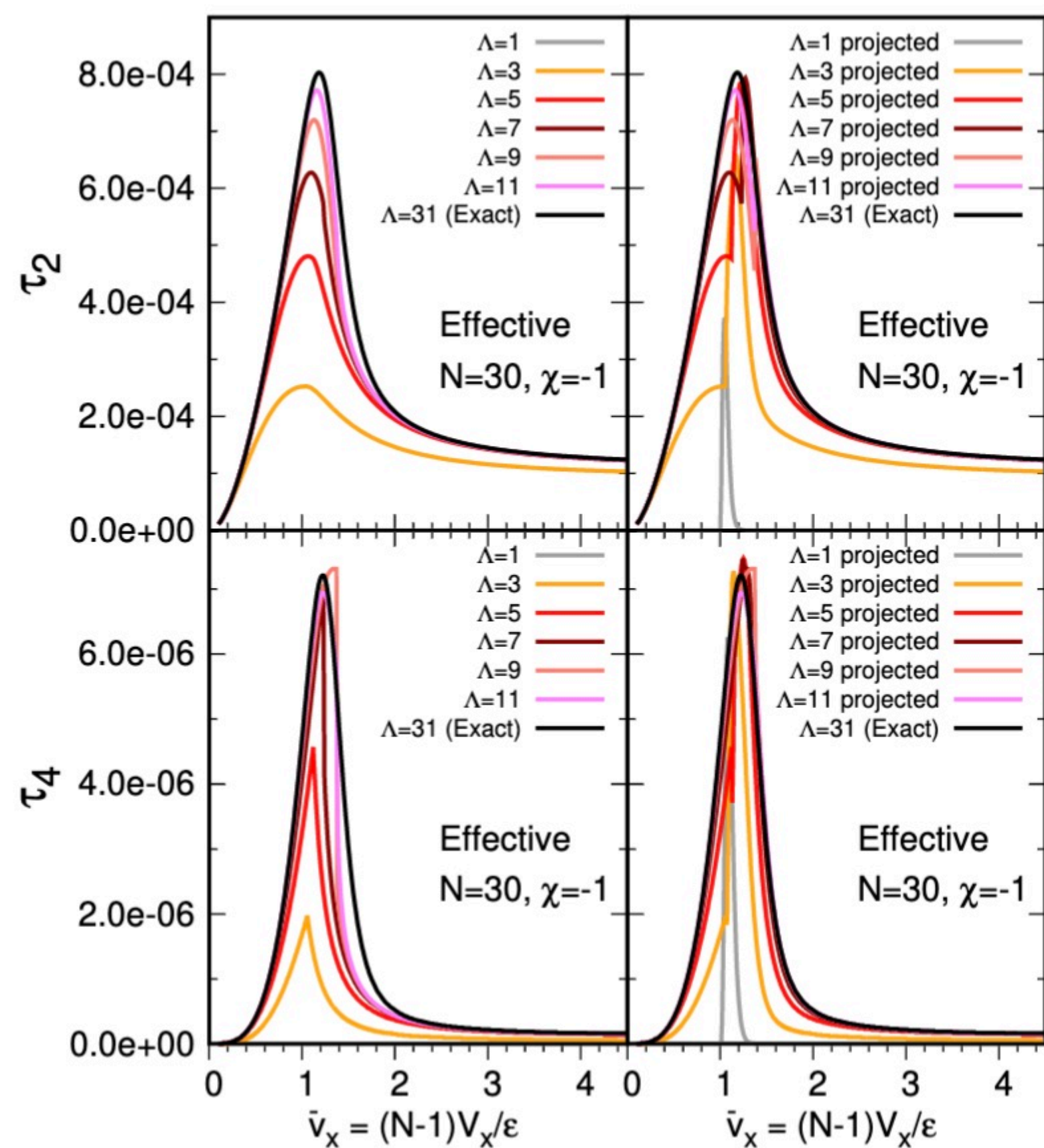
S. Momme Hengstenberg^{1a}, Caroline E. P. Robin^{1,2 b}, and Martin J. Savage^{3c}



Lipkin-Meshkov-Glick Model Generalized

Multi-Body Entanglement and Information Rearrangement in Nuclear Many-Body Systems

S. Momme Hengstenberg^{1a}, Caroline E. P. Robin^{1,2 b}, and Martin J. Savage^{3c}



Possibly

It is starting to become clear that:

techniques from nuclear many-body methods, mean-field methods, building in correlations, renormalization group, effective model spaces and effective field theories

....

likely to be able to advance lattice gauge theory Hamiltonian calculations and dynamics in dense neutrino systems for quantum simulation

....

and maybe vice versa

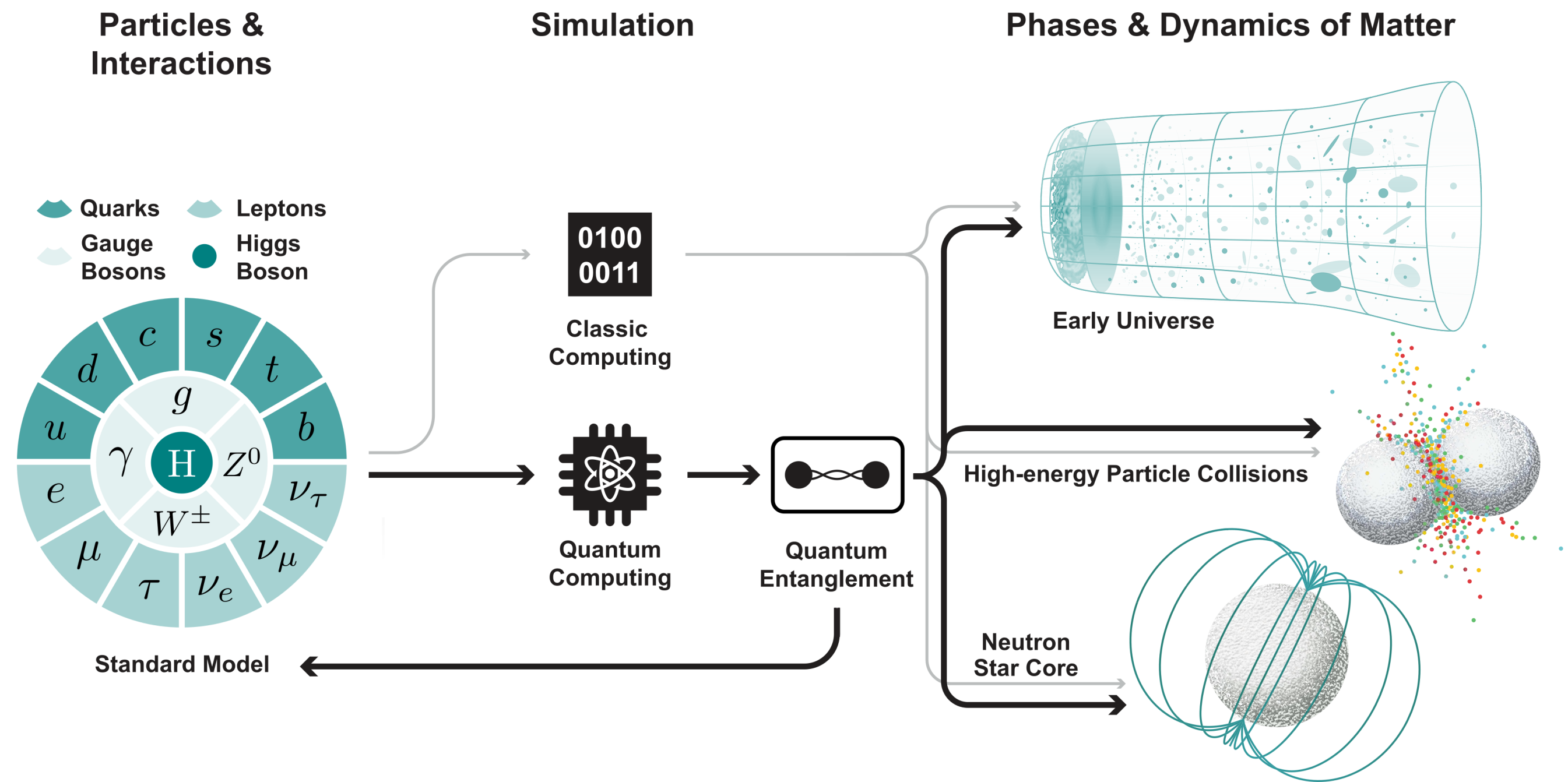
Quantum Computing

Boot Camp

June 20-30, 2023 • Jefferson Lab • Newport News, VA

Summary

- Standard Model dynamics requires quantum simulations
- Early stages in assessing requirements. Significant obstacles remain.
- Encouraging progress in quantum simulations in low-dimensional systems.
- Efforts toward 2+1 and 3+1 simulations.
- Connections within Nuclear Physics are emerging



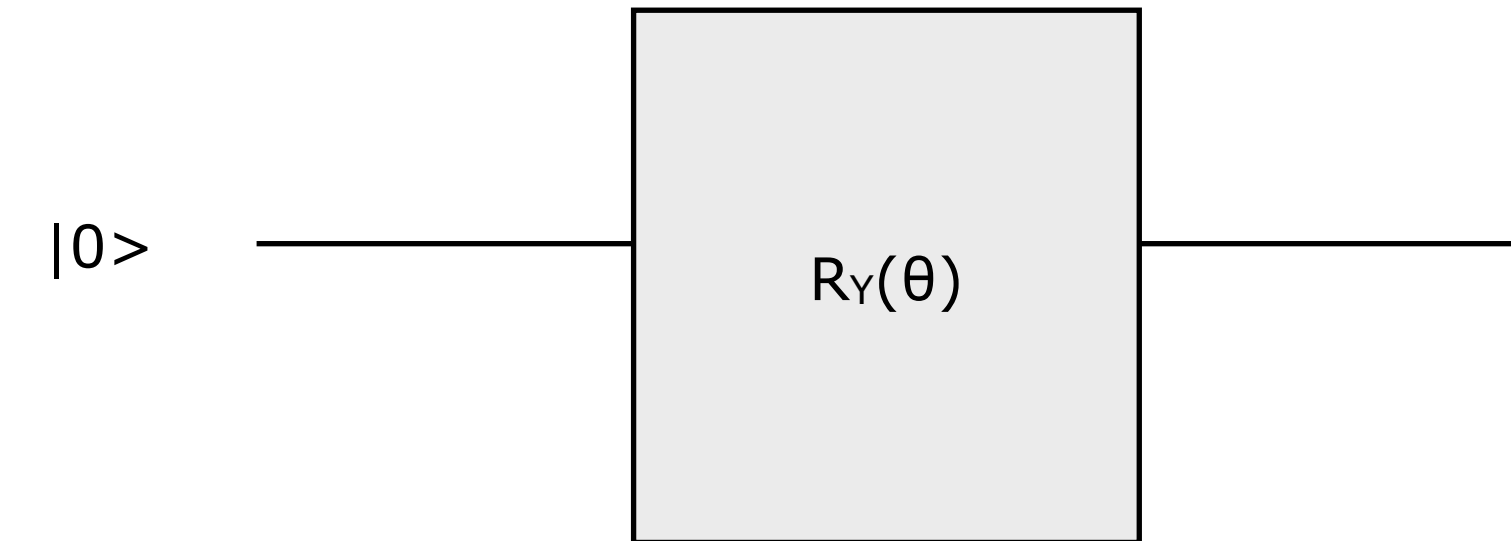


Effective Model Spaces

(Simple) Quantum Circuits for State Preparation

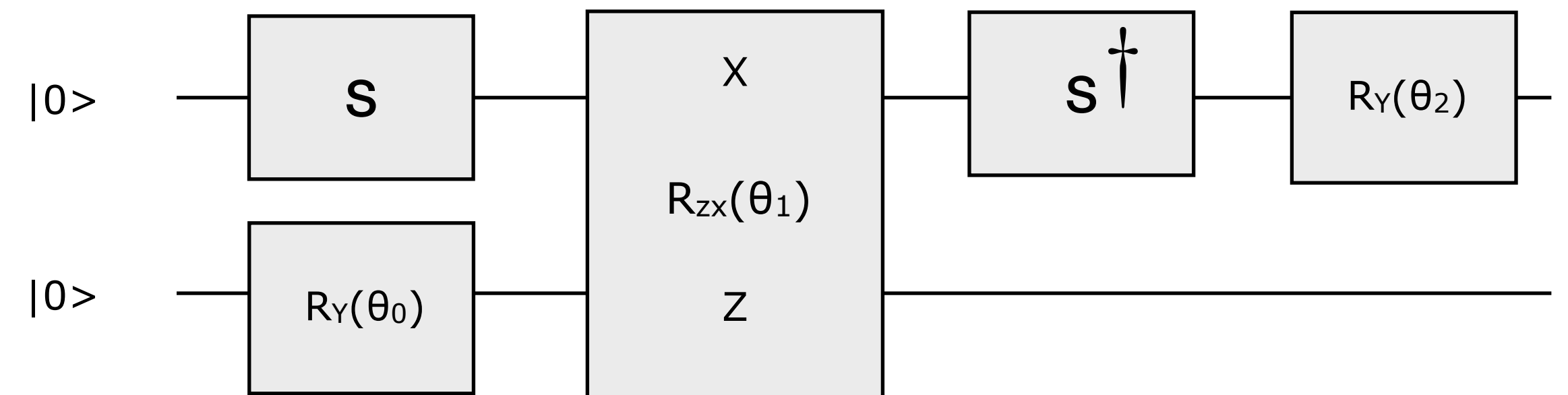
*1 qubit ($\Lambda = 2$):

$$|\Psi(\theta)\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)|1\rangle$$



*2 qubits ($\Lambda = 4$):

$$|\Psi(\theta_0, \theta_1, \theta_2)\rangle$$



$$R_{ZX}(\theta) = e^{-i\frac{\theta}{2}\hat{X}\otimes\hat{Z}}$$