Nucleon Spin Sum Rules and Polarizabilities

Jian-ping Chen, Jefferson Lab, Virginia, USA

JLab User Meeting, June 26-28, 2023

- **Introduction**
- **Nucleon Spin Sum Rules and Polarizabilities**
- **Experimental Extractions of Spin Sum and Polarizabilities (at Low-q)**

  Proton: g2p@Hall A (T) and EG4@Hall B (L)

  Neutron: SAGDH@Hall A with pol. ^{3}He (both L/T)

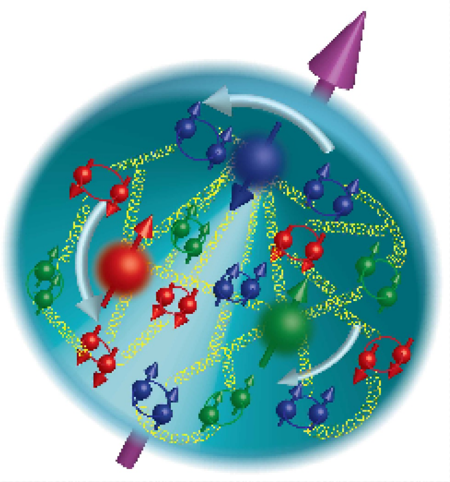
  EG4@Hall B with pol. deuteron (subtract proton) (L)

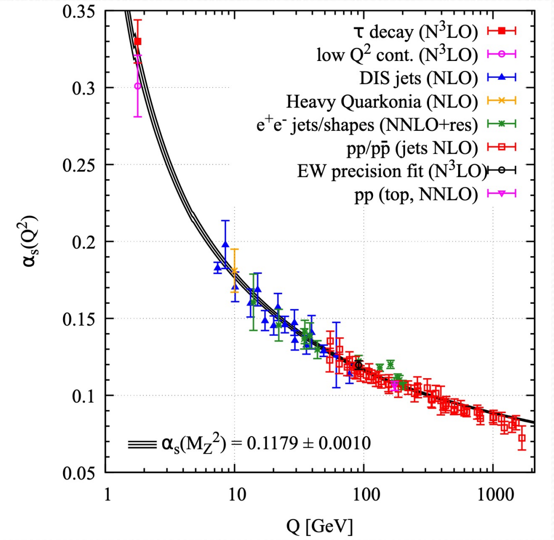
  Bjorken (p-n) Sum and (Effective) Strong Coupling
- **Summary**

Acknowledgment: Thanks to **Alexandre Deur, Karl Slifer** and collaborators for the work in this talk and for providing slides

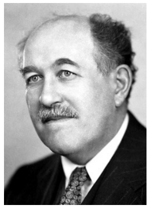
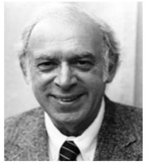
Introduction

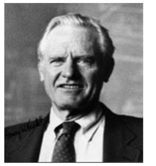
Nucleon Spin Structure and Strong Interaction,



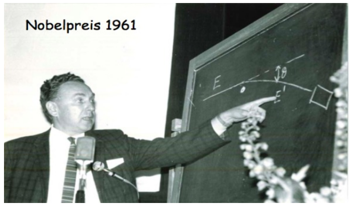


Nucleon Structure and Strong Interaction/QCD

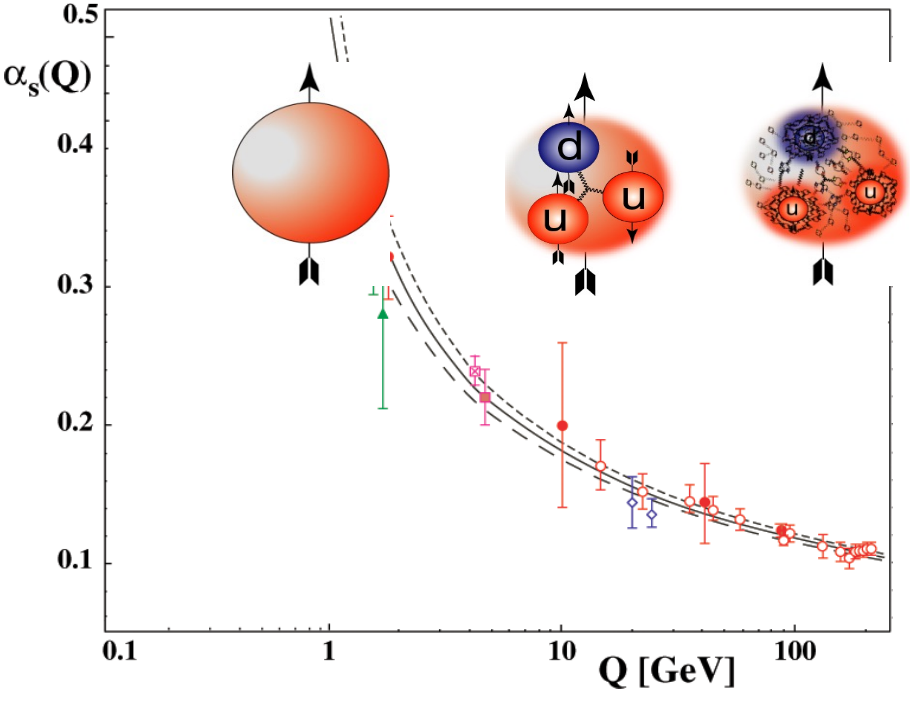


- **Nucleon Structure: discoveries**
  - -- anomalous magnetic moment (1943 Nobel)
  - -- elastic: form factors (1961 Nobel)
  - -- DIS: parton distributions (1990 Nobel)



- **Strong interaction, running coupling ~1**
  - -- asymptotic freedom (2004 Nobel)
  - perturbation calculation works at high energy
  - -- interaction significant at intermediate energy, quark-gluon correlations
  - -- interaction strong at low energy
    confinement

running coupling "constant"

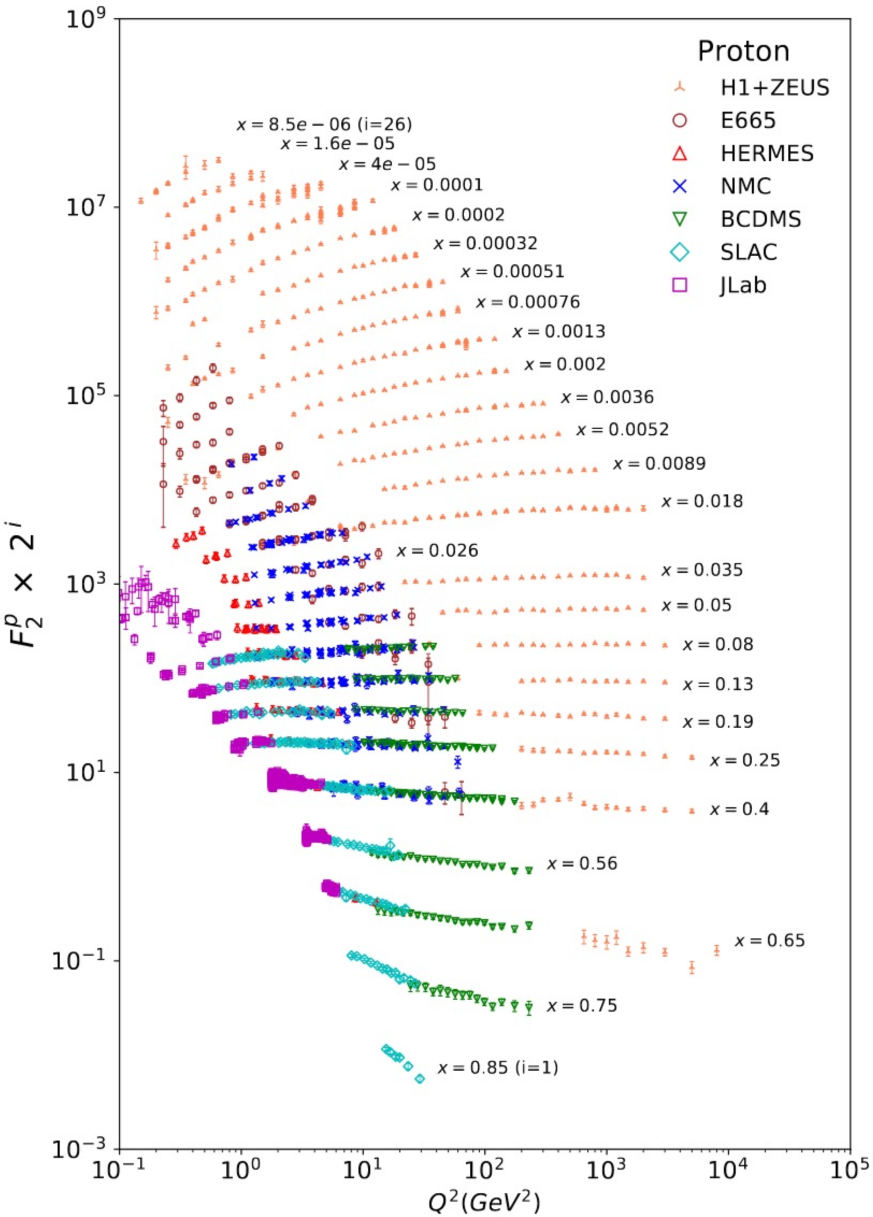


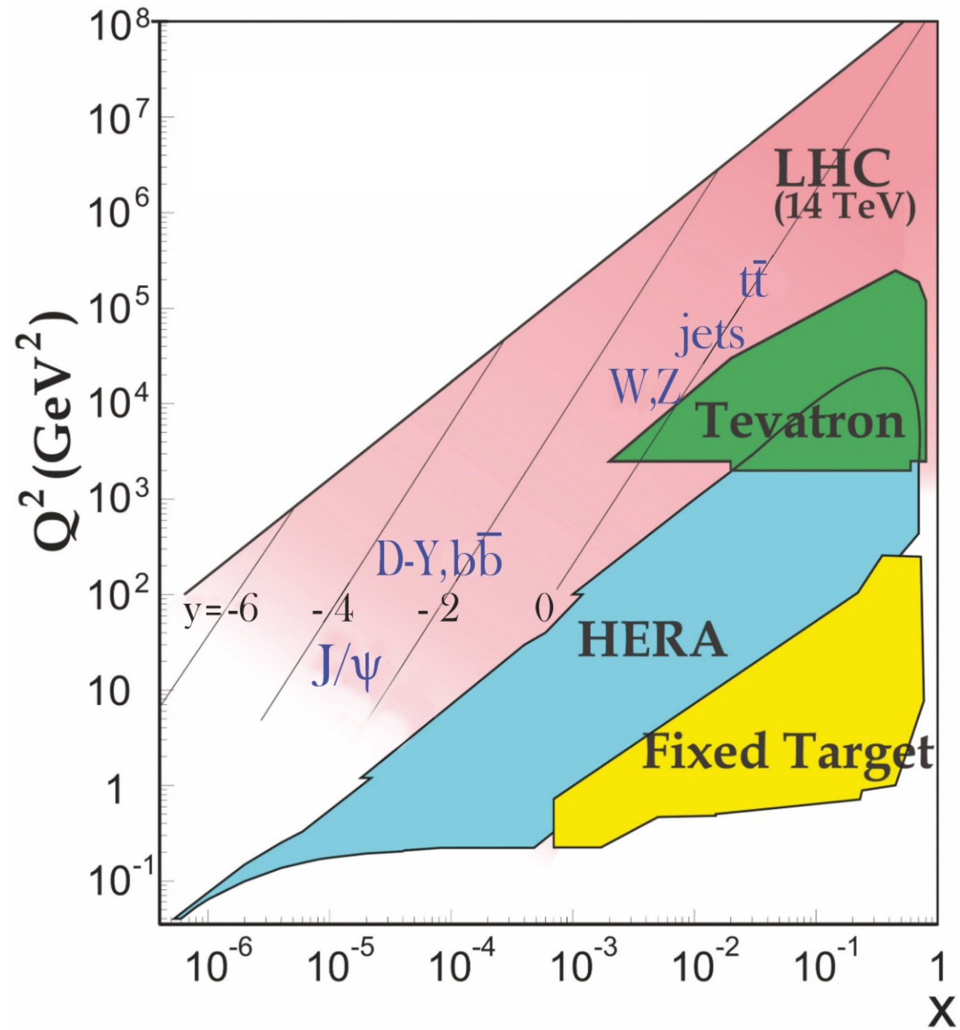
- **A major challenge in fundamental physics:**
  - -- Understand QCD in all regions, including strong (confinement) region

- **Nucleon: most convenient lab to study QCD**

- **Theoretical Tools:**
  pQCD, Lattice QCD, χEFT, Sum Rules, ...

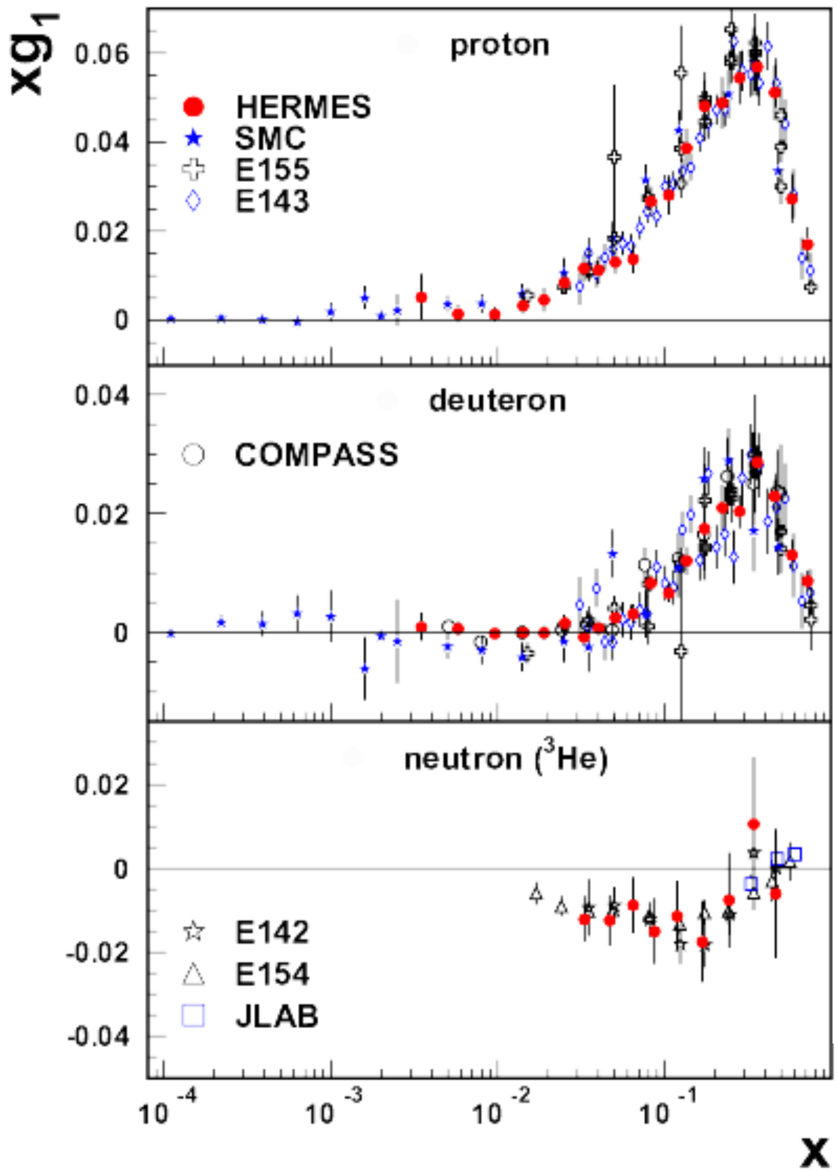
UNPOLARIZED STRUCTURE FUNCTIONS

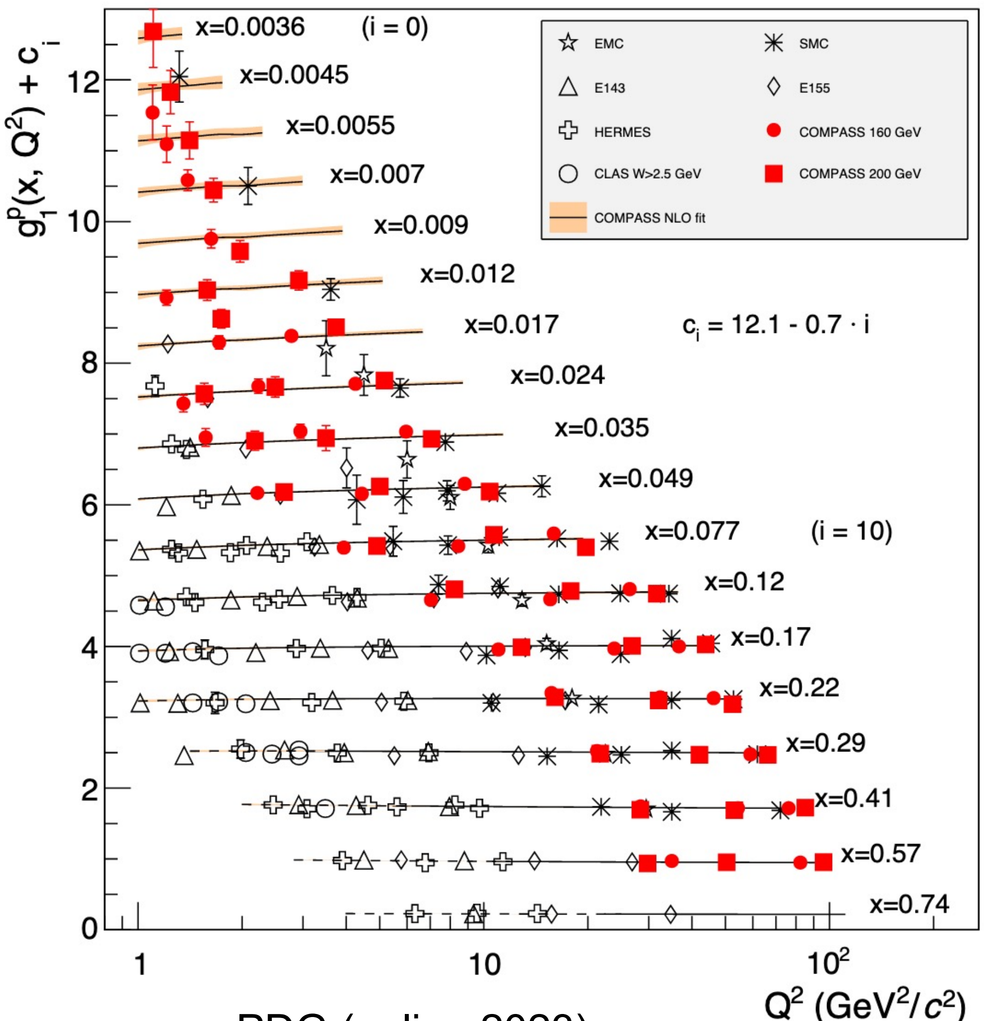




Q^2 evolution: the best test of QCD

POLARIZED STRUCTURE FUNCTIONS





PDG (online 2023)

Spin Sum Rules and Q^2 dependence

Sum Rules

Nucleon Structure ←→ Global Properties

mass, spin, magnetic moment, polarizabilities, …

How the structure is related (gives rise) to the global properties?
How the global properties emerging from the structure?

→ Help understand Strong QCD

Gerasimov-Drell-Hearn Sum Rule

Circularly polarized photon on longitudinally polarized nucleon

$$\boxed{\int_{\nu_{in}}^{\infty}\left(\sigma_{1/2}(\nu)-\sigma_{3/2}(\nu)\right)\frac{d\nu}{\nu}=-\frac{2\pi^2\alpha_{EM}}{M^2}\kappa^2}$$

- A fundamental relation between the nucleon spin structure and its anomalous magnetic moment
- Based on **general** physics principles
  - Lorentz invariance, gauge invariance → low energy theorem
  - unitarity → optical theorem
  - casuality → unsubtracted dispersion relation
    applied to forward Compton amplitude
- Measurements on *proton* up to 800 MeV (Mainz) and up to 3 GeV (Bonn) agree with GDH with assumptions for contributions from un-measured regions

New Hall D GDH experiment/proposal (high energy)

Generalized GDH Sum Rule

- Many approaches: Anselmino, Ioffe, Burkert, Drechsel, …
- Ji and Osborne (J. Phys. G27, 127, 2001):

  Forward Virtual-Virtual Compton Scattering Amplitudes: $S_1(Q^2,\nu)$, $S_2(Q^2, \nu)$

  Same assumptions: no-subtraction dispersion relation
  optical theorem
  (low energy theorem)

- Generalized GDH Sum Rule

$$S_1(Q^2) = 4\int_{el}^{\infty} \frac{G_1(Q^2,\nu)d\nu}{\nu}$$

Bjørken Sum Rule

$$\boxed{\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \int \{g_1^p(x,Q^2) - g_1^n(x,Q^2)\}dx = \frac{1}{6} g_A C_{NS}}$$

g_A: axial charge (from neutron β-decay)
C_{NS}: Q^2-dependent QCD corrections (for flavor non-singlet)

- A fundamental relation relating an integration of spin structure functions to axial-vector coupling constant (axial charge)
- Based on Operator Product Expansion within QCD or Current Algebra
- Valid at large Q^2 (higher-twist effects negligible)
- Data are consistent with the Bjørken Sum Rule at 5-10 % level

(Generalized) Bjørken Sum Rule

$$\Gamma_1^{p-n} = \frac{g_A}{6}\left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3 + \cdots\right] + \sum_{i=2,3\ldots}^{\infty} \frac{\mu_{2i}^{p-n}(Q^2)}{Q^{2i-2}},$$

- A fundamental relation relating an integration of spin structure functions to axial-vector coupling constant (axial charge)
- Based on Operator Product Expansion within QCD or Current Algebra
- Valid at large Q^2 (higher-twist effects negligible)
- Data are consistent with the Bjørken Sum Rule at 5-10 % level

Connecting GDH with Bjorken Sum Rules

- Q^2-evolution of GDH Sum Rule provides **a bridge linking strong QCD to pQCD**
  - Bjorken and GDH sum rules are two limiting cases

    High Q^2, Operator Product Expansion : $S_1(p\text{-}n) \sim g_A$ → Bjorken

    $Q^2 \to 0$, Low Energy Theorem: $S_1 \sim \kappa^2$ → GDH

  - High Q^2 (> ~1 GeV^2): Operator Product Expansion
  - Intermediate Q^2 region: Lattice QCD calculations
  - Low Q^2 region (< ~0.1 GeV^2): Chiral Effective Field Theory (χEFT)

Calculations: BχPT: Ji, Kao,…,Vanderhaeghen,…

Lensky, Alarcon & Pascalutsa

Bernard, Hemmert, Meissner

Spin polarizabilities sum rules

Spin polarizability sum rules involve higher moments:

Generalized forward spin polarizability:

$$\gamma_0 = \frac{4e^2M^2}{\pi Q^6}\int x^2\left(g_1 - \frac{4M^2}{Q^2}x^2 g_2\right)dx$$

Longitudinal-Transverse polarizability:

$$\delta_{LT} = \frac{4e^2M^2}{\pi Q^6}\int x^2(g_1+g_2)dx$$

We do not know how to measure directly generalized spin polarizabilities. The spin polarizability sum rules are used to access them.
They can be calculated with χEFT and Lattice QCD

News: Lattice QCD started calculations with 4-point function on polarizabilities
F. Lee *et al*. (U. George Washington); X. Feng *et al*. (Peking U.)

Low-Q Spin Experiments @ JLab

- **Hall B EG4: proton g_1:** Spokespeople: M. Ripani, M. Battaglieri, A. Deur, R. de Vita
  Students: H. Kang (Seoul U.), K. Kovacs (UVa)

  X. Zheng et al., Nature Physics, vo. 17 736-741 (2021)

- **Hall A g2p: proton g_2**

Spokespeople: K. Slifer, J. P. Chen, A. Camsonne, D. Crabb

Students: D. Ruth (UNH), R. Zielinski (UNH), C. Gu (UVa), M. Allada (Cummings)(W&M), T. Badman(UNH), M. Huang(Duke U.), J. Liu(UVa), P. Zhu(USTC)

D. Ruth et al, Nature Physics 18, 1441 (2022)

- **Hall A SAGDH: neutron g_1 and g_2 with polarized ^{3}He**

Spokespeople: J. P. Chen, A. Deur, F. Garibaldi.

Students: V. Sulkosky (W&M), C. Peng (Duke U.), J. Singh (UVa),V. Laine (Clermont-Fd U.), N. Ton (UVa), J. Yuan (Rutgers U.).

V. Sulkosky et al., Nature Phys., 17 687 (2021)
V. Sulkosky et al., PLB 805 135428 (2020)

Combining EG4 and SAGDH to form Bjorken Sum: **A. Deur et al., Phys. Lett. B 825 (2022) 136878**
Extracting effective coupling a_{g1}: **A. Deur, et al., Particles, 5-171 (2022)**

Summary of Spin Experiments

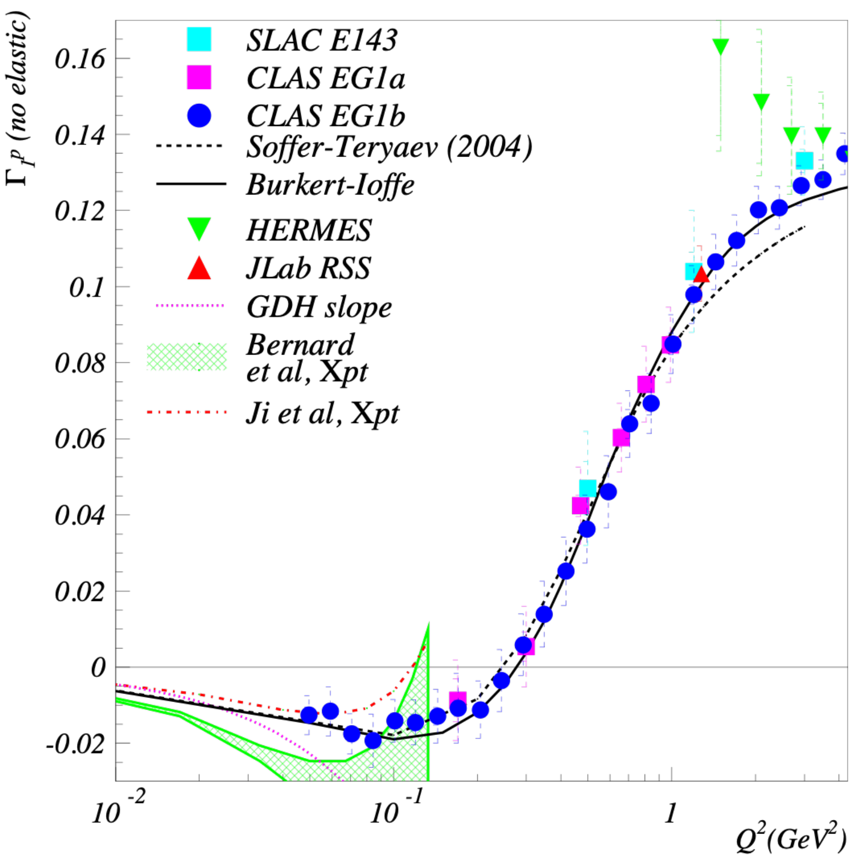
Observable	H target	D target	^{3}He target	
g_1, g_2, Γ_1 & Γ_2 at high Q^2	SLAC JLAB SANE	SLAC	SLAC JLAB E97-117 JLAB E01-012 JLAB E06-014	**JLab12**
g_1 & Γ_1 at high Q^2	SMC HERMES JLAB EG1	SMC HERMES JLAB EG1	HERMES	COMPASS RHIC-Spin
Γ_1 & Γ_2 at low Q^2	JLab RSS	JLab RSS	JLab E94-010 JLab E97-103	
Γ_1 at low Q^2	SLAC HERMES JLAB EG1	SLAC HERMES JLAB EG1	HERMES	
Γ_1, $Q^2 << 1$ GeV^2	JLab EG4	JLab EG4	JLab E97-110	SAGDH
Γ_2, $Q^2 << 1$ GeV^2	JLab E08-027 g2p		JLab E97-110	

Measurement of Low-q Spin Sum Γ_1 and Γ_2 for proton and neutron

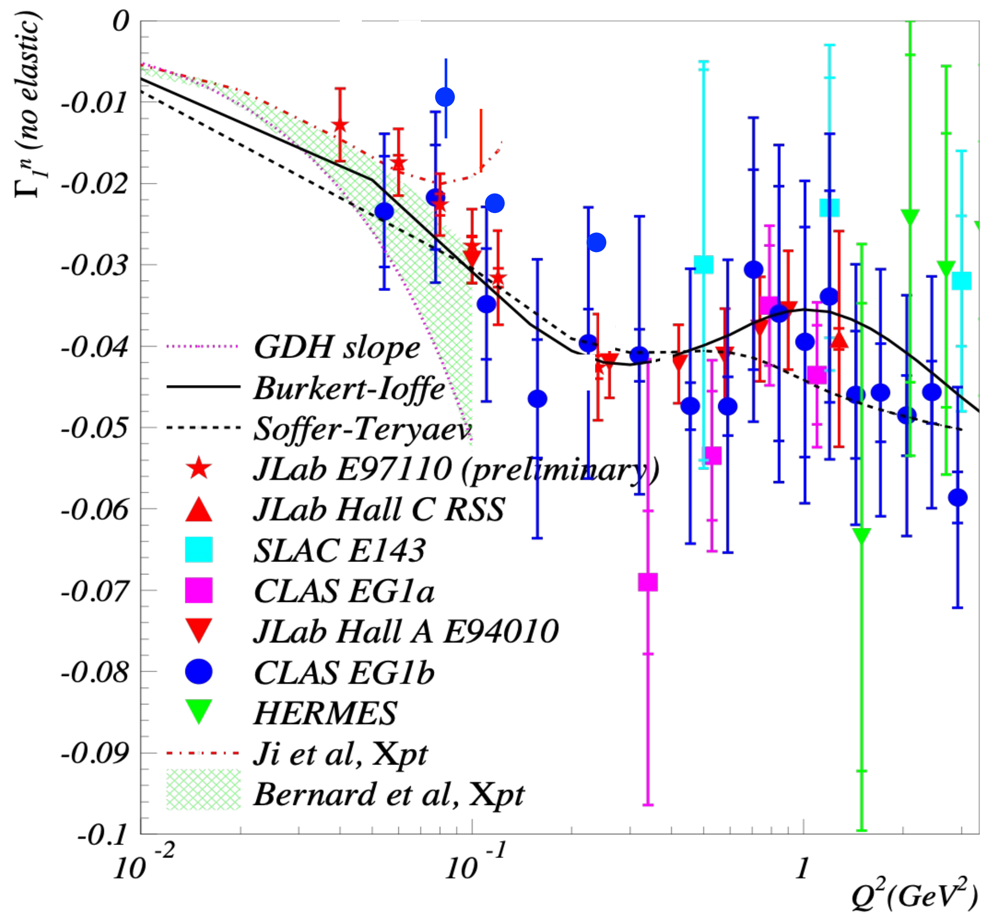
Testing χEFT and study strong QCD

Previous world Γ_1 data before low-Q experiments

Proton



Neutron



Precise mapping of spin structure function moments in intermediate Q^2 region
PQCD, models and data agree.
How about χEFT predictions? Not clear.

EG4: new low-Q data on Γ_1 for proton

X. Zheng et al.,
Nature Physics,
17, 736-741 (2021)

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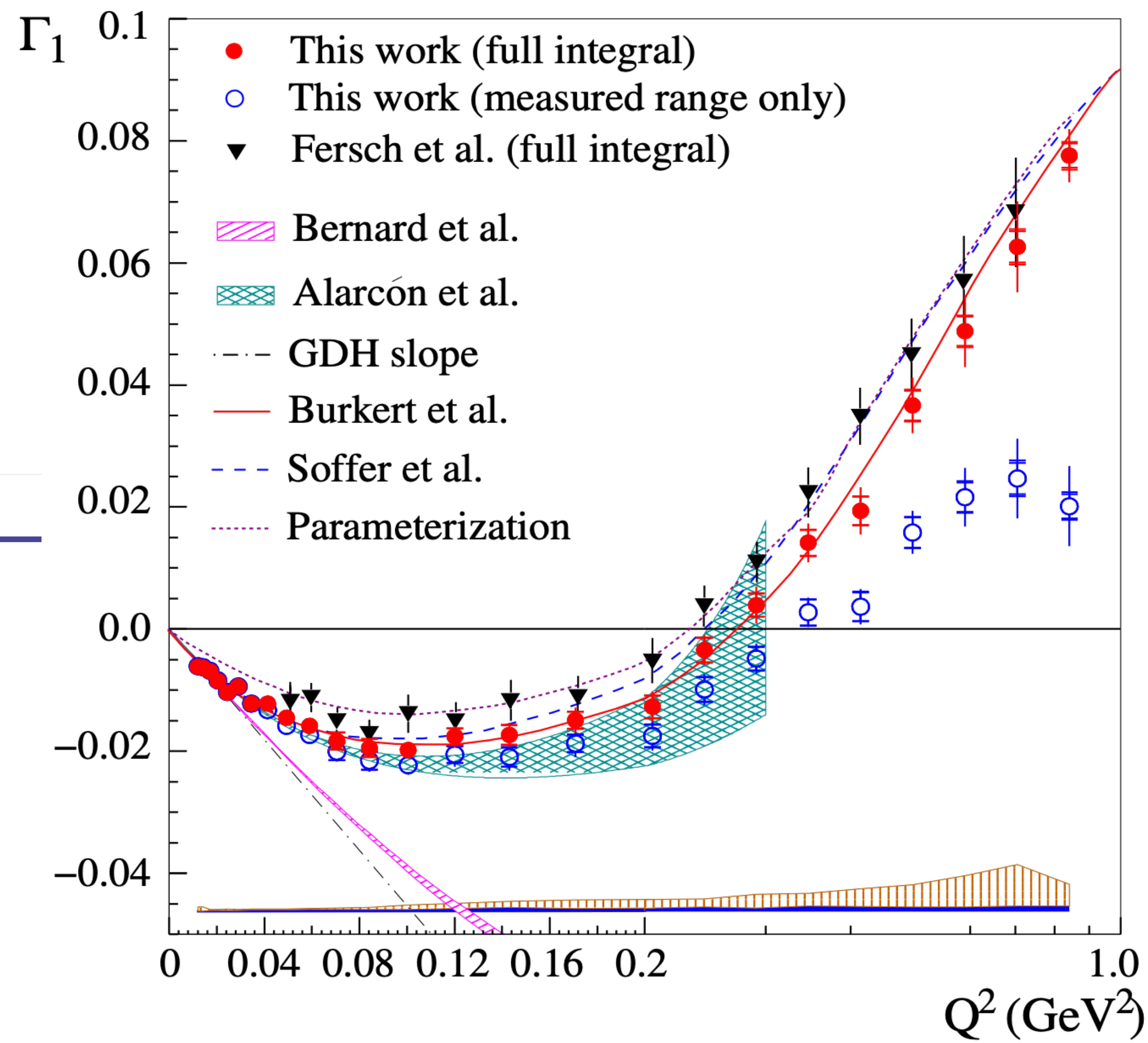
nature > nature physics > articles > article

Article | Published: 12 April 2021

Measurement of the proton spin structure at long distances



•Slight tension between EG4 and EG1 above Q^2 ~0.1 GeV^2.

•EG4 and χEFT agree up to Q^2 ~0.04 GeV^2 (Bernard et al) or Q^2 >0.2 GeV^2 (Alarcón et al.)

•Phenomenological models (Pasechnik et al, Burkert-Ioffe) agree well.

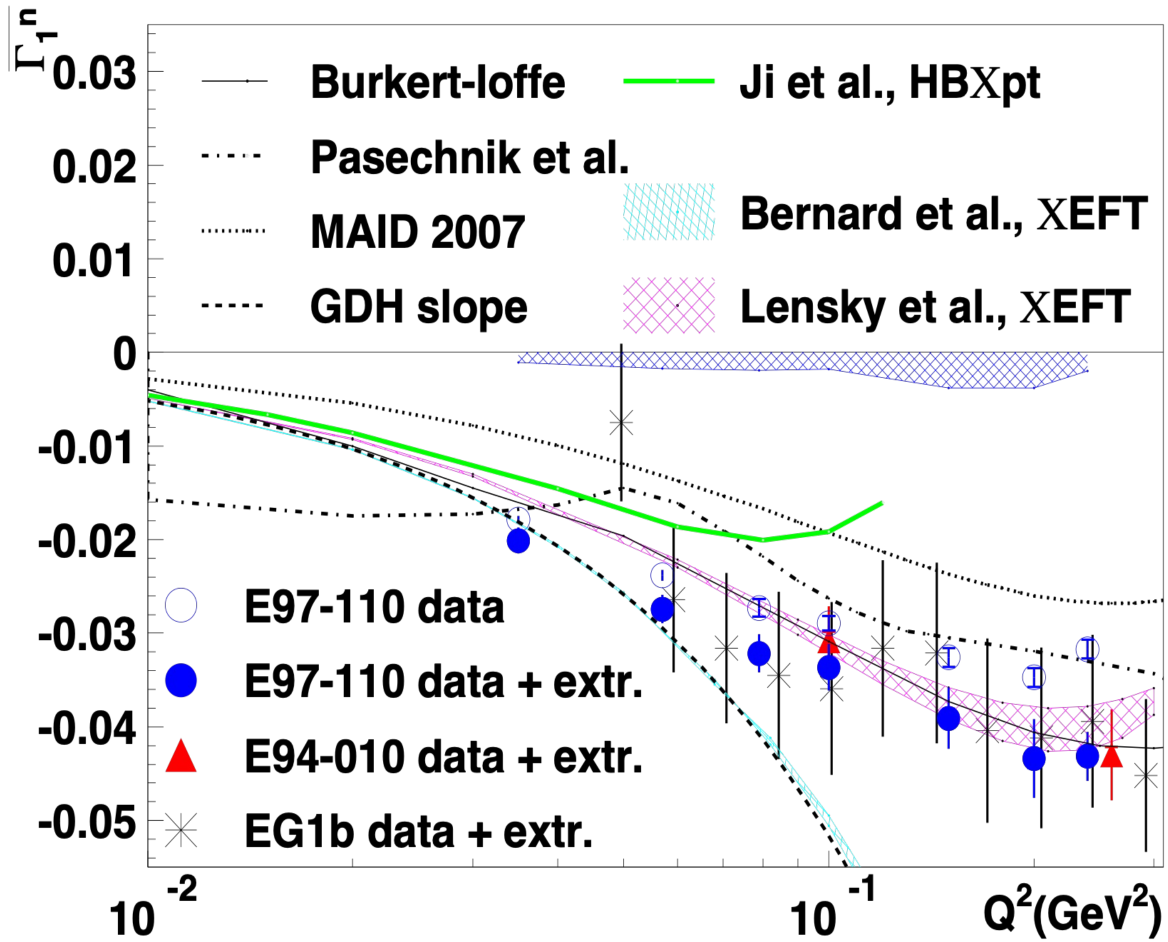
SAGDH: new low-Q data on Γ_1 for neutron

V. Sulkosky et al.,
Physics Letter B
805, 135428 (2020)

ELSEVIER

Physics Letters B
Volume 805, 10 June 2020, 135428

Measurement of the ^{3}He spin-structure functions and of neutron (^{3}He) spin-dependent sum rules at $0.035 \le Q^2 \le 0.24$ GeV^2



- *E97-110 agree with existing data at larger Q^2 (EG1b, E94-010).*
- *E97-110 and χEFT agree up to Q^2 ~0.06 GeV^2 (Bernard et al) or Q^2 >0.08 GeV^2 (Lensky et al.)*
- *Some phenomenological models (Burkert-Ioffe) agree well with data, other (MAID, Pasechnik et al) not as much.*

SAGDH: new Γ_2 data for neutron: Burkhardt–Cottingham sum rule

$$\Gamma_2(Q^2) \equiv \int_0^1 g_2 dx = 0$$

V. Sulkosky et al., Physics Letter B 805, 135428 (2020)

ELSEVIER

Physics Letters B
Volume 805, 10 June 2020, 135428

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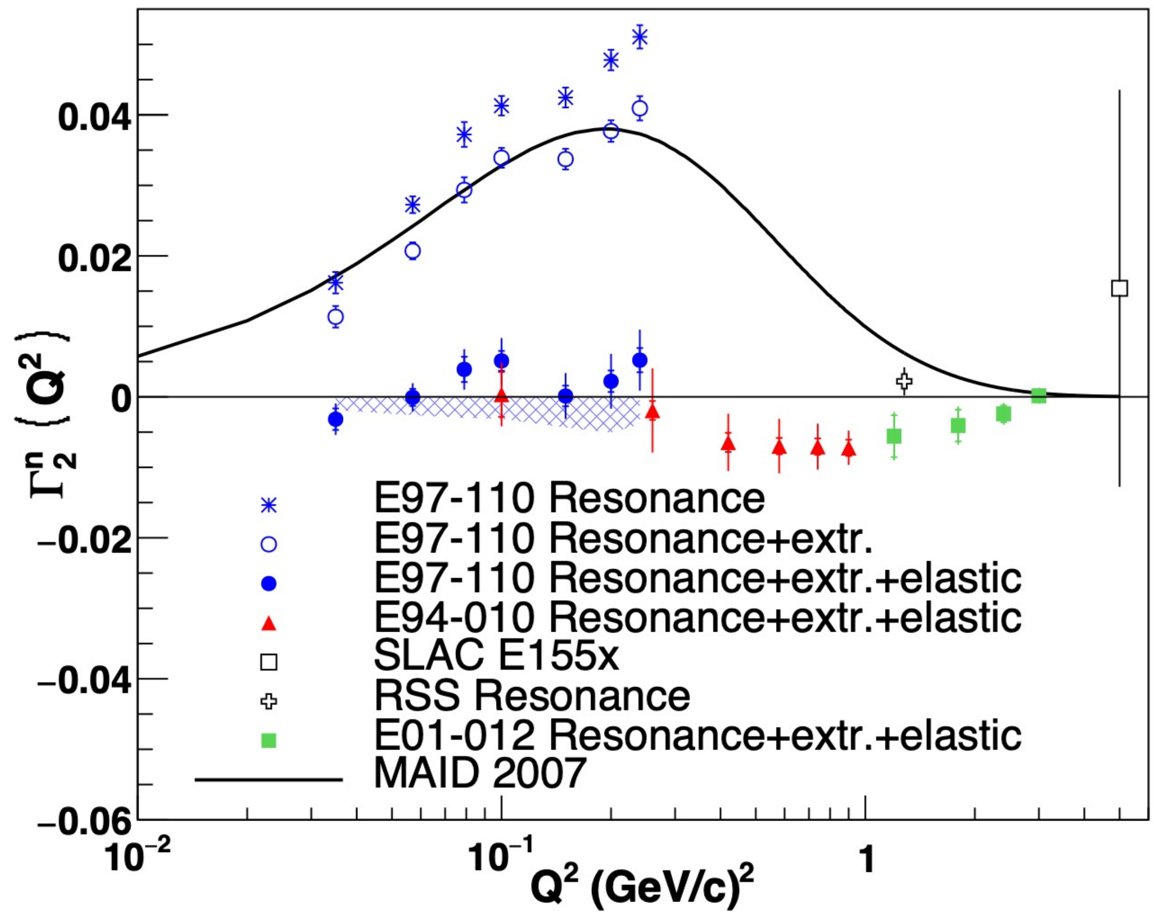
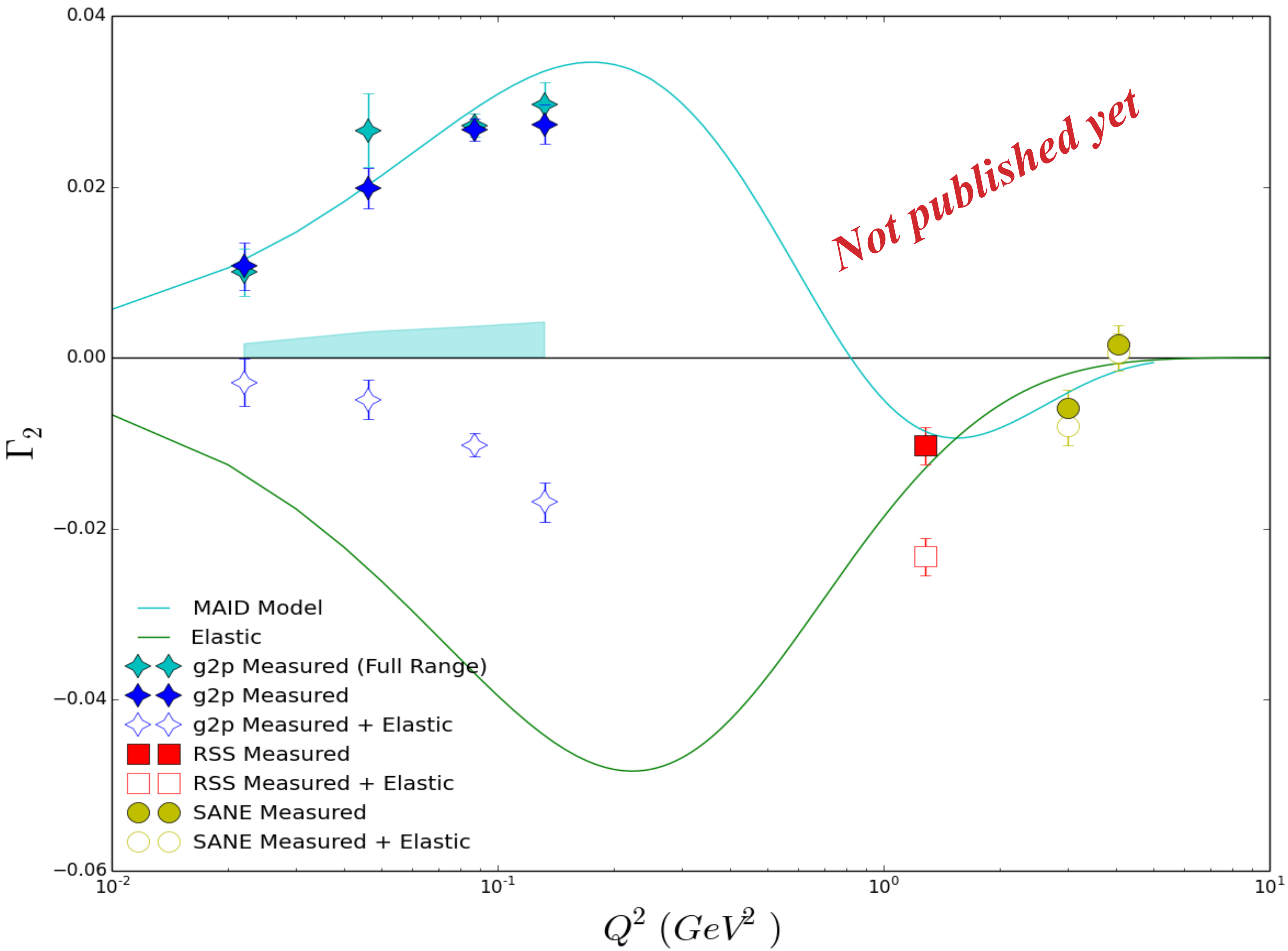


E97-110 verifies the B-C sum rule at low Q^2. Older experiments at higher Q^2 also verify it.

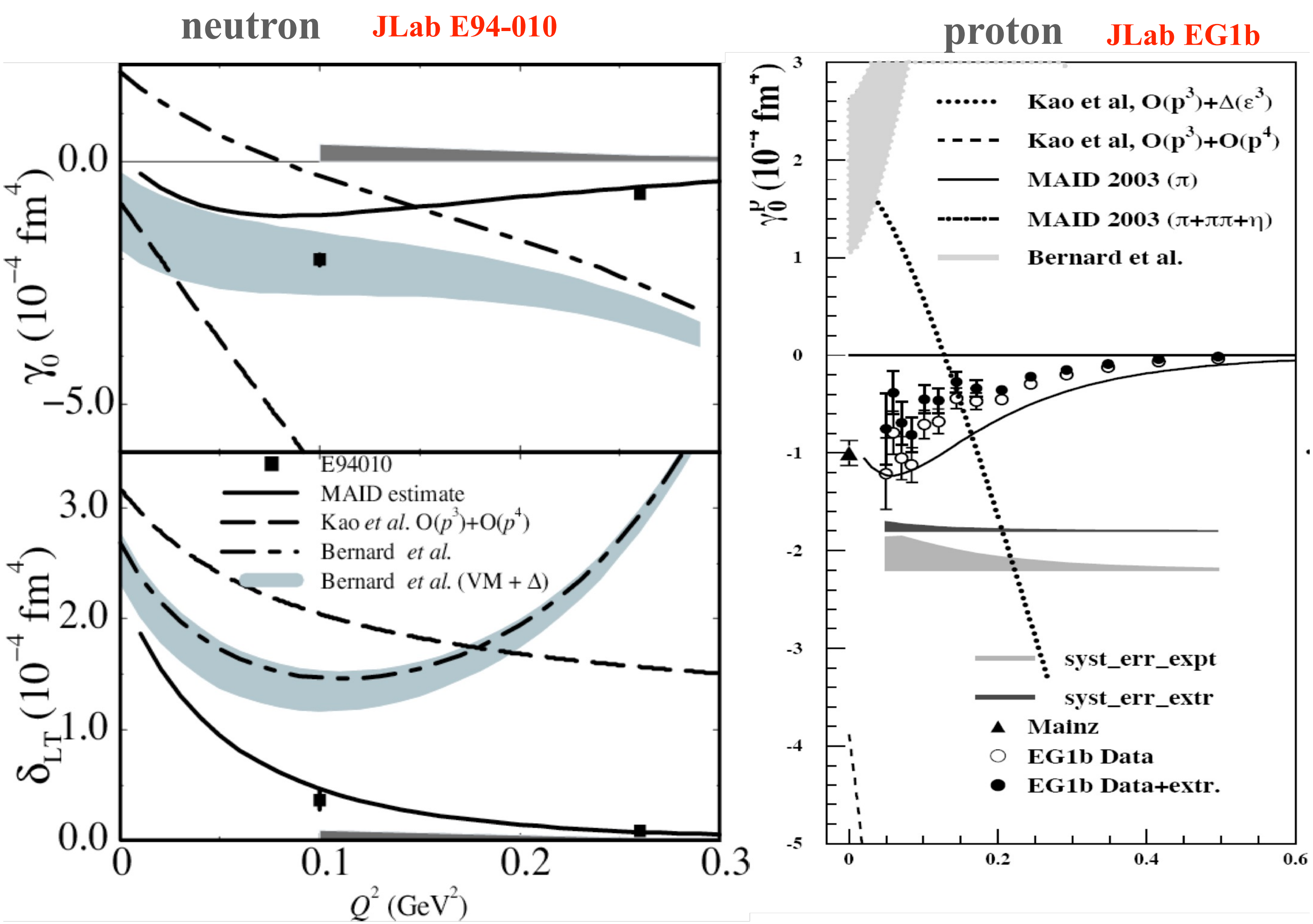
g2p: new Γ_2 data on proton: BC Sum Rule



Generalized Spin Polarizibilities: γ_0 and δ_{LT}

Testing χEFT and study strong QCD

Previous JLab spin polarizabilities data before low-Q experiments



Strong disagreement with χEFT predictions available at that time: **"δ_{LT} puzzle"**

EG4 results on $\gamma_0^p(Q^2)$

X. Zheng et al.,
Nature Physics,
17, 736-741 (2021)

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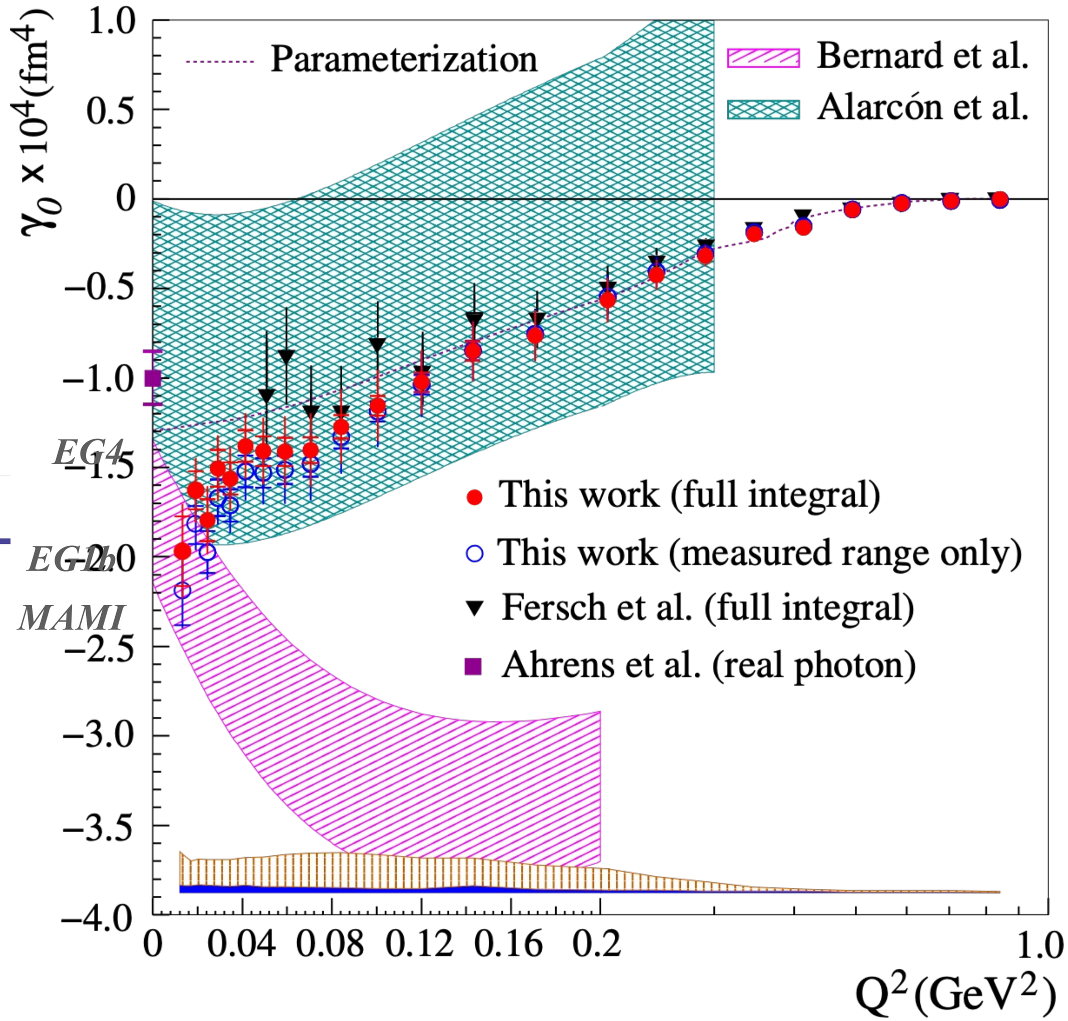

•χEFT result of Alarcón et al agrees with data.
•Bernard et al. χPT calculation agrees for lowest Q^2 points.

Generalized forward spin polarizability γ_0^n from SAGDH

V. Sulkosky et al.,
Nature Physics,
17, 736-741 (2021)

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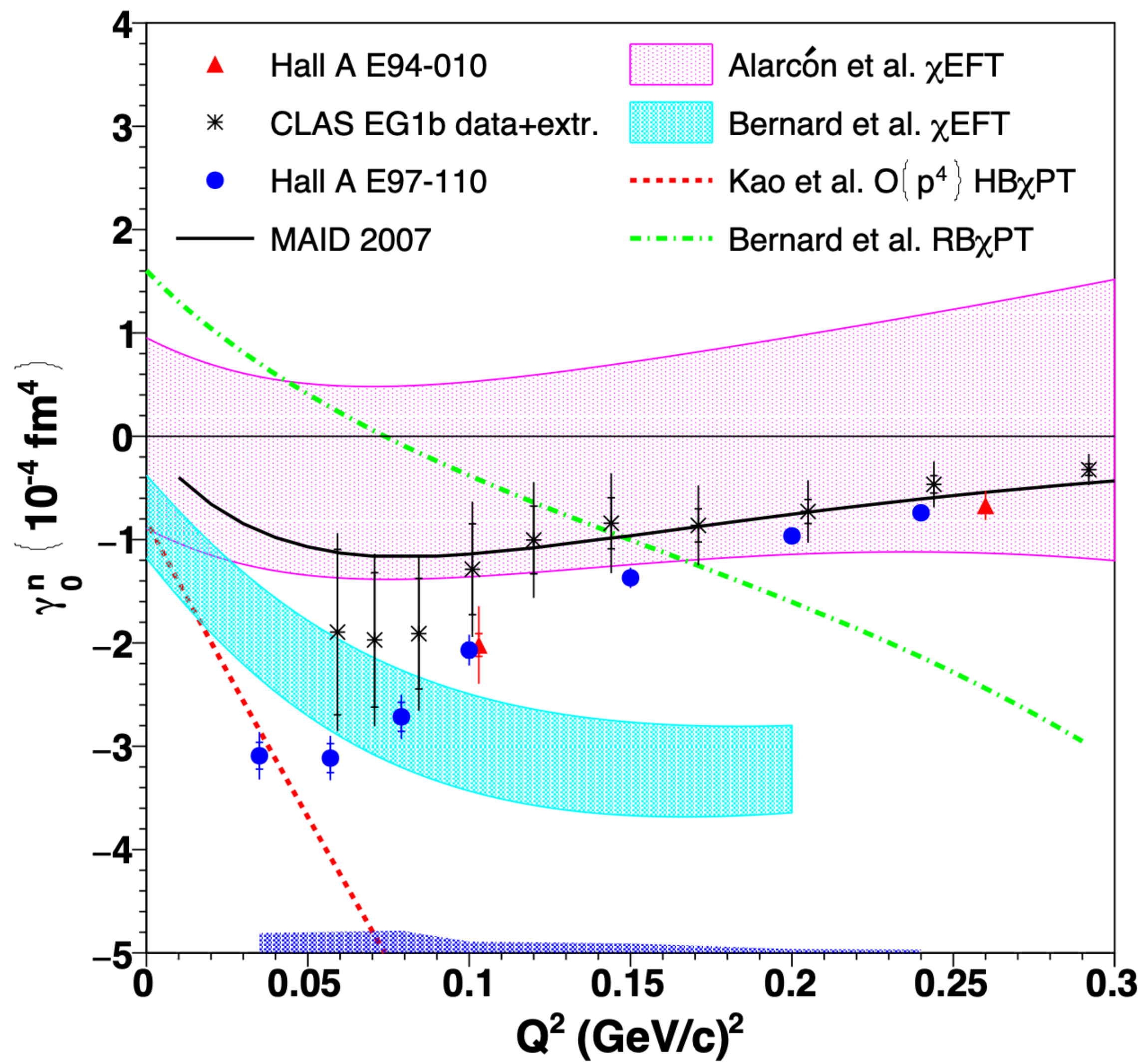
Letter | Published: 31 May 2021

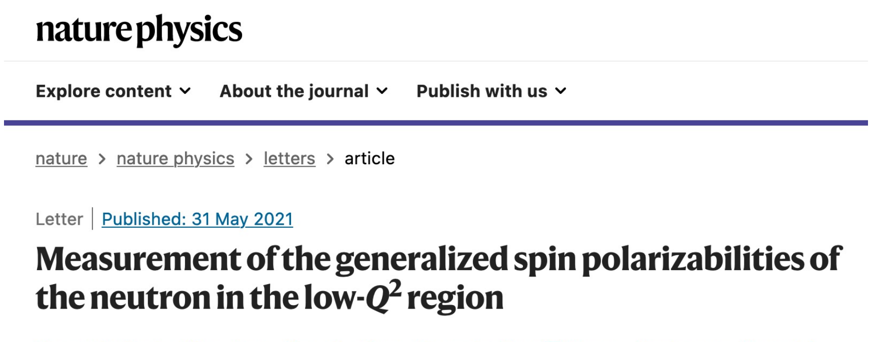
Measurement of the generalized spin polarizabilities of the neutron in the low-Q^2 region



- E97-110 agree with older data at larger Q^2 (EG1b, E94-010). Maid disagrees with the data.
- χEFT result of Alarcón et al disagrees with data.
- Bernard et al. χPT calculation agrees for lowest Q^2 points.

Generalized interference spin polarizability δ_{LT} from SAGDH

V. Sulkosky et al.,
Nature Physics,
17, 736-741 (2021)



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Letter | Published: 31 May 2021

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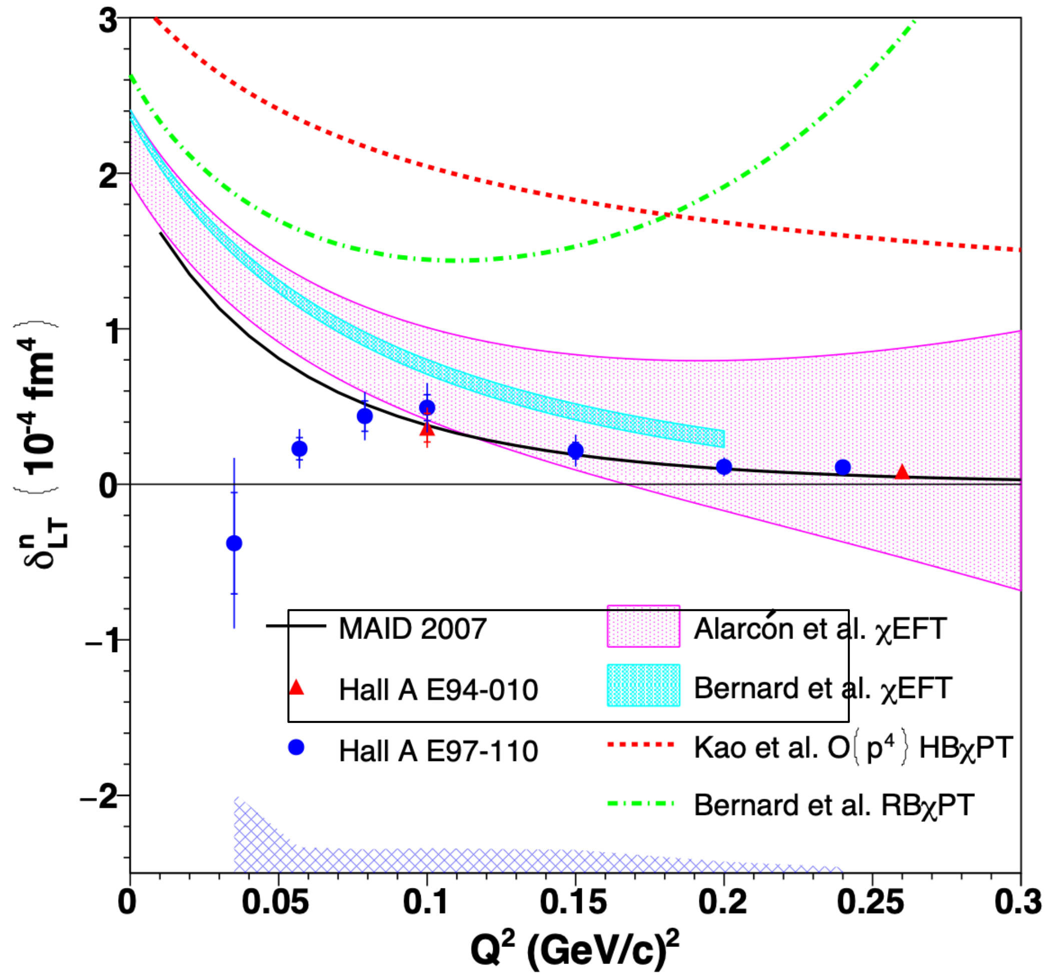
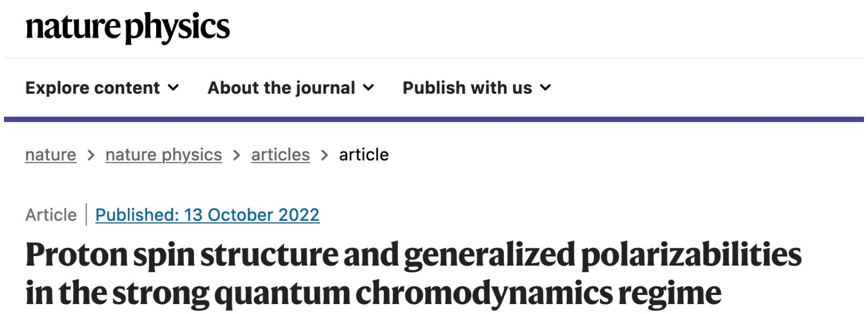


- Good agreement with older data at larger Q^2 and with χEFT & MAID there.
- Disagreement at lower Q^2 (opposite trend)
- “ $\delta^n_{LT}(Q^2)$ puzzle” still remains.

δ_{LT} for Proton from g2p

D. Ruth et al,
Nature Physics
18, 1441 (2022)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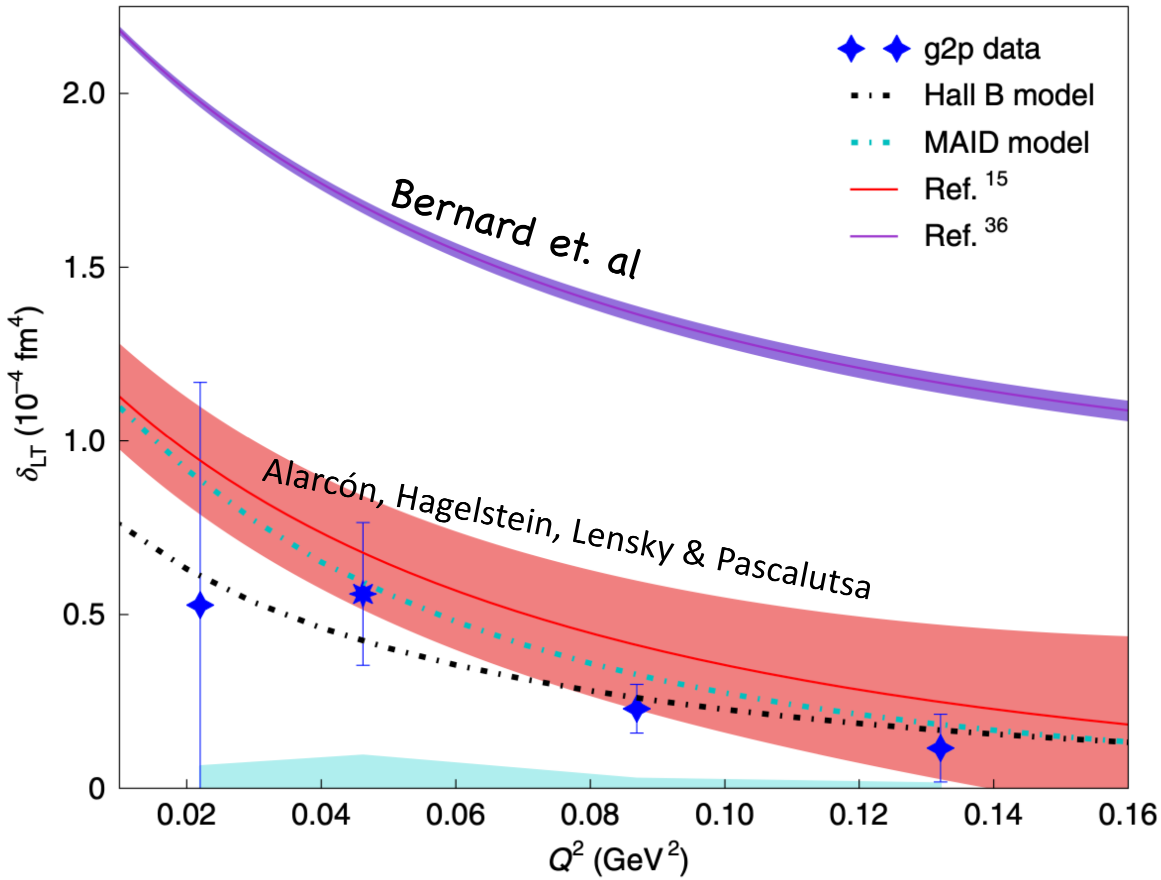
Article | Published: 13 October 2022

Proton spin structure and generalized polarizabilities in the strong quantum chromodynamics regime



- Comparisons with χEFT calculations: favor Alarcon *et al*., strong disagreement with Bernard *et al*.

Bjorken Sum at Low-Q and Effective α_s

Bjorken Sum: Γ_1 of *p-n* (EG4 and SAGDH)

A. Deur, *et al.*
Physics Letter B
825 (2022) 136878

Contents lists available at ScienceDirect
Physics Letters B
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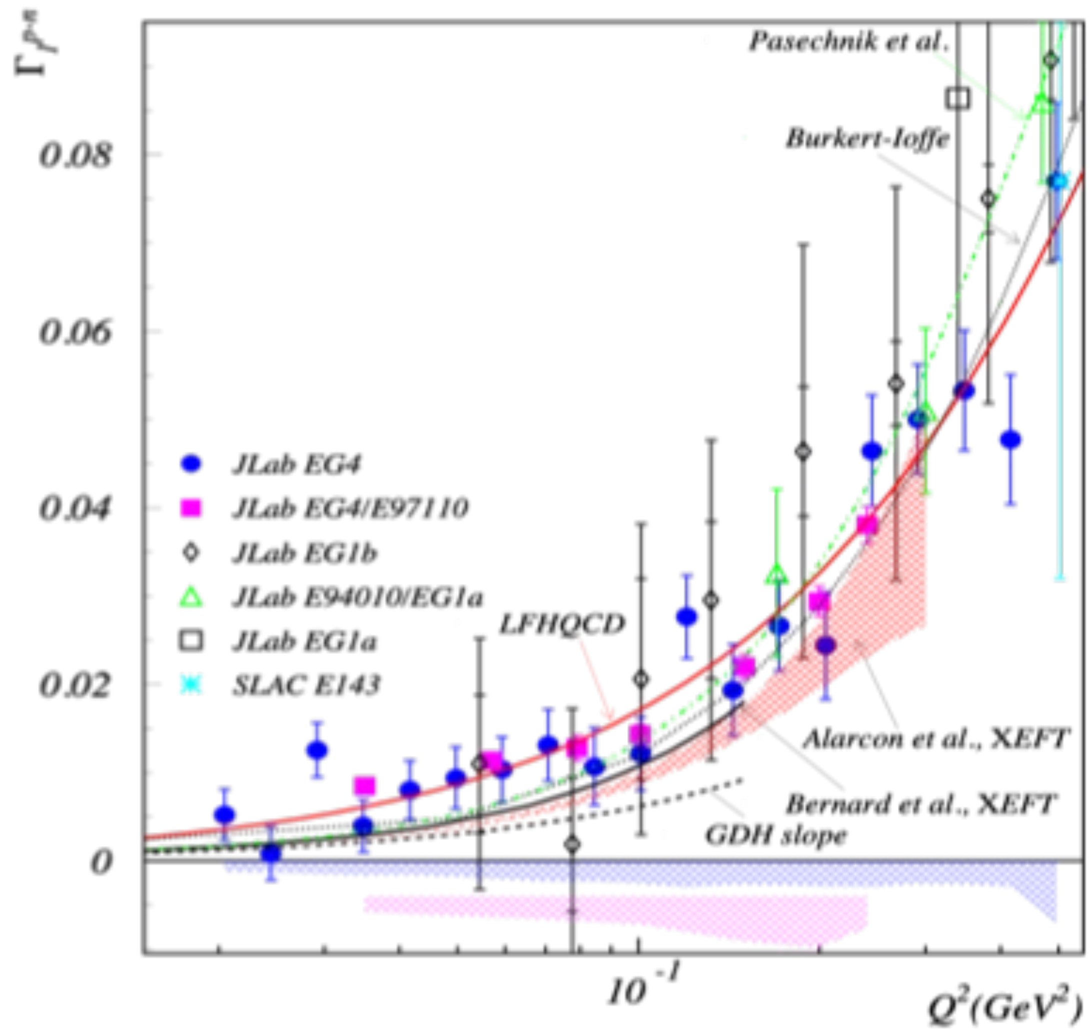
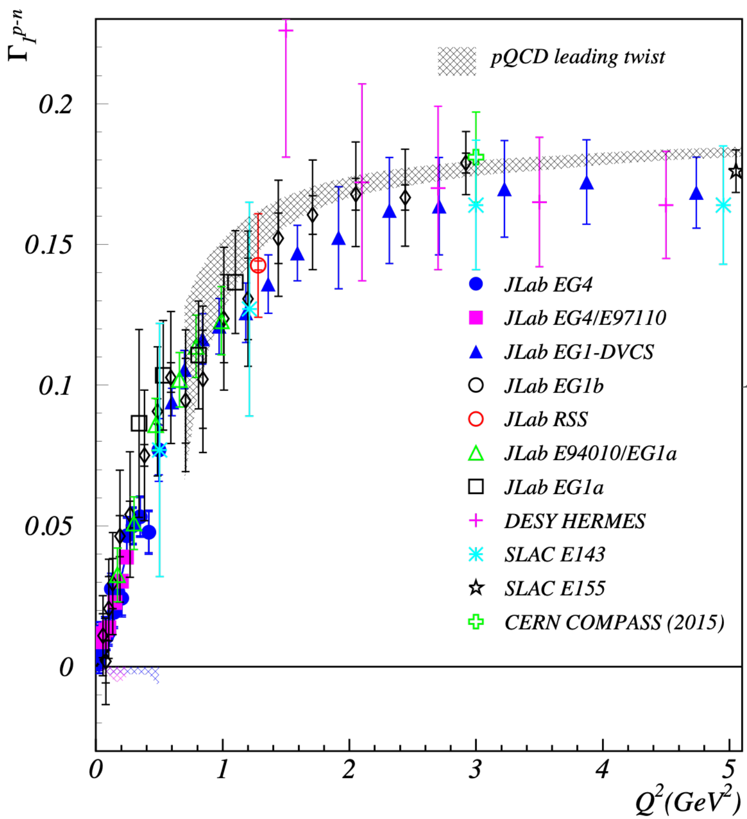
Experimental study of the behavior of the Bjorken sum at very low Q^2

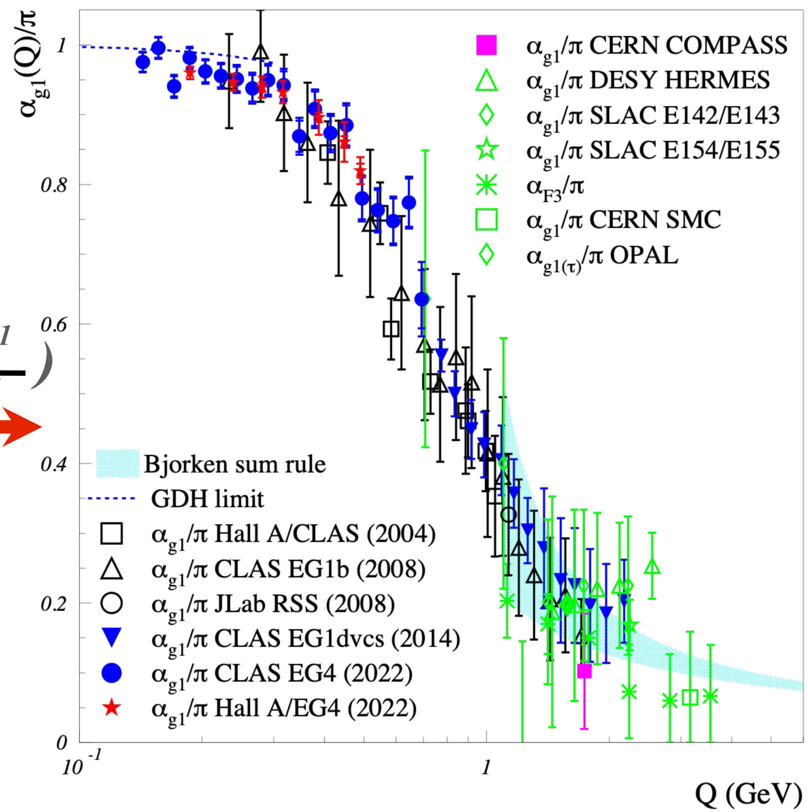


α_{g1} Extracted from the Bjorken Sum data

Bjorken sum Γ_1^{p-n} measurements



$$\Gamma_1^{p-n} \hat{=} \frac{1}{6}\, g_A\left(1-\frac{\alpha_{g1}}{\pi}\right)$$



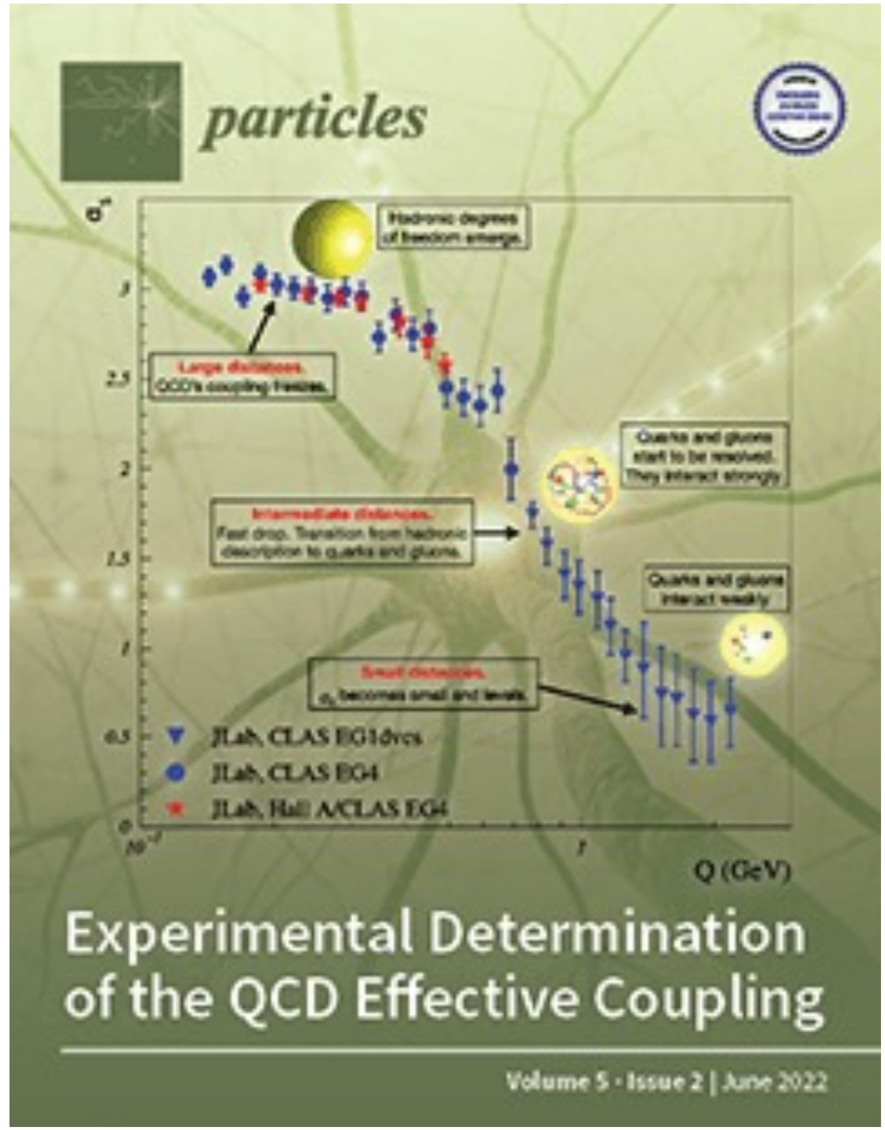
Effective Coupling and Impact

Featured as Cover
Featured in JLab News
https://phys.org/news/2022-08-strength-strong.html
Featured in YouTube
https://www.youtube.com/watch?v=8BTZOz850GI&t=497s
hailed as
"accidental discovery"
"pretty major breakthrough"

Base for understanding of emergence of hadron properties, can have impact on:

hadron spectroscopy
PDFs and GPDs
quark mass functions
pion decay constant
scale of QCD, Λs
QCD Phase/Hot QCD
…

A. Deur, V. Burkert, J. P. Chen and W. Korsch
Particles, 5-171 (2022)



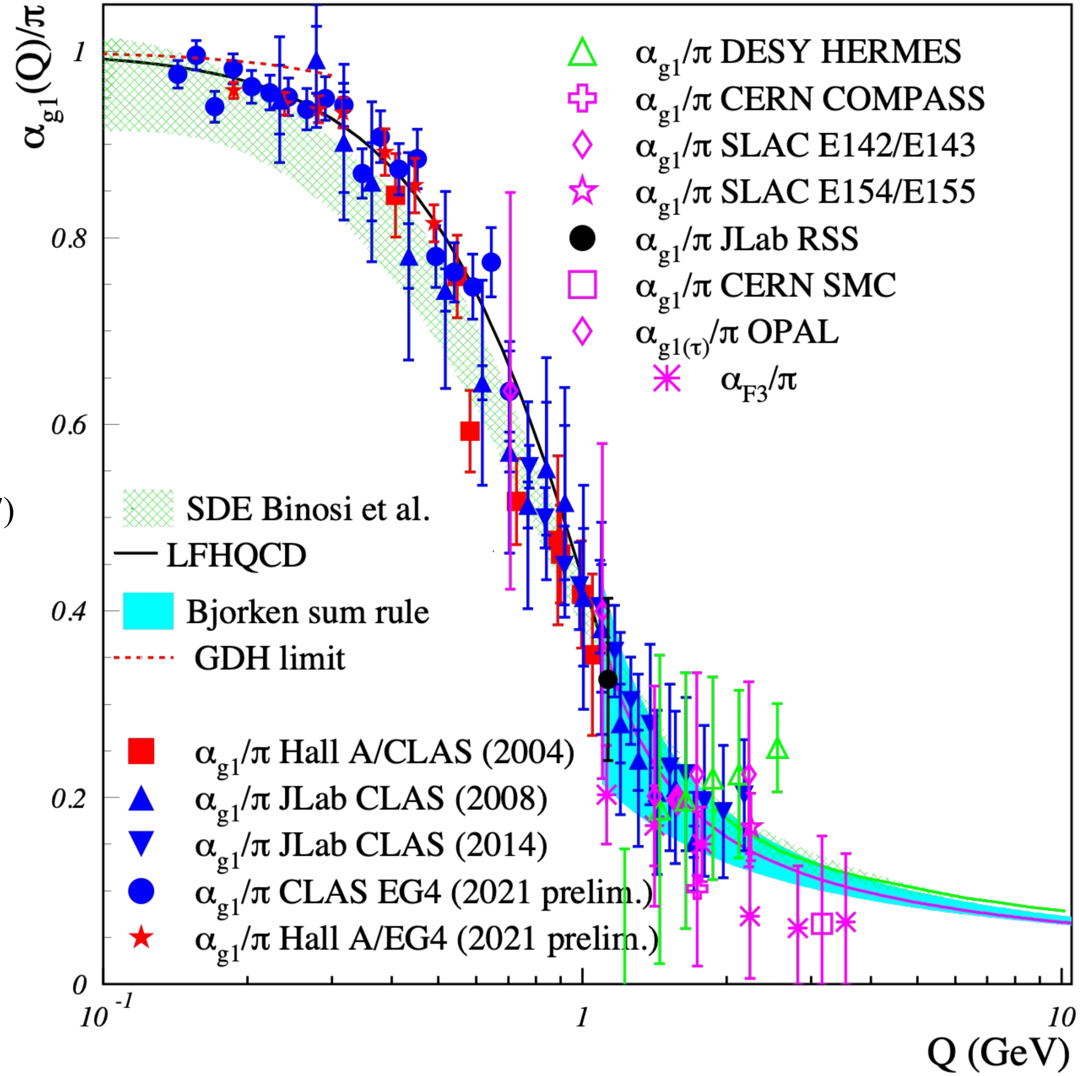
Summary

- Generalized Spin Sum Rules/Polarizabilities
  → clean means to study of QCD over full range of Q2
- Rich results from 3 JLab low-Q spin experiments
  Γ_1, Γ_2, γ_0, δ_{LT} for proton and neutron
  results in 3 *nature physics*, 1 *PRL*, 1 *PLB,* + *more*
  combined results (Bjorken sum) in 1 PLB, α_{g1} extraction in *1 Particle*
- Extensive tests of χEFT calculations
- Lattice QCD predictions becoming available
- Impact in theoretical study of strong QCD
- Spin structure study @ JLab12: A1n/d2n@Hall C, g_1(p/D)@CLAS12
- Future: real photon GDH@Hall D, d2n@SoLID, …
  g2p2 proposal in Hall C (D. Ruth)
  Bjorken sum and α_s extraction @ JLab22 (A. Deur, JLab22 whitepaper)
  …

Comparisons with SDE and LFHQCD Calculations

Binosi et al. PRD 96, 054026 (2017)

Brodsky, de Téramond, Dosch, Lorcé, PLB 759, 171 (2016)



⇒ SDE, LFHQCD and data agree very well.