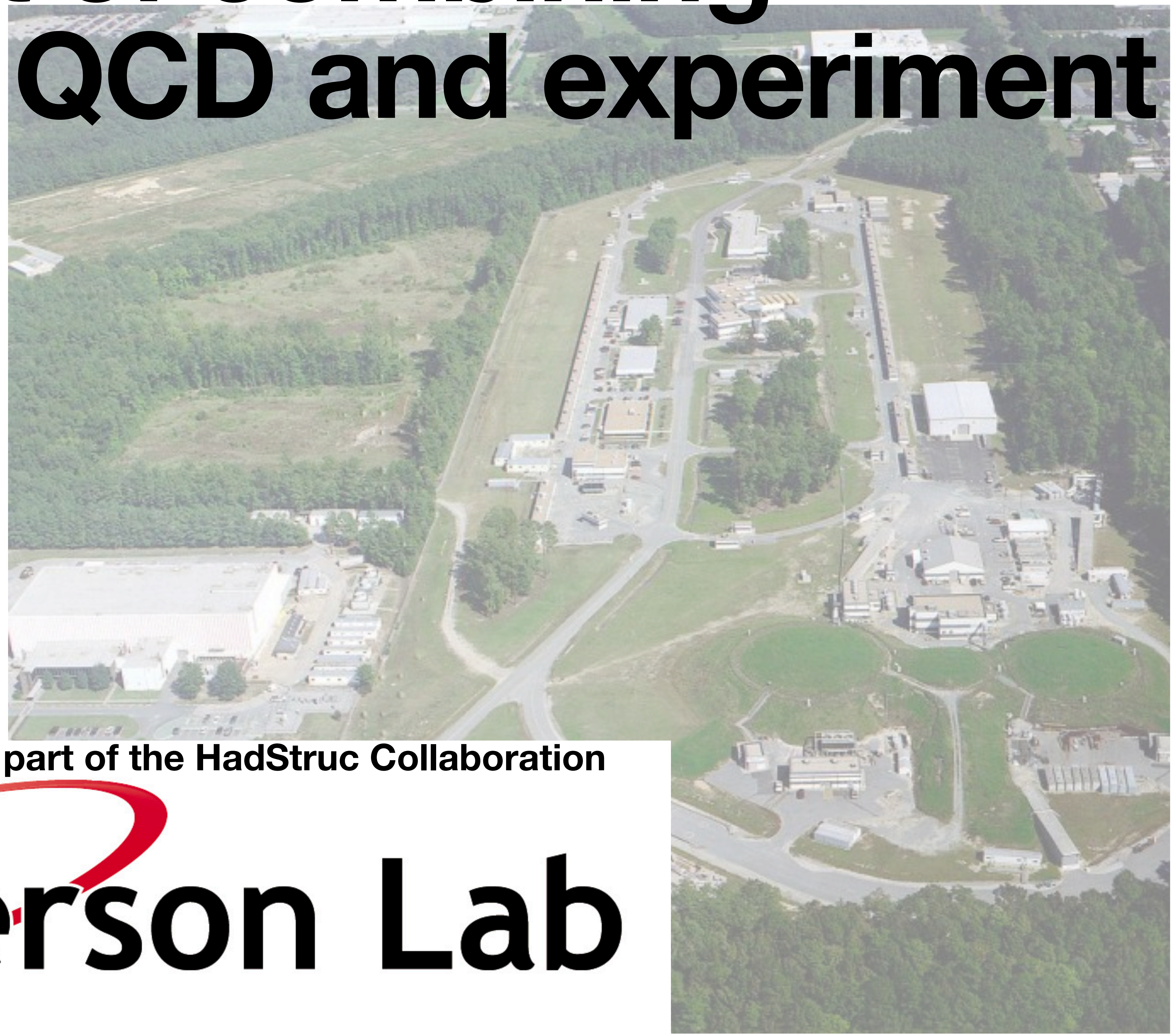
Impact of combining lattice QCD and experiment



Joe Karpie (JLab) part of the HadStruc Collaboration

Jefferson Lab

Parton Structure

For various flavors and spin combinations

Wigner Distribution/
Generalized Transverse Momentum Distribution (GTMD)

$F(x, b_T, k_T)$

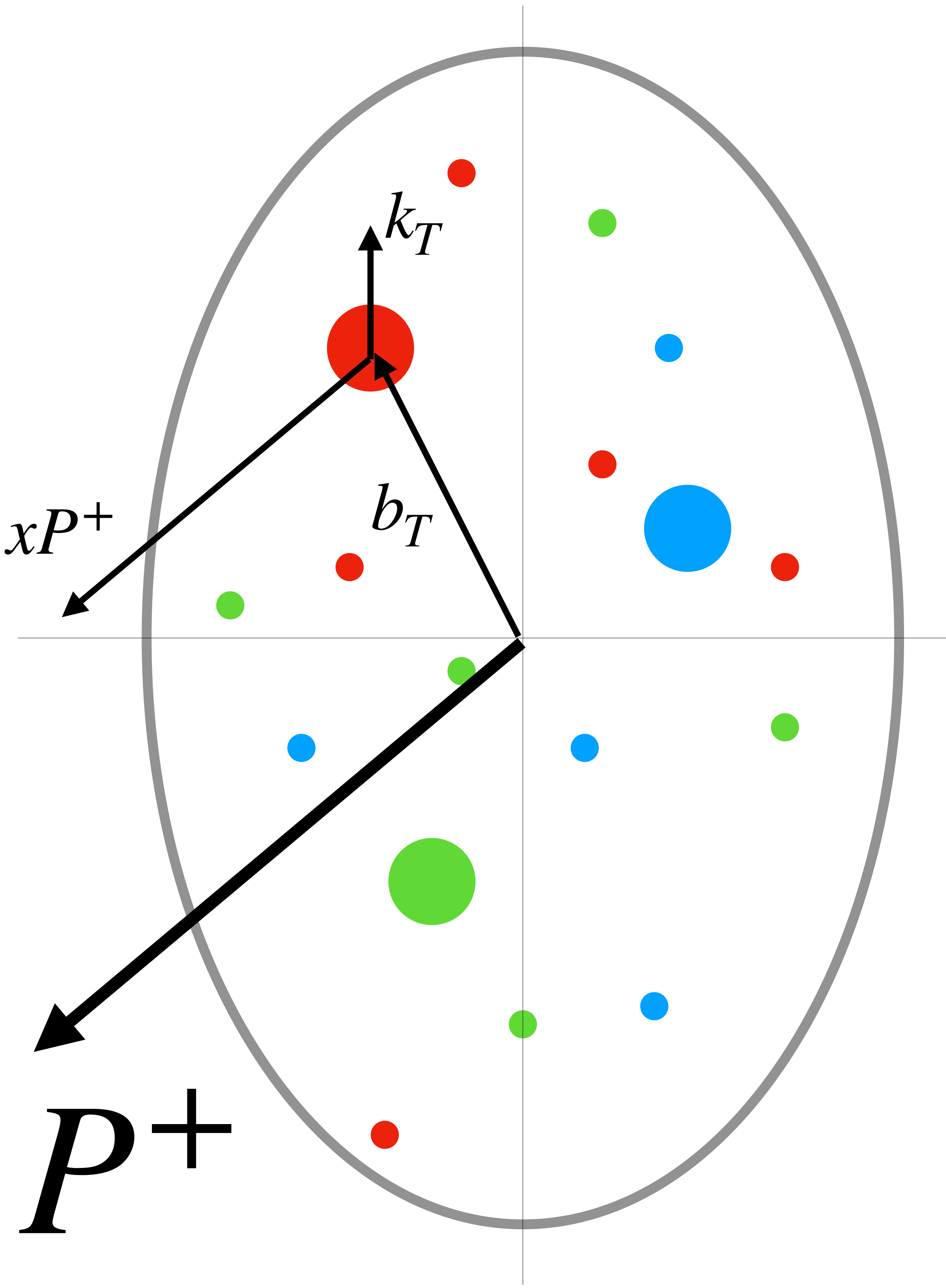
$\int d^2 b_t$ $\qquad$ $\int d^2 k_t$

Transverse Momentum Distribution (TMD)
$f(x, k_T)$

Generalized Parton Distribution (GPD)
$f(x, b_T)$

$\int d^2 k_t$ $\qquad$ $\int d^2 b_t$

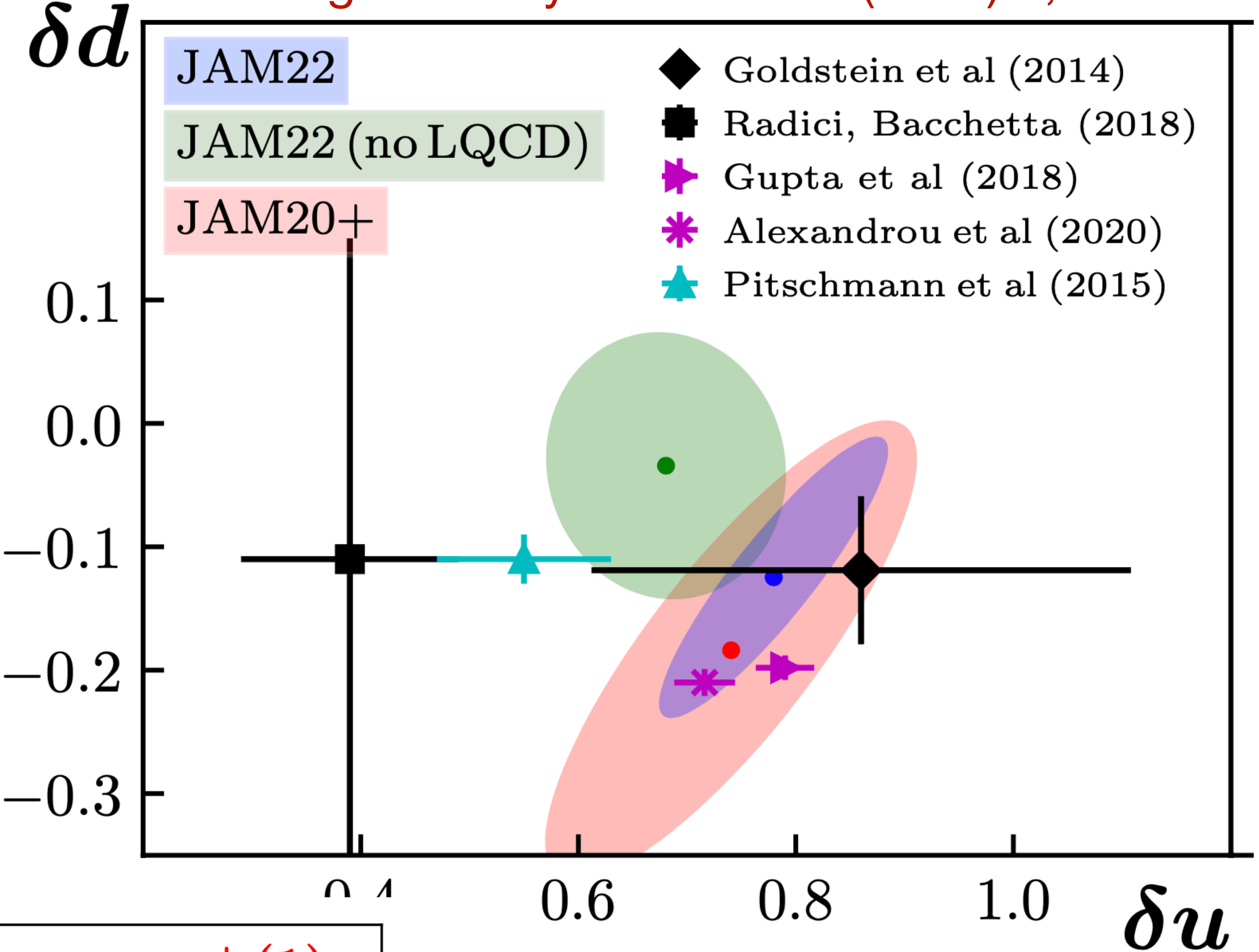
Parton Distribution Function (PDF)
$f(x)$

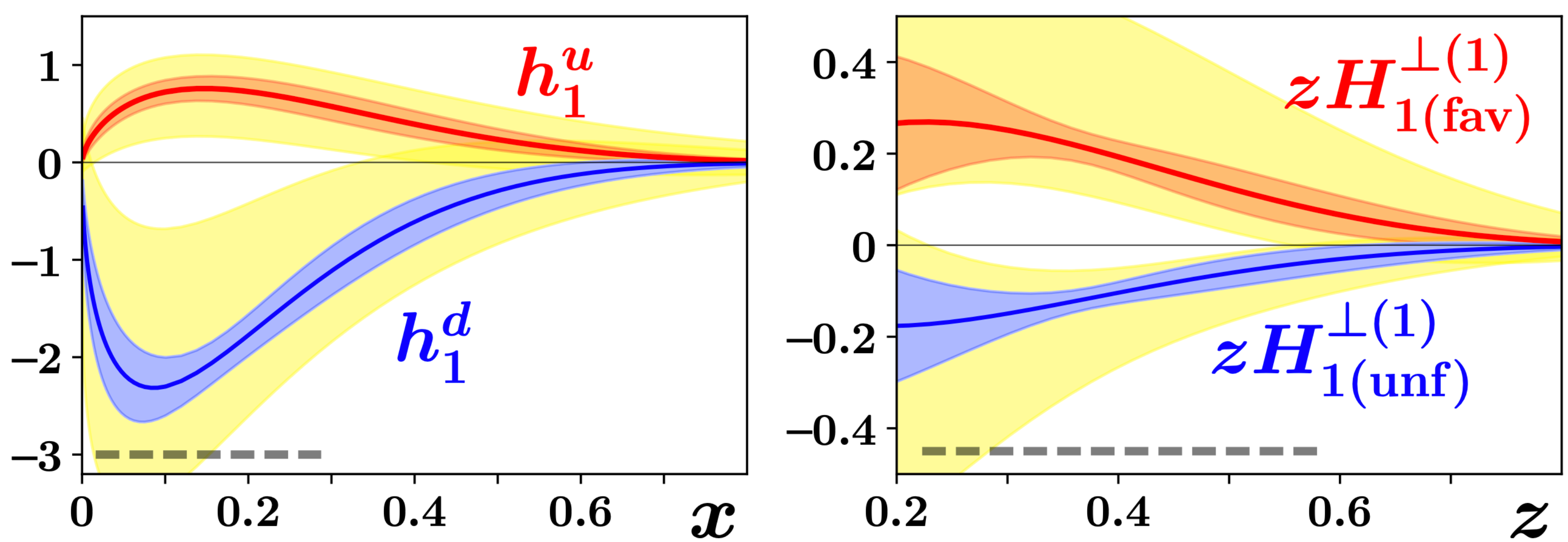


Lattice data can be powerful

- Lattice QCD can calculate matrix elements of many hadrons with numerical methods
- Lattice QCD have cost of theoretical and statistical errors
- Individual local matrix elements added to global analyses already have dramatic impact on final results
- Non local matrix elements can add even more

Gamberg et al Phys Rev D 106 (2022) 3, 034014





Lin et al PRL 120, 152502, (2018)

Parton Distributions and the Lattice

- Parton Distributions are defined by operators with light-like separations

$$M(p,z) = \langle p\,|\,\bar{\psi}(z)\gamma^{\alpha}W(z;0)\psi(0)\,|\,p\rangle$$

- Use space-like separations

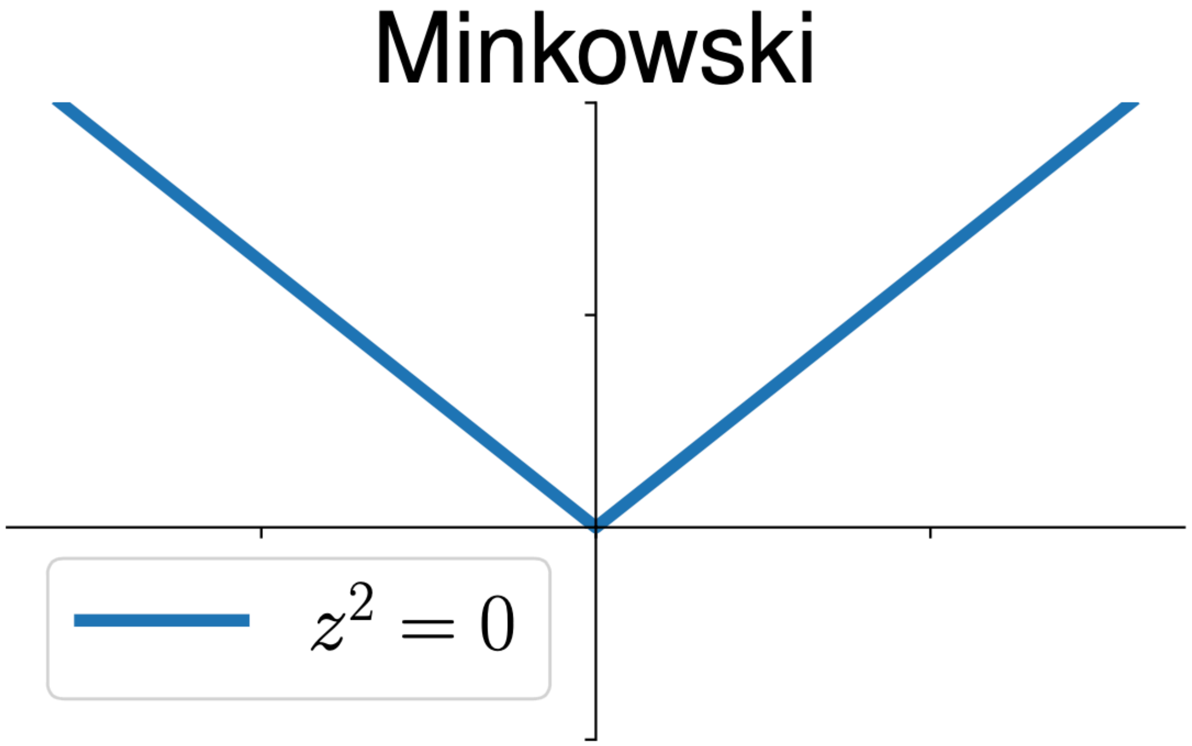
  X. Ji *Phys Rev Lett* 110 (2013) 262002

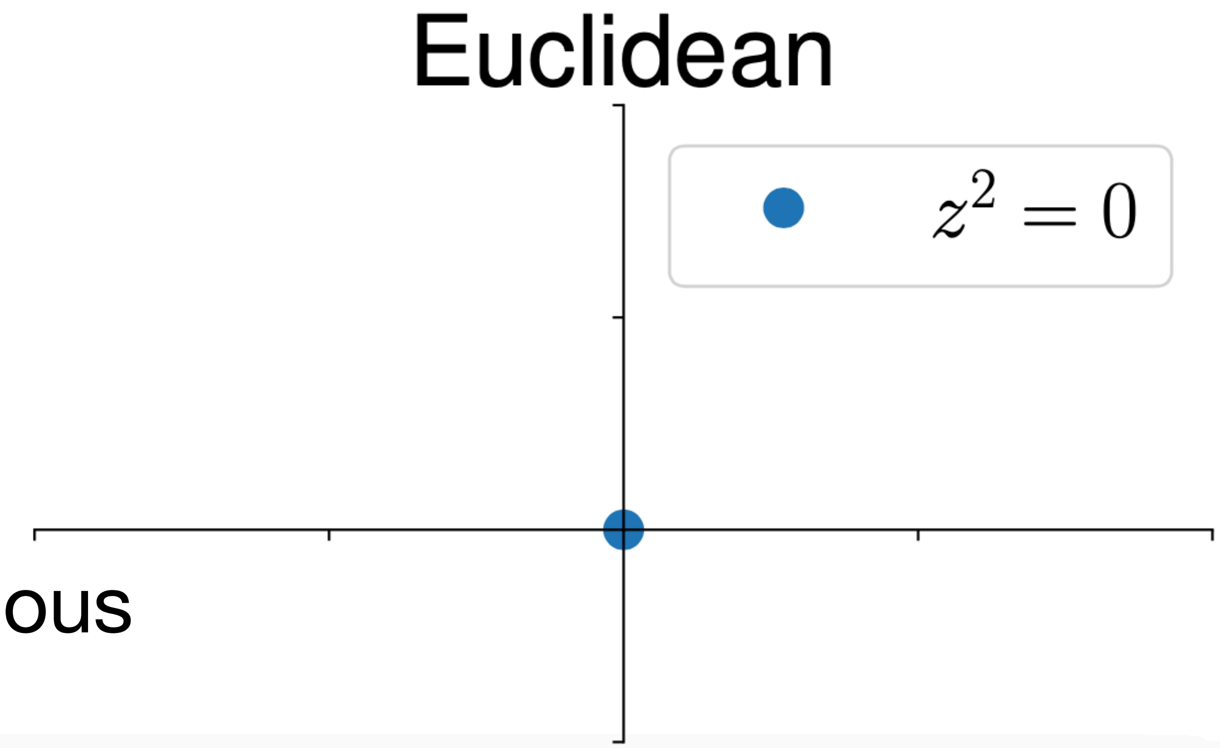
  - Wilson line operators

$$z^2 \neq 0$$

- Fourier transformations of matrix elements give PDF in certain limits

- Factorizable matrix elements analogous to cross sections





Wilson Line Matrix Elements

- In quasi-PDF/LaMET and pseudo-PDF/Short distance, separation and momentum swap roles
    - Matrix element $M(p,z) = \langle p \,|\, \bar{\psi}(z)\gamma^{\alpha} W(z;0)\psi(0) \,|\, p \rangle$

$$= 2p^{\alpha}\mathcal{M}(\nu, z^2) + 2z^{\alpha}\mathcal{N}(\nu, z^2)$$

$$\nu = p \cdot z$$

$$z^2 \neq 0$$

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$\nu = p \cdot z$

$z^2 \neq 0$

  - Quasi-PDF: $\mathcal{M}(z, p_z) = \int_{-\infty}^{\infty} \frac{p_z dy}{2\pi} e^{-iyp_z z}\, \tilde{q}(y, p_z^2)$

  - Large Momentum Effective Theory: X. Ji *Phys. Rev. Lett.* 110 (2013) 262002

$$\tilde{q}(y, p_z^2) = \int \frac{dx}{|x|} K\left(\frac{y}{x}, \frac{\mu^2}{(xp_z)^2}\right) q(x, \mu^2) + O\left(\frac{\Lambda_{\text{QCD}}^2}{(xp_z)^2}, \frac{\Lambda_{\text{QCD}}^2}{((1-x)p_z)^2}\right)$$

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  - Pseudo-ITD:

$$\mathscr{M}(\nu, z^2) = \int dx\, C(x\nu, \mu^2 z^2) q(x, \mu^2) + O(\Lambda_{\text{QCD}}^2 z^2)$$

A. Radyushkin *Phys. Rev. D* 96 (2017) 3, 034025

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  - Pseudo-ITD:

Integral inverse problem like global analysis

$$\mathcal{M}(\nu, z^2) = \int dx\, C(x\nu, \mu^2 z^2) q(x, \mu^2) + O(\Lambda^2_{\rm QCD} z^2)$$

A. Radyushkin *Phys. Rev. D* 96 (2017) 3, 034025

If PDFs are universal, then….

If the same universal PDFs are factorizable from lattice and experiment, and if power corrections can be controlled for both

Why not analyze both simultaneously?

- Factorization of hadronic cross sections

$$d\sigma_h = d\sigma_q \otimes f_{h/q} + P.C.$$

- Factorization of Lattice observables

$$M_h = M_q \otimes f_{h/q} + P.C.$$

Consider Lattice as a theoretical prior to the experimental Global Fit

The Comparison to DIS

- In DIS and pseudo-PDF/Short distance the two variables are analogous

Factorization Scale:

Q^2 / z^2

- Describes scale in hard part
- Scale for factorization to PDF
- Scale in power expansion
- Keep away from $\Lambda^2_{\rm QCD}$
- Technically only requires single value

NEED!

Dynamical variable:

x_B / ν

- Describes interesting partonic structure
- Variable for integration/inverse problems of x
- Can take large or small value
- Want as many as are available
- Wider range improves the inverse problem if you really want x-space

WANT!

Complementarity in Lattice and Experiment

LATTICE

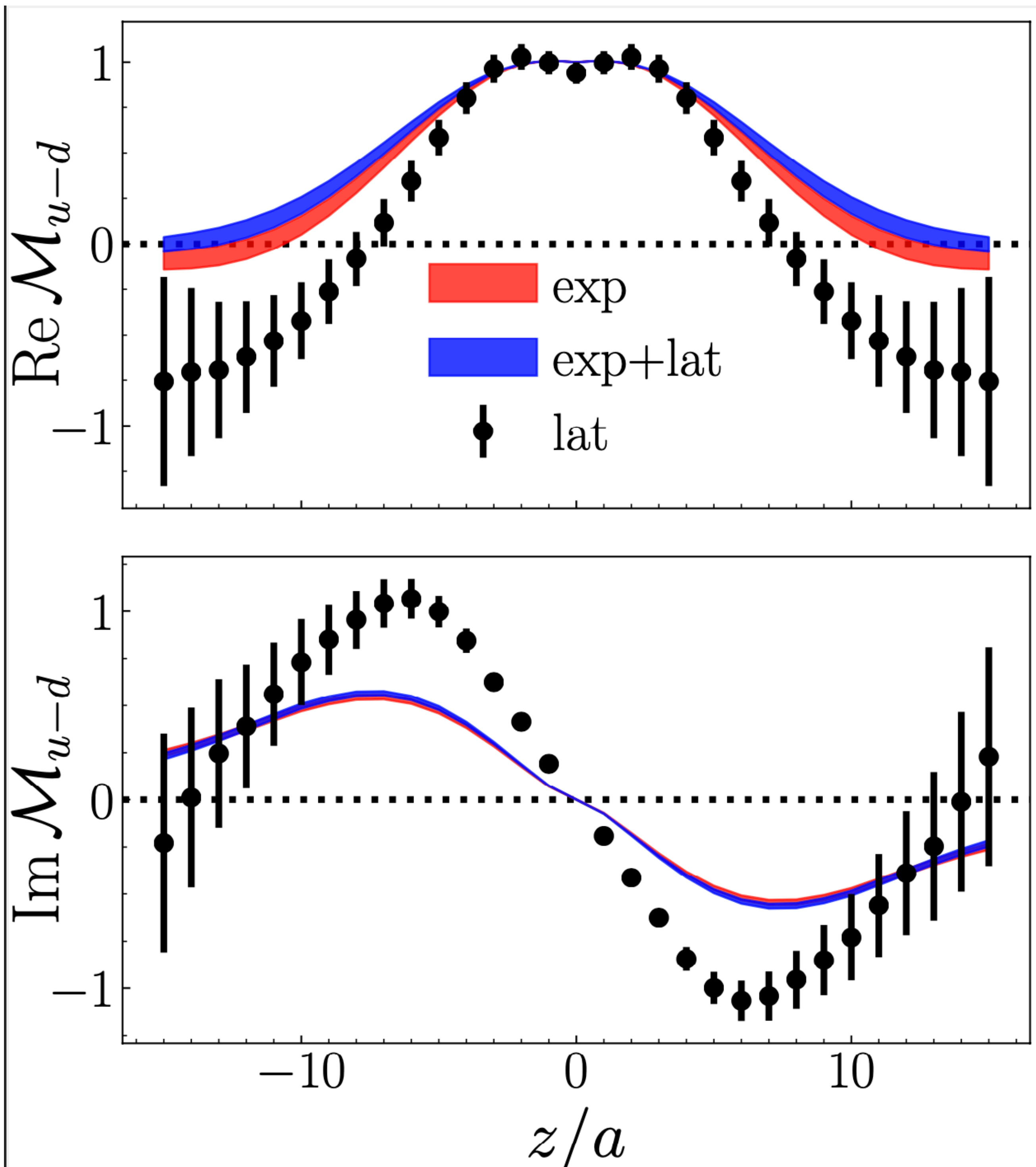
- Lattice limited to low ν, sensitive to $x > \sim 0.2$, but high sensitivity to large x
- Lattice matching relation is integral over all x
- Low p_z data can reach high signal-to-noise compared to experiment
- Lattice can evaluate independently each spin, flavor, and even hadron

EXPERIMENT

- Cross Sections limited to specific max but can reach very low x_B
- Cross Section matching is integral from x_B to 1
  - Creates sensitively to hard kernel in large x region
- Wealth of decades of experimental data outweigh modern lattice

First combined lattice PDF and experiment global analysis (unpol)

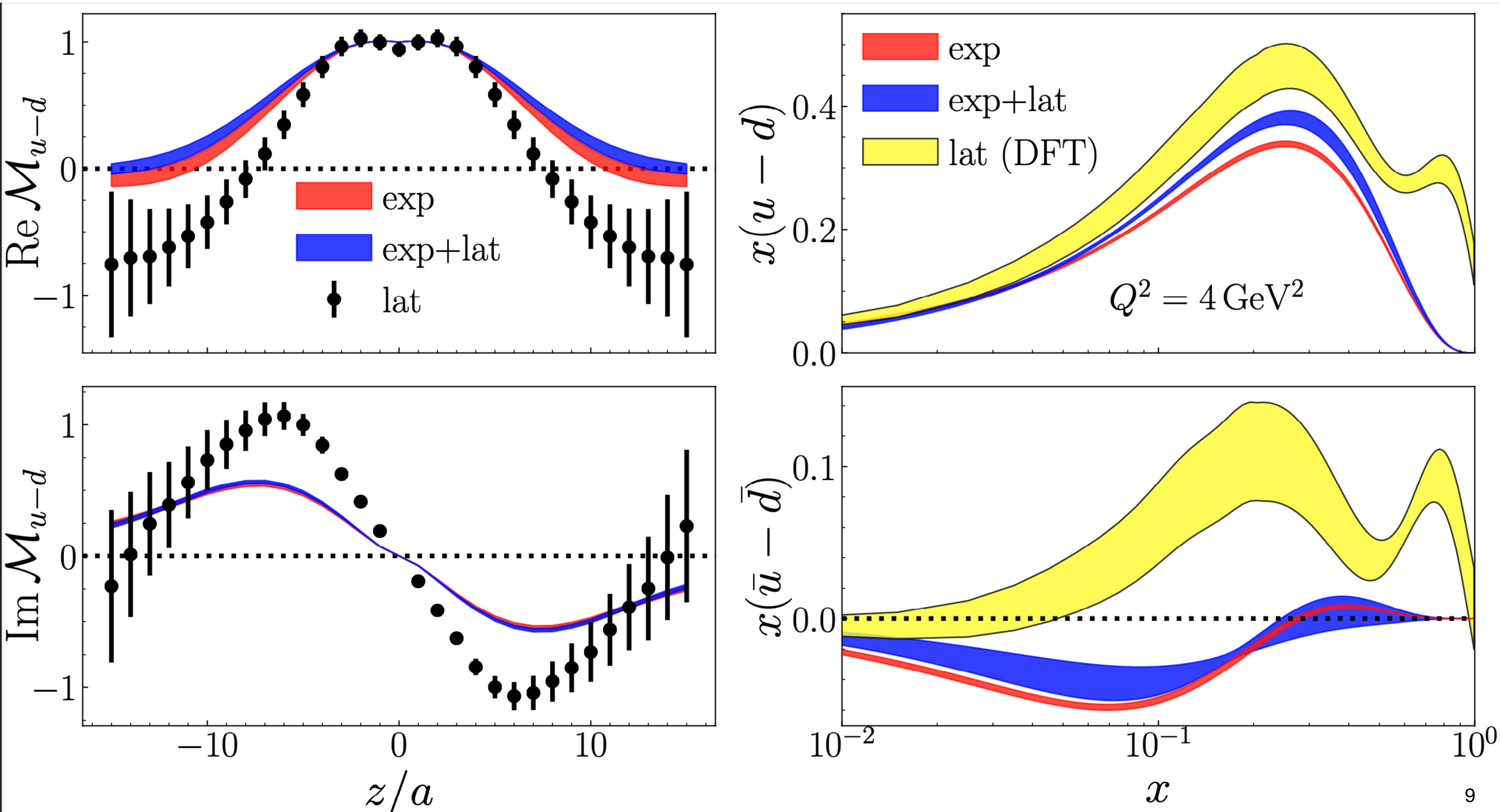
J. Bringewatt et al Phys Rev D 103, 016003 (2021)



- First study by ETMC and JAM collaborations
- Lattice data provide independent measurements of PDFs
- Can study discrepancies to understand systematic errors

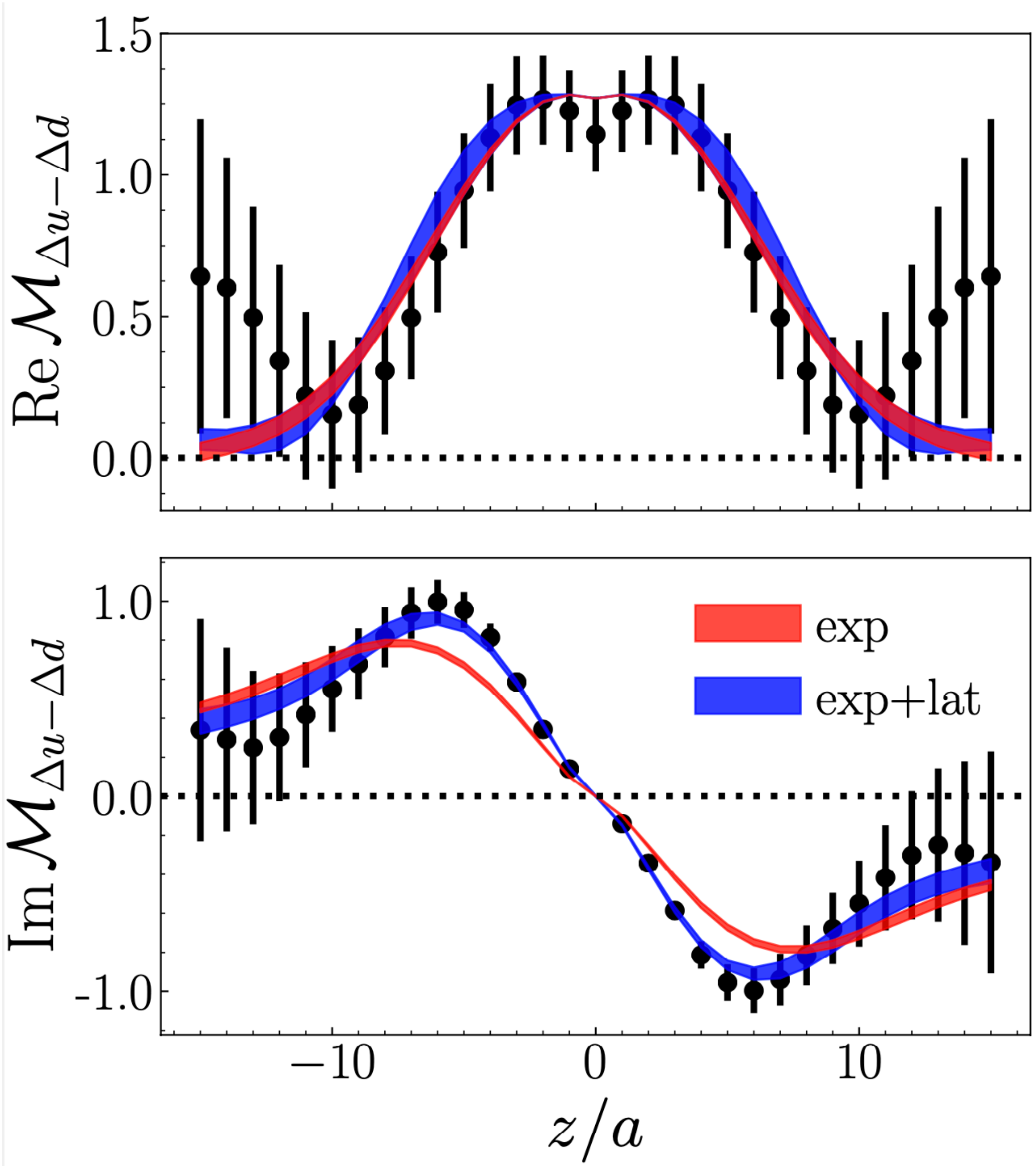
First combined lattice and experiment global analysis (unpol)

J. Bringewatt et al Phys Rev D 103, 016003 (2021)



First combined lattice and experiment global analysis (heli)

J. Bringewatt et al Phys Rev D 103, 016003 (2021)

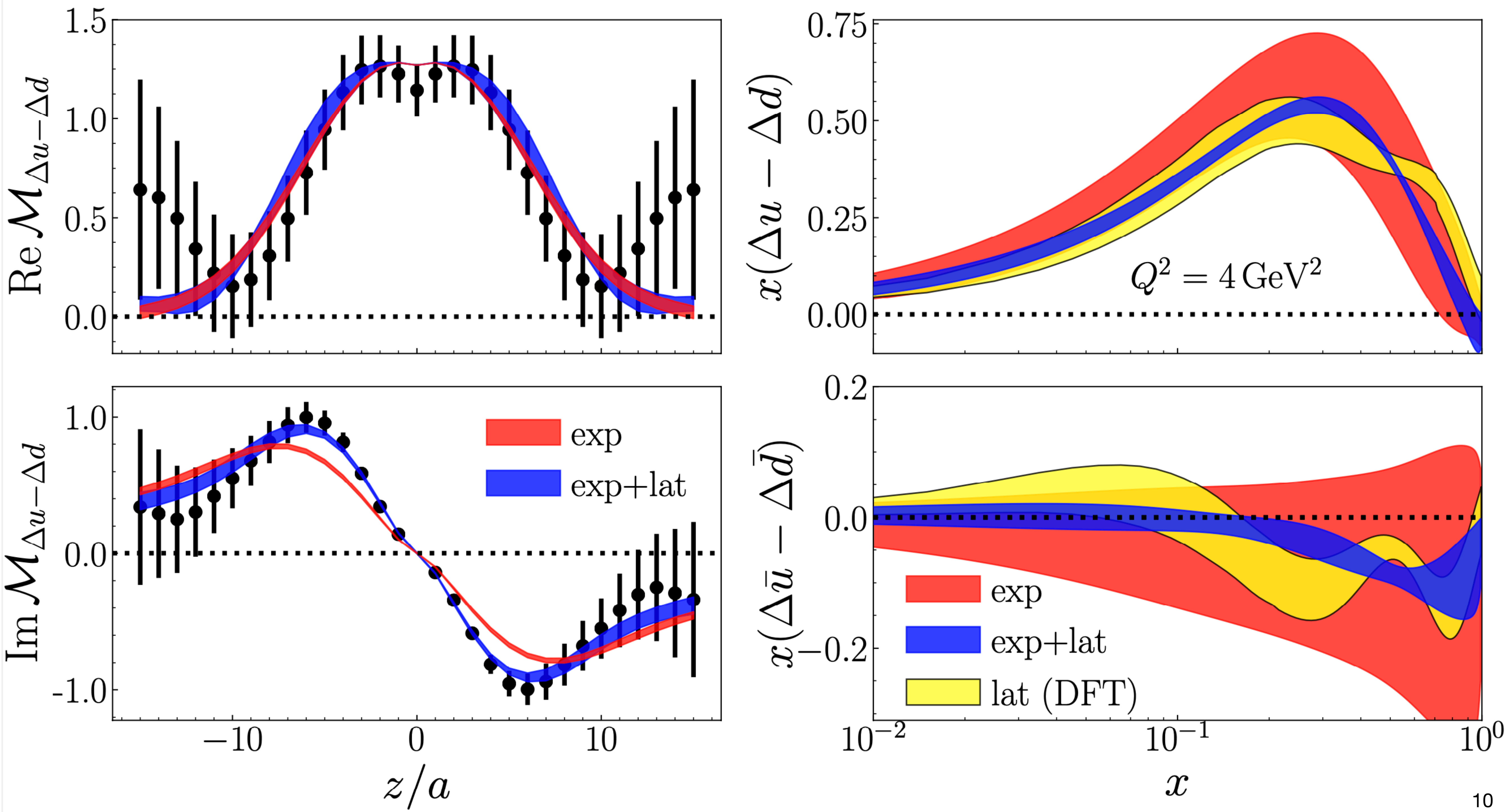


- Lattice matrix elements can give direct independent information on different spins without major modifications

- Some datapoints can be more precise than experiment and give constraining power

First combined lattice and experiment global analysis (heli)

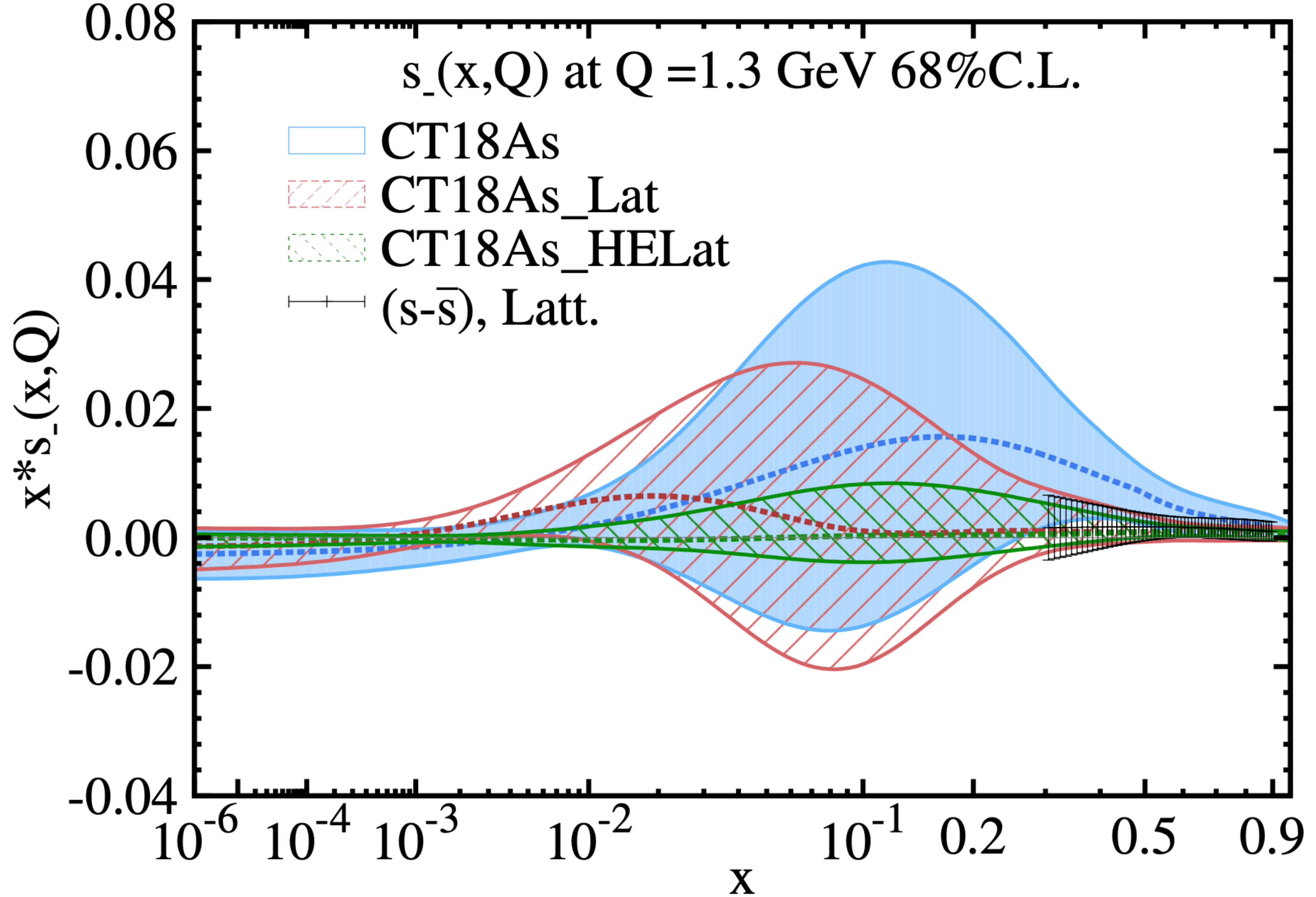
J. Bringewatt et al Phys Rev D 103, 016003 (2021)



Strange quark distributions

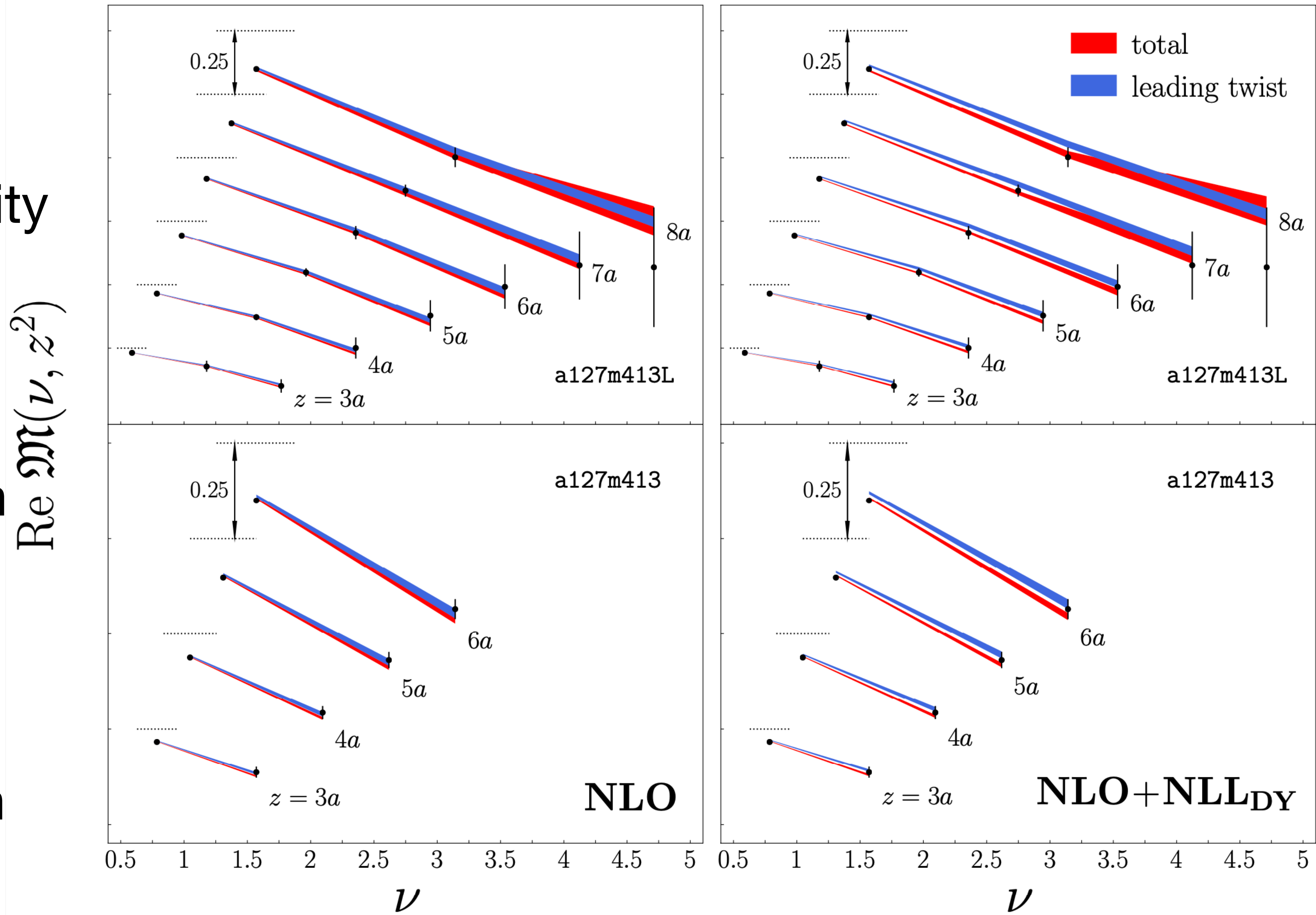
- Lattice can directly access individual quark flavors independently
- Flavor decomposed matrix elements have noisy “disconnected” contributions
- Studies of strange and charm PDFs have begun and give promising precision

Hou et al arXiv 2204.07944



Complementarity in pion PDF

- Lattice can directly access different hadrons
- Lattice lacks sensitivity to threshold logs and can be used to test theoretical kernels
- Improves precision in large x where experimental data does not exist
- Low momentum pion data are extremely precise



P. Barry et al, *Phys. Rev. D* 105 (2022) 11, 114051

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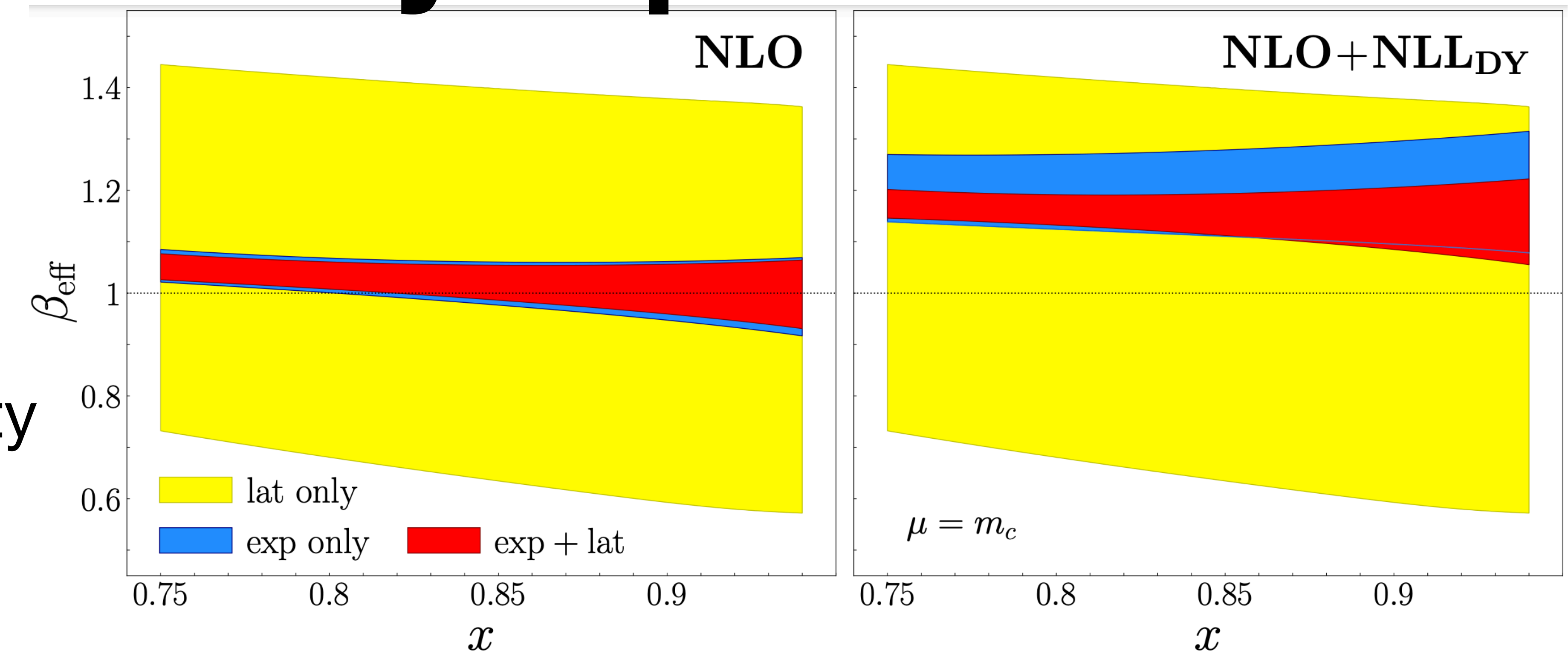
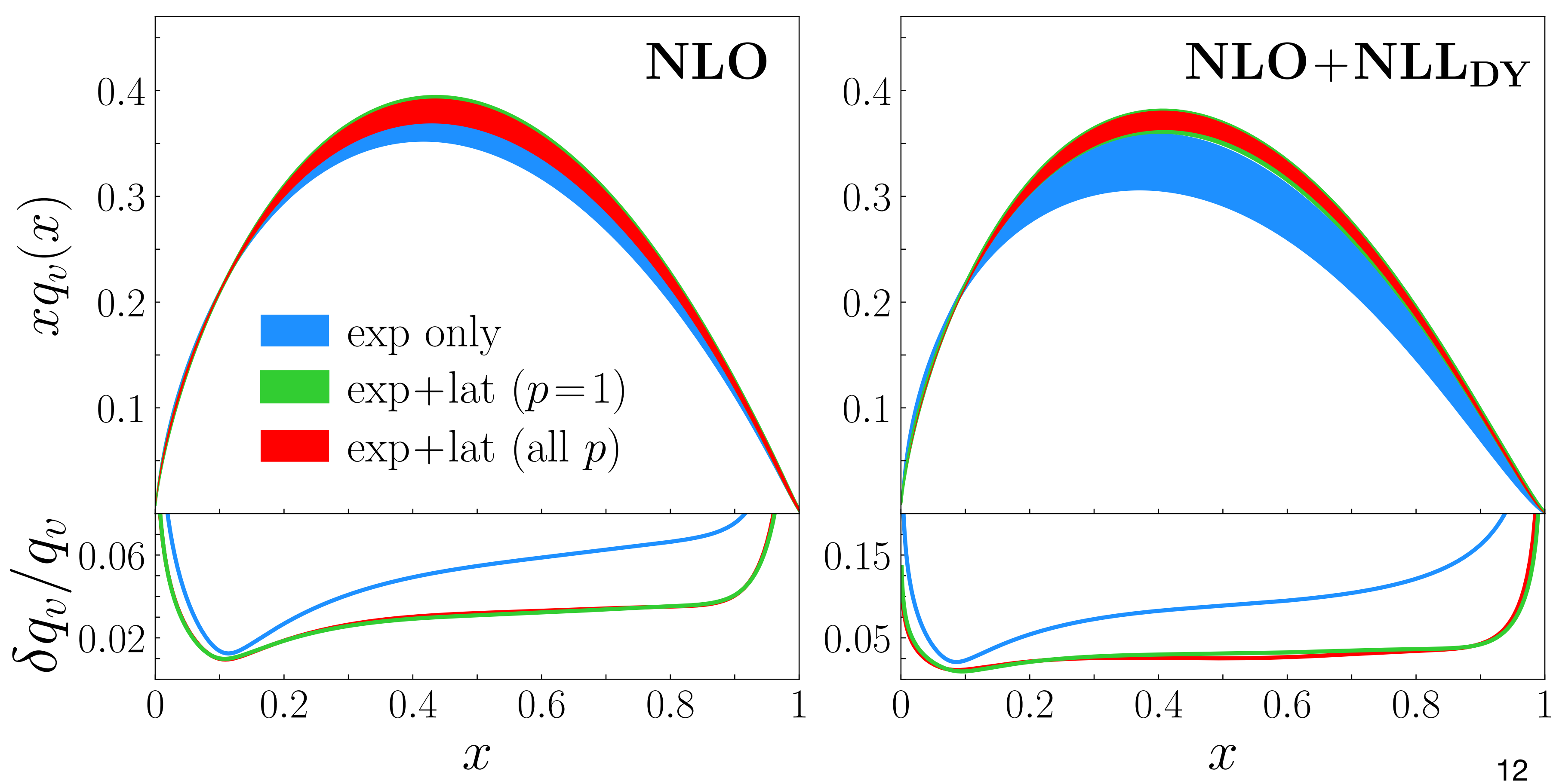


P. Barry et al, *Phys. Rev. D* 105 (2022) 11, 114051



R. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990)

Spinning gluons

$$J = \frac{1}{2}\Delta\Sigma + L_q + L_G + \Delta G$$

$$\Delta G = \int dx\, \Delta g(x)$$

- Positivity constraints are invalid

$$|\Delta g| \leq g$$

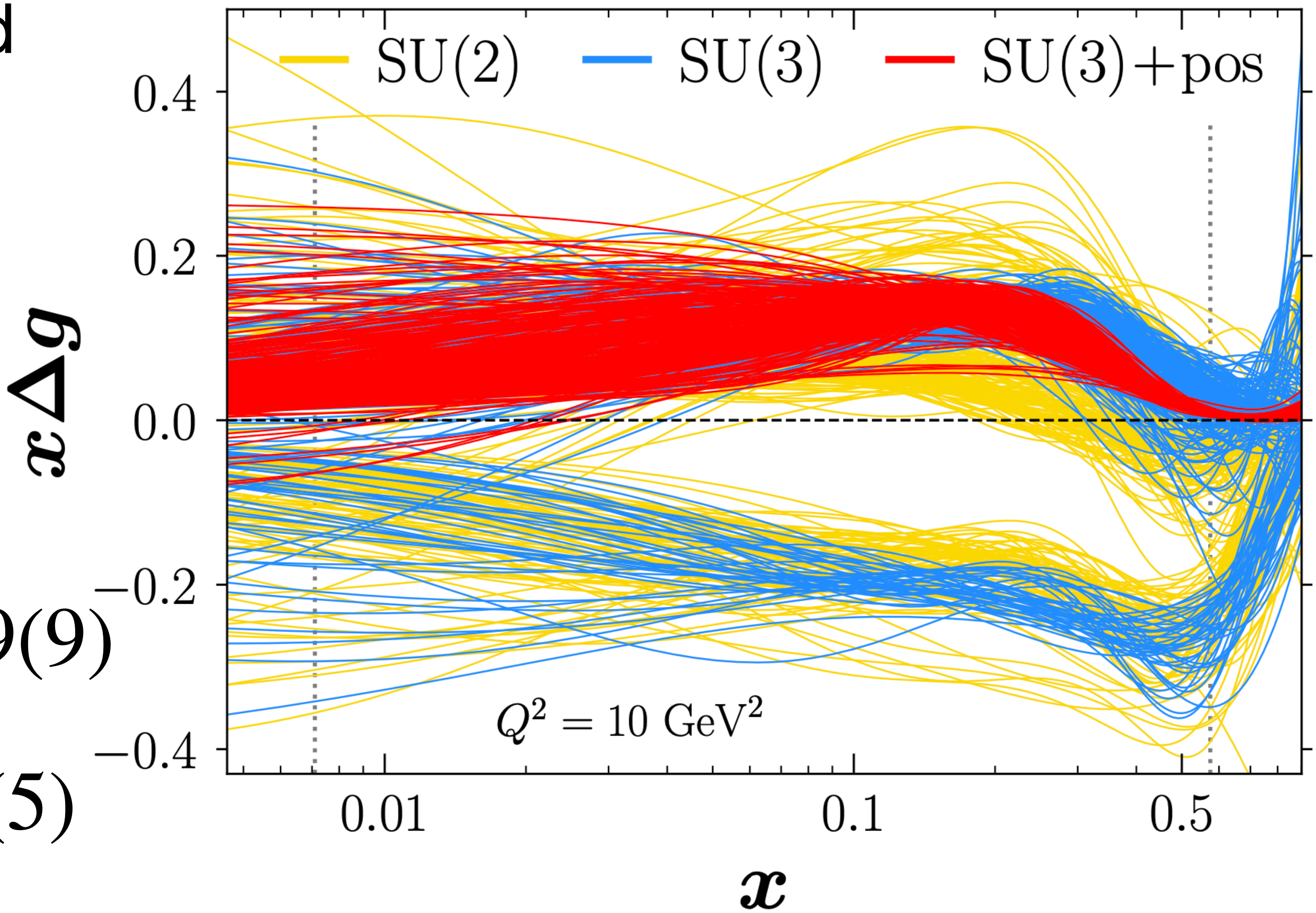
J. Collins, T. Rogers, N. Sato,
Phys Rev D 105 (2022) 7,076010

- Removal reveals new band of solutions

- With constraint: $\Delta G = 0.39(9)$

- Without constraint: $\Delta G = 0.3(5)$

- Local Lattice:
$\Delta G = 0.251(47)(16)$



Y. Zhou et al (JAM) Phys. Rev. D 105, 074022 (2022)

Y-B. Yang et al (χ-QCD) Phys. Rev. Lett. 118, 102001 (2017)
K-F. Liu arXiv: 2112.08416

Helicity Gluon matrix element

I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
C. Egerer et al (HadStruc) arXiv:2207.08733

- Helicity Gluon Matrix Element:

$$\widetilde{M}_{\mu\alpha;\nu\beta}(z,p,s) = \frac{1}{2}\epsilon_{\nu\beta\rho\sigma}M_{\mu\alpha;\rho\sigma} = \langle p,s\,|\,\mathrm{Tr}\left[F^{\mu\alpha}(z)W(z;0)\,\widetilde{F}^{\nu\beta}(0)\right]|\,p,s\rangle$$

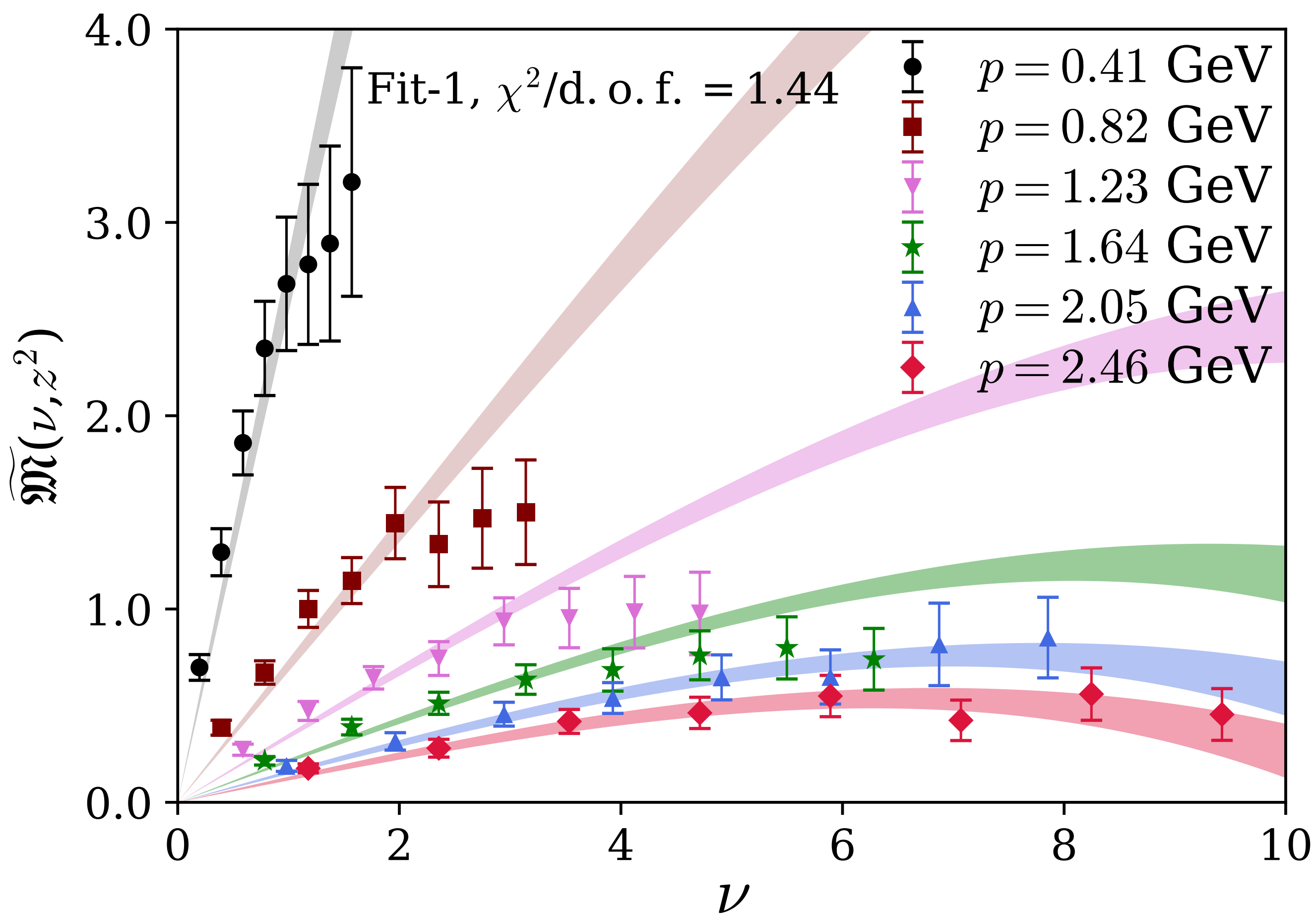
- Gluon matrix elements are typically far noiser than quarks
- Lorentz decomposition has >10 invariant terms.
- Useful Combination $\widetilde{\mathcal{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
  - Gives **two** amplitudes, one has no leading twist contribution
  - Undesired term dominates and must be removed

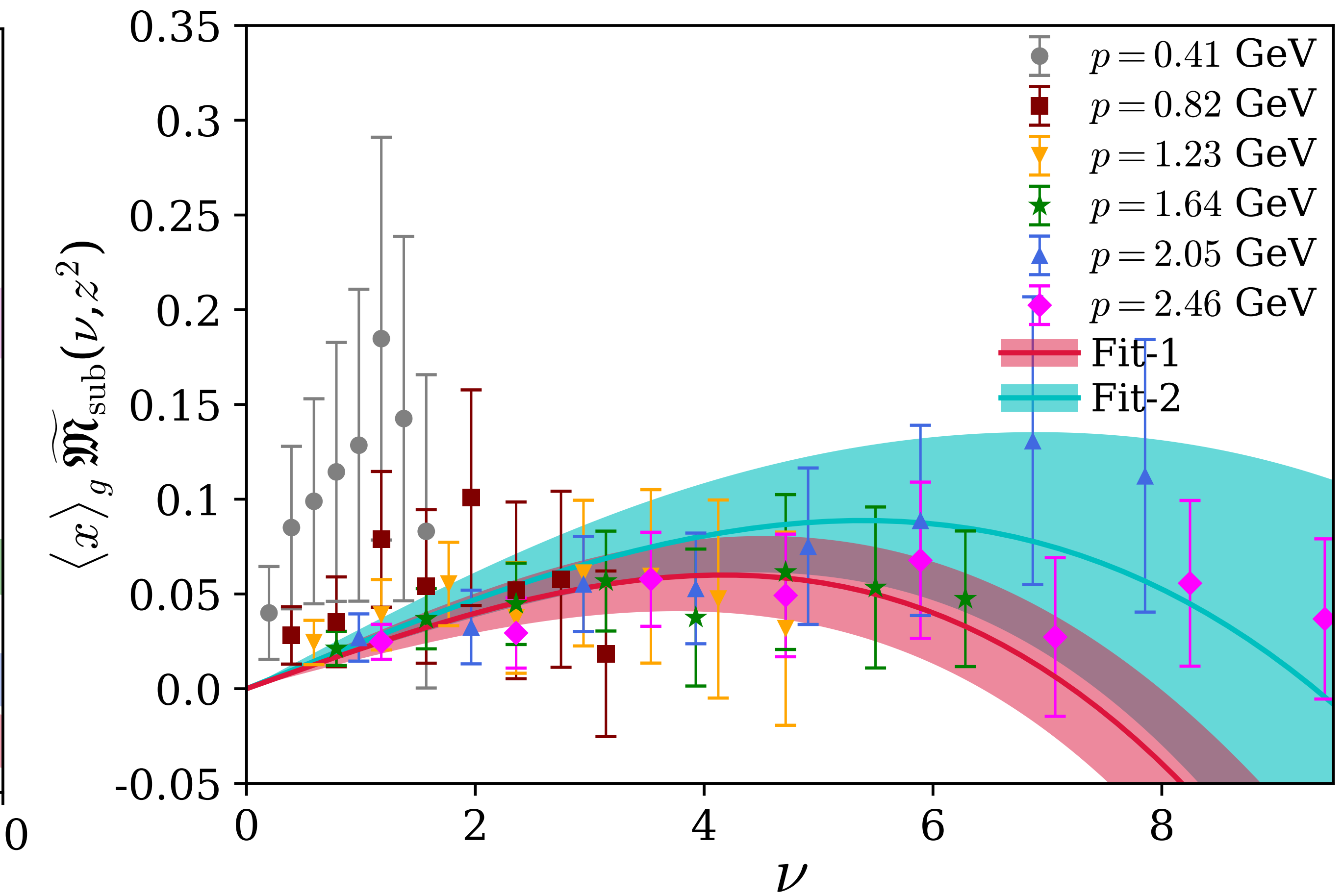
Helicity Gluon Matrix Element

- Large contamination from $\frac{m^2}{p_z^2}\nu\widetilde{\mathscr{M}}_{pp}$ will need to be removed

$$\widetilde{\mathfrak{M}}(z,p) = M_{\Delta g} - \frac{m^2}{p_z^2}\nu\widetilde{\mathscr{M}}_{pp}$$

- Model both terms
- Subtract rest frame



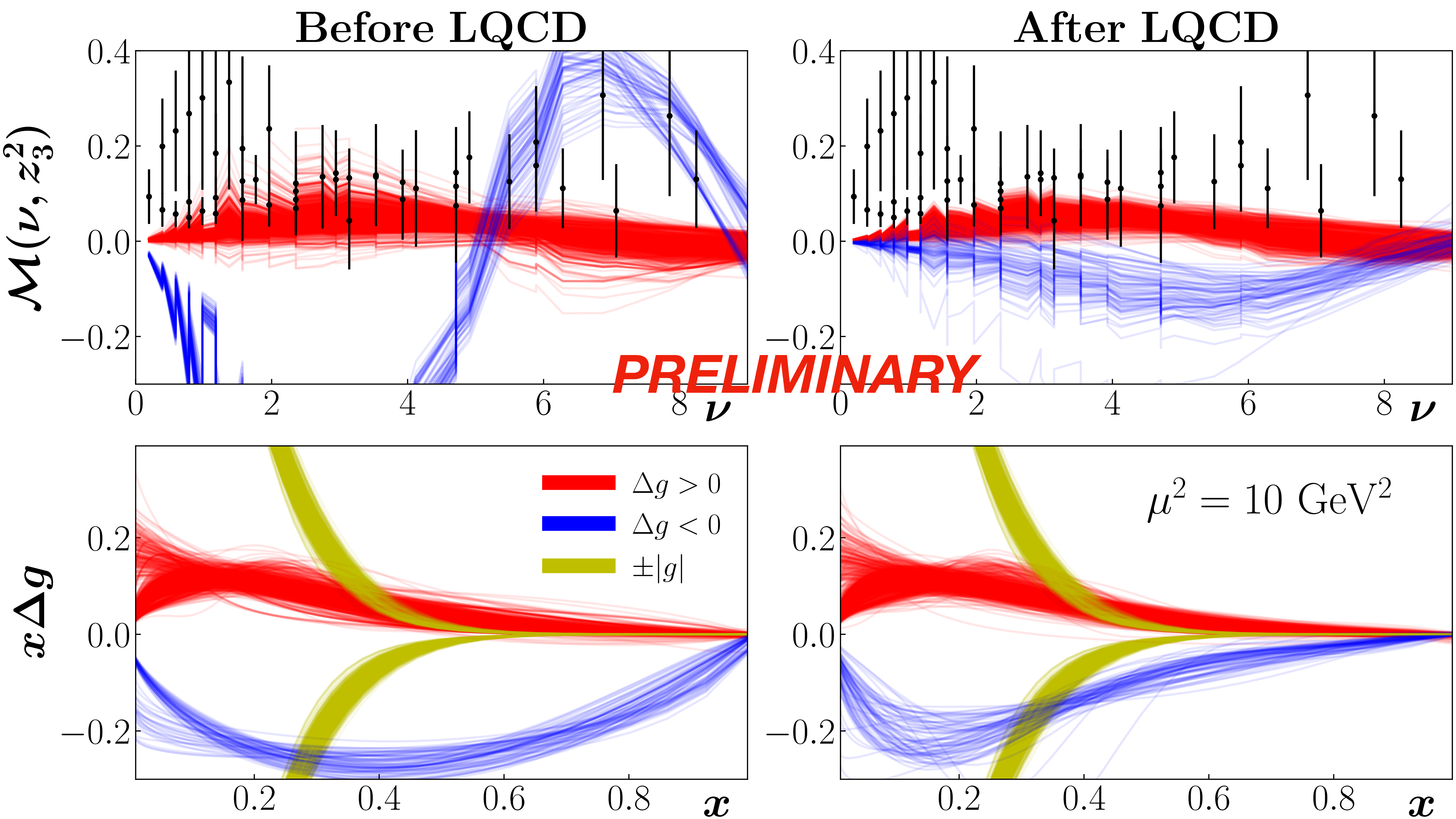


C. Egerer et al (HadStruc) arXiv: 2207.08733

- Model with Neural Network

T. Khan, T. Liu, R. Sufian arXiv:2211.15587

Helicity Gluon PDF with LQCD



Conclusions

- Many PDFs lack the amount of data and the precision which unpolarized quark PDFs obtain
- Lattice data is sensitive to higher x while experiment is sensitive to lower x
- **Combined analysis give better results than either alone**
- Even imprecise lattice data can impact the gluon helicity of the nucleon
  - Though, negative Δg is compatible with modern lattice results
- Future work will need to study lattice systematic errors

Thank you and the organizers!