CFFs to GPDs

Adam Freese
University of Washington
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Statement of the problem

- **Generalized parton distributions** GPDs are four-variable functions.
- **Compton form factors** CFFs are three-variable functions.
- Multiple GPDs exist that give the same CFF.
- How do we deal with it? **Are GPDs actually measurable?**

- GPDs are formally defined using light cone correlators.
  - Amplitude for a quark being at two spacetime locations.
- For quarks (in the light cone gauge):

$$\mathcal{M}^q[\mathcal{O}] = \frac{1}{2}\int \frac{\mathrm{d}z}{2\pi} e^{-i(P\cdot n)zx} \langle p' | \bar{q}\left(\frac{nz}{2}\right) \mathcal{O} q \left(-\frac{nz}{2}\right) | p\rangle$$

- Analogous definitions exist for gluons.
- $\mathcal{O}$ chosen by which GPD we want; e.g.,
  - $\not{n} = \gamma^+$ for helicity-independent, leading twist GPDs.
- Each correlator is decomposed into Lorentz structures; for the proton:

$$\mathcal{M}^q[\not{n}] = \bar{u}' \left[\not{n} H^q(x,\xi,t;Q^2) + \frac{i\sigma^{n\Delta}}{2m_p} E^q(x,\xi,t;Q^2) \right] u$$

- The Lorentz-invariant functions of x, ξ, t, and Q^2 are the GPDs.

GPDs: one kind of partonic structure

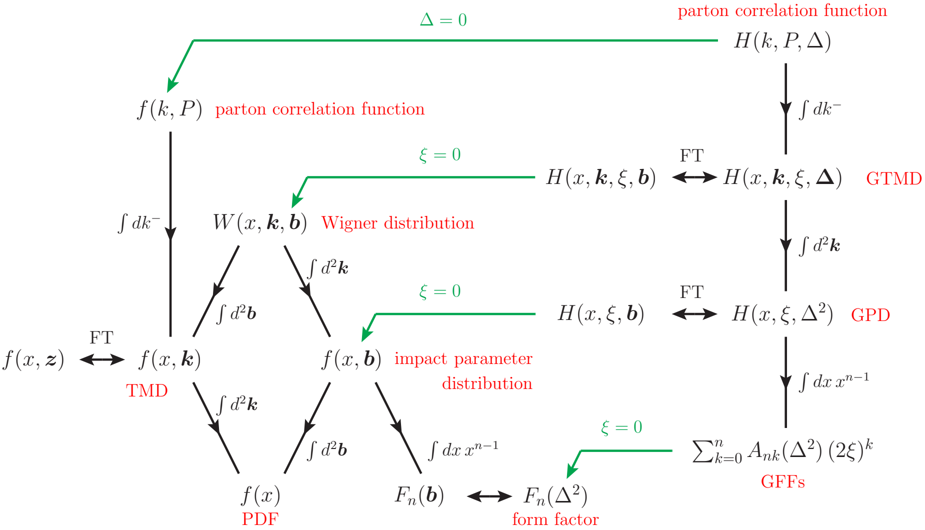
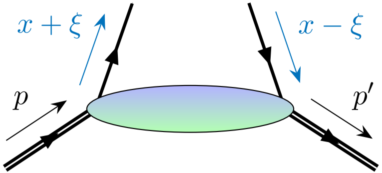


Figure: M. Diehl, arxiv:1512.01328

The GPD variables



$$x = \frac{(k+k')\cdot n}{(p+p')\cdot n}$$

$$\xi = \frac{(p-p')\cdot n}{(p+p')\cdot n}$$

$$t = \Delta^2 = (p'-p)^2$$

n defines the light front, i.e., $n \cdot V \equiv V^+$

- x is *average* momentum fraction of struck parton.
- 2ξ is the **skewness**: momentum fraction lost by struck parton.
- t is the invariant momentum transfer.
- Also depend on resolution scale Q^2—a hard scale in the process (e.g., photon virtuality)

- Several GPDs become PDFs when $p' = p$, i.e., $t = 0$ and $\xi = 0$.
- Definition of light cone correlator:

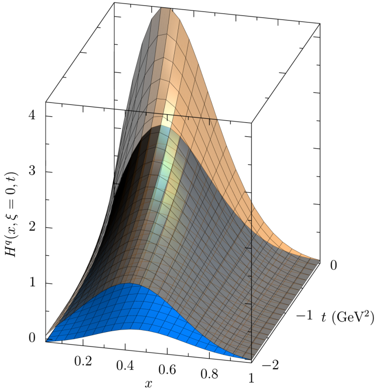
$$\mathcal{M}^q[\mathcal{O}] = \frac{1}{2} \int \frac{\mathrm{d}z}{2\pi} e^{-i(P\cdot n)zx} \langle p' | \bar{q}\left(\frac{nz}{2}\right) \mathcal{O} q \left(-\frac{nz}{2}\right) |p\rangle$$

  - This is how PDFs are formally defined, provided $p' = p$.

    See e.g., Collins's *Foundations of Perturbative QCD*.
- For the proton:

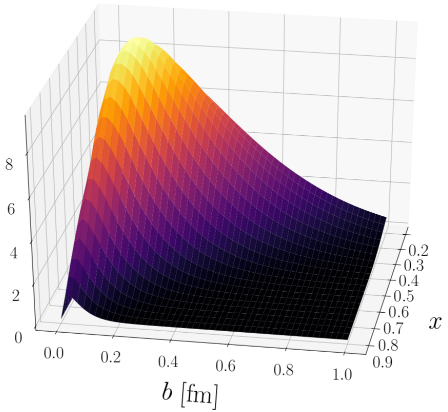
$$H^q(x, 0, 0; Q^2) = q(x; Q^2)$$
$$\tilde{H}^q(x, 0, 0; Q^2) = \Delta q(x; Q^2)$$

Non-skewed GPD ($\xi = 0$)



2D Fourier transform →

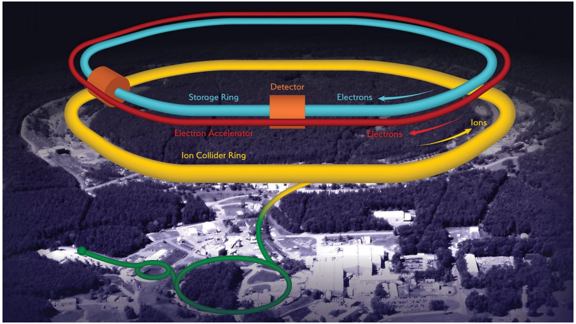
Impact parameter PDF



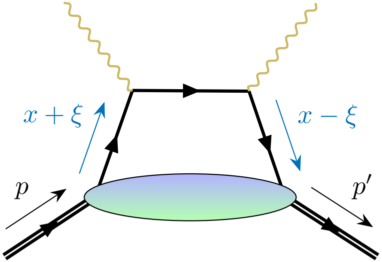
- Partially spatial structure recovered when $\xi = 0$
- 2D Fourier transform gives 2D spatial structure **at fixed light front time**
- Third dimension is **momentum fraction** x

Calculations in figures: AF & Cloët, PRC101 (2020) 035203

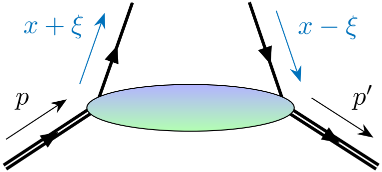
- **Hard exclusive reactions** are used to measure GPDs.
  - **Deeply virtual Compton scattering** (DVCS) to probe quark structure.
  - Deeply virtual meson production (DVMP), *e.g.*, J/ψ or Υ to probe gluon structure.
  - Double DVCS and timelike Compton scattering are other options.
  - …and more!
- Measured at **Jefferson Lab** and the upcoming **Electron Ion Collider**.



DVCS and GPDs



Deeply virtual Compton scattering
$\mathcal{H}(\xi, t; Q^2)$



Generalized parton distribution
$H(x, \xi, t; Q^2)$

- **Deeply virtual Compton scattering** (DVCS) is one method to probe GPDs.
- Loop in diagram: x is integrated out
- Integrated quantities seen in experiment: **Compton form factors**

$$\mathcal{H}(\xi, t; Q^2) = \int_{-1}^{1} \mathrm{d}x \, C(x, \xi) H(x, \xi, t; Q^2) \stackrel{\text{LO}}{=\!=} \int_{-1}^{1} \mathrm{d}x \left[\frac{1}{\xi - x - i0} \mp \frac{1}{\xi + x - i0} \right] H(x, \xi, t; Q^2)$$

Shadow GPDs

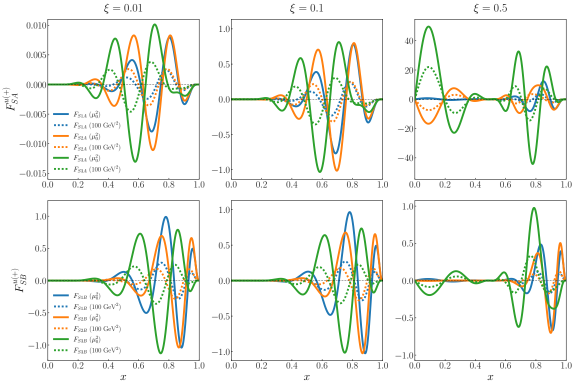
- Need to invert the relationship:

$$\mathcal{H}(\xi, t; Q^2) = \int_{-1}^{1} \mathrm{d}x\, C(x,\xi) H(x,\xi,t;Q^2)$$

- **Shadow GPDs** impose a mighty obstacle:

$$\int_{-1}^{1} \mathrm{d}x\, C(x,\xi) H_s(x,\xi,t;Q_0^2) = 0$$

  - $H(x,\xi,t,Q_0^2) + H_s(x,\xi,t,Q_0^2)$ gives the same DVCS amplitude at $Q^2 = Q_0^2$.
  - Bertone, *et al.*, PRD103 (2021) 114019 (first work on shadows)
  - Moffat *et al.*, arxiv:2303.12006 (plots on right)



Examples of shadow GPDs.

- GPDs are functions of four variables, but …
- …Q^2 dependence is fully determined by **evolution equations**:

$$\frac{\mathrm{d}H(x,\xi,t;Q^2)}{\mathrm{d}\log Q^2} = \frac{\alpha_{\mathrm{QCD}}}{2\pi}\int \frac{\mathrm{d}y}{y} P\left(\frac{x}{y},\frac{\xi}{y}\right) H(y,\xi,t;Q^2)$$

- Should exist a unique map between $\mathcal{H}(\xi,t,Q^2)$ [CFF] and $H(x,\xi,t,Q_0^2)$ [GPD].
  - Both are 3-variable functions.
  - Shadow GPDs evolve into non-shadows!
  - Problem solved with infinite-precision data.
  - Real data have uncertainties.
- **What data do we need?**
  - Over what kinematic domain?
  - With what precision?
  - Is this even feasible with realistic error bars?

Positivity constraints

- Constraints follow from Schwarz inequality:

$$\langle \text{out}|\text{in}\rangle^2 \leq \langle \text{out}|\text{out}\rangle \langle \text{in}|\text{in}\rangle$$

  combined with overlap formalism.
- GPD implication for spin-half case:

$$\left| H^q(x,\xi,t;Q^2) - \frac{\xi^2}{1-\xi^2} E^q(x,\xi,t;Q^2) \right|^2 + \left| \frac{\sqrt{t_{\text{min}} - t}}{2M\sqrt{1-\xi^2}} E^q(x,\xi,t;Q^2) \right|^2 \leqslant \frac{q(x_{\text{in}};Q^2)\, q(x_{\text{out}};Q^2)}{1-\xi^2}$$

- May be spoiled by renormalization, like PDF positivity
  - Collins, Rogers & Sato, PRD105 (2022) 076010
  - Perhaps positivity can be a “soft constraint”?
    i.e., not strictly imposed, but cost function imposed during fit …

Analysis of Bertone *et al.*

- Shadow GPDs discovered, first analyzed by Bertone *et al.*
  - PRD103 (2021) 114019
  - SciPost Phys. Proc. 8 (2022) 107

$$\int_{-1}^{1} \mathrm{d}x \, C(x,\xi) H_s(x,\xi,t;Q_0^2) = 0$$

- They discovered a systematic scheme for generating shadow GPDs
  - via Radon transforms of polynomials with additional constraints
- They found evolution insufficient to constrain shadows

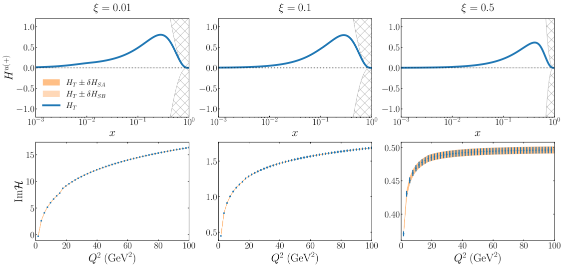
Analysis of Moffat *et al.*

- Construct a GPD as:

$$H = H_{\text{true}} + H_{\text{shadows}}$$

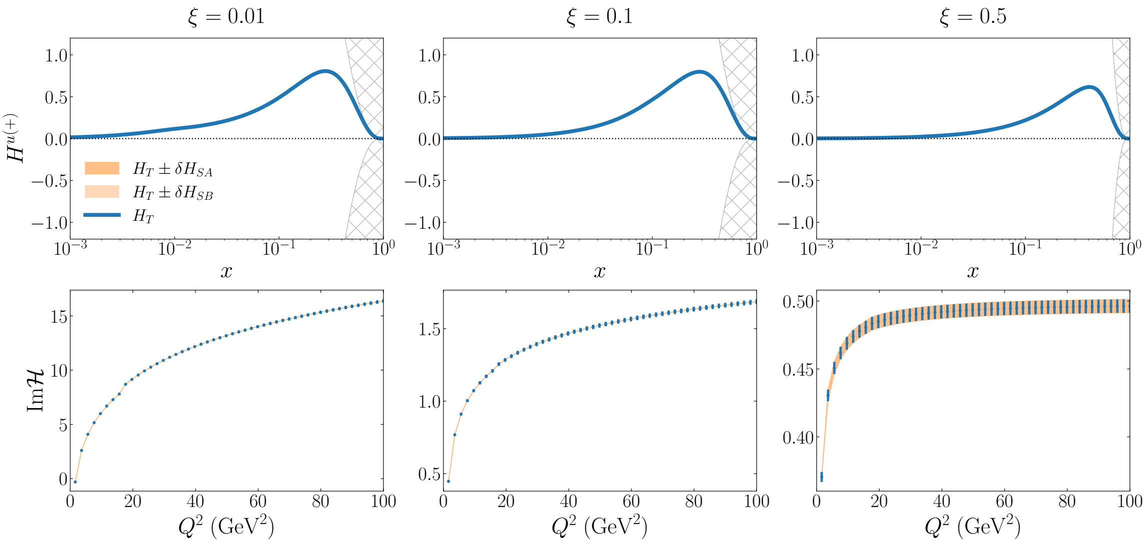
where $H_{\text{shadows}}(x, \xi, t; Q_0^2)$ is a linear sum of possible shadows.

- Generate Monte Carlo CFF data using $H_{\text{true}}(x, \xi, t; Q_0^2)$
  - Use VGG model as proxy
- Fit $H(x, \xi, t; Q_0^2)$ using coefficients of shadows as parameters.
  - We find large uncertainties at large x…



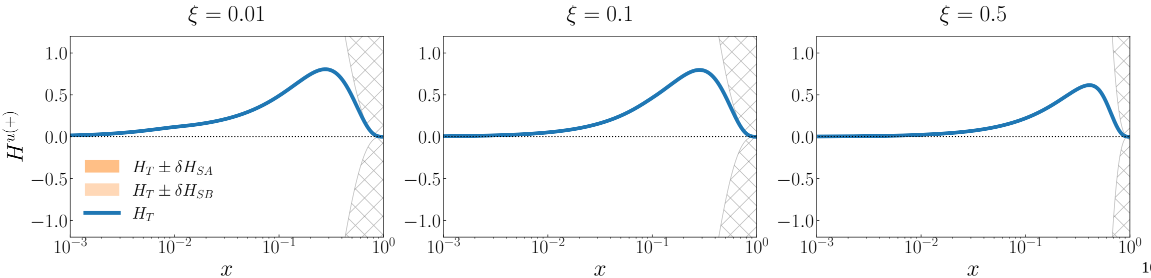
Disclosure: I'm on this paper, maybe not impartial presenter here

A closer look at the plots



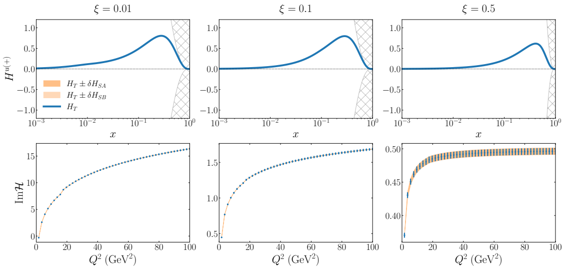
Eric Moffat, AF, *et al.*, arXiv:2303.12006

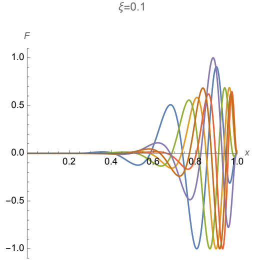
- Dark bands are shadows that vanish at $\xi = 0$.
- We realized forward limit needed $t = 0$ too.
- $t = 0$ will not happen in CFF measurements.
- Light bands don't force shadows to vanish at $\xi = 0$.

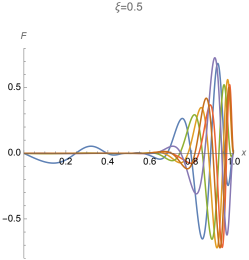


Prognosis

- We analyzed a subset of shadow GPDs.
- *Looks like* data over large (ξ, Q^2) domain could constrain small (x, ξ) GPDs.
  - Add positivity for large x.
- Result depends on shadows being small when x and ξ are small.
- This is true of every shadow GPD we've found to date
  - Examples on bottom right
  - I'm trying to construct a proof (no progress)
  - Eric has tried finding counter-examples (none found)
- **Cautiously optimistic** about GPDs being measurable at small ξ
  - Small ξ relevant to tomography







Other paths

- Stick to observables when extracting
  - e.g., **Compton form factors**
- Use equivalence classes of GPDs
  - $H \sim H + H_{\text{shadow}}$
- Other reactions
  - Double DVCS
  - Single diffractive hard exclusive processes
  - Has different shadows than DVCS
- Use lattice results to further constrain GPDs
- Model-testing paradigm
  - Make CFF predictions via models
  - DVCS etc. serve to *rule out* models
  - More measurements scrutinize survivors