Moments of GPDs from the OPE of nonlocal quark bilinears

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CNF Generalized Parton Distributions and Global Analysis
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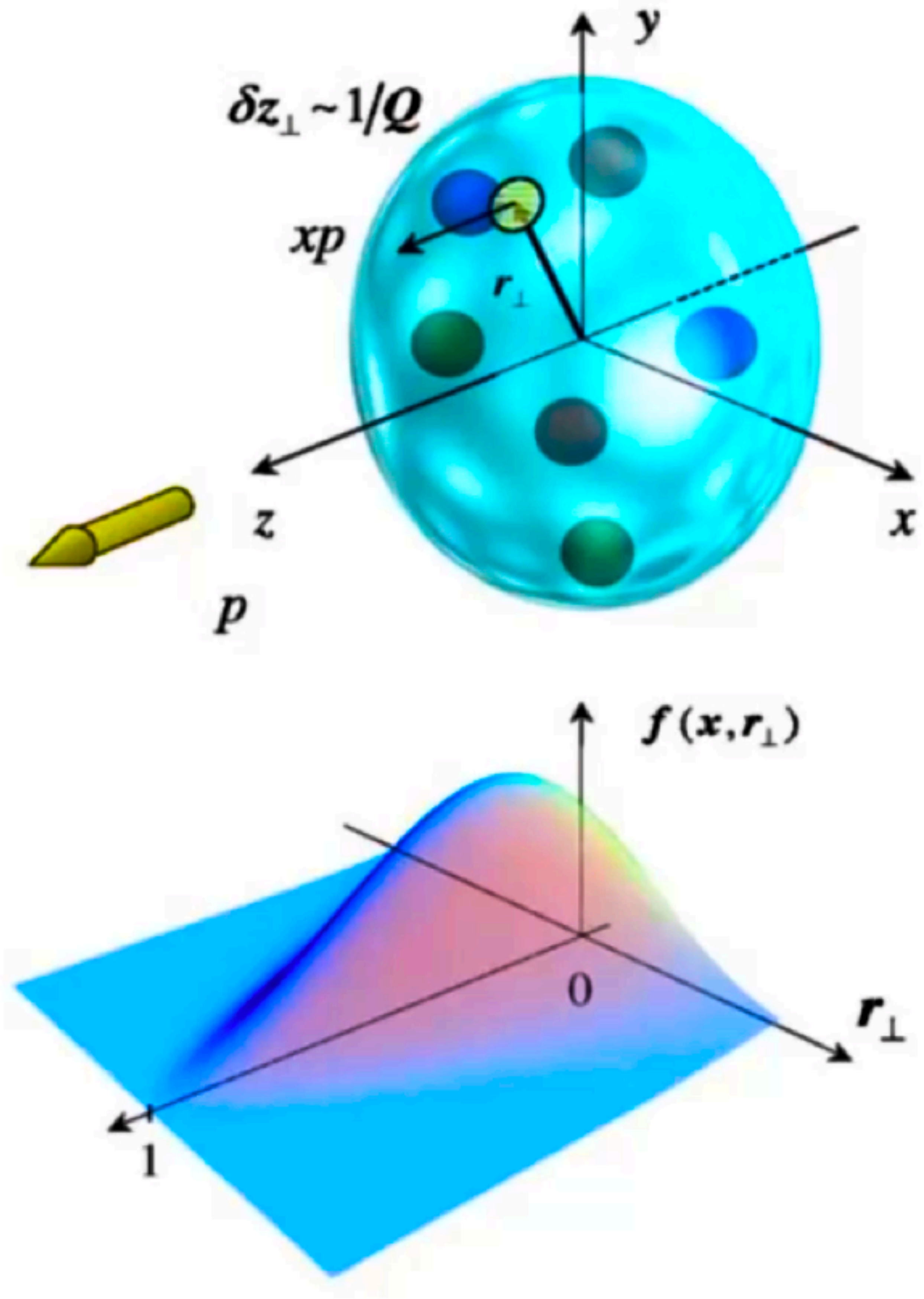
Generalized parton distributions

GPDs goes far beyond the 1D PDFs and the transverse structure encoded in the form factors.



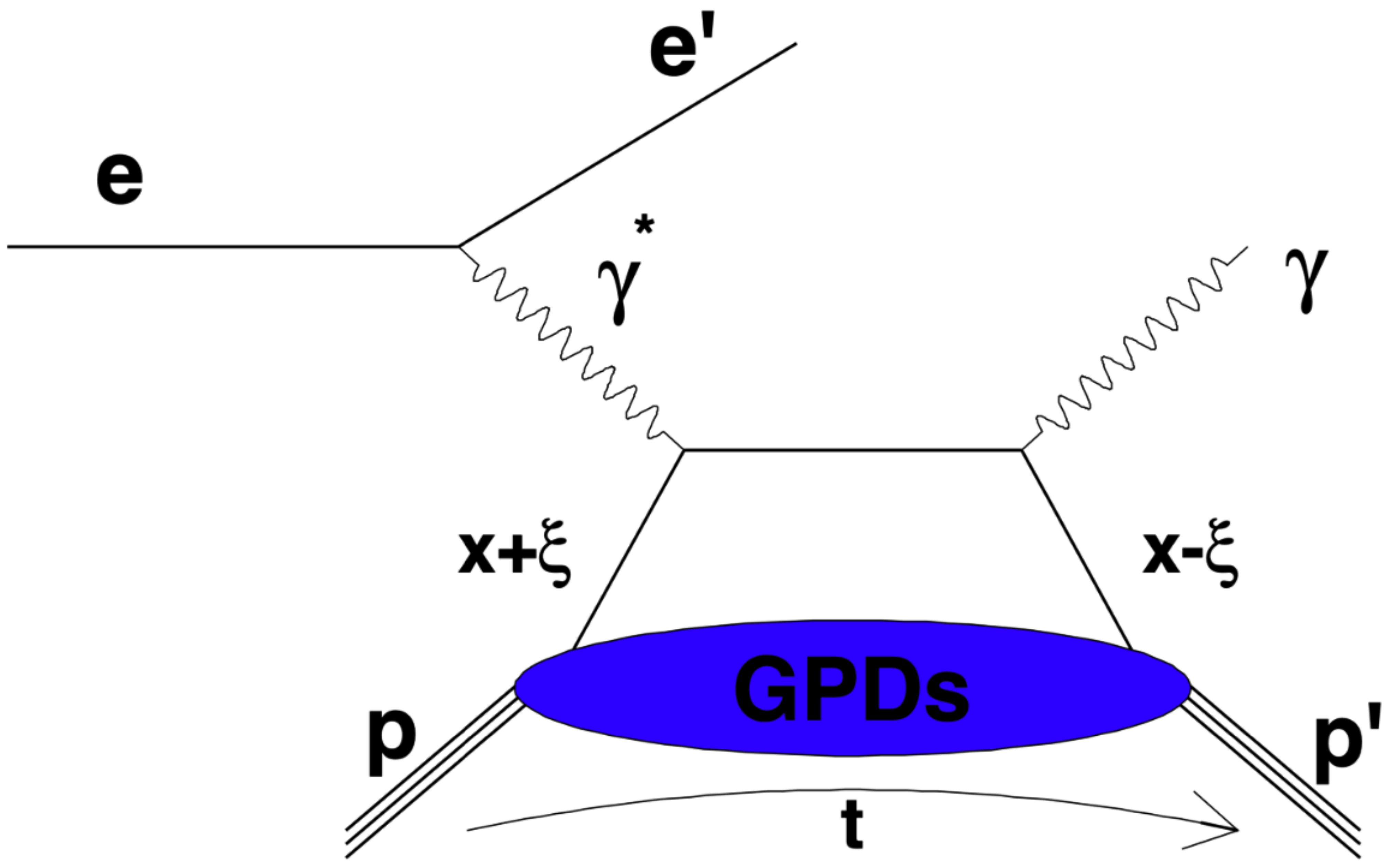
- Offer insights into the 3D image of hadrons.
- Give access to the orbital motion and spin of partons.
- Have a relation to pressure and shear forces inside hadrons.

$$F_{q/g}(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p_f | \bar{q}(-\tfrac{z}{2}) \gamma^\mu \mathscr{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

Belitsky and Radyushkin: Phys.Rept. 418 (2005) 1-387

Generalized parton distributions

DVCS



The golden process to study the quark GPDs is DVCS

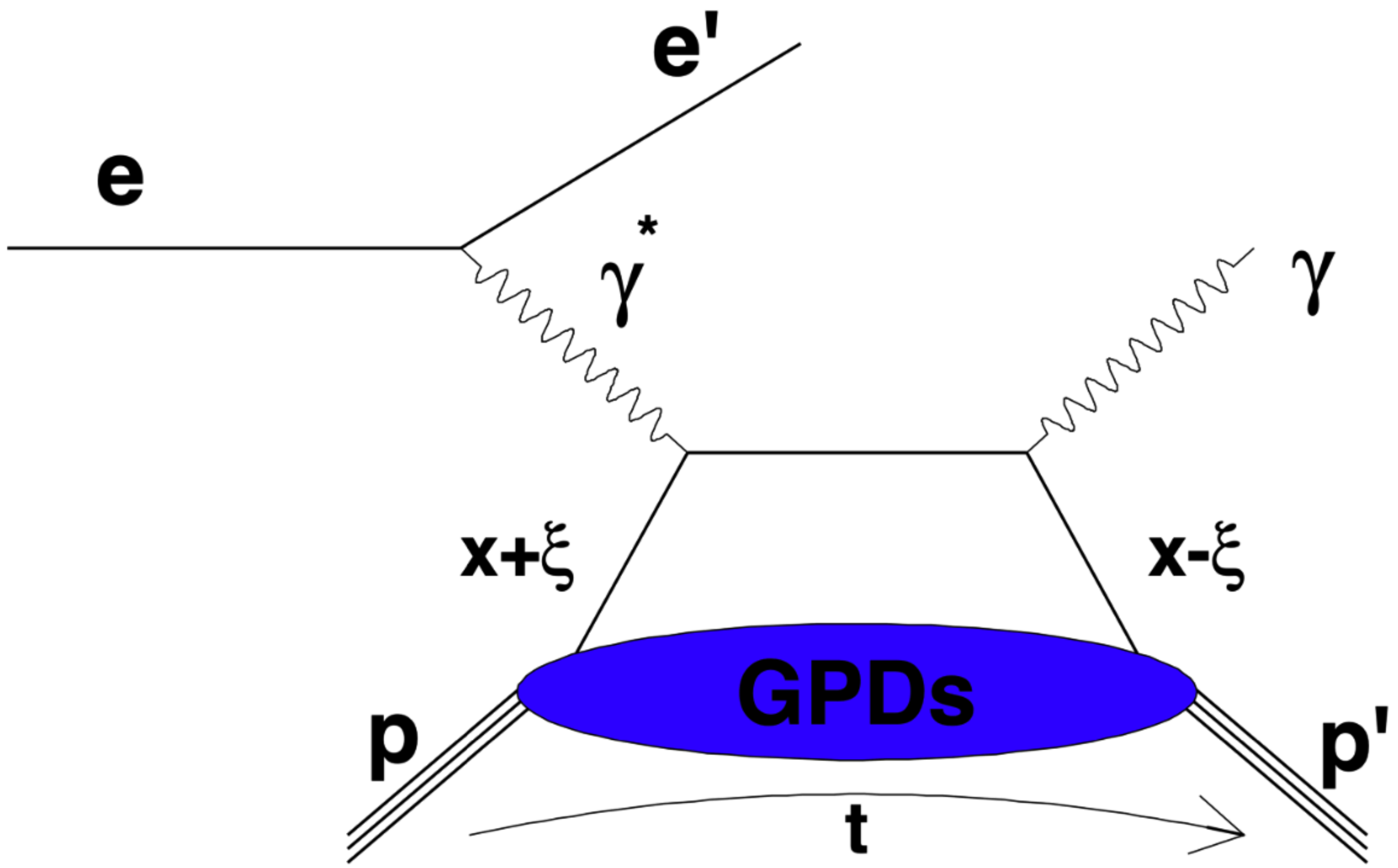
Challenging:

- observables appear at the amplitude level
- multi-dimensionality (x, ξ, t)
- the momentum fraction x is integrated over (Compton Form Factors)

$$\mathcal{F}(\xi, t; Q^2) = \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} \pm \frac{1}{\xi + x - i\epsilon} \right] F(x, \xi, t; Q^2)$$

Generalized parton distributions

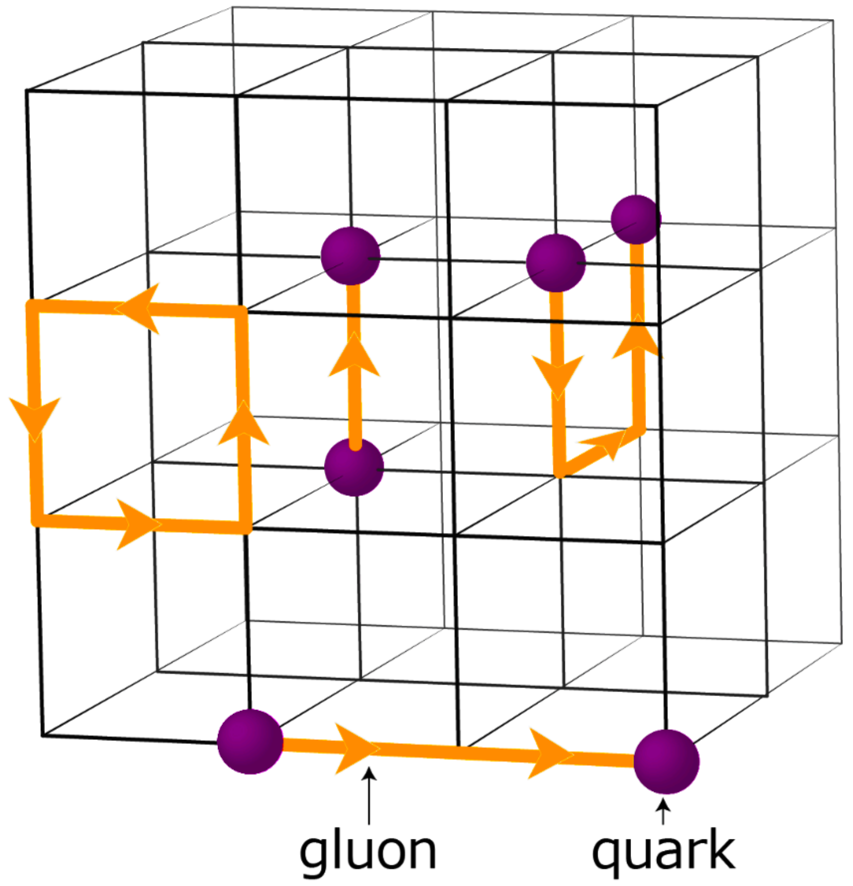
DVCS



Challenging:

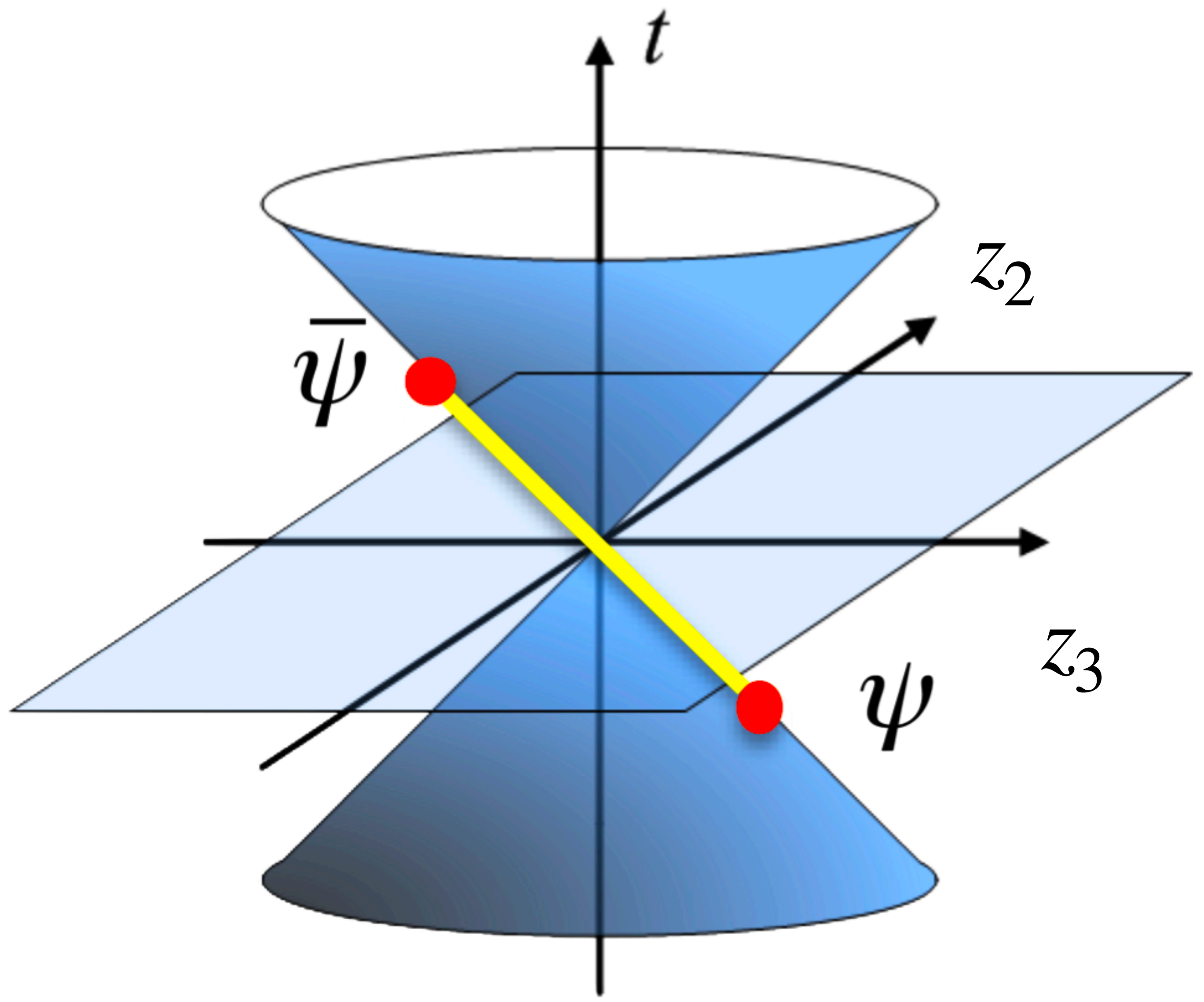
- observables appear at the amplitude level
- multi-dimensionality (x, ξ, t)
- the momentum fraction x is integrated over (Compton Form Factors)

Complementary knowledge from lattice QCD is essential.



Generalized parton distributions

$$z_3 + ct = 0, \quad z_3 - ct \neq 0$$



Light-cone correlation: Cannot be calculated on the lattice

$$\langle p_f | \bar{q}(-\frac{z^-}{2})\gamma^\mu \mathscr{W}(-\frac{z^-}{2}, \frac{z^-}{2}) q(\frac{z^-}{2}) | p_i \rangle$$

$$z^2 = 0$$

- Moments from leading-twist **local operators**.

$$\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1} \ldots \overleftrightarrow{D}^{\alpha_n} q$$

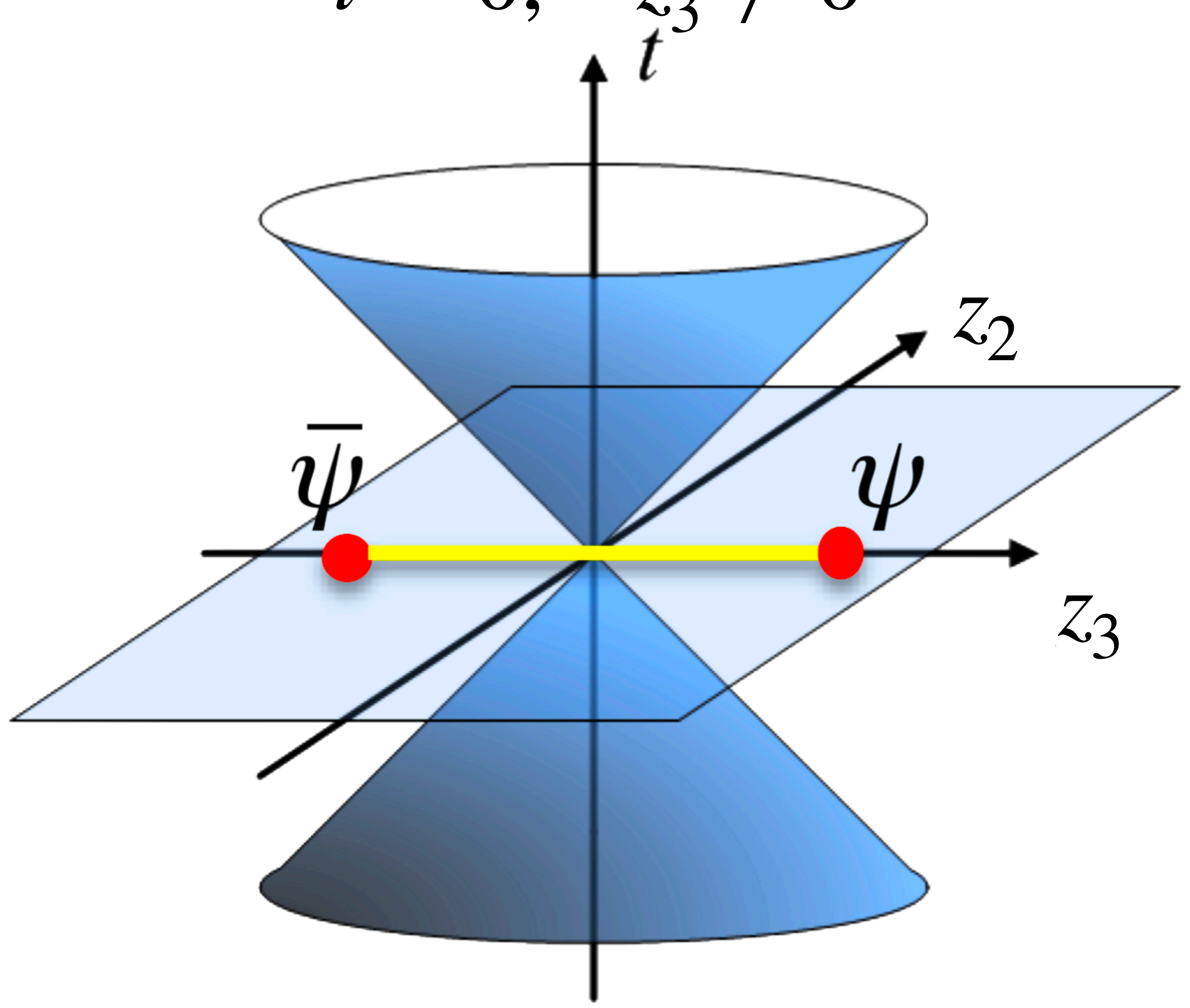
ETMC, PRD 101 (2022)
ETMC, PRD 83 (2011)

High dimensional

Due to signal decay and power-divergent mixing under renormalization, there are no moments beyond the third that exist.

Generalized parton distributions

$t=0, \quad z_3 \neq 0$



$$F^\mu(z, P, \Delta)$$
$$= \langle p_f | \bar{q}(-\tfrac{z}{2}) \gamma^\mu \mathscr{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

$z = (0,0,0,z_3),\ z^2 = z_3^2$

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ETMC, PRD 101 (2022)
ETMC, PRD 83 (2011)

High dimensional

- **Large-momentum effective theory**: x-space matching of quasi-PDF.

X. Ji, PRL 2013
X. Ji, et al, RevModPhys 2021

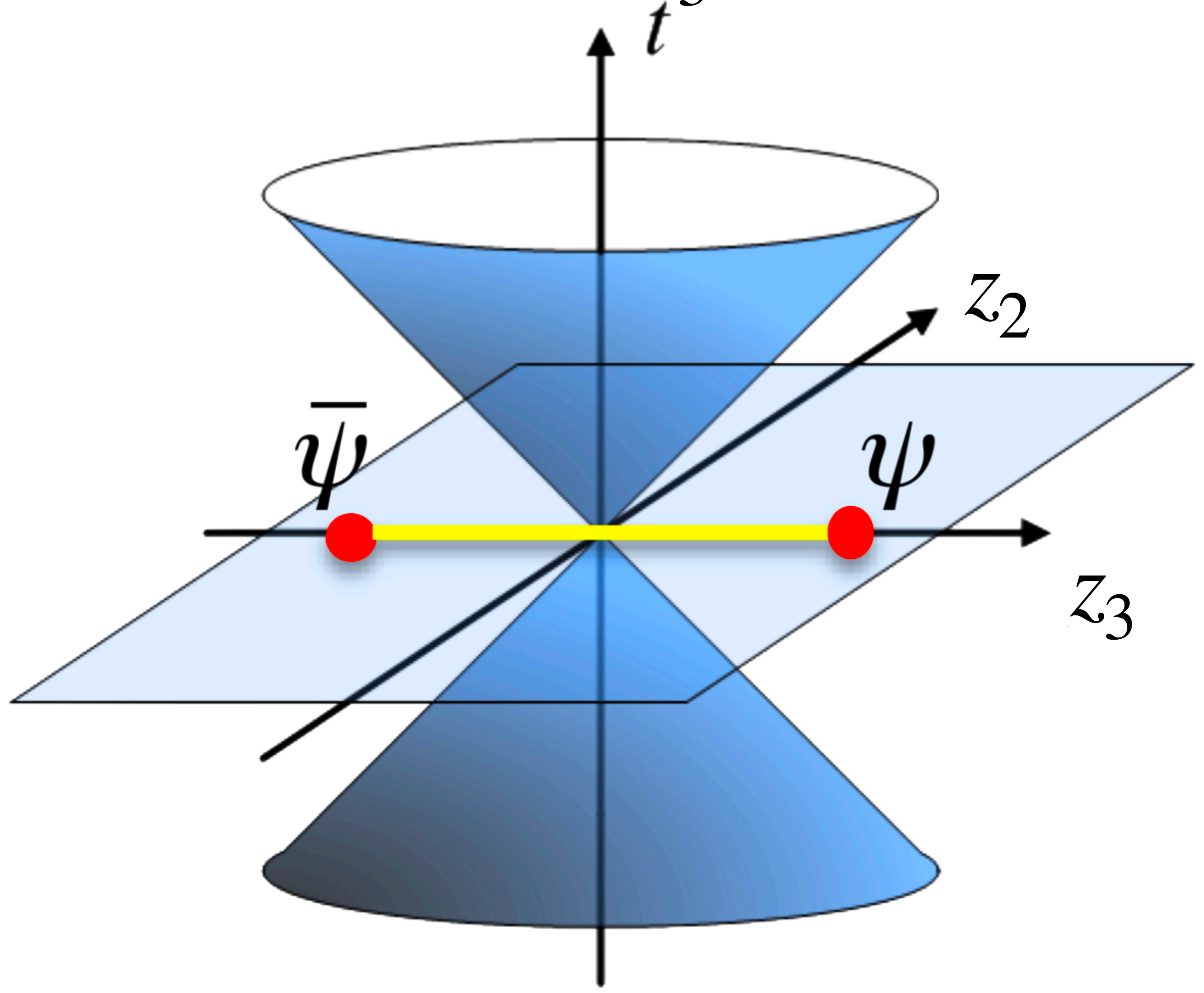
- **Short distance factorization** of the quasi-PDF matrix elements in position space or the pseudo-PDF approach.

A. Radyushkin, PRD 100 (2019)
A. Radyushkin, Int.J.Mod.Phys.A 2020

- …

Short distance factorization

$t = 0, \quad z_3 \neq 0$



$$F^{\mu}(z, P, \Delta) = \langle p_f | \bar{q}(-\tfrac{z}{2})\gamma^{\mu}\mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2})q(\tfrac{z}{2}) | p_i \rangle$$

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SDF of the zero skewness GPD matrix elements:

- V. Braun et al., EPJC 55 (2008)
- A. V. Radyushkin et al., PRD 96 (2017)
- Y. Ma et al., PRL 120 (2018)
- T. Izubuchi et al., PRD 98 (2018)

$$F^{R}(z, P, \Delta) = \int_{-1}^{1} d\alpha \mathcal{C}(\alpha, \mu^2 z^2) \int_{-1}^{1} dy e^{-iy\alpha\lambda} F(x, \xi, \Delta, \mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

$$= \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} C_n(z^2\mu^2) \langle x^n \rangle (t; \mu) + \mathcal{O}(z^2 \Lambda_{\mathrm{QCD}}^2)$$

Perturbative matching

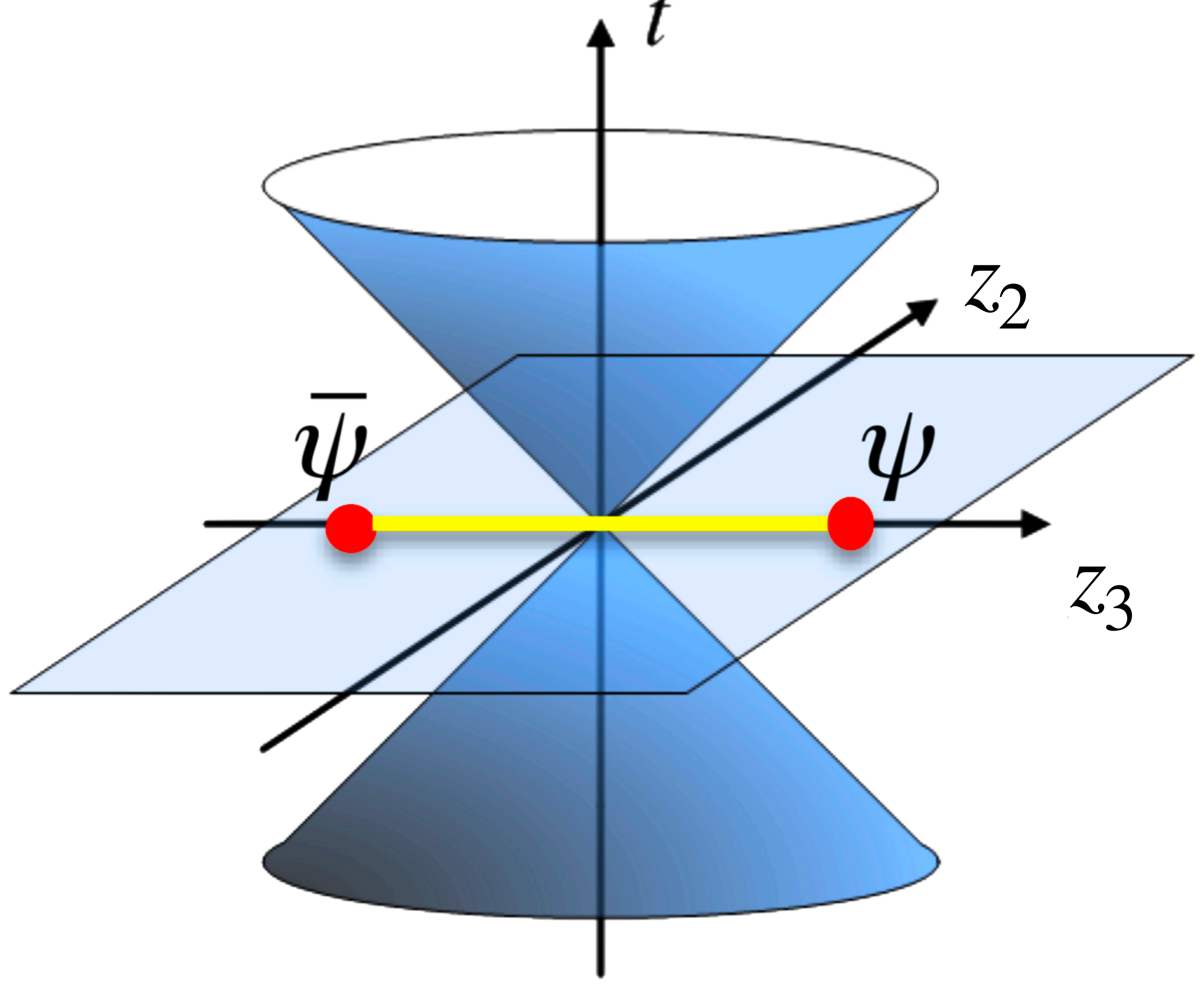
$\lambda = zP$

$$\int_{-1}^{1} dx x^n H^q(x, \xi = 0, t) = A^q_{n+1,0}(t)$$

$$\int_{-1}^{1} dx x^n E^q(x, \xi = 0, t) = B^q_{n+1,0}(t)$$

Short distance factorization

$t = 0, \quad z_3 \neq 0$



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Perturbative matching

$\lambda = zP$

- The perturbative matching is valid in short range of z_3.
- The information that lattice data contains is limited by the range of finite $\lambda = zP$.

quasi-GPD matrix elements

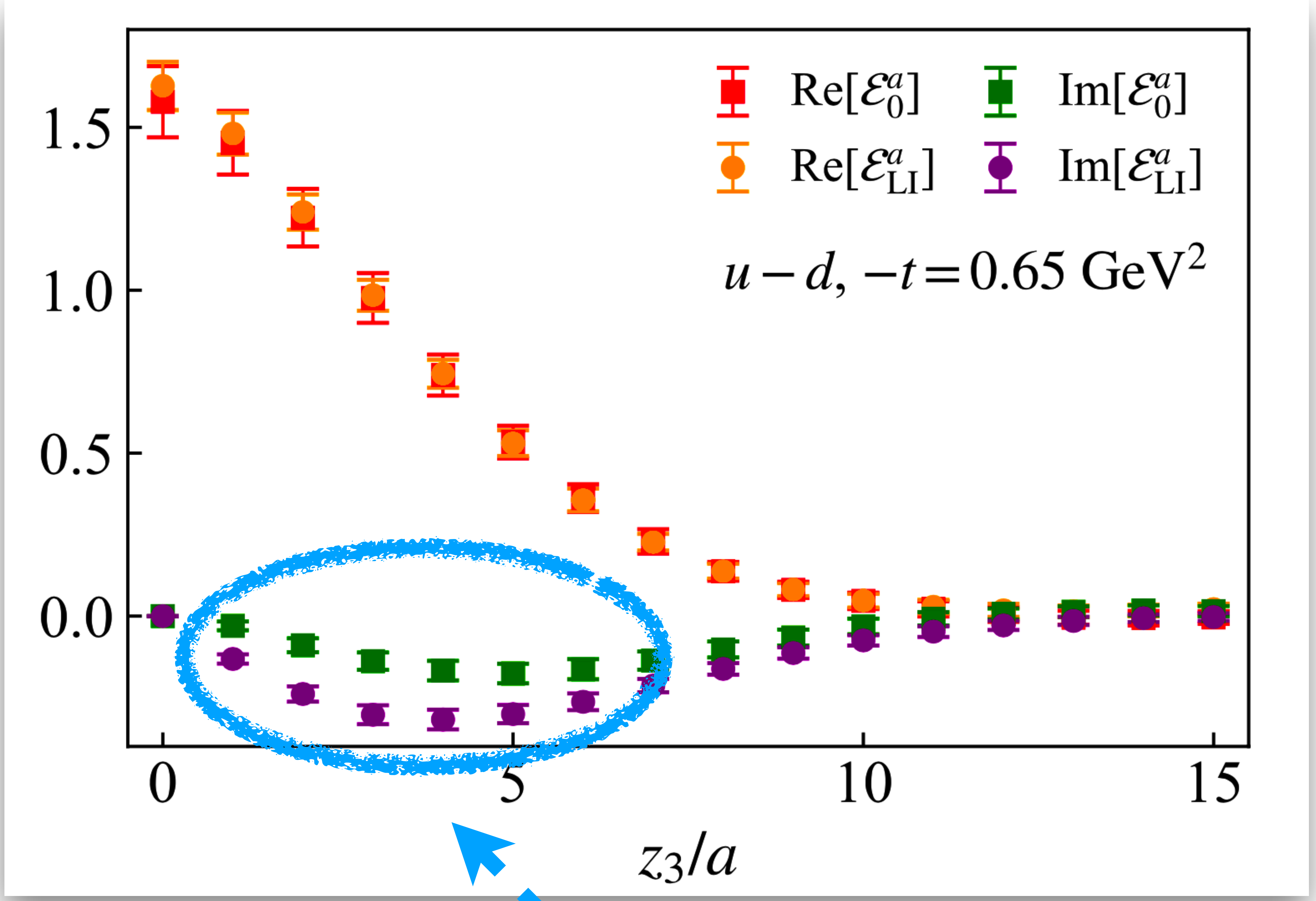
$$F^0(z, P, \Delta)$$
$$= \bar{u}(p_f, \lambda') \left[\gamma^0 \mathscr{H}_0(z, P, \Delta) + \frac{i\sigma^{0\mu}\Delta_\mu}{2m} \mathscr{E}_0(z, P, \Delta) \right] u(p_i, \lambda)$$

- Depend on Lorentz scalars

$$t = \Delta^2 = -0.65\ \mathrm{GeV}^2,\ \xi = 0$$

- Frame dependent at finte momentum

$$P_z = 1.25\ \mathrm{GeV}$$



$$F_0^s \longleftrightarrow \gamma F_0^a - \gamma\beta F_\perp^a$$

power corrections $\sim 1/P$
at the tree level

S. Bhattacharya, et al., PRD 106, 114512 (2022)

quasi-GPD matrix elements

The matrix elements can be parametrized in terms of **8 Lorentz invariant amplitudes:** $A_i\left(z \cdot P, z \cdot \Delta, \Delta^2, z^2\right)$:

$$F^{\mu}(z, P, \Delta) = \bar{u}(p_f, \lambda')\left[\frac{P^{\mu}}{m}A_1 + mz^{\mu}A_2 + \frac{\Delta^{\mu}}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6 + mz^{\mu}i\sigma^{z\Delta}A_7 + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\right]u(p_i, \lambda)$$

- S. Bhattacharya, et al., PRD 106 (2022)

Light-cone or Lorentz invariant quasi-GPD matrix elements

$$\mathcal{H}(z, P, \Delta) = A_1 + \frac{\Delta \cdot z}{P \cdot z}A_3$$

$$\mathcal{E}(z, P, \Delta)$$

$$= -A_1 - \frac{\Delta \cdot z}{P \cdot z}A_3 + 2A_5 + 2P \cdot zA_6 + 2\Delta \cdot zA_8$$

- Solve $A_i(z \cdot P, z \cdot \Delta, \Delta^2, z^2)$ from matrix elements of F^0, F^1, F^2.
- Lorentz invariant, frame independent, computational cheaper for multiple t
- Quasi- differ from light-cone GPDs only by $z^2 \neq 0$

Renormalization

- The operator can be **multiplicatively renormalized**

  - X. Ji, J. H. Zhang and Y. Zhao, PRL120.112001
  - J. Green, K. Jansen and F. Steffens, PRL.121.022004

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B = e^{-\delta m(a)|z|}Z(a)[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

- Short distance factorization with **ratio scheme renormalization**

  - A. V. Radyushkin et al., PRD 96 (2017)
  - BNL, PRD 102 (2020)

$$\mathcal{M}(z^2, zP, \Delta^2) = \frac{\mathcal{H}^R(z,P,\Delta;\mu)}{\mathcal{H}^R(z,P=0,\Delta=0;\mu)} = \frac{\mathcal{H}^B(z,P,\Delta;a)}{\mathcal{H}^B(z,P=0,\Delta=0;a)}$$

$$= \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle(\Delta^2;\mu) + \mathcal{O}(z^2\Lambda^2_{\mathrm{QCD}})$$

$C_n^{\overline{MS}}(\mu^2 z^2) = 1 + \alpha_s C^{(1)}(\mu^2 z^2) + \ldots$ up to NNLO

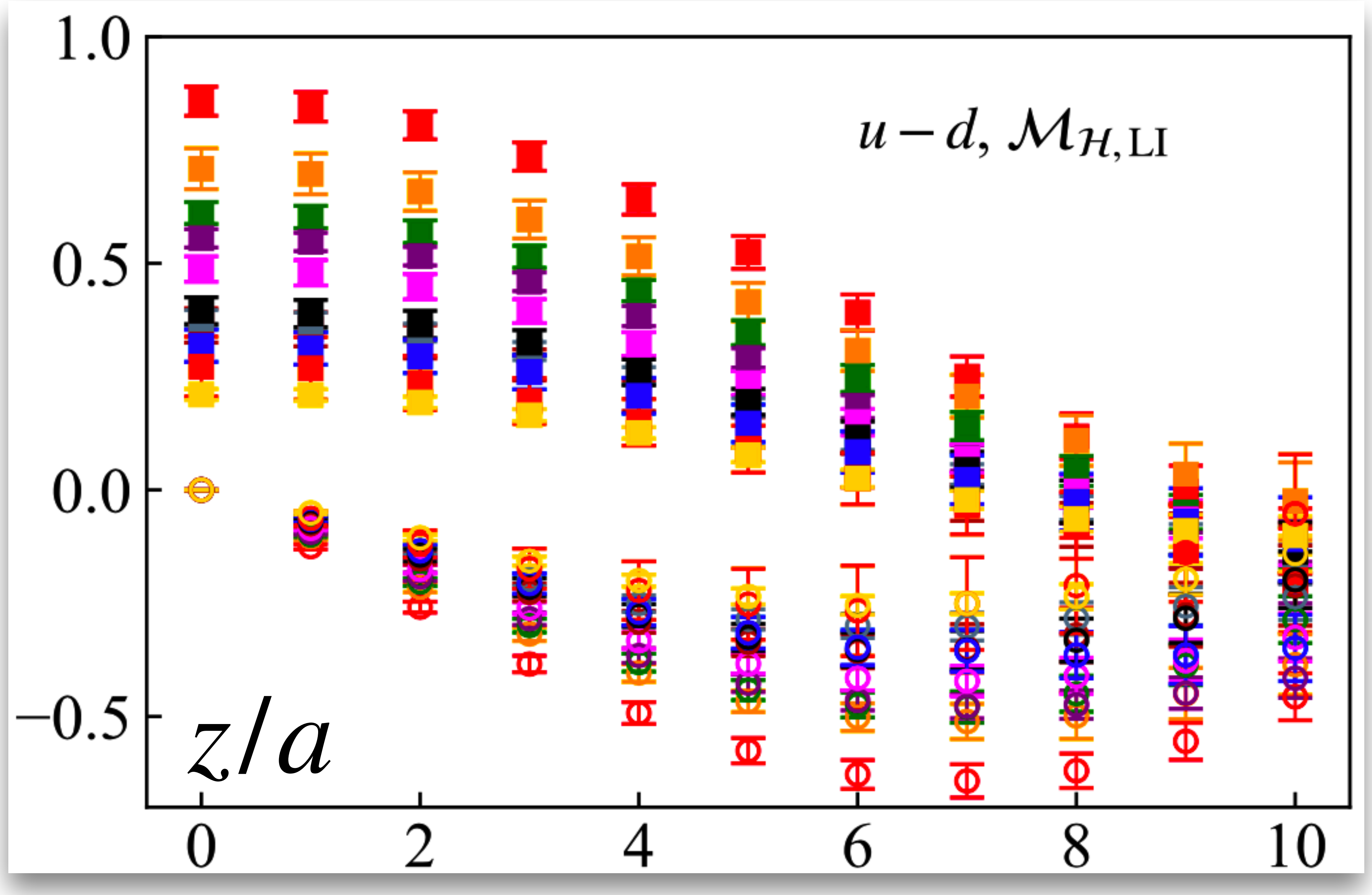
- **Lattice setup**

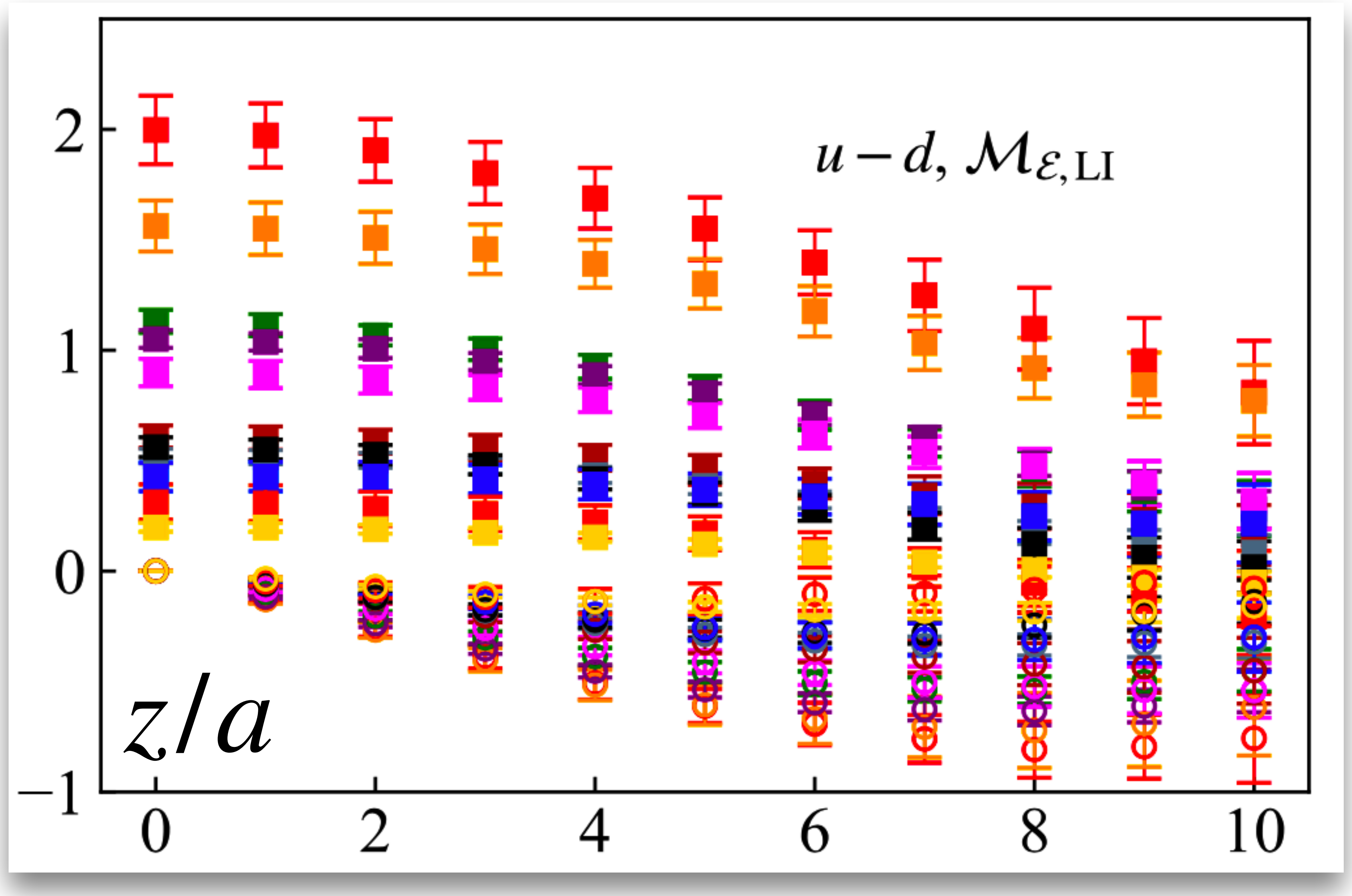
$m_\pi = 260$ MeV, $a = 0.093$ fm, $32^3 \times 64$, $N_f = 2+1+1$ twisted mass fermions

Renormalized matrix elements

- arXiv: 2305.11117

$-t =$ 0.17GeV^2, 0.34GeV^2, 0.66GeV^2, 0.69GeV^2, 0.81GeV^2, 1.26GeV^2, 1.39GeV^2, 1.40GeV^2, 1.54GeV^2, 2.33GeV^2, 2.78GeV^2

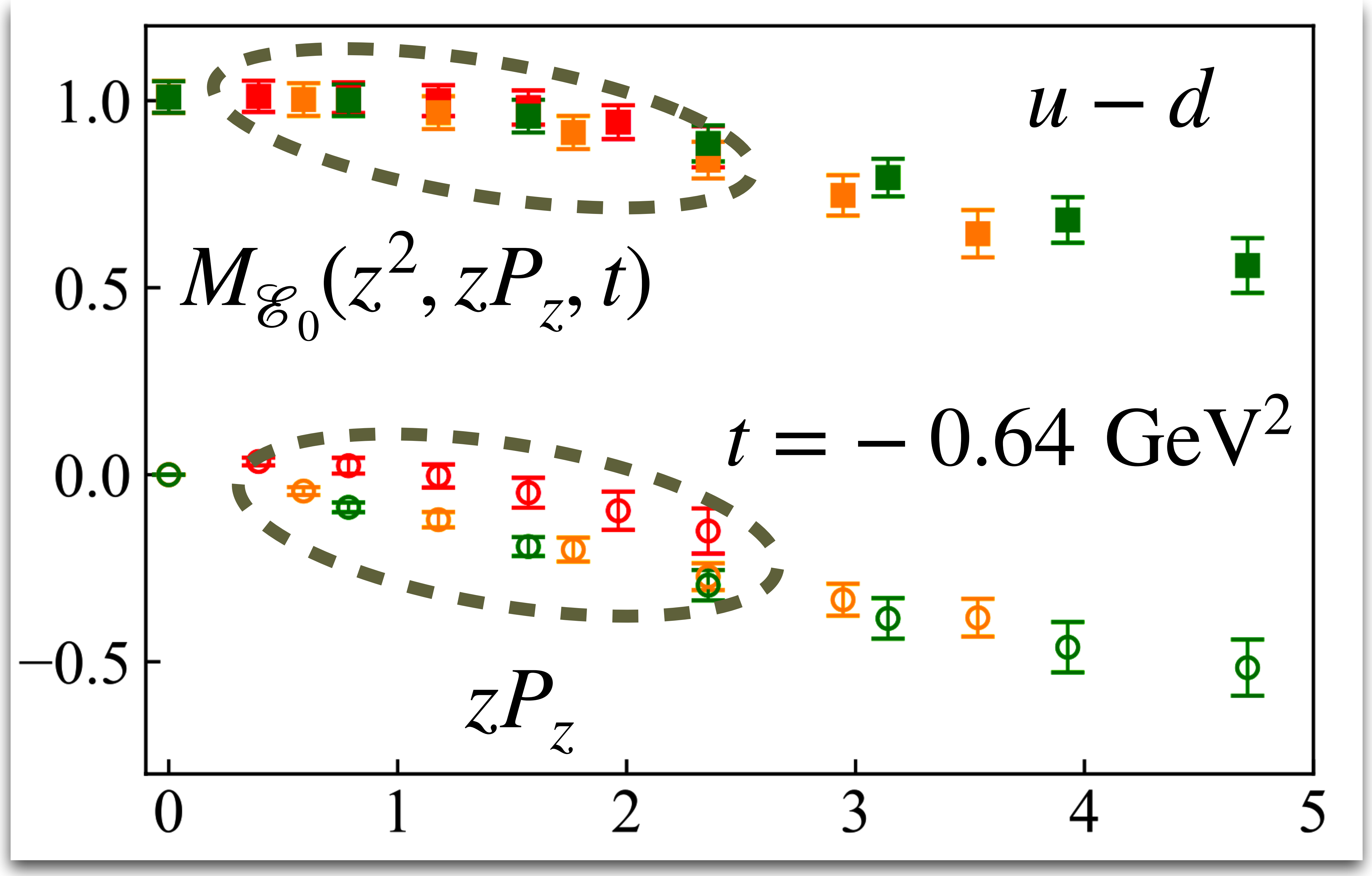


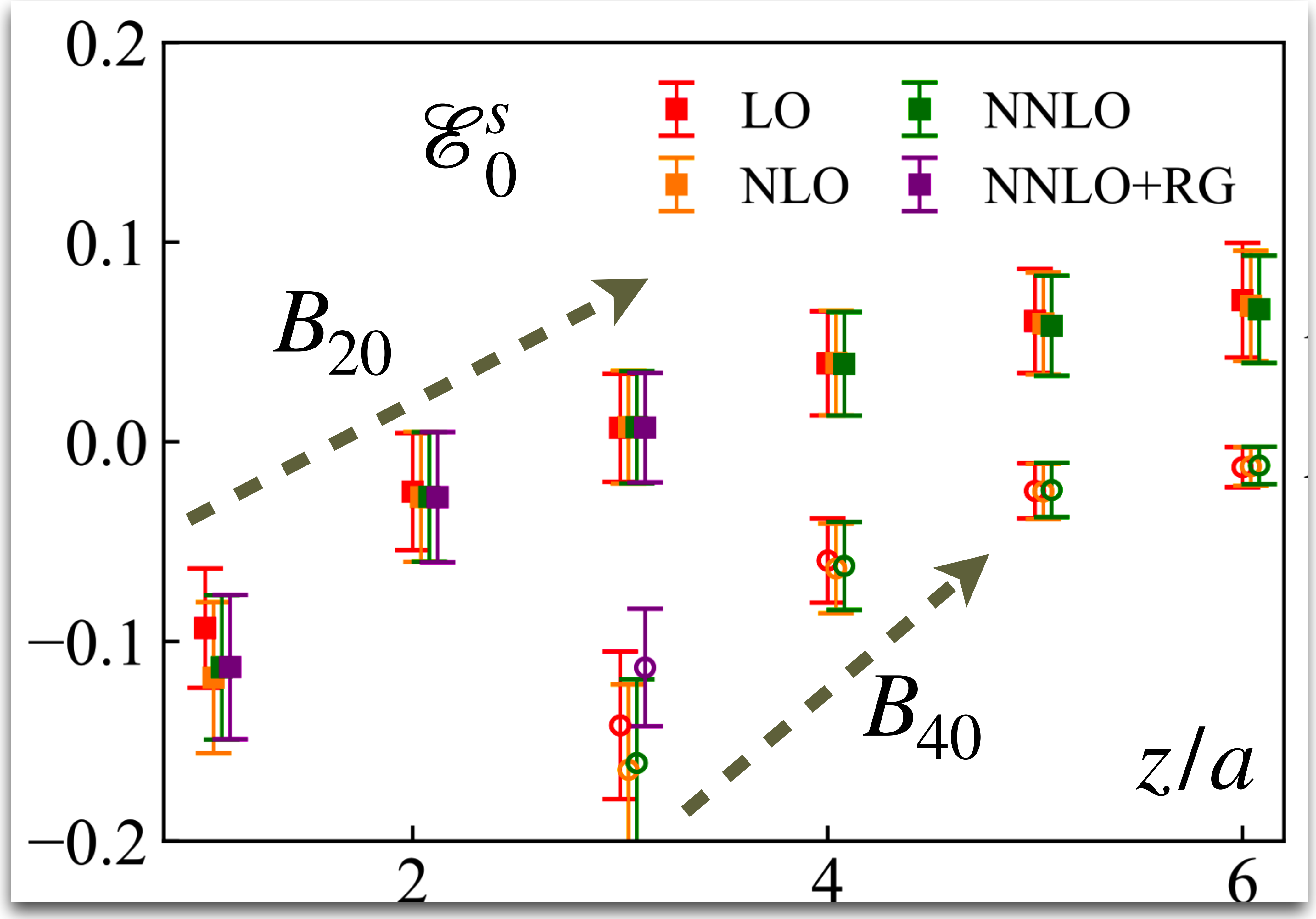


- filled symbols: real part, sensitive to even moments
- ◉ unfilled symbols: imaginary part, sensitive to odd moments

$P_z = 1.25\ \mathrm{GeV},\ a = 0.093\ \mathrm{fm}$

Mellin moments of GPDs: γ_0 definition





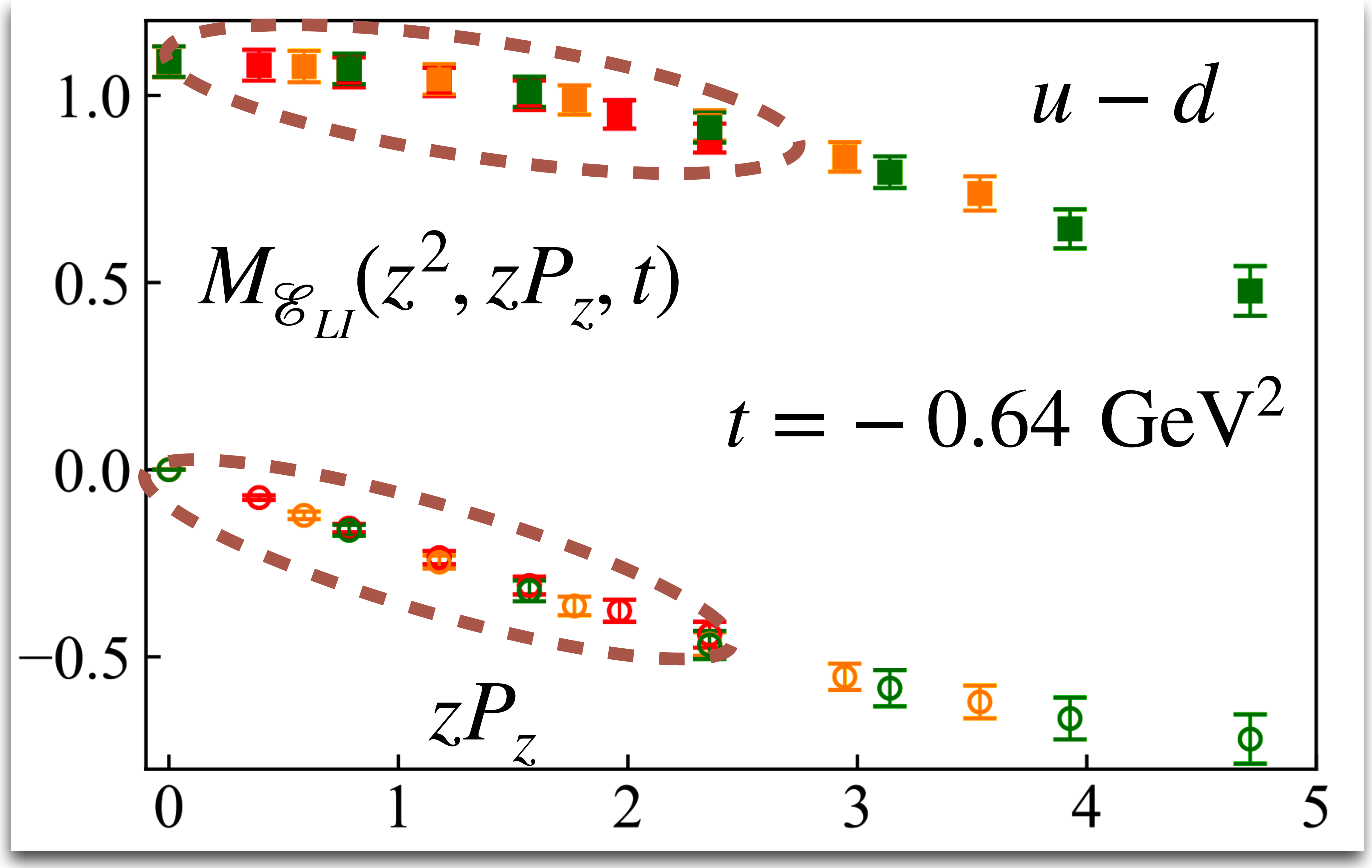
- no scaling with zP_z
- not constant in z

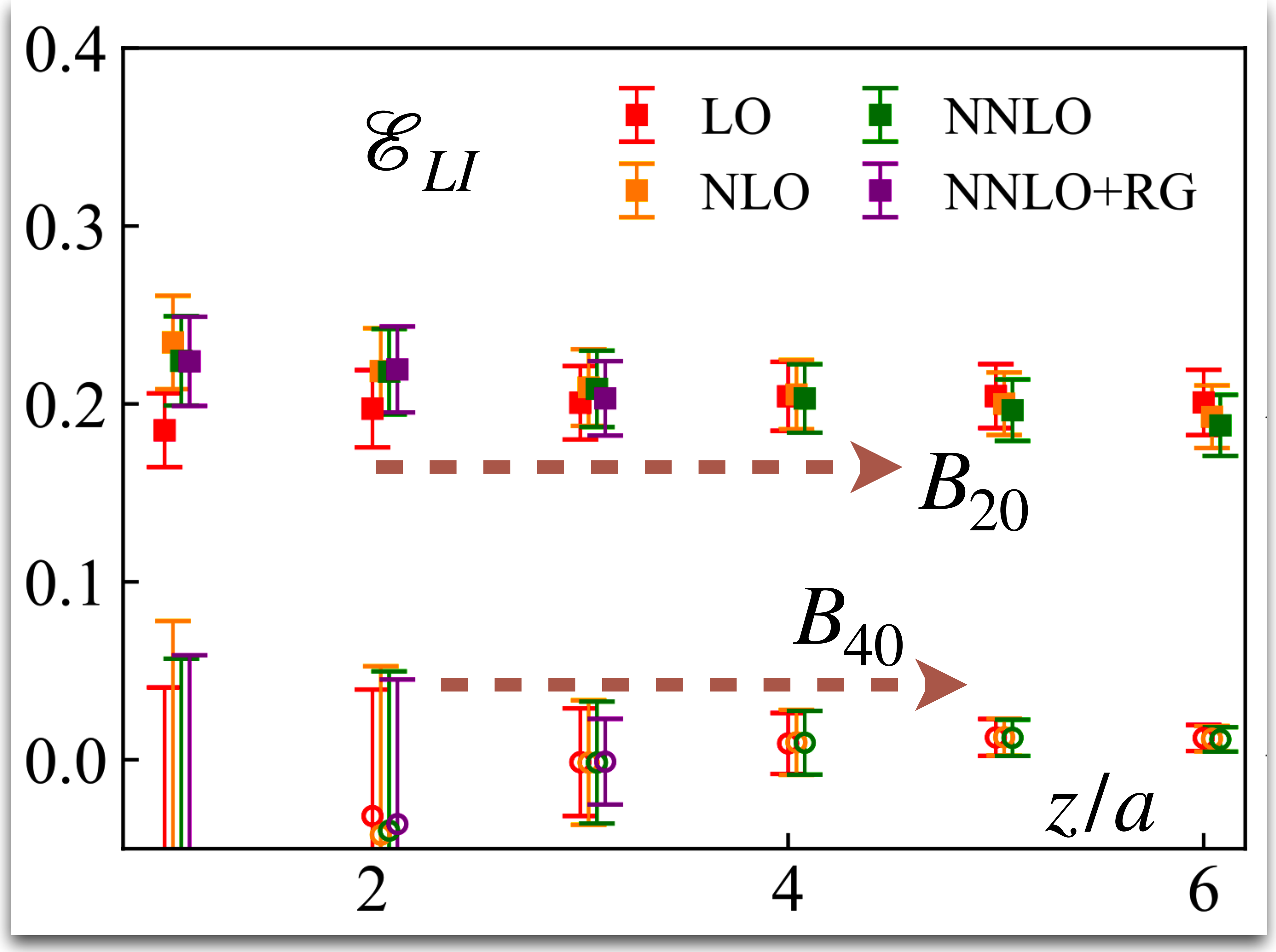
$$\mathscr{M}(z^2, zP, \Delta^2) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle(\mu) + \mathscr{O}(z^2 \Lambda^2_{\text{QCD}})$$

$$F^0(z, P, \Delta) = \bar{u}(p_f, \lambda') \left[\gamma^0 \mathscr{H}_0(z, P, \Delta) + \frac{i\sigma^{0\mu}\Delta_\mu}{2m} \mathscr{E}_0(z, P, \Delta) \right] u(p_i, \lambda)$$

$\mu = 2$ GeV $\quad P_z = 0.83,\ 1.25,\ 1.67$ GeV

Mellin moments of GPDs: LI definition





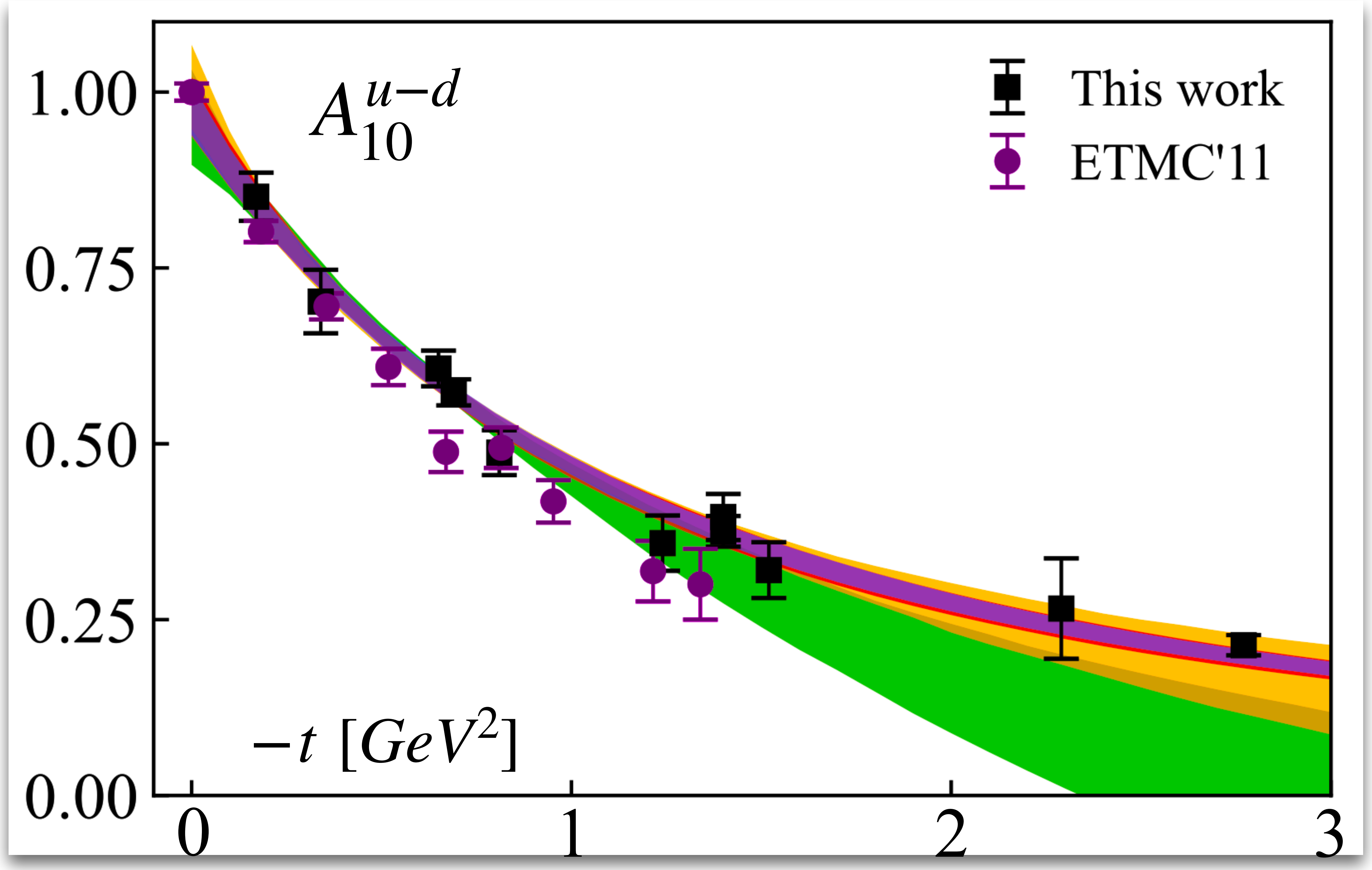
- Perturbative corrections $C_n(z^2\mu^2) = 1 + \mathcal{O}(\alpha_s)$
- Stable moments $\langle x^n \rangle(\mu)$

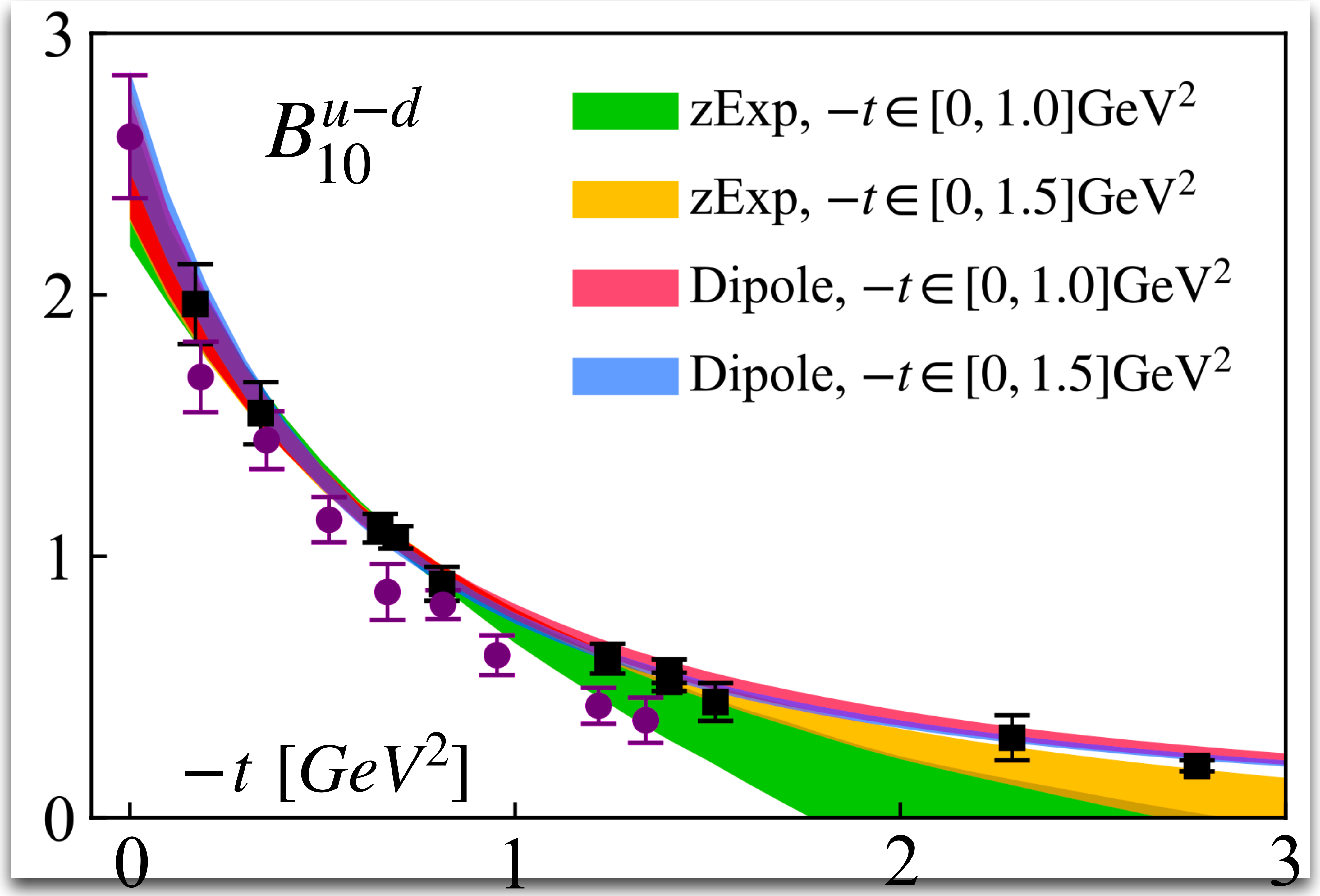
$$\mathscr{M}(z^2, zP, \Delta^2) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle(\mu) + \mathcal{O}(z^2\Lambda^2_{\rm QCD})$$

$$\mathscr{E}(z, P, \Delta) = -A_1 - \frac{\Delta \cdot z}{P \cdot z} A_3 + 2A_5 + 2P \cdot zA_6 + 2\Delta \cdot zA_8$$

$\mu = 2$ GeV $\quad P_z = 0.83,\ 1.25,\ 1.67$ GeV

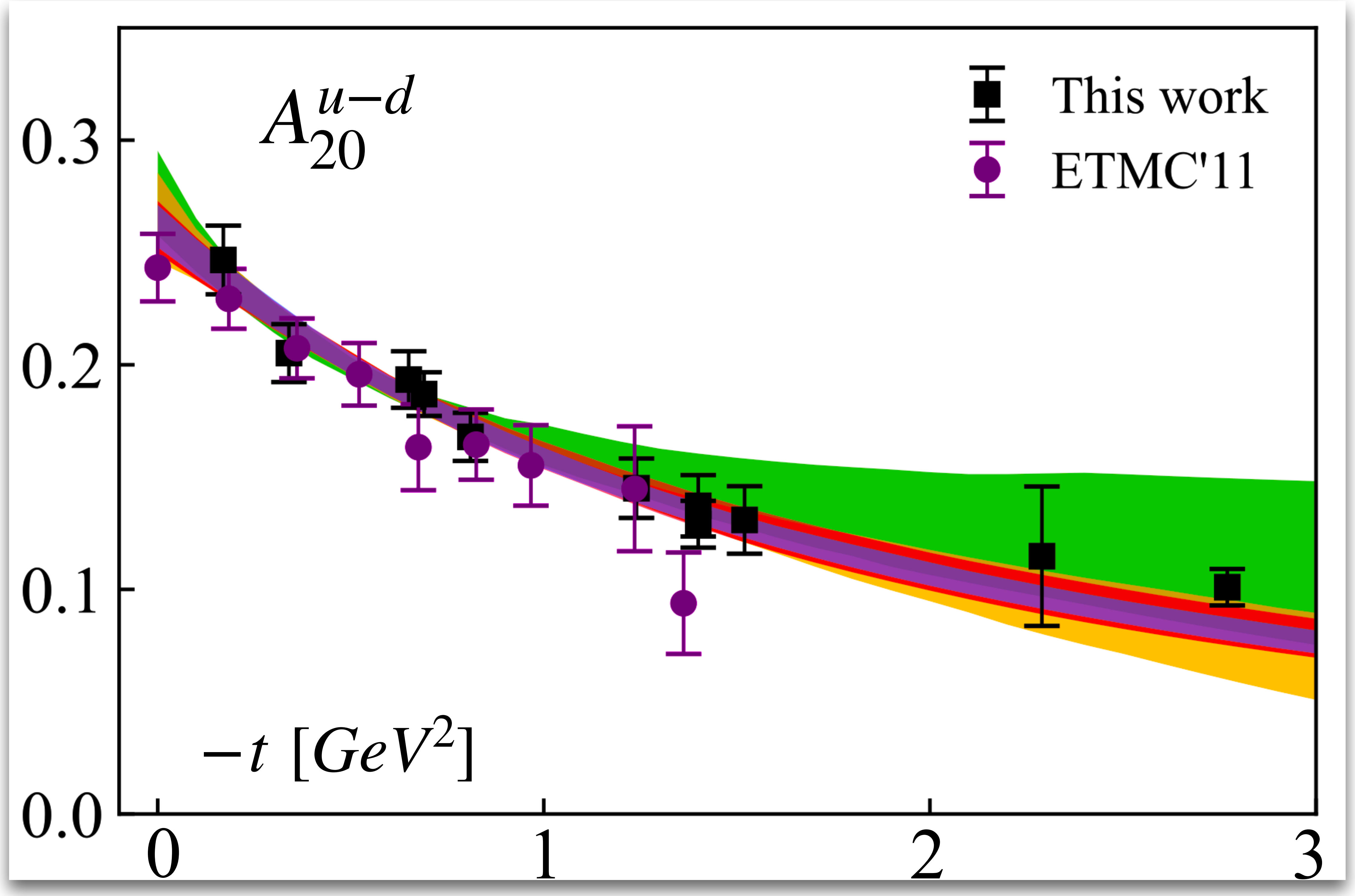
Mellin moments of GPDs

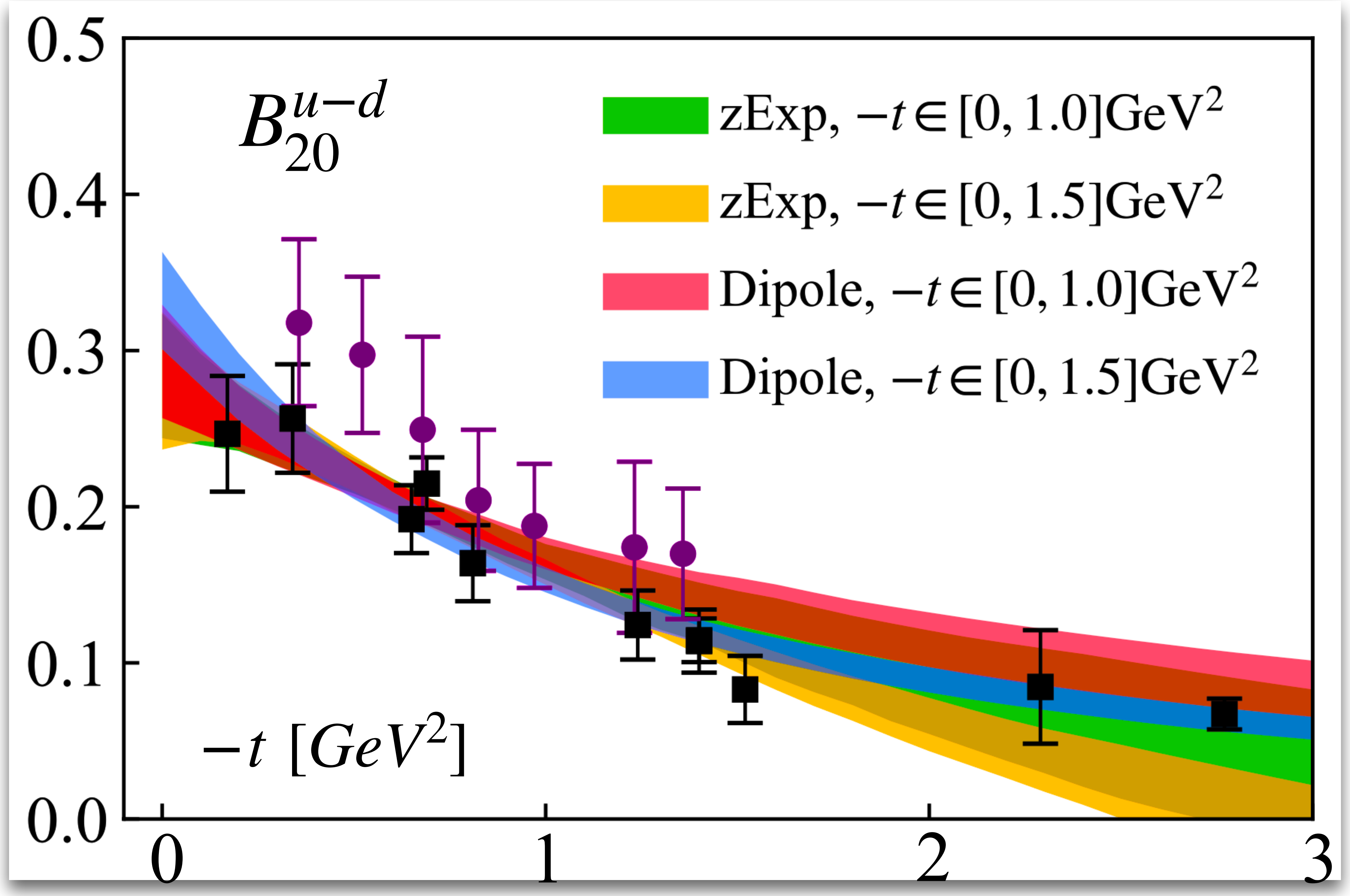




- Good agreement with available traditional lattice QCD calculations of GPD moments using local operators (ETMC'11)

Mellin moments of GPDs





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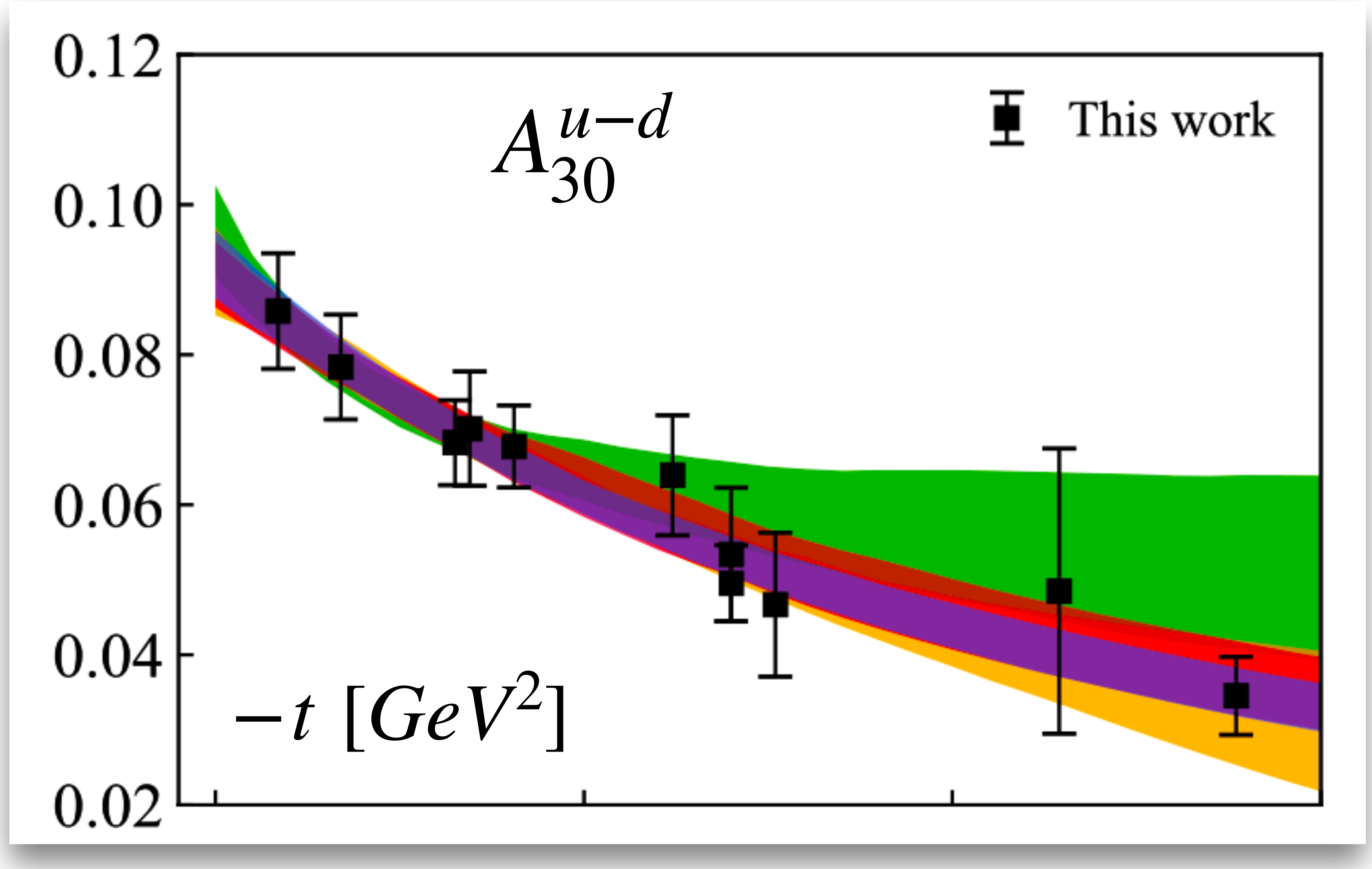
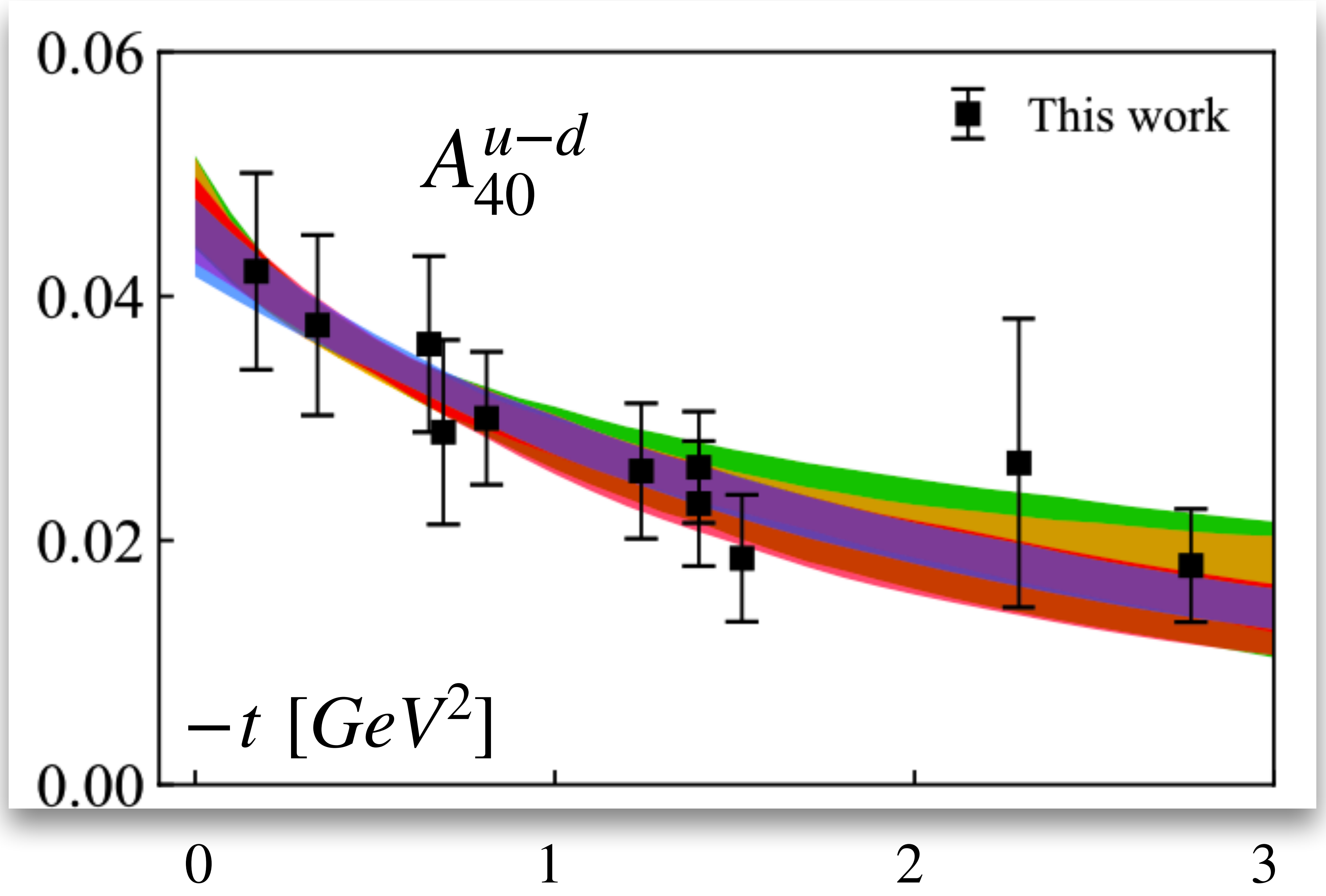
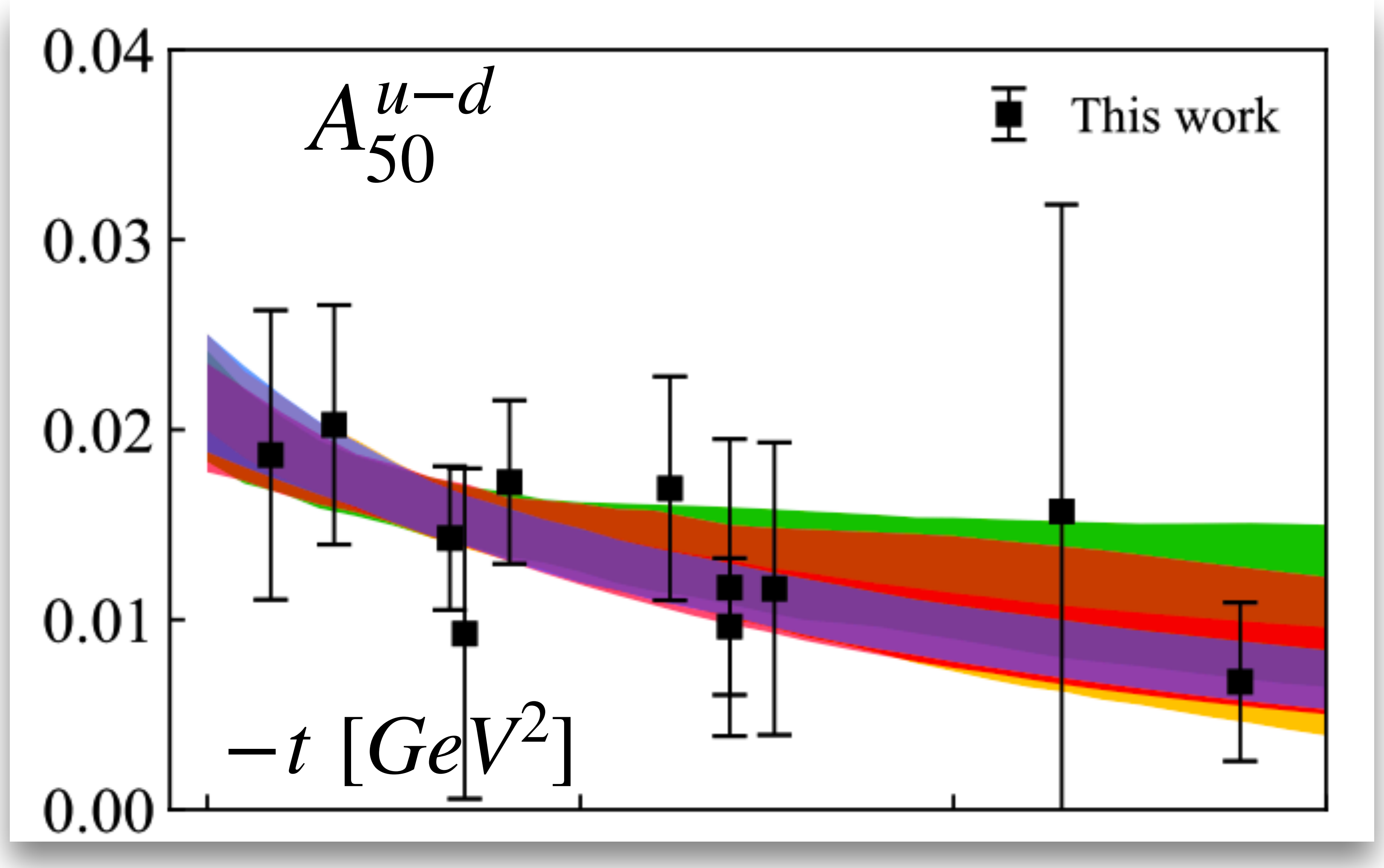
Mellin moments of GPDs

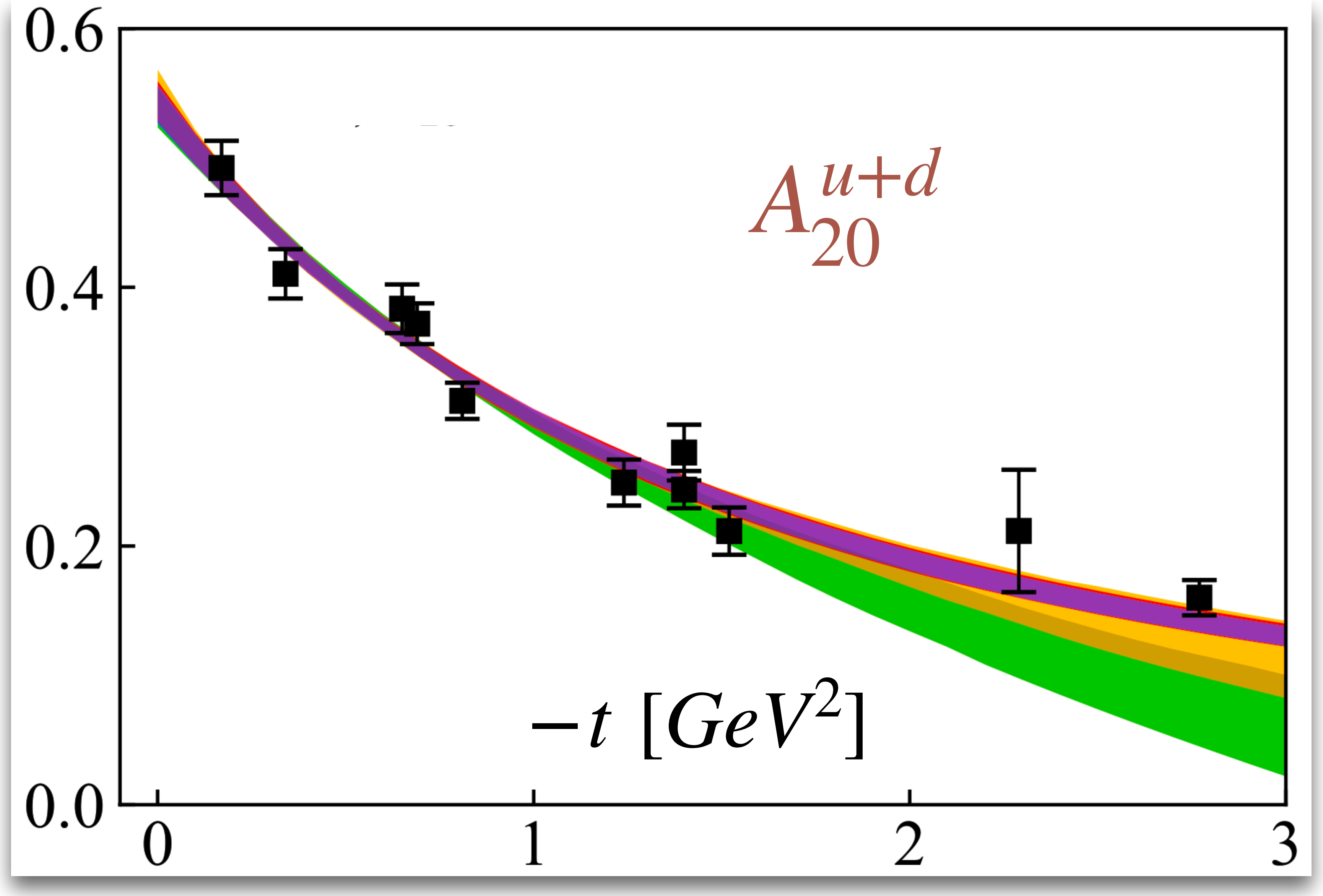


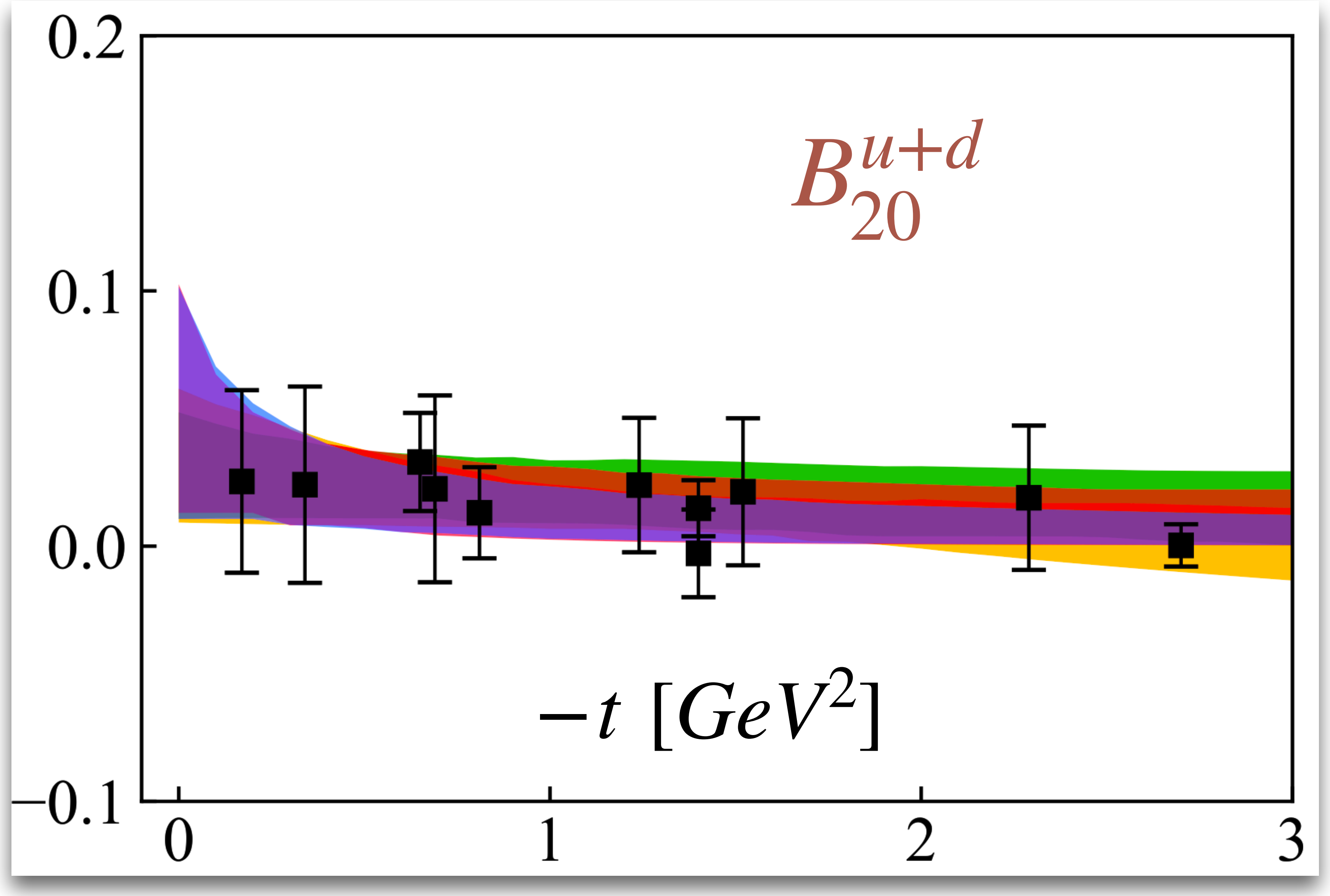




- Up to 5th moments of GPDs show reasonable signals and smooth $-t$ dependence.
- Higher moments can be constrained by increasing the hadron momentum.

Mellin moments of GPDs





- 2nd moments: Gravitational form factors

Ji sum rule: $$J^q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$$

$$J^{u-d} = 0.281(21)(11)$$

$$J^{u+d} = 0.296(22)(33)$$

- $m_\pi = 260$ MeV, $a = 0.093$ fm
- Disconnected diagrams neglected

Impact parameter space interpretation

- Unpolarized quark inside unpolarized nucleon

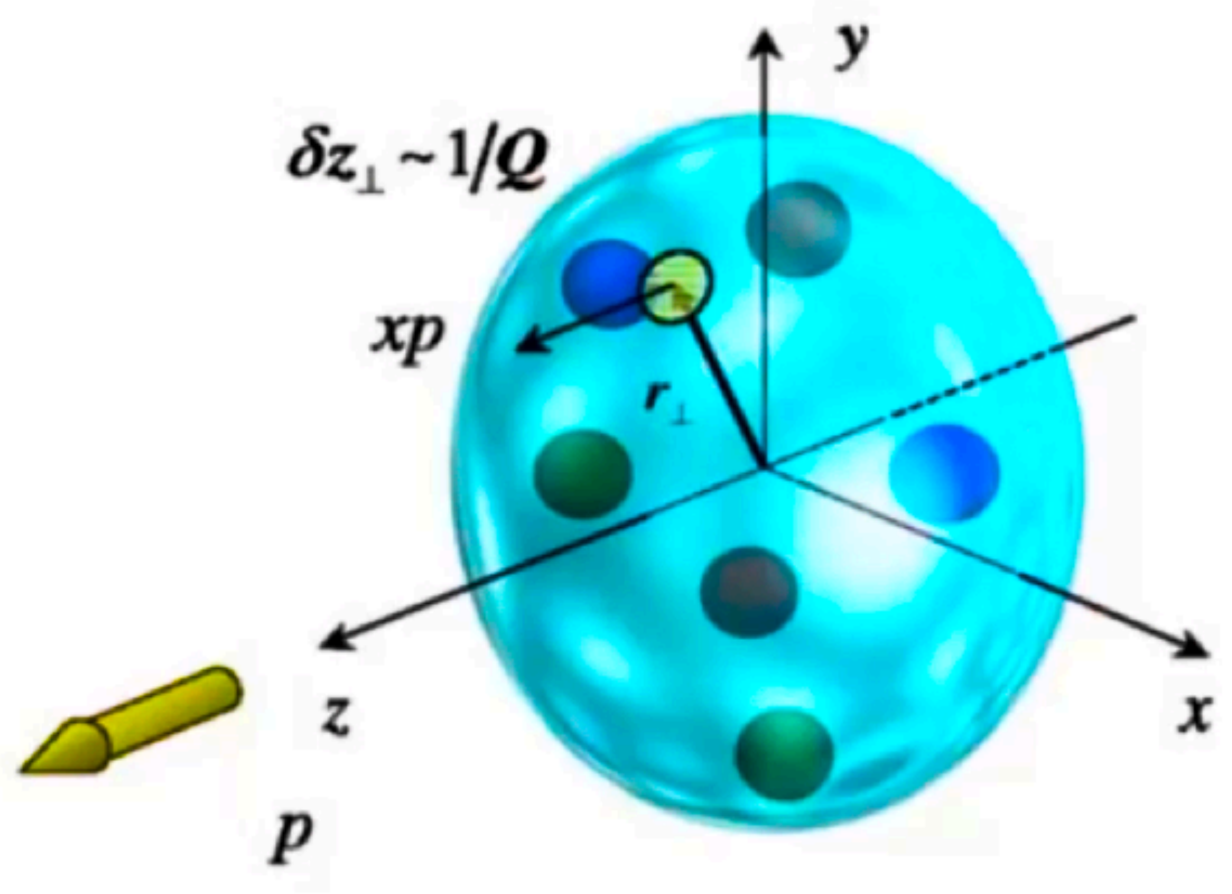
$$q(x,\vec{b}_\perp)=\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}H(x,-\vec{\Delta}_\perp^2)e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}$$

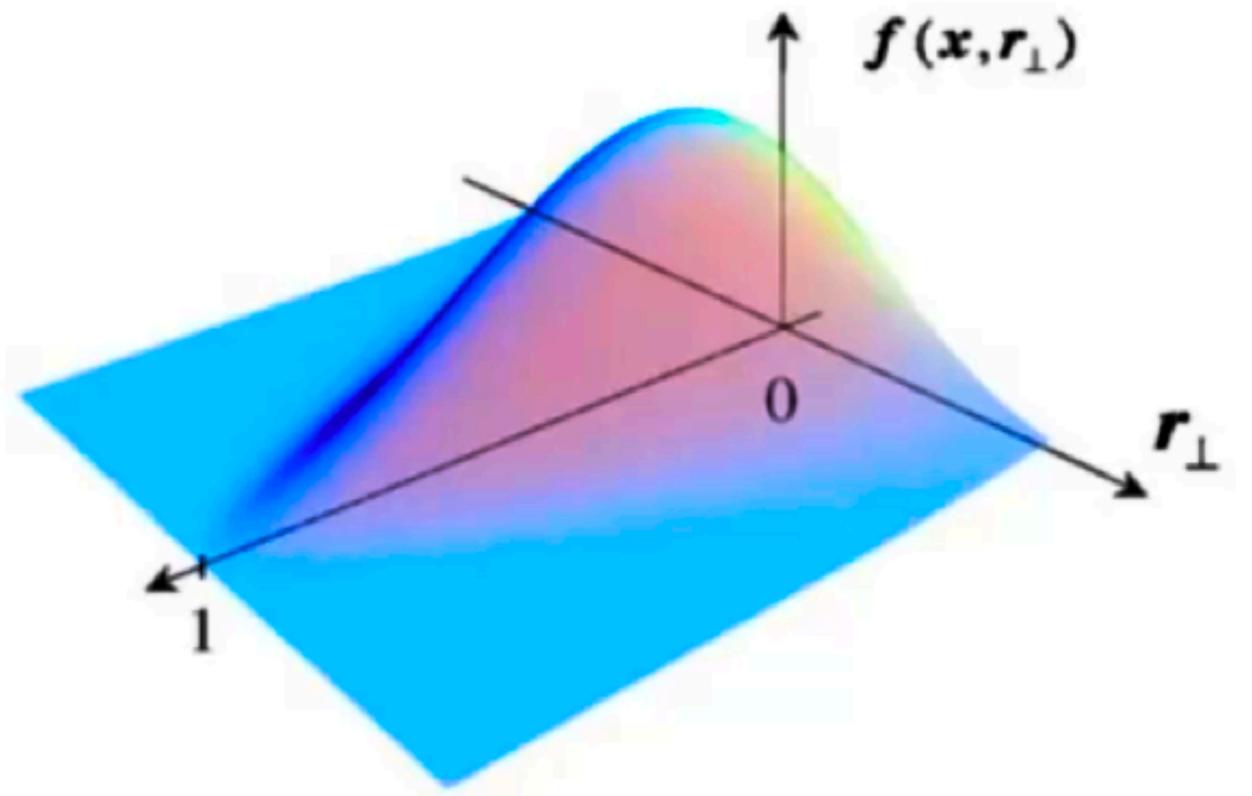
$$\rho_{n+1}(\vec{b}_\perp)=\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}A_{n+1,0}(-\vec{\Delta}_\perp^2)e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}$$

- Unpolarized quark inside transversely polarized nucleon

$$q^T(x,\vec{b}_\perp)=\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\left[H(x,-\vec{\Delta}_\perp^2)+\frac{i\Delta_y}{2M}E(x,-\vec{\Delta}_\perp^2)\right]e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}$$

$$\rho^T_{n+1}(\vec{b}_\perp)=\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\left[A_{n+1,0}(-\vec{\Delta}_\perp^2)+\frac{i\Delta_y}{2M}B_{n+1,0}(-\vec{\Delta}_\perp^2)\right]e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}$$

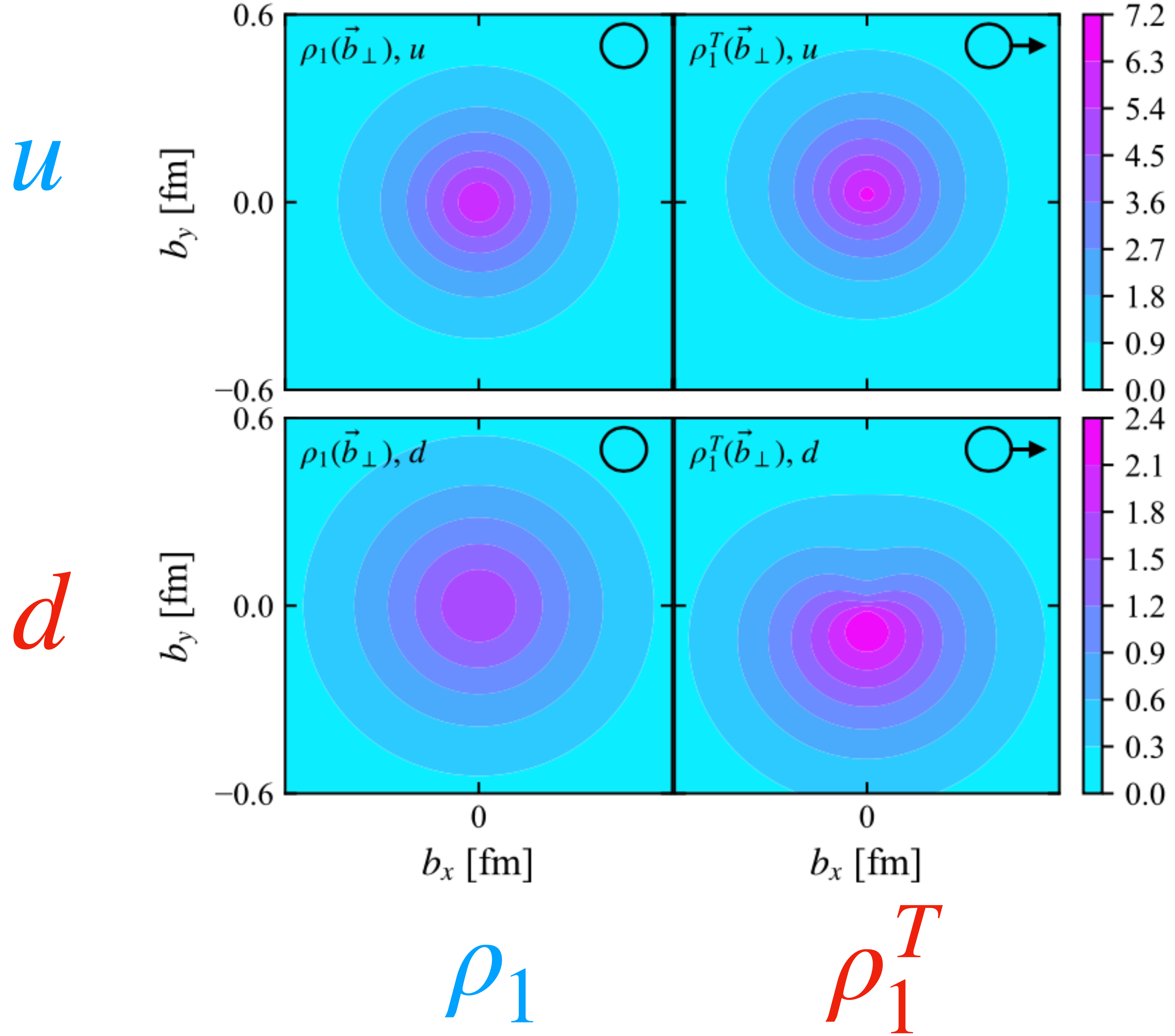




Belitsky and Radyushkin: Phys.Rept. 418 (2005) 1-387

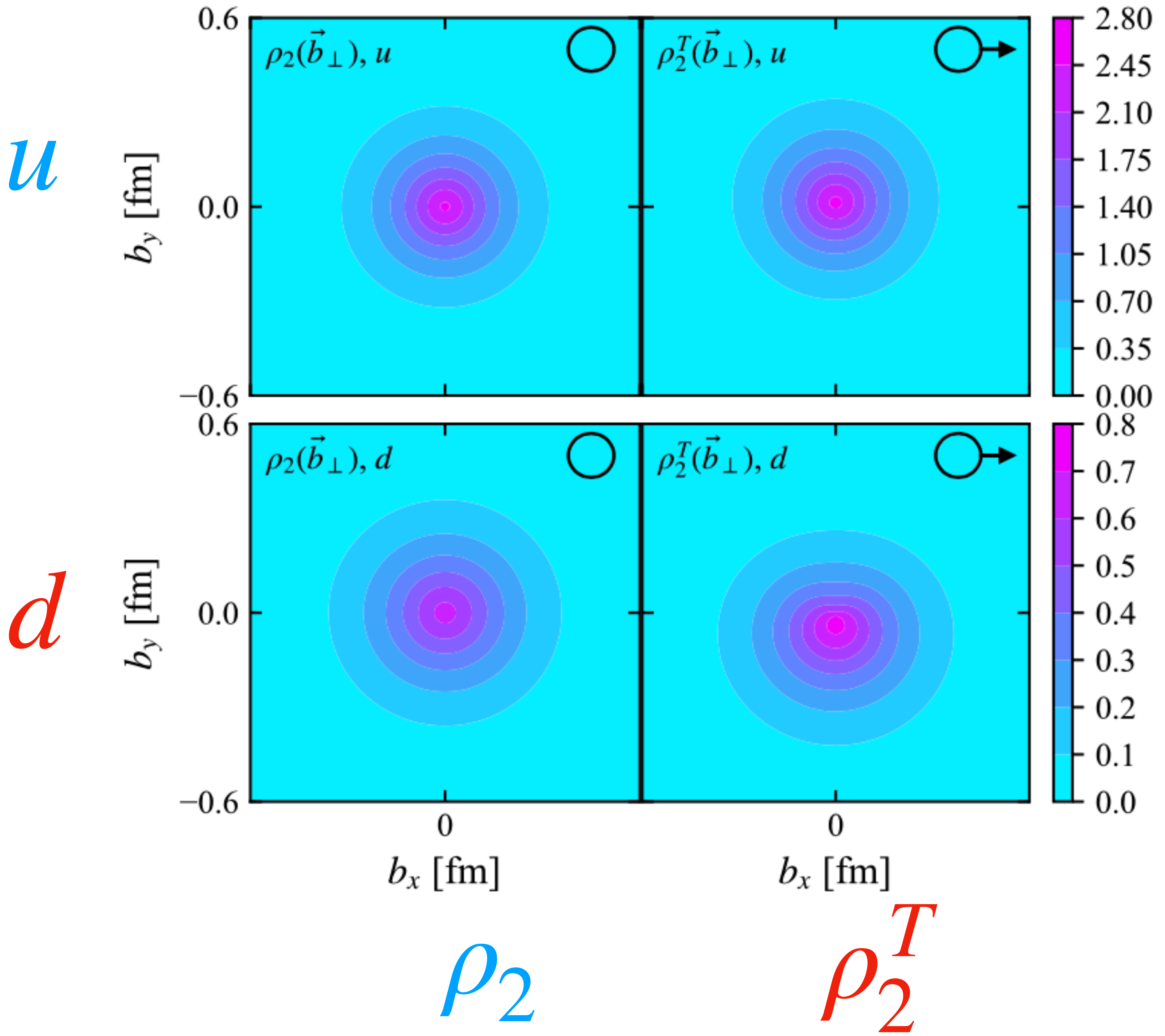
- Matthias Burkardt, Int.J.Mod.Phys.A 18 (2003) 173-208

Impact parameter space interpretation



- The **1st** moment: charge distribution
- **d** quark exhibits a broader distribution and smaller amplitudes
- When transversely polarized, the **u** and **d** quarks shift in different directions, with the **d** quarks showing larger distortion.

Impact parameter space interpretation

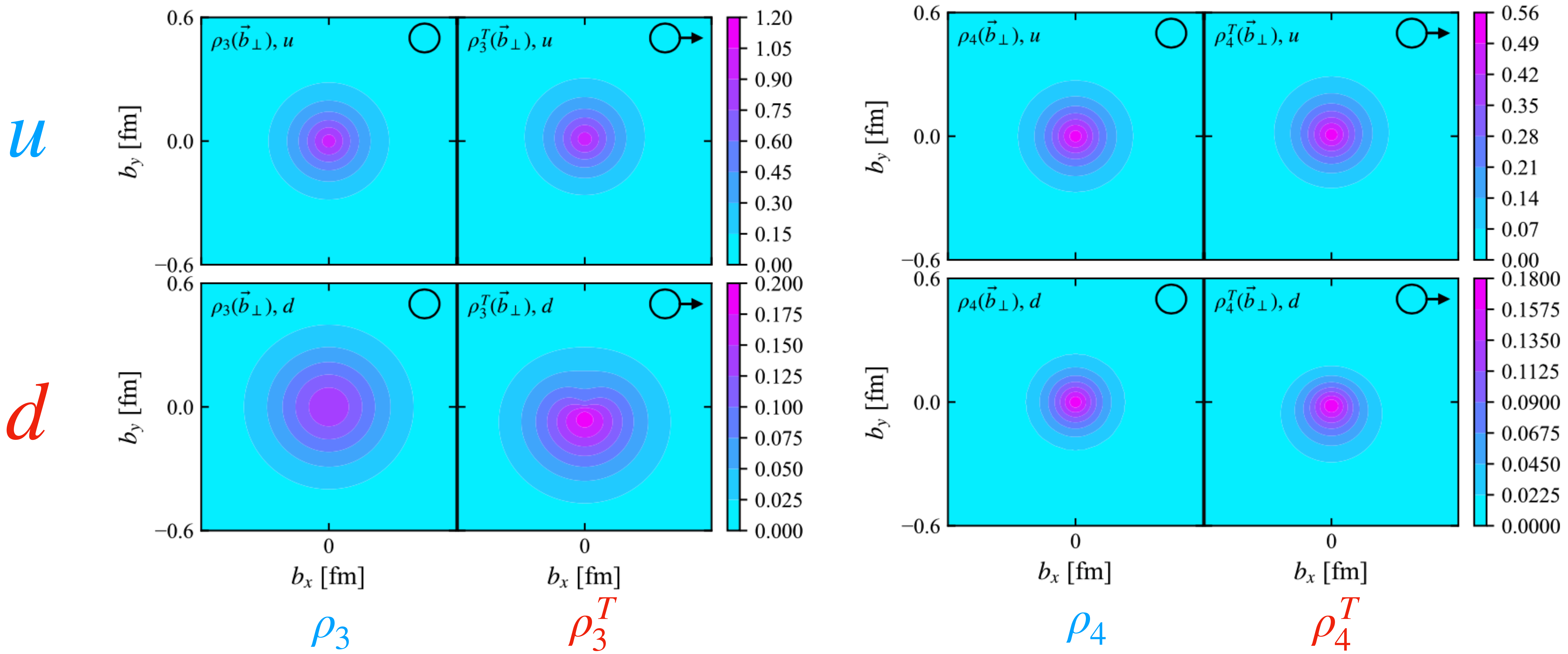


- The **2nd** moment: energy distribution
- The total contribution of **u** and **d** quarks to the transverse center of energy is small.

$$\sum_{u,d} \int d^2\vec{b}_\perp\ \vec{b}_\perp \rho_2^T(\vec{b}_\perp) = 1/(2M) B_{20}^{u+d}(0)$$

$$B_{20}^{u+d}(0) = 0.047(33)(65)$$

Impact parameter space interpretation



- Higher moments are weighted by x^n, exhibiting a sharper drop in the transverse distance.

Summary and outlook

- We carried out lattice calculation of the quasi-GPD matrix elements of proton using the Lorentz invariant definition.
- The matrix elements are renormalized in ratio scheme and the Mellin moments up to the 5th ones were extracted using the leading-twist short distance factorization frame work.
  - Higher moments can be better constrained with higher momentum and statistics, and the methods can be extended to non-zero skewness GPDs.
  - Calculations with physical quark masses and smaller lattice spacings are needed to address the lattice artifacts.

Thanks for your attention!