Proton GPDs from lattice QCD

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Temple University

CNF Workshop on
Generalized Parton Distributions and Global Analysis

June 13, 2023

Outline

PHYSICAL REVIEW LETTERS **125,** 262001 (2020)

Unpolarized and Helicity Generalized Parton Distributions of the Proton within Lattice QCD

Constantia Alexandrou,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[4] Kyriakos Hadjiyiannakou,[1] Karl Jansen,[5] Aurora Scapellato,[3] and Fernanda Steffens[6]

PHYSICAL REVIEW D **105,** 034501 (2022)

Transversity GPDs of the proton from lattice QCD

Constantia Alexandrou,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[4] Kyriakos Hadjiyiannakou,[1,2] Karl Jansen,[5] Aurora Scapellato,[4] and Fernanda Steffens[6]

★ **Twist-2 GPDs: "traditional" calculations**

★ **Twist-3 GPDs**

arXiv:2306.05533v1 [hep-lat] 8 Jun 2023

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya[1,2], Krzysztof Cichy[3], Martha Constantinou[1], Jack Dodson[1], Andreas Metz[1], Aurora Scapellato[1], Fernanda Steffens[4]

PHYSICAL REVIEW D **106,** 114512 (2022)

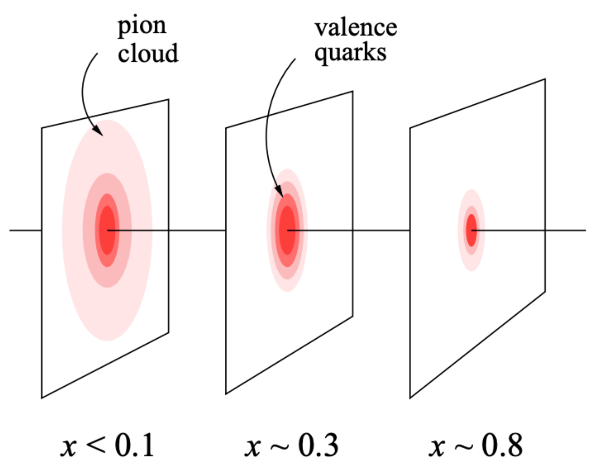
Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

+ Joshua Miller

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

★ **Twist-2 GPDs: new approach**

Motivation for GPDs studies



[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]

$1_{\mathrm{mom}} + 2_{\mathrm{coord}}$ tomographic images of quark distribution in nucleon at fixed longitudinal momentum

3-D image from FT of the longitudinal mom. transfer

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs

- GPDs phenomenology more complicated than PDFs (multi-dimensionality)

- and more challenges …

Essential to complement the knowledge on GPD from lattice QCD

Twist-classification of PDFs, GPDs, TMDs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

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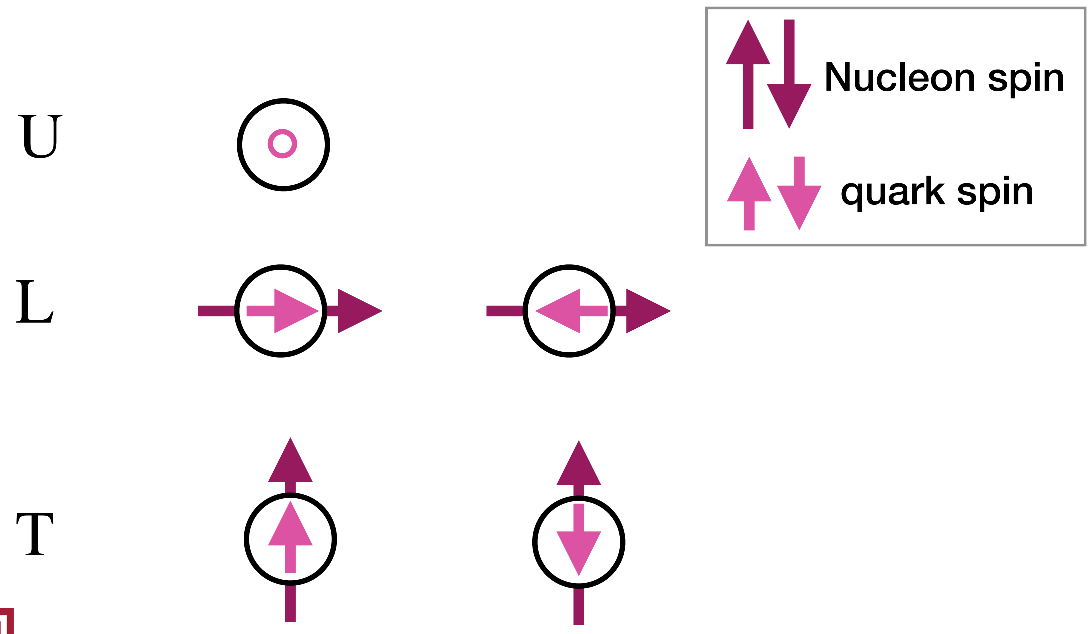
Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L $(\gamma^+\gamma^5)$	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity

Probabilistic interpretation



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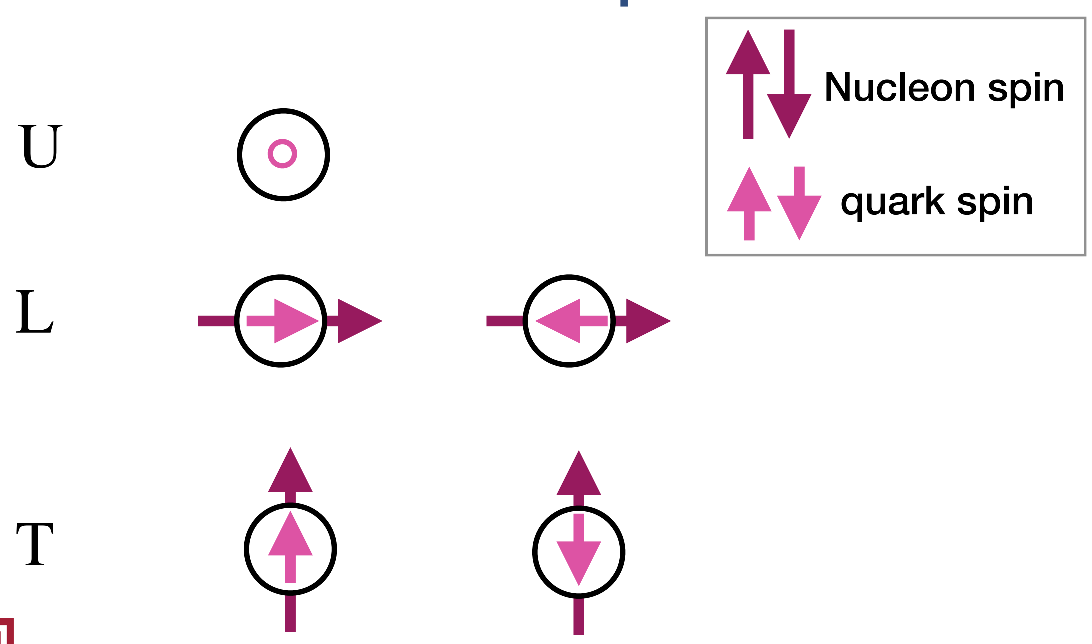
Twist-3 $(f_i^{(1)})$

Selected

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

Probabilistic interpretation



- ★ Lack density interpretation, but can be sizable
- ★ Kinematically suppressed
  Difficult to isolate experimentally
- ★ Theoretically: contain $\delta(x)$ singularities
- ★ Contain info on quark-gluon-quark correlators

Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\underbrace{\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]}_{\text{local operators}}$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big)\Big\}$$

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★ **Matrix elements of non-local operators**
(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\underbrace{\mathcal{W}(z,0)}_{\text{Wilson line}}\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\langle N(P')|O_V^{\mu}(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^{\mu}(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

Accessing information on GPDs

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local operators

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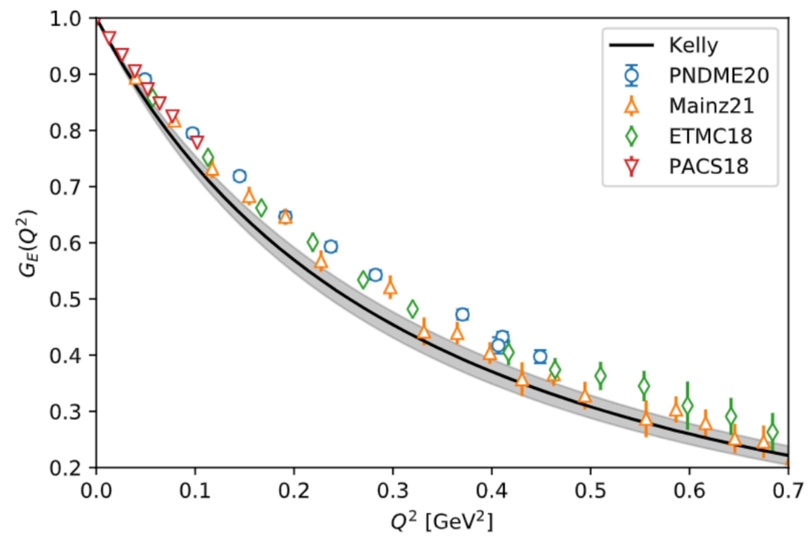

Wide -t range that comes at the cost of 1

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GPDs

Through non-local matrix elements

of fast-moving hadrons

Access of GPDs on a Euclidean Lattice

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]

Matrix elements of nonlocal (equal-time) operators with fast moving hadrons

$$\tilde{q}_{\Gamma}^{\mathrm{GPD}}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

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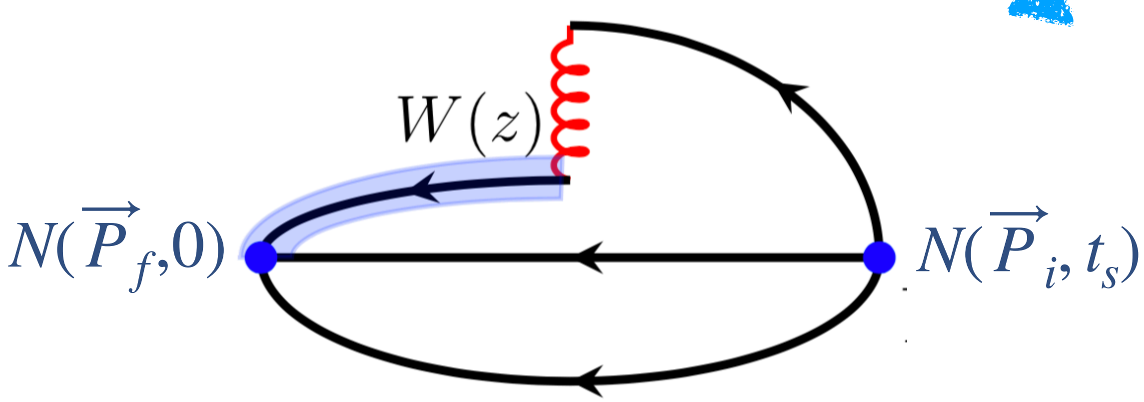

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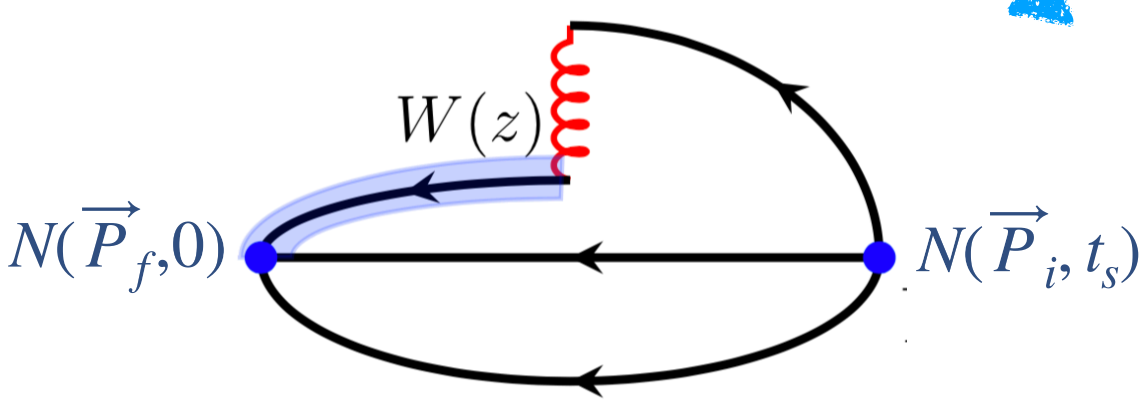

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Variables of the calculation:

- length of the Wilson line (z)
- nucleon momentum boost (P_3)
- momentum transfer (t)
- skewness (ξ)

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

frame	P_3 [GeV]	$\boldsymbol{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

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Symmetric frame
very expensive
computationally

Traditional calculations of GPDs

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PHYSICAL REVIEW D **105,** 034501 (2022)

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First lattice calculation of x-dependent GPDs

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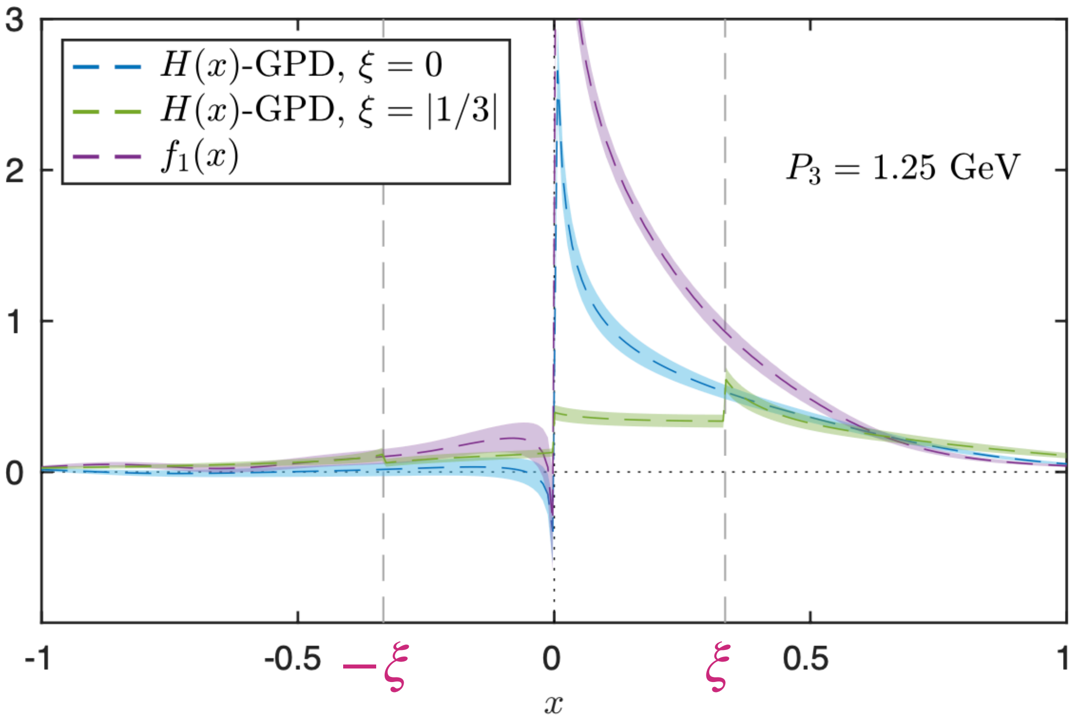

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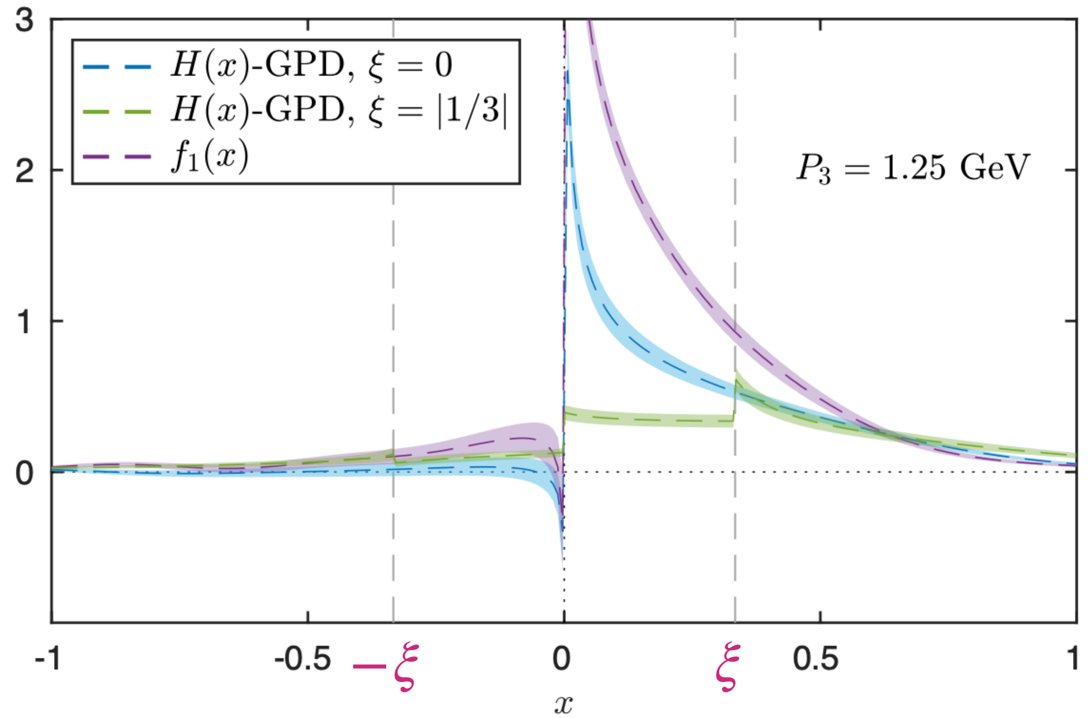

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★ ERBL/DGLAP: Qualitative differences

★ $\xi = \pm x$ inaccessible (formalism breaks down)

★ $x \to 1$ region: qualitatively comparison with power counting analysis [F. Yuan, PRD69 (2004) 051501, hep-ph/0311288]

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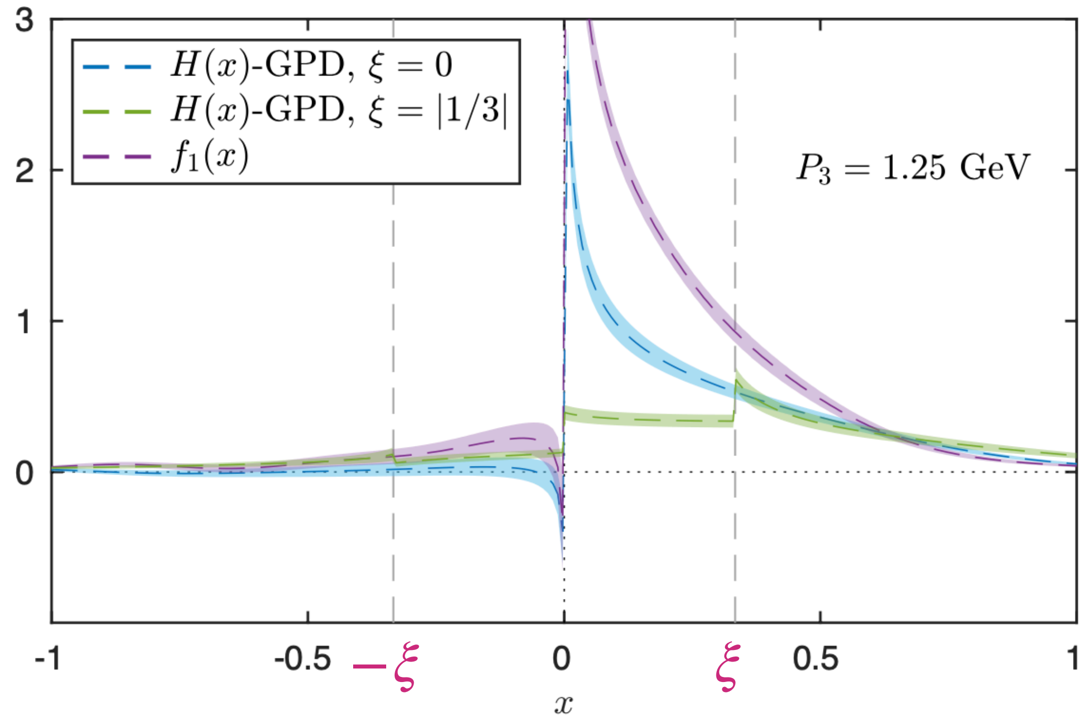

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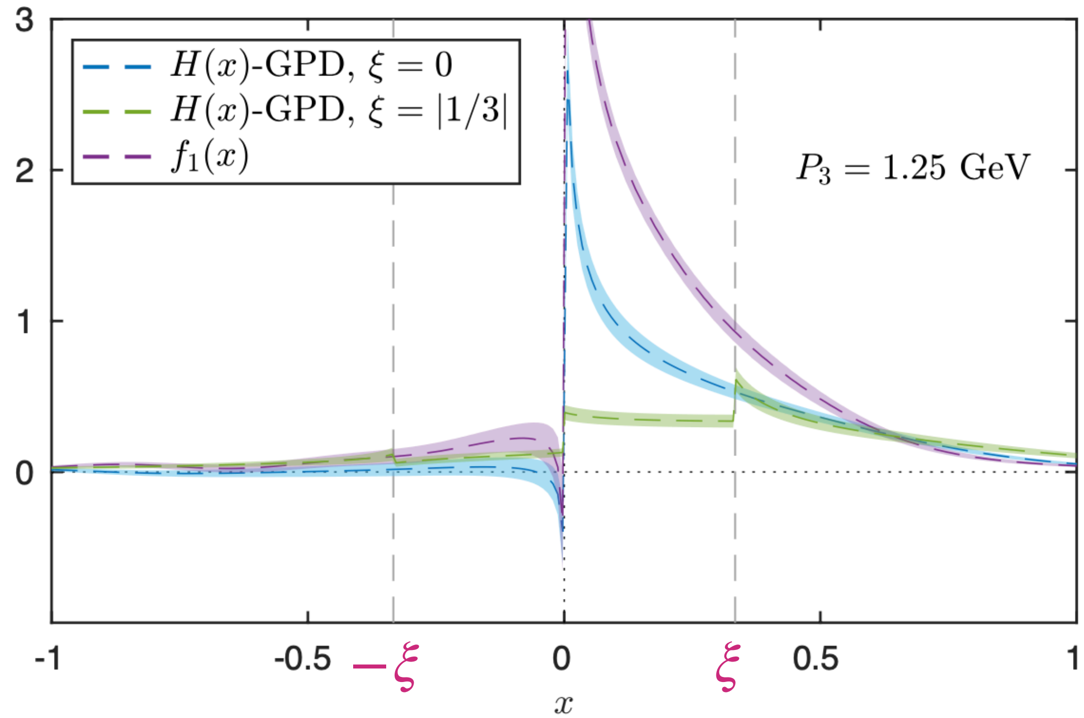

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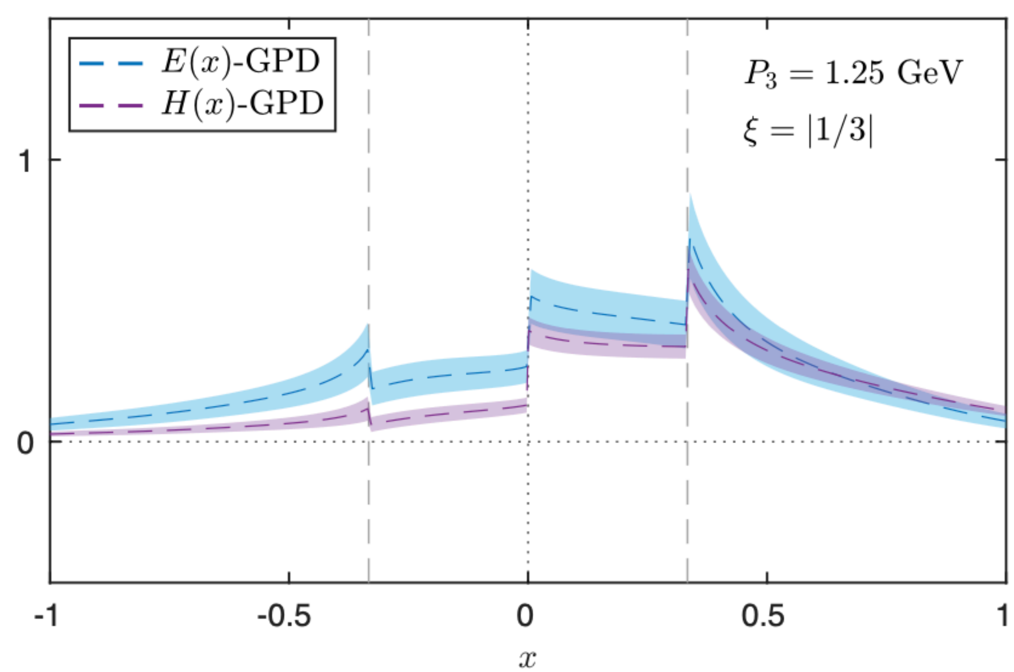
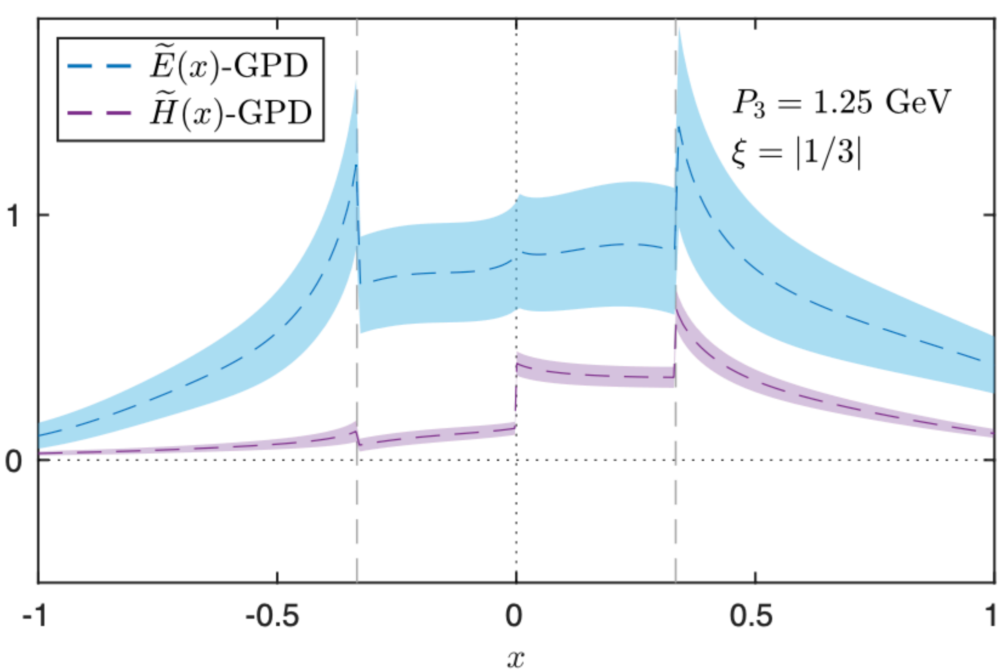
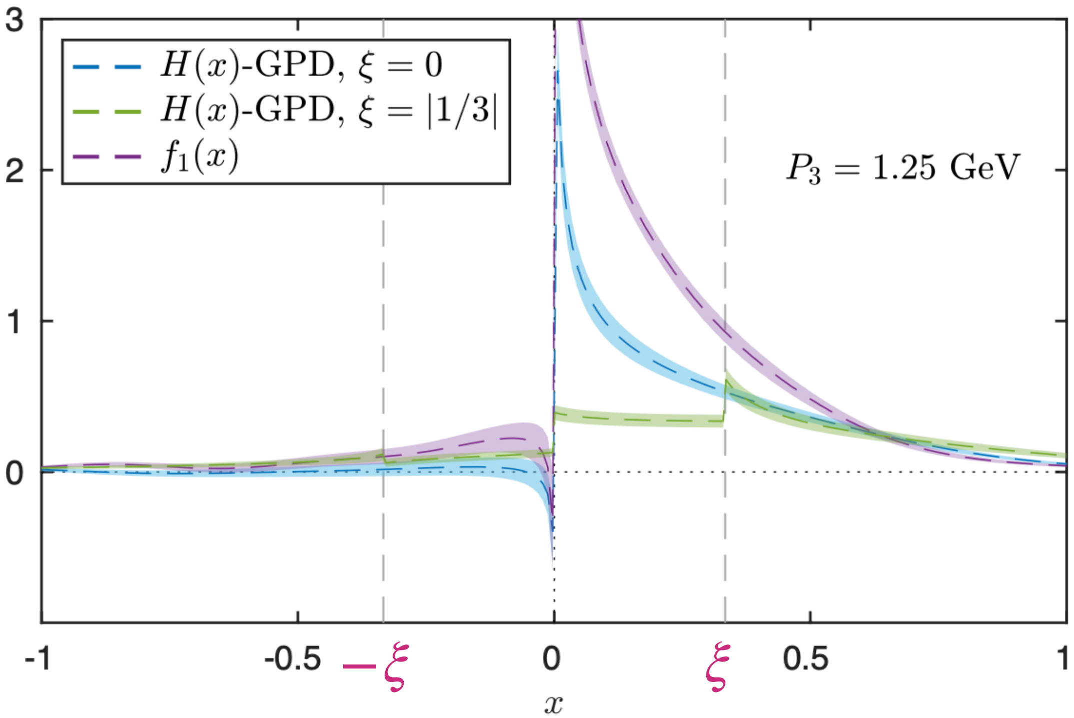




First lattice calculation of x-dependent GPDs



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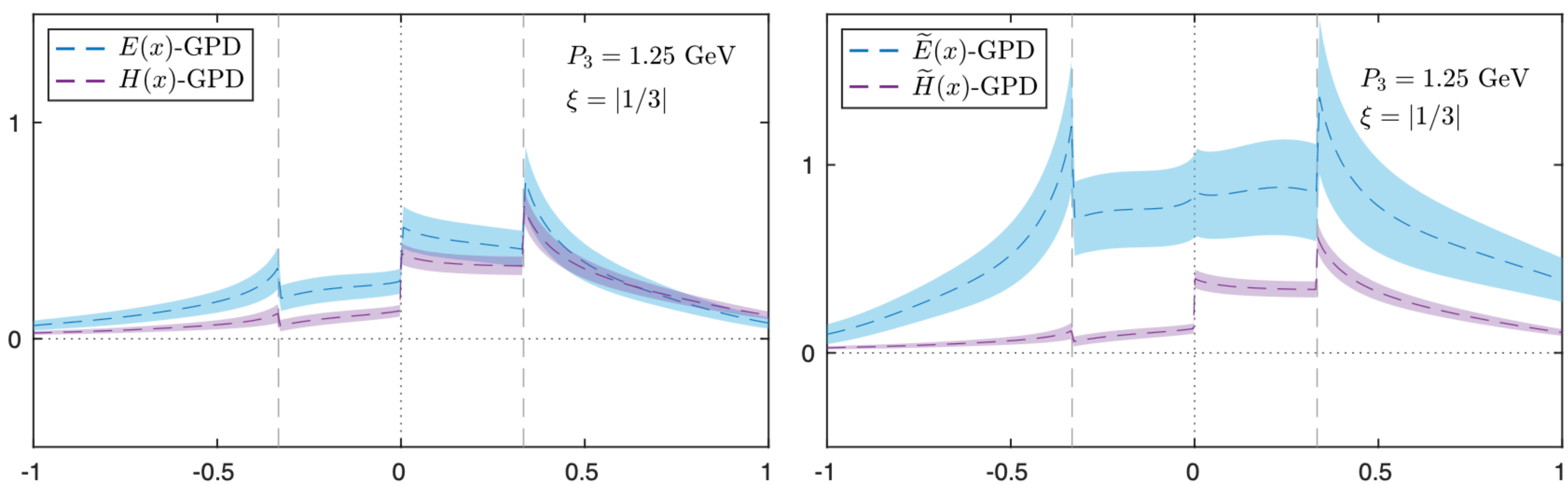


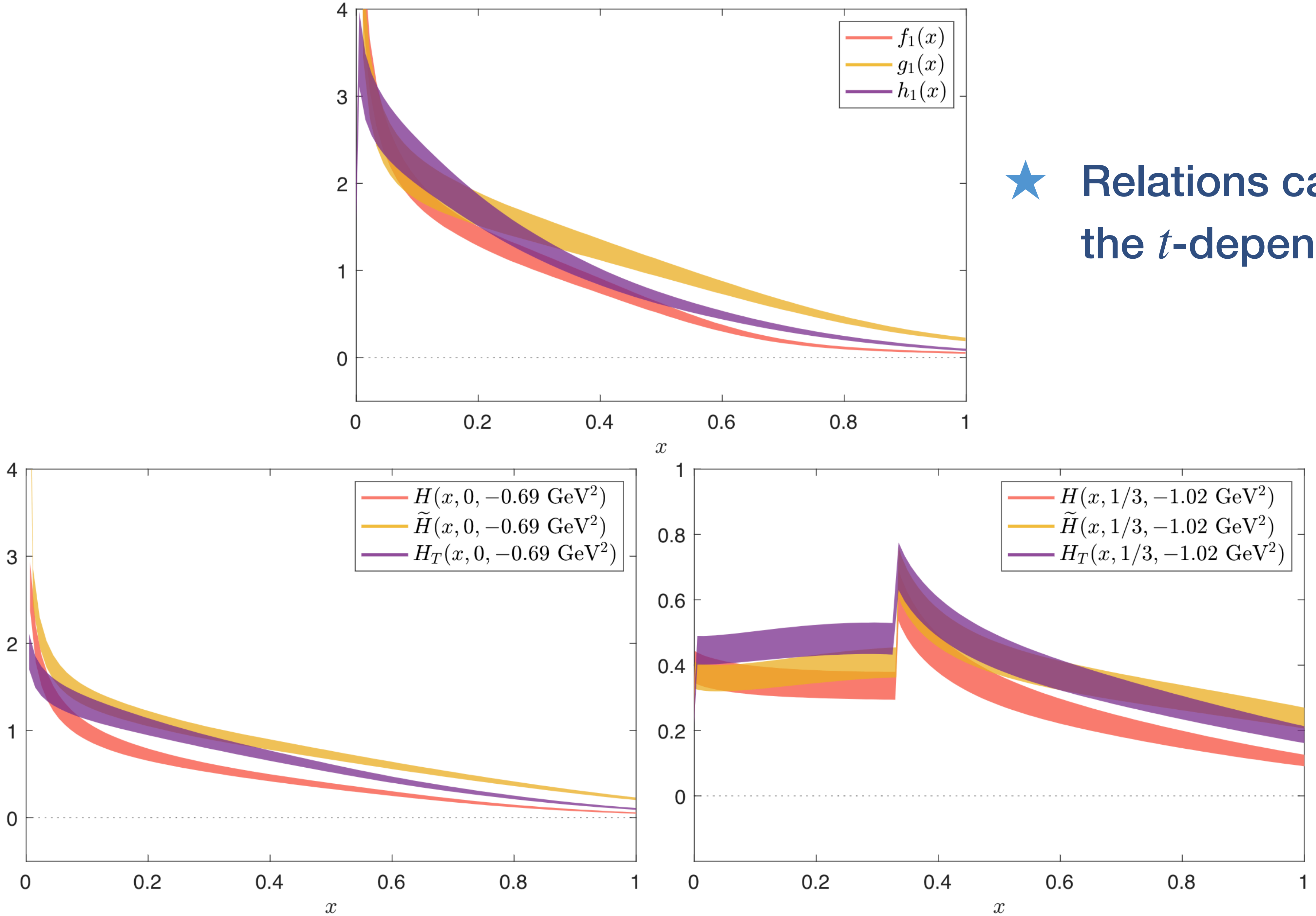
- ★ important contribution in the proton spin

$$\int_{-1}^{+1} dx\, x^2 H^q(x,\xi,t) = A_{20}^q(t) + 4\xi^2 C_{20}^q(t)\,, \qquad \int_{-1}^{+1} dx\, x^2 E^q(x,\xi,t) = B_{20}^q(t) - 4\xi^2 C_{20}^q(t)$$

First lattice calculation of x-dependent GPDs

★ Qualitative understanding of GPDs and their relations

★ Qualitative understanding of ERBL and DGLAP regions



★ Relations can be identified for the t-dependence of GPDs

New parametrization of GPDs

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

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GPDs on the lattice

★ γ^+ inspired parametrization is prohibitively expensive

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q}(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

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★ Lorentz invariant parametrization

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu M A_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

A_i : - Lorentz invariant amplitudes
- have definite symmetries

GPDs on the lattice

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★ Two decompositions can be related

$$\mathcal{H}_0^s(A_i^s;z) = A_1 + \frac{z(\Delta_1^2+\Delta_2^2)}{2P_3}A_6\,,$$

$$\mathcal{E}_0^s(A_i^s;z) = -A_1 - \frac{m^2 z}{P_3}A_4 + 2A_5 - \frac{z\left(4E^2+\Delta_1^2+\Delta_2^2\right)}{2P_3}A_6$$

GPDs on the lattice

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Light-cone GPDs using lattice correlators in non-symmetric frames

Theoretical setup

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu M A_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Goals

(A) A_i are to the standard H, E GPDs $\quad \mathcal{H}_0^s(A_i^s; z) = A_1 + \frac{z(\Delta_1^2 + \Delta_2^2)}{2P_3}A_6$

(B) Extraction of standard GPDs using A_i obtained from any frame

(C) quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone:

Theoretical setup

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Goals

(A) A_i are to the standard H, E GPDs $\quad \mathcal{H}^s_0(A^s_i; z) = A_1 + \frac{z(\Delta_1^2 + \Delta_2^2)}{2P_3}A_6$

(B) Extraction of standard GPDs using A_i obtained from any frame

(C) quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone:

$$H(z\cdot P, z\cdot\Delta, t = \Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3$$

$$E(z\cdot P, z\cdot\Delta, t = \Delta^2, z^2) = -A_1 - \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3 + 2A_5 + 2P_{avg,s/a}\cdot zA_6 + 2\Delta_{s/a}\cdot zA_8$$

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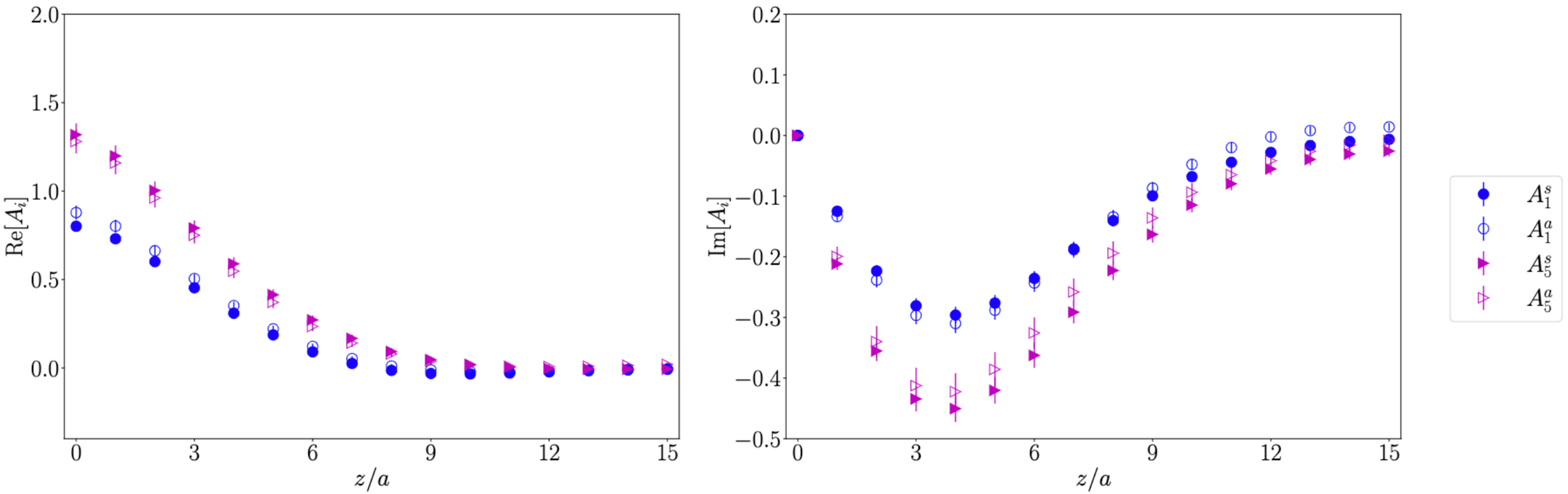
(A) **Proof-of-concept calculation ($\xi = 0$):**

- **symmetric frame:** $\quad \vec{p}^{\,s}_f = \vec{P} + \frac{\vec{Q}}{2}, \quad \vec{p}^{\,s}_i = \vec{P} - \frac{\vec{Q}}{2} \quad -t^s = \vec{Q}^2 = 0.69\, GeV^2$

- **asymmetric frame:** $\quad \vec{p}^{\,a}_f = \vec{P}, \quad \vec{p}^{\,a}_i = \vec{P} - \vec{Q} \quad t^a = -\vec{Q}^2 + (E_f - E_i)^2 = 0.65\, GeV^2$

Comparison of A_i in two frames



★ A_1, A_5 dominant contributions

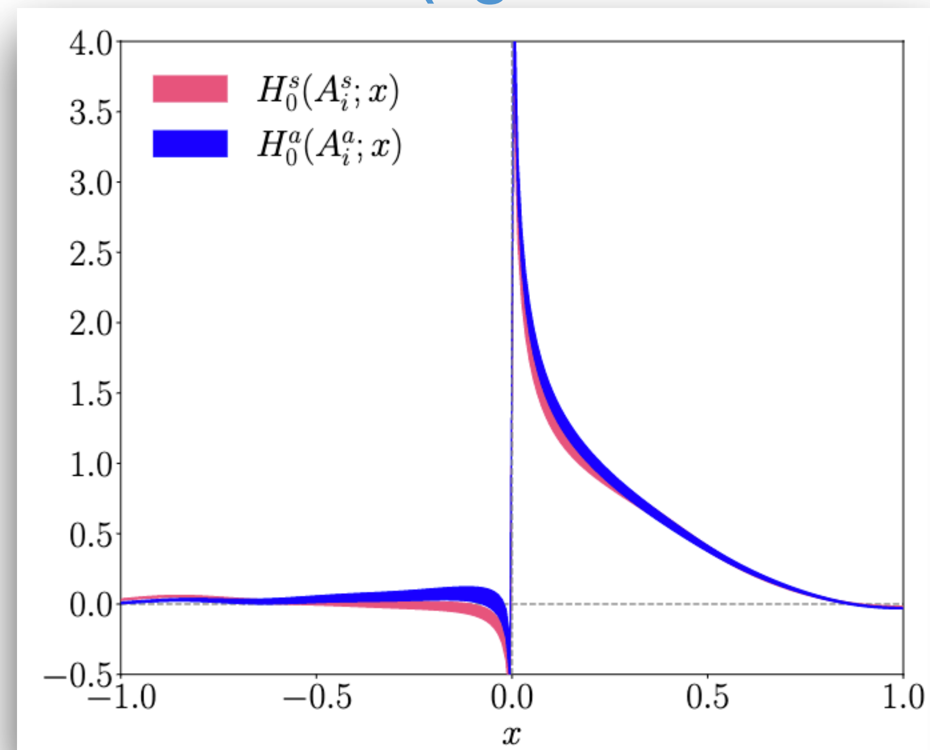
★ Full agreement in two frames for both Re and Im parts of A_1, A_5

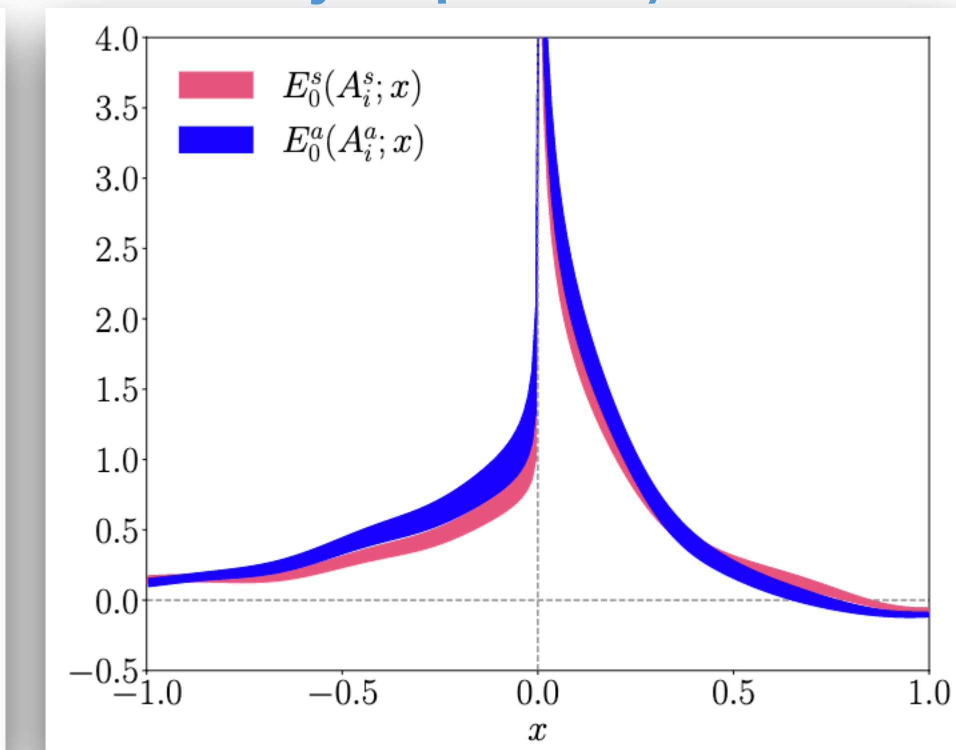
★ A_3, A_4, A_8 zero at $\xi = 0$

★ A_2, A_6, A_7 suppressed (at least for this kinematic setup and $\xi = 0$)

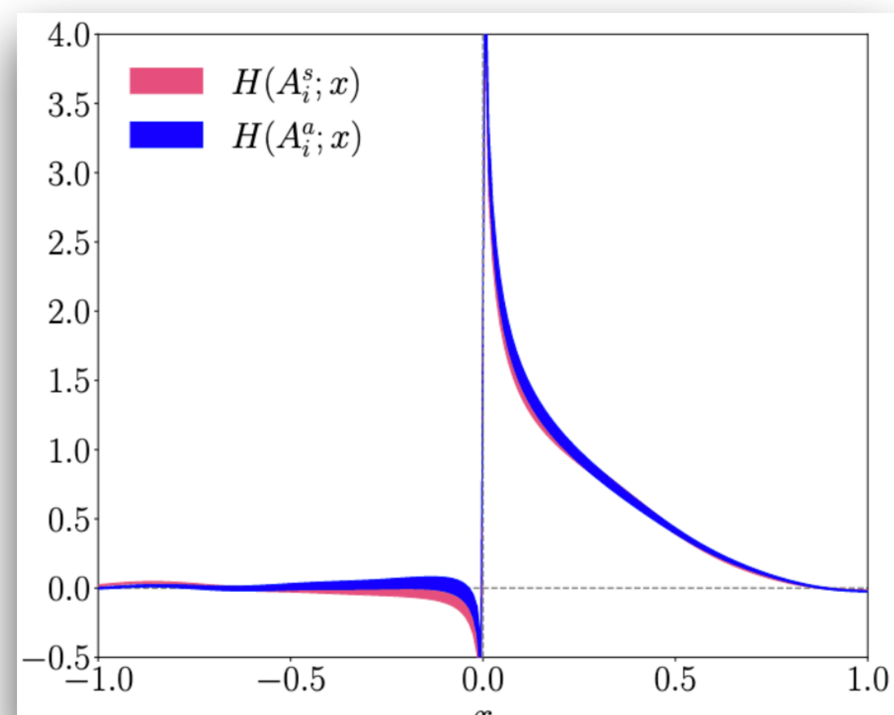
GPDs in terms of A_i

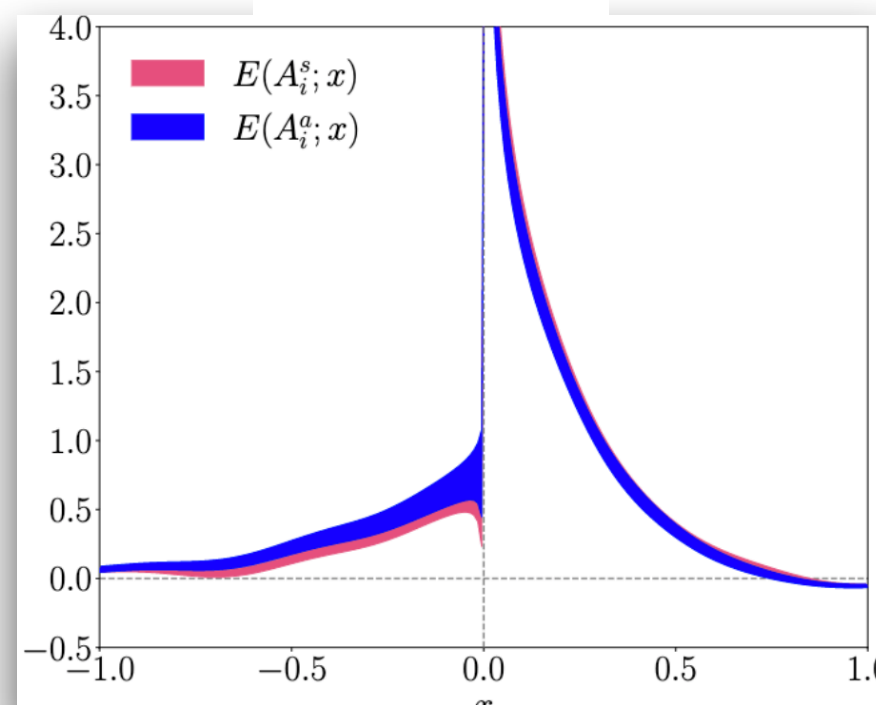
Non LI definitions (agreement not theoretically expected)





LI definition (agreement anticipated theoretically)





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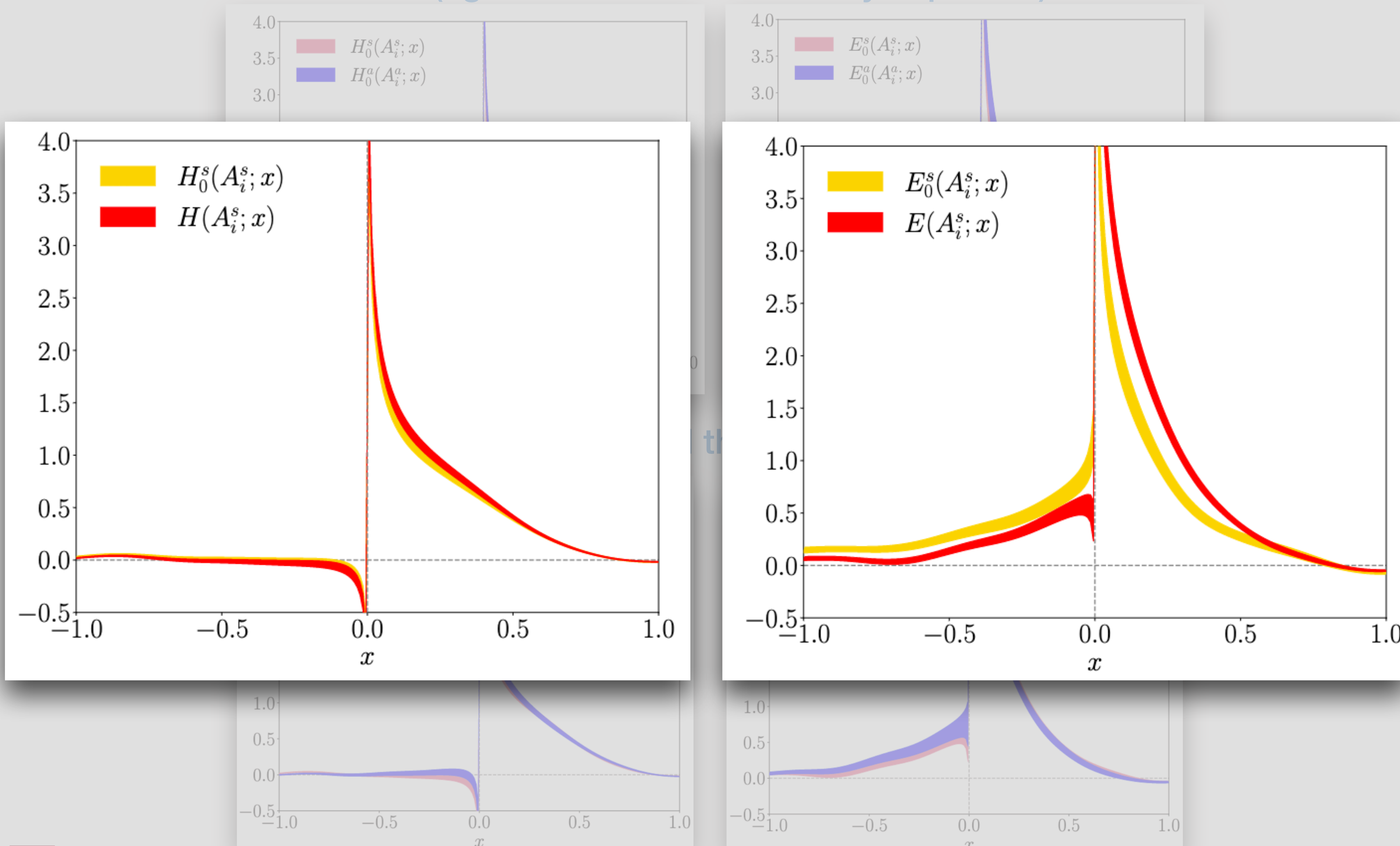
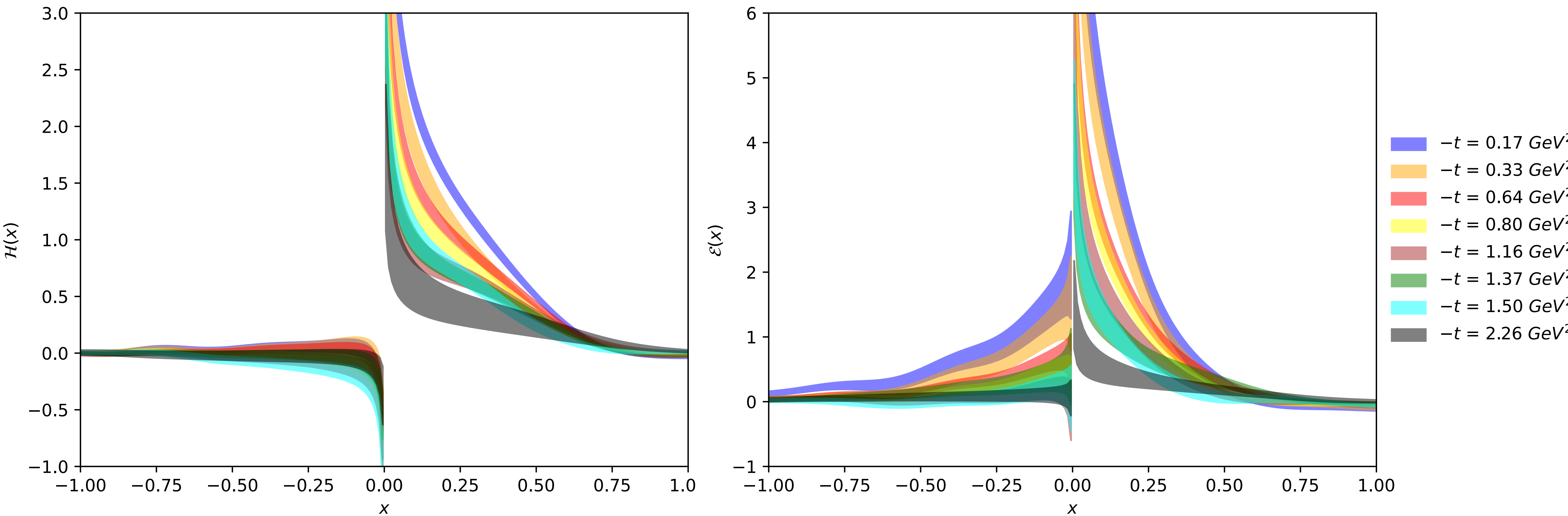


H, E light-cone GPDs

- ★ quasi-GPDs transformed to momentum space
- ★ Matching formalism to 1 loop accuracy level
- ★ +/-x correspond to quark and anti-quark region
- ★ Anti-quark region susceptible to systematic uncertainties.



- ★ Similar analysis for helicity GPDs

H, E light-cone GPDs

Several values of -t accessible at once

- ★ quasi-GPDs transformed to momentum space
- ★ Matching formalism to 1 loop accuracy level
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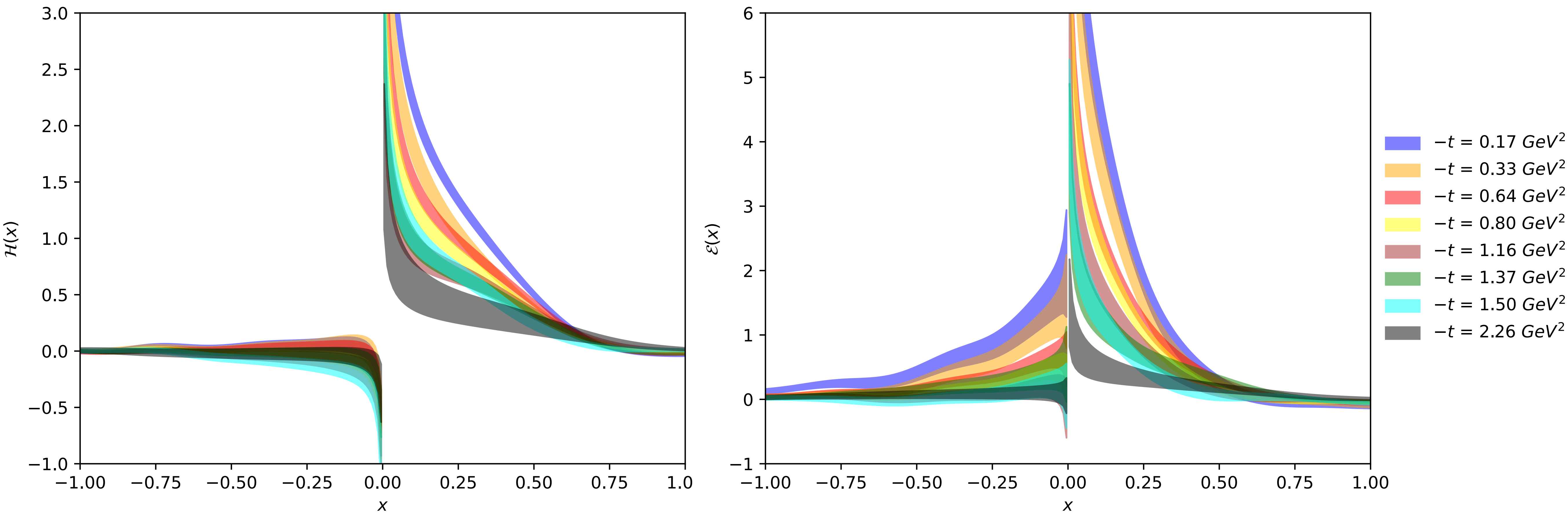
- ★ Anti-quark region susceptible to systematic uncertainties.



- ★ Similar analysis for helicity GPDs

Exploration of twist-3 GPDs

arXiv:2306.05533v1 [hep-lat] 8 Jun 2023

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya[1,2], Krzysztof Cichy[3], Martha Constantinou[1], Jack Dodson[1], Andreas Metz[1], Aurora Scapellato[1], Fernanda Steffens[4]

First lattice calculation of twist-3 GPDs

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Bigg[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+ \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+ \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Bigg]u(p_i,\lambda)$$

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

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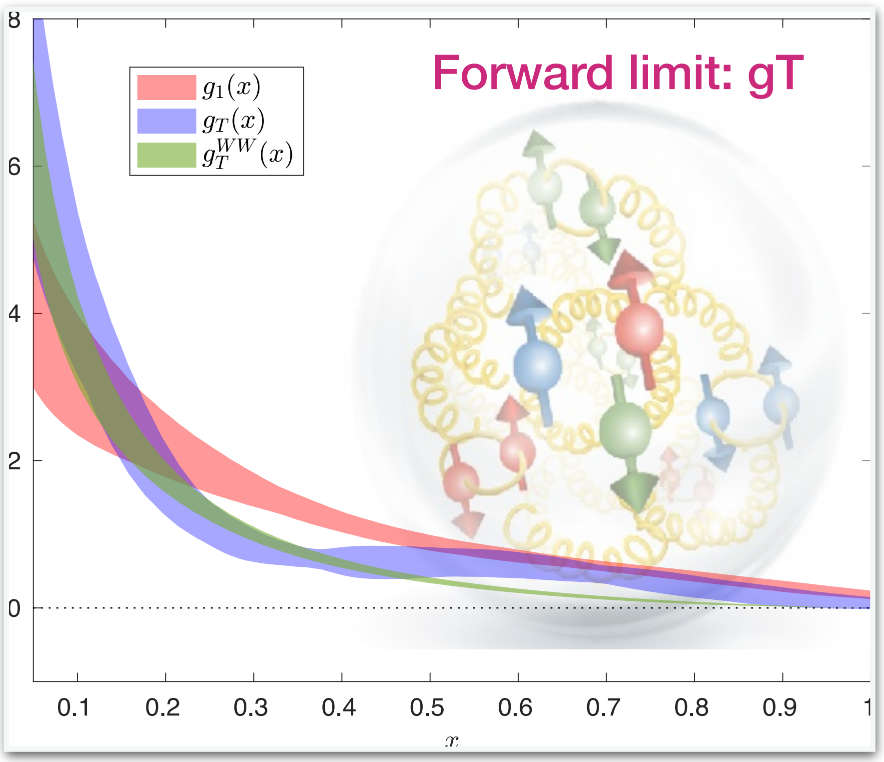

[S. Bhattacharya et al., PRD 102 (2020) 11]
(Editors Highlight)

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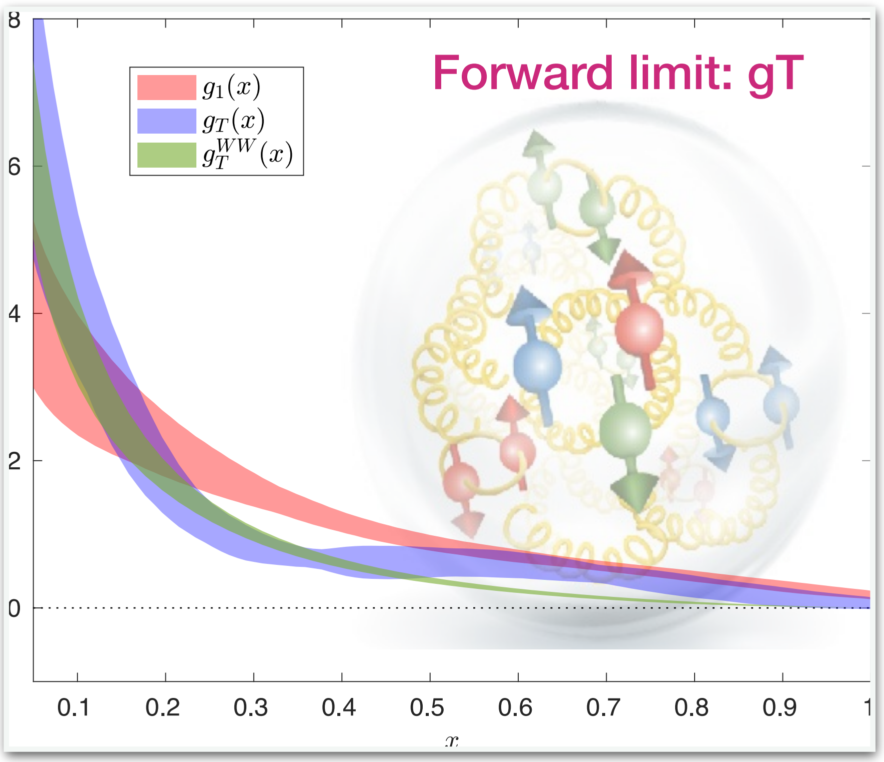

[S. Bhattacharya et al., PRD 102 (2020) 11]
(Editors Highlight)

★ Consistency checks: sum rules

$$\int_{-1}^{1} dx\,\widetilde{H}(x,\xi,t) = G_A(t)\,, \quad \int_{-1}^{1} dx\,\widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\,\widetilde{G}_i(x,\xi,t) = 0\,, \quad i=1,2,3,4$$

$$\int dx\,x\,\widetilde{G}_3 = \frac{\xi}{4}G_E(t)$$

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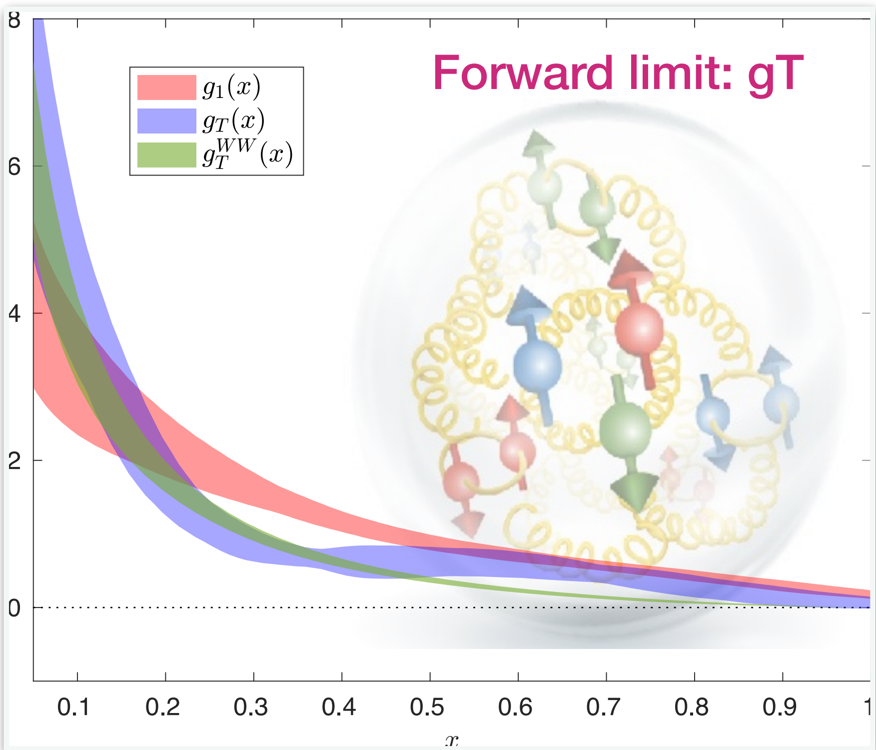

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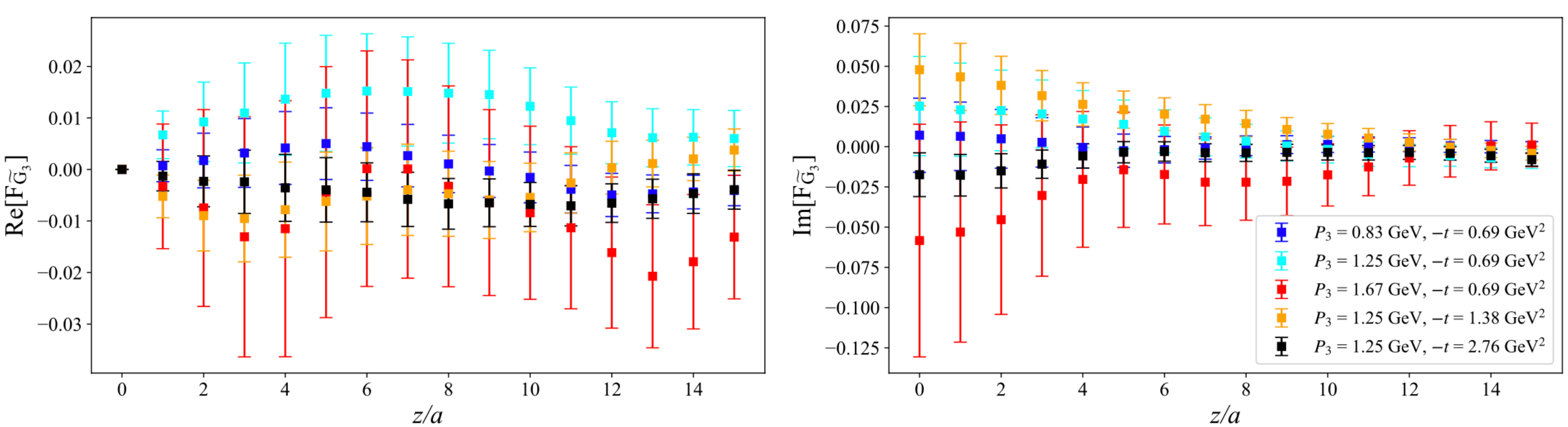
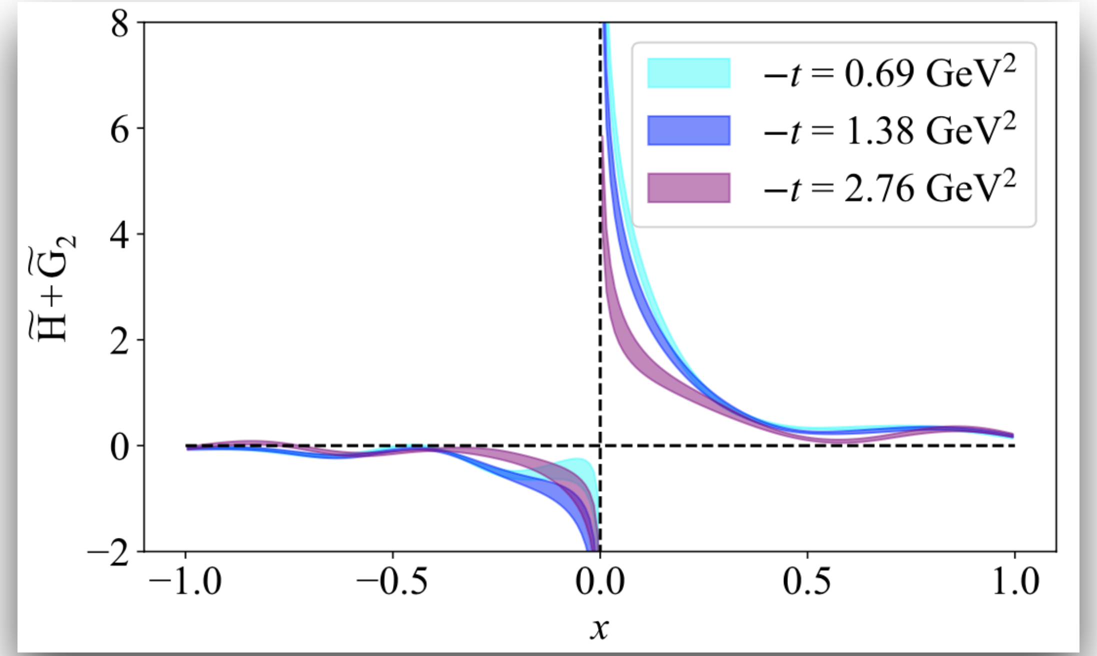


Indeed, numerically found to be zero within uncertainties at ξ=0

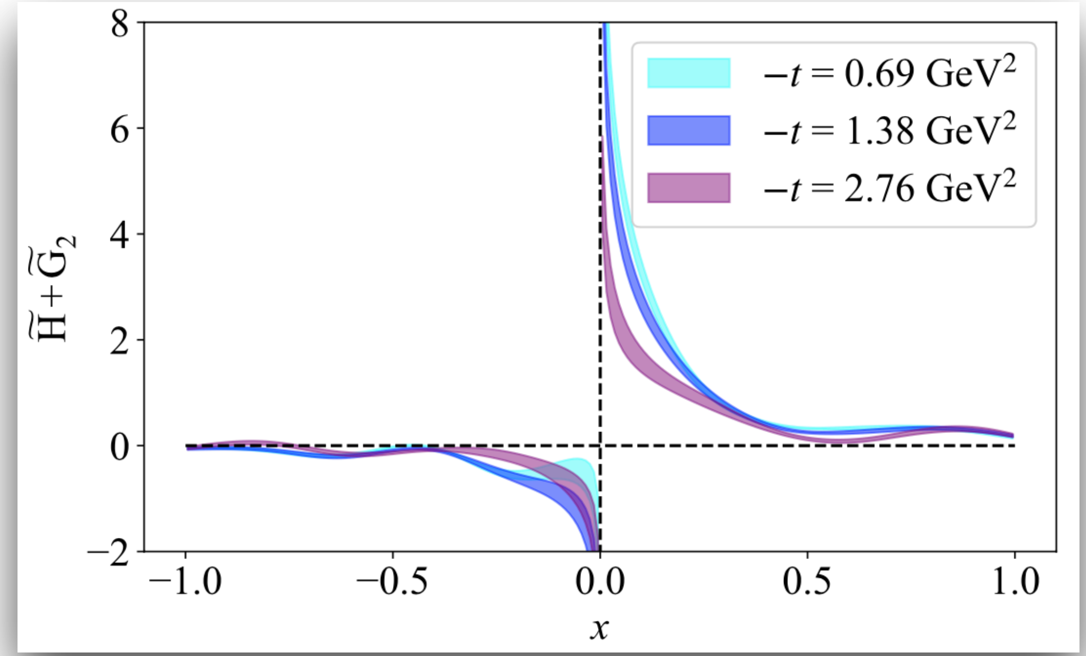
First lattice calculation of twist-3 GPDs



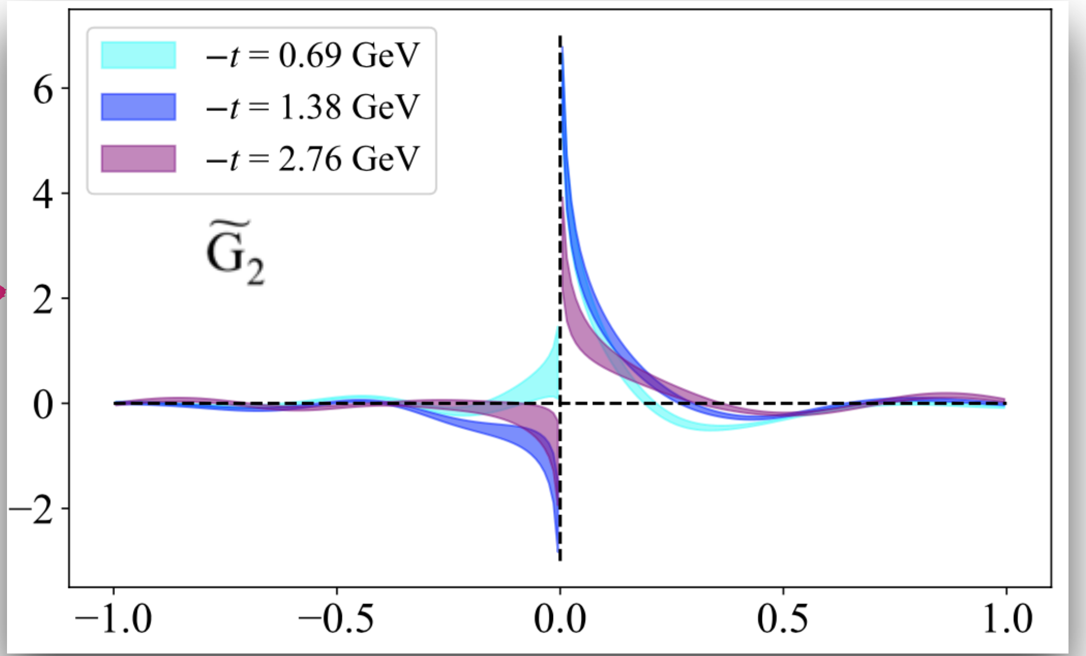
Isolating $\widetilde{G}_2$

using $\widetilde{H}$

First lattice calculation of twist-3 GPDs



Isolating $\widetilde{G}_2$ using $\widetilde{H}$



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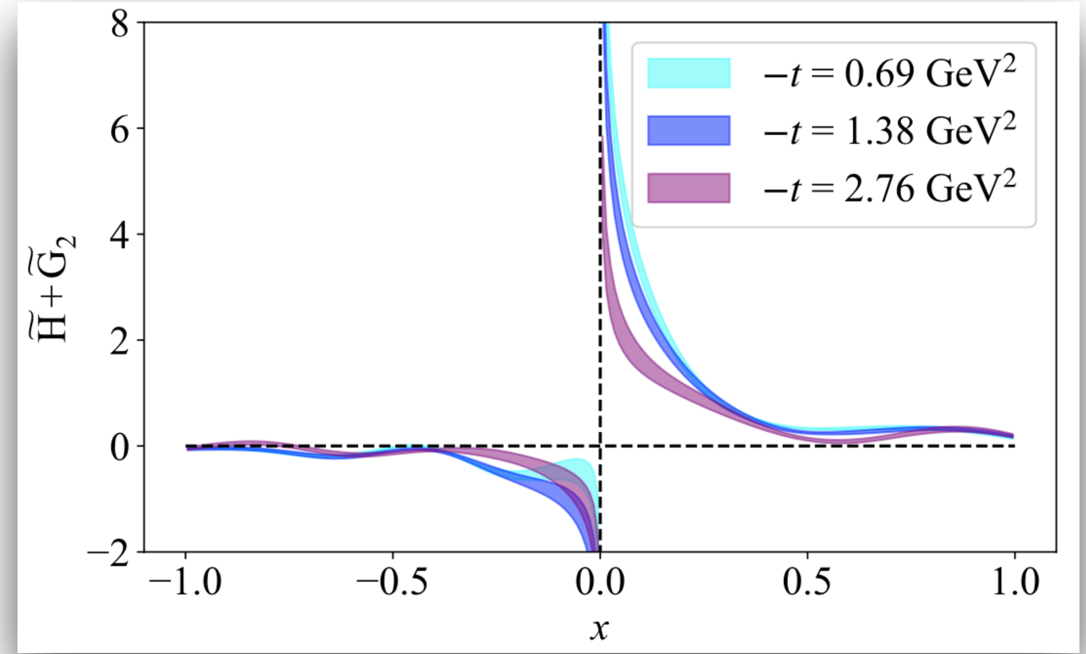

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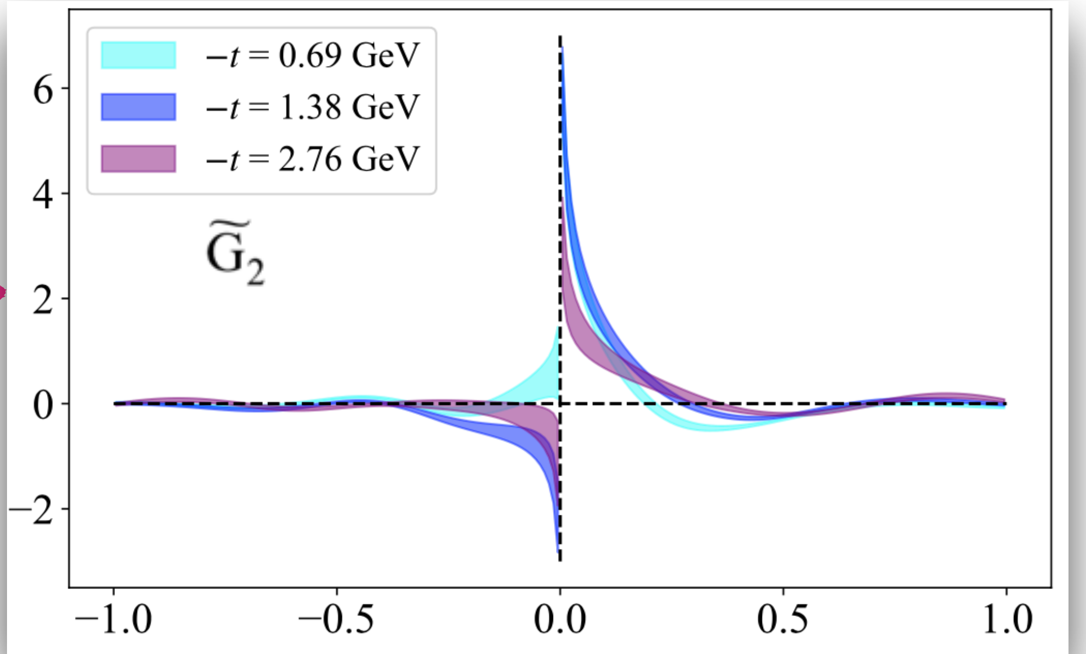

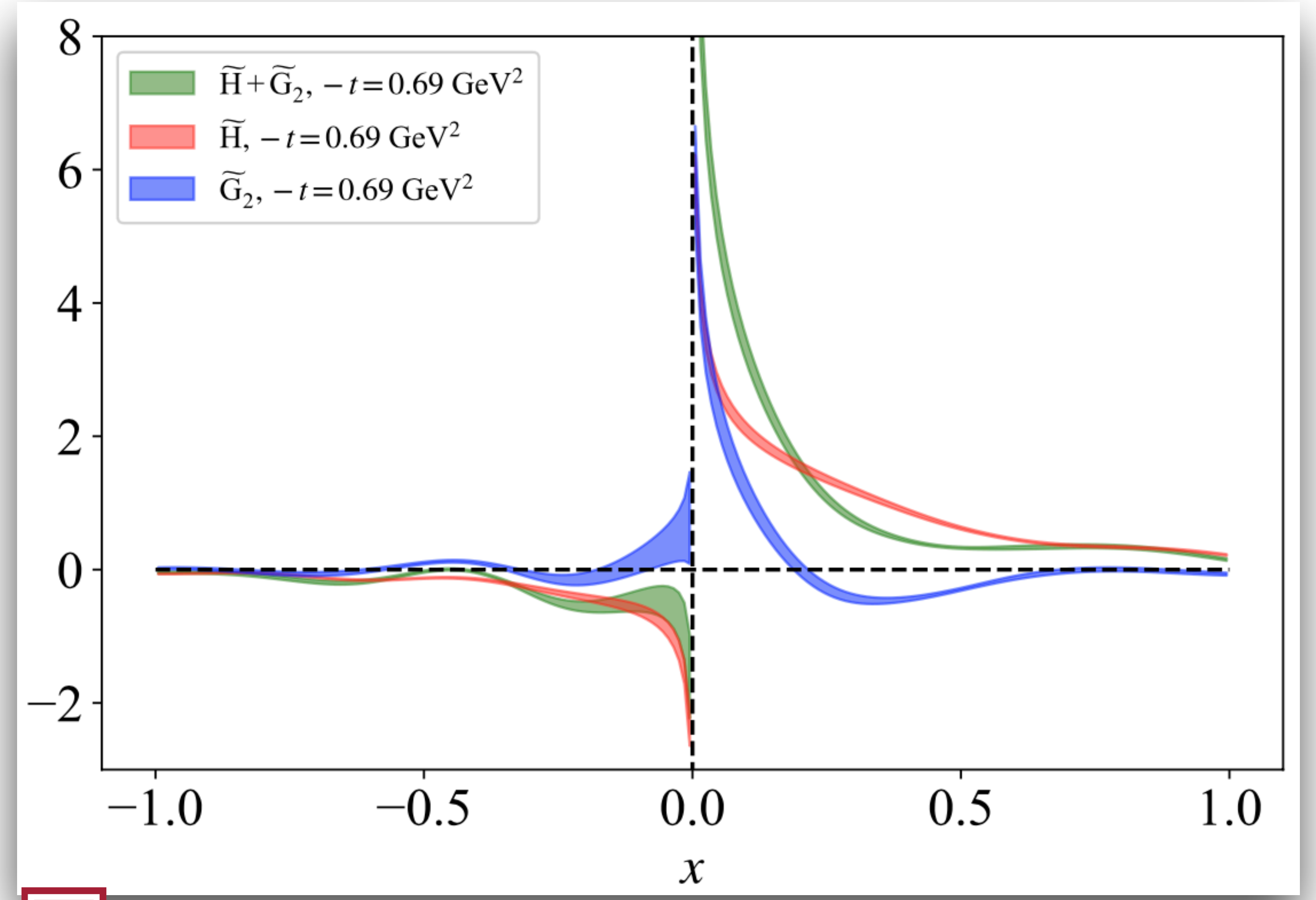

Negative areas in $\widetilde{G}_2$ theoretically anticipated:

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

First lattice calculation of twist-3 GPDs

- ★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness
- ★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

First lattice calculation of twist-3 GPDs

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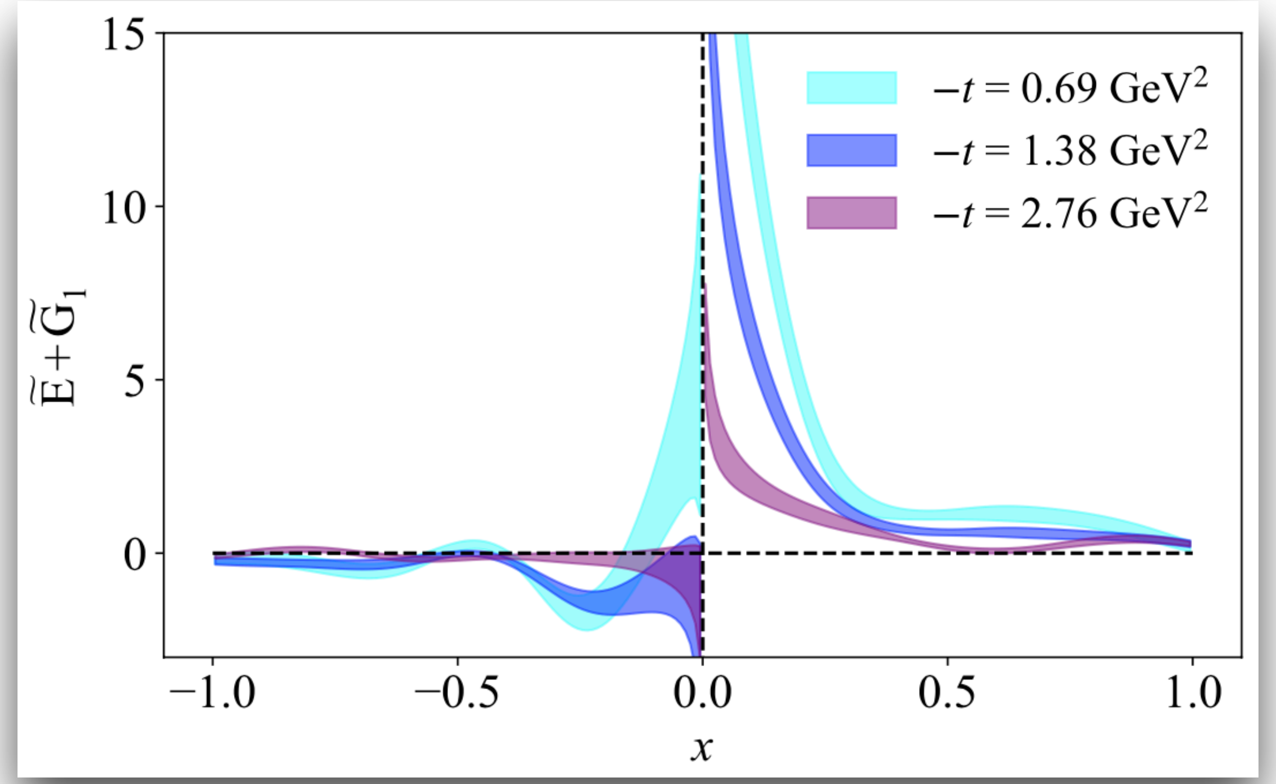

★ Sizable contributions as expected

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

First lattice calculation of twist-3 GPDs

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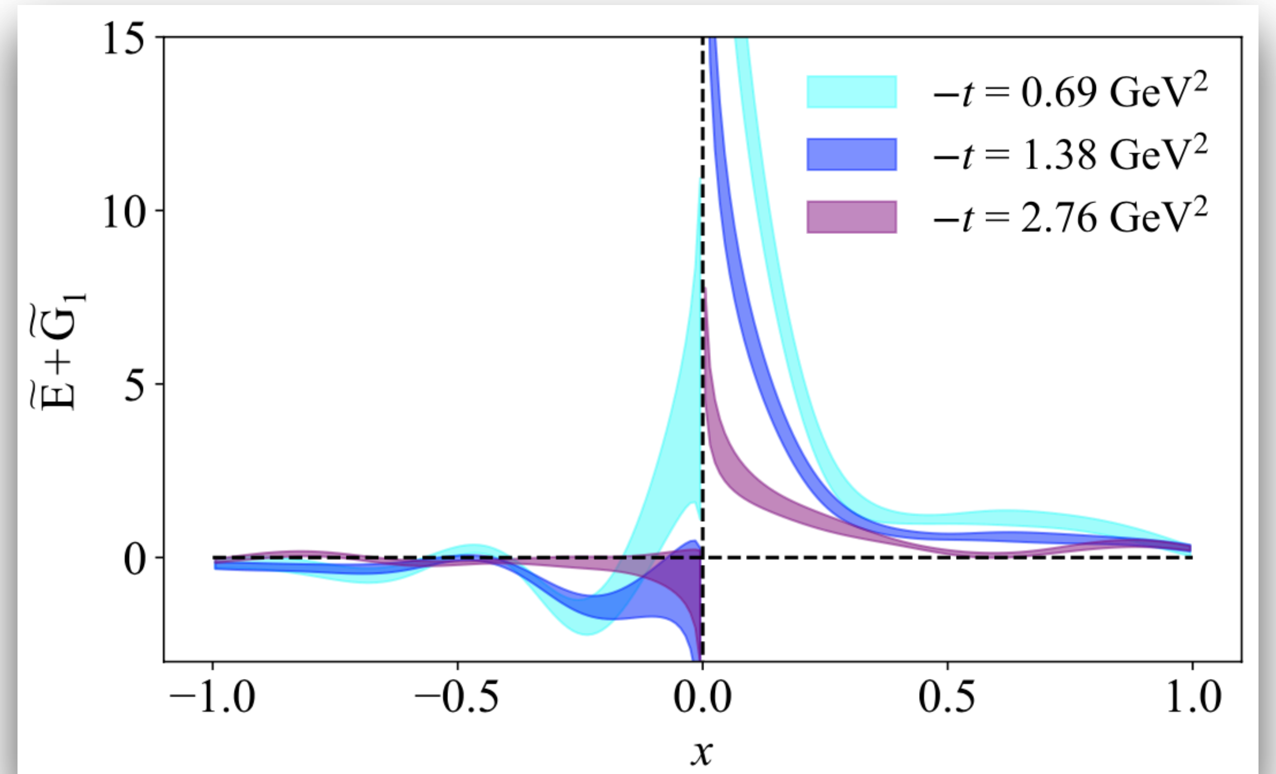

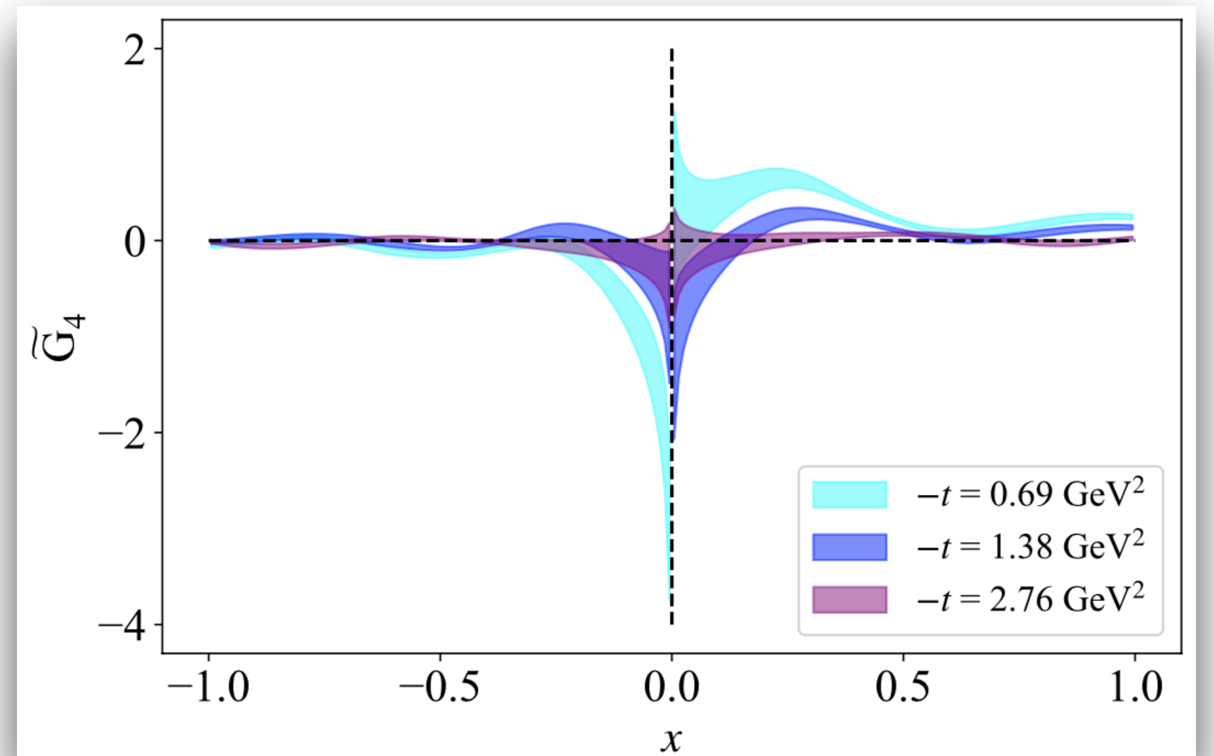

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★ $\widetilde{G}_4$ very small; no theoretical argument to be zero

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E$$

First lattice calculation of twist-3 GPDs

Consistency checks

★ Norms satisfied

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

★ Alternative decomposition (LI) numerically confirmed

[Fernanda Steffens]

$$F_{\widetilde{H}+\widetilde{G}_2} = \frac{1}{2m^2}\frac{z_3 P_0^2(\Delta_\perp)^2}{P_3} + A_2$$

$$F_{\widetilde{G}_3} = \frac{1}{2m^2}\Big(z_3 P_0^2 \Delta_3 - z_3 P_3 P_0 \Delta_0\Big) A_1 - z_3 P_3 A_8$$

$$F_{\widetilde{E}+\widetilde{G}_1} = \frac{2 z_3 P_0^2}{P_3} + 2A_5$$

$$F_{\widetilde{G}_3} = \frac{1}{m^2}\Big(z_3 P_0 P_3^2 - z_3 P_0^3\Big) A_1$$

First lattice calculation of twist-3 GPDs

Consistency checks

★ Norms satisfied

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

★ Alternative decomposition (LI) numerically confirmed

[Fernanda Steffens]

$$F_{\widetilde{H}+\widetilde{G}_2} = \frac{1}{2m^2}\frac{z_3 P_0^2 (\Delta_\perp)^2}{P_3} + A_2$$

$$F_{\widetilde{G}_3} = \frac{1}{2m^2}\Big(z_3 P_0^2 \Delta_3 - z_3 P_3 P_0 \Delta_0\Big) A_1 - z_3 P_3 A_8$$

$$F_{\widetilde{E}+\widetilde{G}_1} = \frac{2 z_3 P_0^2}{P_3} + 2A_5$$

$$F_{\widetilde{G}_3} = \frac{1}{m^2}\Big(z_3 P_0 P_3^2 - z_3 P_0^3\Big) A_1$$

Consistency checks show encouraging results

How to lattice QCD data fit into the overall effort for hadron tomography

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★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

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PHENOMENOLOGY
LATTICE QCD
THEORY

QUARK-GLUON TOMOGRAPHY
COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number:
DE-SC0023646

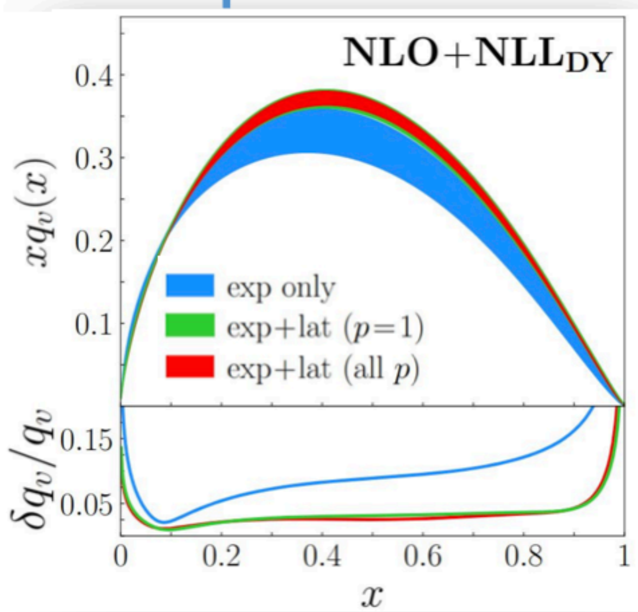
1. **Theoretical studies** of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties

2. **Lattice QCD** calculations of GPDs and related structures

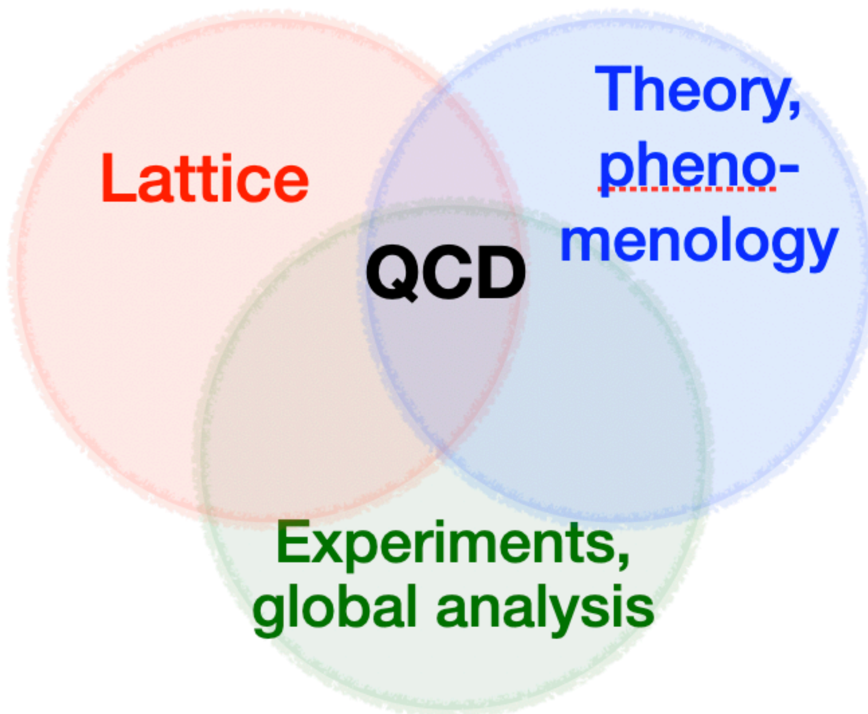
3. **Global analysis** of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

Synergies: constraints & predictive power of lattice QCD

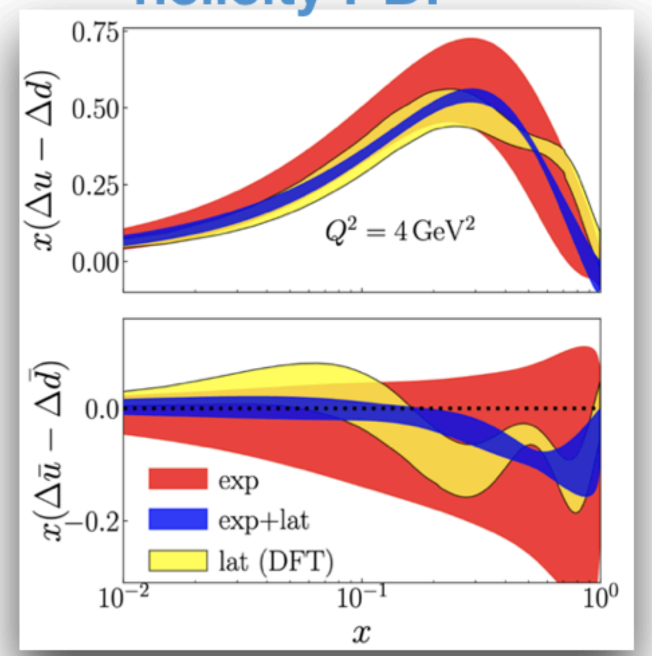
pion PDF



[JAM/HadStruc, PRD105 (2022) 114051]

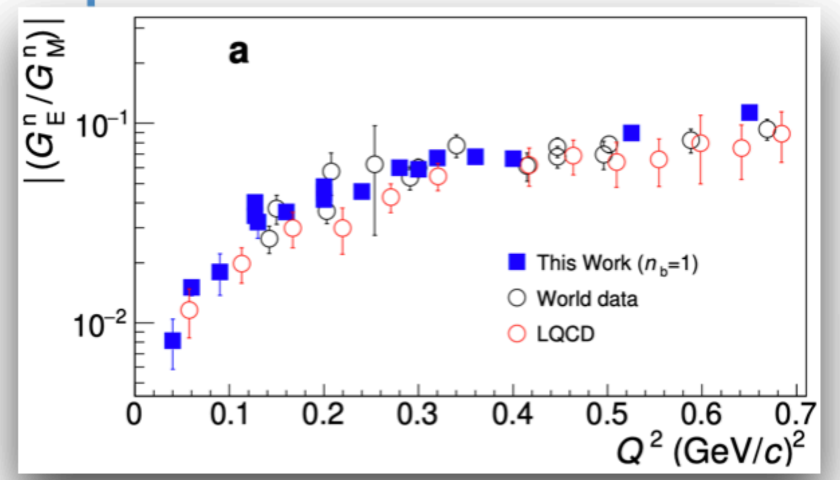


helicity PDF



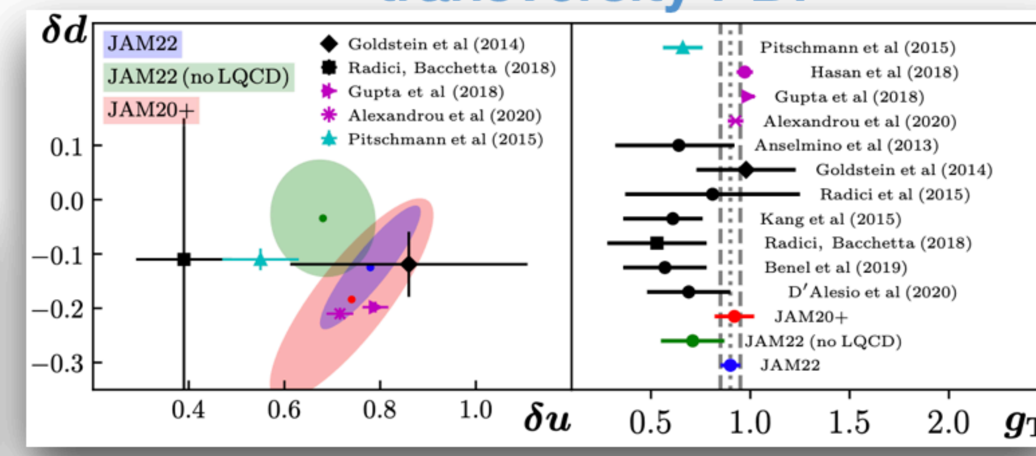
[JAM & ETMC, PRD 103 (2021) 016003]

proton & neutron radius



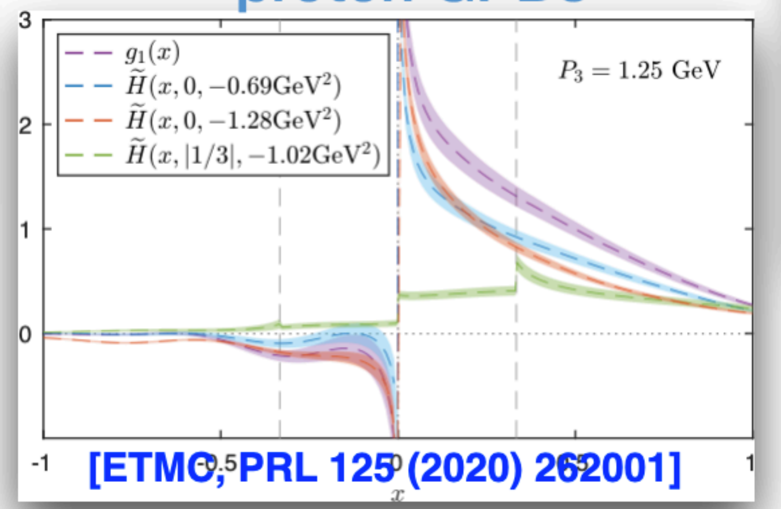
[Atac et al., Nature Comm. 12, 1759 (2021)]

transversity PDF



[JAM, PRD 106 (2022) 3, 034014]

proton GPDs



[ETMC, PRL 125 (2020) 262001]

And many more!

Summary

★ Lattice QCD data on GPDs will play an important role in the pre-EIC era and can complement experimental efforts of JLab@12GeV

★ New proposal for Lorentz invariant decomposition has great advantages:
- significant reduction of computational cost
- access to a broad range of t and ξ

★ Future calculations have the potential to transform the field of GPDs

★ Mellin moments can be extracted utilizing quasi-GPDs data

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Thank you

U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING

U.S. DEPARTMENT OF ENERGY | Office of Science

DOE Early Career Award (NP)
Grant No. DE-SC0020405

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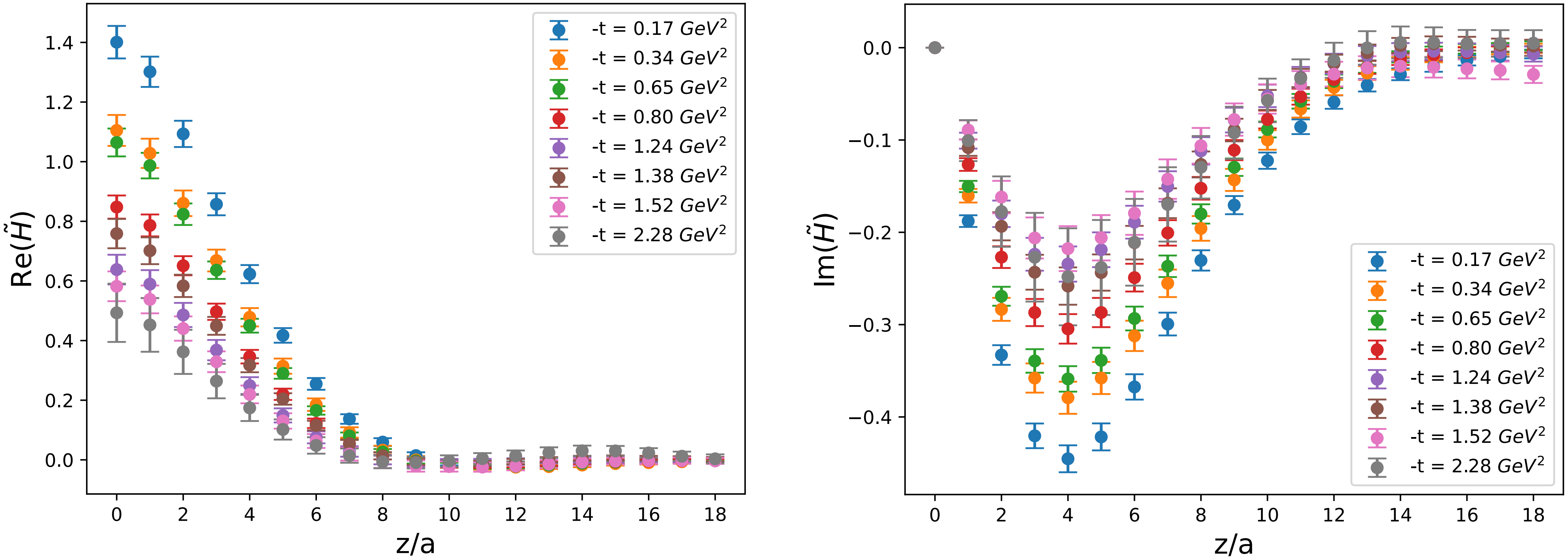

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[S. Bhattacharya et al., PRD 102, 054021 (2020)]

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★ Lattice data on transversity GPDs

$$\int_{-2}^{2} dx H_{Tq}(x,0,-0.69\,\mathrm{GeV}^2,P_3)=\{0.65(4),\,0.64(6),\,0.81(10)\}\,, \qquad \int_{-2}^{2} dx H_{Tq}(x,\tfrac{1}{3},-1.02\,\mathrm{GeV}^2,1.25\,\mathrm{GeV})=0.49(5)\,,$$

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Sum rules not imposed in calculation

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★ GPDs: off-forward matrix elements of non-local light-cone operators

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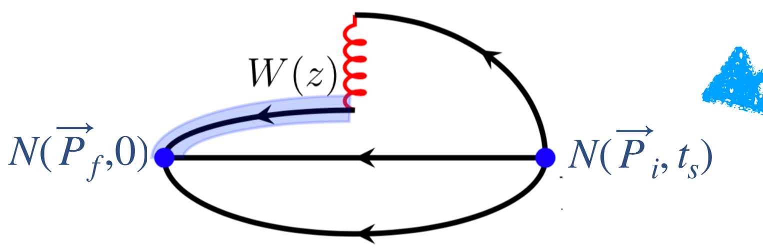

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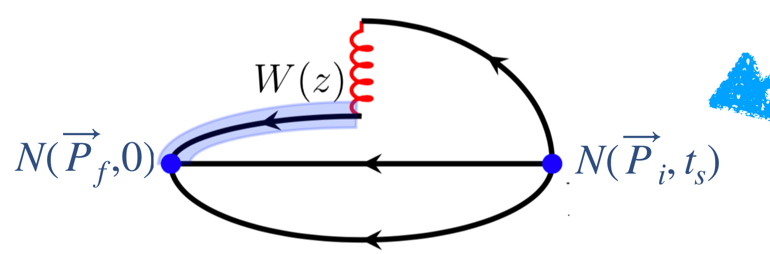

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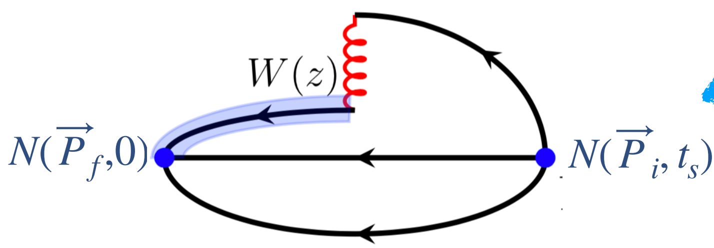

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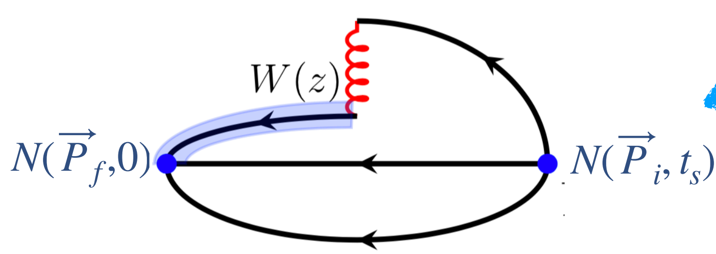

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→ reduction of power corrections in fwd limit [Radyushkin, PLB 767, 314, 2017]

$$F^{[\gamma^3]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^3 H_{Q(0)}(x,\xi,t;P^3) + \frac{i\sigma^{3\mu}\Delta_\mu}{2M}E_{Q(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

→ finite mixing with scalar [Constantinou & Panagopoulos (2017)]

★ Symmetric frame ($\vec{p}^{\,s}_f = \vec{P} + \vec{Q}/2\,, \vec{p}^{\,s}_i = \vec{P} - \vec{Q}/2$): separate calculations at each t