

Modeling and extraction of GPDs from exclusive processes

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- **PARTONS** (PARton Tomography Of Nucleon Software – <https://partons.cea.fr/>) software framework for the **phenomenology of 3D hadron structure** (GPDs and TMDs) [Berthou et al, 2018]
- Developed since **2012**, **open source** and readily available in a virtual machine environment. Users can run **XML scenarios** to compute directly observables from already implemented GPD models, or develop their own modules in C++. **Plenty of examples and documentation! GPD models** (GK, VGG, Vinnikov, ...), **evolution, observables** (DVCS, TCS, DVMP, ...), **neural network fits of CFFs, event generator EpIC, ...**
- See also **Gepard** (<https://gepard.phy.hr/>)



- GPDs at small ξ
 - t dependence
 - ξ dependence
- Deconvolution problem for $x < \xi$
- Deconvolution problem at moderate x and ξ

Generalized parton distributions

GPDs generalize usual **parton distribution functions** (PDFs) as lightcone projections of non-local matrix elements with **possible four-momentum transfer between initial and final hadron state**. For a spin-1/2 hadron, quark GPDs H^q and E^q are defined in the lightcone gauge as [Müller et al, 1994], [Radyushkin, 1996], [Ji, 1997]

$$\begin{aligned} & \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle p_2 \left| \bar{\psi}^q \left(-\frac{z}{2} \right) \gamma^+ \psi^q \left(\frac{z}{2} \right) \right| p_1 \right\rangle \Big|_{z_\perp=0, z^+=0} \\ &= \frac{1}{2P^+} \left(H^q(x, \xi, t) \bar{u}(p_2) \gamma^+ u(p_1) + E^q(x, \xi, t) \bar{u}(p_2) \frac{i\sigma^{+\mu} \Delta_\mu}{2M} u(p_1) \right) \end{aligned} \quad (1)$$

where

$$p_2 - p_1 = \Delta, \quad t = \Delta^2, \quad P = \frac{1}{2}(p_1 + p_2), \quad \xi = -\frac{\Delta^+}{2P^+}. \quad (2)$$

GPDs at small $\xi - t$ dependence

- When $x \gg \xi$, negligible asymmetry between incoming ($x - \xi$) and outgoing ($x + \xi$) parton longitudinal momentum fraction \rightarrow **smooth limit of GPDs**

$$H(x, \xi, t, \mu^2) \approx H(x, 0, t, \mu^2) \quad \text{for } x \gg \xi. \quad (3)$$

- Extraction of the t -dependent PDF $H(x, 0, t, \mu^2)$?
 - Forward limit gives ordinary PDFs

$$H(x, 0, t = 0, \mu^2) = f(x, \mu^2). \quad (4)$$

- First Mellin moment gives elastic form factors

$$\int dx H(x, 0, t) = F_1(t). \quad (5)$$

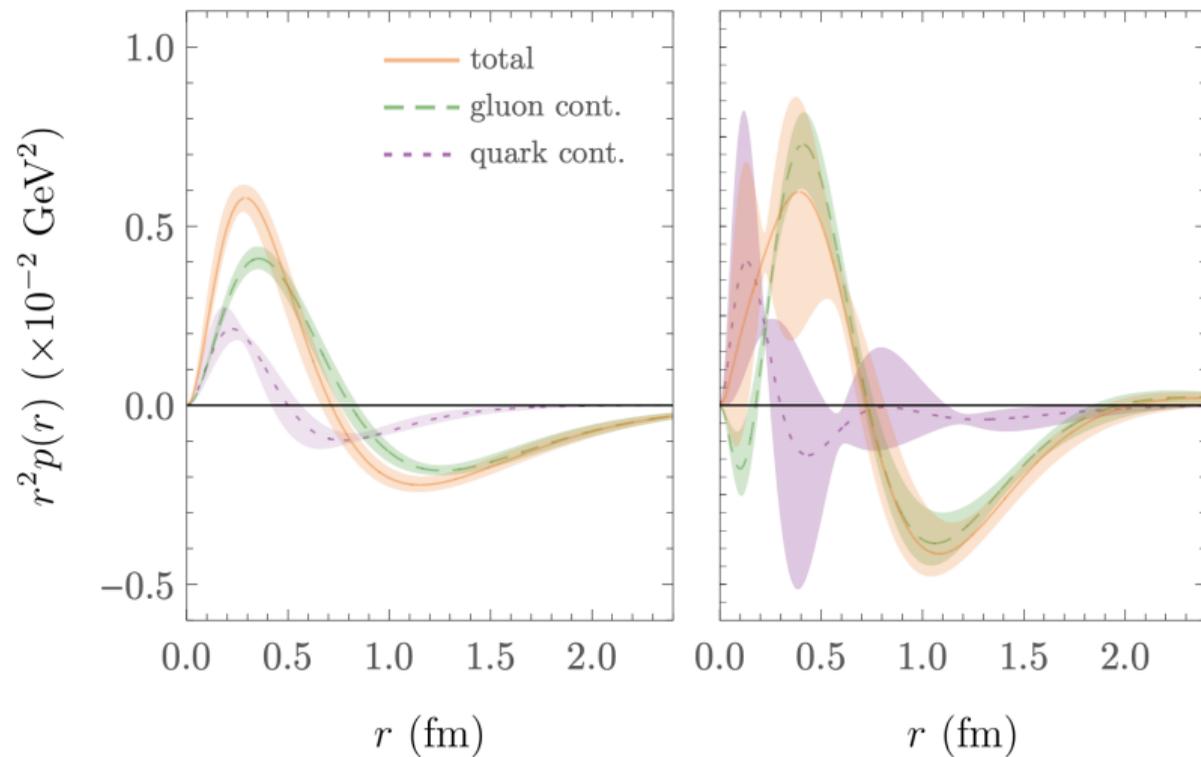
Impact parameter distribution (IPD) [Burkardt, 2000]

$$I_a(x, \mathbf{b}_\perp, \mu^2) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} F^a(x, 0, t = -\Delta_\perp^2, \mu^2) \quad (6)$$

is the density of partons with plus-momentum x and transverse position \mathbf{b}_\perp from the center of plus momentum in a hadron \rightarrow **hadron tomography**

- Better modelling of the t -dependent PDF requires more data, more difficult to obtain with larger systematic uncertainty
 - x -dependence at $\xi = 0$ computed on the lattice from the **non-local euclidean matrix elements** (LaMET [Ji, 2013], short-distance factorization [Radyushkin, 2017], ...)
 - Higher order Mellin moments of GPDs (**generalized form factors**) computed on the lattice with local operators (limited by operator mixing to the first 3)
 - Experimental data from exclusive processes: **most of these data have a particular sensitivity to the region $x \approx \xi$, so precisely not $x \gg \xi$!**
- How can one leverage the experimental data to constrain t -dependent PDFs?

GPDs at small $\xi - t$ dependence



From [Shanahan, Detmold, 2019]

Difference on the uncertainty of the extraction of GFFs from lattice data depending on the t -parametrization: tripole on the left, z-expansion on the right.

GPDs at small ξ – ξ dependence

- Why don't we just assume

$$H(x, \xi, t, \mu^2) \approx H(x, 0, t, \mu^2) \quad \text{for } \xi \ll 1 \text{ even if } x \approx \xi? \quad (7)$$

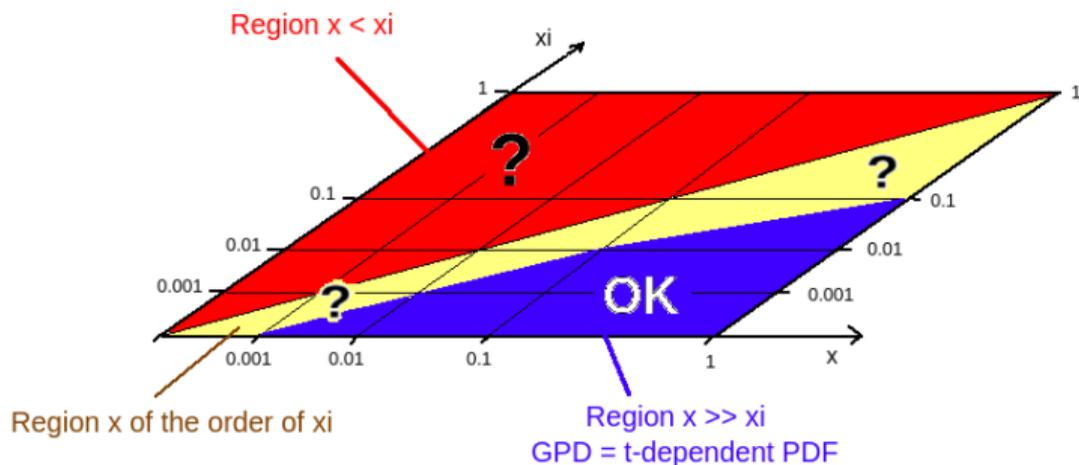
Because significant asymmetry between incoming and outgoing ($x + \xi \gg x - \xi$) parton momentum means very different dynamics, materialized e.g. by a very different behavior under evolution.

No reason for the ξ dependence to be negligible even at very small ξ .

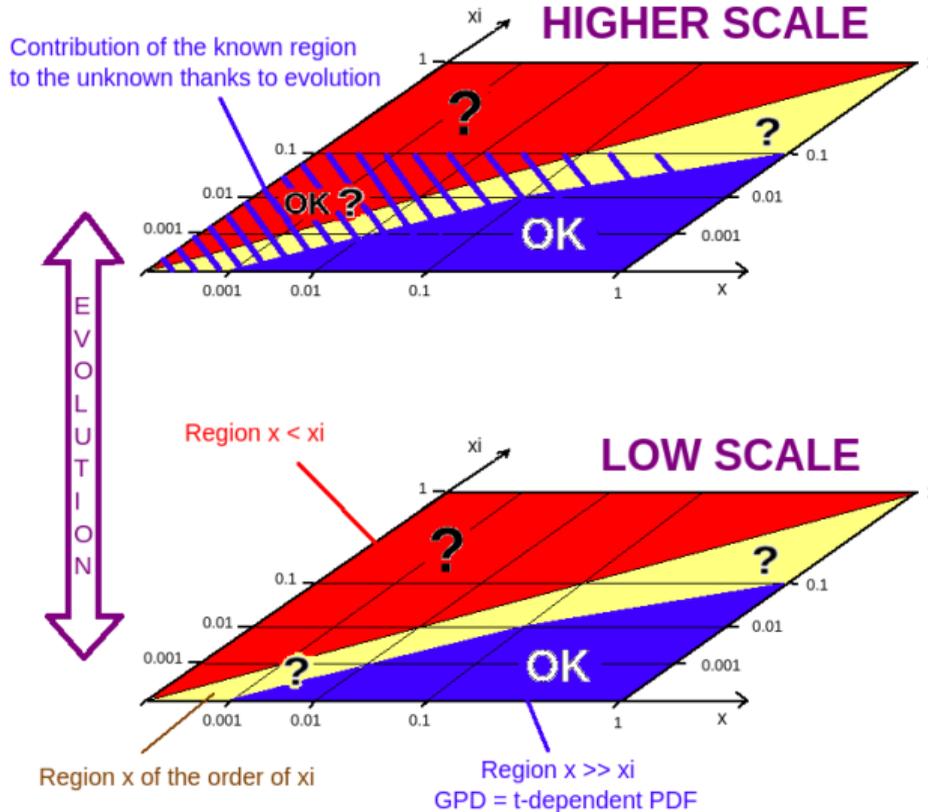
Skewness ratios $\frac{H(x,x)}{H(x,0)}$ as large as 1.6 have

been advocated at small x . [Frankfurt et al, 1998]

[Shuvaev et al, 1999]



GPDs at small ξ – ξ dependence



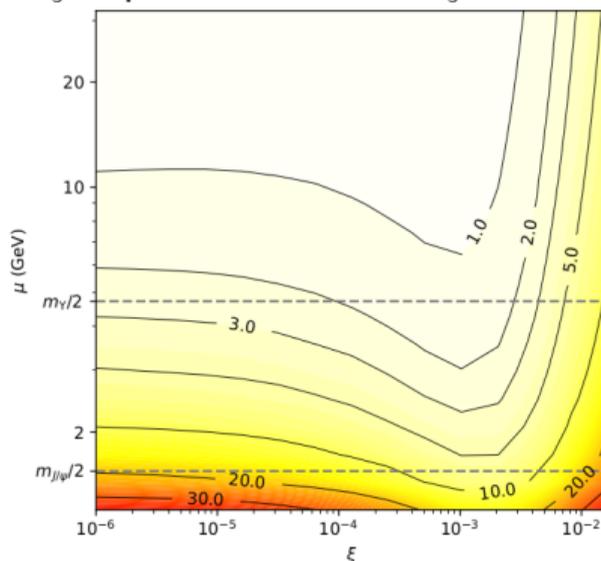
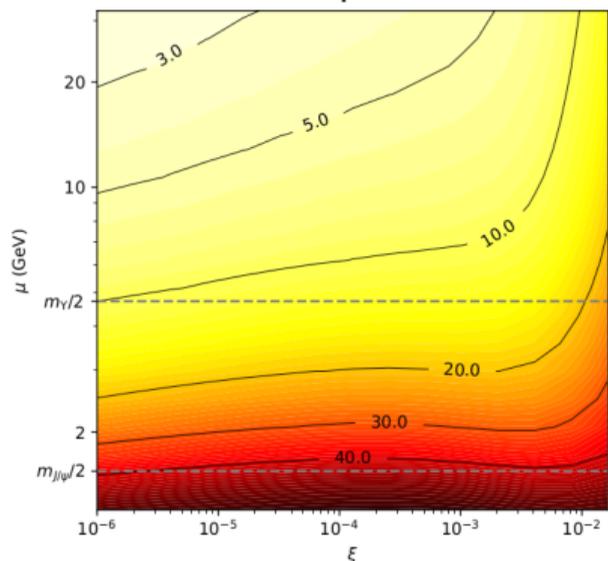
- Evolution displaces the GPD from the large x to the small x region
- Significant ξ dependence arises perturbatively in the small x and ξ region
- But how does it compare to the unknown ξ dependence at initial scale?

Obviously depends on the range of evolution, value of x and ξ , and profile of the known t -dependent PDF.

GPDs at small ξ – ξ dependence

Example: working at $t = 0$, with the MMHT2014 PDF [Harland-Lang et al, 2015] at 1 GeV (**prior knowledge of t -dependent PDF**). We want to assess the dominance of the region $x \gg \xi$ at initial scale in the value of the GPD on the diagonal as scale increases.

Pessimistic assumption on unknown ξ dependence at $x = \xi$ for 1 GeV: 60%.



Uncertainty on the diagonal of the light sea quarks (left) and gluons (right) depending on $x = \xi$ and μ .
Stronger μ effect for gluons, divergence of PDFs at small x visible.

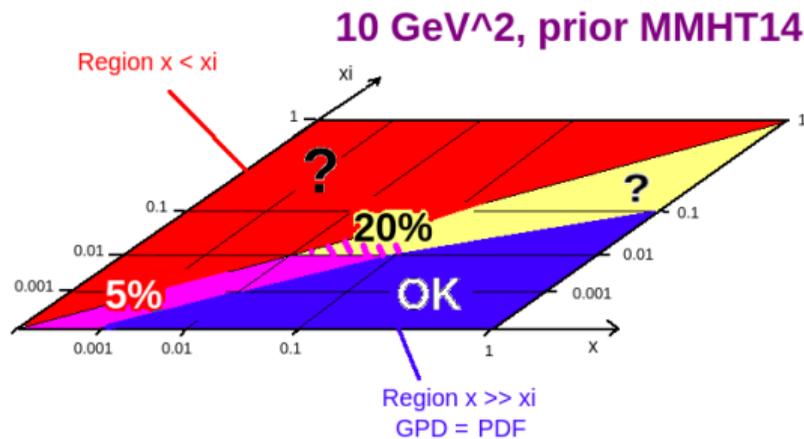
[HD, Winn, Bertone, hep-ph:2302.07861 (2023)]

Generating perturbatively the ξ dependence offers a well defined functional space for GPDs at small ξ which verifies the main theoretical constraints (polynomiality of Mellin moments, positivity, limits, ...)

By subtracting the degree of freedom of the ξ dependence, we have regularized the deconvolution problem, and we have an evaluation of the uncertainty associated to this regularization.

Better modelling: include missing higher order corrections by varying the scales \rightarrow use higher order evolution

Deconvolution problem for $x < \xi$



- Summary of the situation for H^G at $t = 0$ with MMHT2014 PDFs as prior
- What is happening for $x < \xi$, and what is the **deconvolution problem**?
- GPDs satisfy a **polynomiality property** arising from Lorentz covariance: [Ji, 1998], [Radyushkin, 1999]

$$\int_{-1}^1 dx x^n H^q(x, \xi, t, \mu^2) = \sum_{k=0 \text{ even}}^n A_{n,k}^q(t, \mu^2) \xi^k + \text{mod}(n, 2) \xi^{n+1} C_n^q(t, \mu^2). \quad (8)$$

red contribution: if a function $D^q(\alpha, t, \mu)$ is odd in α , [Polyakov, Weiss, 1999]

$$\int_{-1}^1 dx x^n \Theta\left(1 - \frac{|x|}{|\xi|}\right) \text{sgn}(\xi) D^q\left(\frac{x}{\xi}, t, \mu^2\right) = \text{mod}(n, 2) \xi^{n+1} \int_{-1}^1 d\alpha \alpha^n D^q(\alpha, t, \mu^2). \quad (9)$$

DVCS dispersion relation [Anikin, Teryaev, 2007], [Diehl, Ivanov, 2007]

$$C_H(t, Q^2) = \text{Re } \mathcal{H}(\xi, t, Q^2) - \frac{1}{\pi} \int_0^1 d\xi' \text{Im } \mathcal{H}(\xi', t, Q^2) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \quad (10)$$

$$\stackrel{\text{LO}}{=} 2 \sum_q e_q^2 \int_{-1}^1 dz \frac{D^q(z, t, Q^2)}{1 - z} \quad (11)$$

Since z is integrated out, only hope comes from the knowledge of the LO scale dependence of the D-term (ERBL equation). How effective is evolution to constrain it?

Shadow distributions

Find a distribution with reasonable shape such that it gives no experimental contribution at one scale, and check how big its contribution becomes as you move from the initial scale \rightarrow measures worst case uncertainty propagation from experiment to fit

Deconvolution problem for $x < \xi$

- [HD, Lorcé, Moutarde, Sznajder, Trawinski, Wagner, Eur.Phys.J.C 81 (2021) 4, 300]: a very simple shadow D-term cause causes an **inflation of uncertainty by a factor 20 with full correlation between fitted parameters** over a range of $Q^2 \in [1.5, 4] \text{ GeV}^2$
- Preliminary prediction (EIC): over range in $Q^2 \in [1.5, 50] \text{ GeV}^2$, **inflation of uncertainty reduced to a factor 7 thanks to sole increase of range in Q^2** [HD, Ph.D. thesis (2022)]

Deconvolution problem at moderate x and ξ

General deconvolution problem: Compton form factors (CFFs) given by [Radyushkin, 1997], [Ji, Osborne, 1998], [Collins, Freund, 1999]

$$\mathcal{H}^q(\xi, t, Q^2) = \int_{-1}^1 \frac{dx}{\xi} T^q\left(\frac{x}{\xi}, \alpha_s, \frac{Q^2}{\mu^2}\right) H^q(x, \xi, t, \mu^2). \quad (12)$$

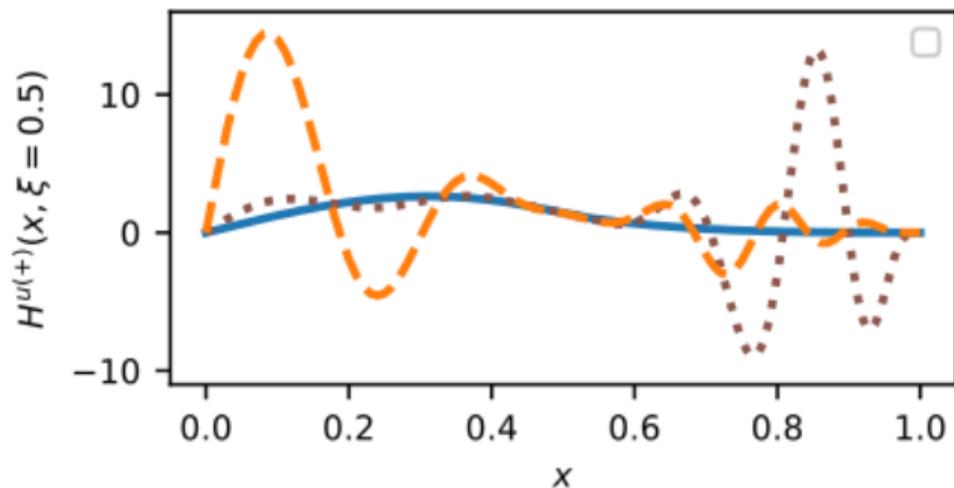
What is a reasonable shape for shadow GPD?

- 1 **Double distributions** [Radyushkin, 1997] as polynomials in their two variables (α, β) ?
- 2 **Neural network model** of double distributions?

Deconvolution problem at moderate x and ξ

Double distributions as polynomials in their two variables (α, β) [Bertone, HD, Mezrag, Moutarde, Sznajder, Phys.Rev.D 103 (2021) 11, 114019]

- Enforces polynomiality by construction
- Analytical computation of the CFF \rightarrow exact cancellation possible at least up to NLO
- Precise test of the accuracy of evolution: at NLO, should vary as $\mathcal{O}(\alpha_s^2)$



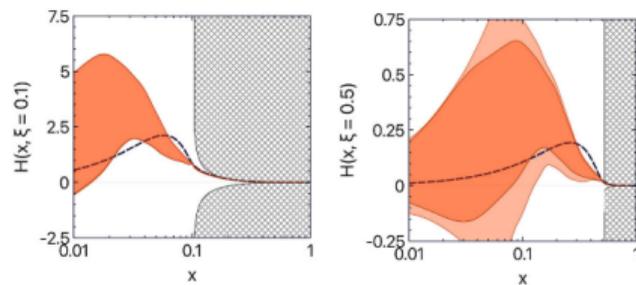
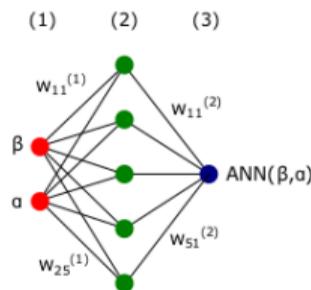
- **Result:** the three models give CFFs that vary by $\approx 10^{-5}$ at moderate ξ over a range of $[1, 100]$ $\text{GeV}^2 \rightarrow$ **enormous inflation of uncertainty from experimental data at moderate ξ**
- **Limitation:** large fluctuations at large x unphysical, incompatible with positivity constraints

Deconvolution problem at moderate x and ξ

Neural network model of double distributions [HD, Grocholski, Moutarde, Sznajder, Eur.Phys.J.C 82 (2022) 3, 252]

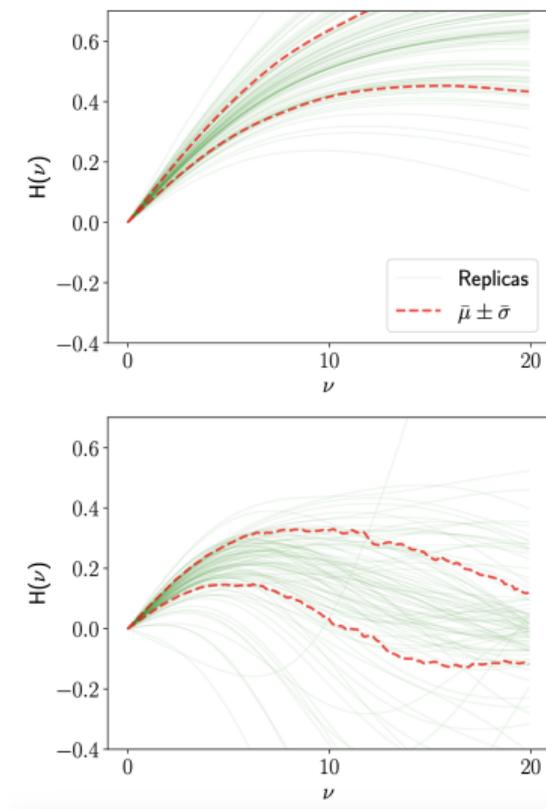
- Enforces polynomiality by construction
- More flexible without the need of very large polynomial powers (precision issue for floating point computation)
- More flexible framework to implement positivity constraint: mock constraint

$$|H^q(x, \xi, t)| \leq \sqrt{f^q \left(\frac{x + \xi}{1 + \xi} \right) f^q \left(\frac{x - \xi}{1 - \xi} \right) \frac{1}{1 - \xi^2}} \quad (13)$$



- Proof of concept – closure test :

Deconvolution problem at moderate x and ξ



In loffe-time,

$$H^q(\nu, \xi, t) = \int_{-1}^1 dx e^{ix\nu} H^q(x, \xi, t). \quad (14)$$

$\xi = 0.1$ (top) and $\xi = 0.9$ (bottom): the positivity constraint we have set reduces considerably the amount of information contained at small loffe times (very large auto-correlation of uncertainty at small ξ and ν).

[Riberdy, HD, Mezrag, Szajder, hep-ph:2306.01647 (2023)]

- Other exclusive processes can be expressed in terms of GPDs. Close parent to DVCS is **time-like Compton scattering** (TCS) [Berger et al, 2002]. Although its measurement will reduce the uncertainty, especially on $\text{Re } \mathcal{H}$ [Jlab proposal PR12-12-001], and produce a valuable check of the universality of the GPD formalism, the similar nature of its convolution (see [Müller et al, 2012]) makes it subject to the same shadow GPDs.
- **Deeply virtual meson production** (DVMP) [Collins et al, 1997] is also an important source of knowledge on GPDs, with currently a larger lever arm in Q^2 . The process involves form factors of the general form

$$\mathcal{F}(\xi, t) = \int_0^1 du \int_{-1}^1 \frac{dx}{\xi} \phi(u) T\left(\frac{x}{\xi}, u\right) F(x, \xi, t) \quad (15)$$

where $\phi(u)$ is the leading-twist meson distribution amplitude (DA).

- At LO, the GPD and DA parts of the integral factorize and shadow GPDs cancel the form factor.
- Situation at NLO remains to be clarified, it is foreseeable new shadow GPDs (dependent on the DA) could be generated also for this process.

Conclusion

- Different regions of GPDs and different sources of data let themselves to different modelling strategies, **but can be implemented in a common framework!**
- Flexible modelling of the t -dependent PDF
- Generate small ξ dependence perturbatively **but with uncertainty!**
- When uncertainty too large, switch to a flexible modelling at intermediate ξ (neural network)
- **What has not been evoked:** flavor and helicity! Having u, d, g twist-two GPDs $H, E, \tilde{H}, \tilde{E}$, etc. \rightarrow proliferation of functions to fit
- **What has not been evoked:** systematic errors in the data! We have discussed the uncertainty related to the modelling (**unknow ξ dependence at small scale, deconvolution of experimental data, ...**) but there is the significant systematical uncertainty of the data itself (higher twists, ...)

Thank you for your attention!

Deeply virtual Compton scattering and the structure of hadrons

- Remarkably, GPDs allow access to gravitational form factors (GFFs) of the **energy-momentum tensor (EMT)** [Ji, 1997] defined for parton of type a

Gravitational form factors [Lorcé et al, 2017]

$$\langle p', s' | T_a^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \left\{ \frac{P^\mu P^\nu}{M} A_a(t, \mu^2) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C_a(t, \mu^2) + M \eta^{\mu\nu} \bar{C}_a(t, \mu^2) \right. \\ \left. + \frac{P^{\{\mu} i \sigma^{\nu\} \rho} \Delta_\rho}{4M} [A_a(t, \mu^2) + B_a(t, \mu^2)] + \frac{P^{[\mu} i \sigma^{\nu] \rho} \Delta_\rho}{4M} D_a(t, \mu^2) \right\} u(p, s) \quad (16)$$

where

$$\Delta = p' - p, \quad t = \Delta^2, \quad P = \frac{p + p'}{2} \quad (17)$$

Deeply virtual Compton scattering and the structure of hadrons

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Energy density: T^{00}
 Momentum density: T^{0i}
 Energy flux: T^{i0}
 Momentum flux: T^{ij}
 Shear stress: T^{12}, T^{21}
 Normal stress: T^{11}, T^{22}, T^{33}

from C. Lorcé

In the Breit frame ($\vec{P} = 0$, $t = -\vec{\Delta}^2$), radial distributions of energy and momentum in the proton are described by Fourier transforms of the **GFFs** w.r.t. variable $\vec{\Delta}$ [Polyakov, 2003].

- Example of such distribution: radial pressure anisotropy profile

$$s_a(r, \mu^2) = -\frac{4M}{r^2} \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \frac{t^{-1/2}}{M^2} \frac{d^2}{dt^2} \left[t^{5/2} C_a(t, \mu^2) \right] \quad (18)$$

- This pressure profile can be extracted from **GPDs** thanks to e.g. for quarks

$$\int_{-1}^1 dx x H^q(x, \xi, t, \mu^2) = A_q(t, \mu^2) + 4\xi^2 C_q(t, \mu^2) \quad (19)$$

$$\int_{-1}^1 dx x E^q(x, \xi, t, \mu^2) = B_q(t, \mu^2) - 4\xi^2 C_q(t, \mu^2) \quad (20)$$

Extraction of GFFs

- At this stage, we don't need to fully extract the GPDs H or E to conveniently access the GFF $C_q(t, \mu^2)$. The **polynomiality property** gives that the GFF $C_q(t, \mu^2)$ only depends on the D -term via

$$\int_{-1}^1 dz z D^q(z, t, \mu^2) = 4C_q(t, \mu^2) \quad (21)$$

- The experimental data is sensitive to the D -term through the **subtraction constant** defined by the **dispersion relation** (see e.g. [Diehl, Ivanov, 2007])

DVCS dispersion relation

$$C_H(t, Q^2) = \text{Re } \mathcal{H}(\xi, t, Q^2) - \frac{1}{\pi} \int_0^1 d\xi' \text{Im } \mathcal{H}(\xi', t, Q^2) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \quad (22)$$

The subtraction constant $C_H(t, Q^2)$ is a function of the D -term given at LO by

$$C_H(t, Q^2) = 2 \sum_q e_q^2 \int_{-1}^1 dz \frac{D^q(z, t, Q^2)}{1-z} \quad (23)$$

Extraction of GFFs

- How do we get from

$$\int_{-1}^1 dz \frac{D^q(z, t, \mu^2)}{1-z} \quad \text{to} \quad \int_{-1}^1 dz z D^q(z, t, \mu^2) ? \quad (24)$$

- This is a prototype of the more complicated GPD extraction problem we will face later on. The known solution is through evolution.
- Let's expand the D -term on a basis of Gegenbauer polynomials

$$D^q(z, t, \mu^2) = (1-z^2) \sum_{\text{odd } n} d_n^q(t, \mu^2) C_n^{3/2}(z) \quad (25)$$

Then

GFF C_a extraction

$$\int_{-1}^1 dz \frac{D^q(z, t, \mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t, \mu^2) \quad \text{and} \quad \int_{-1}^1 dz z D^q(z, t, \mu^2) = \frac{4}{5} d_1(t, \mu^2) \quad (26)$$

Extraction of GFFs

- Since the LO subtraction constant reads

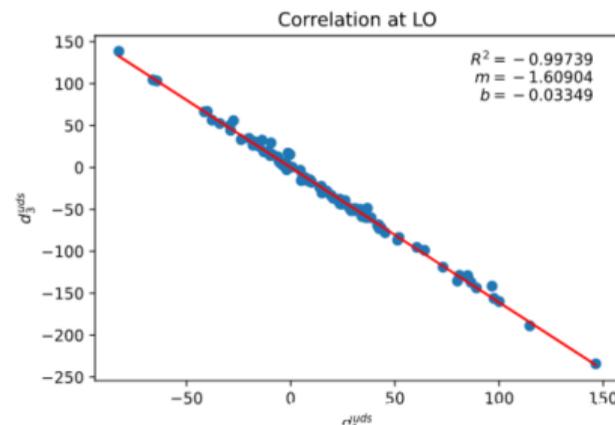
$$\int_{-1}^1 dz \frac{D^q(z, t, \mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t, \mu^2) \quad (27)$$

if we allow d_3^q to be non-zero, at some scale μ_0^2 , we can have $d_1^q(\mu_0^2) = -d_3^q(\mu_0^2)$, so a **vanishing subtraction constant, but non-zero GFF** $C_q(\mu_0^2)$. If the effect of evolution is not significant enough, these configurations are not ruled out and add a considerable uncertainty.

$$d_1^{uds}(\mu_F^2) \quad -0.5 \pm 1.2$$



$$\begin{array}{ll} d_1^{uds}(\mu_F^2) & 11 \pm 25 \\ d_3^{uds}(\mu_F^2) & -11 \pm 26 \end{array}$$



Deconvoluting a Compton form factor

- Question was raised 20 years ago. Evolution was proposed as a crucial element in [Freund, 1999], but the question has remained essentially open.
- We show that GPDs exist which bring contributions to the LO and NLO CFF of only subleading order even under evolution. We call them **LO and NLO shadow GPDs**.

Definition of an NLO shadow GPD

For a given scale μ_0^2 ,

$$\forall \xi, \forall t, T_{NLO}^q(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0 \quad \text{and} \quad H^q(x, \xi = 0, t = 0, \mu_0^2) = 0 \quad (28)$$

$$\text{so for } Q^2 \text{ and } \mu^2 \text{ close enough to } \mu_0^2, T_{NLO}^q(Q^2, \mu^2) \otimes H^q(\mu^2) = \mathcal{O}(\alpha_s^2(\mu^2)) \quad (29)$$

- Let H^q be an NLO shadow GPD, and G^q be any GPD. Then G^q and $G^q + H^q$ have the same forward limit, and the same NLO CFF up to a numerically small and theoretically subleading contribution.

Shadow GPDs at leading order

- Complete details in [Bertone, HD, Mezrag, Moutarde, Sznajder, Phys.Rev.D 103 (2021) 11, 114019]
- We search for our shadow GPDs as simple **double distributions (DD)** $F(\beta, \alpha, \mu^2)$ to respect polynomiality, with a zero D-term. Then, thanks to dispersion relations, we can restrict ourselves to the imaginary part only $\text{Im } T^q(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0$.
- We search our DD as a polynomial of order N in (β, α) , characterised by $\sim N^2$ coefficients c_{mn} :

$$F(\beta, \alpha, \mu_0^2) = \sum_{m+n \leq N} c_{mn} \alpha^m \beta^n \quad (30)$$

Shadow GPDs at next-to-leading order

- **First study beyond leading order:** Apart from the **LO** part, the NLO CFF is composed of a **collinear part** (compensating the α_s^1 term resulting from the convolution of the LO coefficient function and the evolved GPD) and a genuine **1-loop NLO** part.

$$\mathcal{H}^q(\xi, Q^2) = C_0^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2) C_1^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2) C_{coll}^q \otimes H^{q(+)}(\mu_0^2) \log\left(\frac{\mu^2}{Q^2}\right) \quad (31)$$

An explicit calculation of each term for our polynomial double distribution gives that

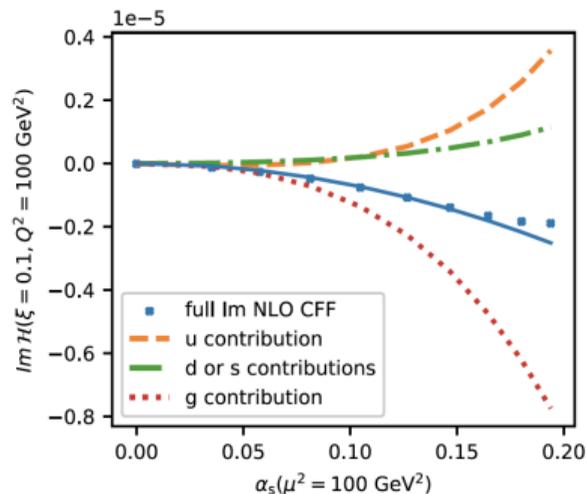
$$\text{Im } T_{coll}^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto \alpha_s(\mu^2) \log\left(\frac{\mu^2}{Q^2}\right) \left[\left(\frac{3}{2} + \log\left(\frac{1-\xi}{2\xi}\right)\right) \text{Im } T_{LO}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N+1} \frac{k_w^{(coll)}}{(1+\xi)^w} \right] \quad (32)$$

and assuming $\text{Im } T_{LO}^q \otimes H^q(\mu^2) = 0$,

$$\text{Im } T_1^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto \alpha_s(\mu^2) \left[\log\left(\frac{1-\xi}{2\xi}\right) \text{Im } T_{coll}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N-1} \frac{k_w^{(1)}}{(1+\xi)^w} \right]$$

Shadow GPDs at next-to-leading order

- By linearity of both the CFF convolution and the evolution equation, we can evaluate separately the contribution to the CFF of a quark shadow NLO GPD under evolution.
- We probe the prediction of evolution as $\mathcal{O}(\alpha_s^2(\mu^2))$ with our previous NLO shadow GPD on a lever-arm in Q^2 of $[1, 100]$ GeV² (typical collider kinematics) using APFEL++ code.



- The fit by $\alpha_s^2(\mu^2)$ is very good up to values of α_s of the order of its \overline{MS} values. For larger values, large logs and higher orders slightly change the picture.
- The numerical effect of evolution remains very small. For a GPD of order 1, the NLO CFF is only of order 10^{-5} .

- Other exclusive processes can be expressed in terms of GPDs. Close parent to DVCS is **time-like Compton scattering** (TCS) [Berger et al, 2002]. Although its measurement will reduce the uncertainty, especially on $\text{Re } \mathcal{H}$ [Jlab proposal PR12-12-001], and produce a valuable check of the universality of the GPD formalism, the similar nature of its convolution (see [Müller et al, 2012]) makes it subject to the same shadow GPDs.
- **Deeply virtual meson production** (DVMP) [Collins et al, 1997] is also an important source of knowledge on GPDs, with currently a larger lever arm in Q^2 . The process involves form factors of the general form

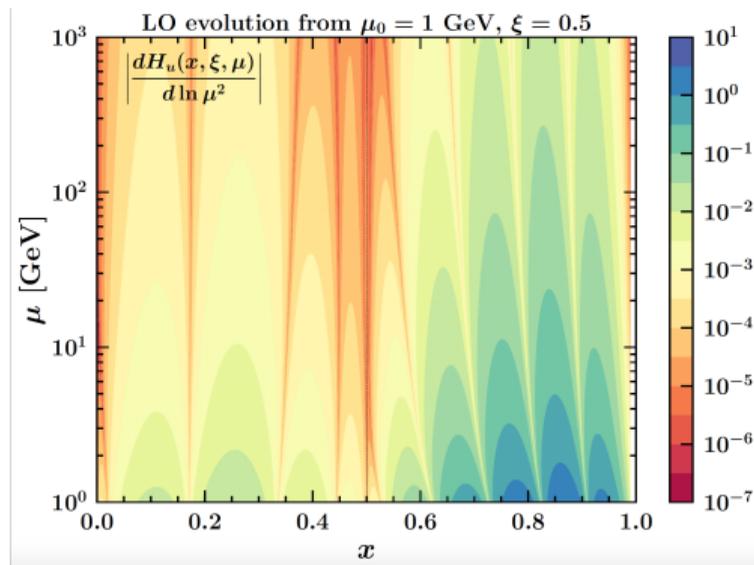
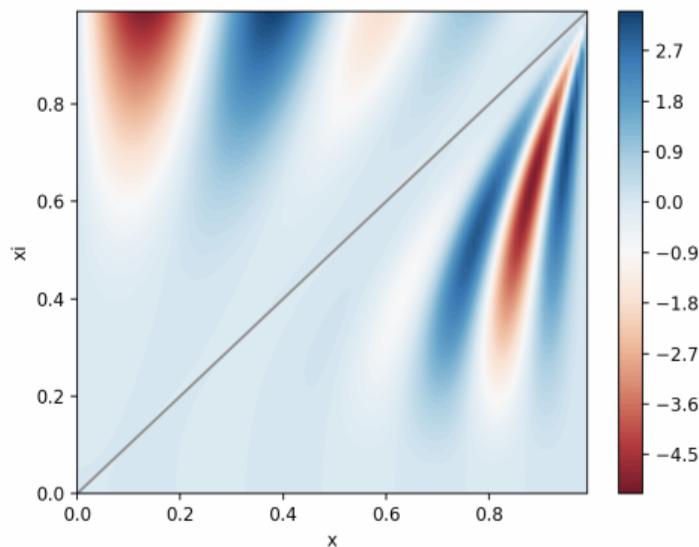
$$\mathcal{F}(\xi, t) = \int_0^1 du \int_{-1}^1 \frac{dx}{\xi} \phi(u) T\left(\frac{x}{\xi}, u\right) F(x, \xi, t) \quad (34)$$

where $\phi(u)$ is the leading-twist meson distribution amplitude (DA).

- At LO, the GPD and DA parts of the integral factorize and shadow GPDs cancel the form factor.
- Situation at NLO remains to be clarified, it is foreseeable new shadow GPDs (dependent on the DA) could be generated also for this process.

Shadow GPDs at next-to-leading order

- Cancelling both terms gives rise to two additional systems with a linear number of equations. The first NLO shadow GPD is found for $N = 21$, and adding the condition that the DD vanishes at the edges of its support gives a first solution for $N = 25$ (see below).



Color plot of an NLO shadow GPD at initial scale 1 GeV^2 , and its evolution for $\xi = 0.5$ up to 10^6 GeV^2 via APFEL++ and PARTONS [Bertone].

Evolution of GPDs

GPD's dependence on scale is given by **renormalization group equations**. In the limit $\xi = 0$, usual DGLAP equation:

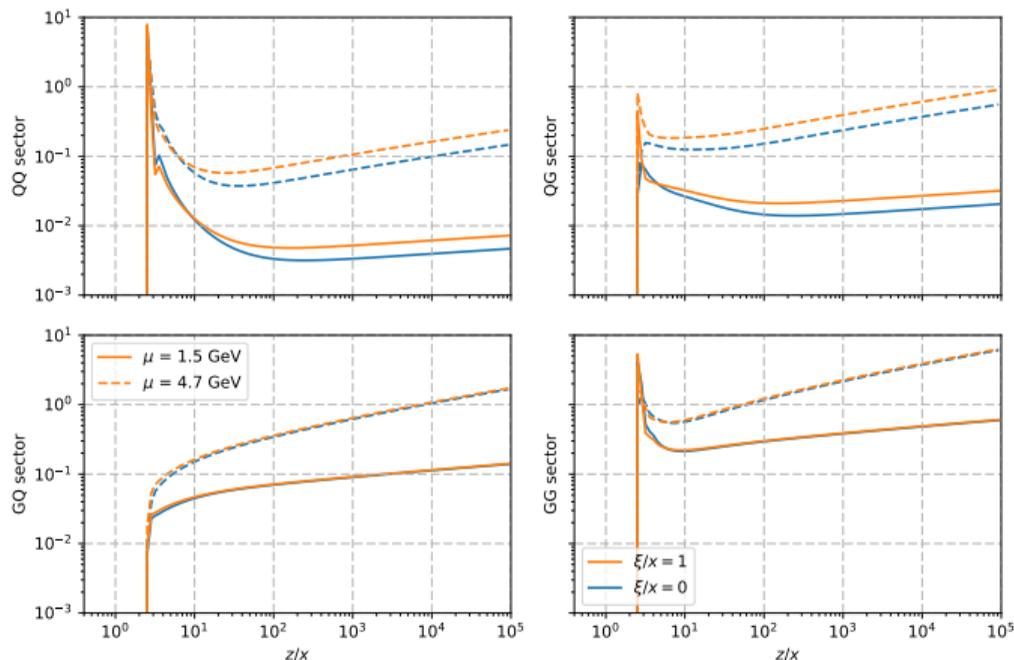
$$\frac{df^{q+}}{d\mu}(x, \mu) = \frac{C_F \alpha_s(\mu)}{\pi\mu} \left\{ \int_x^1 dy \frac{f^{q+}(y, \mu) - f^{q+}(x, \mu)}{y - x} \left[1 + \frac{x^2}{y^2} \right] + f^{q+}(x, \mu) \left[\frac{1}{2} + x + \log \left(\frac{(1-x)^2}{x} \right) \right] \right\} \quad (35)$$

But in the limit $x = \xi$:

$$\frac{dH^{q+}}{d\mu}(x, x, \mu) = \frac{C_F \alpha_s(\mu)}{\pi\mu} \left\{ \int_x^1 dy \frac{H^{q+}(y, x, \mu) - H^{q+}(x, x, \mu)}{y - x} + H^{q+}(x, x, \mu) \left[\frac{3}{2} + \log \left(\frac{1-x}{2x} \right) \right] \right\} \quad (36)$$

Assuming that GPD = t -dependent PDF at small ξ and $x \approx \xi$ is incompatible with evolution, which generates an intrinsic ξ dependence!

Evolution of GPDs



Behaviour of $\Gamma^{ab}(z/x, \xi/x; \mu_0 = 1 \text{ GeV}, \mu)$ – **provided x and ξ are small enough, evolution range is large enough and PDF not too singular at small x , the large z region at initial scale with $z \gg \xi$ dominates evolution**

The Shuvaev transform

- Remarkably,

$$\Gamma^{ga}(x, \xi, z; \mu_0, \mu) \approx S^g(x, \xi, n) \star \mathcal{M}(n, y) \star \Gamma^{ga}(y, 0, z; \mu_0, \mu) \quad (37)$$

The **GPD evolution operator** is nicely approximated by the Shuvaev transform of the Mellin transform of its **limit for $\xi = 0$ (DGLAP evolution operator)**. The approximation is excellent as soon as $z > 4\xi$.

