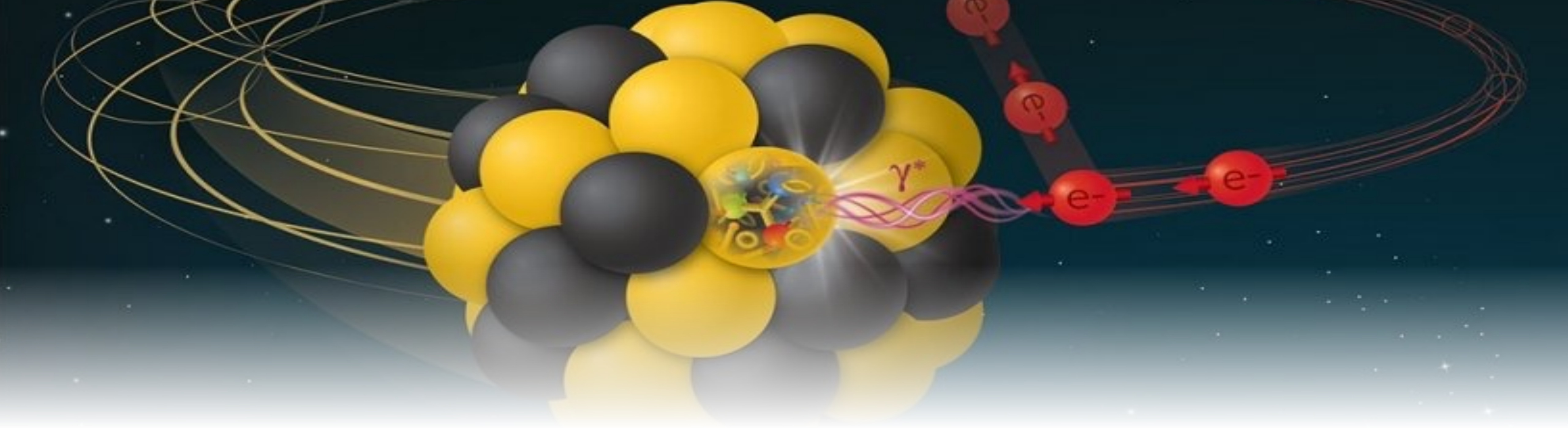


Moment parameterization of GPDs and global analysis

Yuxun Guo

University of Maryland, College Park

Lawrence Berkeley National Laboratory

CNF GPD global analysis workshop

Jun. 12-14th, 2023

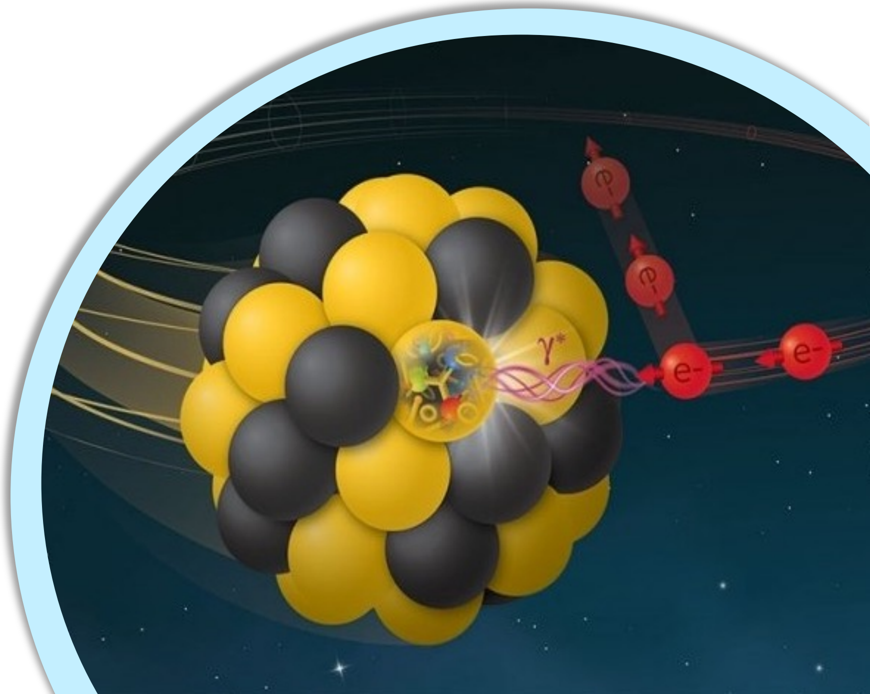
Outline

» Intro – GPD and nucleon 3D structure

» Moment param. of GPDs and global analysis

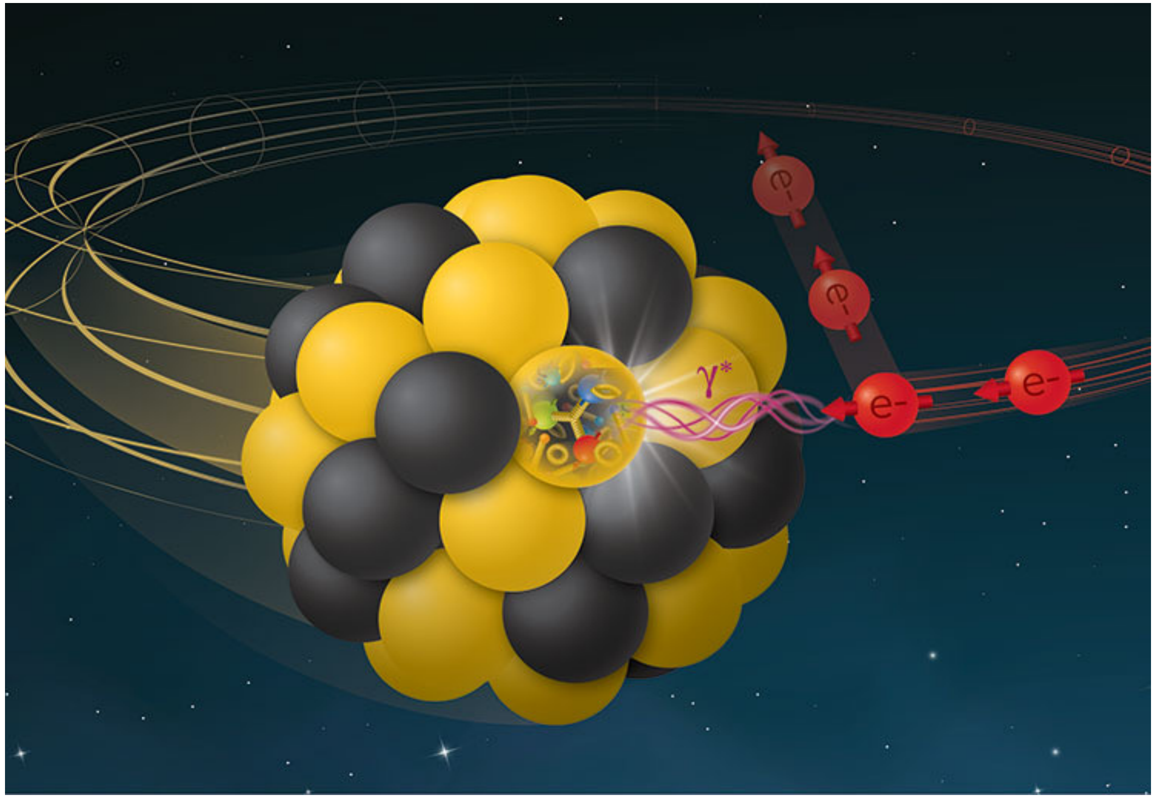
» DA-terms of GPD with non-zero skewness

» Summary and outlook



Nucleon spin and 3D structure

With asymptotic freedom, we can access the nucleon structure by higher-energy knocked out processes.



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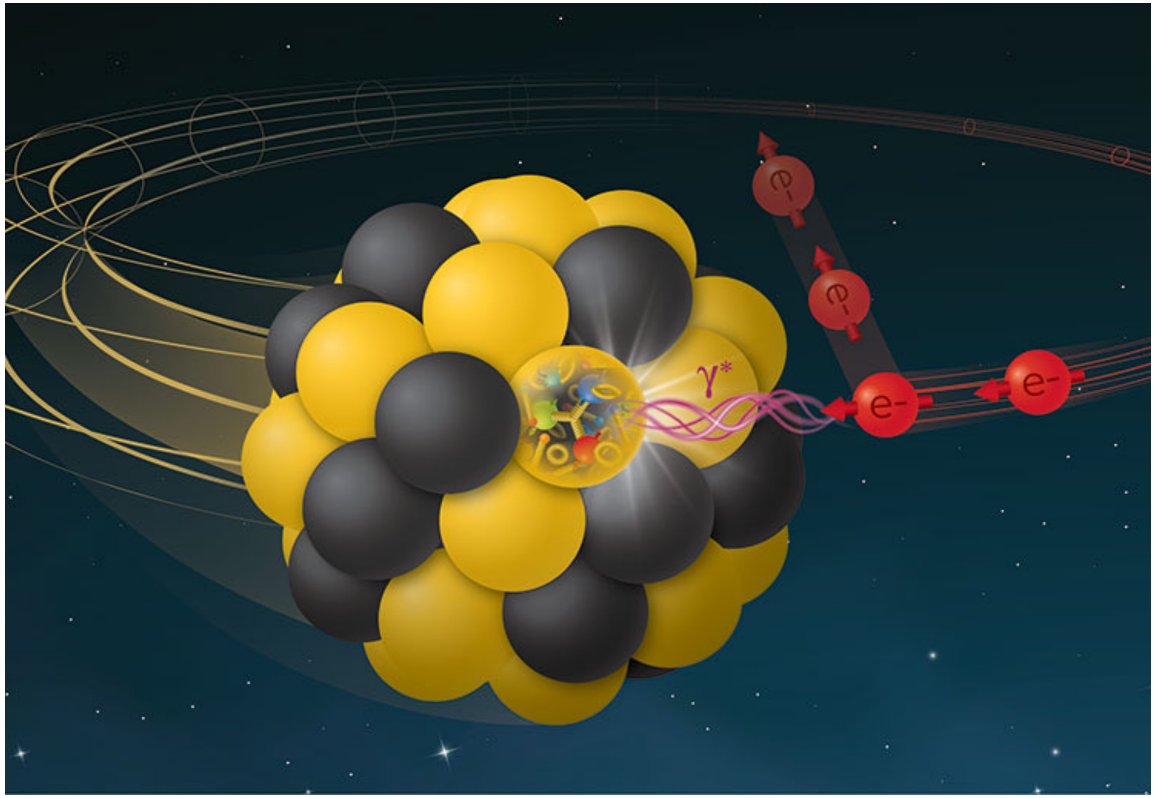

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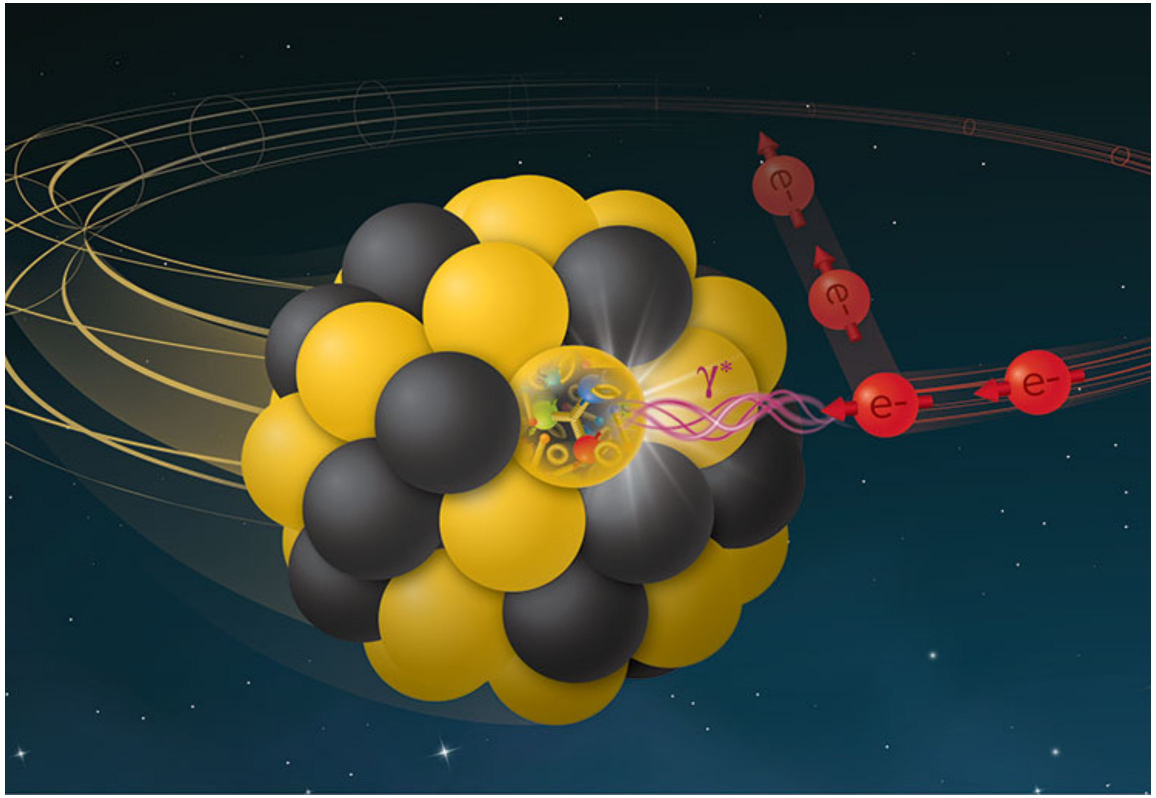

Generalized Parton Distributions (GPDs)

X. Ji, Phys. Rev. Lett. 78 610-613 (1997)

3D mass & spin structures with GPDs

GPDs are 3D distributions unifying parton distributions and form factors

$$F(x,\Delta^\mu) = F\left(x,\xi,t\right)$$

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We cannot easily access GFFs in experiment, but we can access GPDs!

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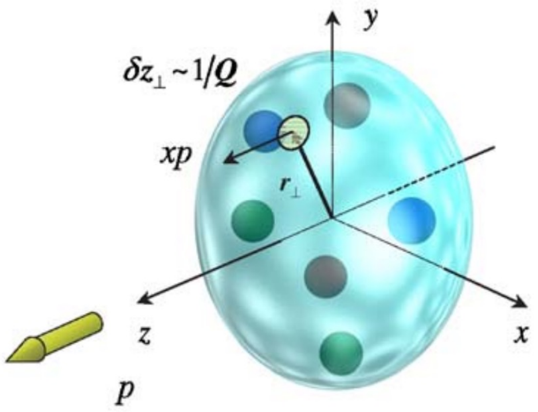

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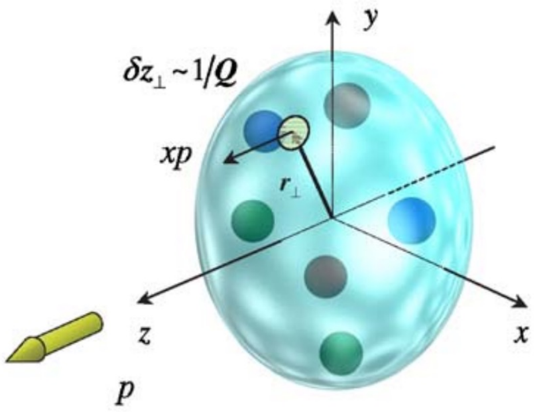

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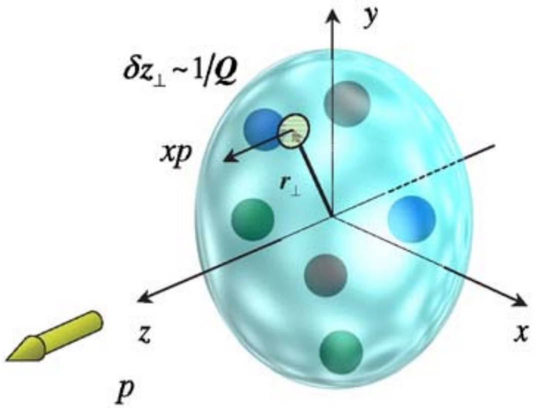

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$$J_q^T(x) = \int \mathrm{d}^2\boldsymbol{b}(b^y \times xP^+)\rho_q^T(x, \boldsymbol{b})$$

Y. Guo et. al. Nucl. Phys. B 969 115440 (2021)

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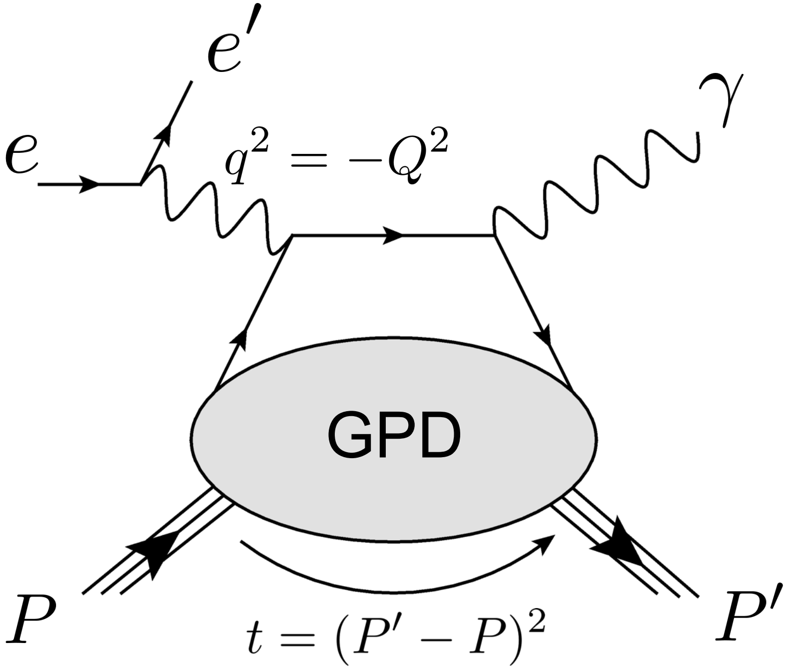

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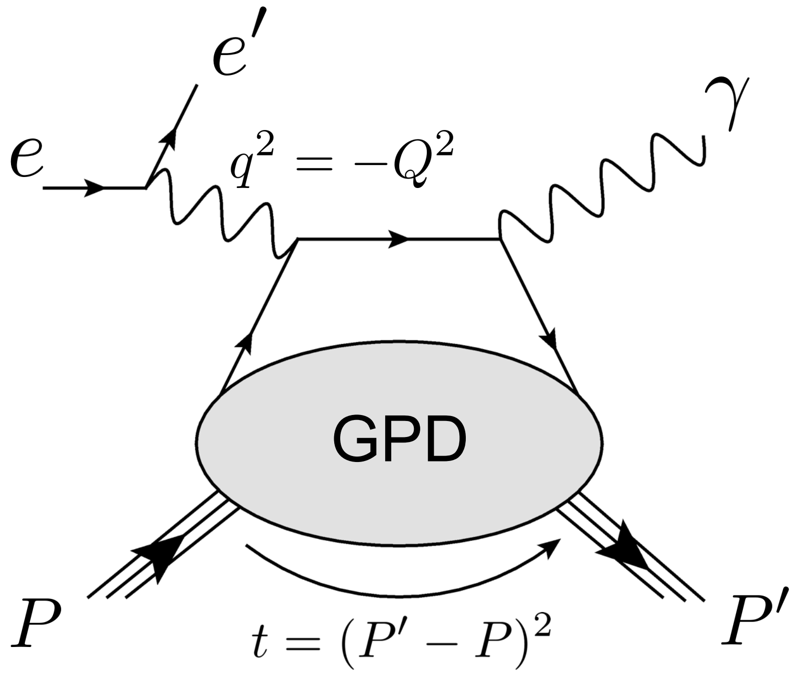

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$$\mathcal{H}_{CFF}(\xi, t) = -\sum_q Q_q^2 \int_{-1}^{1} \mathrm{d}x \left(\frac{1}{x - \xi + i0} + \frac{1}{x + \xi - i0} \right) H_q(x, \xi, t) \,,$$

General strategy of GPD global analysis

- Parameterization of GPDs
- Compute GPD observables
- Inputs (constraints) on GPDs
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  - Computation efficiency required!

Parameterization of GPD

The conformal moment parameterization of GPD is helpful

$$F(x,\xi,t)=\sum_{j=0}^{\infty}(-1)^{j}p_{j}(x,\xi)\mathcal{F}_{j}(\xi,t)$$

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GPDs through Universal Moment Parameterization (GUMP)

Collaborators: Xiangdong Ji, Kyle Shiells, Gabriel Santiago, Jinghong Yang

Y. Guo et. al. JHEP 09 215 (2022)
Y. Guo et. al. JHEP 05 150 (2023)

Inputs for the global analysis

Experiments

- PDFs from global analysis
  JAM, Phys. Rev. D 106 3, L031502 (2022)
  - Polarized and unpolarized PDFs from JAM

Z. Ye et. Al., Phys. Lett. B 777 8-15 (2018)

- Charge form factors from global analysis
  - YAHL global analysis of EM form factors
  - Flavor separation combing proton and neutron data

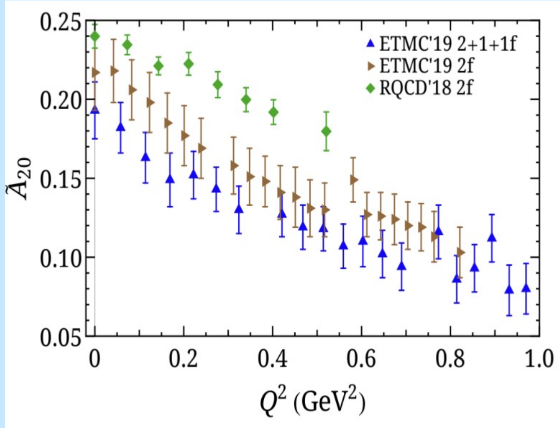
CLAS, Phys. Rev. Lett. 123 3, 032502 (2019)
Jlab Hall A, PoS Hadron2017 170 (2018)

- DVCS cross-section measurements
  - Combined data from CLAS and Hall A (UU and LU)
  - H1 experiments at HERA

H1, Phys. Lett. B 681 391-399 (2009)

Lattice

- Lattice results themselves have tensions



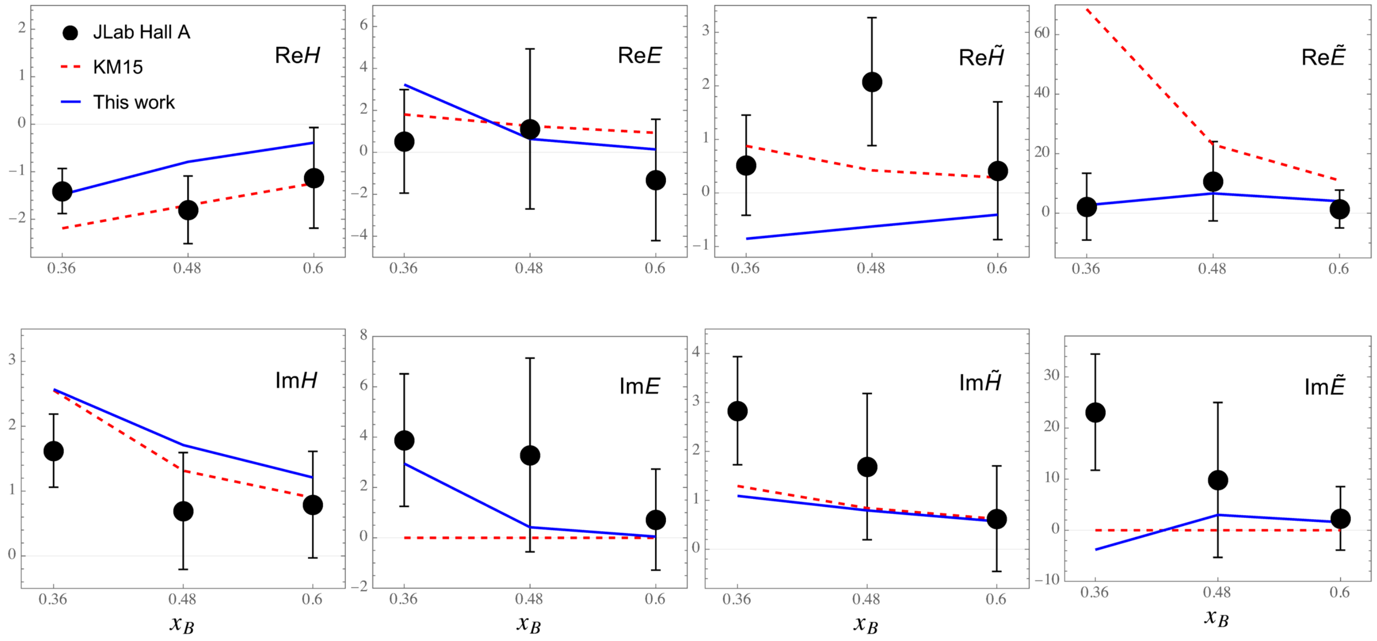
M. Constantinou et. al. Prog. Part. Nucl. Phys. 121 103908 (2021)

- Lattice form factors and GPDs from a single group.

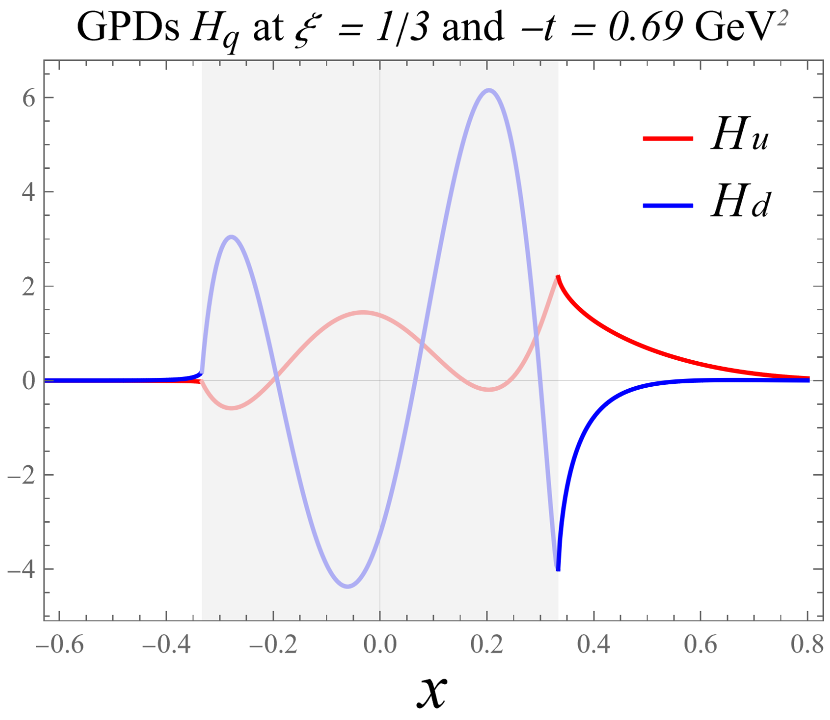
C. Alexandrou et. al. Phys. Rev. Lett. 125 26, 262001 (2020)
C. Alexandrou et. al. PoS LATTICE2021 250 (2022)

Extracted CFFs and GPDs

Extracted CFFs close to the local extraction



Example of extracted GPD



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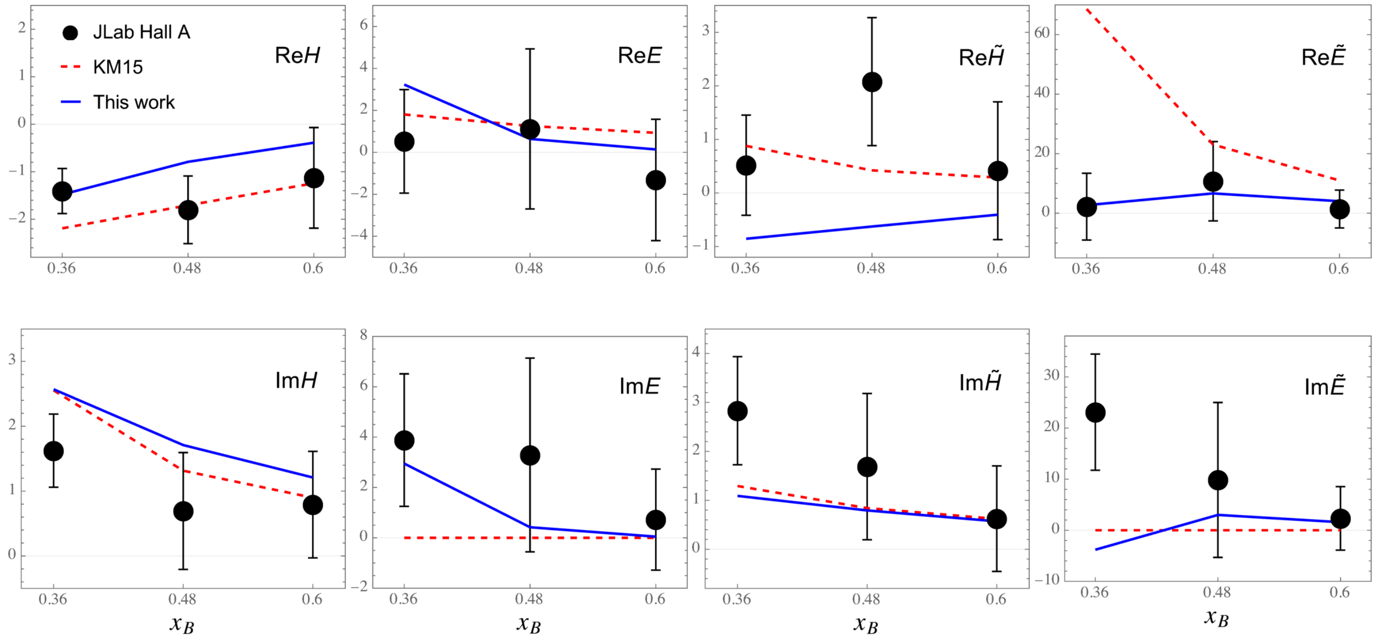

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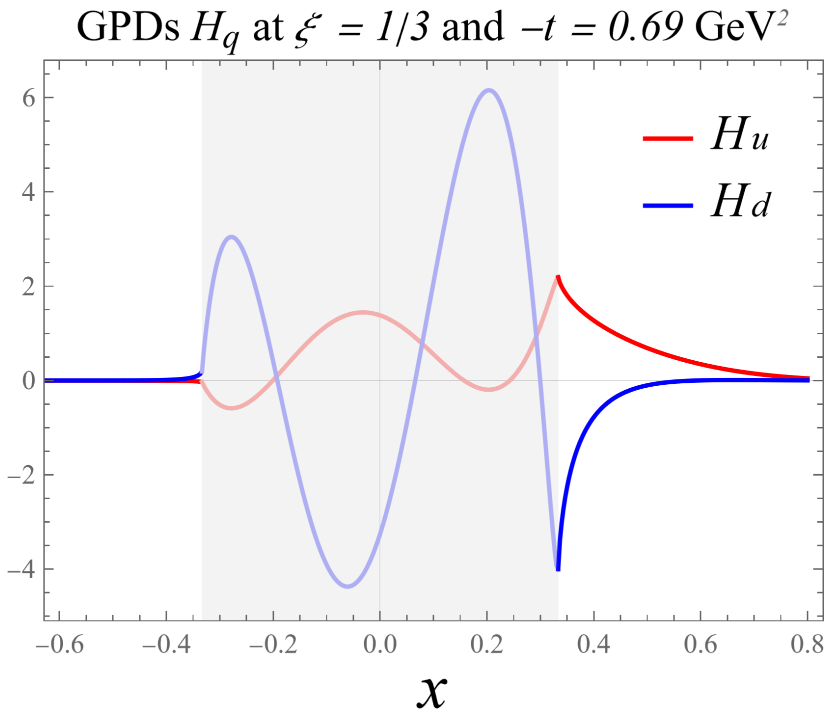
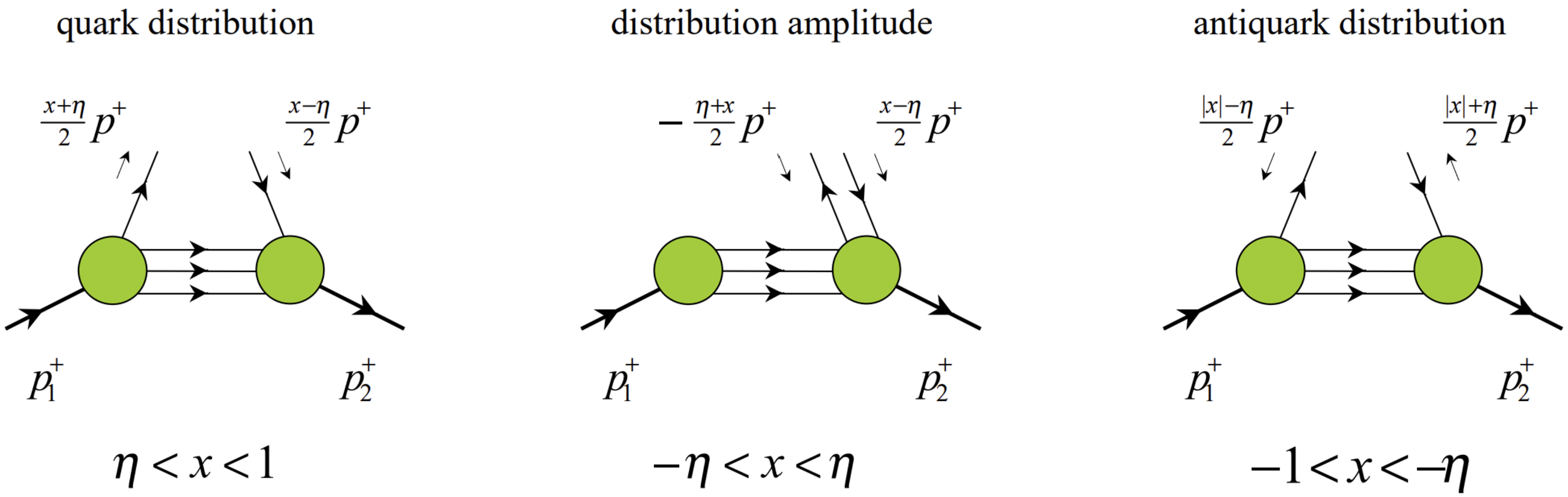


Caveats: ansatzes and empirical constraints used

Partonic interpretations of GPDs

GPDs involve different partonic interpretations



A. Belitsky, Phys. Rept. 418 1-387 (2005)

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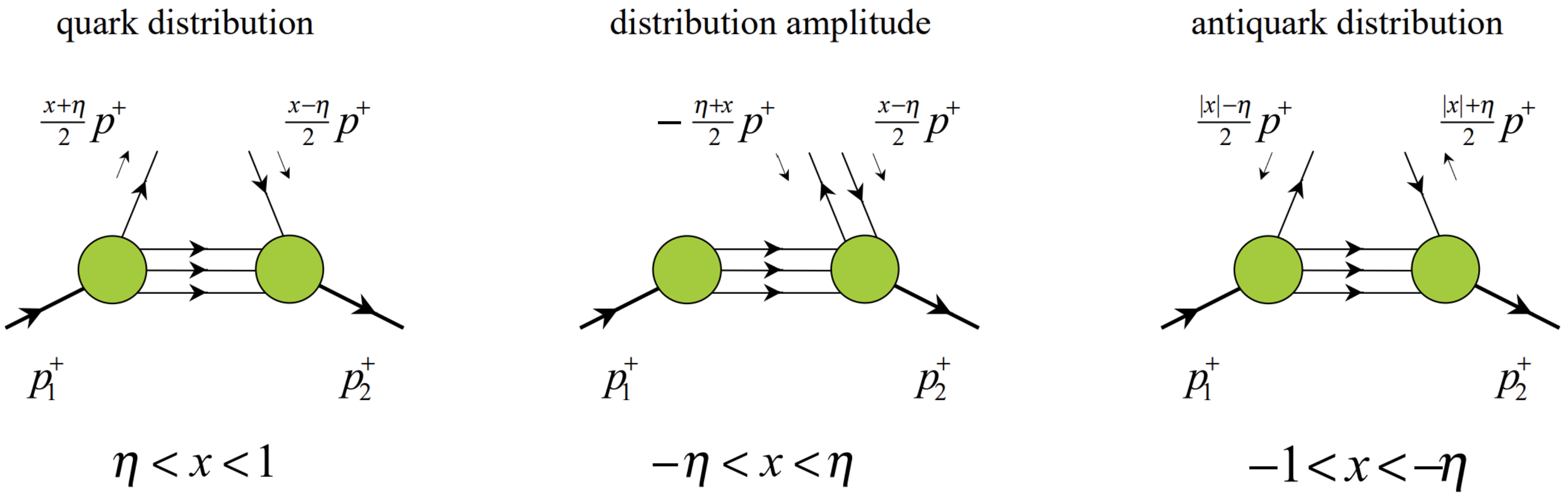

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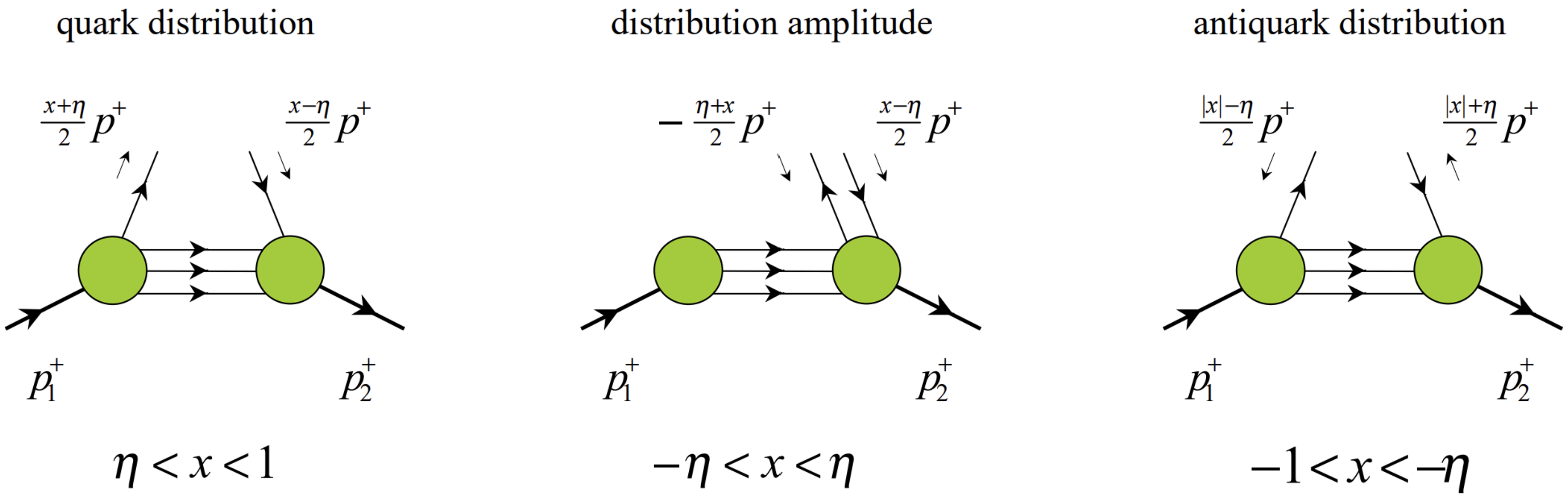

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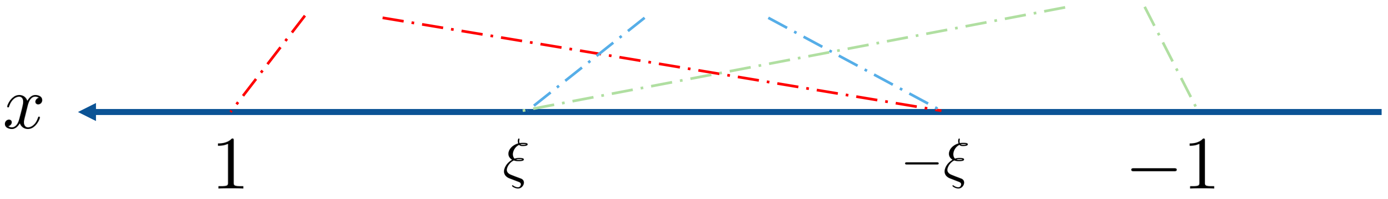

D-terms and DA-terms

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- D-terms are one kind of DA-terms
- Each generalized form factors could have corresponding DA-terms

GPD in DA-like region

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The DA-terms become non-trivial as xi increases

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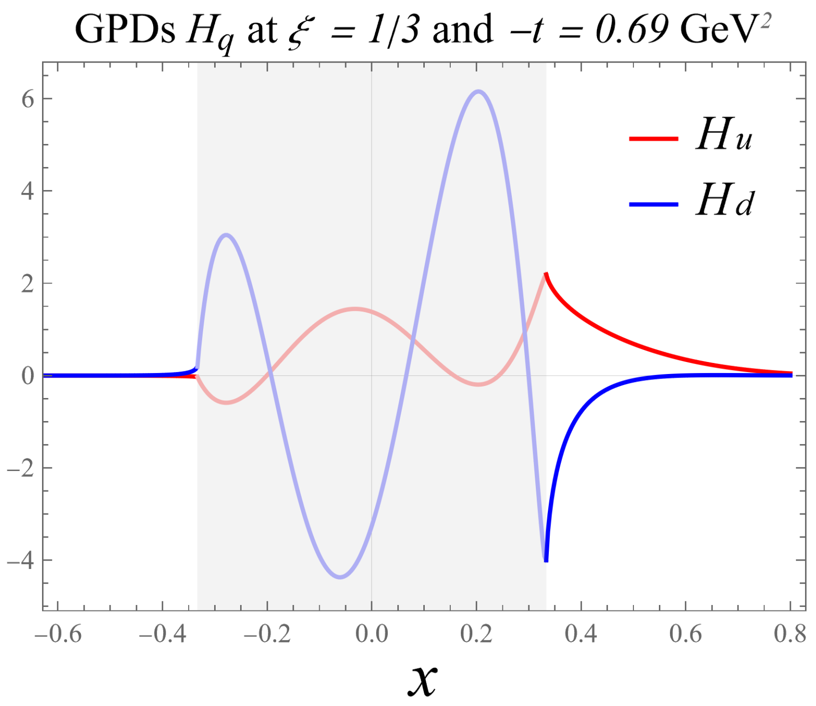

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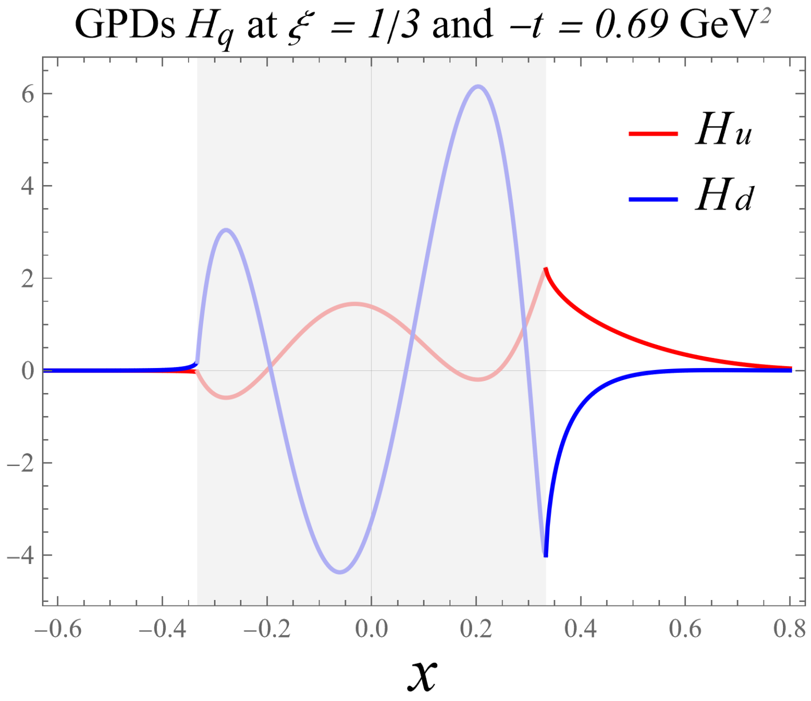
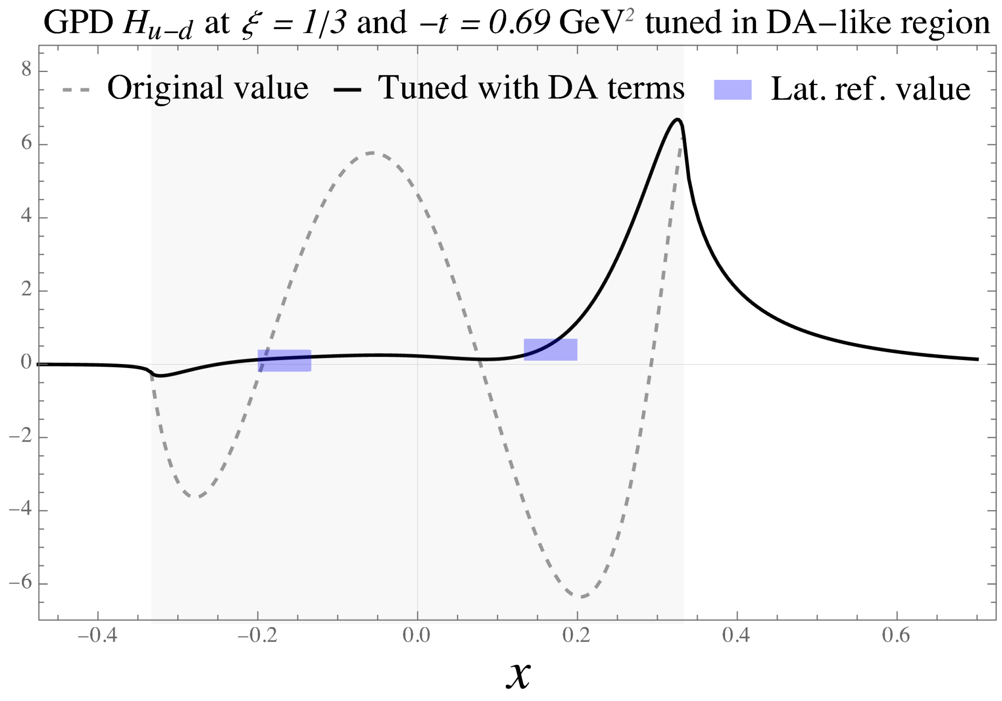
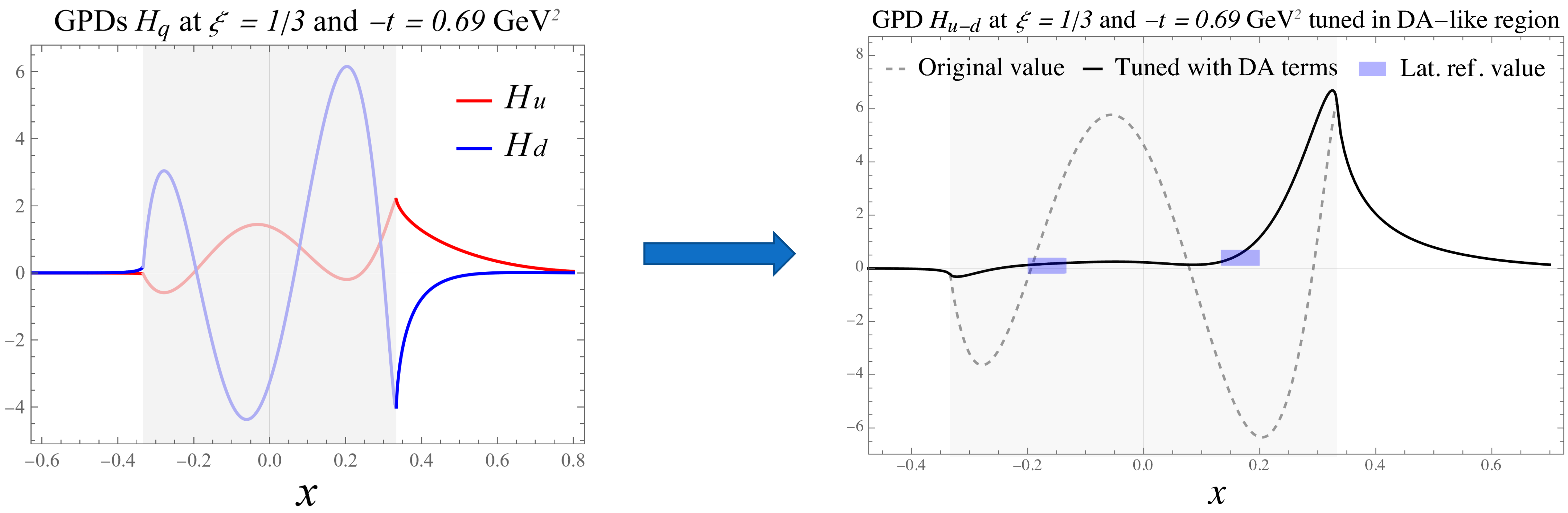




GPD in DA-like region

The DA-terms become non-trivial as xi increases



Extra inputs crucial to determine the shape of GPDs in the middle regions.

Some implications on phenomenology

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Potential extraction of leading moments from CFFs with medium/large xi?

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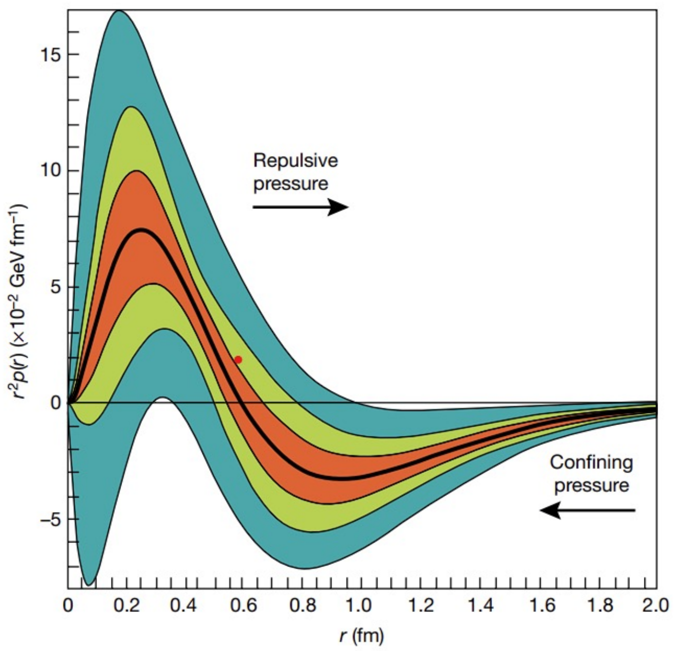
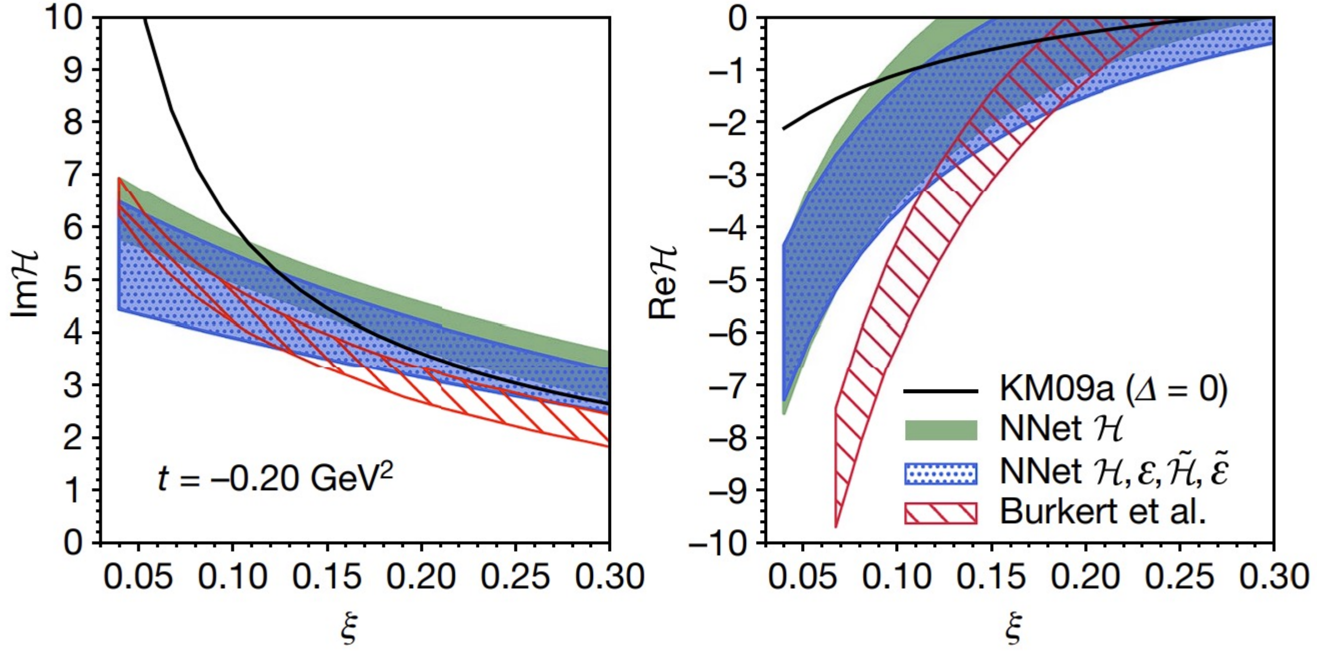
Extraction of the D-terms from DVCS analysis





V. D. Burkert et. al., Nature 557 7705, 396-399 (2018)

K. Kumericki, Nature 570 7759, E1-E2 (2019)

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Extraction of the GFFs from threshold heavy quarkonium production

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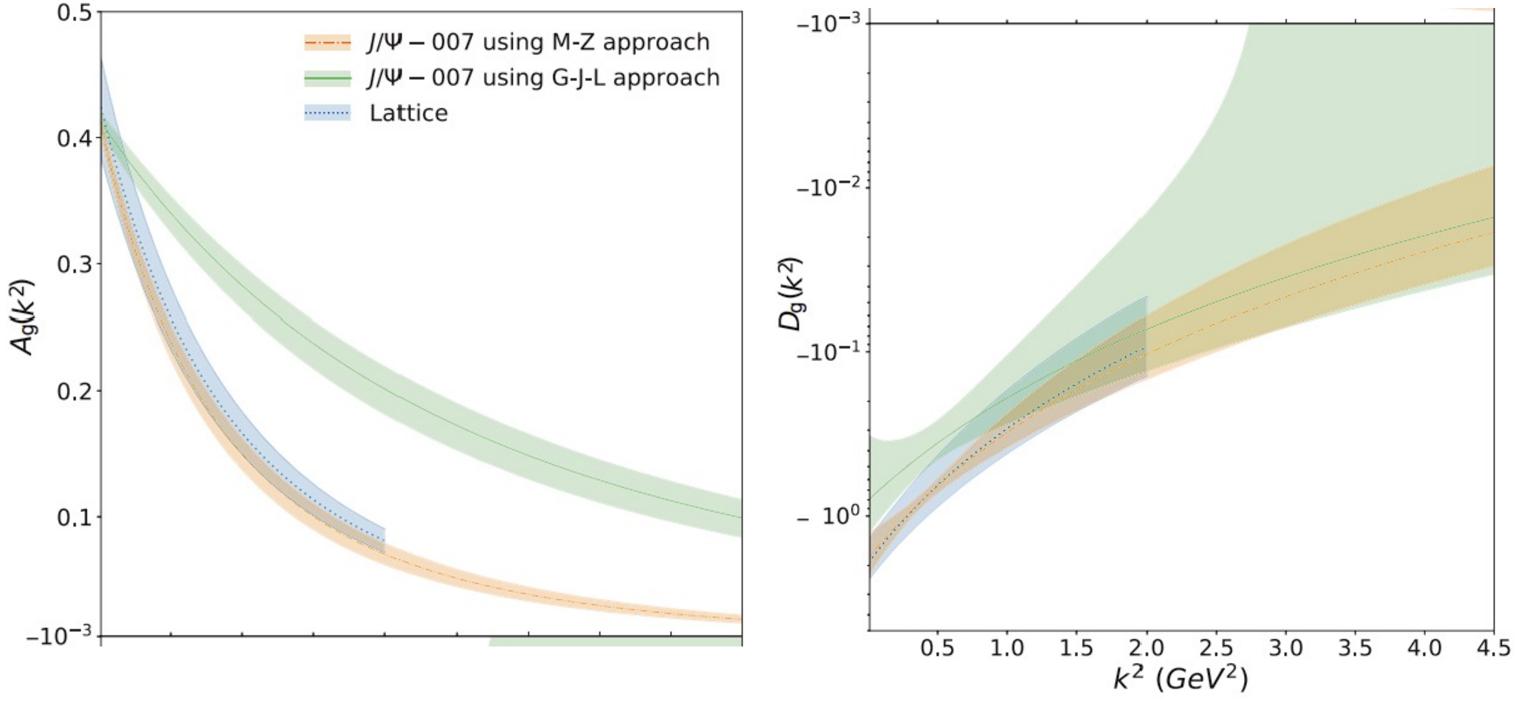

B. Duran et. al., Nature 615, 813-816 (2023)

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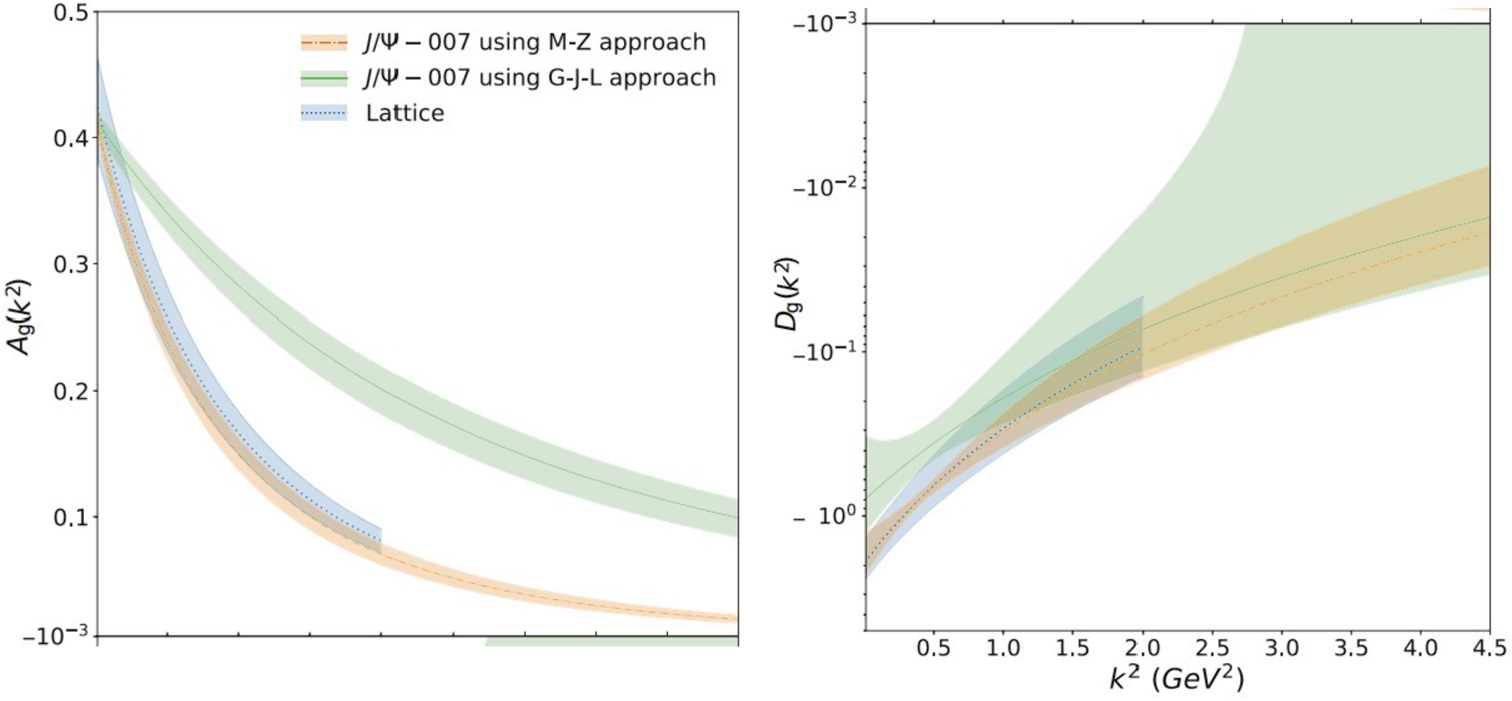
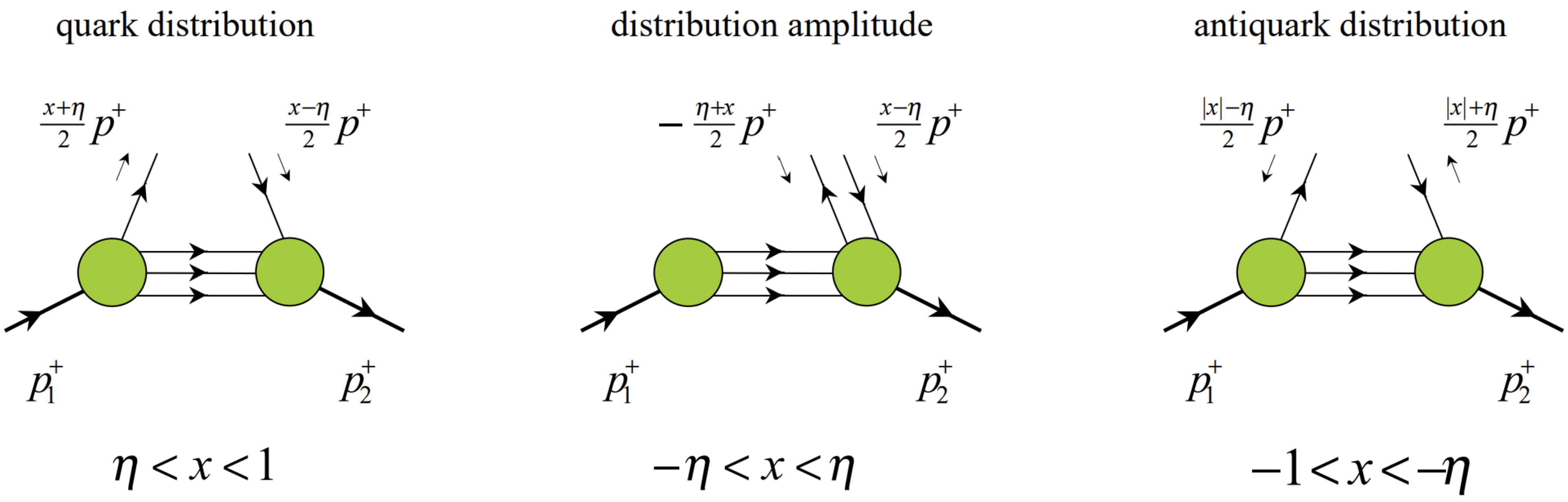


Y. Guo et. al. Phys. Rev. D 103, 096010 (2021)
Y. Guo et. al. arxiv: 2305. 06992
Y. Guo, X Ji, and F. Yuan. in progress

B. Duran et. al., Nature 615, 813-816 (2023)

Some final comments

The different interpretations of GPDs in different regions do have physical relevance.



To implement such picture is easier with moment param. for which the results appear encouraging.

Summary and outlook

Summary

- GPDs reveals the nucleon 3D structures including mass and spin.
- Global analysis program by parameterization moments of GPDs.
- Interesting developments utilizing the skewness effects and DA-terms

Outlook

- Global analysis for gluon GPDs with meson production
- Higher order evolutions/corrections and more quark flavor
- More developments on the skewness effects of GPDs

Thank you!