Introduction to GPDs (and TMDs)

Hervé Dutrieux
College of William and Mary

2023 Summer Hall A/C Meeting

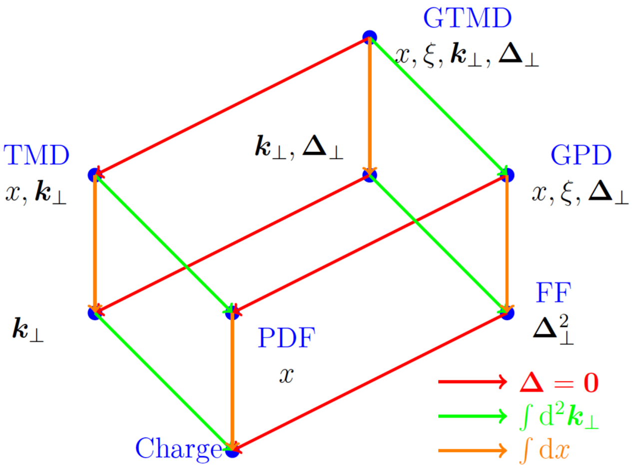
June 29th, 2023 – hldutrieux@wm.edu

WILLIAM & MARY
CHARTERED 1693

Outlook

- GPDs and TMDs: the bare minimum
- Phenomenology of GPDs

GPDs and TMDs: the bare minimum



[Lorcé, Pasquini, Vanderhaeghen, 2011]

GPDs and TMDs: the bare minimum

- **EFFs**: spatial distribution of charge in the Breit frame ($\vec{P}=0,\ t=-\vec{\Delta}^2$) or IMF

$$\langle p_2|\bar{\psi}^q(0)\gamma^\mu\psi^q(0)|p_1\rangle = \bar{u}(p_2)\left[F_1^q(t)\gamma^\mu + F_2^q(t)\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}\right]u(p_1) \tag{1}$$

  The impact parameter is the Fourier transform of the momentum transfer to the target in the transverse plane $\Delta_\perp$.

- **PDFs**: distribution of longitudinal momentum in the hadron (light-cone gauge)

$$p^+\int\frac{\mathrm{d}z^-}{2\pi}e^{ixp^+z^-}\left\langle p\middle|\bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+\psi^q\left(\frac{z}{2}\right)\middle|p\right\rangle\Big|_{z_\perp=0,\ z^+=0} = \bar{u}(p)f^q(x)\gamma^+u(p)$$

  The longitudinal momentum xp^+ is the Fourier transform of the light-like separation z between the field operators.

GPDs and TMDs: the bare minimum

GPDs encompass both EFFs and PDFs: non-zero separation between the field operators and momentum transfer to the target **[Müller et al, 1994]**, **[Radyushkin, 1996]**, **[Ji, 1997]** (lightcone gauge)

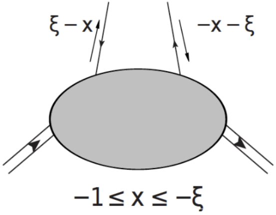
$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \left\langle p_2 \middle| \bar{\psi}^q \left(-\frac{z}{2}\right) \gamma^+ \psi^q \left(\frac{z}{2}\right) \middle| p_1 \right\rangle \Big|_{z_\perp=0,\ z^+=0}$$
$$= \frac{1}{2P^+}\left(H^q(x,\xi,t)\bar{u}(p_2)\gamma^+ u(p_1) + E^q(x,\xi,t)\bar{u}(p_2)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}u(p_1)\right) \tag{2}$$

where

$$p_2 - p_1 = \Delta,\ \ t = \Delta^2,\ \ P = \frac{1}{2}(p_1 + p_2),\ \ \xi = -\frac{\Delta^+}{2P^+}\,. \tag{3}$$

ξ is called **skewness** and characterizes the longitudinal momentum transfer, whereas $\Delta_\perp$ characterises the transverse momentum transfer.

GPDs and TMDs: the bare minimum



GPDs and TMDs: the bare minimum

Forward limit of GPDs

$$\begin{cases} H^q(x,\xi=0,t=0) & = q(x)\Theta(x) - \bar{q}(-x)\Theta(-x) \\ H^g(x,\xi=0,t=0) & = xg(x)\Theta(x) - xg(-x)\Theta(-x) \end{cases} \tag{4}$$

where $\Theta(x)$ is the Heaviside step function.

Elastic form factors

$$\int_{-1}^{1} \mathrm{d}x\, H^q(x,\xi,t) = F_1^q(t), \qquad \int_{-1}^{1} \mathrm{d}x\, E^q(x,\xi,t) = F_2^q(t) \tag{5}$$

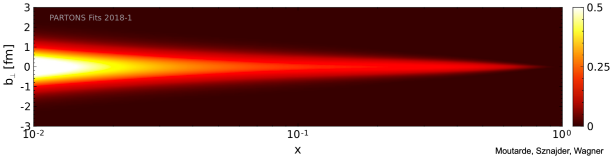
$\rightarrow$ **independent of ξ !**

GPDs and TMDs: the bare minimum

Impact parameter distribution (IPD) [Burkardt, 2000]

$$I_a(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} F^a(x, 0, t = -\Delta_\perp^2) \tag{6}$$

is the density of partons with longitudinal momentum x and transverse position $\mathbf{b}_\perp$ from the center of longitudinal momentum in a hadron $\rightarrow$ **hadron tomography**



Density of up quarks (valence GPD) in an unpolarized proton from a parametric fit to DVCS data in the PARTONS framework [Moutarde et al, 2018].

GPDs and TMDs: the bare minimum

Moments of higher-order in x of the GPD define generalized form factors. Due to Lorentz covariance, [Ji, 1998], [Radyushkin, 1999]

$$\int_{-1}^{1} \mathrm{d}x\, x^n H^q(x, \xi, t) = \sum_{k=0 \text{ even}}^{n+1} H^q_{n,k}(t)\, \xi^k \tag{7}$$

The moments of order 0 are linked to EFFs, the moments of order 1 to **gravitational form factors (GFFs)** of the energy-momentum tensor [Ji, 1997]

Gravitational form factors [Lorcé *et al*, 2017]

$$\langle p', s' | T^{\mu\nu}_a | p, s \rangle = \bar{u}(p', s') \Bigg\{ \frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C_a(t) + M \eta^{\mu\nu} \bar{C}_a(t)$$
$$+ \frac{P^{\{\mu} i\sigma^{\nu\}\rho} \Delta_\rho}{4M} [A_a(t) + B_a(t)] + \frac{P^{[\mu} i\sigma^{\nu]\rho} \Delta_\rho}{4M} D^{GFF}_a(t) \Bigg\} u(p, s) \tag{8}$$

GPDs and TMDs: the bare minimum

- Link between **GFFs** and **GPDs** thanks to *e.g.* for quarks

$$\int_{-1}^{1} \mathrm{d}x\, x\, H^q(x, \xi, t, \mu^2) = A_q(t, \mu^2) + 4\xi^2 C_q(t, \mu^2) \tag{9}$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, E^q(x, \xi, t, \mu^2) = B_q(t, \mu^2) - 4\xi^2 C_q(t, \mu^2) \tag{10}$$

- Ji's sum rule **[Ji, 1997]**

$$J^q = \frac{1}{2}\left(A_q(0) + B_q(0)\right) \tag{11}$$

- Radial distributions of hadron matter properties **[Polyakov, 2003]**: in the Breit frame ($\vec{P} = 0,\ t = -\vec{\Delta}^2$), radial pressure anisotropy profile

$$s_a(r) = -\frac{4M}{r^2}\int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}}\, \frac{t^{-1/2}}{M^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[t^{5/2}\, C_a(t)\right] \tag{12}$$

GPDs and TMDs: the bare minimum

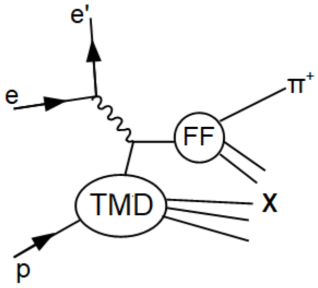
- GPDs are **collinear** distributions like PDFs: the initial transverse momentum of partons in the hadron is not integrated over. This lack of precision falls short for several important processes, among others
  1. low-q_T Drell-Yann and semi-inclusive DIS
  2. high-energy factorization, notably DIS

  $\rightarrow$ requires a more general theorem than collinear factorization: **TMD factorization** **[Collins, Soper, Sterman, 1985]**: for instance collinear factorization at low q_T diverges, whereas TMD factorization allows to resum $\log(q_T)$ contributions.
- The transverse momentum of partons is NOT the Fourier transform of the impact parameter.

Naive factorization of SIDIS

$$\int \mathrm{d}^2 k_{1T}^2 \mathrm{d}^2 k_{2T}^2 \, TMD\ PDF(x, k_{1T})\, TMD\ FF(z, zk_{2T})\, \delta^{(2)}(k_{1T} + q_T - k_{2T}) \otimes \text{hard scattering} \tag{13}$$

- Aside from the ordinary UV divergences, there are also "light-cone" or "rapidity" divergences linked to gluons moving with infinite rapidity in the direction opposite the hadron.
- Soft gluons must be taken into account by an additional factor.



[S. Diehl et al, 2022]

GPDs and TMDs: the bare minimum

TMD factorization of SIDIS **[Collins, 2011]**, **[Aybat, Rogers, 2011]**

$$\int \mathrm{d}^2 k_{1T}^2 \mathrm{d}^2 k_{2T}^2 \, \mathit{TMD\ PDF}(x, k_{1T}; \mu, \zeta_F) \, \mathit{TMD\ FF}(z, z k_{2T}; \mu, \zeta_D) \, \delta^{(2)}(k_{1T} + q_T - k_{2T}) \otimes \text{hard scattering} + Y(Q, q_T) + \mathcal{O}(\Lambda/Q)^a \tag{14}$$

μ is the ordinary renormalization scale that regulate UV divergences, and $\zeta_{F,D}$ depend on an arbitrary rapidity scale to cure the light-cone divergences. $Y(Q, q_T)$ accounts for the large q_T behavior where collinear factorization is relevant. Schematically,

$$\mathit{TMD\ PDF}(x, b_T; \mu, \zeta) = F^{\text{unsub}}(x, b_T; \mu) S(b_T, \zeta) \tag{15}$$

where a remaining rapidity divergence is cancelled by the interplay of the soft and unsubtracted terms. b_T **is the Fourier transform of** q_T**, not the impact parameter!**

GPDs and TMDs: the bare minimum

The UV divergence in both collinear and TMD factorizations can be solved by introducing the usual dimnsional regulator and its associated μ scale in $\overline{MS}$, giving rise to renormalization-group equations (RGEs):

- For GPDs, the evolution equation generalizes the DGLAP limit at $\xi \to 0$, and reproduces the ERBL equation at $\xi \to 1$ **[Müller et al, 1994]**

$$\frac{\mathrm{d}}{\mathrm{d}\log(\mu)} H(x,\xi,t,\mu) = \int_{-1}^{1} \frac{\mathrm{d}y}{y} K\left(\frac{x}{y}, \frac{\xi}{x}, \alpha_s(\mu)\right) H(y,\xi,t,\mu) \tag{16}$$

- TMD PDFs have two scales for UV and rapidity divergences

$$\frac{\mathrm{d}\log F(x,b_T;\mu,\zeta)}{\mathrm{d}\log\sqrt{\zeta}} = K(b_T,\mu)\,, \quad \frac{\mathrm{d}\log F(x,b_T;\mu,\zeta)}{\mathrm{d}\log\mu} = \gamma(\mu,\zeta) \tag{17}$$

GPDs and TMDs: the bare minimum

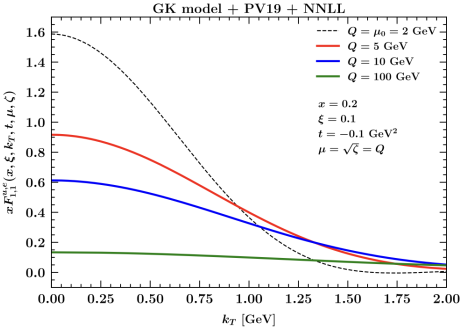
At small b_T (large q_T), TMDs can be computed from collinear factorization

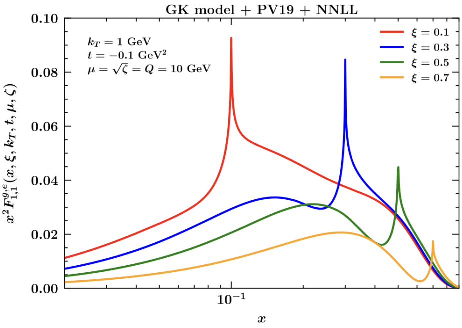
$$F(x, b_T; \mu, \zeta) = \int_x^1 \frac{\mathrm{d}y}{y} C\left(\frac{x}{y}, b_T; \zeta, \mu\right) f(x, \mu) + \mathcal{O}(\Lambda b_T)^a \tag{18}$$

One requires a consistent matching procedure to relate the perturbative calculation at large q_T to a non-perturbative parametrization at small q_T **[Collins, Soper, 1982]**.

GPDs and TMDs: the bare minimum

What about GTMDs? Combine both the picture in impact parameter space and transverse momentum dependence, closely related to the Wigner distribution of a parton in a hadron. At small b_T, GTMDs can be perturbatively computed from GPDs, and introducing non-perturbative ingredients from small b_T TMDs gives a sense of what a GTMD might look like **[Bertone, 2022]**.





Outlook

- GPDs and TMDs: the bare minimum
- Phenomenology of GPDs

Phenomenology of GPDs

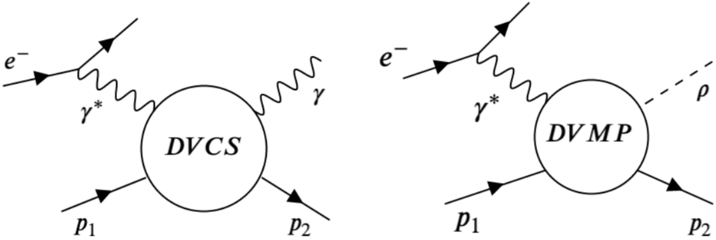
- **PARTONS** (PARton Tomography Of Nucleon Software – https://partons.cea.fr/) software framework for the **phenomenology of 3D hadron structure** (GPDs and TMDs) **[Berthou et al, 2018]**

- Developed since **2012**, **open source** and readily available in a virtual machine environment. Users can run **XML scenarios** to compute directly observables from already implemented GPD models, or develop their own modules in C++. **Plenty of examples and documentation! GPD models** (GK, VGG, Vinnikov, ...), **evolution**, **observables** (DVCS, TCS, DVMP, ...), **neural network fits of CFFs**, **event generator EpIC**, ...
- See also **Gepard** (https://gepard.phy.hr/))

Phenomenology of GPDs

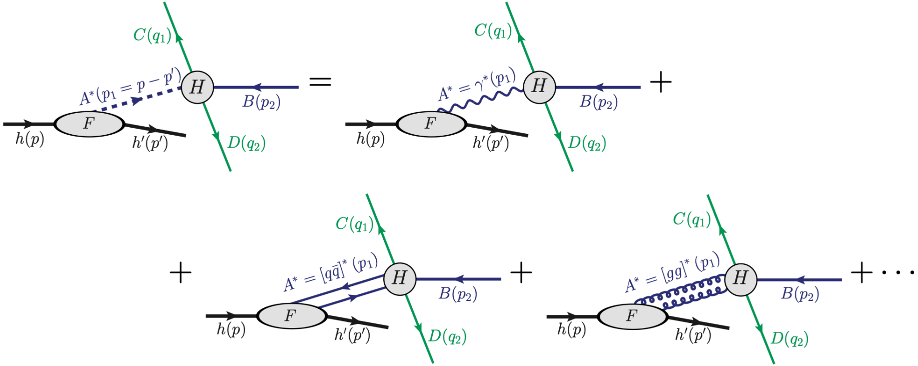
Experimental access to GPDs has first been envisioned through deeply virtual Compton scattering (DVCS) and deeply virtual meson production (DVMP). DVCS has long been considered as a golden channel as DVMP requires an additional non-perturbative object (DA). However, DVCS interferes coherently with the Bethe-Heitler process, expressed purely in terms of EFFs:

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x_B \mathrm{d}t \mathrm{d}Q^2 \mathrm{d}\phi \mathrm{d}\phi_S} = |\mathcal{T}_{DVCS}|^2 + |\mathcal{T}_{BH}|^2 + \mathcal{I} \quad (19)$$



Phenomenology of GPDs

Single Diffractive Hard Exclusive Process [Qiu, Yu, 2022]



Phenomenology of GPDs

The DVCS amplitude is parametrized in terms of Compton form factors (CFFs), which are related to GPDs thanks to a collinear factorization theorem

CFF convolution (leading twist) [Radyushkin, 1997], [Ji, Osborne, 1998], [Collins, Freund, 1999]

$$\mathcal{F}(\xi, t, Q^2) = \sum_a \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} C\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu)\right) \frac{F^a(x, \xi, t, \mu)}{|x|^{p_a}} \tag{20}$$

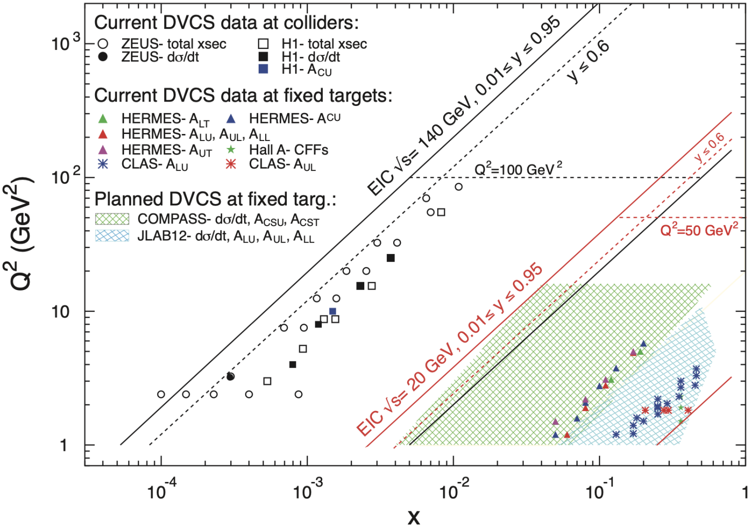
where $p_q = 0$ and $p_g = 1$.

At LO,

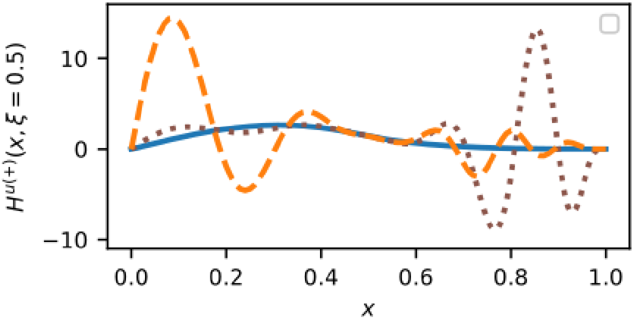
$$\mathrm{Im}\,\mathcal{H}(\xi, t, Q^2) = \pi \sum_q e_q^2 H^q(\xi, \xi, t, \mu^2 = Q^2) \tag{21}$$

The data has a smaller dimensionality than the distribution to extract. One has to rely on the knowledge of the scale dependence from perturbation theory to extract the GPD.

Phenomenology of GPDs



Let us play a game: construct models of GPDs designed to yield exactly the same CFF at a given scale at NLO, and see how well evolution separates them evolving from 1 to 100 GeV^2. [Bertone et al, 2021]



The NLO CFF generated by these three curves differ by 10^{-5}: they are indiscernable in experimental data $\rightarrow$ **shadow GPDs**

Phenomenology of GPDs

Similar issue for extraction of gravitational form factors. How to go beyond the issue?

- Collect more experimental data, especially those who either have several hard scales (DDVCS, multiple particle production ...) or an enhanced dependence on the hard scale (like pion exclusive photoproduction [Qiu, Yu, 2022]) $\rightarrow$ GPD Global analysis (CNF workshop `https://indico.jlab.org/event/713/`)
- Implement more theoretical constraints: end points behaviors, lattice QCD priors, positivity?
- DVCS remains a crucial process as it is probing a crucial region for the phenomenology of GPDs, and one that other methods seem to have trouble resolving!

Phenomenology of GPDs

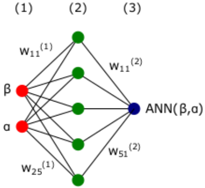
Similar issue for extraction of gravitational form factors. How to go beyond the issue?

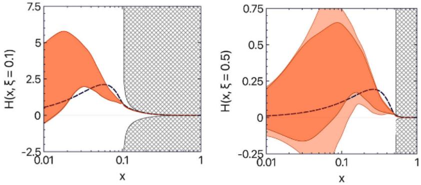
- Collect more experimental data, especially those who either have several hard scales (DDVCS, multiple particle production ...) or an enhanced dependence on the hard scale (like pion exclusive photoproduction **[Qiu, Yu, 2022]**) $\rightarrow$ GPD Global analysis (CNF workshop `https://indico.jlab.org/event/713/`)
- Implement more theoretical constraints: end points behaviors, lattice QCD priors, positivity?
- DVCS remains a crucial process as it is probing a crucial region for the phenomenology of GPDs, and one that other methods seem to have trouble resolving!

Neural network model of double distributions **[HD et al, 2022]**

- Enforces polynomiality by construction
- More flexible without the need of very large polynomial powers (precision issue for floating point computation)
- More flexible framework to implement positivity constraint: mock constraint

$$|H^q(x,\xi,t)| \leq \sqrt{f^q\left(\frac{x+\xi}{1+\xi}\right) f^q\left(\frac{x-\xi}{1-\xi}\right) \frac{1}{1-\xi^2}} \tag{22}$$





- Proof of concept – closure test :

Photon polarization asymmetry results

Definition

$$A_{\odot U} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} \propto \frac{\frac{L_0}{L}\sin\phi \frac{(1+\cos^2\theta)}{\sin(\theta)} \mathrm{Im}\mathcal{H}}{d\sigma_{BH}}$$

Experimentally:

$$A_{\odot U}(-t, E\gamma, M; \phi) = \frac{1}{P_b}\frac{N^+ - N^-}{N^+ + N^-}$$

- A sizeable asymmetry is measured, above the expected vanishing asymmetry predicted for BH.
- Results have been compared to 2 model predictions:
  1. VGG model
  2. GK model
- The size of the asymmetry is well reproduced by both models, giving a hint for the universality of GPDs.

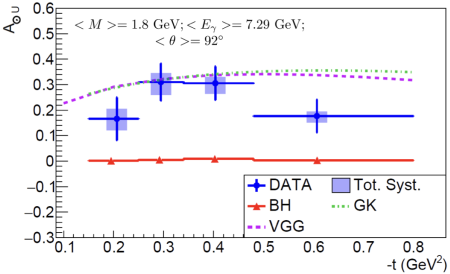
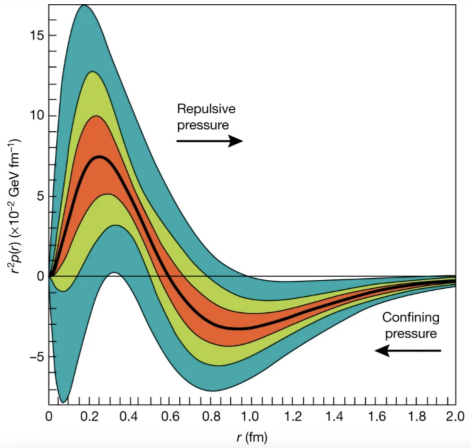


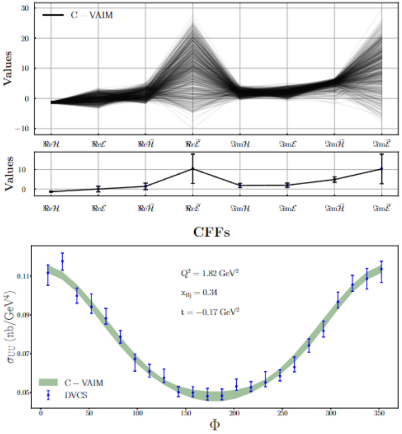
Figure in First Measurement of Timelike Compton Scattering, P. Chatagnon *et al.* (CLAS Collaboration), Phys. Rev. Lett. 127, 262501 (2021)

Jefferson Lab clas

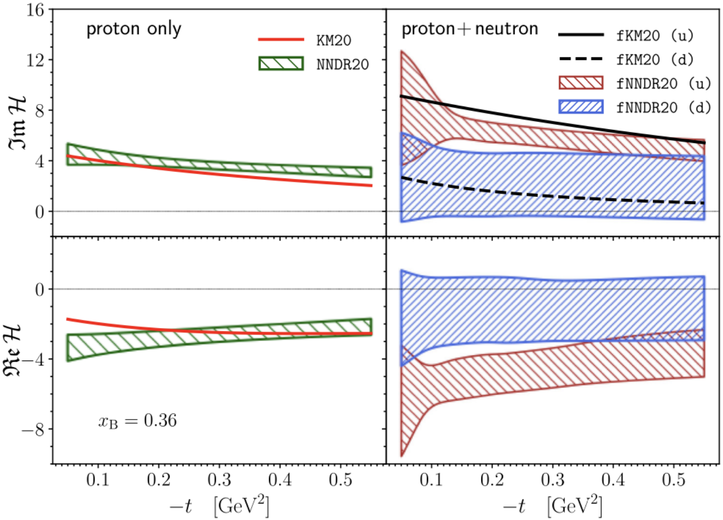
Prediction of CFFs



[Burkert, Elouadrhiri, Girod, 2018]

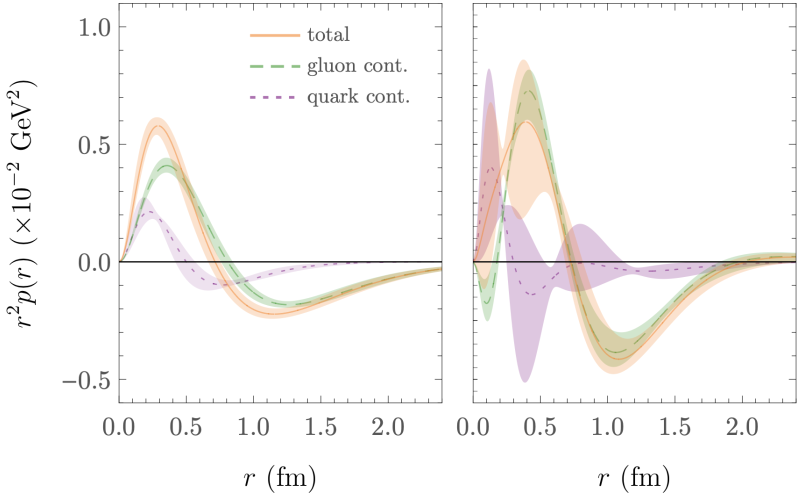


[Almaeen et al]



[Cuic, Kumericki, Schafer, 2020]

GPDs at small ξ – t dependence



From **[Shanahan, Detmold, 2019]**
Difference on the uncertainty of the extraction of GFFs from lattice data depending on the t-parametrization: tripole on the left, z-expansion on the right.

Phenomenology of GPDs

Many interesting phenomenological aspects of GPDs are encompassed in the limit $\xi \to 0$, or t-dependent PDF. **[Moffat et al, 2022]** demonstrated that the type of shadow GPDs introduced so far do not contribute significantly to the uncertainty at small x and ξ.

- When $x \gg \xi$, negligible asymmetry between incoming $(x-\xi)$ and outgoing $(x+\xi)$ parton longitudinal momentum fraction $\to$ **smooth limit of GPDs**

$$H(x,\xi,t,\mu^2) \approx H(x,0,t,\mu^2) \quad \text{for } x \gg \xi \,. \tag{23}$$

- Extraction of the t-dependent PDF $H(x,0,t,\mu^2)$?
  - Forward limit gives ordinary PDFs

$$H(x,0,t=0,\mu^2) = f(x,\mu^2) \,. \tag{24}$$

  - First Mellin moment gives elastic form factors

$$\int \mathrm{d}x \, H(x,0,t) = F_1(t) \,. \tag{25}$$

Phenomenology of GPDs

- x-dependence at $\xi = 0$ computed on the lattice from the **non-local euclidean matrix elements** (LaMET **[Ji, 2013]**, short-distance factorization **[Radyushkin, 2017]**, ...)
- Higher order Mellin moments of GPDs (**generalized form factors**) computed on the lattice with local operators (limited by operator mixing to the first 3)
- Experimental data from exclusive processes: **most of these data have a particular sensitivity to the region $x \approx \xi$, so precisely not $x \gg \xi$!**

- How can one leverage the experimental data to constrain t-dependent PDFs?

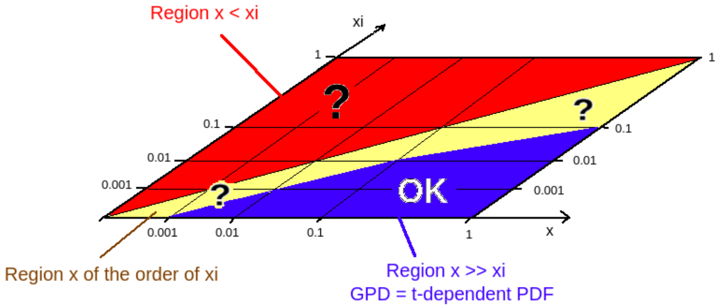
Phenomenology of GPDs

- Why don't we just assume

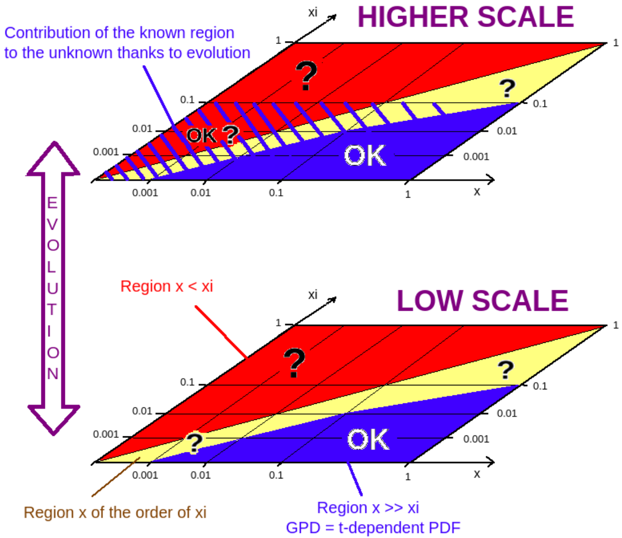
$$H(x, \xi, t, \mu^2) \approx H(x, 0, t, \mu^2) \ \text{ for } \xi \ll 1 \text{ even if } x \approx \xi? \tag{26}$$

Because significant asymmetry between incoming and outgoing $(x + \xi \gg x - \xi)$ parton momentum means very different dynamics, materialized *e.g.* by a very different behavior under evolution.

No reason for the ξ dependence to be negligible even at very small ξ. Skewness ratios $\frac{H(x,x)}{H(x,0)}$ as large as 1.6 have been advocated at small x. **[Frankfurt et al, 1998]** **[Shuvaev et al, 1999]**



Phenomenology of GPDs

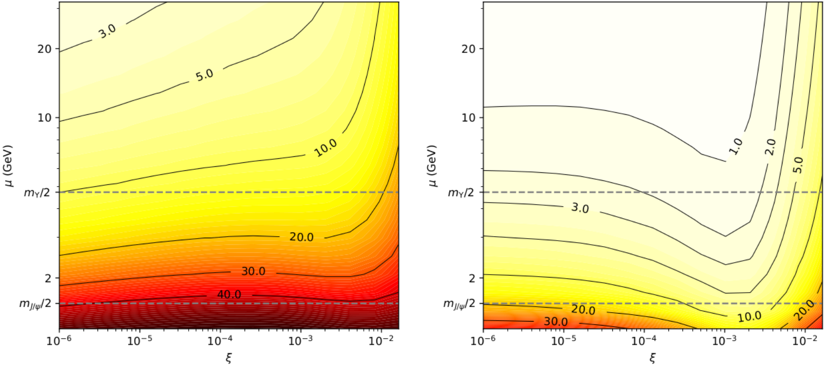


- Evolution displaces the GPD from the large x to the small x region
- Significant ξ dependence arises perturbatively in the small x and ξ region
- But how does it compare to the unknown ξ dependence at initial scale?

Obviously depends on the range of evolution, value of x and ξ, and profile of the known t-dependent PDF.

Phenomenology of GPDs

Example: working at $t = 0$, with the MMHT2014 PDF **[Harland-Lang et al, 2015]** at 1 GeV (**prior knowledge of t-dependent PDF**). We want to assess the dominance of the region $x \gg \xi$ at initial scale in the value of the GPD on the diagonal as scale increases.
Pessimistic assumption on unknown ξ dependence at $x = \xi$ for 1 GeV: 60%.



Uncertainty on the diagonal of the light sea quarks (left) and gluons (right) depending on $x = \xi$ and μ.
Stronger μ effect for gluons, divergence of PDFs at small x visible.

[HD, Winn, Bertone, hep-ph:2302.07861 (2023)]

Phenomenology of GPDs

Generating perturbatively the ξ dependence offers a well defined functional space for GPDs at small ξ which verifies the main theoretical constraints (polynomiality of Mellin moments, positivity, limits, ...)

By subtracting the degree of freedom of the ξ dependence, we have regularized the deconvolution problem, and we have an evaluation of the uncertainty associated to this regularization.

Better modelling: include missing higher order corrections by varying the scales $\rightarrow$ use higher order evolution

And it only works at small ξ!

Conclusion

- 3D hadron structure is a difficult but extremely dynamical field!
- New experimental data, future facilities, more refined statistical treatments, nescent first-principles calculations set the stage for compelling new developments
- Increasing need for benchmarking and rationalization of the efforts!

Thank you for your attention!

Deeply virtual Compton scattering and the structure of hadrons

- Remarkably, GPDs allow access to gravitational form factors (GFFs) of the **energy-momentum tensor (EMT)** [Ji, 1997] defined for parton of type a

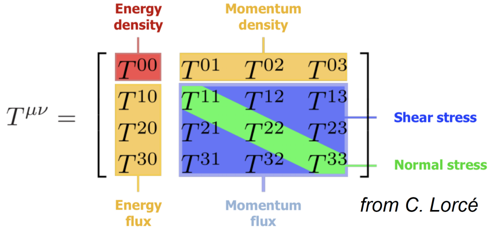
Gravitational form factors [Lorcé et al, 2017]

$$\langle p', s' | T_a^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \Bigg\{ \frac{P^\mu P^\nu}{M} A_a(t, \mu^2) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C_a(t, \mu^2) + M \eta^{\mu\nu} \bar{C}_a(t, \mu^2)$$
$$+ \frac{P^{\{\mu} i\sigma^{\nu\}\rho} \Delta_\rho}{4M} \left[A_a(t, \mu^2) + B_a(t, \mu^2) \right] + \frac{P^{[\mu} i\sigma^{\nu]\rho} \Delta_\rho}{4M} D_a(t, \mu^2) \Bigg\} u(p, s) \tag{27}$$

where

$$\Delta = p' - p, \;\; t = \Delta^2, \;\; P = \frac{p + p'}{2} \tag{28}$$

Deeply virtual Compton scattering and the structure of hadrons



from C. Lorcé

In the Breit frame ($\vec{P} = 0$, $t = -\vec{\Delta}^2$), radial distributions of energy and momentum in the proton are described by Fourier transforms of the **GFFs** w.r.t. variable $\vec{\Delta}$ **[Polyakov, 2003]**.

- Example of such distribution: radial pressure anisotropy profile

$$s_a(r, \mu^2) = -\frac{4M}{r^2} \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \frac{t^{-1/2}}{M^2} \frac{\mathrm{d}^2}{\mathrm{d}t^2} \left[t^{5/2} C_a(t, \mu^2) \right] \tag{29}$$

- This pressure profile can be extracted from **GPDs** thanks to *e.g.* for quarks

$$\int_{-1}^{1} \mathrm{d}x\, x\, H^q(x, \xi, t, \mu^2) = A_q(t, \mu^2) + 4\xi^2 C_q(t, \mu^2) \tag{30}$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, E^q(x, \xi, t, \mu^2) = B_q(t, \mu^2) - 4\xi^2 C_q(t, \mu^2) \tag{31}$$

Extraction of GFFs

- At this stage, we don't need to fully extract the GPDs H or E to conveniently access the GFF $C_q(t,\mu^2)$. The **polynomiality property** gives that the GFF $C_q(t,\mu^2)$ only depends on the D-term via

$$\int_{-1}^{1} \mathrm{d}z\, zD^q(z,t,\mu^2) = 4C_q(t,\mu^2) \tag{32}$$

- The experimental data is sensitive to the D-term through the **subtraction constant** defined by the **dispersion relation** (see *e.g.* [Diehl, Ivanov, 2007])

DVCS dispersion relation

$$\mathcal{C}_H(t,Q^2) = \mathrm{Re}\,\mathcal{H}(\xi,t,Q^2) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi'\,\mathrm{Im}\,\mathcal{H}(\xi',t,Q^2)\left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right) \tag{33}$$

The subtraction constant $\mathcal{C}_H(t,Q^2)$ is a function of the D-term given at LO by

$$\mathcal{C}_H(t,Q^2) = 2\sum_q e_q^2 \int_{-1}^{1} \mathrm{d}z\, \frac{D^q(z,t,Q^2)}{1-z} \tag{34}$$

Extraction of GFFs

- How do we get from

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} \quad \text{to} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) \text{ ?} \tag{35}$$

- This is a prototype of the more complicated GPD extraction problem we will face later on. The known solution is through evolution.
- Let's expand the D-term on a basis of Gegenbauer polynomials

$$D^q(z,t,\mu^2) = (1-z^2) \sum_{\text{odd } n} d_n^q(t,\mu^2) \, C_n^{3/2}(z) \tag{36}$$

Then

GFF C_a extraction

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \quad \text{and} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) = \frac{4}{5} \, d_1(t,\mu^2) \tag{37}$$

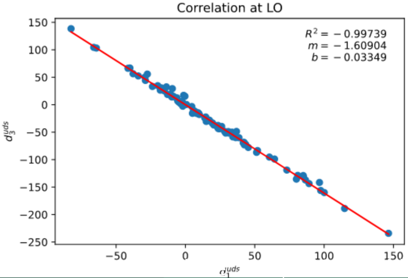
- Since the LO subtraction constant reads

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \tag{38}$$

if we allow d_3^q to be non-zero, at some scale μ_0^2, we can have $d_1^q(\mu_0^2) = -d_3^q(\mu_0^2)$, so a **vanishing subtraction constant, but non-zero GFF** $C_q(\mu_0^2)$. If the effect of evolution is not significant enough, these configurations are not ruled out and add a considerable uncertainty.

$$\boxed{d_1^{uds}(\mu_F^2) \qquad -0.5 \pm 1.2}$$

$$\longrightarrow \boxed{\begin{array}{ll} d_1^{uds}(\mu_F^2) & 11 \pm 25 \\ d_3^{uds}(\mu_F^2) & -11 \pm 26 \end{array}}$$



Deconvoluting a Compton form factor

- Question was raised 20 years ago. Evolution was proposed as a crucial element in **[Freund, 1999]**, but the question has remained essentially open.
- We show that GPDs exist which bring contributions to the LO and NLO CFF of only subleading order even under evolution. We call them **LO and NLO shadow GPDs**.

Definition of an NLO shadow GPD

For a given scale μ_0^2,

$$\forall \xi, \forall t, \; T^q_{NLO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0 \quad \text{and} \quad H^q(x, \xi = 0, t = 0, \mu_0^2) = 0 \tag{39}$$

$$\text{so for } Q^2 \text{ and } \mu^2 \text{ close enough to } \mu_0^2, \; T^q_{NLO}(Q^2, \mu^2) \otimes H^q(\mu^2) = \mathcal{O}(\alpha_s^2(\mu^2)) \tag{40}$$

- Let H^q be an NLO shadow GPD, and G^q be any GPD. Then G^q and $G^q + H^q$ have the same forward limit, and the same NLO CFF up to a numerically small and theoretically subleading contribution.

Shadow GPDs at leading order

- Complete details in **[Bertone, HD, Mezrag, Moutarde, Sznajder, Phys.Rev.D 103 (2021) 11, 114019]**
- We search for our shadow GPDs as simple **double distributions (DD)** $F(\beta, \alpha, \mu^2)$ to respect polynomiality, with a zero D-term. Then, thanks to dispersion relations, we can restrict ourselves to the imaginary part only $\mathrm{Im}\ T^q(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0$.
- We search our DD as a polynomial of order N in (β, α), characterised by $\sim N^2$ coefficients c_{mn}:

$$F(\beta, \alpha, \mu_0^2) = \sum_{m+n \leq N} c_{mn}\, \alpha^m \beta^n \tag{41}$$

Shadow GPDs at next-to-leading order

- **First study beyond leading order:** Apart from the **LO** part, the NLO CFF is composed of a **collinear part** (compensating the α_s^1 term resulting from the convolution of the LO coefficient function and the evoluted GPD) and a genuine **1-loop NLO** part.

$$\mathcal{H}^q(\xi, Q^2) = C_0^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_1^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_{coll}^q \otimes H^{q(+)}(\mu_0^2) \log\left(\frac{\mu^2}{Q^2}\right) \tag{42}$$

An explicit calculation of each term for our polynomial double distribution gives that

$$\mathrm{Im}\ T_{coll}^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto$$

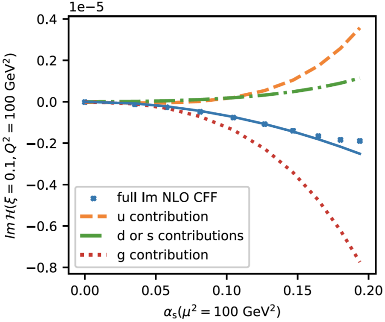
$$\alpha_s(\mu^2) \log\left(\frac{\mu^2}{Q^2}\right)\left[\left(\frac{3}{2} + \log\left(\frac{1-\xi}{2\xi}\right)\right) \mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N+1} \frac{k_w^{(coll)}}{(1+\xi)^w}\right] \tag{43}$$

and assuming $\mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) = 0$,

$$\mathrm{Im}\ T_1^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto \alpha_s(\mu^2)\left[\log\left(\frac{1-\xi}{2\xi}\right) \mathrm{Im}\ T_{coll}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N-1} \frac{k_w^{(1)}}{(1+\xi)^w}\right]$$

Shadow GPDs at next-to-leading order

- By linearity of both the CFF convolution and the evolution equation, we can evaluate separately the contribution to the CFF of a quark shadow NLO GPD under evolution.
- We probe the prediction of evolution as $\mathcal{O}(\alpha_s^2(\mu^2))$ with our previous NLO shadow GPD on a lever-arm in Q^2 of $[1, 100]$ GeV^2 (typical collider kinematics) using APFEL++ code.



- The fit by $\alpha_s^2(\mu^2)$ is very good up to values of α_s of the order of its $\overline{MS}$ values. For larger values, large logs and higher orders slightly change the picture.
- The numerical effect of evolution remains very small. For a GPD of order 1, the NLO CFF is only of order 10^{-5}.

Perspectives

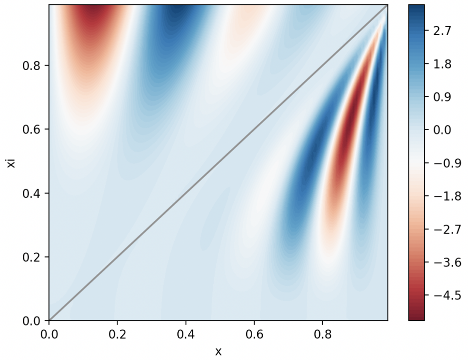
- Other exclusive processes can be expressed in terms of GPDs. Close parent to DVCS is **time-like Compton scattering** (TCS) **[Berger et al, 2002]**. Although its measurement will reduce the uncertainty, especially on $\mathrm{Re}\,\mathcal{H}$ **[Jlab proposal PR12-12-001]**, and produce a valuable check of the universality of the GPD formalism, the similar nature of its convolution (see **[Müller et al, 2012]**) makes it subject to the same shadow GPDs.
- **Deeply virtual meson production** (DVMP) **[Collins et al, 1997]** is also an important source of knowledge on GPDs, with currently a larger lever arm in Q^2. The process involves form factors of the general form

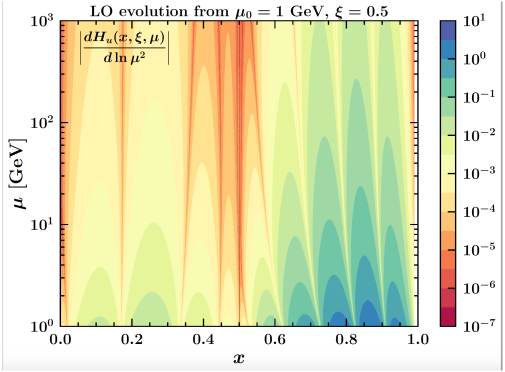
$$\mathcal{F}(\xi, t) = \int_0^1 \mathrm{d}u \int_{-1}^1 \frac{\mathrm{d}x}{\xi}\, \phi(u) T\left(\frac{x}{\xi}, u\right) F(x, \xi, t) \tag{45}$$

where $\phi(u)$ is the leading-twist meson distribution amplitude (DA).
- At LO, the GPD and DA parts of the integral factorize and shadow GPDs cancel the form factor.
- Situation at NLO remains to be clarified, it is foreseeable new shadow GPDs (dependent on the DA) could be generated also for this process.

Shadow GPDs at next-to-leading order

- Cancelling both terms gives rise to two additional systems with a linear number of equations. The first NLO shadow GPD is found for $N = 21$, and adding the condition that the DD vanishes at the edges of its support gives a first solution for $N = 25$ (see below).





Color plot of an NLO shadow GPD at initial scale 1 GeV^2, and its evolution for $\xi = 0.5$ up to 10^6 GeV^2 via APFEL++ and PARTONS **[Bertone]**.

Evolution of GPDs

GPD's dependence on scale is given by **renormalization group equations**. In the limit $\xi = 0$, usual DGLAP equation:

$$\frac{\mathrm{d}f^{q+}}{\mathrm{d}\mu}(x,\mu) = \frac{C_F\alpha_s(\mu)}{\pi\mu}\left\{\int_x^1 \mathrm{d}y\, \frac{f^{q+}(y,\mu) - f^{q+}(x,\mu)}{y-x}\left[1+\frac{x^2}{y^2}\right] + f^{q+}(x,\mu)\left[\frac{1}{2} + x + \log\left(\frac{(1-x)^2}{x}\right)\right]\right\} \tag{46}$$

But in the limit $x = \xi$:

$$\frac{\mathrm{d}H^{q+}}{\mathrm{d}\mu}(x,x,\mu) = \frac{C_F\alpha_s(\mu)}{\pi\mu}\left\{\int_x^1 \mathrm{d}y\, \frac{H^{q+}(y,x,\mu) - H^{q+}(x,x,\mu)}{y-x} + H^{q+}(x,x,\mu)\left[\frac{3}{2} + \log\left(\frac{1-x}{2x}\right)\right]\right\} \tag{47}$$

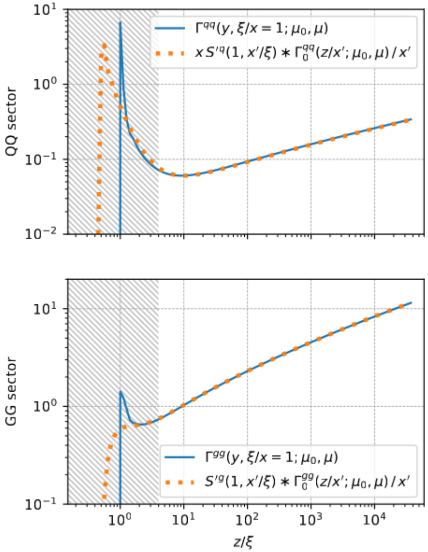
Assuming that GPD = t-dependent PDF at small ξ and $x \approx \xi$ is incompatible with evolution, which generates an intrinsic ξ dependence!

The Shuvaev transform

- Remarkably,

$$\Gamma^{ga}(x,\xi,z;\mu_0,\mu) \approx S^{g}(x,\xi,n) \star \mathcal{M}(n,y) \star \Gamma^{ga}(y,0,z;\mu_0,\mu) \quad (48)$$

The GPD evolution operator is nicely approximated by the Shuvaev transform of the Mellin transform of its limit for $\xi = 0$ (DGLAP evolution operator). The approximation is excellent as soon as $z > 4\xi$.



Perspectives

- Other exclusive processes can be expressed in terms of GPDs. Close parent to DVCS is **time-like Compton scattering** (TCS) **[Berger et al, 2002]**. Although its measurement will reduce the uncertainty, especially on $\mathrm{Re}\,\mathcal{H}$ **[Jlab proposal PR12-12-001]**, and produce a valuable check of the universality of the GPD formalism, the similar nature of its convolution (see **[Müller et al, 2012]**) makes it subject to the same shadow GPDs.
- **Deeply virtual meson production** (DVMP) **[Collins et al, 1997]** is also an important source of knowledge on GPDs, with currently a larger lever arm in Q^2. The process involves form factors of the general form

$$\mathcal{F}(\xi, t) = \int_0^1 \mathrm{d}u \int_{-1}^1 \frac{\mathrm{d}x}{\xi} \phi(u) T\left(\frac{x}{\xi}, u\right) F(x, \xi, t) \tag{49}$$

  where $\phi(u)$ is the leading-twist meson distribution amplitude (DA).
- At LO, the GPD and DA parts of the integral factorize and shadow GPDs cancel the form factor.
- Situation at NLO remains to be clarified, it is foreseeable new shadow GPDs (dependent on the DA) could be generated also for this process.