Triaxial deformation of nuclei probed by Lambda particle

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Grand challenges of hypernuclear physics

Interaction: "baryon-baryon interaction"

- **2 body interaction between baryons (Y: hyperon, N: nucleon)**
  - hyperon-nucleon (YN)
  - hyperon-hyperon (YY)

Major issues in hypernuclear physics

Structure: "many-body system of nucleons and hyperon"

- **Addition of hyperon as an impurity in (hyper)nuclei**
  - No Pauli exclusion between N and Y
  - YN interaction is different from NN

Structure changes
Unique structure, ⋯ etc.

Today: "deformation of hypernuclei"

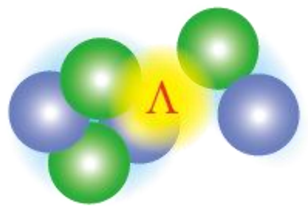
Structure of Λ hypernuclei

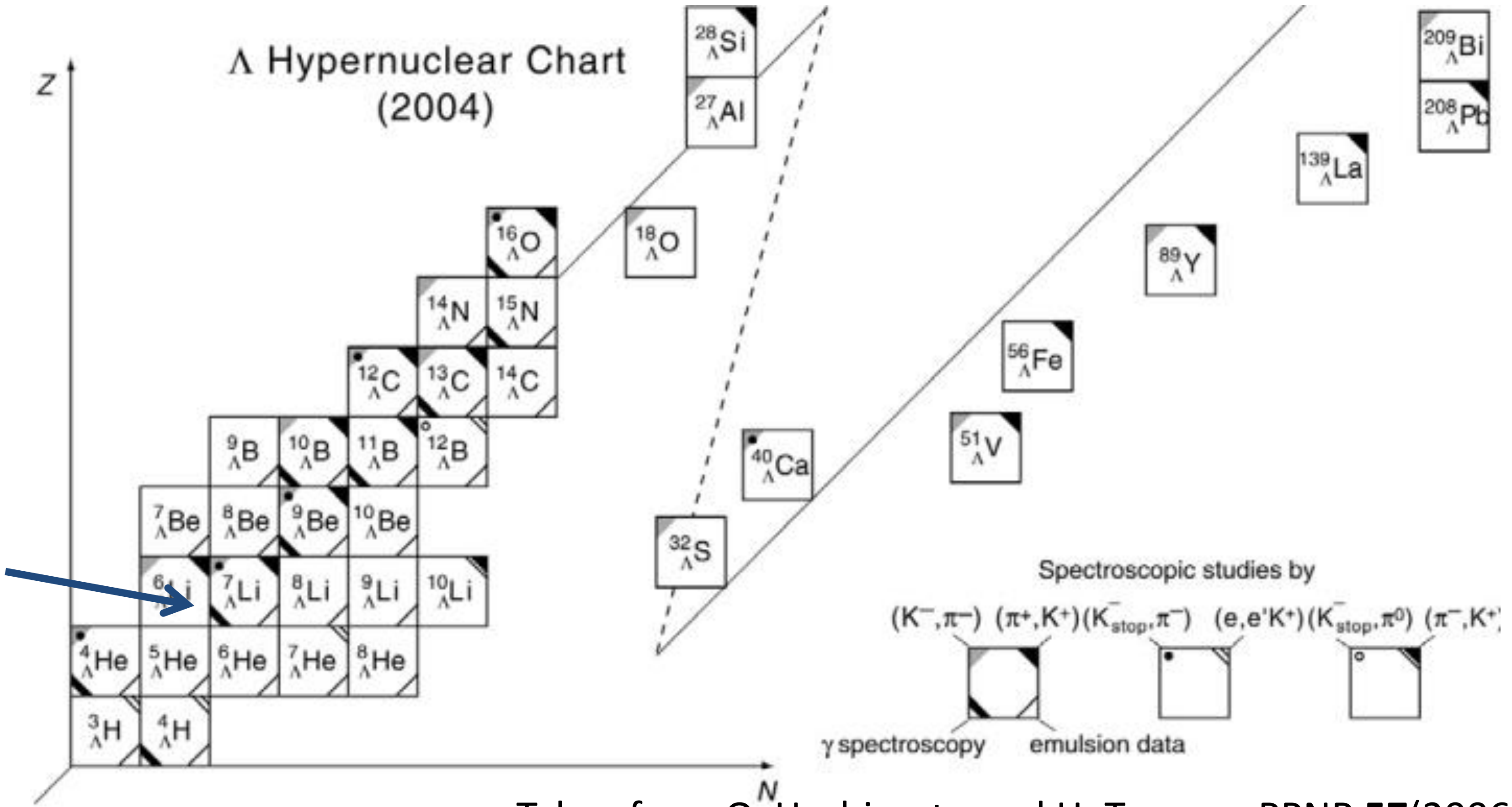
◆ Λ hypernuclei observed so far

- Concentrated in light Λ hypernuclei
- Most have well-developed cluster structure

Light Λ hypernuclei

Developed cluster

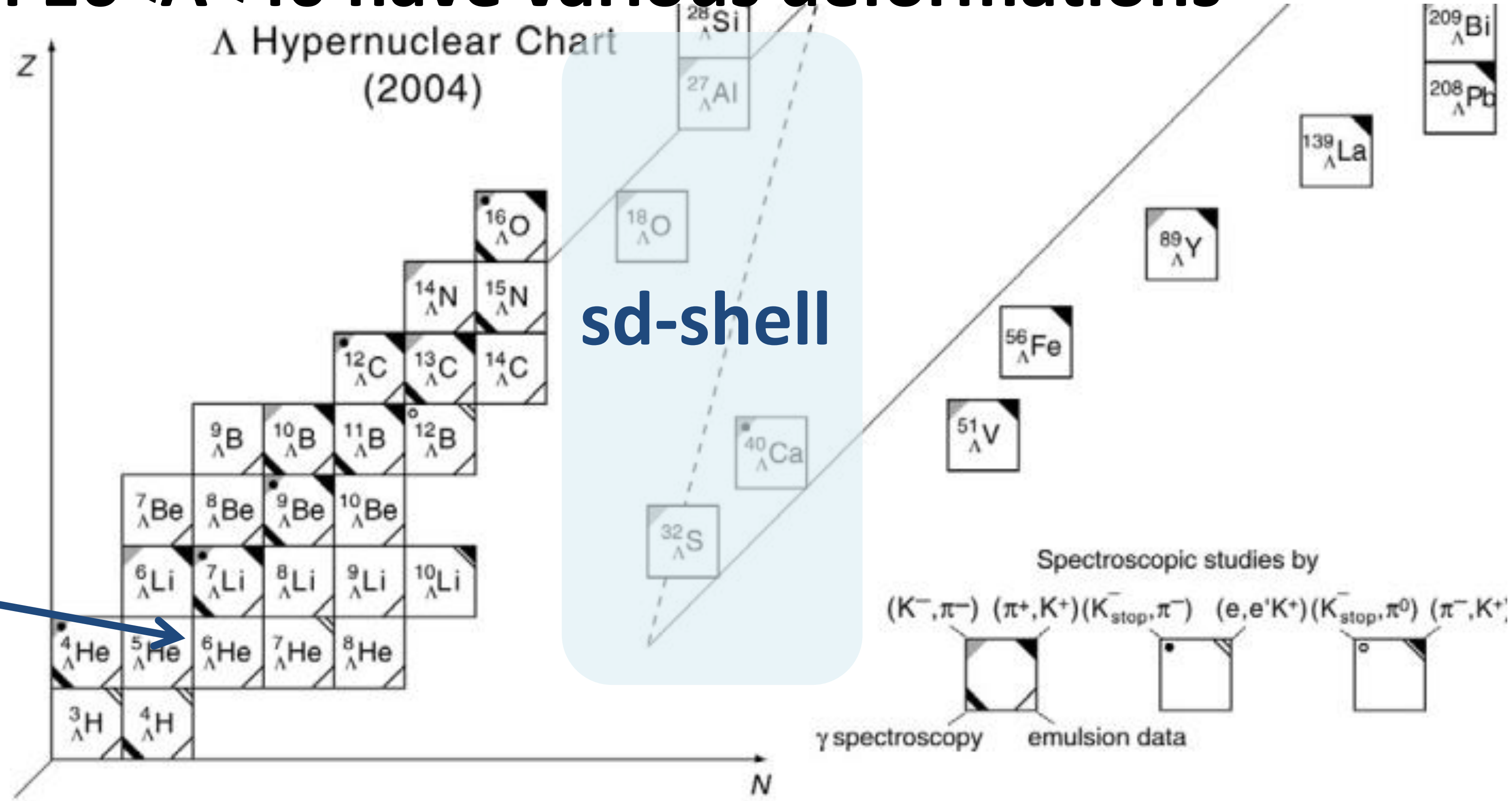




Taken from O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

Toward heavier and exotic Λ hypernuclei

- **Experiments at JLab, J-PARC, *etc*.**
  - **Heavier (*sd*-shell and more) hypernuclei can be produced**
  - **sd-shell nuclei with 16<A<40 have various deformations**



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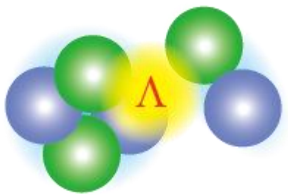

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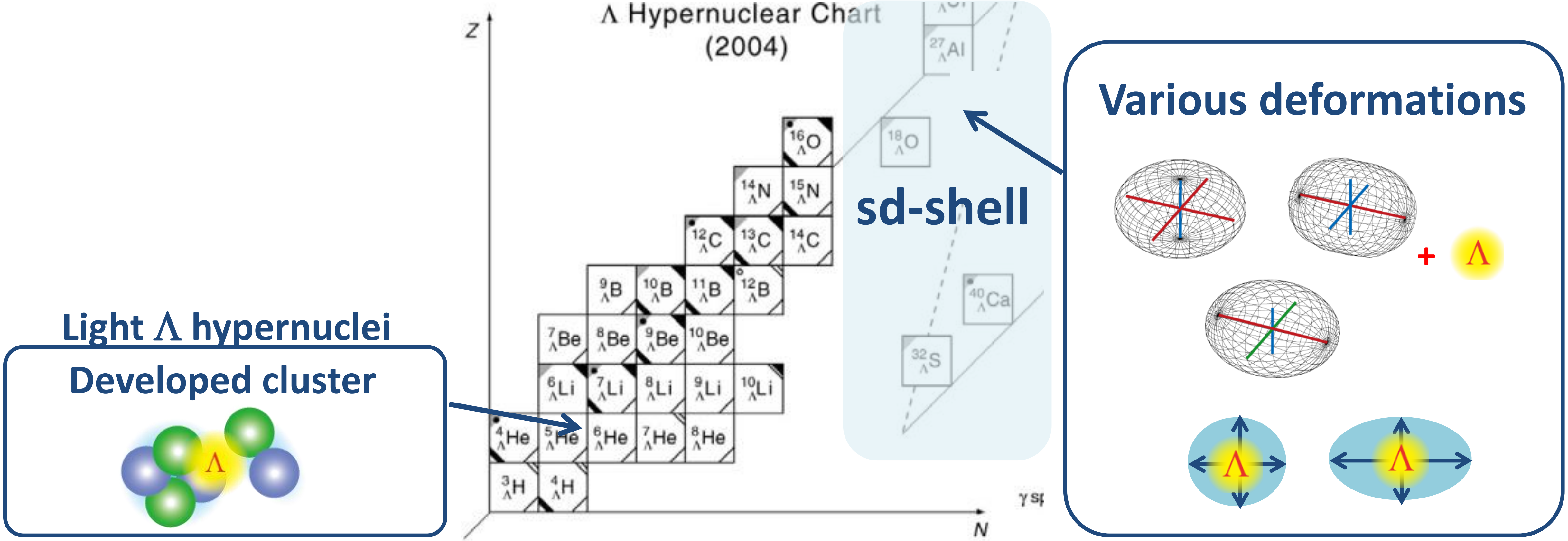
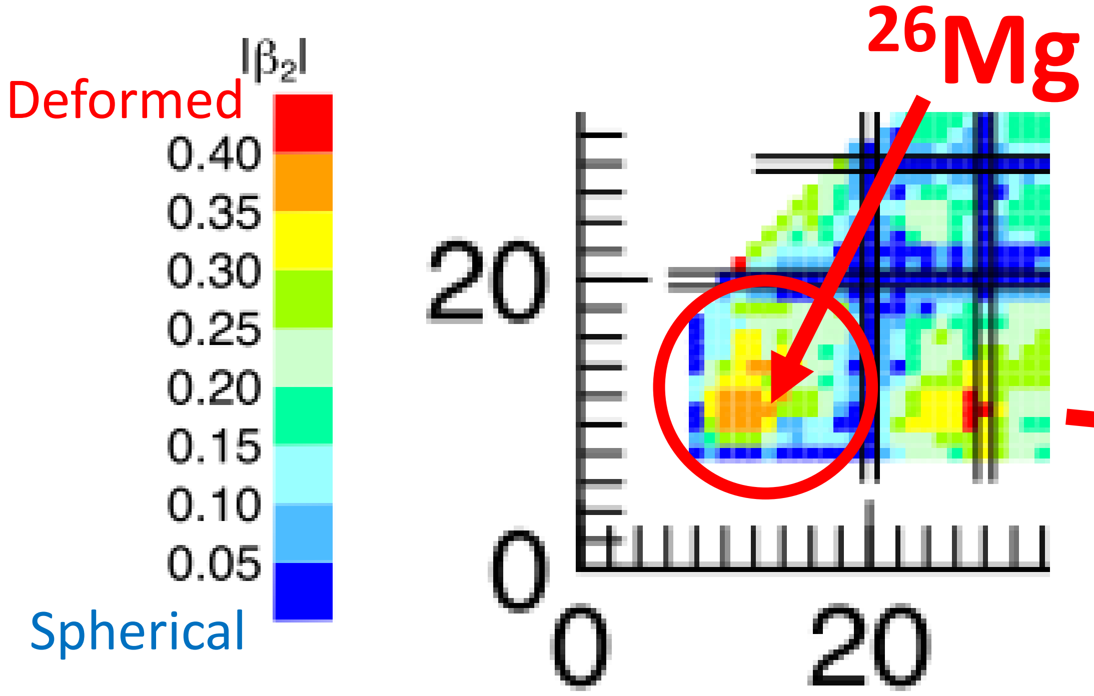


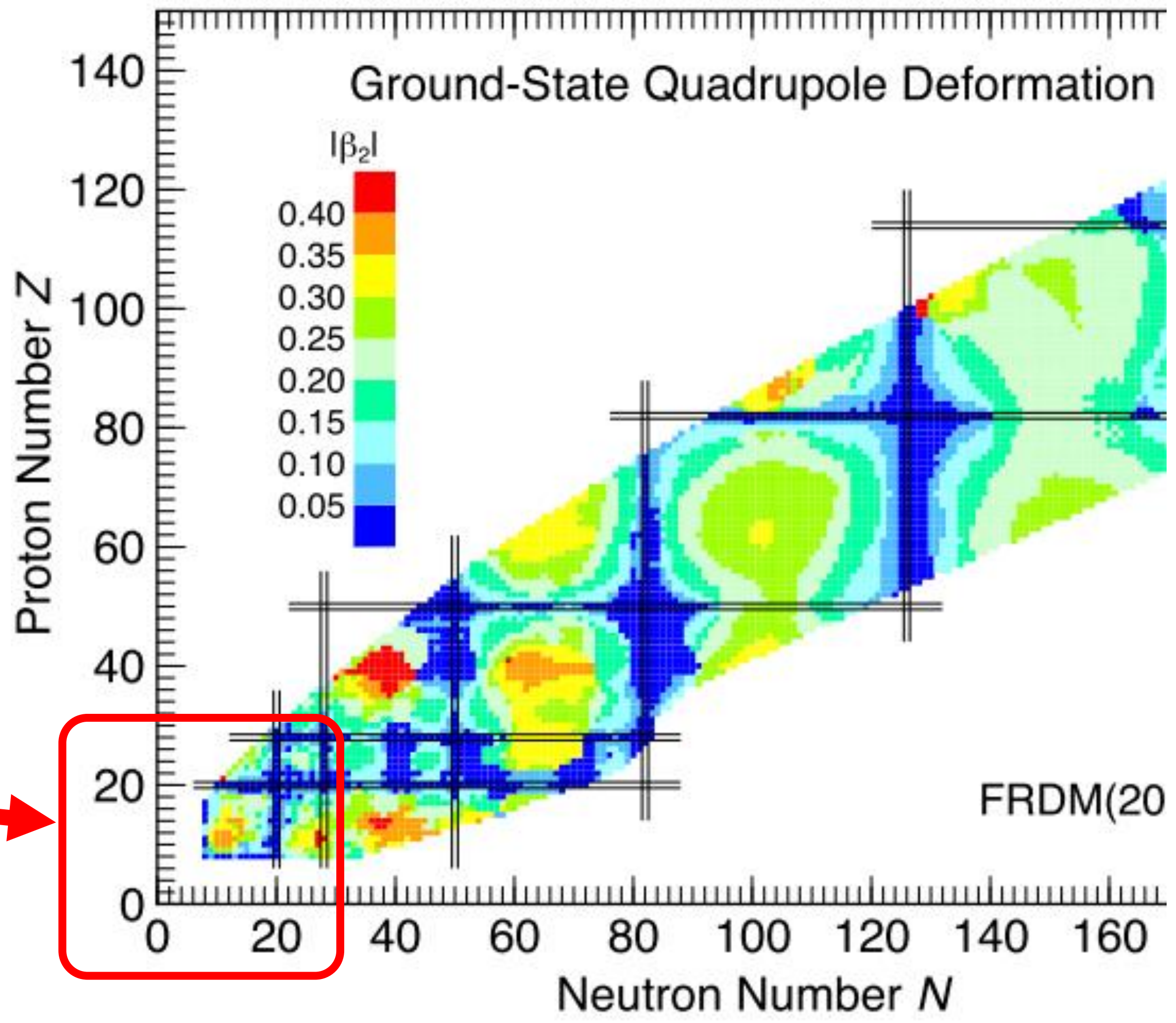
Structure study of deformed hypernuclei

Deformation of nuclei

◆Most of nuclei are deformed

- Deformation can be confirmed through B(E2) measurement
- Mid of *sd*-shell region with 20≲A≲30 is the lightest having large deformation



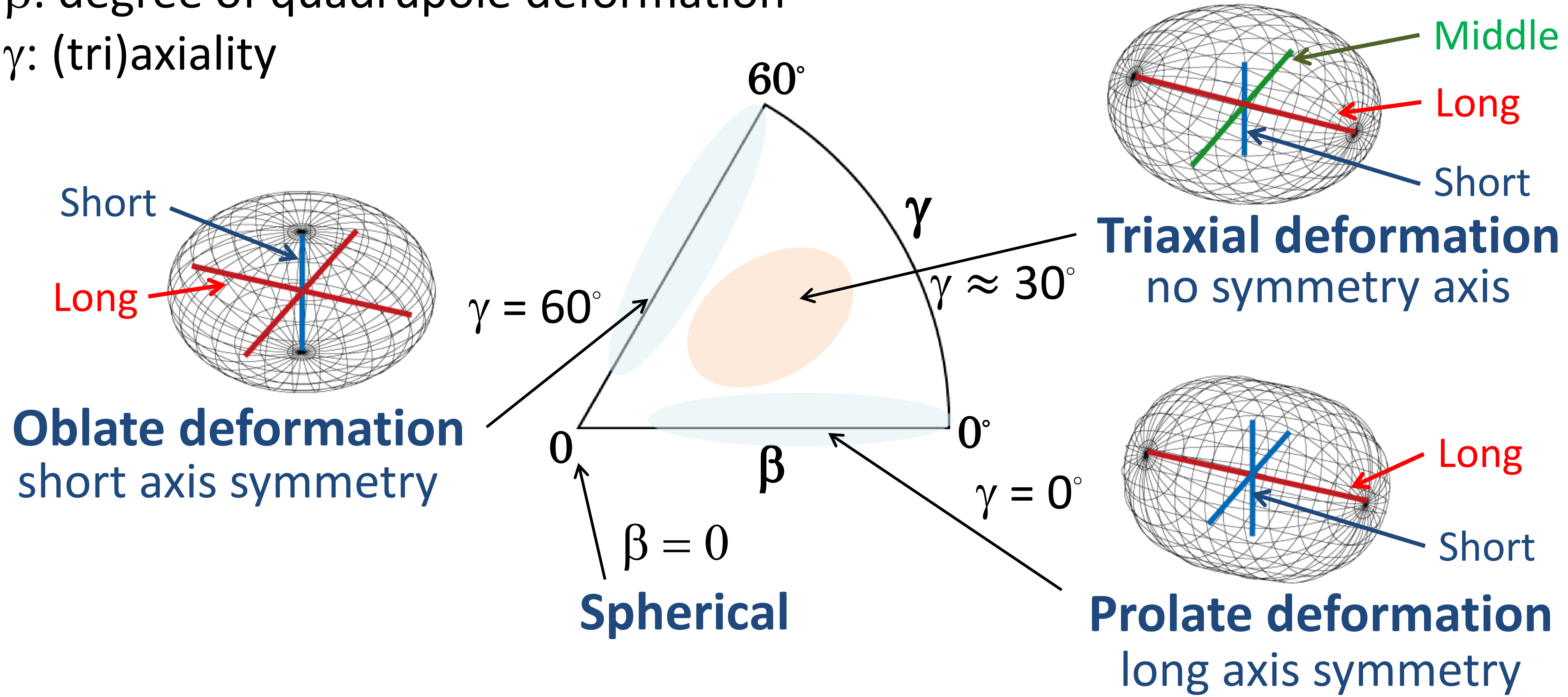


P. Möller, et al., Atomic Data and Nuclear Data Tables **109**(2016)

Description of nuclear deformation

◆Nuclear quadrupole deformation (β,γ)

- β: degree of quadrupole deformation
- γ: (tri)axiality



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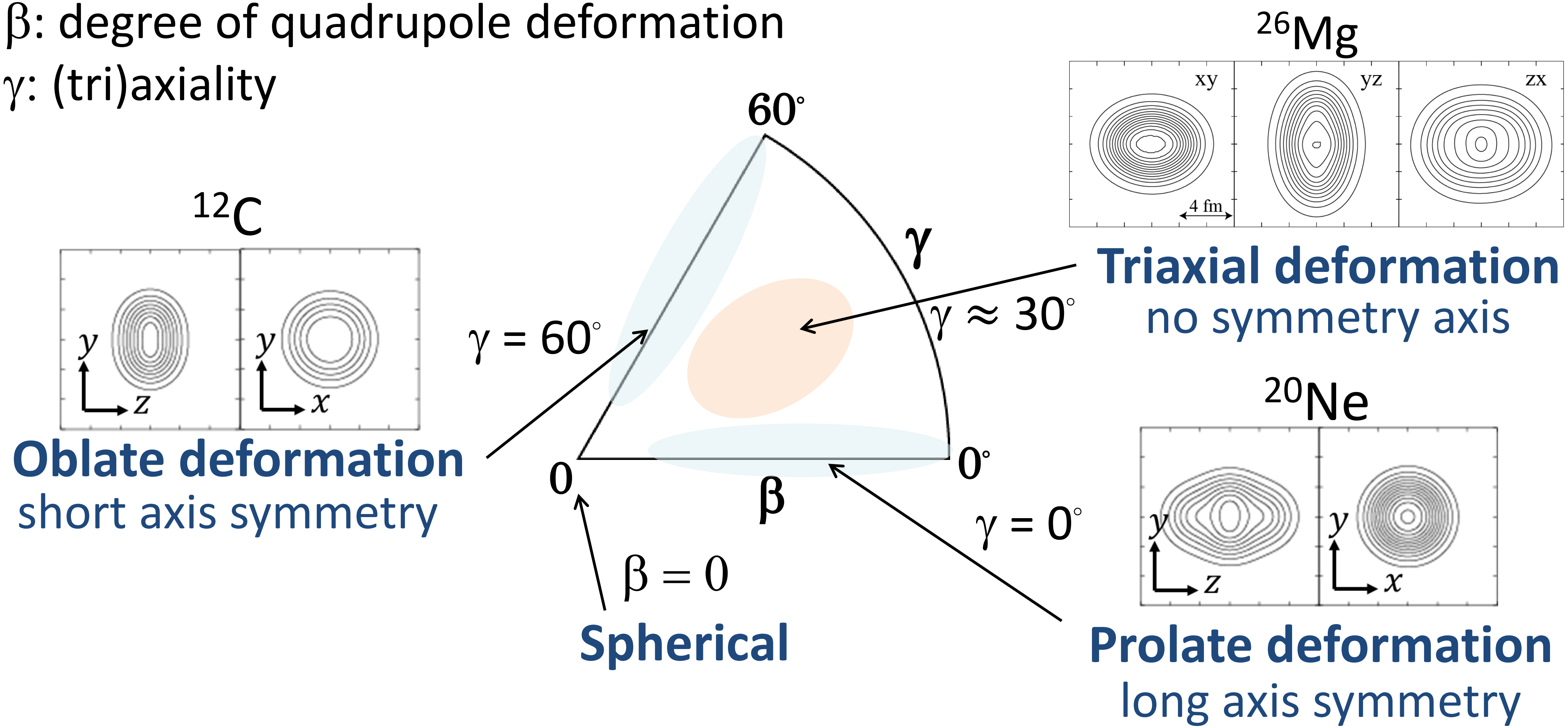
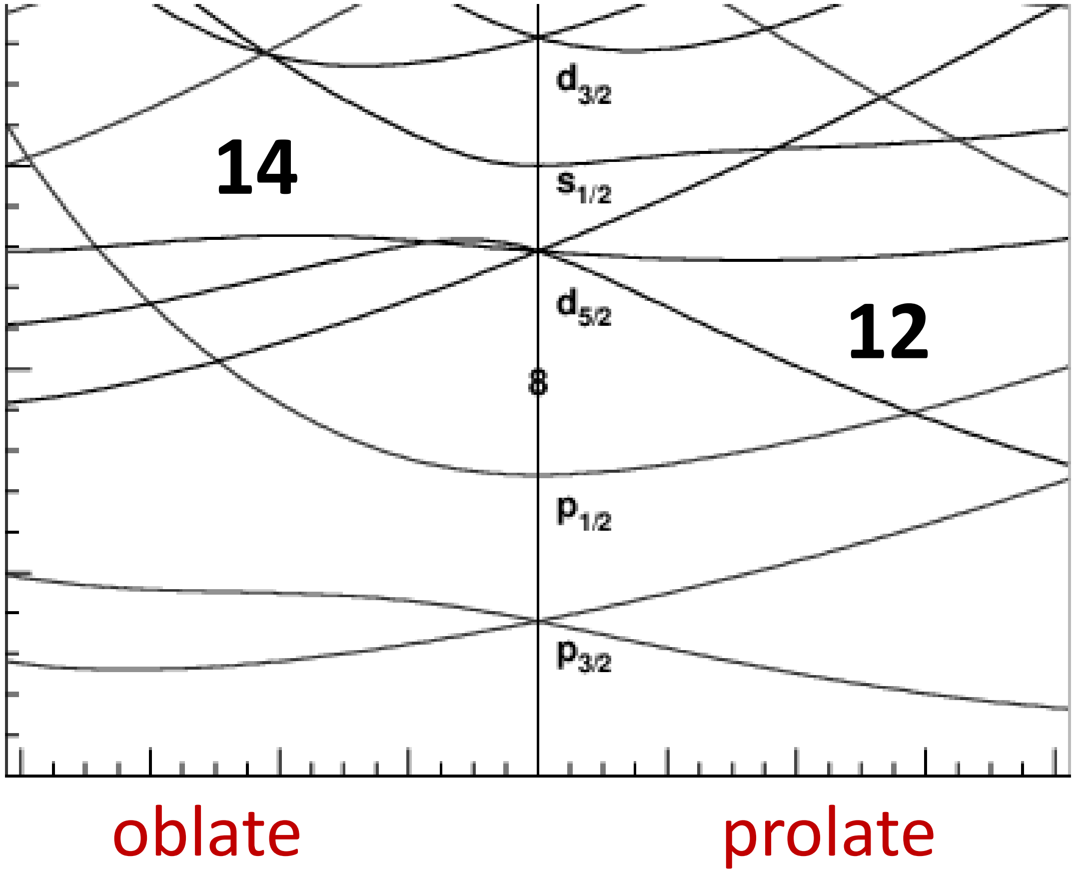


Deformation of ^{26}Mg

- Large B(E2): at least proton distribution is deformed
- In Nilsson diagram, large shell gaps appear with Z=12 in prolate side and N=14 with oblate deformations

Interest:
What is deformation property?
Triaxially deformed?

Nilsson diagram



Davydov model: rigid rotor with triaxial deformation

- Low-lying $K = 2^+$ band built on 2_2^+ state appears due to γ vibration
- Ratios of excitation energies and B(E2) values depend on γ

$E_x(2_2^+)/E_x(2_1^+)$

$B(E2; 2_2^+ \to 2_1^+)/B(E2; 2_2^+ \to 0_1^+)$

Axially symmetric

Triaxial

γ deg	$\dfrac{\varepsilon_2(2)}{\varepsilon_1(2)}$	$b(E2;\ 21 \to 0)$	$b(E2;\ 22 \to 0)$	$b(E2;\ 22 \to 21)$	$\dfrac{b(E2;\ 22 \to 21)}{b(E2;\ 22 \to 0)}$
0	∞	1.000	0	0	1.43
5	64.2	0.993	0.0074	0.011	1.49
10	15.9	0.972	0.028	0.051	1.70
15	6.85	0.947	0.053	0.143	2.70
20	3.73	0.933	0.067	0.357	5.35
22.5	2.93	0.937	0.0625	0.563	19.02
24	2.59	0.948	0.052	0.782	15.1
25	2.41	0.955	0.0425	0.865	20.6
26	2.26	0.968	0.0324	1.01	31.2
28	2.07	0.99	0.010	1.28	126
29	2.01	0.996	0.004	1.41	363
30	2.00	1.000	0	1.43	∞

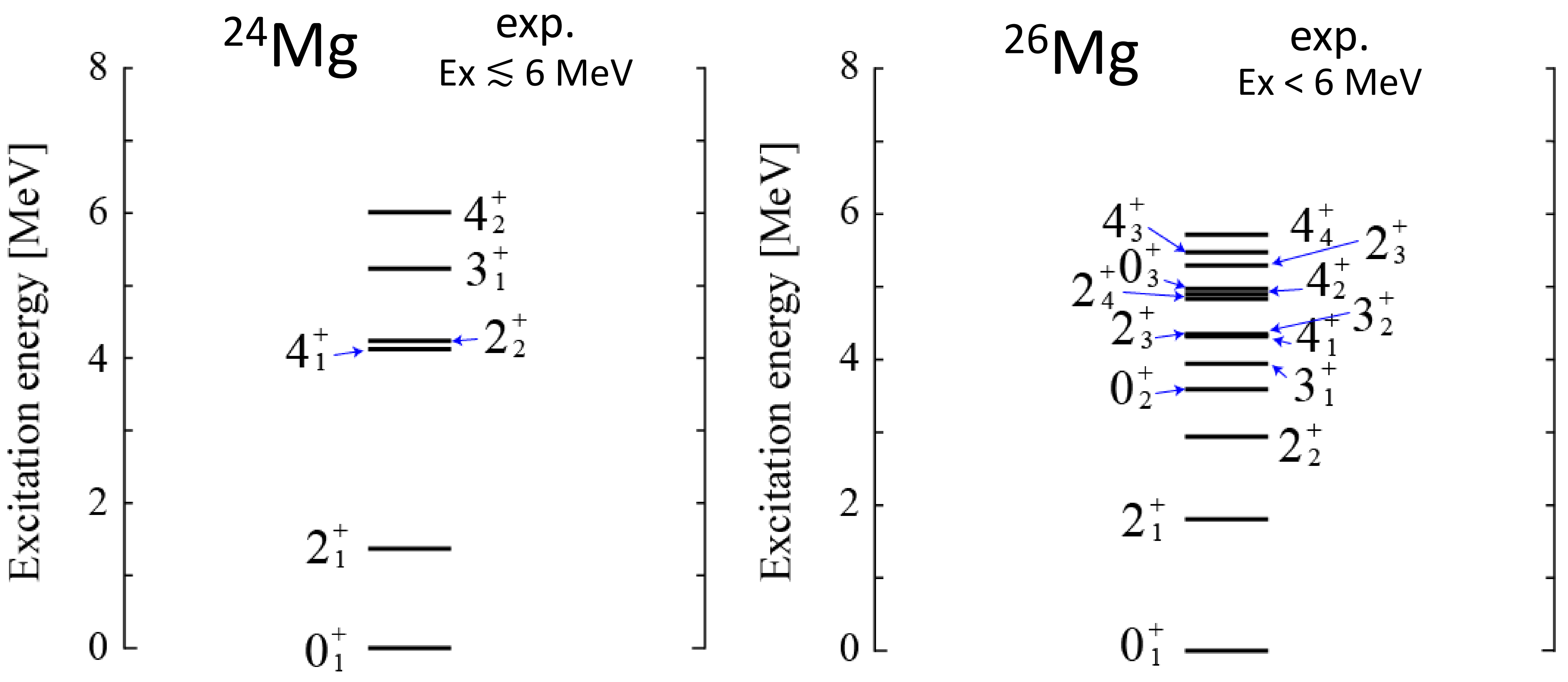
decreased

increased

Davydov and Filippov, Nucl. Phys.**8**,237(1958)

Situation of Mg nuclei

^{24}Mg and ^{26}Mg: candidates of triaxially deformed nuclei



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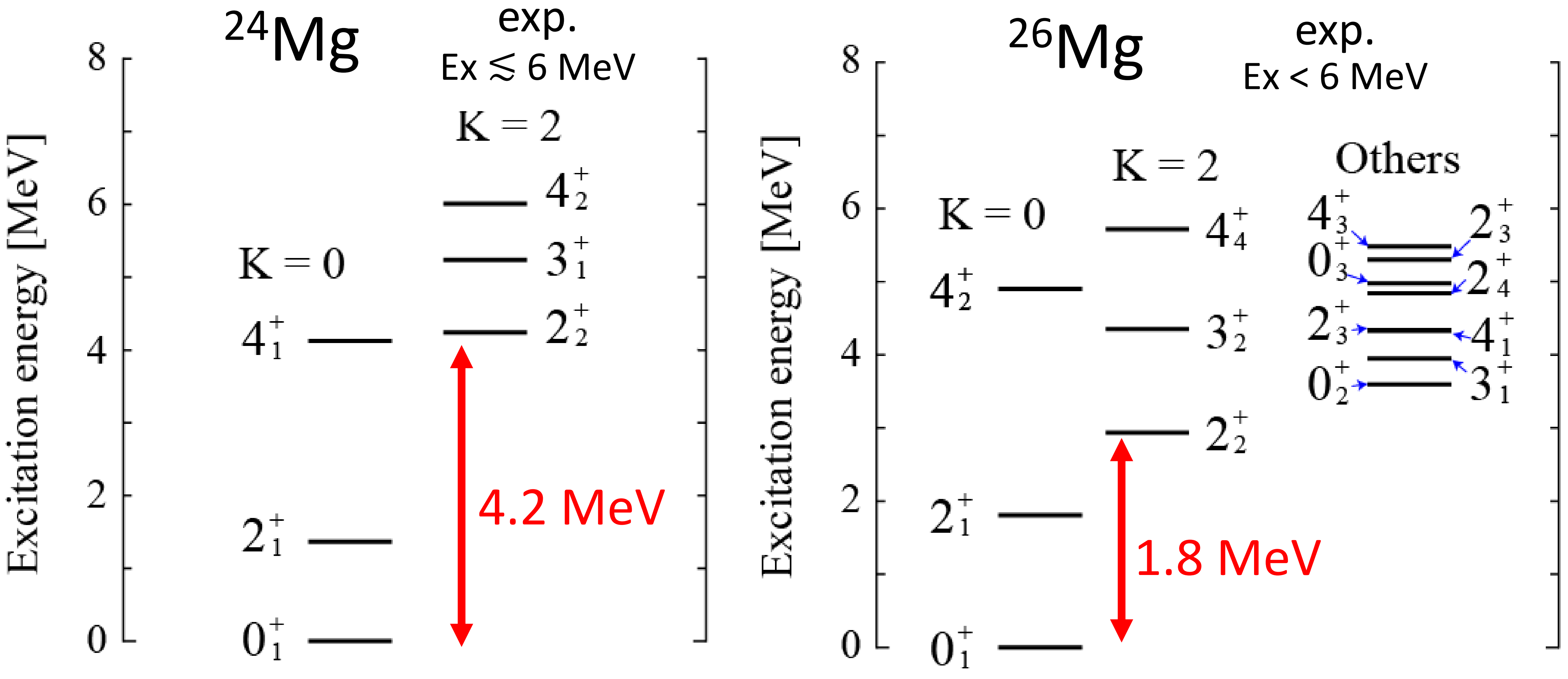


Comparison of exp. data with Davydov model

^{24}Mg and ^{26}Mg: candidates of triaxially deformed nuclei

^{24}Mg

$$\frac{E_x(2_2^+)}{E_x(2_1^+)} = \frac{4.24\ \mathrm{MeV}}{1.37\ \mathrm{MeV}} \fallingdotseq 3.1$$

→ $\gamma \fallingdotseq 22°$ by Davydov model

$$\frac{B(E2; 2_2^+ \to 2_1^+)}{B(E2; 2_2^+ \to 0_1^+)} \fallingdotseq 1.9$$

→ $\gamma \fallingdotseq 10°$ by Davydov model

^{26}Mg

$$\frac{E_x(2_2^+)}{E_x(2_1^+)} = \frac{2.94\ \mathrm{MeV}}{1.81\ \mathrm{MeV}} \fallingdotseq 1.6$$

→ Smaller than minimum value of Davydov model (2.0 at $\gamma = 30°$)

$$\frac{B(E2; 2_2^+ \to 2_1^+)}{B(E2; 2_2^+ \to 0_1^+)} \fallingdotseq 18$$

→ $\gamma \fallingdotseq 25°$ by Davydov model

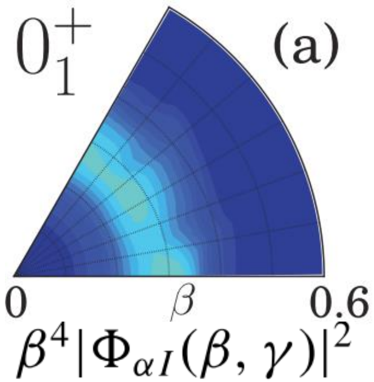
Recent theoretical studies on ^{26}Mg

Interest: deformation property because Z=12 (prolate) vs. N=14 (oblate)

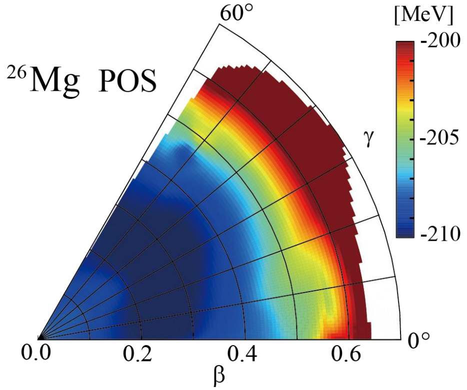
- **Coexistence of deformations/γ-softness of energy surface**

Terasaki et al. NPA**621**(1997)
Rodrıguez-Guzman et al. NPA**709** (2002)
Peru et al PRC**77** (2008)
Hinohara, Kanada-En'yo PRC**83** (2011)

CHB+LQRPA



AMD (present)



- **Difference of isospin character of 2_1^+ and 2_2^+**

Kanada-En’yo, et al., Phys. Rev. C**102**,014607(2020)

- **Triaxial deformation and importance of γ degree-of-freedom**
- **Difference from rigid-rotor model**

Short summary: deformation of ^{26}Mg

- Shell gap in Nilsson diagram at Z=12 (prolate) vs. N=14 (oblate)

Interest: $\left\{ \begin{array}{l} \text{What is deformation property of } ^{26}\text{Mg?} \\ \text{Candidate of triaxially deformed nuclei} \end{array} \right.$

- Large B(E2): proton distribution is deformed
  - No information on neutron distribution and triaxiality

- Comparison with theoretical studies
  - Low-lying 2_2^+ is a sign of triaxial deformation in rigid rotor model
  - Difference from rigid rotor: γ-softness, isospin character

To identify triaxial deformation using a Λ particle as a probe

What is expected in deformed Λ hypernuclei

- **Deformation change**
  - Λ particle can change nuclear deformation
- **Difference of B_Λ depending on nuclear deformation**
  - Energy shifts in excitation spectra
- **Coupling of Λ to deformed nuclei shows unique structure**
  - For example, rotational band, mixing of configuration, … etc.

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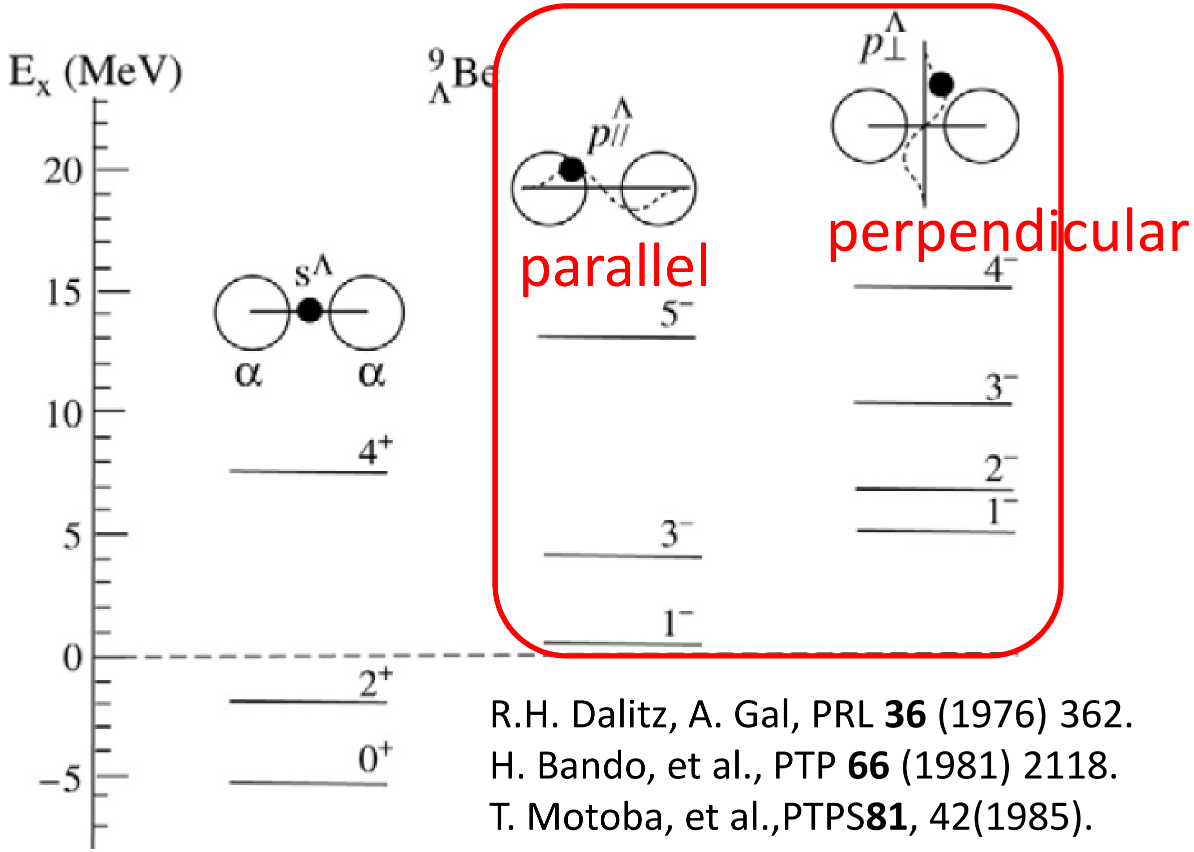
Coupling of Λ in *p*-orbit: *p*-states of $^{9}_{\Lambda}$Be

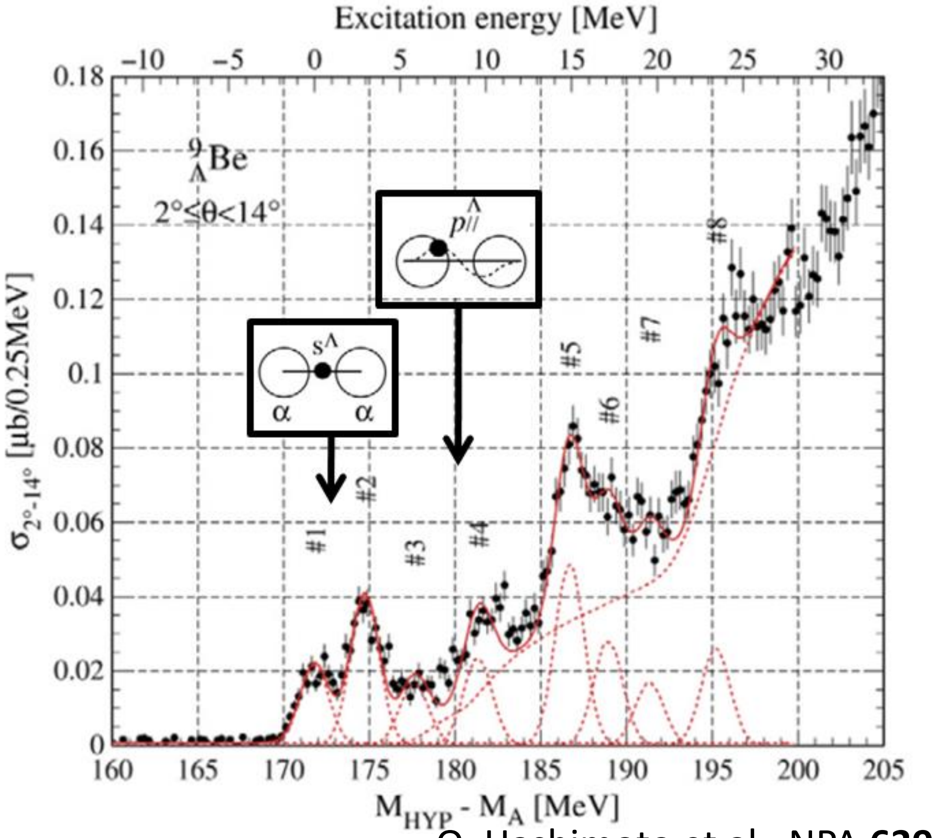
$^{9}_{\Lambda}$Be: axially symmetric 2α clustering

Two rotational bands as *p*-states
- Anisotropic *p* orbit of Λ hyperon
- Axial symmetry of 2α clustering

→ ***p*-orbit parallel to/perpendicular to the 2α clustering**



R.H. Dalitz, A. Gal, PRL **36** (1976) 362.
H. Bando, et al., PTP **66** (1981) 2118.
T. Motoba, et al.,PTPS**81**, 42(1985).



O. Hashimoto et al., NPA **639** (1998) 93c.

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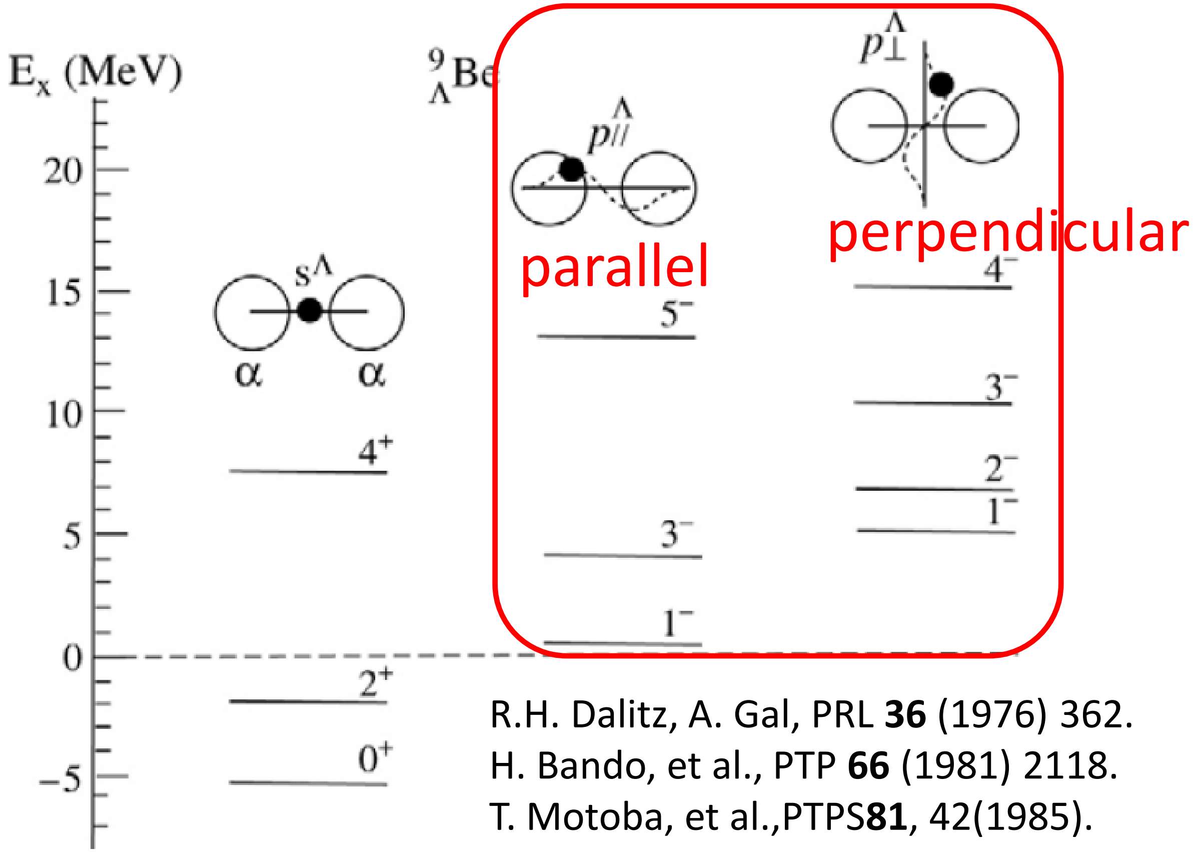

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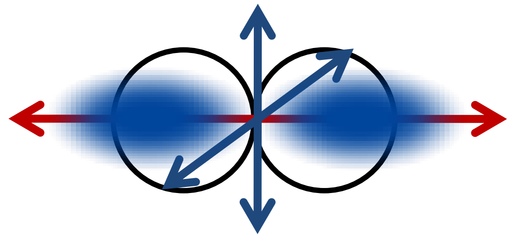
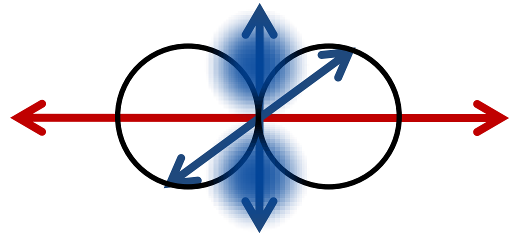
***p* orbit parallel to 2α (long axis)**



Large overlap
Deeply bound

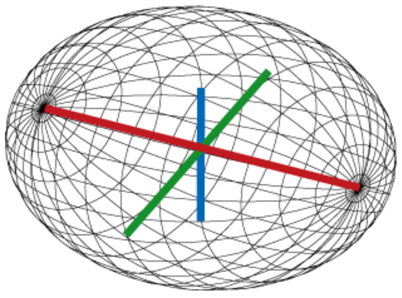
***p* orbit perpendicular to 2α (short axes)**



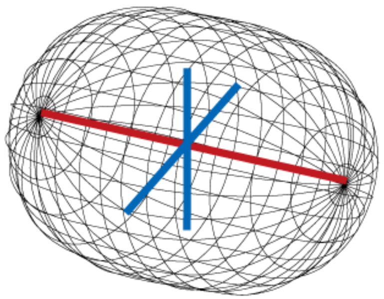
Small overlap
Shallowly bound

Triaxial deformation

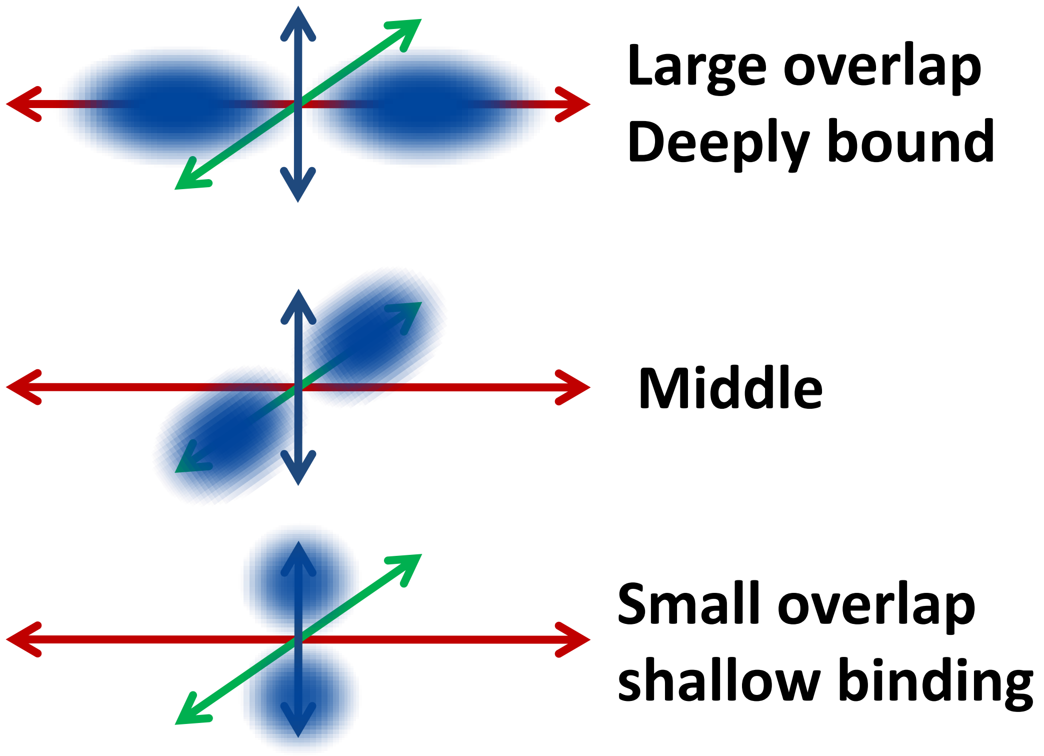
If nucleus is triaxially deformed,
***p*-states can split into 3 different state**



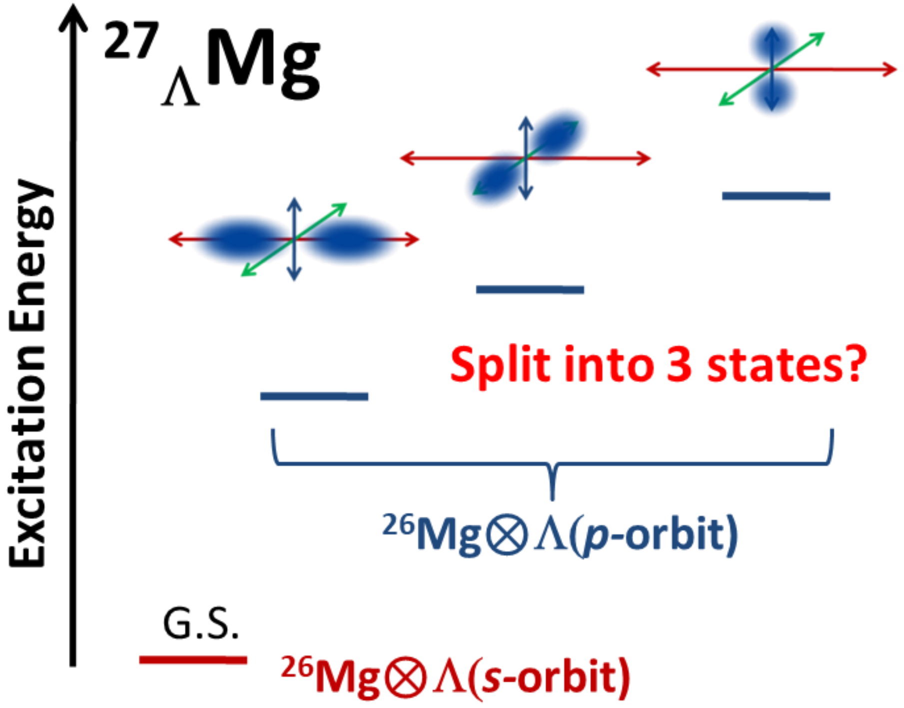
Triaxial deformation



Prolate deformation



Candidate: Mg hypernuclei



Observing the 3 different *p*-states is strong evidence of triaxial deformation

Theoretical Framework: HyperAMD

M.I., *et al.*, PRC**83**(2011) 044323
M. I., *et al.*, PRC**83**(2011) 054304

HyperAMD: Antisymmetrized Molecular Dynamics for hypernuclei

◆Hamiltonian

$$\hat{H} = \hat{T}_N + \hat{V}_{NN} + \hat{T}_\Lambda + \hat{V}_{\Lambda N} - \hat{T}_g$$

NN : Gogny D1S

ΛN : YNG interaction

◆Wave function

- **Nucleon part : Slater determinant**

  Spatial part of s.-p. w.f. is described as Gaussian packets

$$\varphi_N(\vec{r}) = \frac{1}{\sqrt{A!}} \det\left[\varphi_i(\vec{r}_j)\right]$$

$$\varphi_i(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \nu_\sigma (r - Z_i)_\sigma^2\right] \chi_i \eta_i$$

$$\chi_i = \alpha_i \chi_\uparrow + \beta_i \chi_\downarrow$$

- **Single-particle w.f. of Λ hyperon:**

  Superposition of Gaussian packets

$$\varphi_\Lambda(r) = \sum_m c_m \varphi_m(r)$$

$$\varphi_m(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \mu\nu_\sigma (r - z_m)_\sigma^2\right] \chi_m$$

$$\chi_m = a_m \chi_\uparrow + b_m \chi_\downarrow$$

- **Total w.f. :**

$$\psi(\vec{r}) = \sum_m c_m \varphi_m(r_\Lambda) \otimes \frac{1}{\sqrt{A!}} \det\left[\varphi_i(\vec{r}_j)\right]$$

Theoretical Framework: HyperAMD

M.I., *et al.*, PRC**83**(2011) 044323
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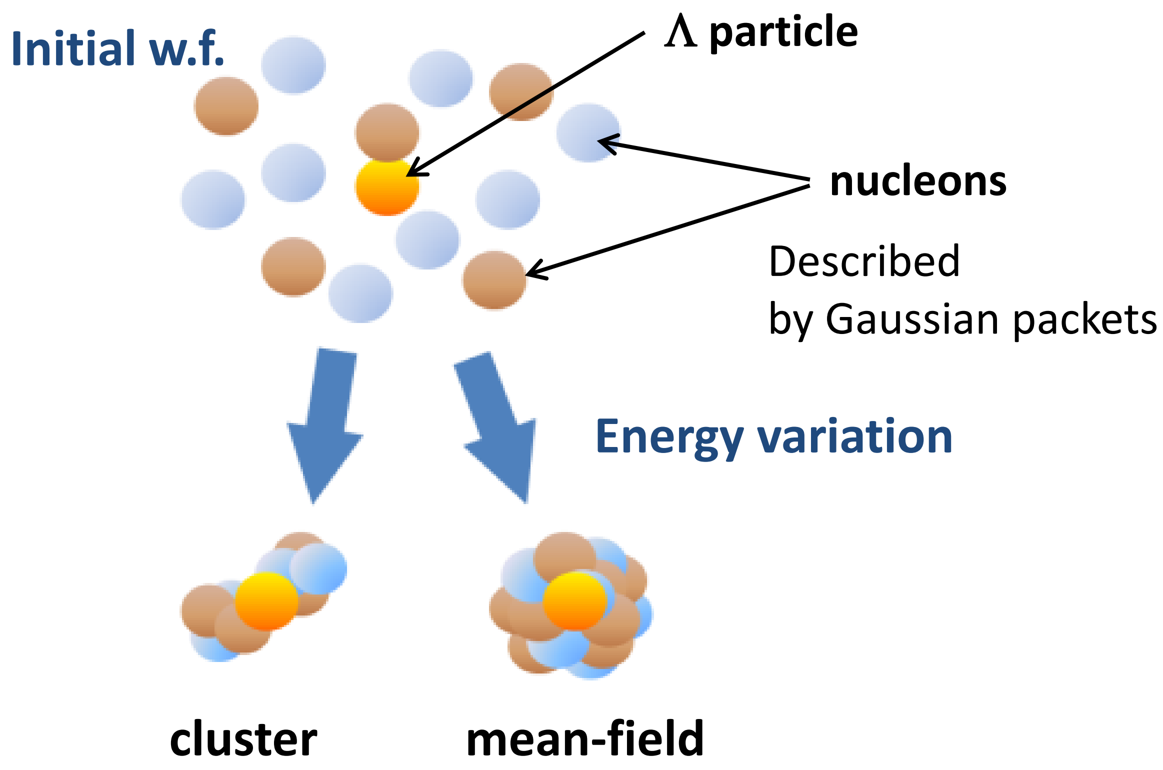
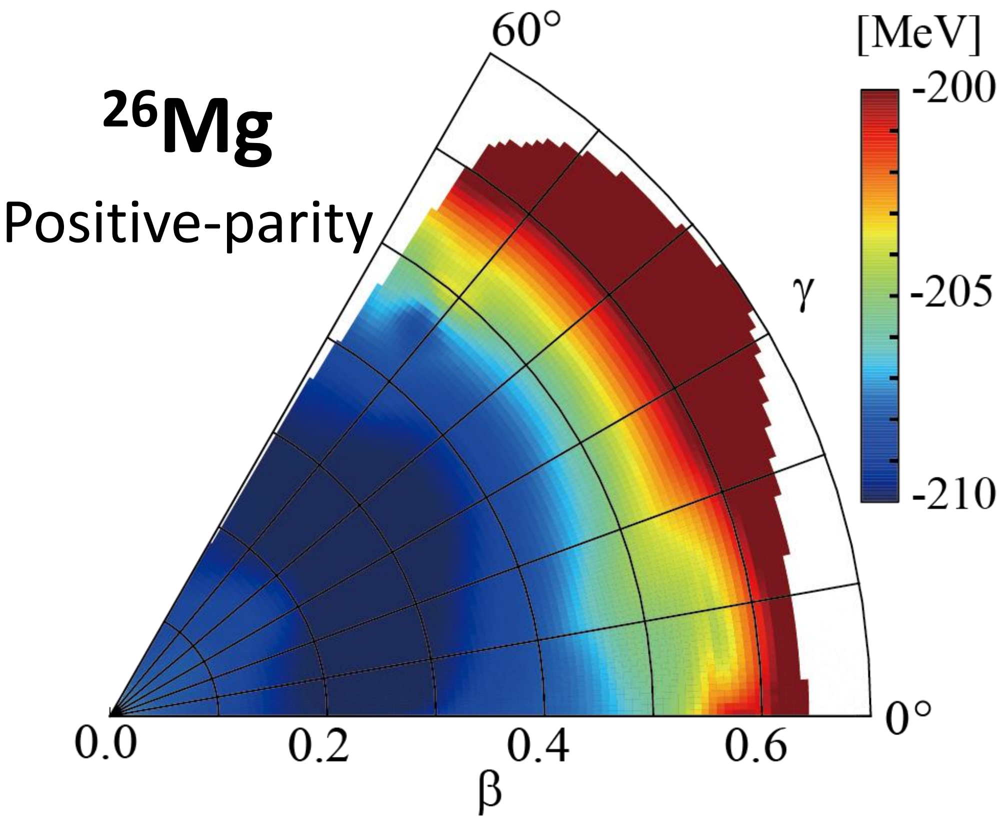
◆Procedure of the numerical calculation

Variation

- Imaginary time development method: $\frac{dX_i}{dt} = \frac{\kappa}{\hbar}\frac{\partial H^{\pm}}{\partial X_i^*}$ $\quad \kappa < 0$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$





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Angular Momentum Projection

$$\left|\Phi_K^s; JM\right\rangle = \int d\Omega D_{MK}^{J*}(\Omega) R(\Omega) \left|\Phi^{s+}\right\rangle$$

Generator Coordinate Method (GCM)

- Superposition of intrinsic wave functions with different configuration
- Diagonalization of $H_{sK,s'K'}^{J\pm}$ and $N_{sK,s'K'}^{J\pm}$

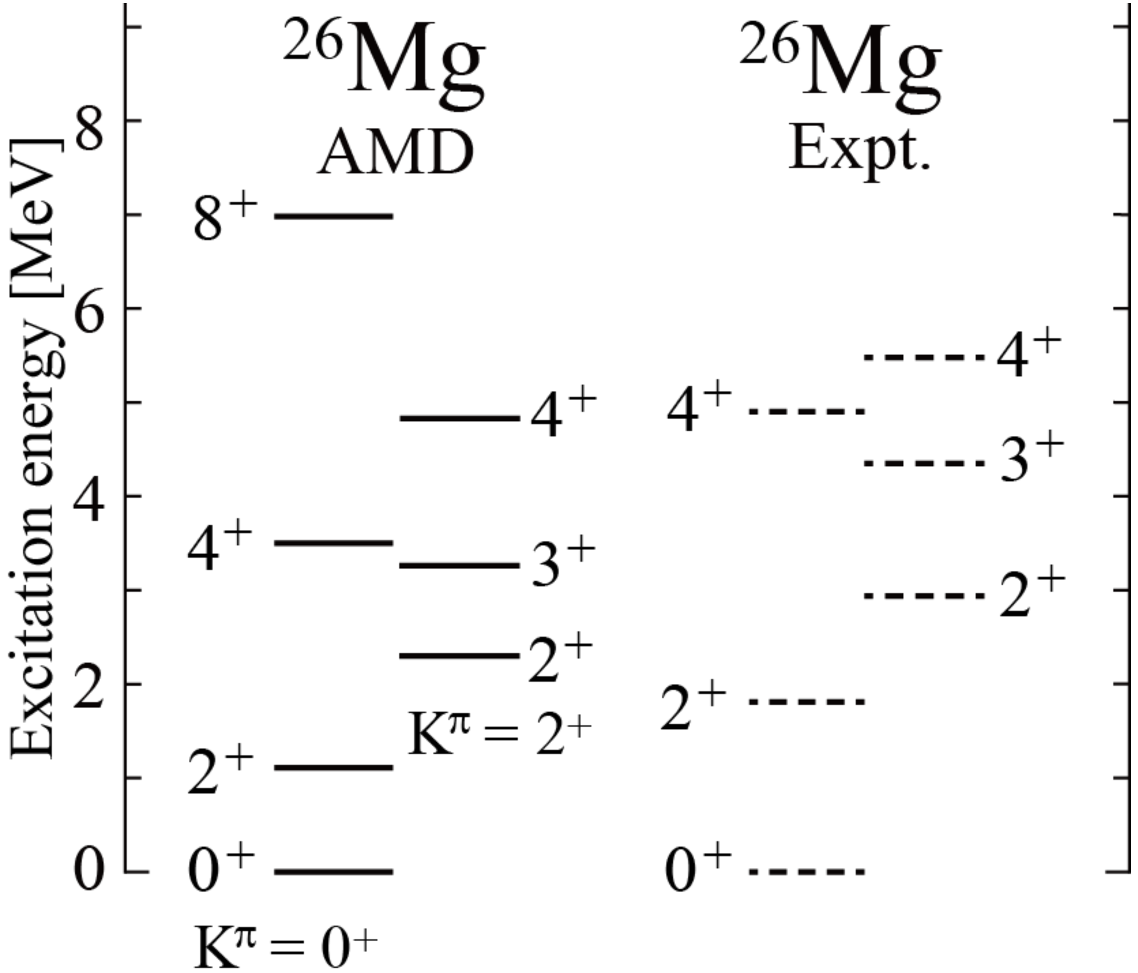
$$H_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s; J^{\pm}M \left| \hat{H} \right| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$

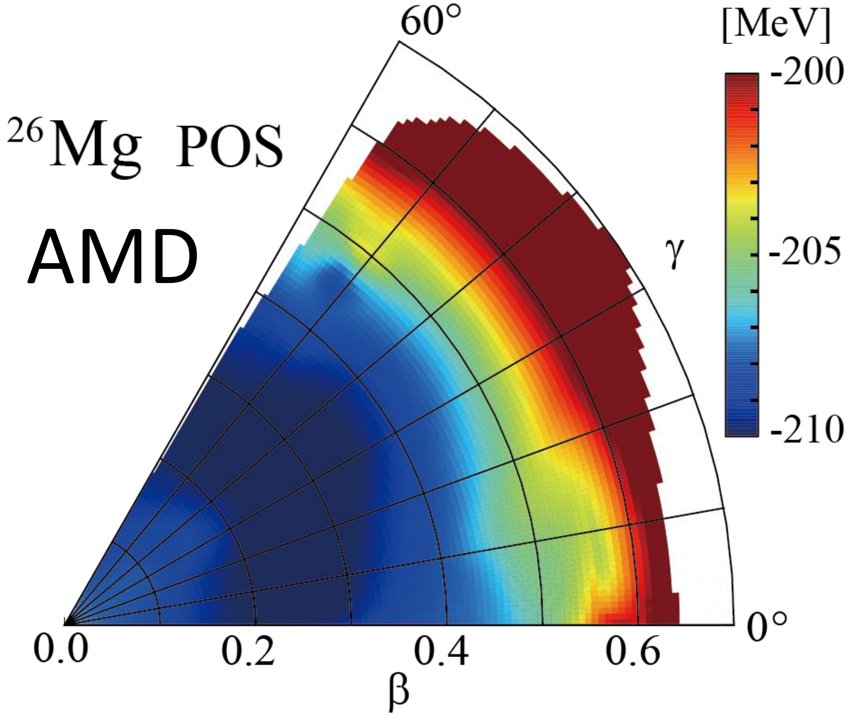
$$N_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s; J^{\pm}M \middle| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$

$$\left|\Psi^{J\pm M}\right\rangle = \sum_{sK} g_{sK} \left|\Phi_K^s; J^{\pm}M\right\rangle$$

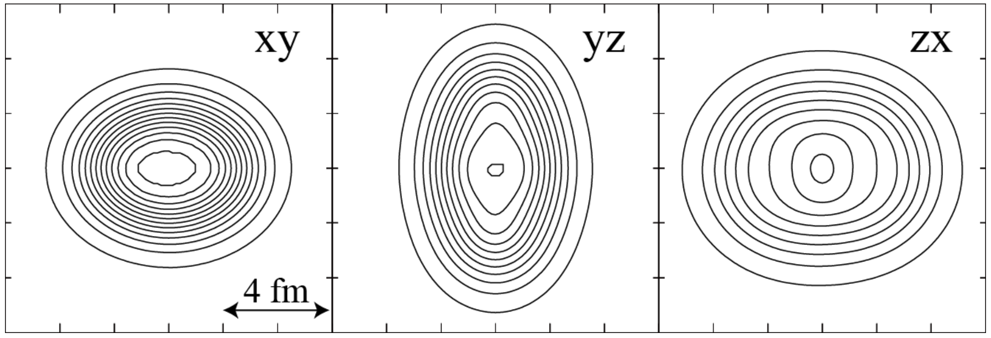
Results of ^{26}Mg

- Low-lying K=2 band is obtained
- Triaxially deformed and γ-soft energy surface



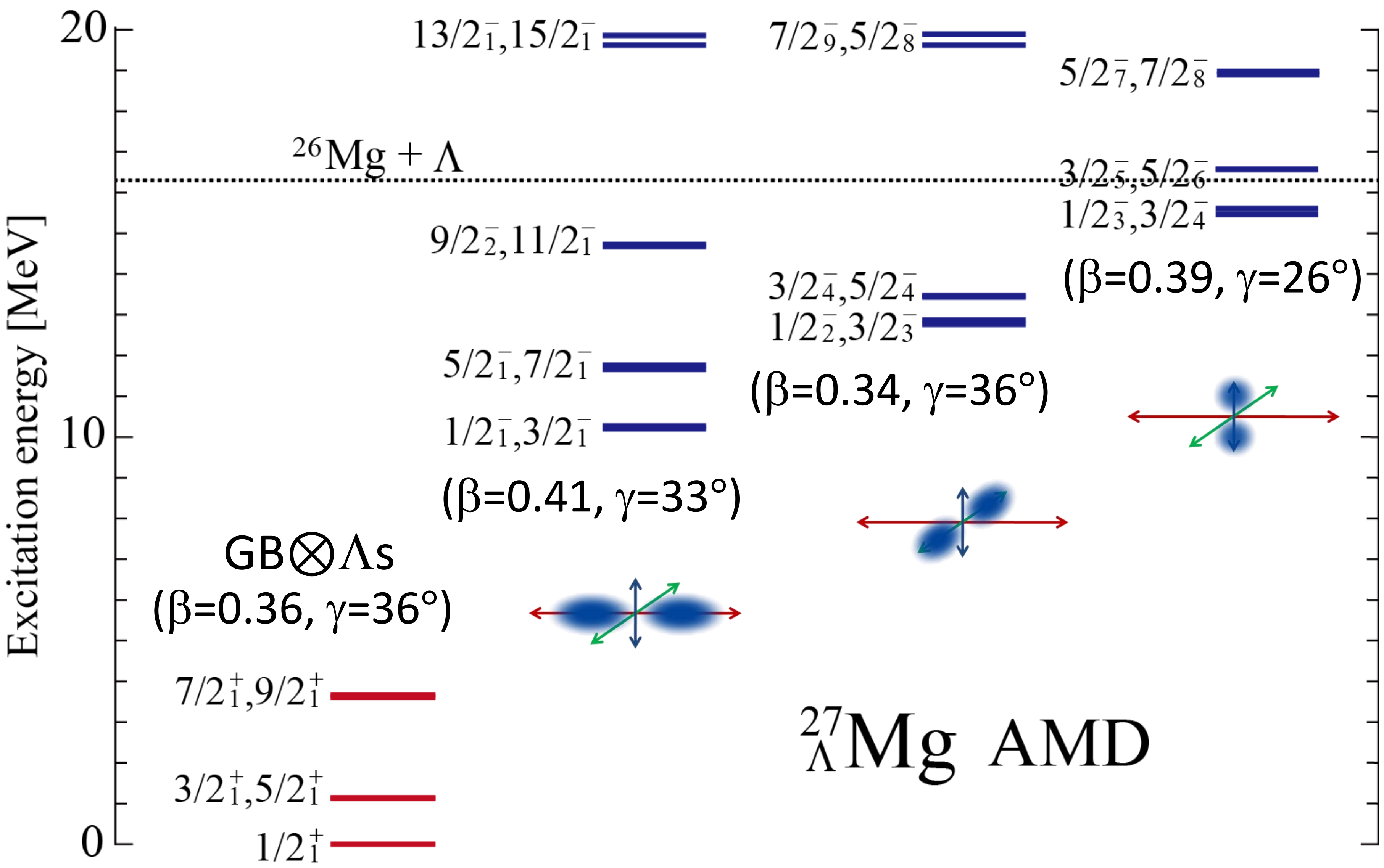


^{26}Mg ground state: β=0.41, γ=33°



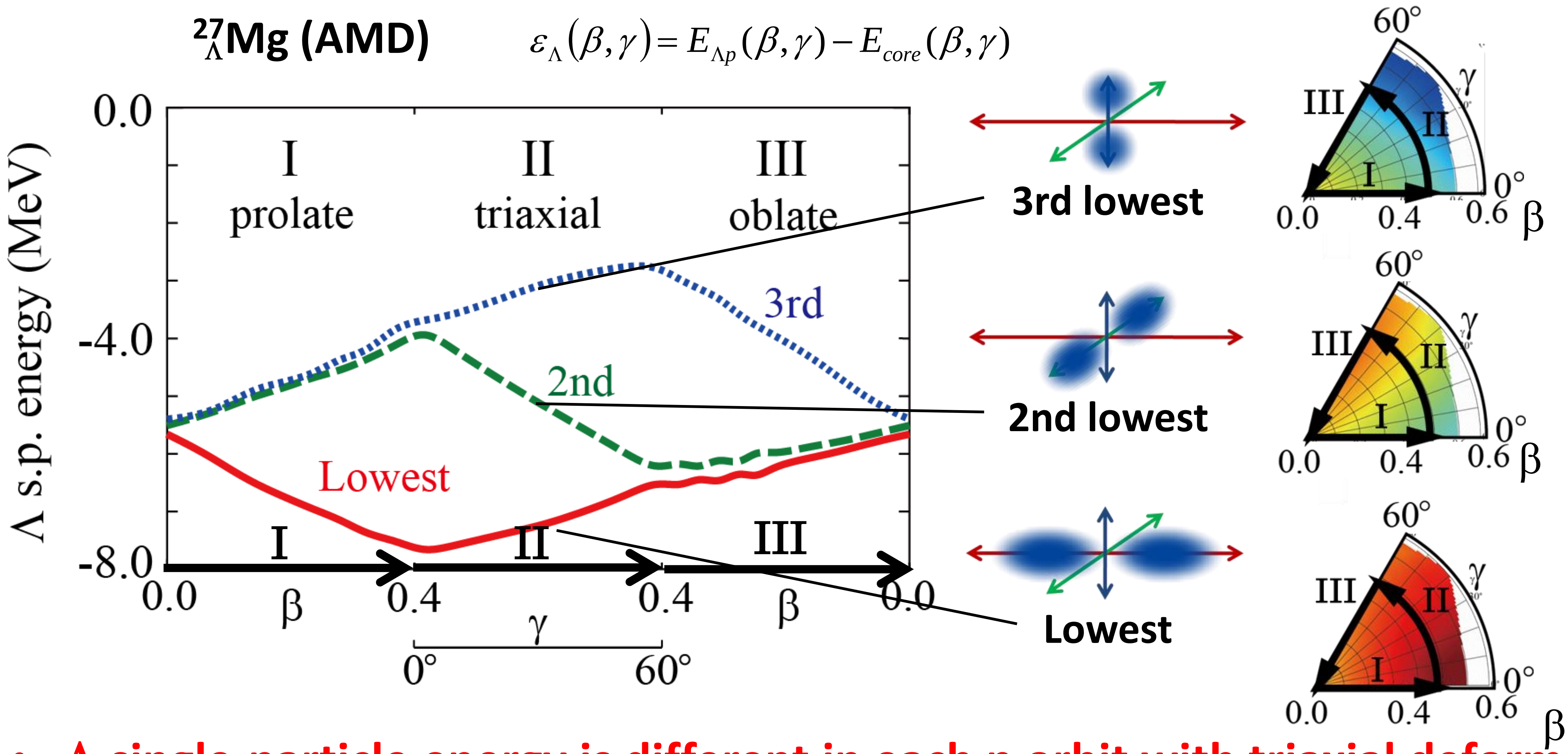
Results: $^{27}{}_{\Lambda}$Mg

- **3 bands are obtained by Λ in *p*-orbit** → **Splitting of the *p* states**



cf. ^{26}Mg ground state:
β=0.41, γ=33°

Results : Single particle energy of Λ hyperon ε_Λ



- **Λ single-particle energy is different in each p orbit with triaxial deformation**
- **3 different p-states appear if the core nucleus is triaxially deformed**

Summary

Deformation of the core nucleus ^{26}Mg

- *sd*-shell nuclei such as ^{26}Mg are lightest ones with large deformation
- Deformation property of ^{26}Mg is of particular interest, where Z = 12 and N = 14 favor different deformations
- Large B(E2): proton distribution is deformed
  - No information on neutron distribution and triaxiality
- Comparison with theoretical studies
  - Low-lying 2_2^+ is a sign of triaxial deformation in rigid rotor model
  - Difference from rigid rotor: γ-softness, isospin character

Summary

$^{27}{}_{\Lambda}$Mg: a candidate of triaxially deformed hypernucleus

- Splitting of *p*-states in $^{27}{}_{\Lambda}$Mg is predicted due to triaxial deformation
- Λ in *p*-states does not change nuclear deformation significantly

➜ Λ can be a unique probe of triaxial deformation

Future plans

- Detailed and further analysis of structure of $^{27}{}_{\Lambda}$Mg and ^{26}Mg: $\beta\gamma$-dependence, rotational bands, B(E2), … etc.

- Production cross section of $^{27}{}_{\Lambda}$Mg: How to identify?