Nucleon Energy-Energy Correlator in Lepton-Nucleon Collisions

Haotian Cao

Beijing Normal University

2023 EIC User Group Early Career Workshop (Warsaw University)
July 23-24, 2023

Liu, Zhu, Phys.Rev.Lett. 130 (2023)
HC, Liu, Zhu, 2303.01530
Liu, Liu, Pan, Yuan, Zhu, 2301.01788

Outline

Why studying Nucleon structure

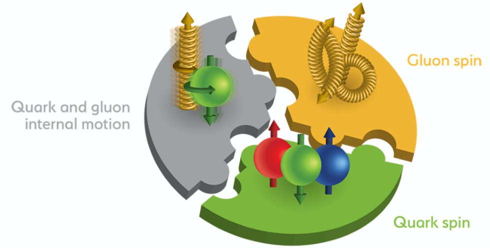
Still many question yet to answer about Proton:

Proton Mass decomposition

Proton Spin components

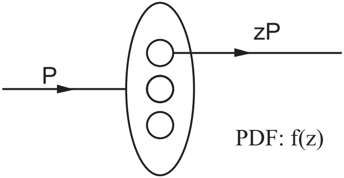
...

Need more information of the Nucleon structure, which is the future focus of EIC.



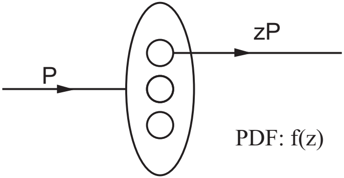
Conventional approach to nucleon structure

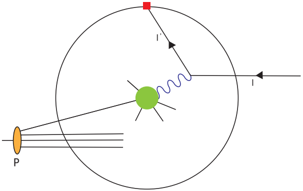
Parton distribution function(PDF): probability of finding partons in a hadron with longitudinal momentum fraction z



Conventional approach to nucleon structure

Parton distribution function(PDF): probability of finding partons in a hadron with longitudinal momentum fraction z.



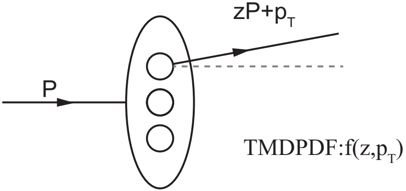


Deep inelastic scattering(DIS)

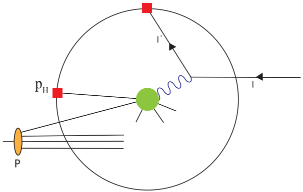
Inclusive
Clean in both experiment and theory
Lose information

Conventional approach to nucleon structure

Transverse momentum dependent (TMD) PDF: probability of finding partons in a hadron with longitudinal momentum fraction z and the parton transverse momentom p_T



Conventional approach to nucleon structure



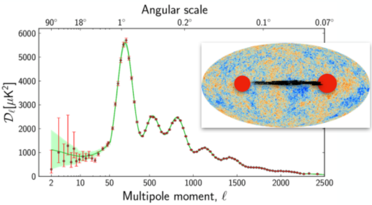
Semi-Inclusive DIS

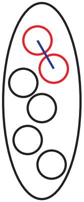
TMDPDF Include transverse momentum information.

Usually needs 2 non-pert object:

$\sigma = \hat{\sigma}(z,x) D(z,\vec{q}_T) \otimes f(z,\vec{p}_T)$

Loses information: correlation between partons in momentum space



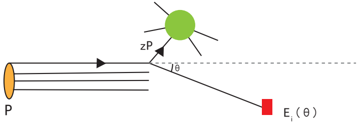


Nucleon Energy-Energy Correlator(NEEC)

Follow the idea of Energy-Energy Correlator[Dixon, Moult, Zhu...], NEEC was proposed[Liu,Zhu(2023)]

$$f_{q,\mathrm{EEC}}(z,\theta) = \int \frac{dy^-}{4\pi} e^{-izP^+\frac{y^-}{2}} \langle P|\bar{\chi}_n(y^-)\frac{\gamma^+}{2}\hat{\mathcal{E}}(\theta)\chi_n(0)|P\rangle$$

$$\hat{\mathcal{E}}(\theta)|X\rangle = \sum_{i\in X}\frac{E_i}{E_P}\Theta(\theta-\theta_i)|X\rangle$$



We do not constrain on the parton transverse momentum. The transverse dynamics are encoded in $\hat{\mathcal{E}}(\theta)$

Nucleon Energy-Energy Correlator(NEEC)

Follow the idea of Energy-Energy Correlator[Dixon, Moult, Zhu...], NEEC was proposed[Liu,Zhu(2023)] as an example of extending EEC to the proton form factor.

$$f_{q,\mathrm{EEC}}(z,\theta) = \int \frac{dy^-}{4\pi} e^{-izP^+\frac{y^-}{2}} \langle P|\bar{\chi}_n(y^-)\frac{\gamma^+}{2}\hat{\mathcal{E}}(\theta)\chi_n(0)|P\rangle$$

$$\hat{\mathcal{E}}(\theta)|X\rangle = \sum_{i\in X}\frac{E_i}{E_P}\Theta(\theta-\theta_i)|X\rangle$$

Compare it with pdf

$$f_q(z) = \int \frac{dy^-}{4\pi} e^{-izP^+\frac{y^-}{2}} \langle P|\bar{\chi}_n(y^-)\frac{\gamma^+}{2}\chi_n(0)|P\rangle$$

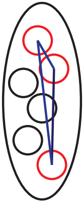
the only difference is the energy density operator.

Higher-Point correlator

Multi-point correlation is straightforwardly generalized with

$$f_{q,\mathrm{ENC}}(z,\theta) = \int \frac{dy^-}{4\pi} e^{-izP^+\frac{y^-}{2}} \langle P|\bar{\chi}_n(y^-)\frac{\gamma^+}{2}\hat{\mathcal{E}}(\theta_1)\hat{\mathcal{E}}(\theta_2)...\hat{\mathcal{E}}(\theta_N)\chi_n(0)|P\rangle$$

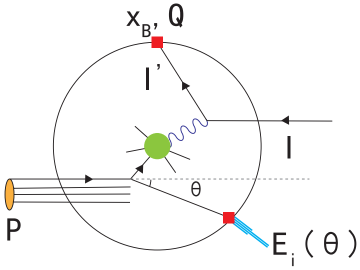
Nucleon internal dynamics will be imprinted in the detailed structure of these correlation functions.



How to probe NEEC

We claim NEEC can be probed in this way

$$\Sigma(Q^2, x_B, \theta) = \sum_i \int d\sigma(x_B, Q^2, p_i) \frac{E_i}{E_P} \Theta(\theta - \theta_i)$$
$$= \frac{\alpha^2}{Q^4} L_{\mu\nu}(Q^2, x_B) \int d^4x e^{iq\cdot x} \langle P| j^{\mu\dagger}(x)\, \hat{\mathcal{E}}(\theta)\, j^\nu(0)|P\rangle$$



Inclusive measurement weighted by E_i. No jet or hadrons.

Factorization theorem

When $\theta \ll 1$ possible leading contribution $p_h \sim Q(1,1,1)$, $p_c \sim Q(1,\theta^2,\theta)$ and $p_s \sim Q(\theta^a,\theta^a,\theta^a)$, $a > 1$. Before showing the factorization theorem we study more about $\hat{\mathcal{E}}(\theta)$, recall $\hat{\mathcal{E}}(\theta)|X\rangle = \sum_{i\in X} \frac{E_i}{E_P}\Theta(\theta-\theta_i)|X\rangle$
Decompose the final state

$$\hat{\mathcal{E}}(\theta)|X\rangle = \frac{1}{E_P}\sum_{i\in X}\Big(E_{H,i}\Theta(\theta-\theta_{H,i}) + E_{C,i}\Theta(\theta-\theta_{C,i}) + E_{S,i}\Theta(\theta-\theta_{S,i})\Big)|X_H,X_C,X_S\rangle$$

$E_{S,i} \sim Q\theta^a$, $\Theta(\theta-\theta_{H,i}) \sim 0$

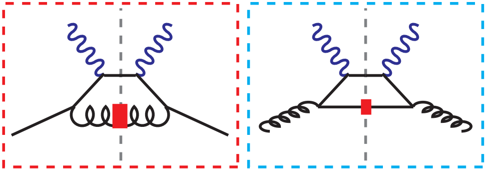
$$\hat{\mathcal{E}}(\theta)|X\rangle = \frac{1}{E_P}\sum_{i\in X} E_{C,i}\Theta(\theta-\theta_{C,i})|X_H,X_C,X_S\rangle$$

Factorization theorem

Match $j^{\mu\dagger}\mathcal{E}(\theta)j^{\nu}$ to the SCET operators

$$\langle P|j^{\mu\dagger}(x)\hat{\mathcal{E}}(\theta)j^{\nu}(0)|P\rangle = C_q^{\mu\nu}\langle P|\bar{\chi}_n(x)Y^{\dagger}(x)\frac{\gamma^+}{2}\hat{\mathcal{E}}(\theta)Y(0)\chi_n(0)|P\rangle + C_g^{\mu\nu}\langle P|\mathcal{B}_{\perp}(x)\mathcal{Y}^{\dagger}(x)\hat{\mathcal{E}}(\theta)\mathcal{Y}(0)\mathcal{B}_{\perp}(0)|P\rangle$$

We have both quark and gluon contribution



Factorization theorem

Match $j^{\mu\dagger}\mathcal{E}(\theta)j^{\nu}$ to the SCET operators

$$\langle P|j^{\mu\dagger}(x)\hat{\mathcal{E}}(\theta)j^{\nu}(0)|P\rangle = C_q^{\mu\nu}\langle P|\bar{\chi}_n(x)Y^{\dagger}(x)\frac{\gamma^+}{2}\hat{\mathcal{E}}(\theta)Y(0)\chi_n(0)|P\rangle + C_g^{\mu\nu}\langle P|\mathcal{B}_{\perp}(x)\mathcal{Y}^{\dagger}(x)\hat{\mathcal{E}}(\theta)\mathcal{Y}(0)\mathcal{B}_{\perp}(0)|P\rangle$$

$$\Sigma(Q^2, x_B, \theta) = \frac{\alpha^2}{Q^4}L_{\mu\nu}(Q^2, x_B)\int d^4x e^{iq\cdot x}\langle P|j^{\mu\dagger}(x)\,\hat{\mathcal{E}}(\theta)\,j^{\nu}(0)|P\rangle$$

1.When replaceing $\hat{\mathcal{E}}(\theta)$ to 1, NEEC will be exactly the same as PDF, the weighted cross section will be recovered to the cross section of inclusive DIS

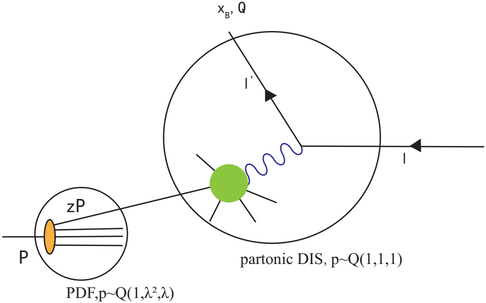
2.Hard coefficients are independent of the details of the collinear sector.

Factorization theorem

Factorization theorem in collinear limit

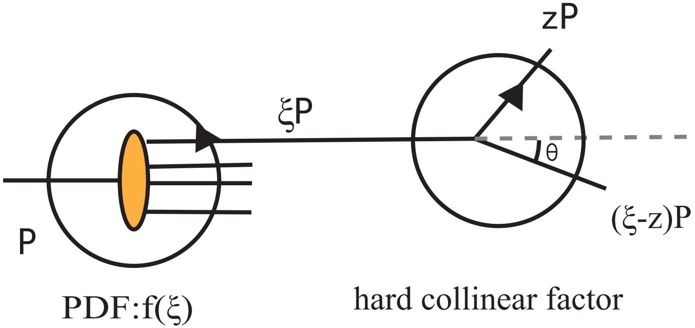
$$\Sigma(Q^2, x_B, \theta) = \int_{x_B}^{1} \frac{dz}{z} \hat{\sigma}_i\left(\frac{x_B}{z}\right) f_{i,\mathrm{EEC}}(z, P^+\theta)\,.$$

The only thing to do is to change $f_i(z)$ in inclusive DIS to $f_{i,\mathrm{EEC}}(z,\theta)$



Factorization theorem

When $\theta \ll 1$ but it is large enough that $\Lambda_{QCD} \ll \theta Q$. The collinear modes can be further split into the hard collinear modes (C_1) where $p_{C_1} \sim (Q, \theta^2 Q, \theta Q)$, and the C_2 modes in $SCET_{II}$ where $p_{C_2} \sim (Q, \Lambda_{QCD}^2/Q, \Lambda_{QCD})$. We can further match NEEC to PDF



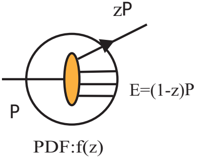
Factorization theorem

Decompose the final state

$$\hat{\mathcal{E}}(\theta)|X\rangle = \frac{1}{E_P}\sum_{i\in C_1,\, j\in C_2}\Big(E_{C_1,i}\Theta(\theta-\theta_{C_1,i}) + E_{C_2,j}\Theta(\theta-\theta_{C_2,j})\Big)|X_{C_1}X_{C_2}\rangle$$

For particles in C_2 modes $\theta_{c,i}\sim\Lambda_{QCD}/Q$ the Θ can be replaced by 1, then we use $\Theta(\theta-\theta_{C_1,j}) = 1-\Theta(\theta_{C_1,j}-\theta)$

$$\hat{\mathcal{E}}(\theta)|X\rangle = \frac{1}{E_P}\sum_{i\in X}\Big(E_X - E_{C_1,i}\Theta(\theta_{C_2,i}-\theta)\Big)|X_{C_1}X_{C_2}\rangle$$



The first term will countribute as

$$f^{(0)}_{i,\mathrm{EEC}}(z,\theta) = f_i(z) - zf_i(z)$$

Factorization theorem

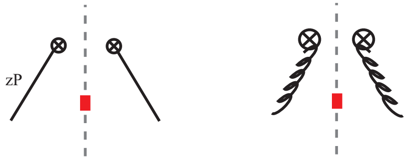
The second term will countribute as

$$f_{\mathrm{EEC}}^{(1)}(z,\theta) = -\int_z^1 \frac{d\xi}{\xi} I'\left(\frac{z}{\xi},\theta\right)\xi f(\xi)$$

The matching coefficient can be get by calculating the difference between the NEEC and the collinear PDF, using the SCET Feynman rules.

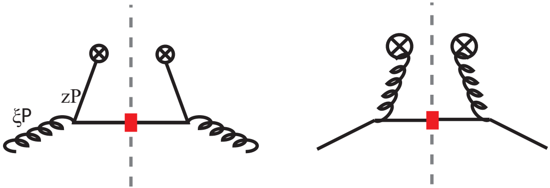
Factorization theorem

The LO contribution in the second term will be zero, which can be noticed by the feynman graphs



The NLO contribution are calculated with graphs like these. The result in Mellin space is

$$f^{(1)}_{\mathrm{EEC}}(N,\theta) = \frac{\alpha_s}{2\pi}\left[-\ln\frac{Q\theta}{2u\mu}\left(2P^{(0)}(N) - 2P^{(0)}(N+1)\right) + d(N) - d(N+1)\right]$$

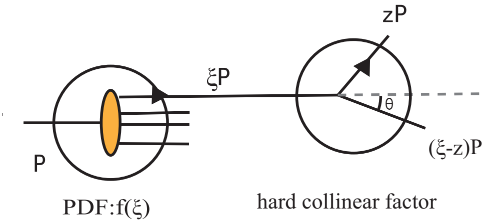


Factorization theorem

We can understand why this object is called NEEC, which is obvious from the fix order calculation.

$$\frac{df^{(1)}_{\mathrm{EEC}}(z,\theta)}{d\theta} \propto \left[\left(1-\frac{z}{\xi}\right)\frac{1}{\theta}P\left(\frac{z}{\xi}\right)\right]\xi f(\xi)\,,$$

initial energy density:$\xi f(\xi)$,
final energy density:$\left(1-\frac{z}{\xi}\right)\frac{1}{\theta}P$
This manifests the angular correlation between final energy-initial energy density



Evolution Equation

Before giving the evolution equation we have to add the scale dependence to NEEC ,the only scale that enters NEEC is $P^+\theta$, then the θ dependence should have the form $P^+\theta$. Since NEEC is dimensionless θP^+ will only show up in the form $ln\frac{\theta P^+}{\mu}$. define $u = \frac{x_B}{z}$. In breit frame, $P^+ = \frac{Q}{zu}$

$$\Sigma(Q^2, x_B, \theta) = \int \frac{du}{u}\,\hat{\sigma}_i\,(u)\, f_{i,\mathrm{EEC}}(\frac{x_B}{u}, ln\frac{Q\theta}{u\mu})$$

Evolution Equation

From the consistency relation, NEEC will satisfy the similar equation as pdf

$$\frac{d}{d\ln\mu^2} f_{i,\mathrm{EEC}}(N, \frac{Q\theta}{u\mu}) = \int d\xi\, \xi^{N-1} P(\xi)\, f_{\mathrm{EEC}}(N, \ln\frac{Q\theta}{\xi u\mu})$$

We can solve the Evolution Equation recursively

$$\begin{aligned} f_{\mathrm{EEC}}(N, \ln\frac{Q\theta}{u\mu}) =& f_{\mathrm{EEC}}(N, \ln\frac{Q\theta}{u\mu_0}) \\ &+ \int_{\mu_0}^{\mu} d\ln\mu'^2 \int d\xi \xi^{N-1} P(\xi)\, f_{\mathrm{EEC}}(N, \ln\frac{Q\theta}{\xi u\mu'}) \end{aligned}$$

Using the ansatz solution

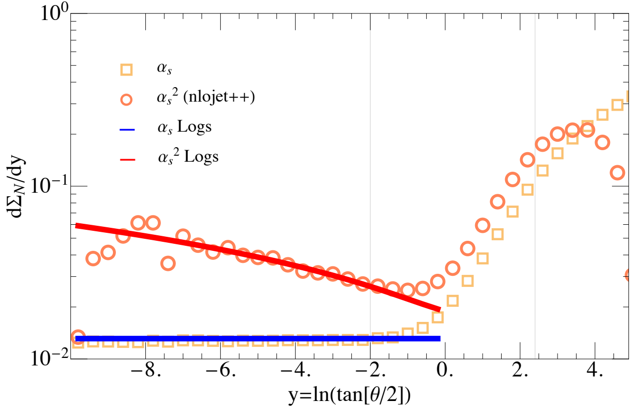
$$f_{\mathrm{EEC}}(N, \ln\frac{Q\theta}{u\mu}) = D(\mu,\mu_0) f_{\mathrm{EEC}}(N, \ln\frac{Q\theta}{u\mu_0}) + R(\mu,\mu_0)$$

$D(\mu_0,\mu_0) = 1$ and $R(\mu_0,\mu_0) = 0$

Consistency check

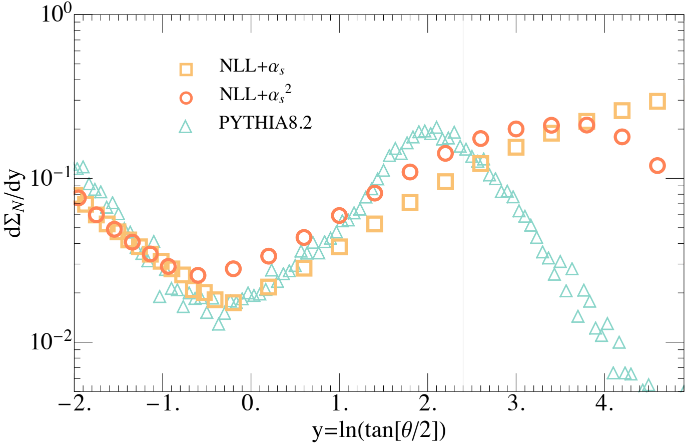
Compare the complete fixed order result with NLL resummed result expanded to $O(\alpha_s)$ and $O(\alpha_s^2)$

$$\Sigma_N(Q^2,\theta) = \sum_i \int d\sigma(x_B, Q^2, p_i) x_B^{N-1} \frac{E_i}{E_P} \Theta(\theta - \theta_i)$$



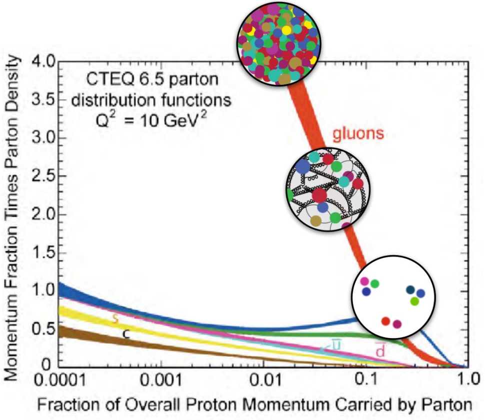
$N = 3$
$E_P = 275\,GeV$
$E_l = 18\,GeV$
$Q = 10\,GeV$

Comparison of the $\mathrm{NLL} + \alpha_s$, $\mathrm{NLL} + \alpha_s^2$ and the `Pythia` simulation at partonic level. Reasonable agreement is found in the small θ (y) region(near-side)



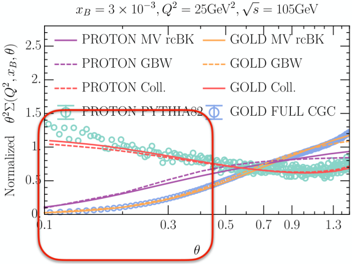
Application to the gluon saturation

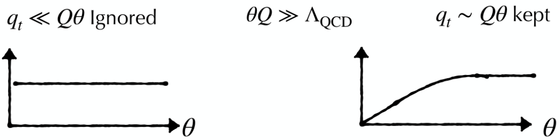
Gluon saturation at small x



Application to the gluon saturation

NEEC as evident portal to the onset of gluon saturation. We can define a turning point around which the slope of the distribution starts to switch its monotonicity.





Conclusion

NEEC is a brand new description of the nucleon structures and QCD dynamics.

The factorization theorem and NLL result makes the first step towards a precise measurement of NEEC

A lot to study in the future.

Thank you!