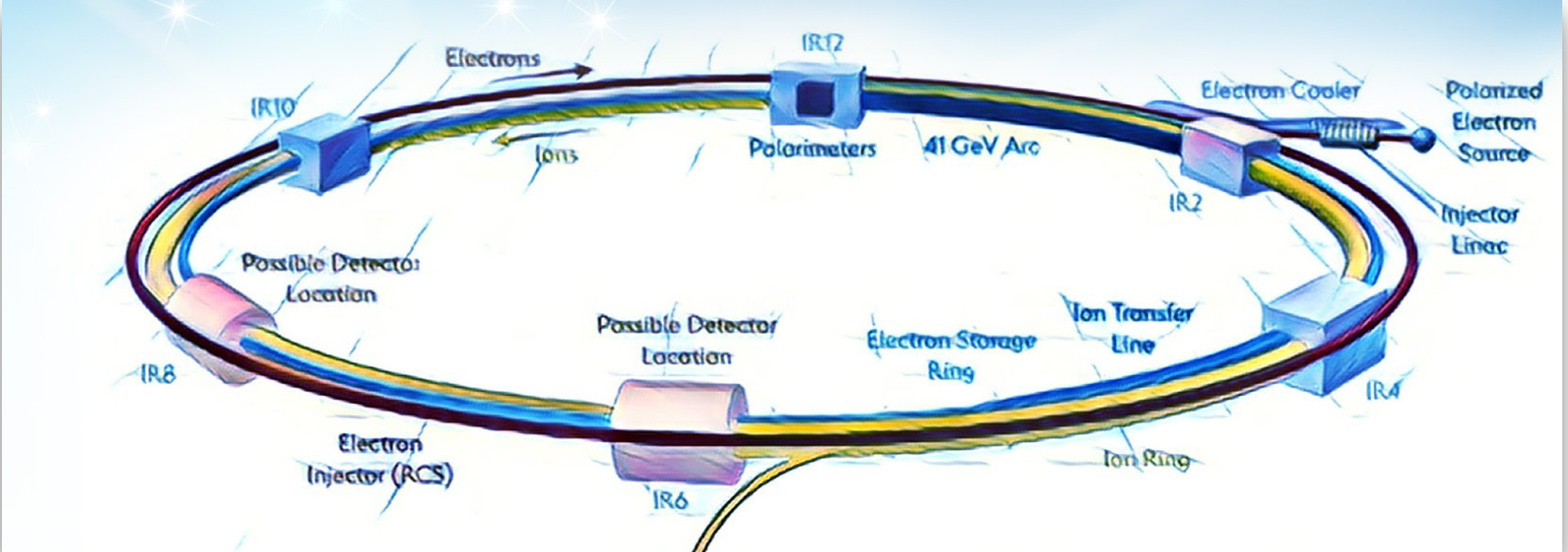


GLUON TMD MODELING FOR PROTON 3D IMAGING

Francesco Giovanni Celiberto, UAH Madrid

EARLY
CAREER
WORKSHOP

July 23-24 • University of Warsaw • Poland



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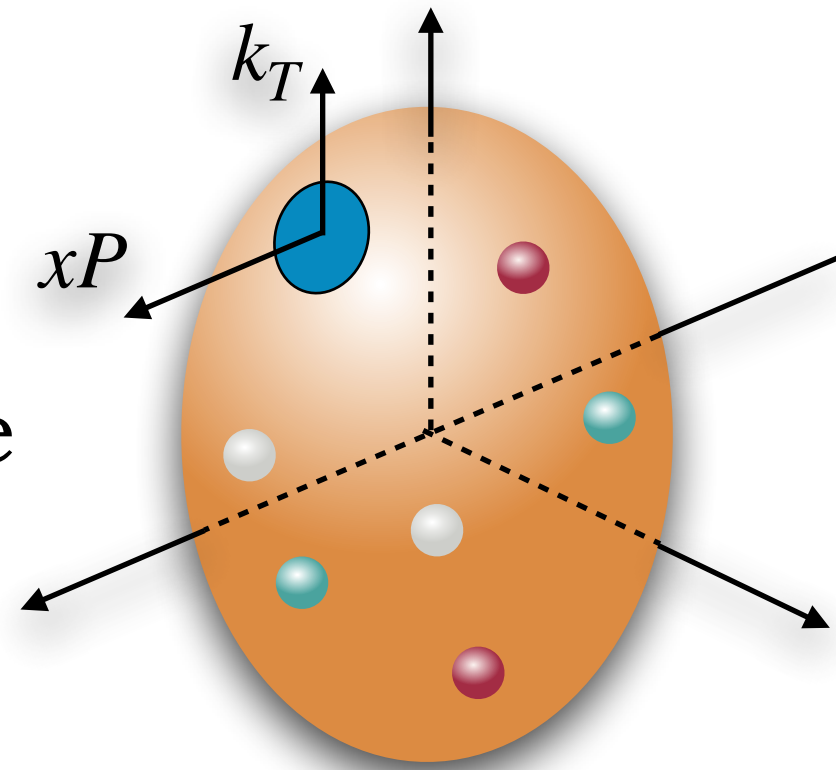
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Parton densities: an incomplete family tree

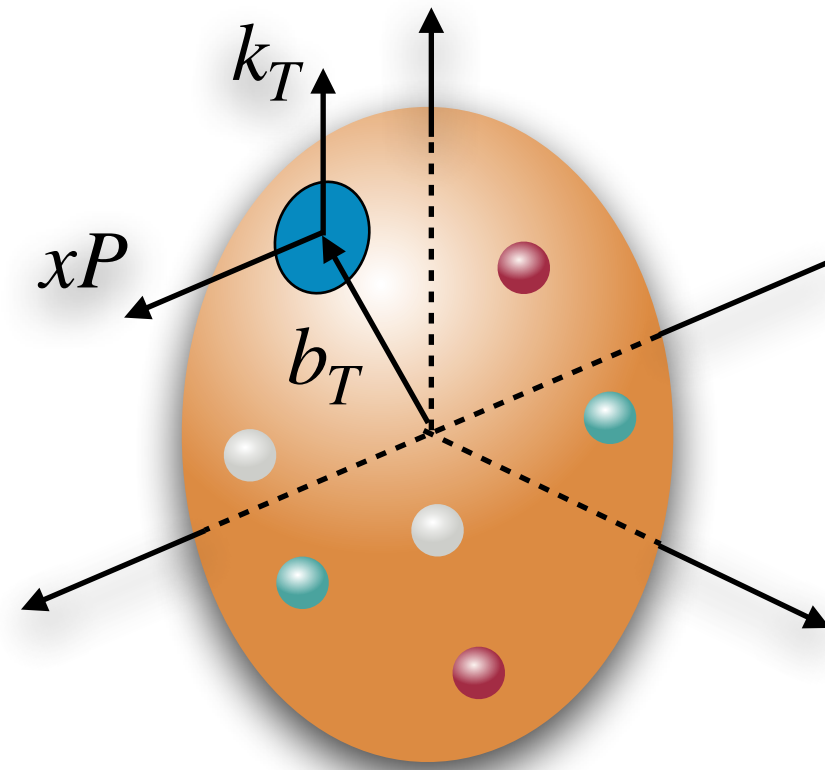
3D

TMDs
(semi-)inclusive



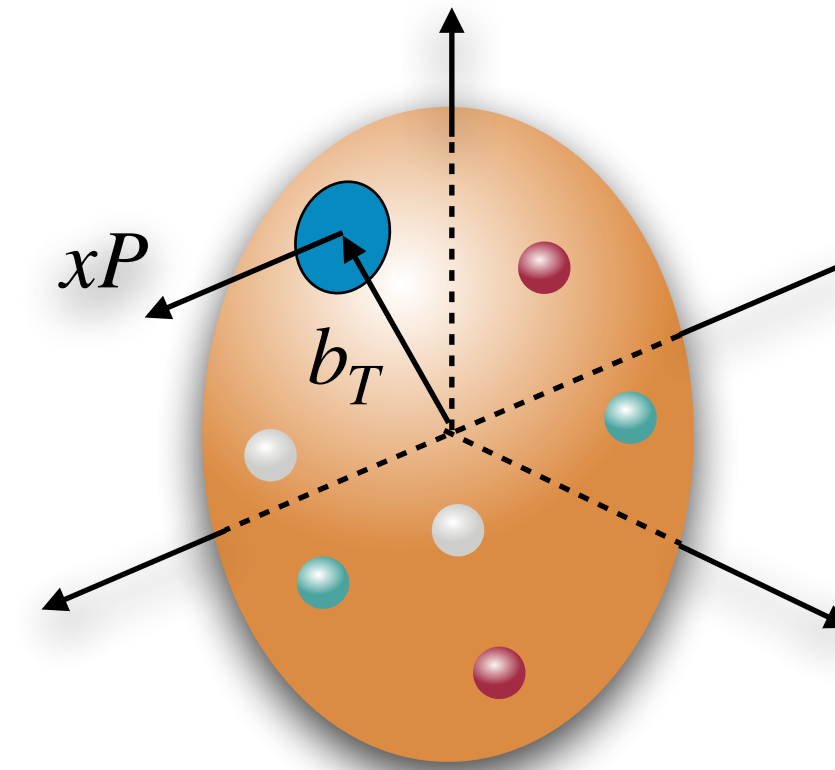
5D

Wigner distributions

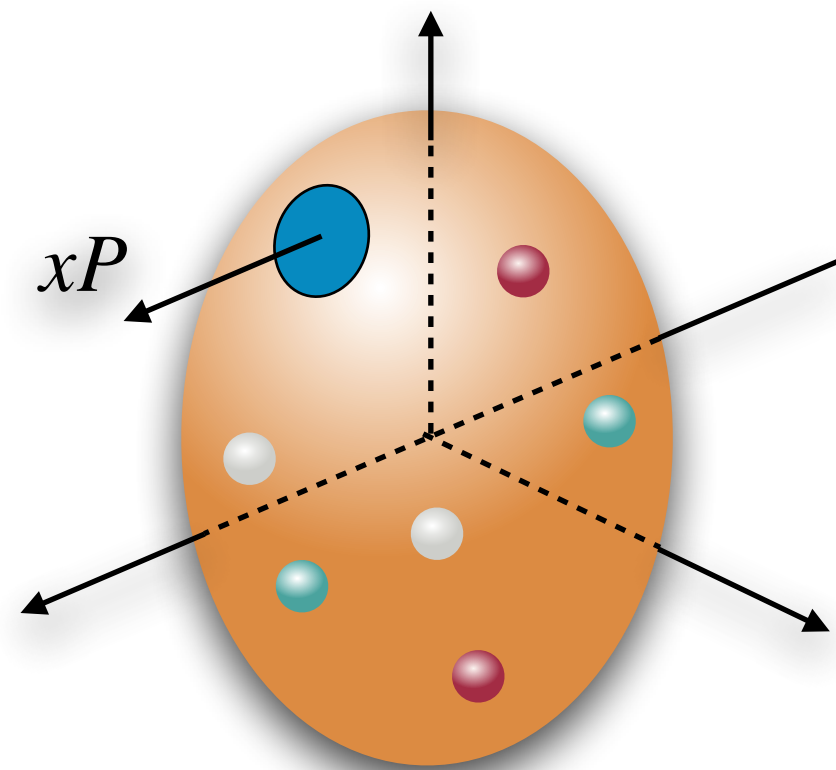


3D

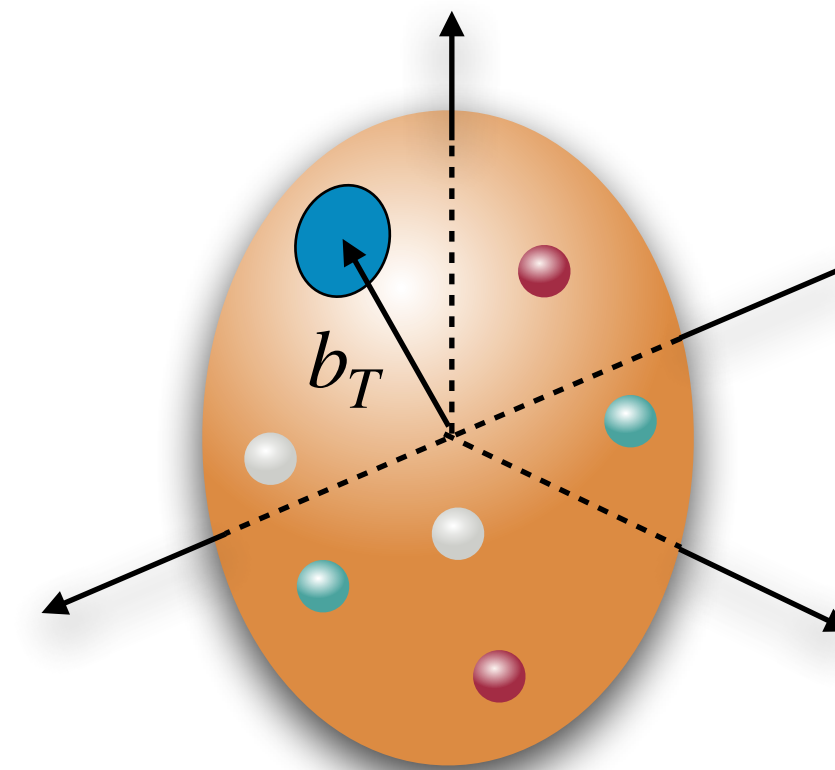
FT of GPDs
exclusive



PDFs
(semi-)inclusive



FT of Form Factors



1D

→ \vec{b}_\perp dependence
→ \vec{k}_\perp dependence



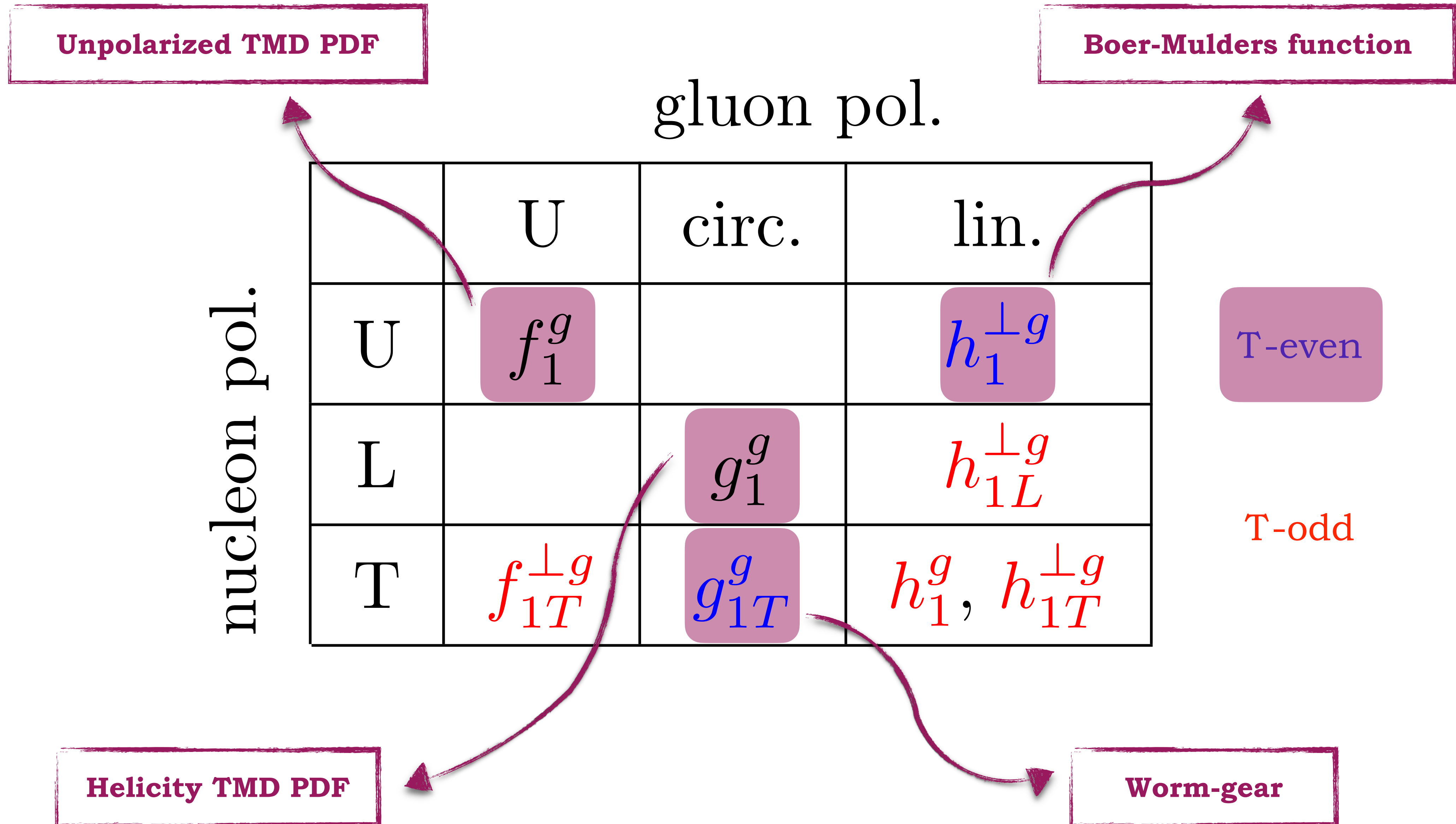
these two variables are NOT Fourier conjugate

2D

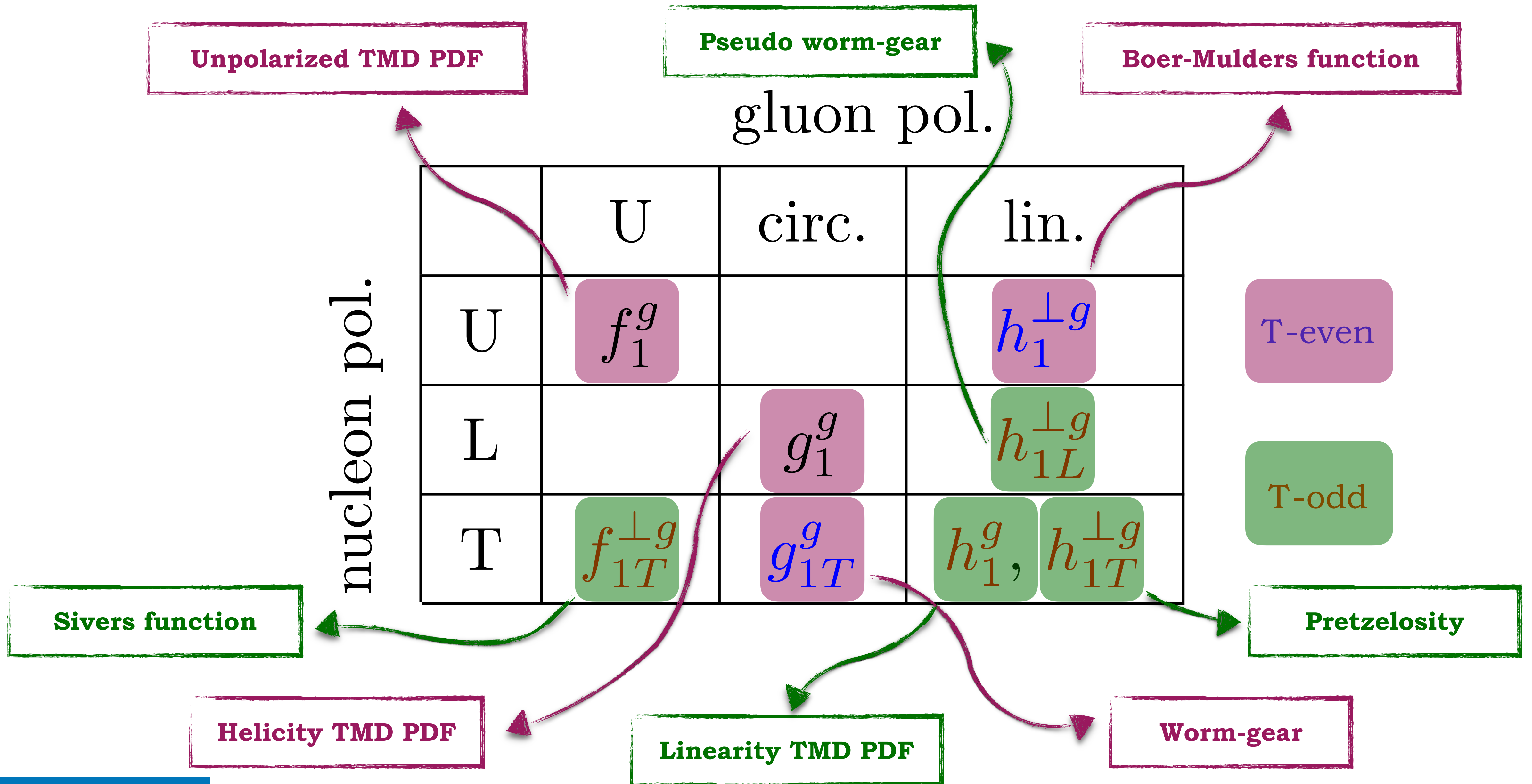
Gluon TMD PDFs at twist-2

		gluon pol.			
		U	circ.	lin.	
nucleon pol.	U	f_1^g		$h_1^{\perp g}$	T-even
	L		g_1^g	$h_{1L}^{\perp g}$	T-odd
	T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g, h_{1T}^{\perp g}$	

Gluon TMD PDFs at twist-2



Gluon TMD PDFs at twist-2



An Early Career Story

ECW 2021 online


16:55

Towards twist-2 T-odd TMD gluon distributions

🕒 15m 

We present explorative analyses of the 3D gluon content of the proton via a study of polarized T-odd gluon TMDs at twist-2, calculated in a spectator model for the parent nucleon. Our approach encodes a flexible parameterization for the spectator-mass density, suited to describe both moderate and small-x effects. All these prospective developments are relevant in the investigation of the gluon dynamics inside nucleons and nuclei, which constitutes one of the major goals of the EIC program.

Speaker: Francesco Giovanni **Celiberto** (ECT*/FBK Trento & INFN-TIFPA)

 talk_EICUG_ECW_2...

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Speaker: Francesco Giovanni **Celiberto** (ECT*/FBK Trento & INFN-TIFPA)

 talk_EICUG_ECW_2...


ECW 2022 Stony Brook

16:00

An overview of gluon TMD PDFs and polarization

🕒 20m 

Speaker: Francesco Giovanni **Celiberto** (ECT*/FBK Trento & INFN-TIFPA)

 talk_EICUG_ECW_2...

An Early Career Story

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16:55

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 talk_EICUG_ECW_2...

ECW 2022 Stony Brook


ECW 2023 Warsaw

16:00

An overview of gluon TMD PDFs and polarization

🕒 20m 

Speaker: Francesco Giovanni **Celiberto** (ECT*/FBK Trento & INFN-TIFPA)

 talk_EICUG_ECW_2...

10:50

Gluon TMD modeling for proton 3D imaging

🕒 20m 

We present exploratory analyses of the 3D gluon content of the proton via a study of polarized gluon TMDs at leading-twist, calculated in a spectator model for the parent nucleon. Our approach encodes a flexible parameterization for the spectator-mass density, suited to describe both moderate- and small-x effects. Particular attention will be paid phenomenological applications to angular modulations and spin asymmetries, such as the Boer-Mulders and the Sivers effects. These studies relevant in the investigation of gluon dynamics inside nucleons and nuclei, which represents a major goal of new-generation colliding machines.

Speaker: Francesco Giovanni **Celiberto** (UAH Madrid)

¿ Why so much enthusiasm in
Gluon TMDs ?



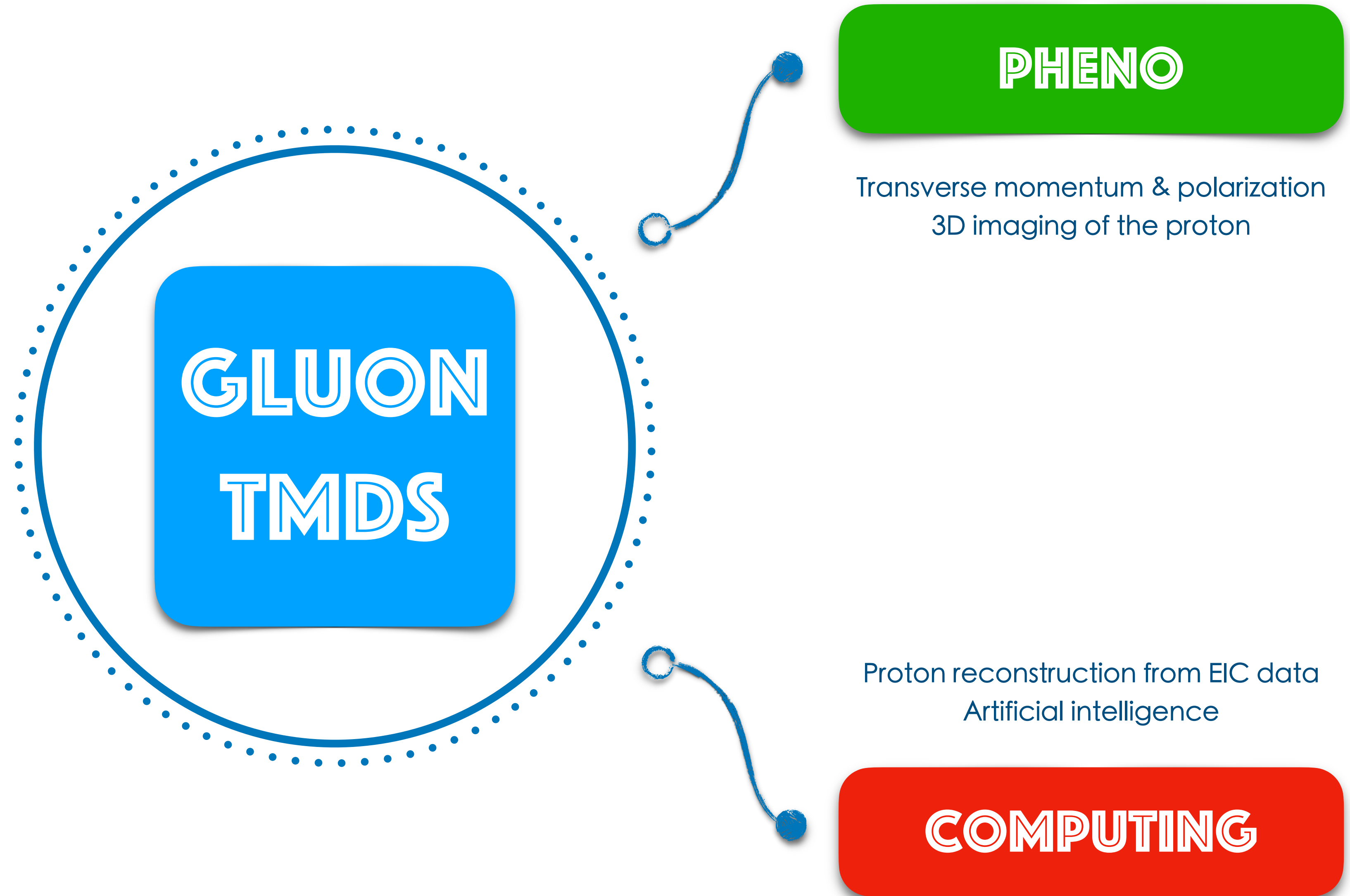
A window of opportunities



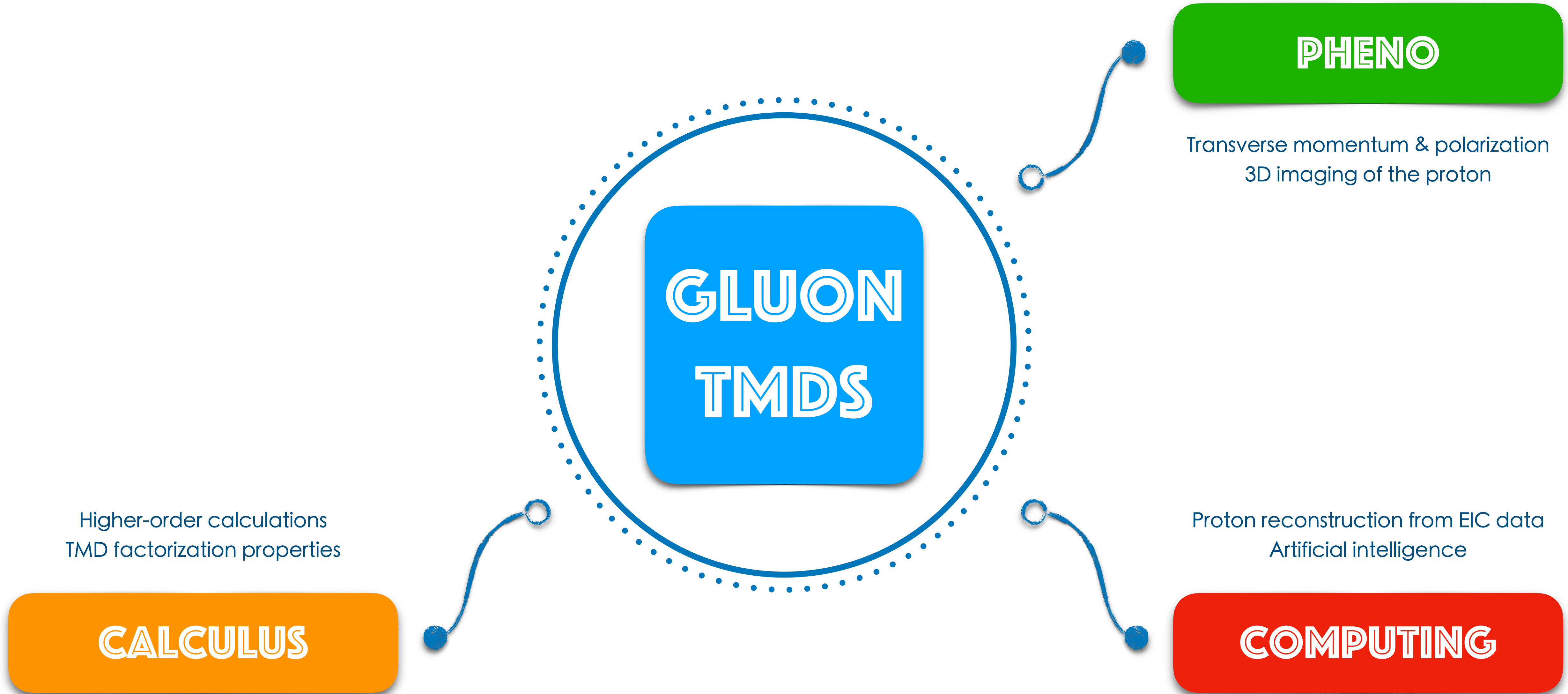
PHENO

Transverse momentum & polarization
3D imaging of the proton

A window of opportunities



A window of opportunities



A window of opportunities

EXPERIMENT

Design of new-generation detectors
Synergies between EIC and LHC

PHENO

Transverse momentum & polarization
3D imaging of the proton

GLUON
TMDS

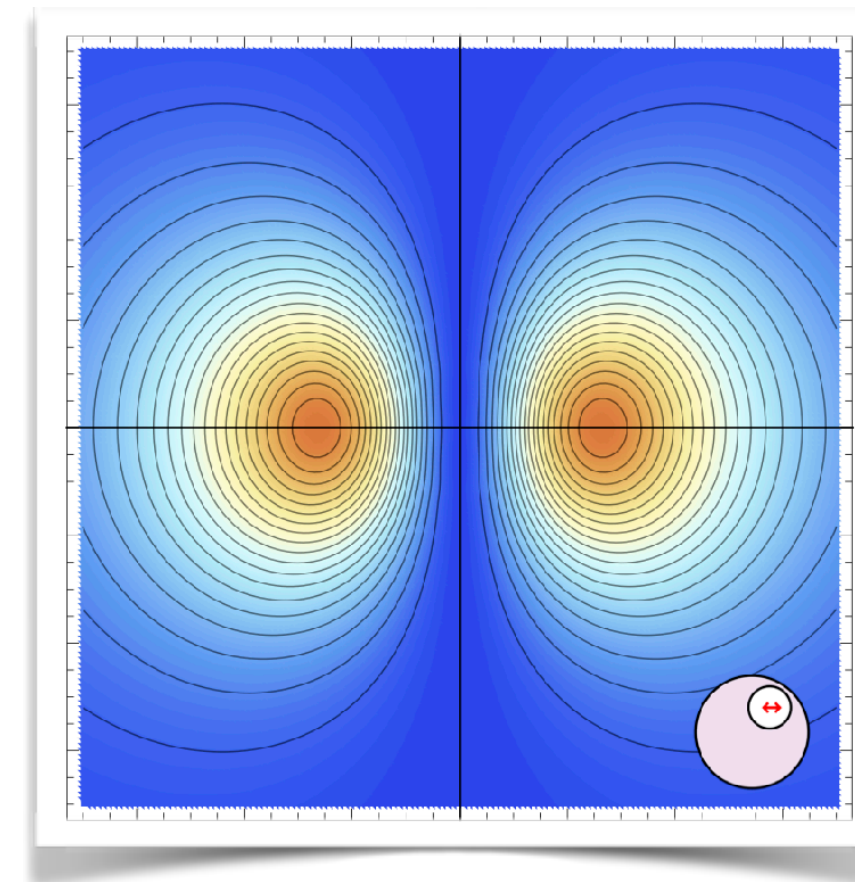
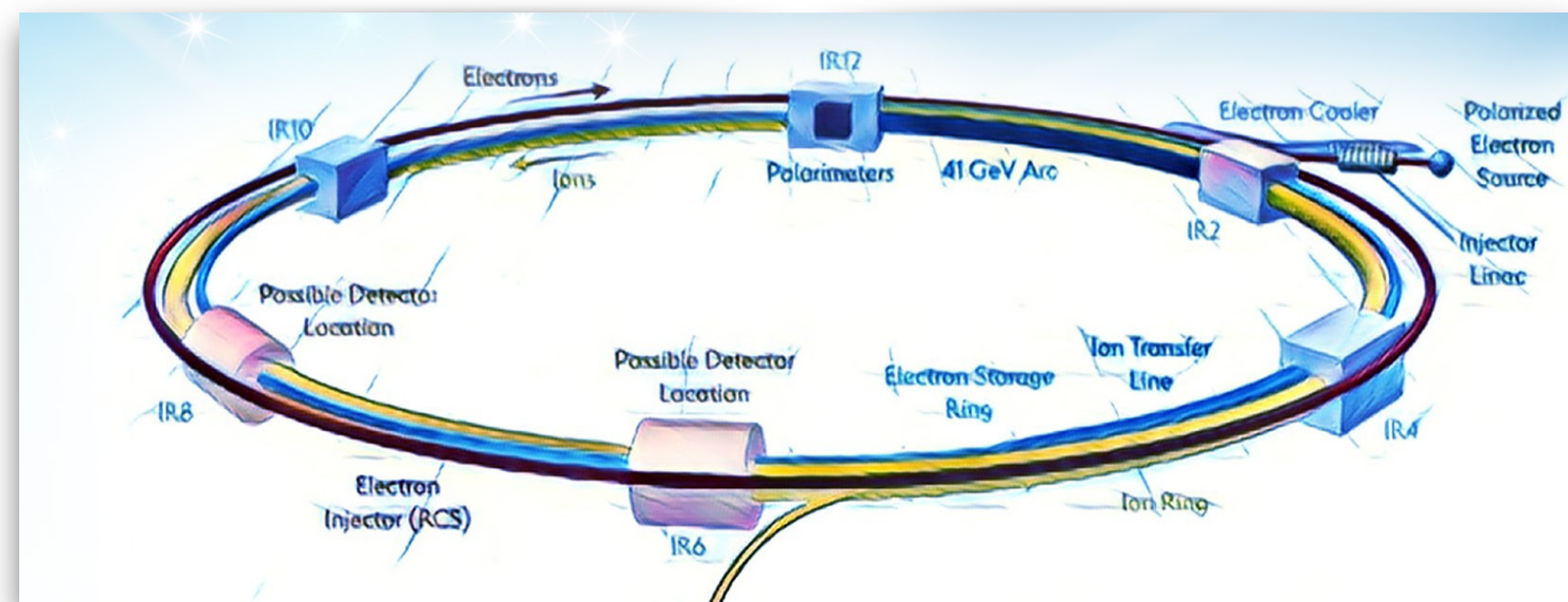
Higher-order calculations
TMD factorization properties

Proton reconstruction from EIC data
Artificial intelligence

CALCULUS

COMPUTING

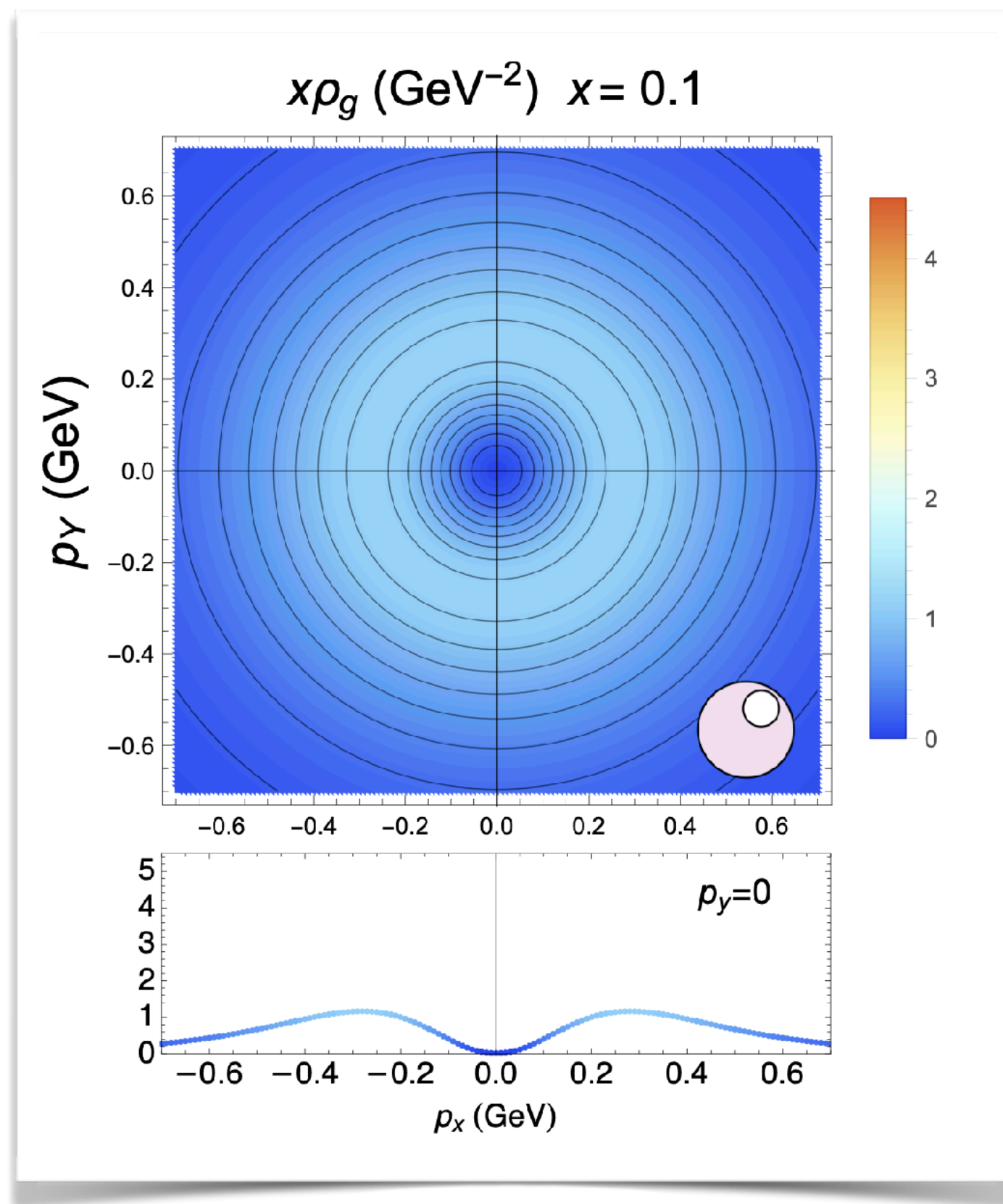
PHENO



3D proton imaging: Tomographic reconstruction & TMDs

[A. Bacchetta, F.G.C., M. Radici, P. Tael, Eur. Phys. J. C 80 (2020) no.8]

Unpolarized

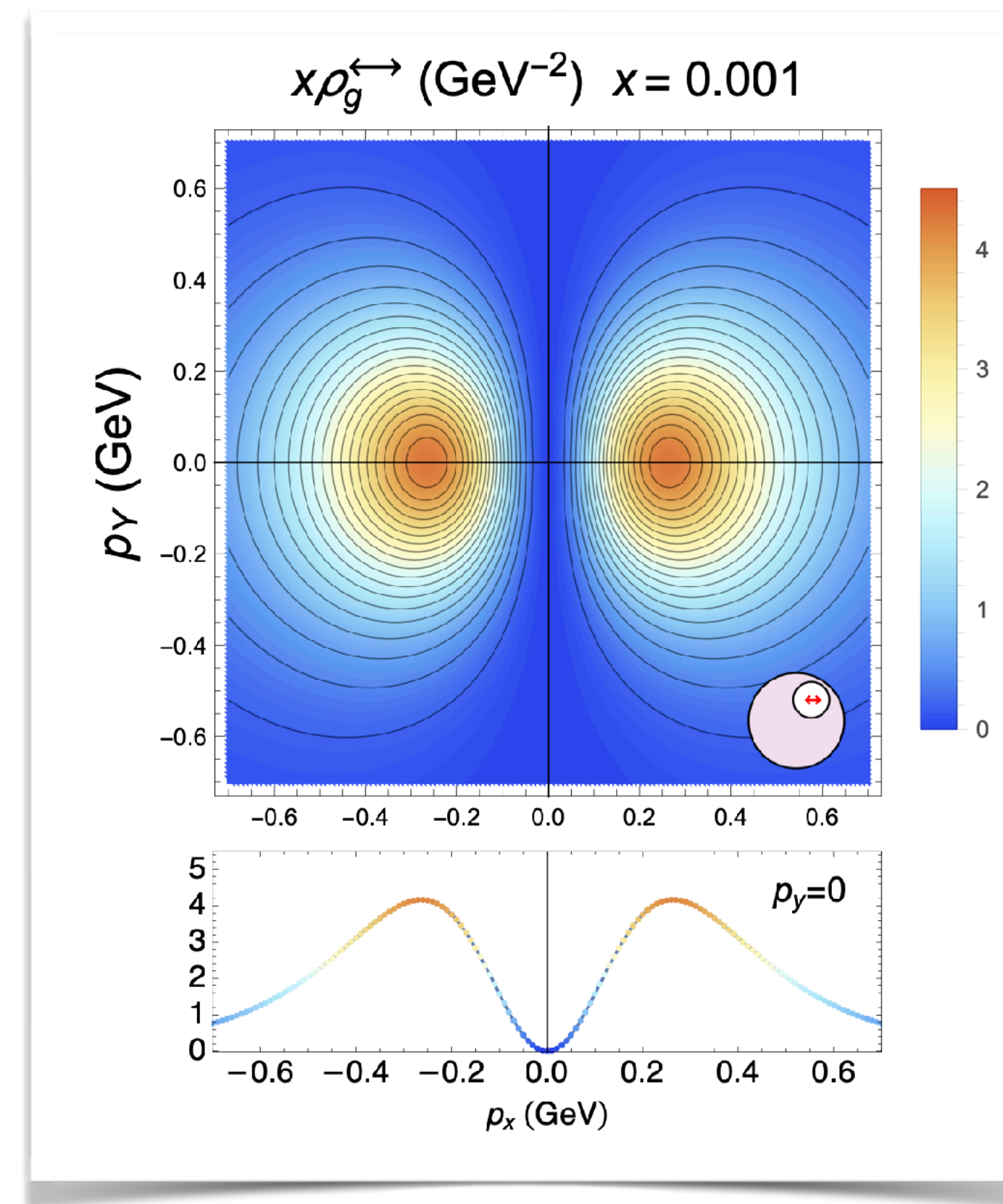
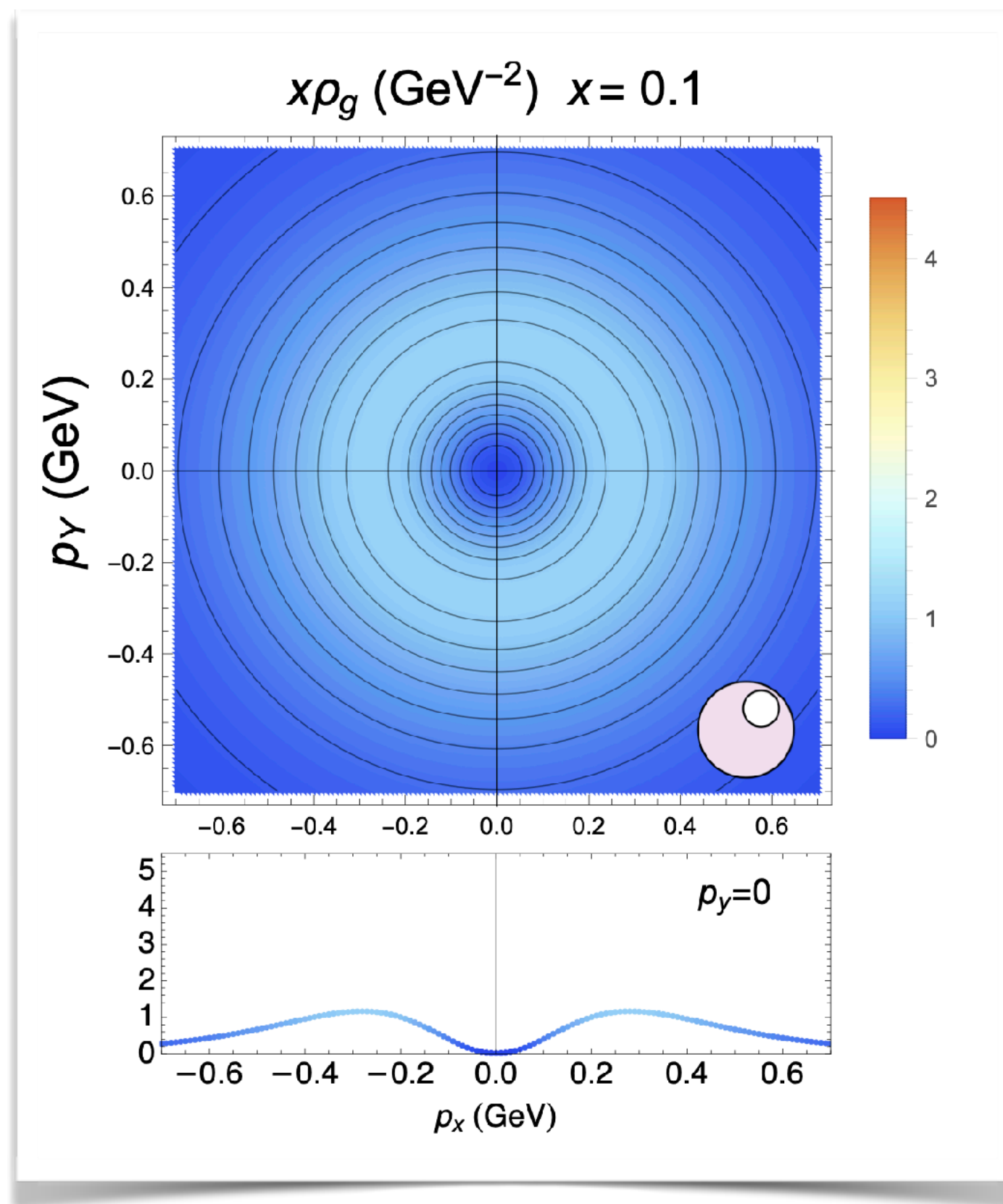


3D proton imaging: Tomographic reconstruction & TMDs

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Unpolarized

Boer-Mulders



$$x\rho^{\leftrightarrow}(x, p_x, p_y) = \frac{1}{2} \left[x f_1^g(x, \mathbf{p}_T^2) + \frac{p_x^2 - p_y^2}{2M^2} x h_1^{\perp g}(x, \mathbf{p}_T^2) \right]$$

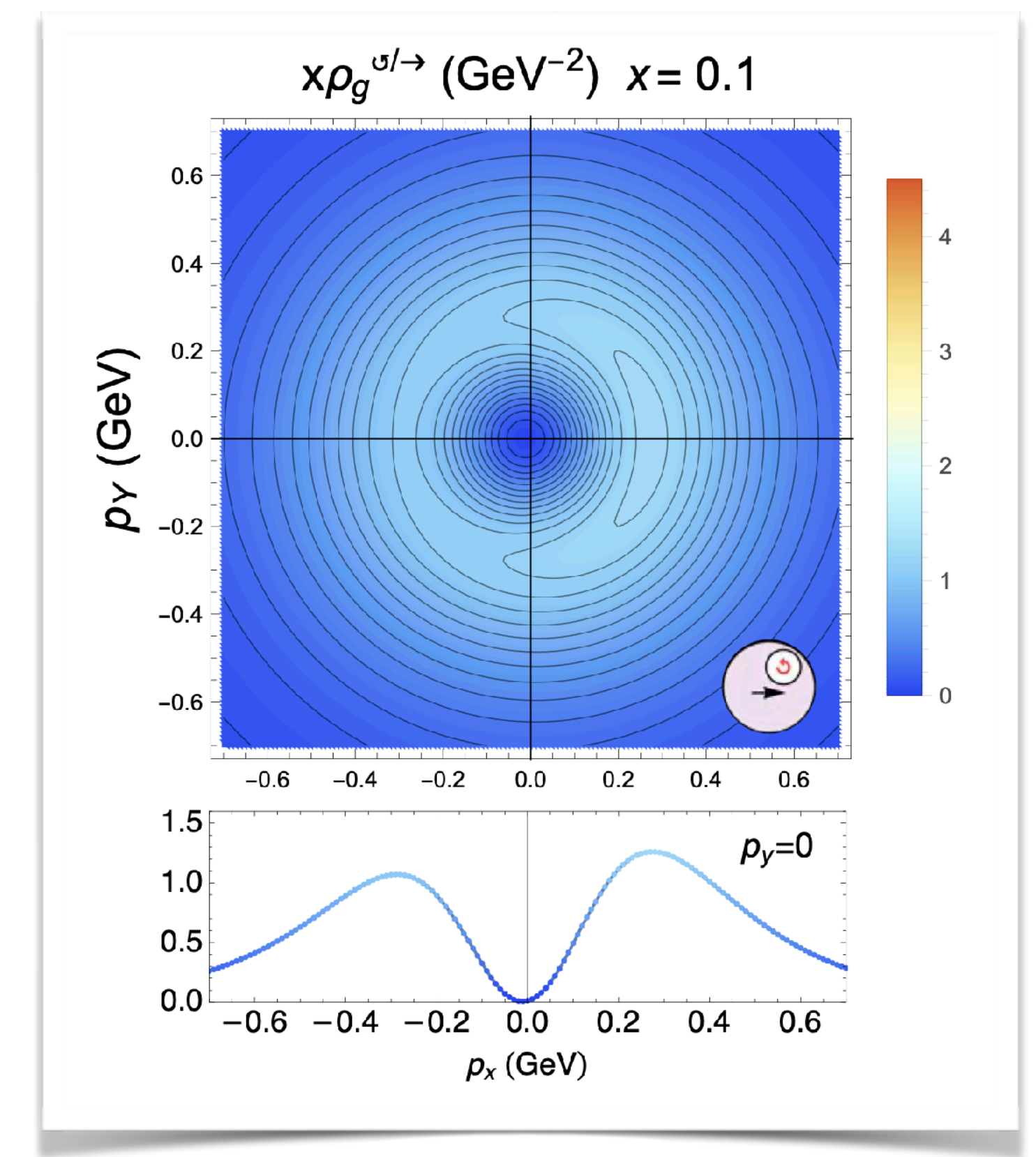
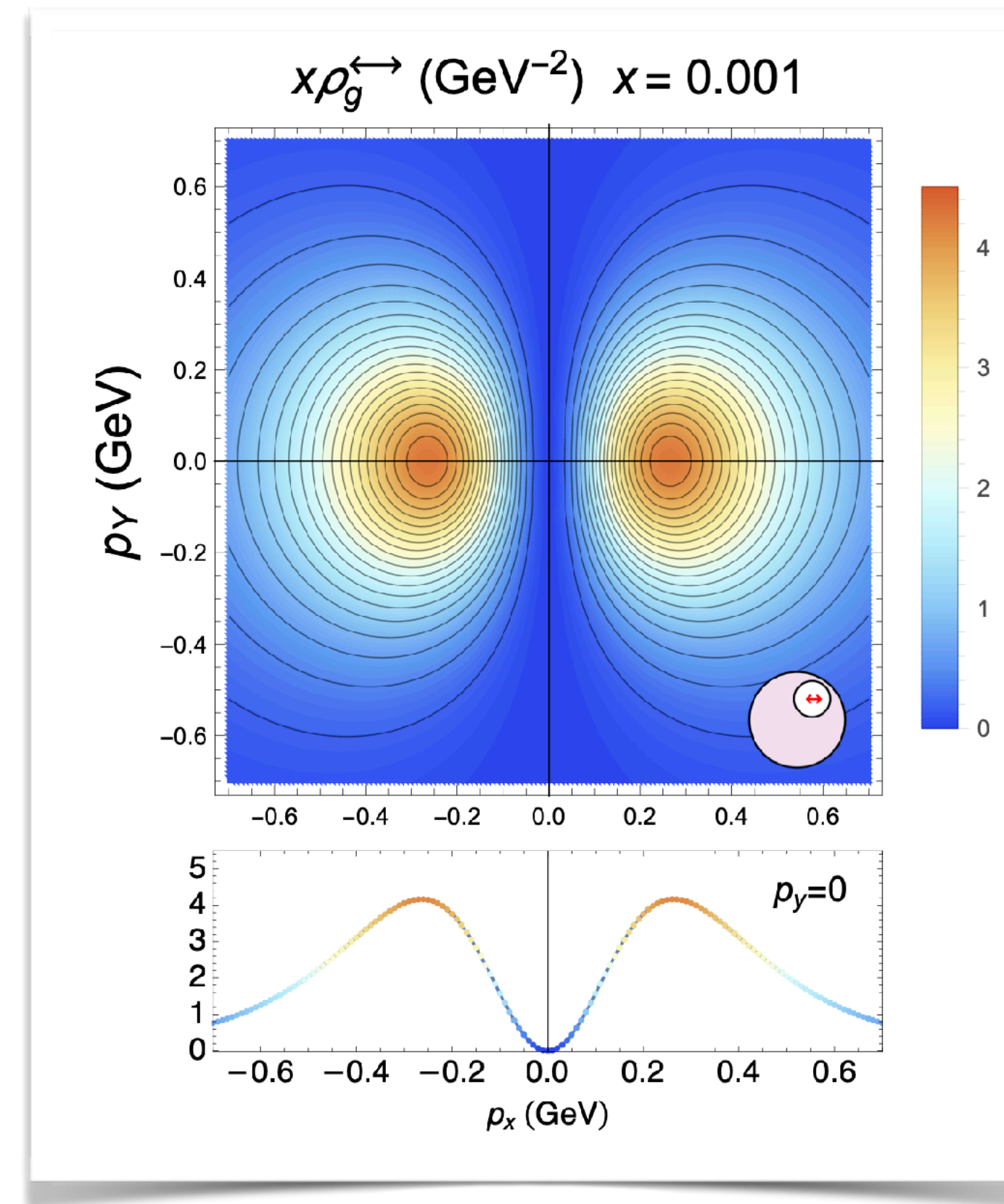
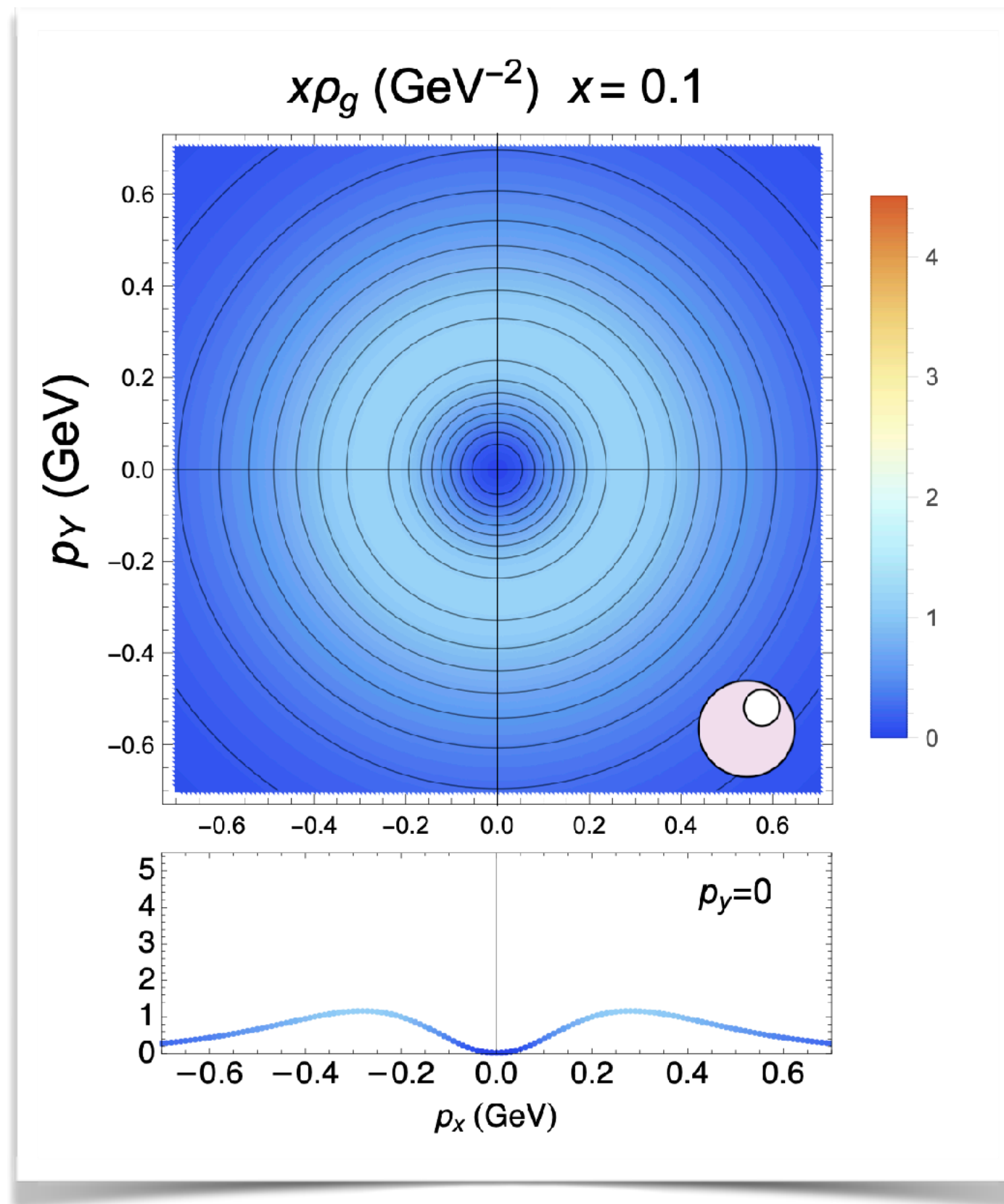
3D proton imaging: Tomographic reconstruction & TMDs

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Unpolarized

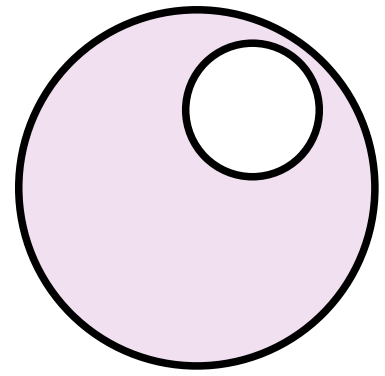
Boer-Mulders

Worm-gear

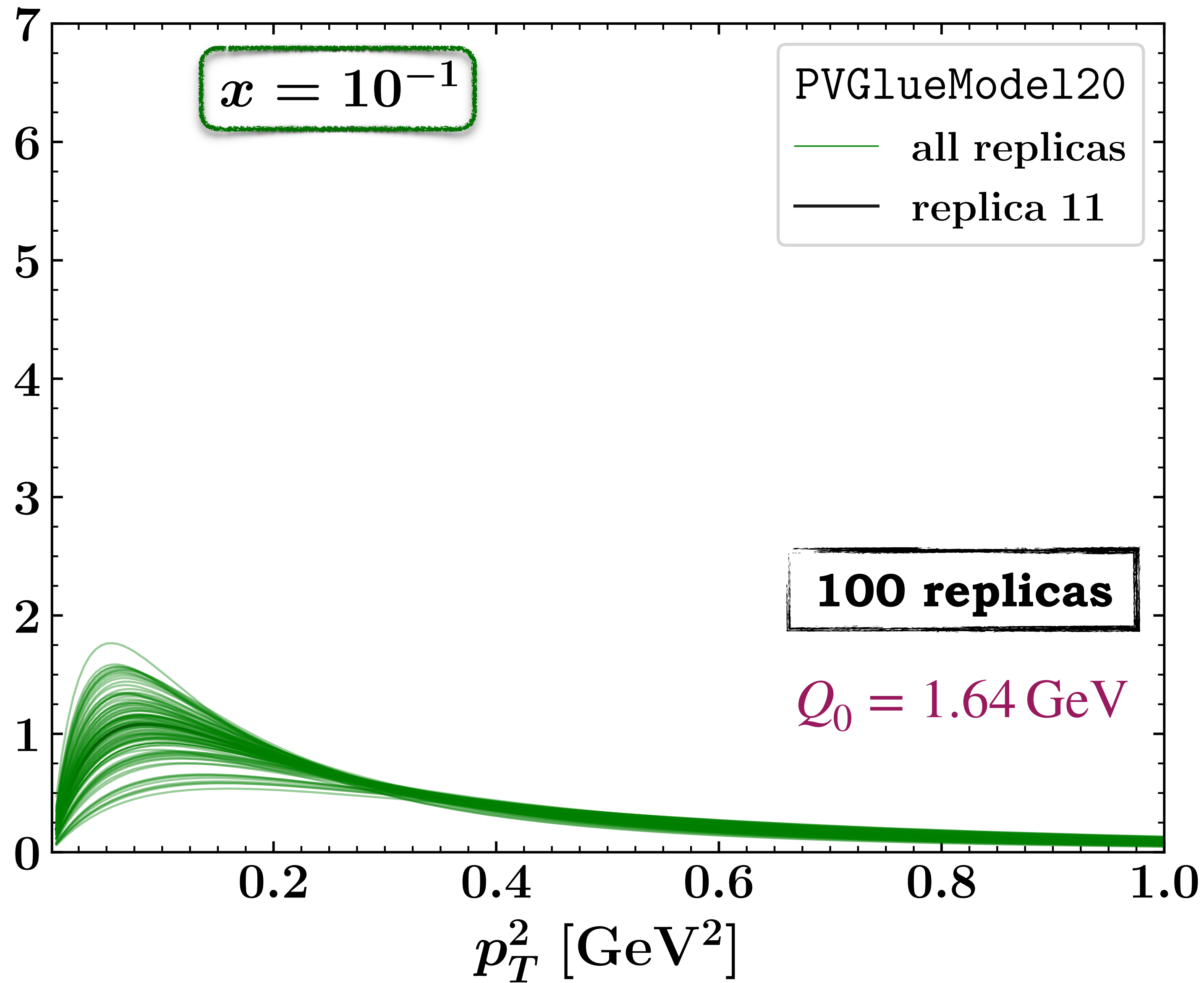


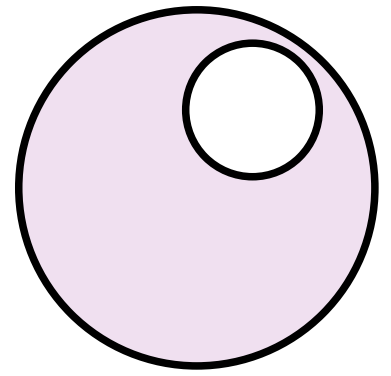
$$x\rho^{\leftrightarrow}(x, p_x, p_y) = \frac{1}{2} \left[x f_1^g(x, \mathbf{p}_T^2) + \frac{p_x^2 - p_y^2}{2M^2} x h_1^{\perp g}(x, \mathbf{p}_T^2) \right]$$

$$x\rho^{\ominus/\rightarrow}(x, p_x, p_y) = x f_1^g(x, \mathbf{p}_T^2) - \frac{p_x}{M} x g_{1T}^g(x, \mathbf{p}_T^2)$$

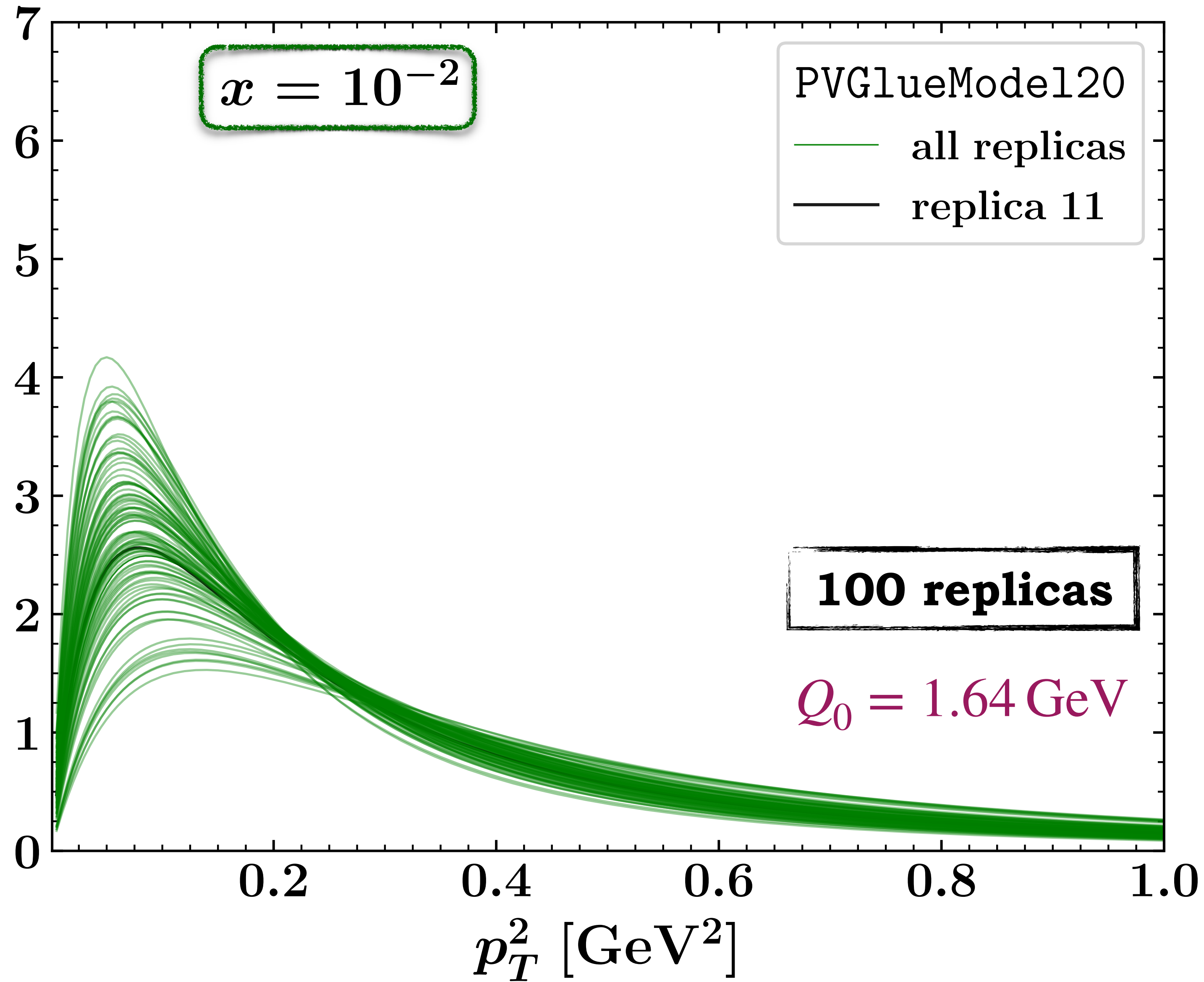


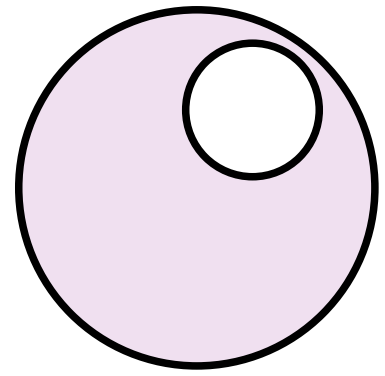
$$x f_1(x, p_T^2)$$



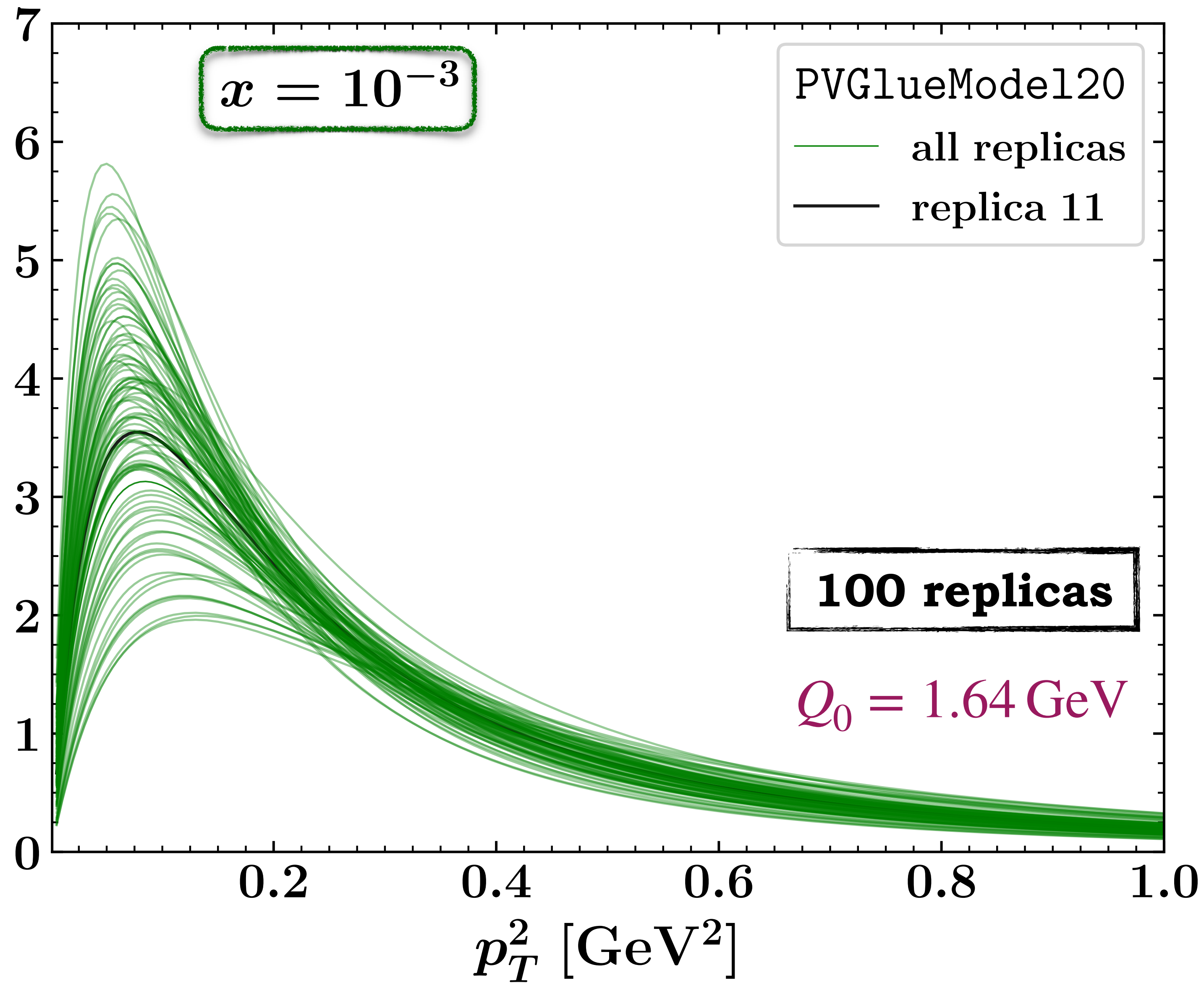


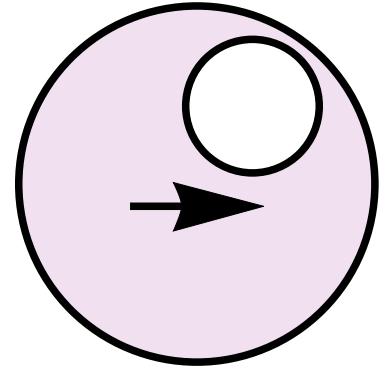
$$x f_1(x, p_T^2)$$



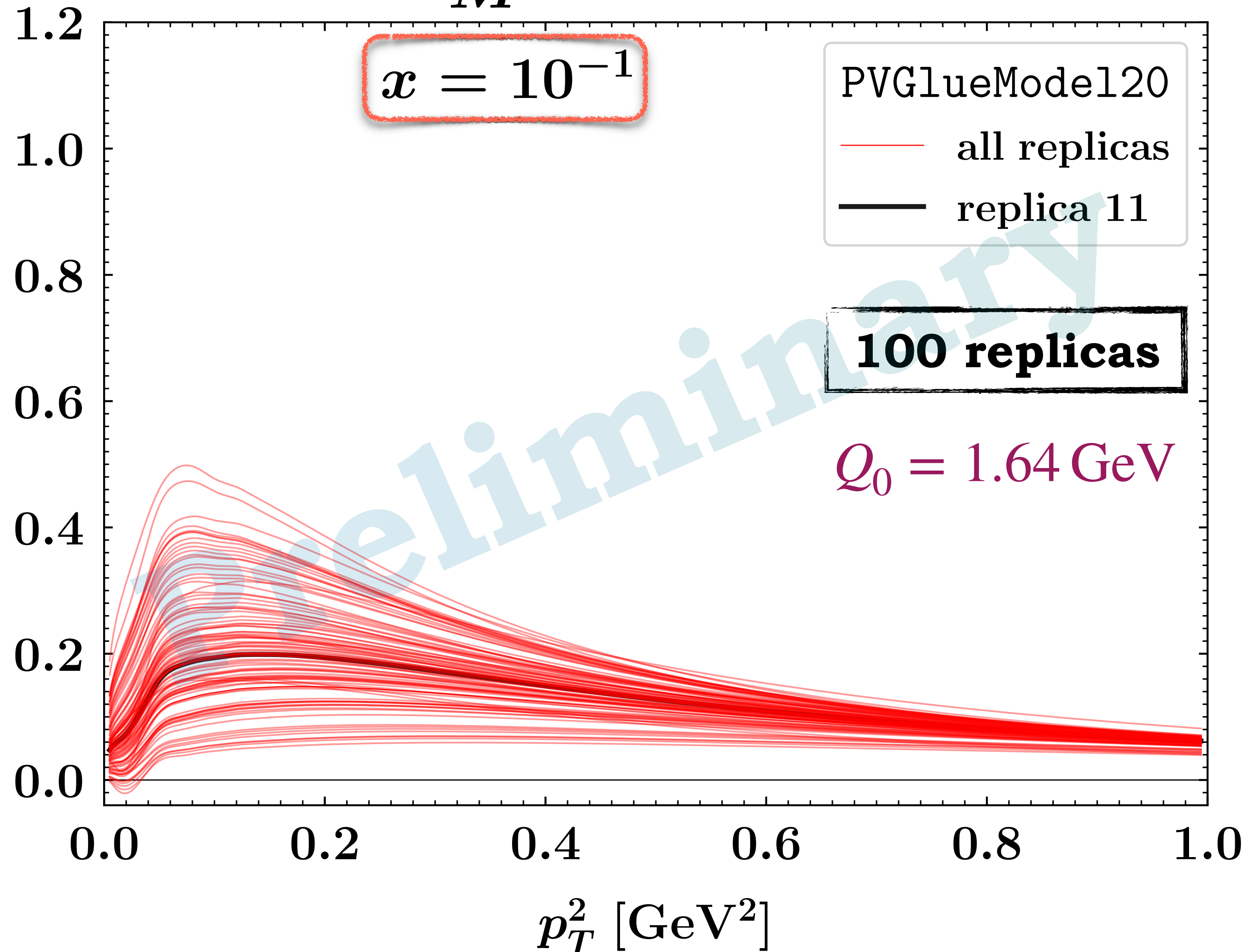


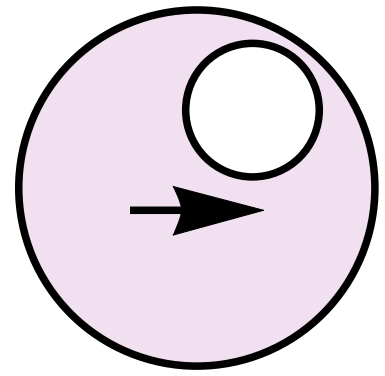
$$x f_1(x, p_T^2)$$



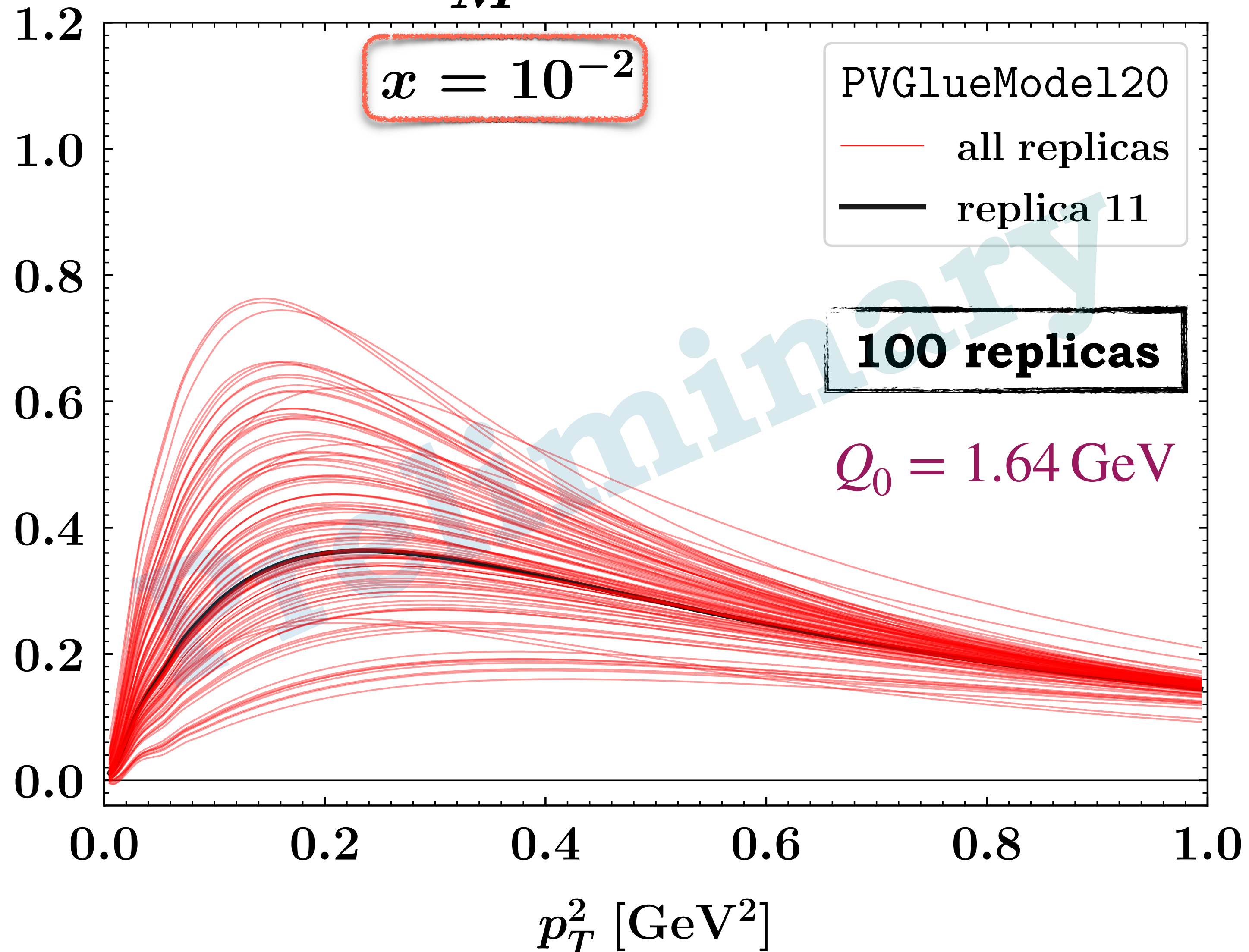


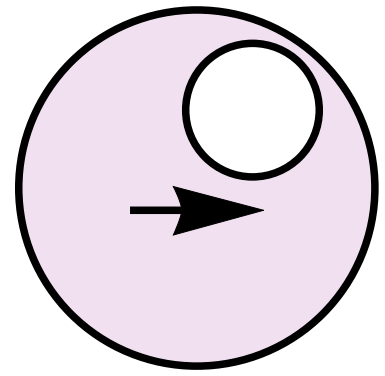
$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



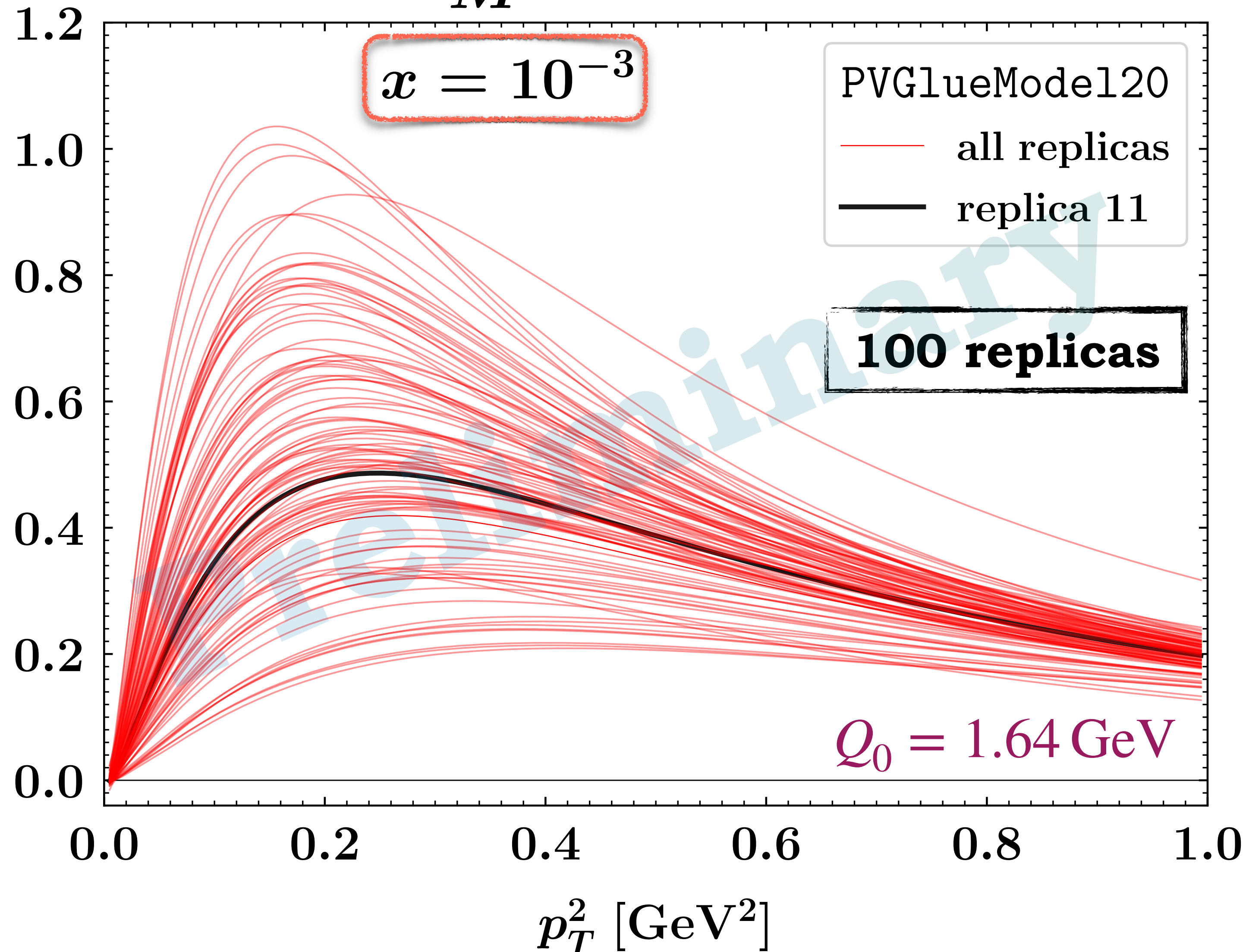


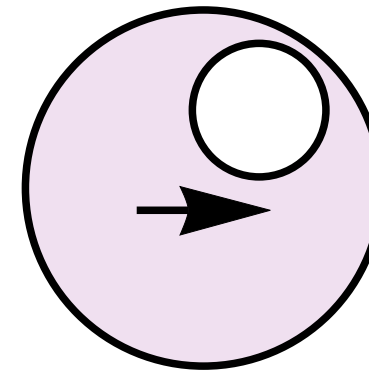
$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



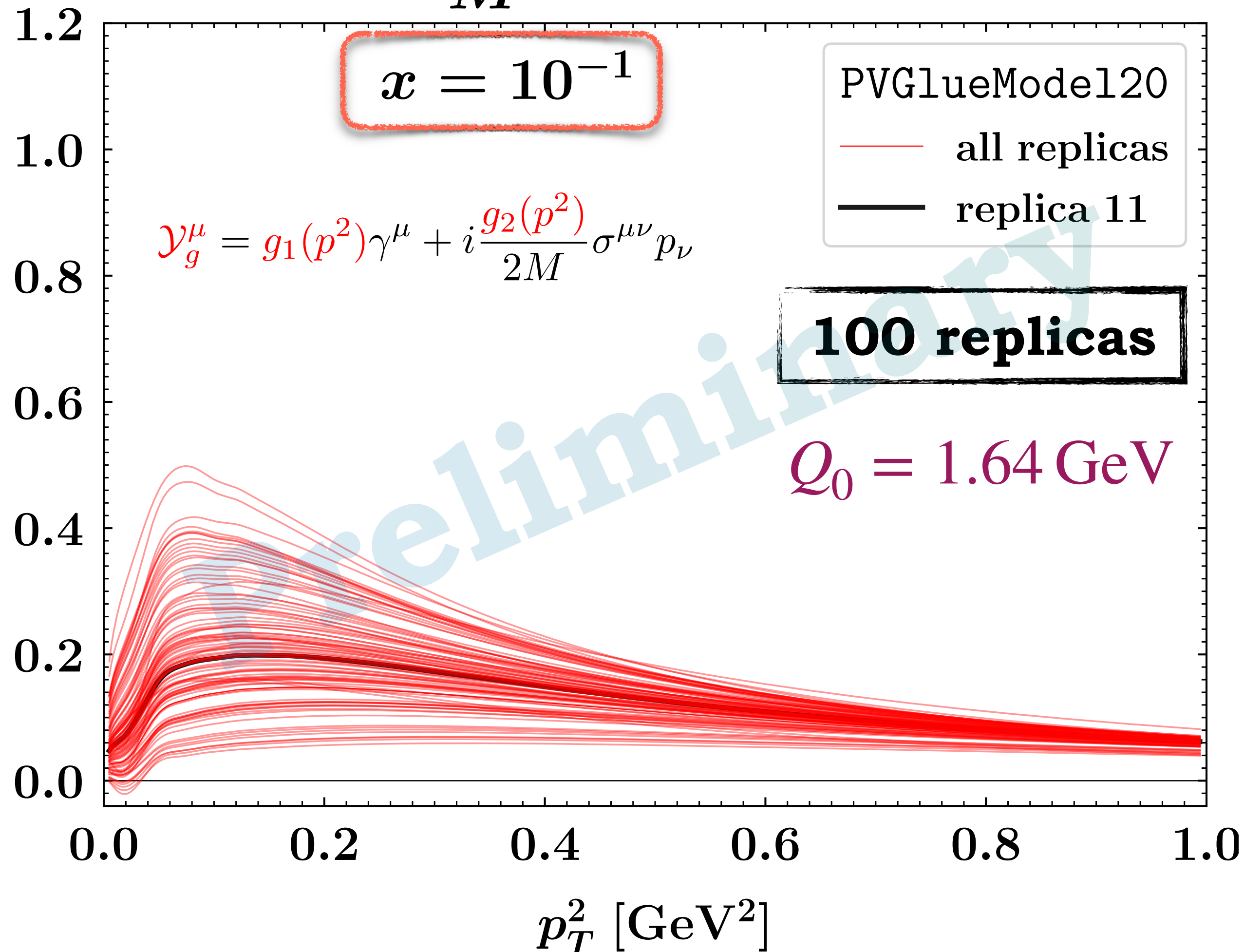


$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$

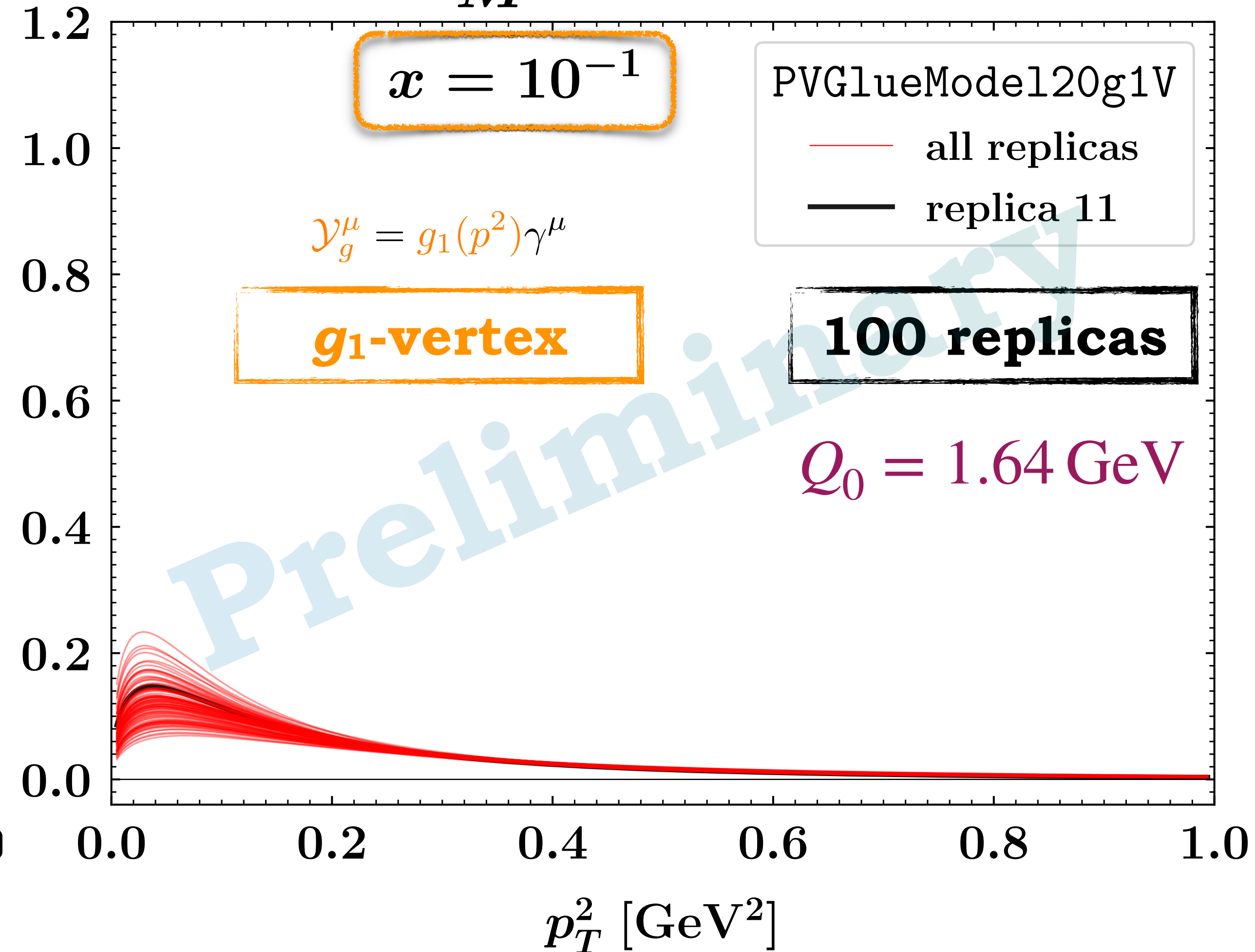


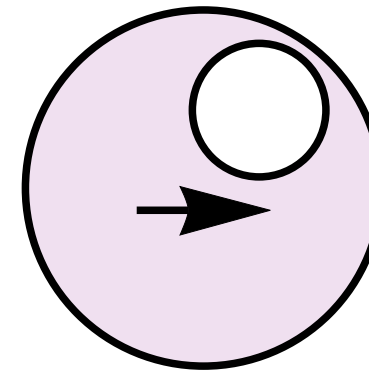


$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$

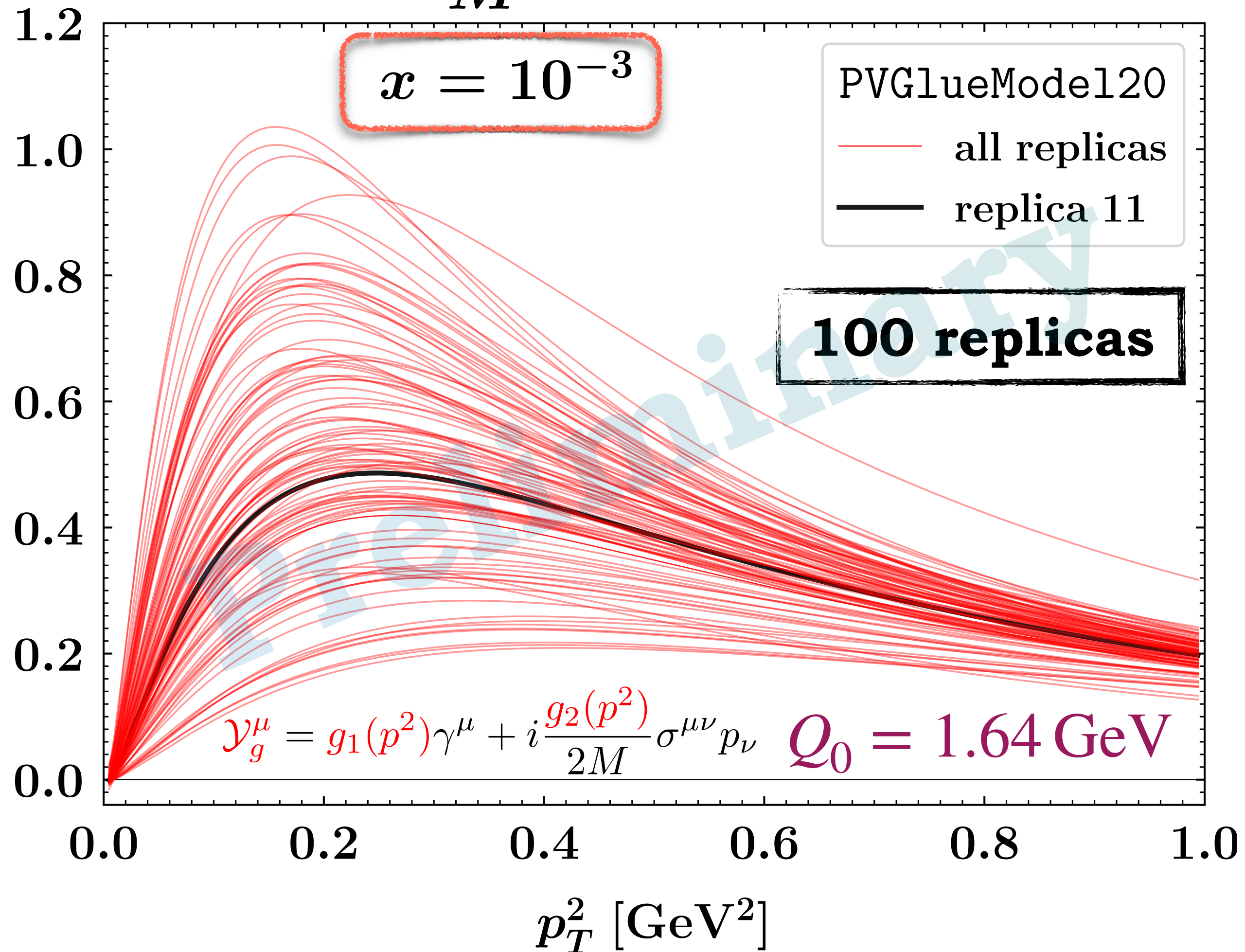


$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$

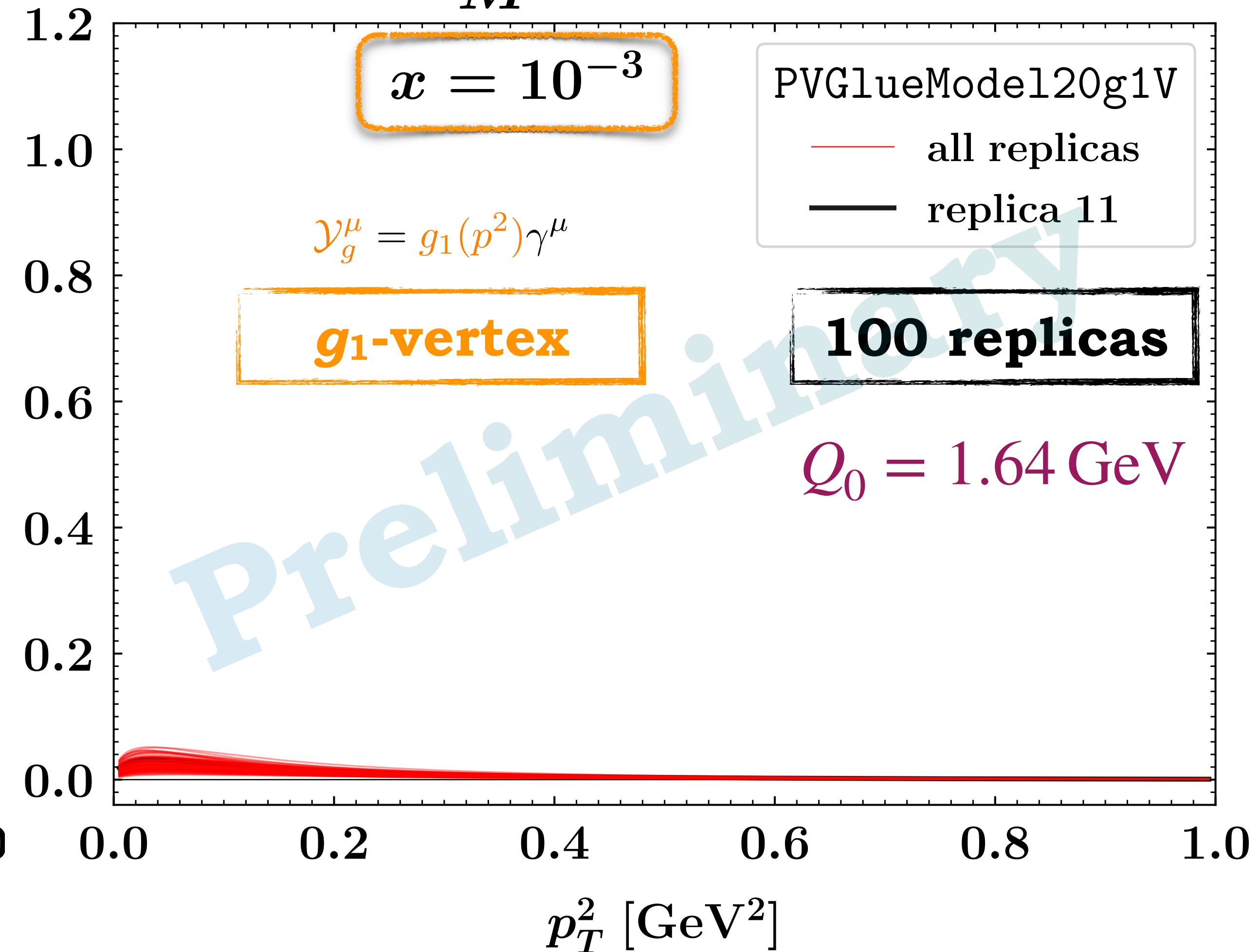




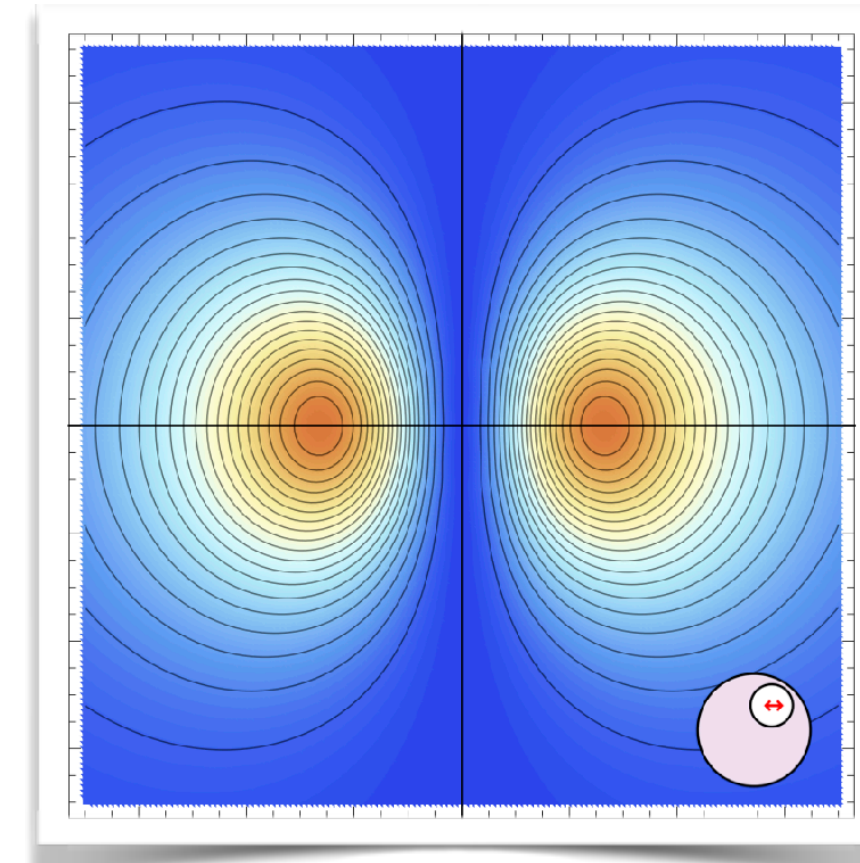
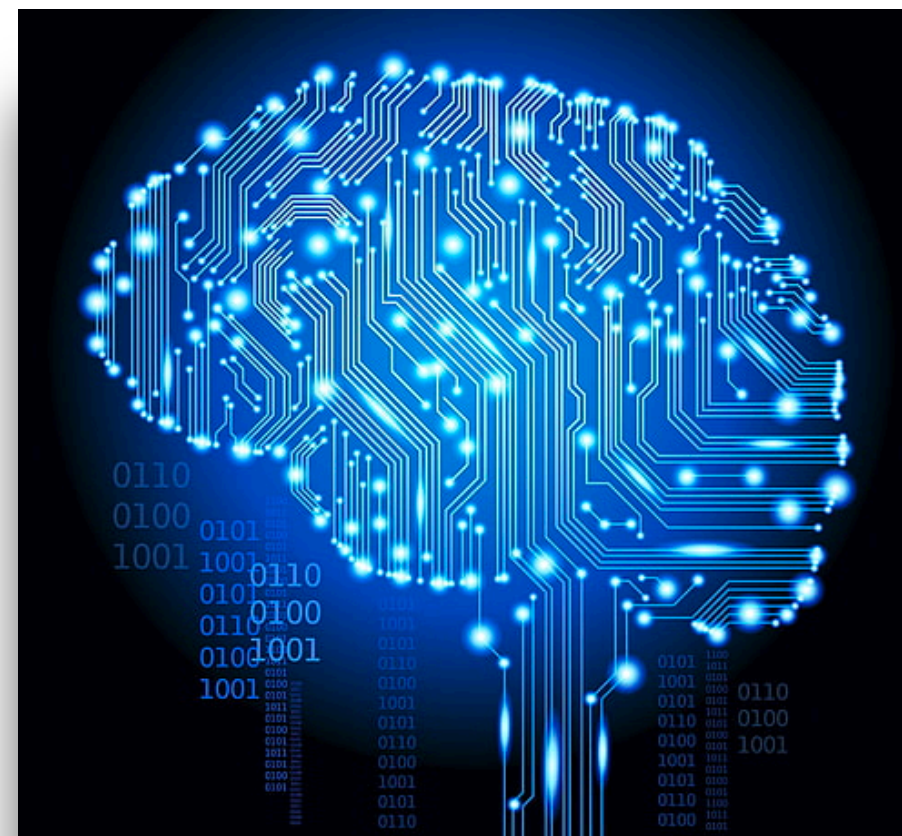
$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



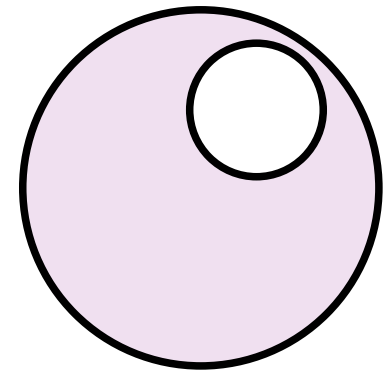
$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



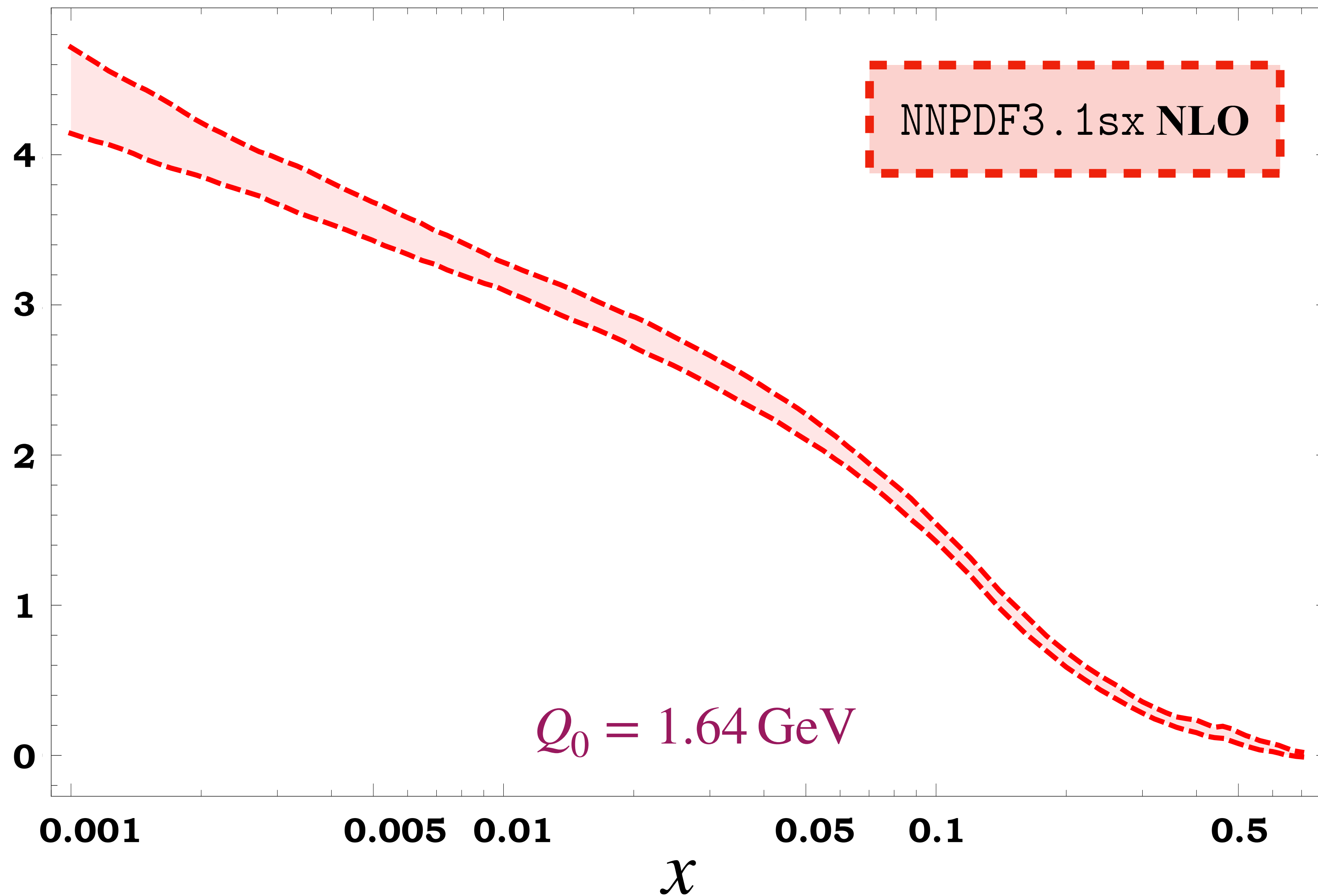
COMPUTING



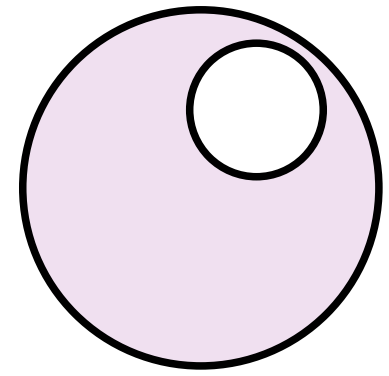
Unpolarized gluon collinear PDF



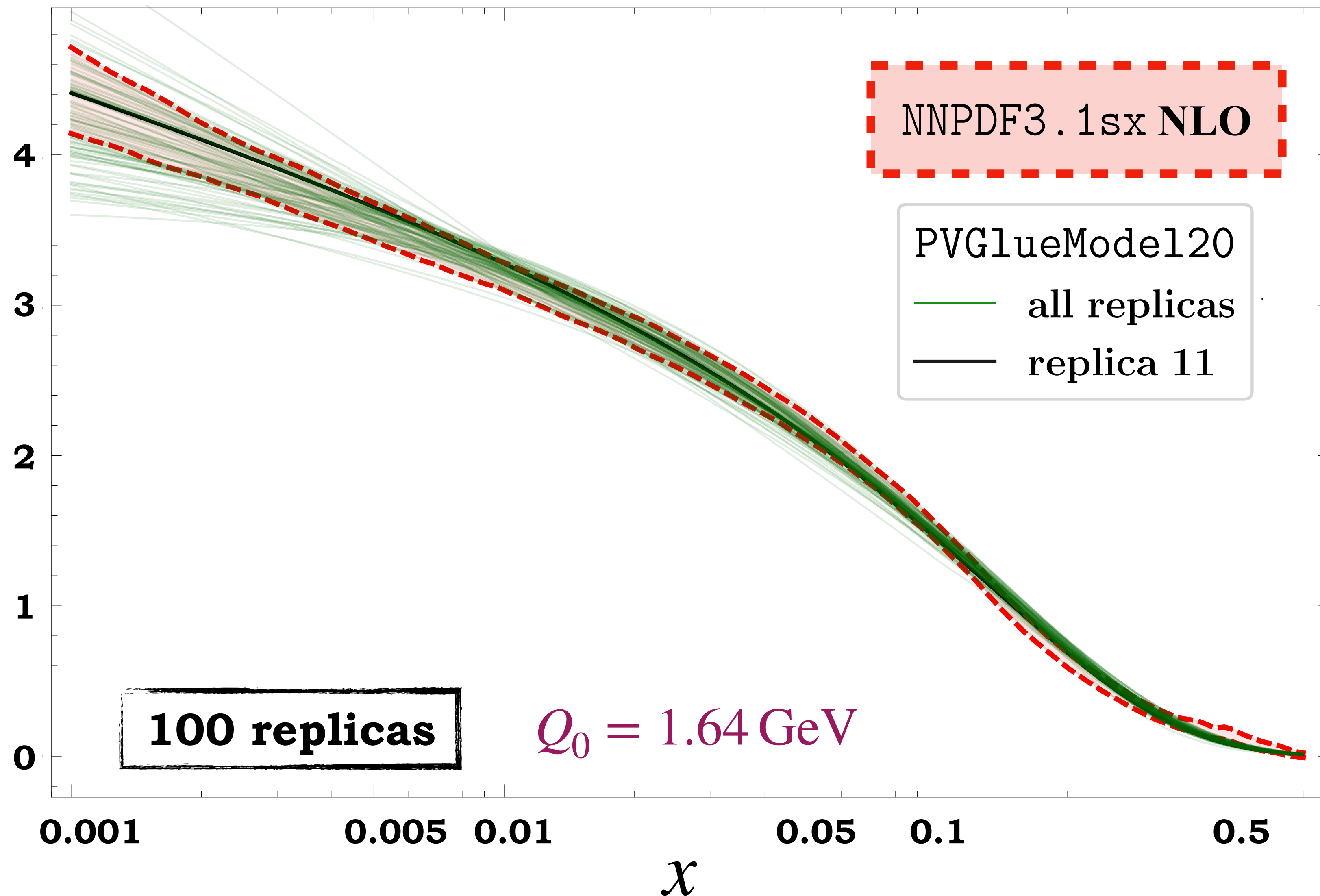
$$x f_1(x)$$



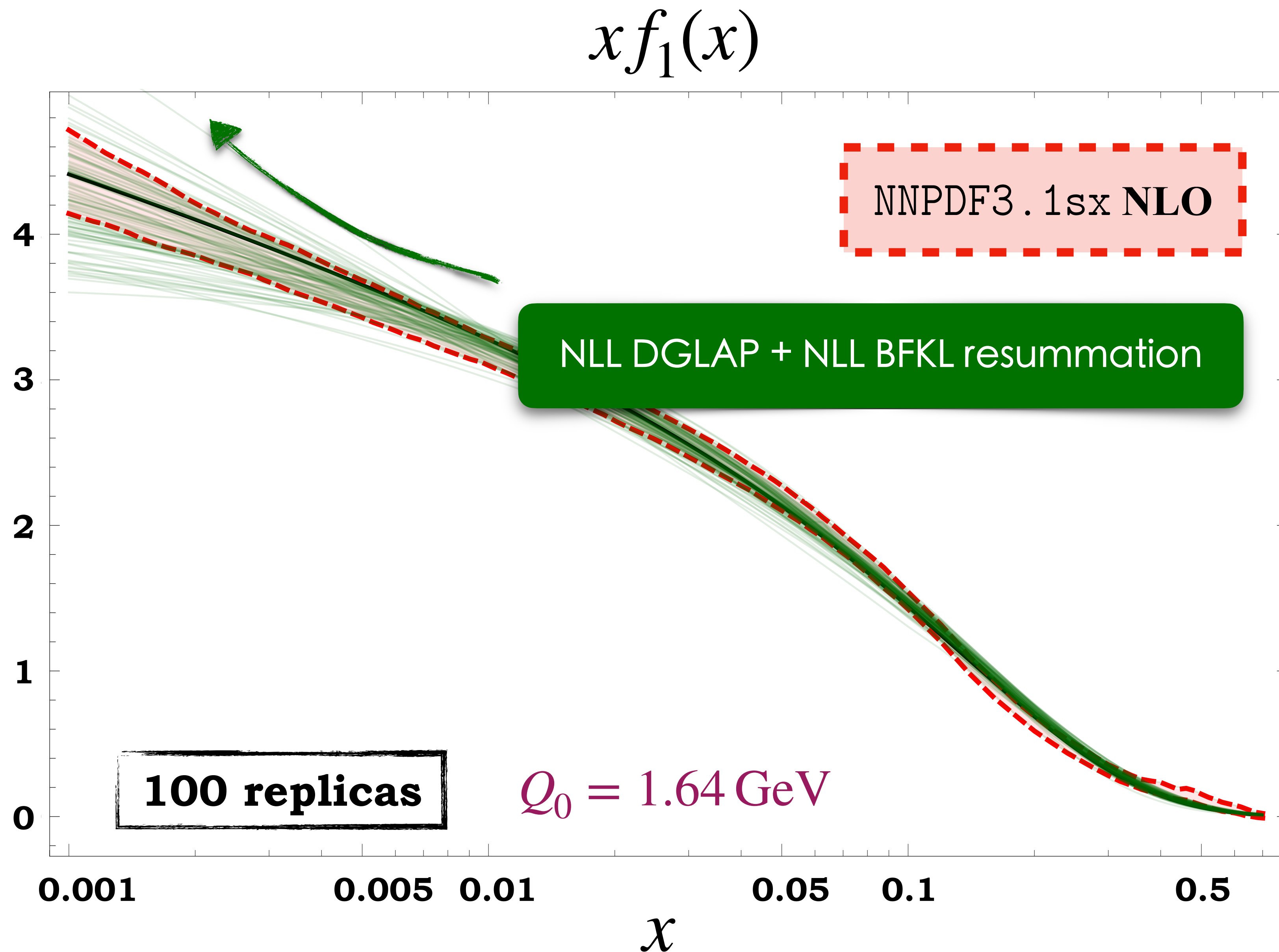
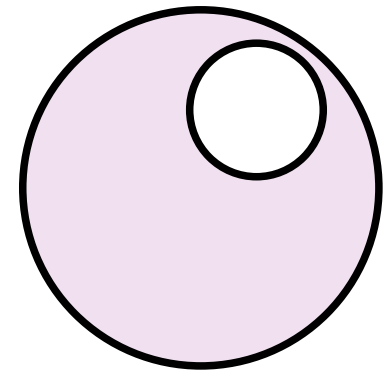
Unpolarized gluon collinear PDF



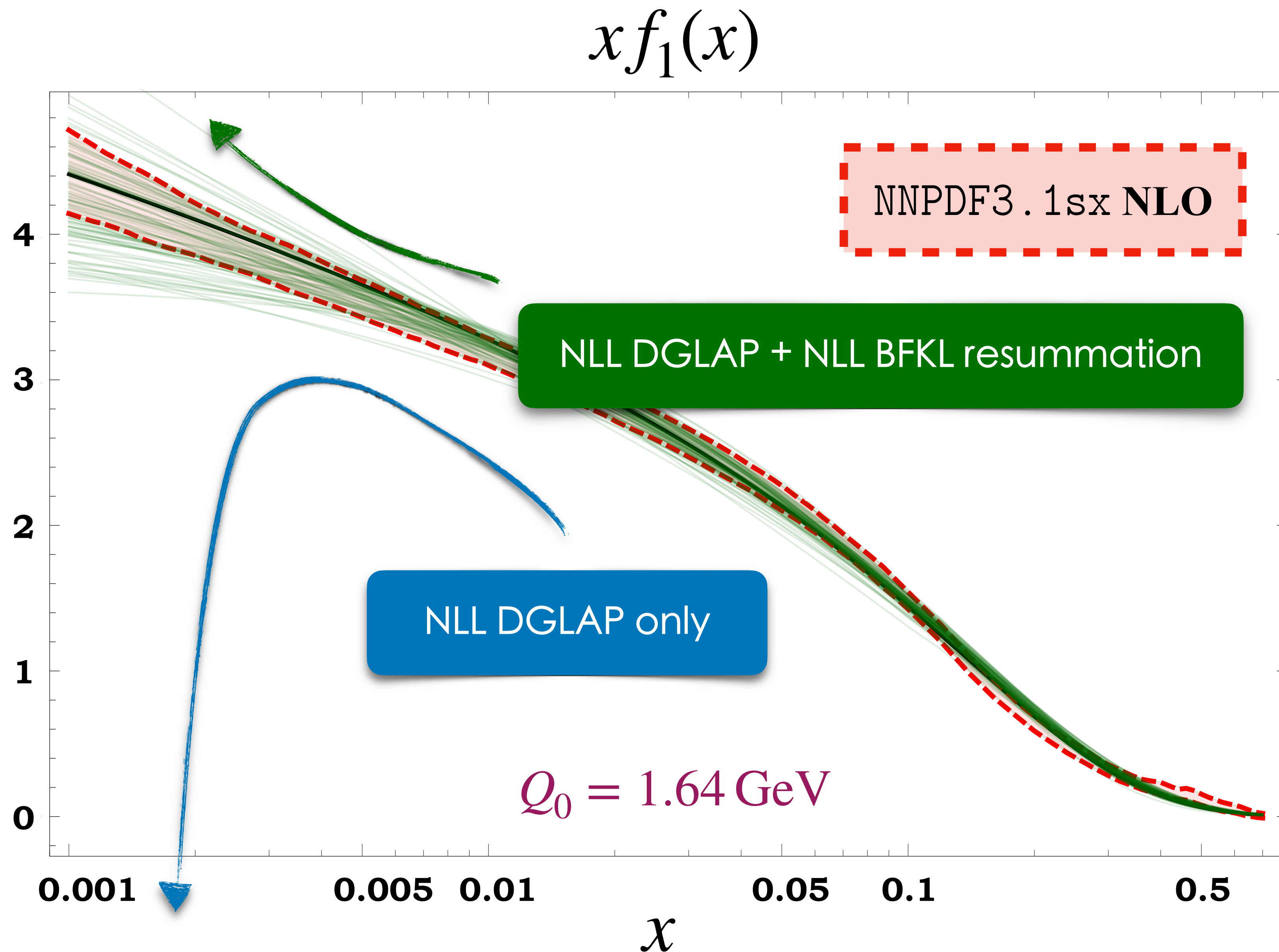
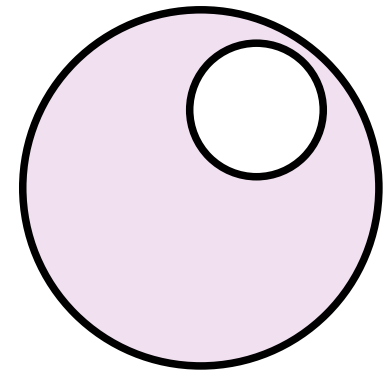
$$x f_1(x)$$



Unpolarized gluon collinear PDF



Unpolarized gluon collinear PDF



j C'mon, this is a traditional fit !



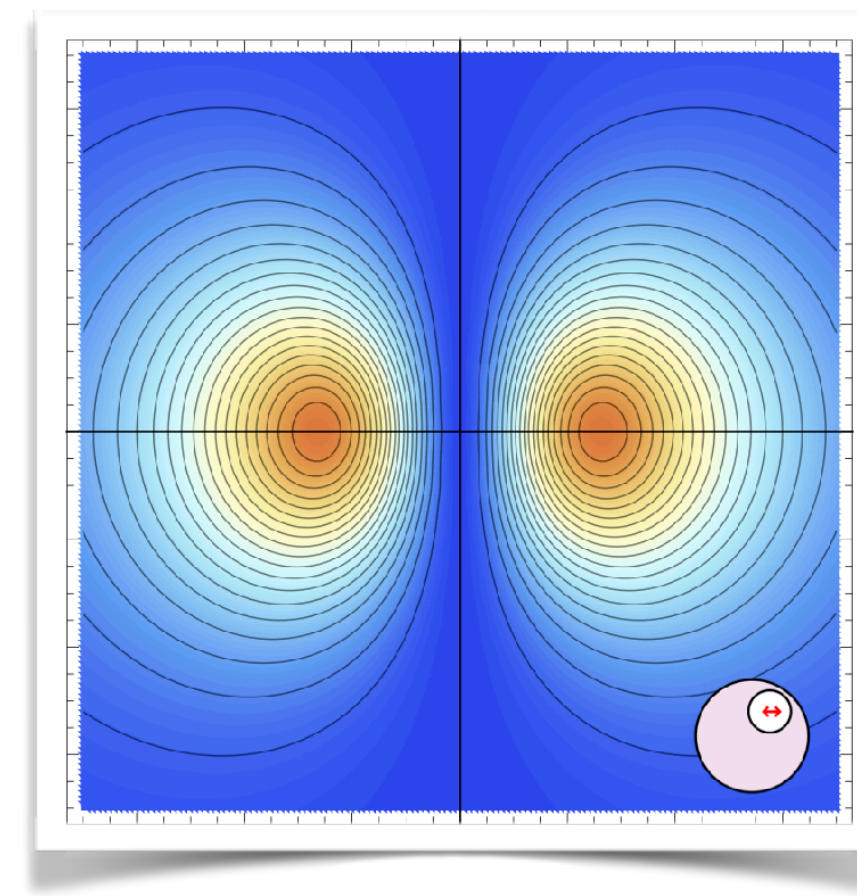
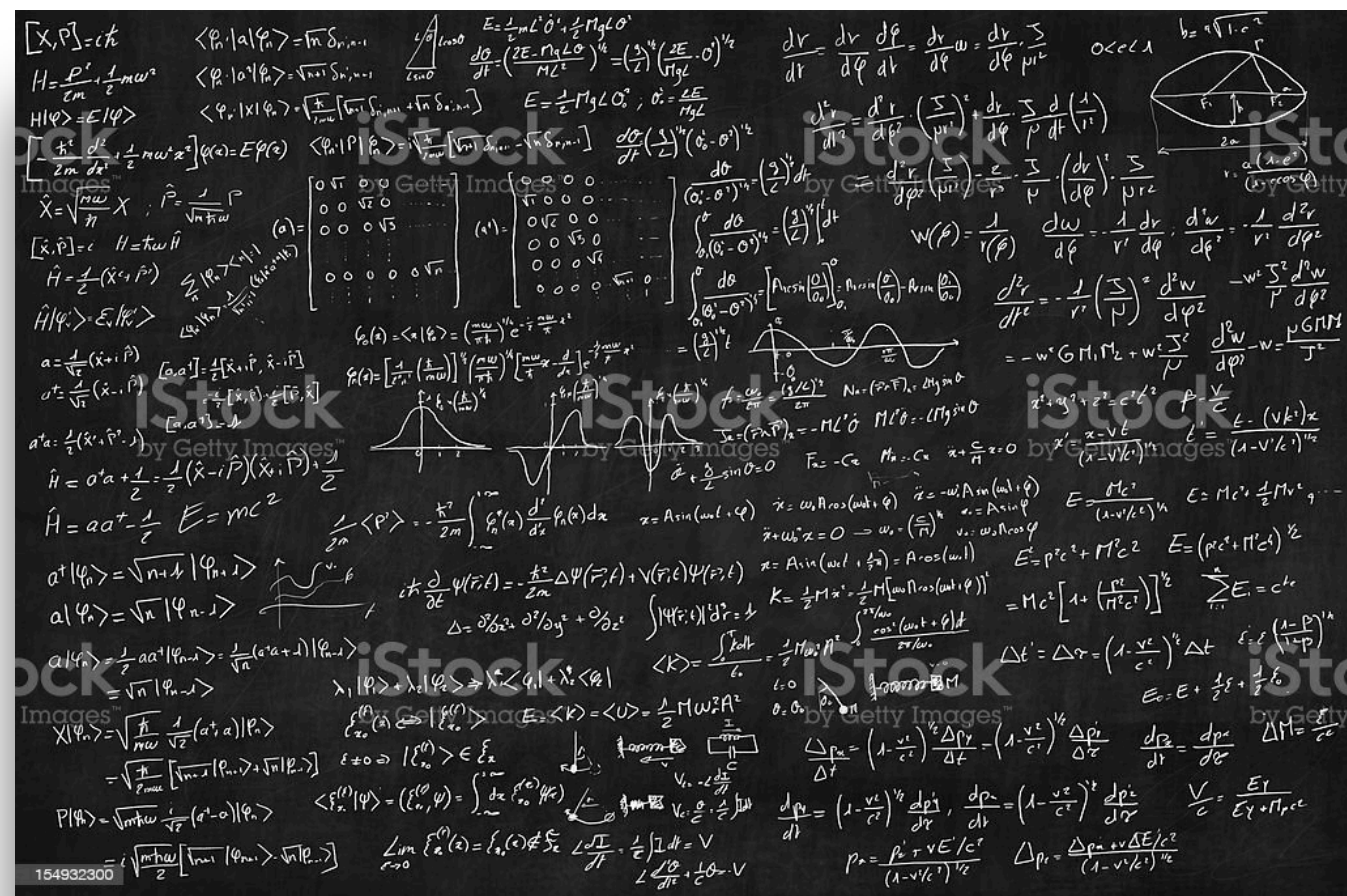
j C'mon, this is a traditional fit !



 [Tomorrow's lecture by [Stefano Forte](#)]

Yes, that fit works for a model...
...but EIC data will need AI

CALCULUS



Gluon TMD PDFs: Gauge links & modified universality

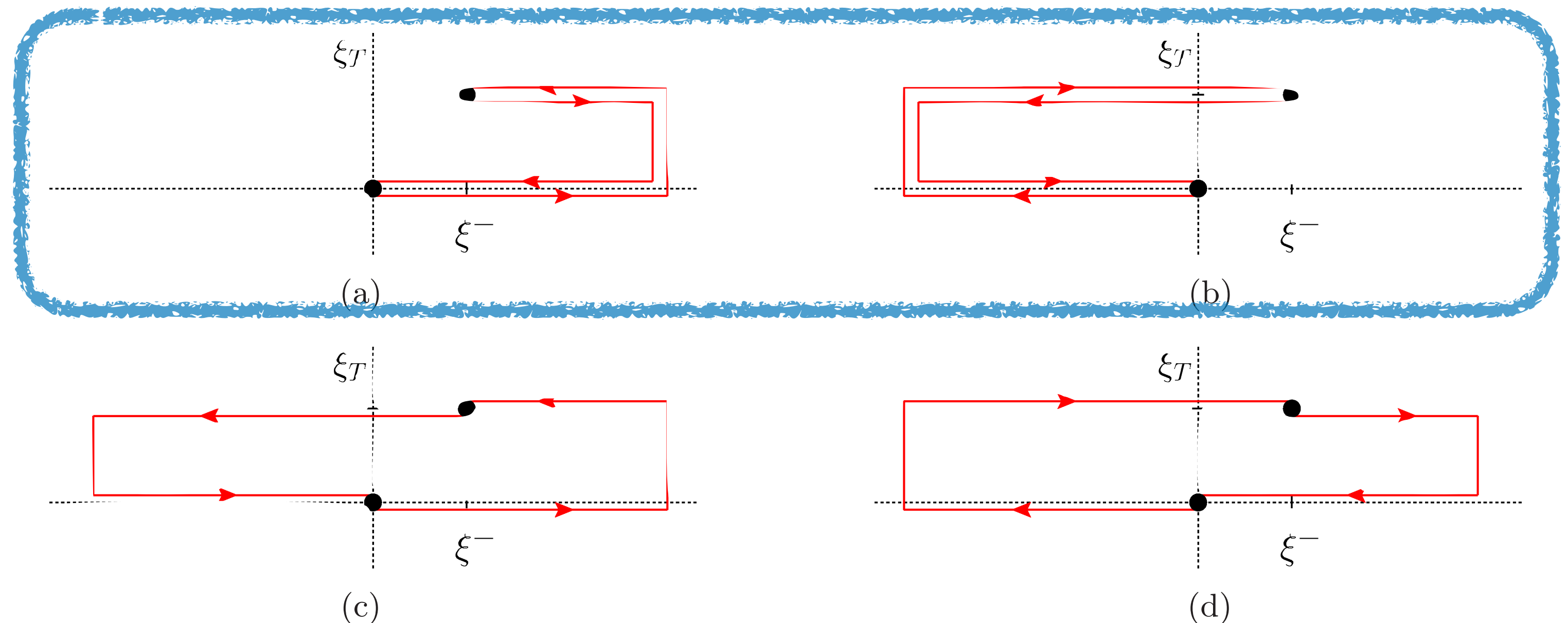
- * Gluon TMDs → more complicated structure with respect to quark staple links
- * Factorization-preserving processes → two main kinds of modified universality
- * Different classes of processes → distinct gluon TMDs, not related to each other

Gluon TMD PDFs: Gauge links & modified universality

- * Gluon TMDs \rightarrow more complicated structure with respect to quark staple links
- * Factorization-preserving processes \rightarrow two main kinds of modified universality
- * Different classes of processes \rightarrow distinct gluon TMDs, not related to each other

f-type (WW)

(a) [+ , +] or (b) [- , -]

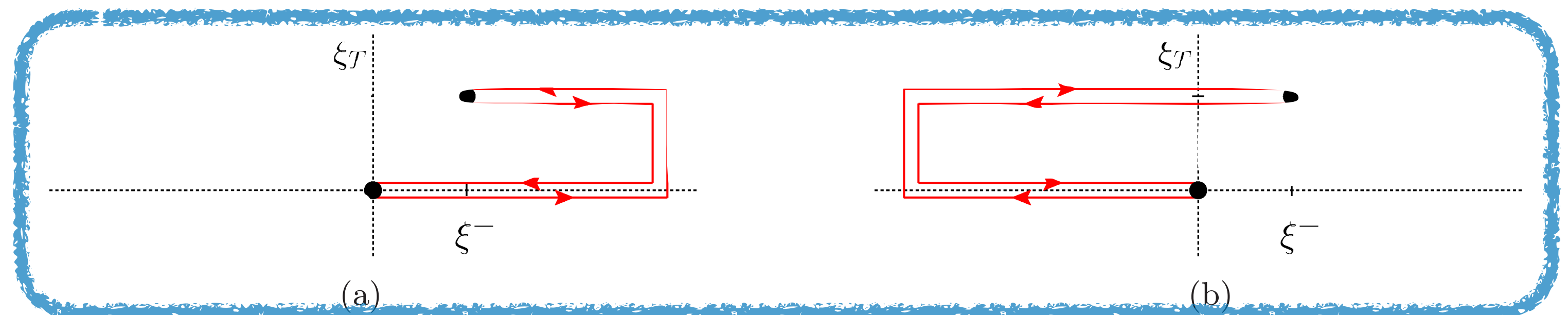


Gluon TMD PDFs: Gauge links & modified universality

- * Gluon TMDs → more complicated structure with respect to quark staple links
- * Factorization-preserving processes → two main kinds of modified universality
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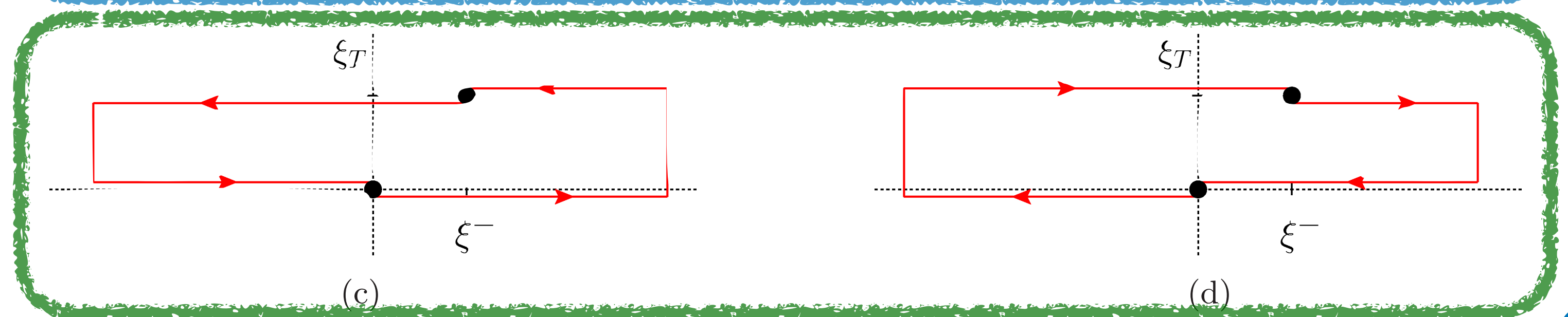
f-type (WW)

(a) [+ , +] or (b) [- , -]



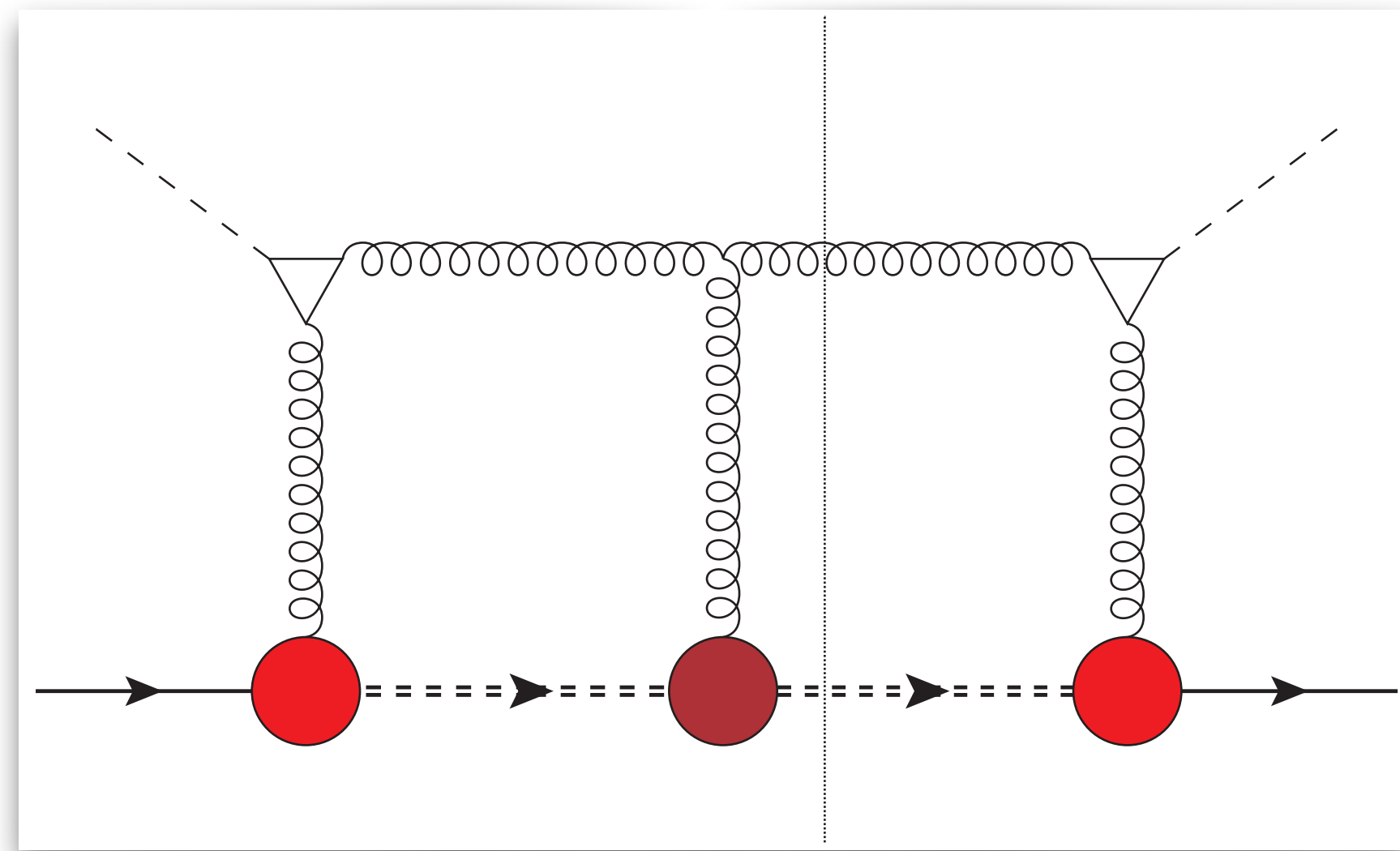
d-type (dipole)

(c) [+ , -] or (d) [- , +]

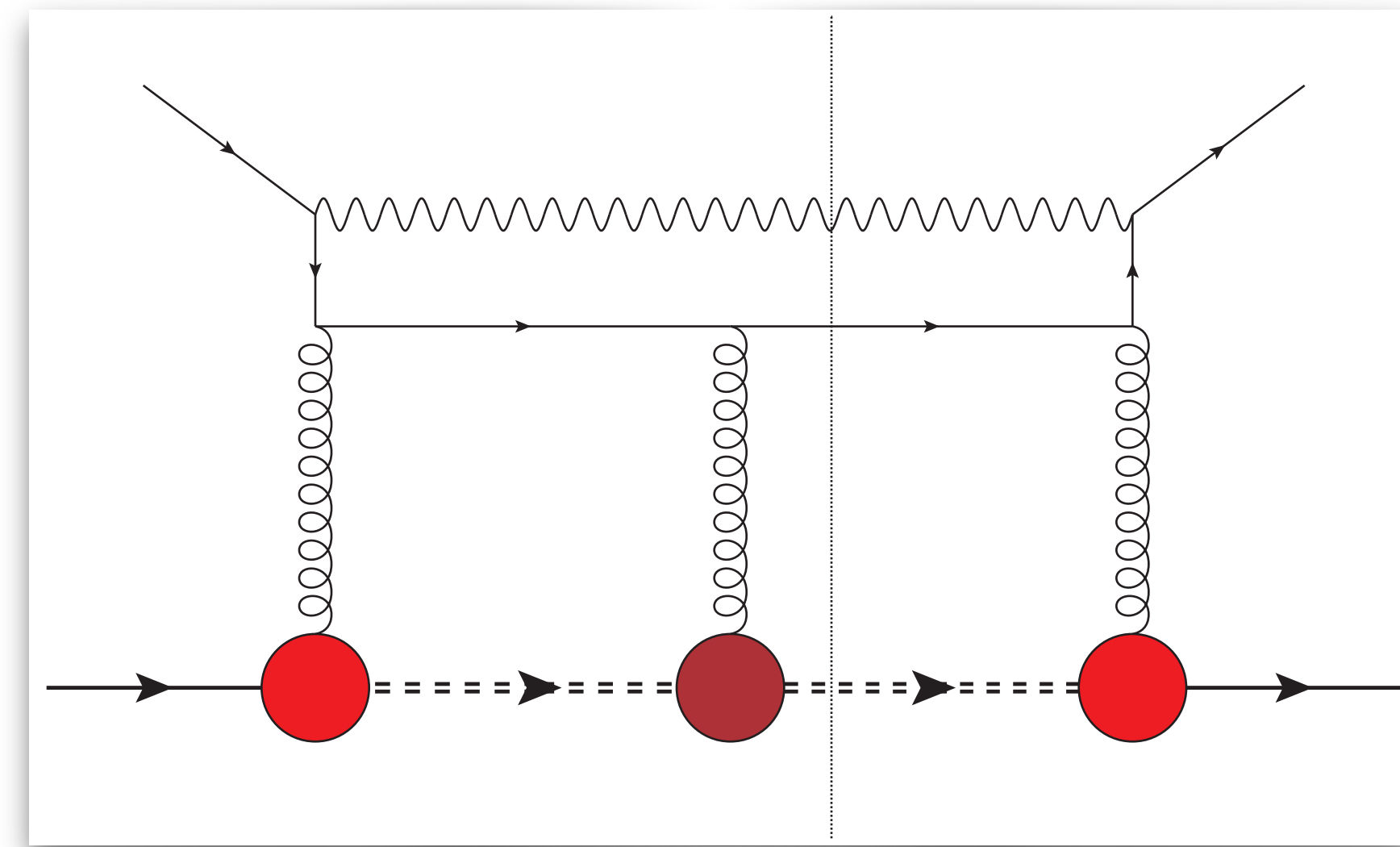


T-odd gluon TMDs in a spectator model

Higgs-gluon fusion \Rightarrow f-type [+ , +]



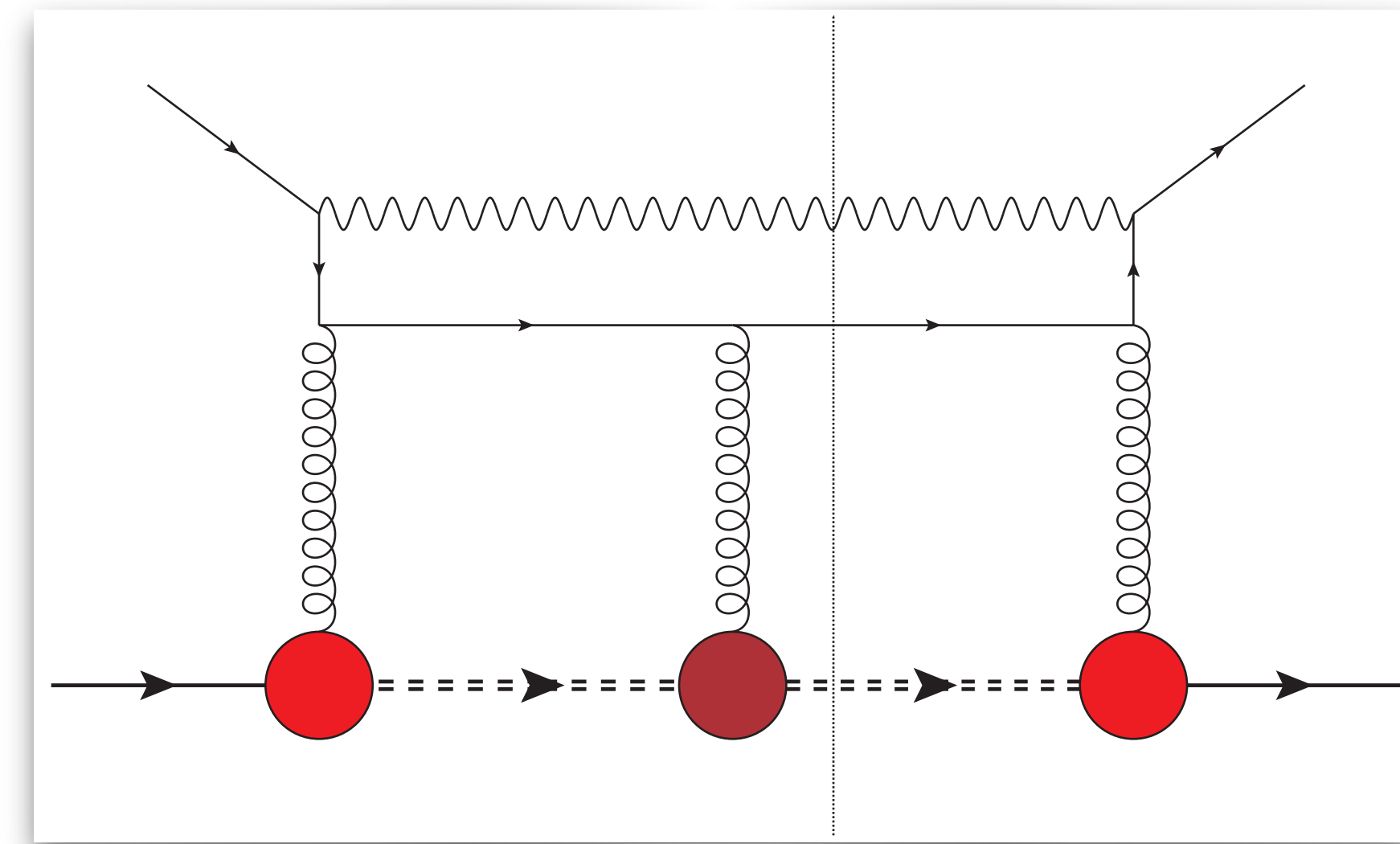
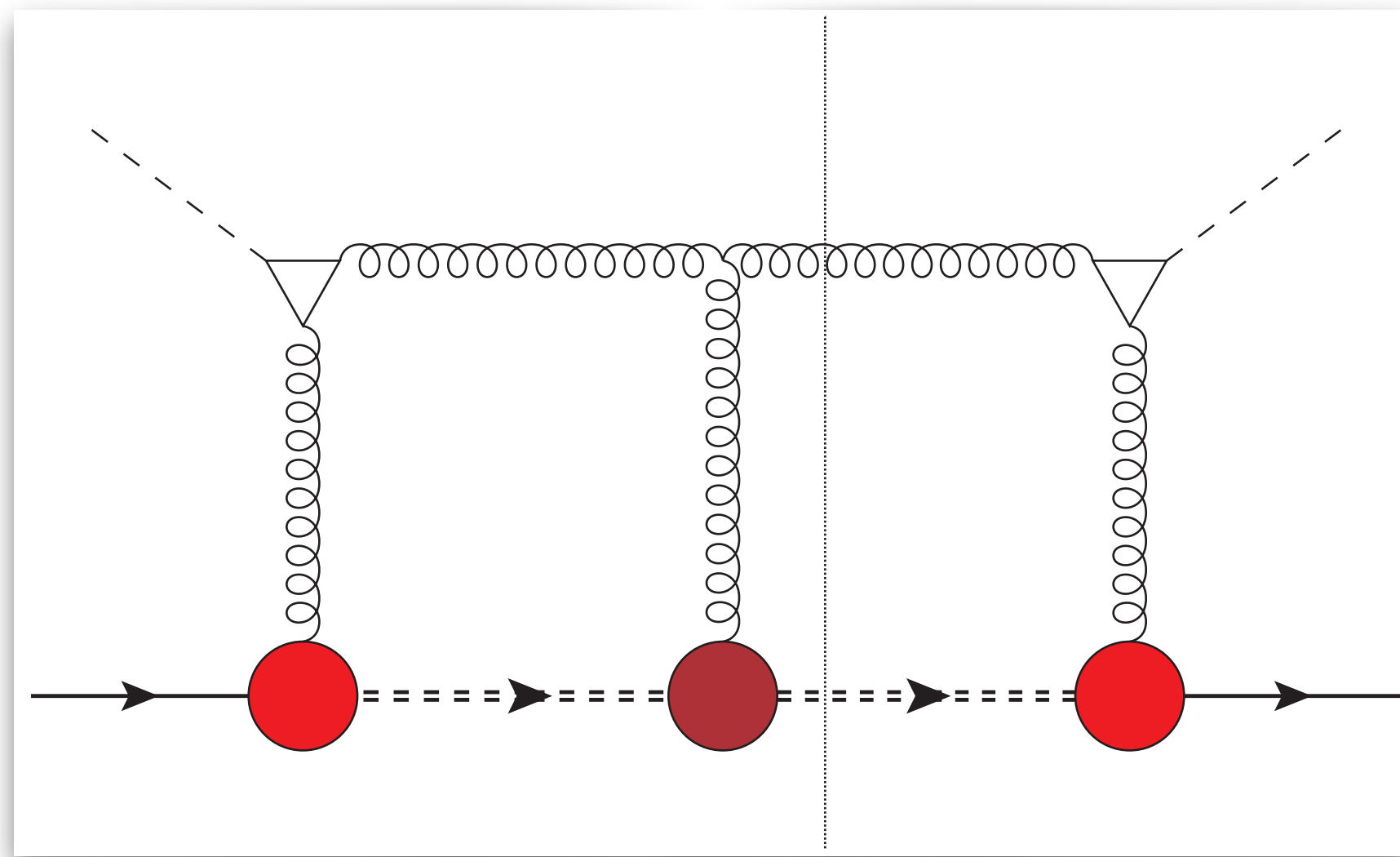
Photon-jet emission \Rightarrow d-type [+ , -]



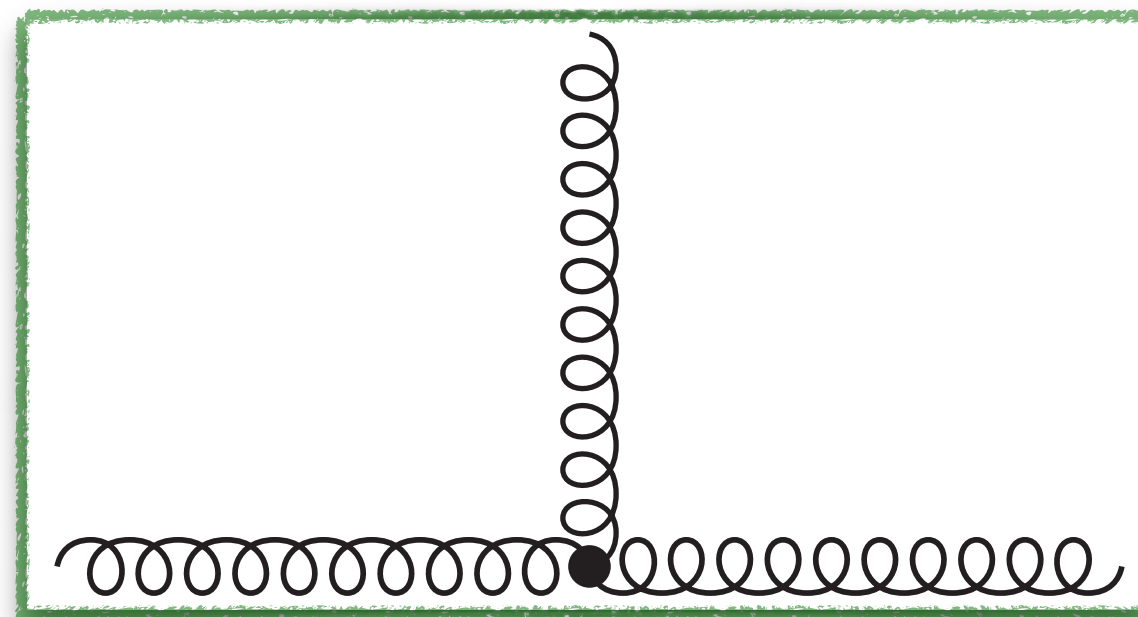
T-odd gluon TMDs in a spectator model

Higgs-gluon fusion \Rightarrow f-type [+ , +]

Photon-jet emission \Rightarrow d-type [+ , -]



* If the model were pQCD, say a gluon-target model...

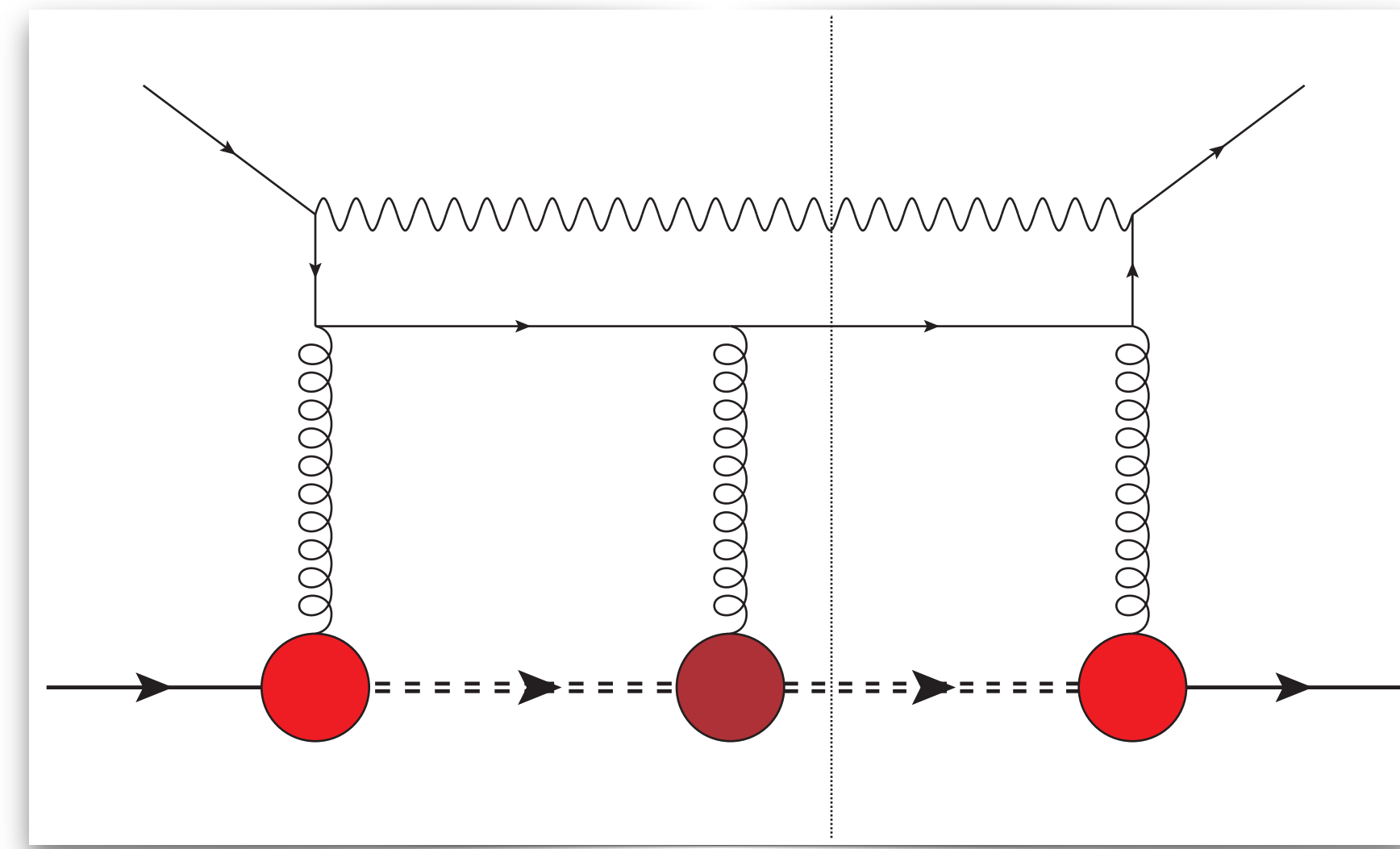
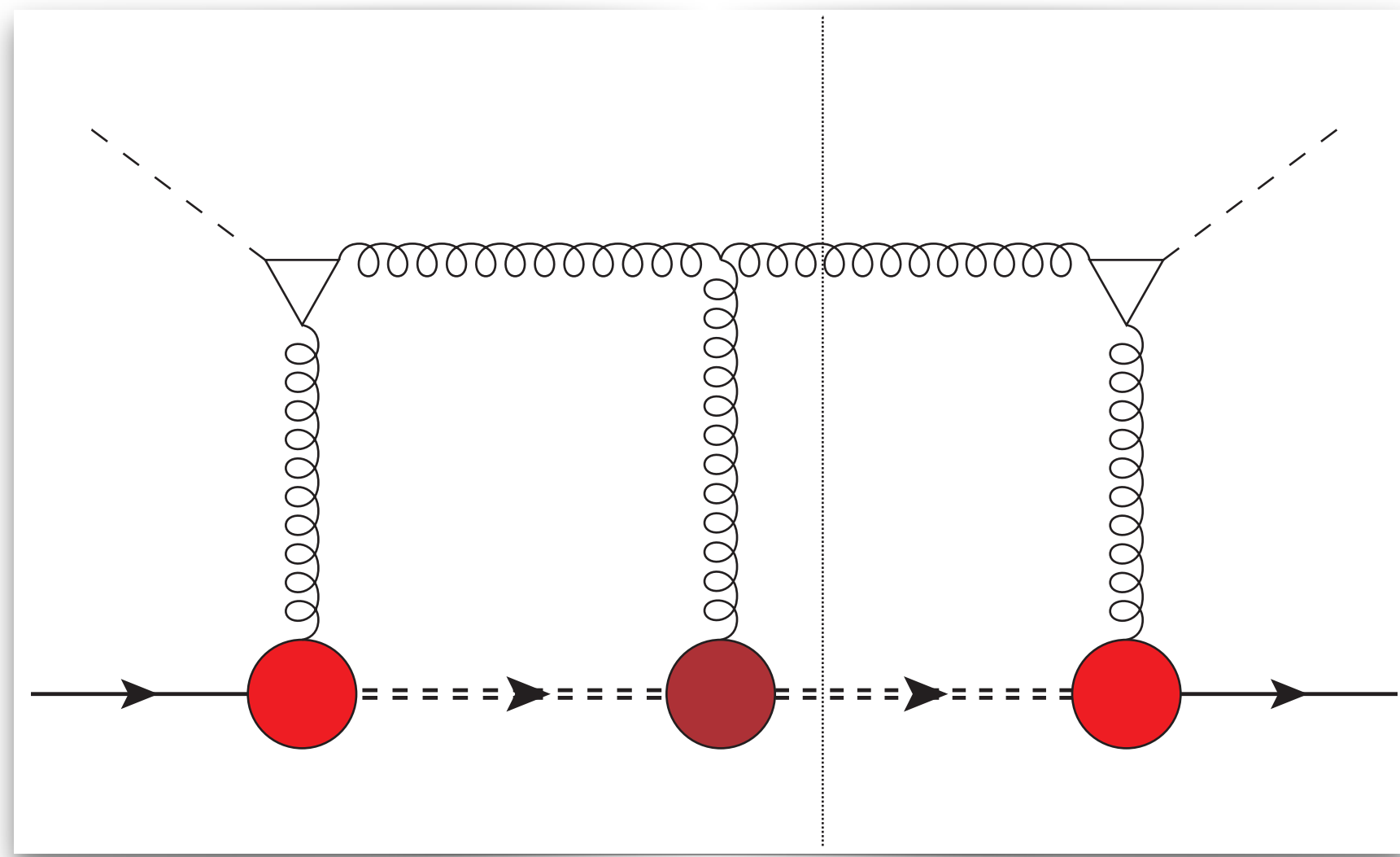


$$- g_s f_{abc}$$

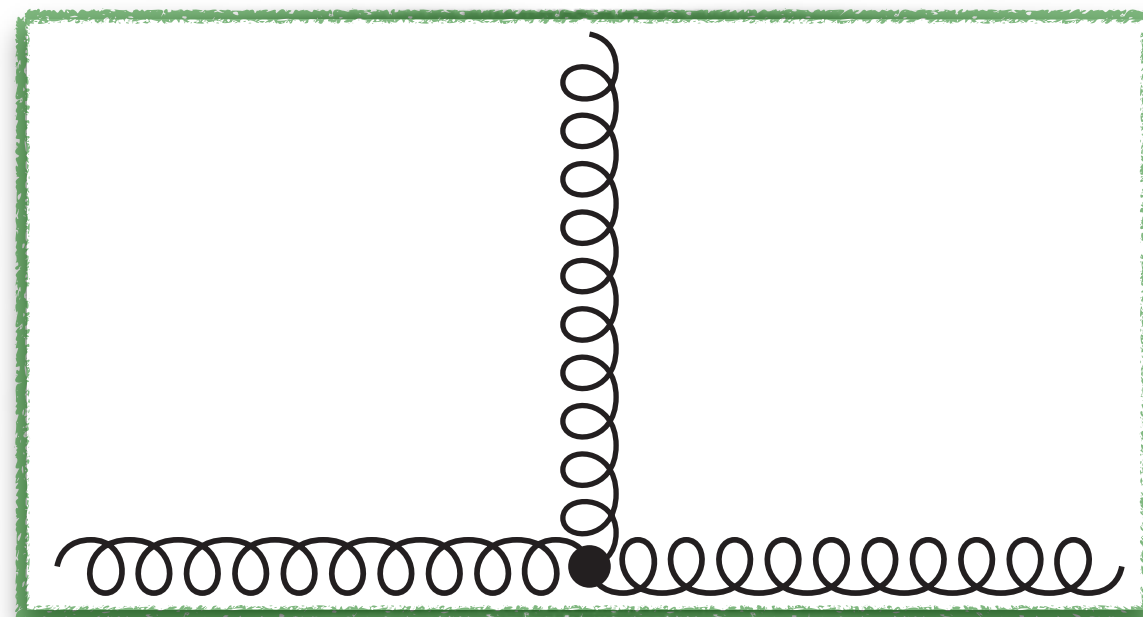
T-odd gluon TMDs in a spectator model

Higgs-gluon fusion \Rightarrow f-type [+ , +]

Photon-jet emission \Rightarrow d-type [+ , -]

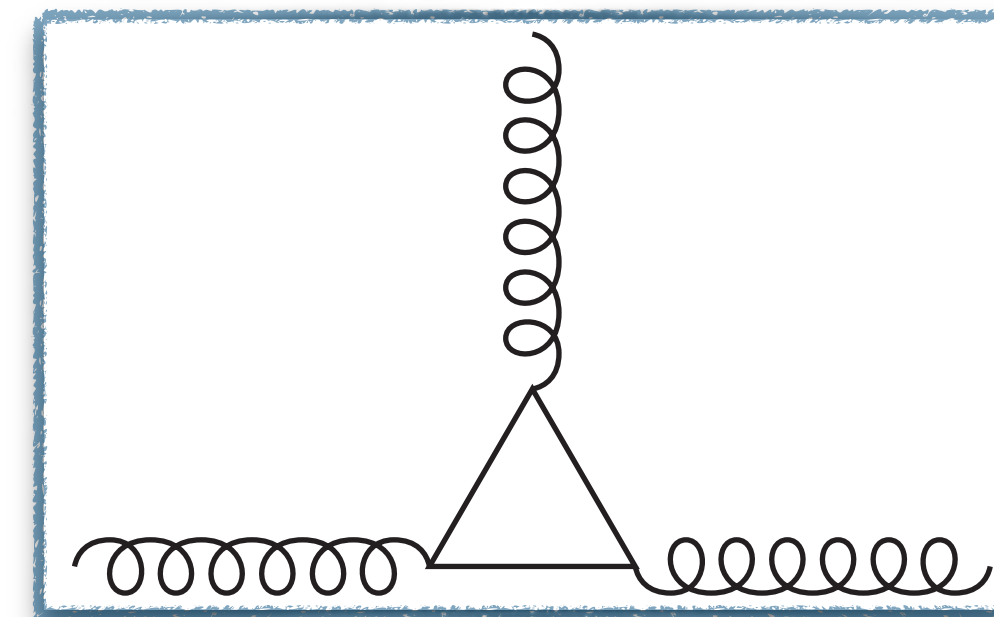


* If the model were pQCD, say a gluon-target model...



$$-g_s f_{abc}$$

$$i g_s^3 d_{abc}$$



$$-i t_c$$

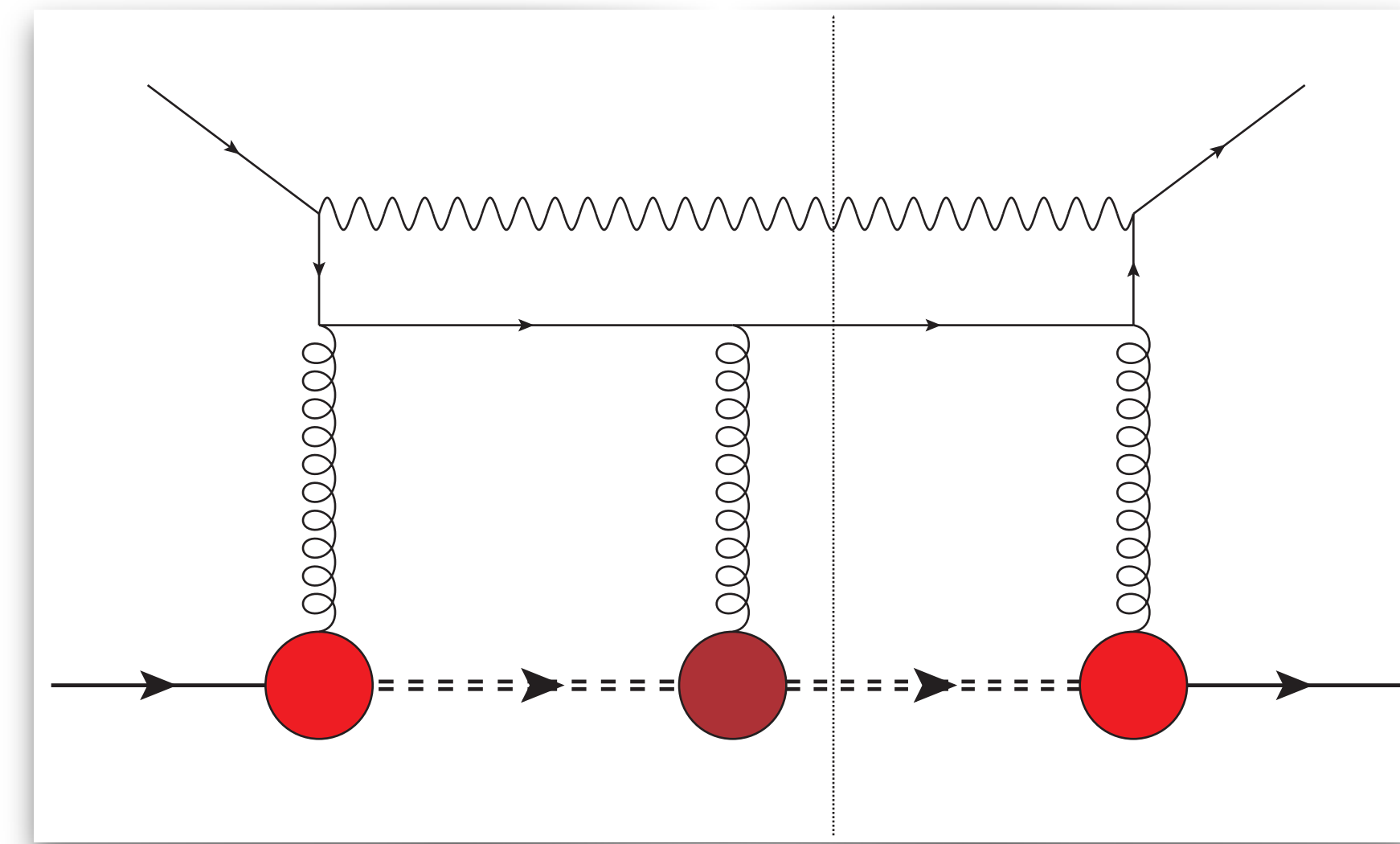
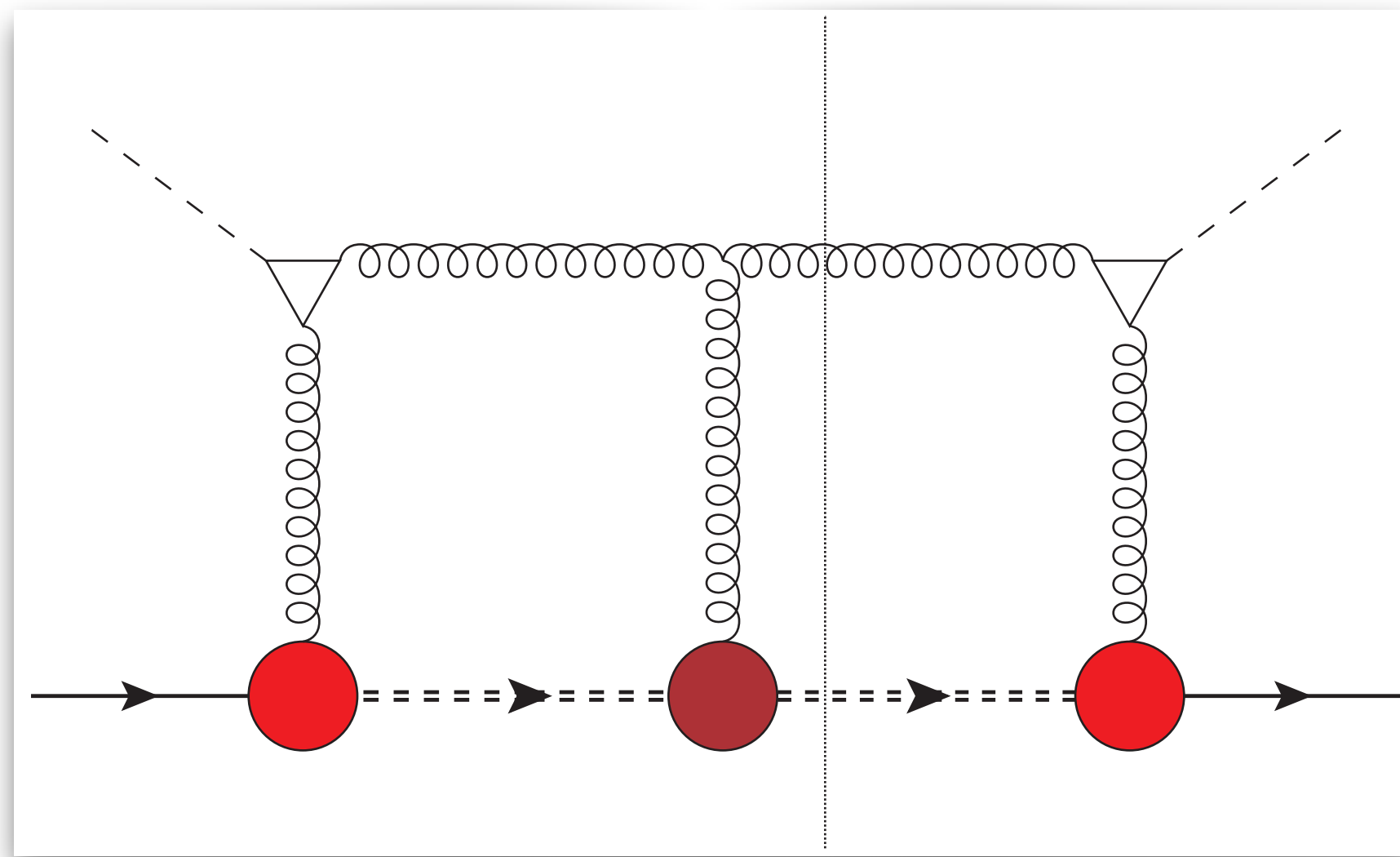
$$-i t_a$$

$$-i t_b$$

T-odd gluon TMDs in a spectator model

Higgs-gluon fusion \Rightarrow f-type [+ , +]

Photon-jet emission \Rightarrow d-type [+ , -]



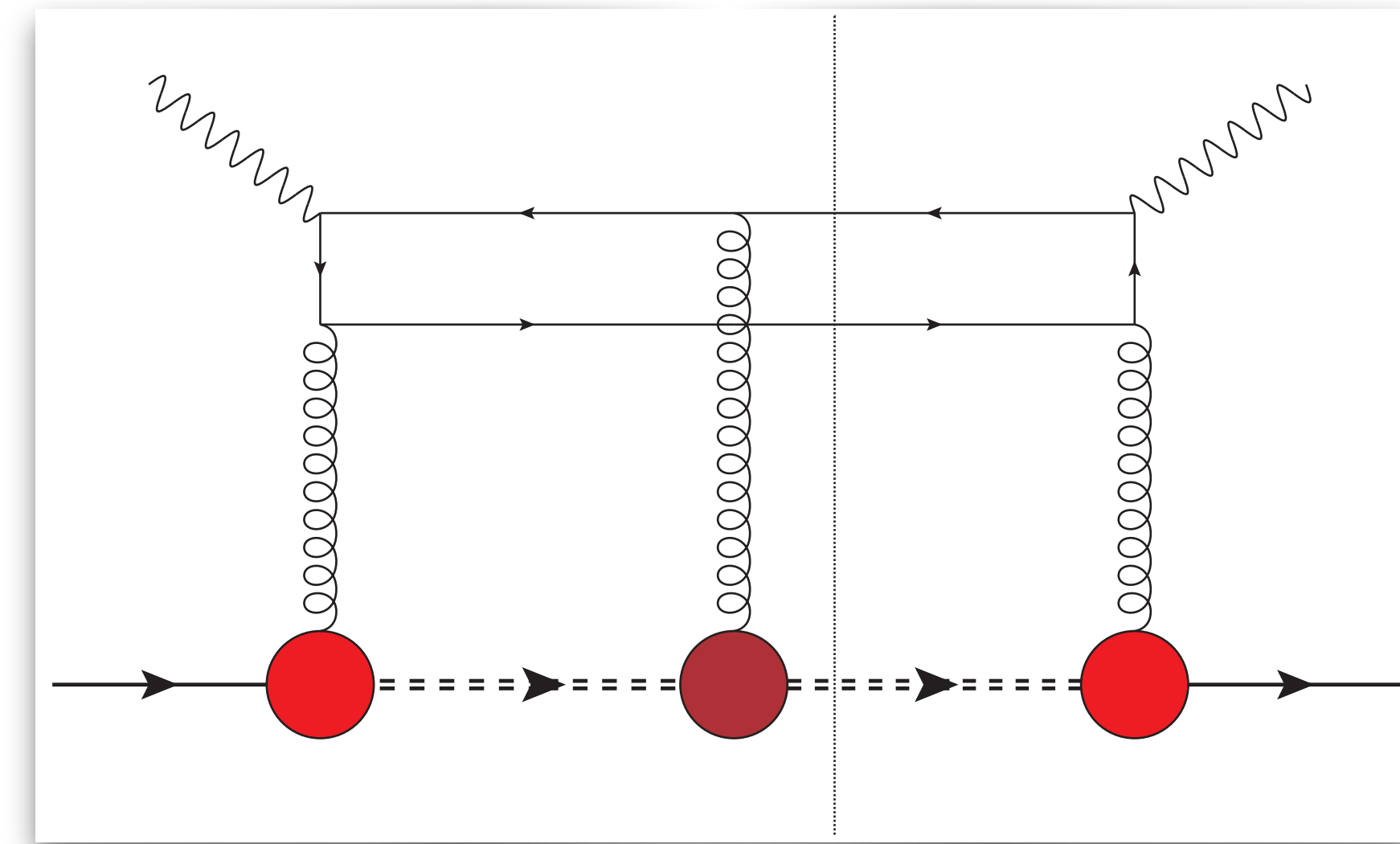
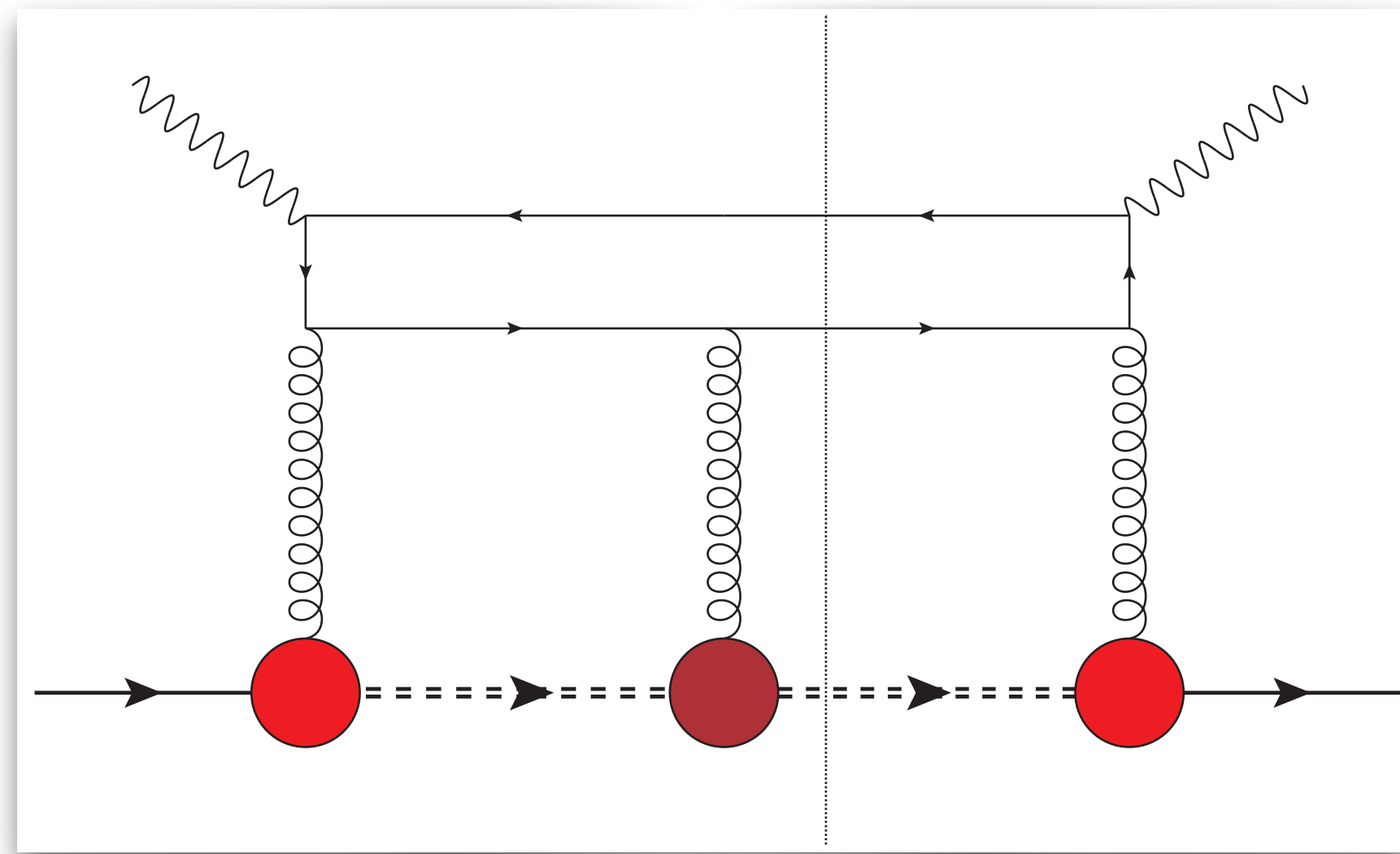
* If the model were pQCD, say a gluon-target model...

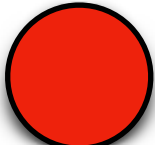
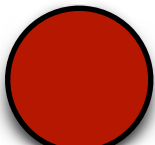
! ...but the model is not pQCD !

We want to model the nonperturbative content of T-odd TMD PDFs

Analytic structure of T-odd gluon TMDs

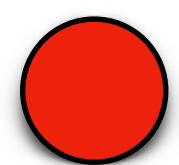
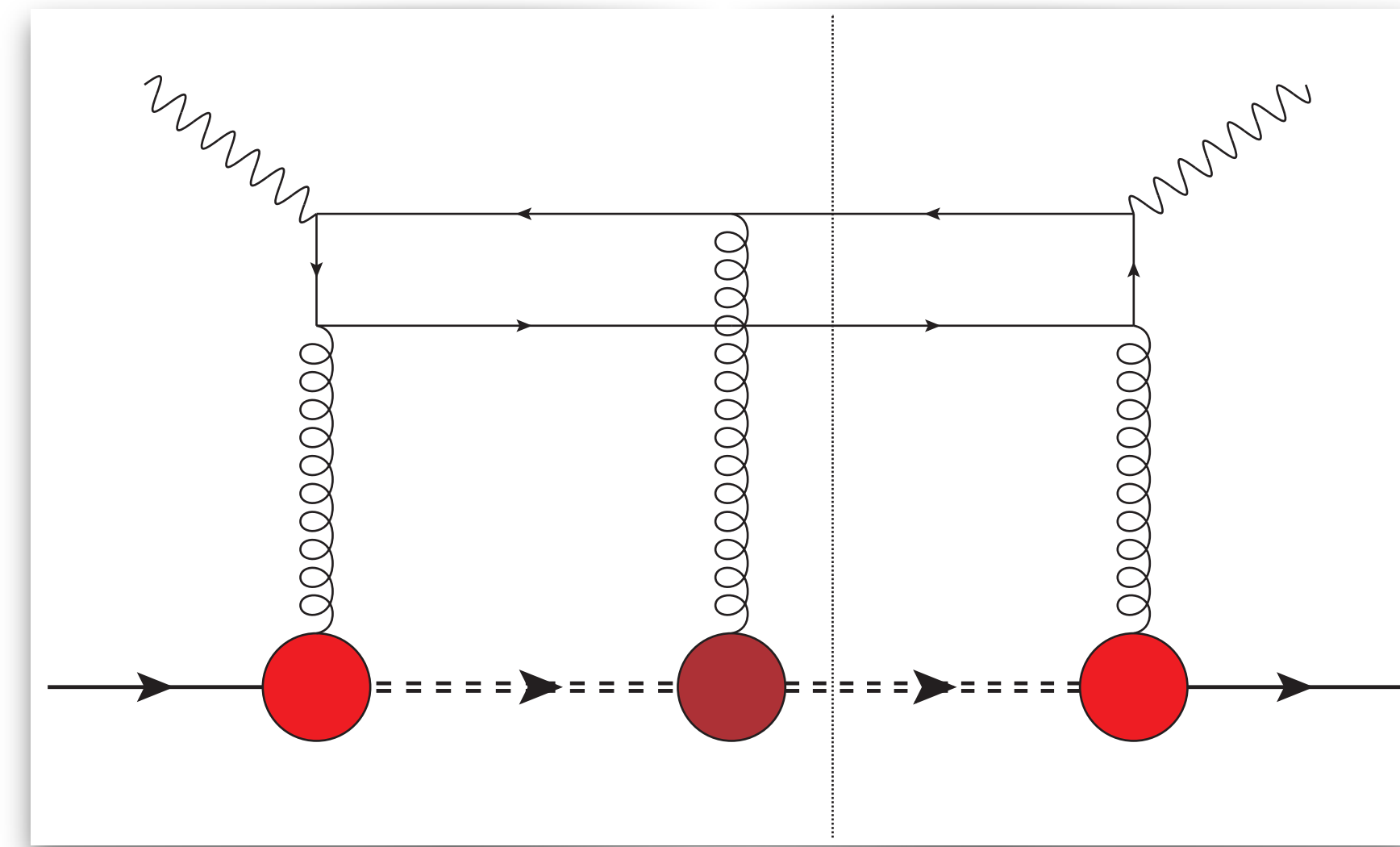
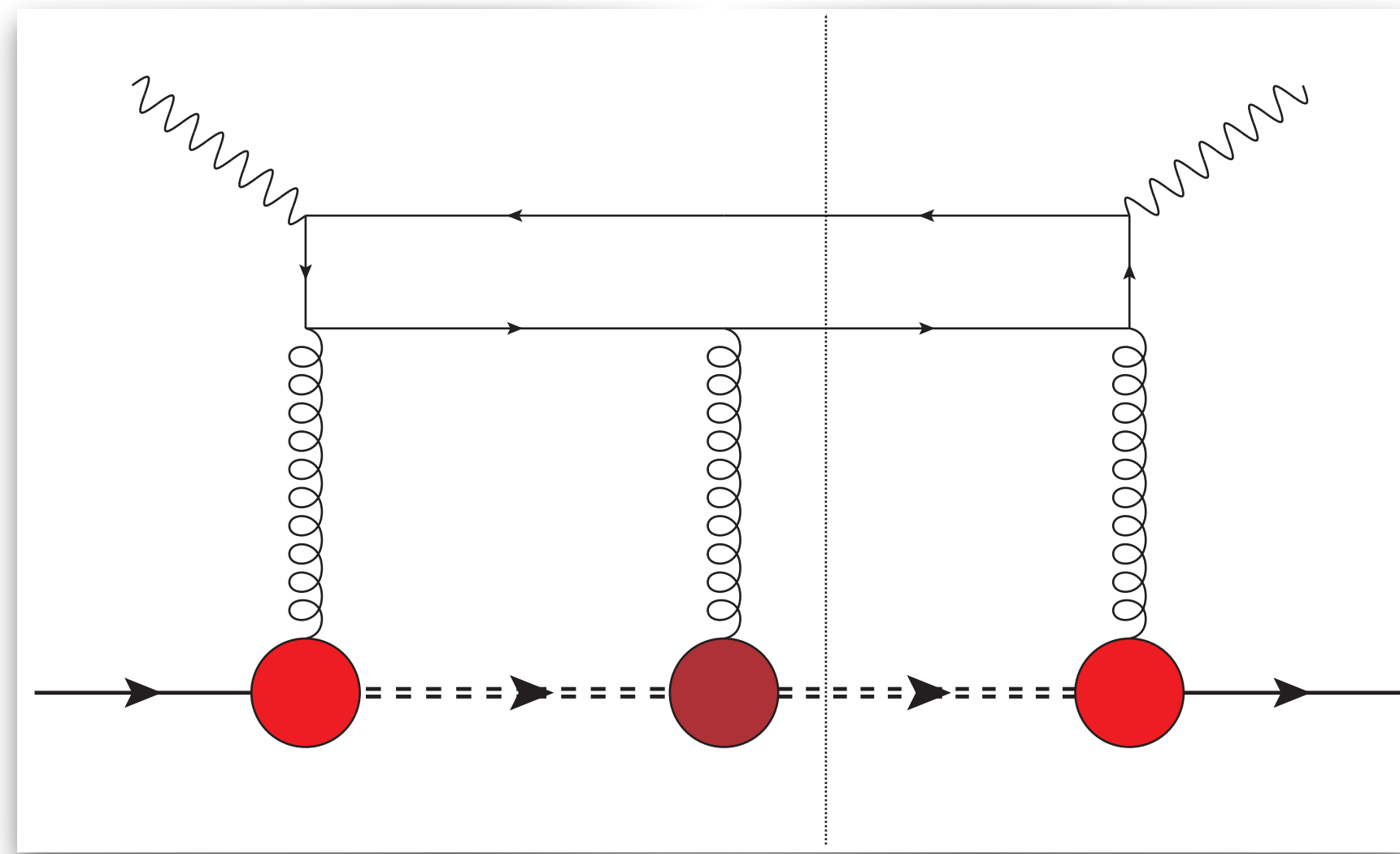
Two-jet SIDIS \Rightarrow f-type [+ , +]



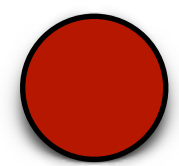
-  nucleon-gluon-spectator
-  spectator-gluon-spectator

Analytic structure of T-odd gluon TMDs

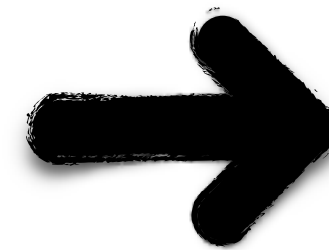
Two-jet SIDIS \Rightarrow f-type [+ , +]



nucleon-gluon-spectator



spectator-gluon-spectator



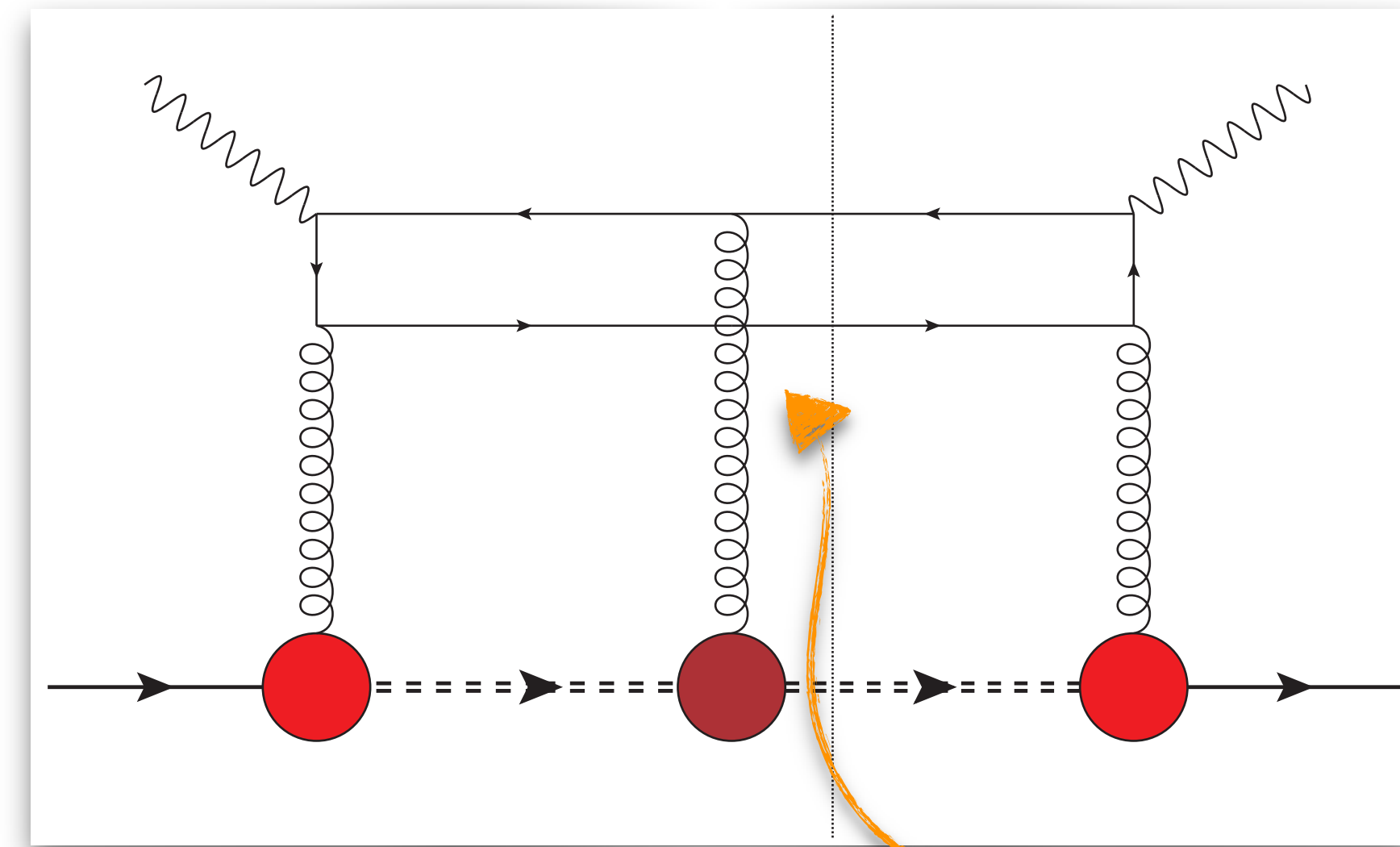
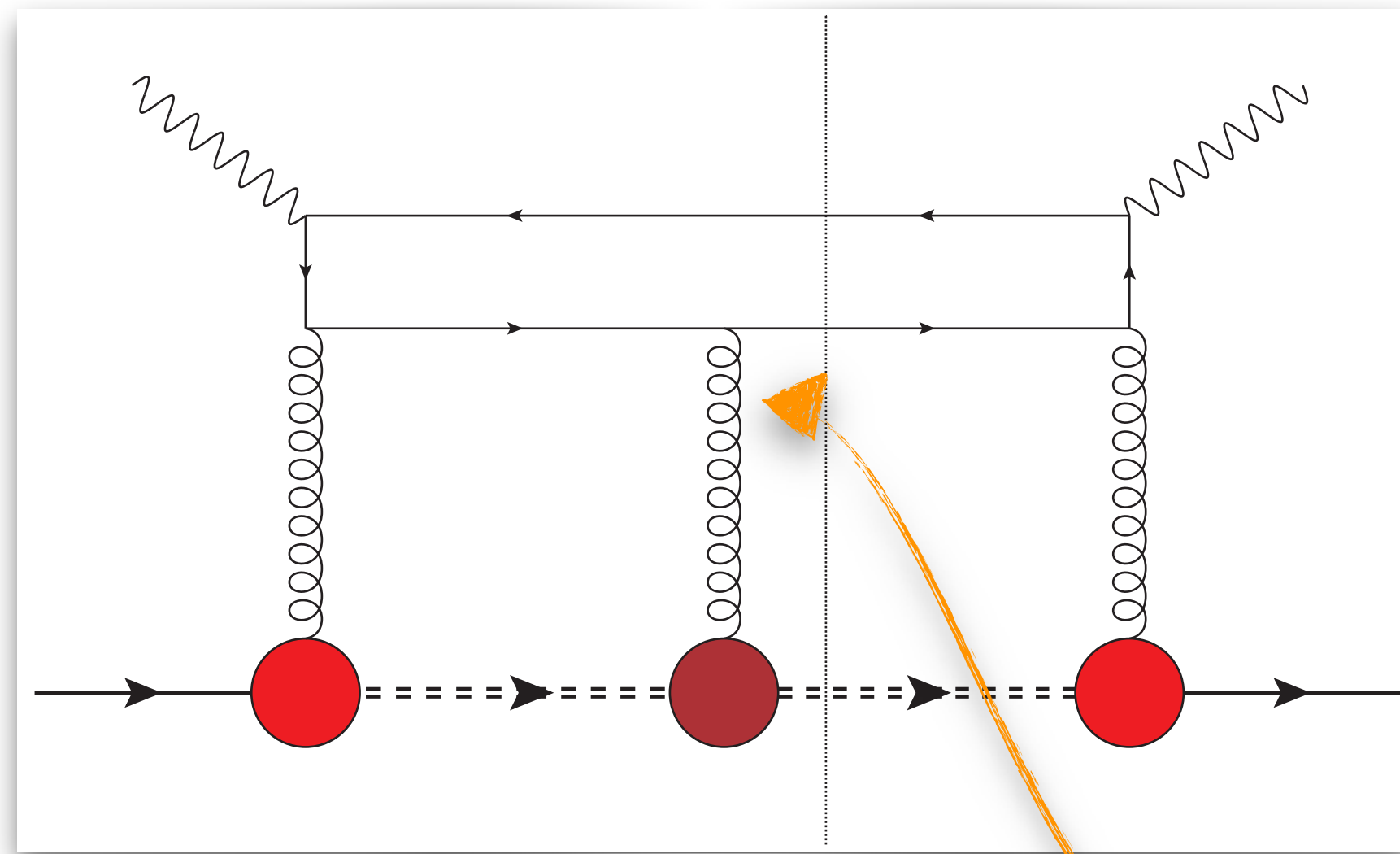
$$8 \times 8 \times 4$$

$$F(x, \mathbf{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x, \mathbf{p}_T^2) g_i(\mathbf{p}_T^2) g_j(\mathbf{p}_T^2) g_k(\mathbf{p}_T^2)$$

$$C_{ijk}^{(F)}(x, \mathbf{p}_T^2) = \sum_{l=1}^7 C_{ijk}^{(F),l}(x, \mathbf{p}_T^2) \mathcal{D}_l(x, \mathbf{p}_T^2)$$

Analytic structure of T-odd gluon TMDs

Two-jet SIDIS \Rightarrow f-type [+ , +]



- nucleon-gluon-spectator
- spectator-gluon-spectator

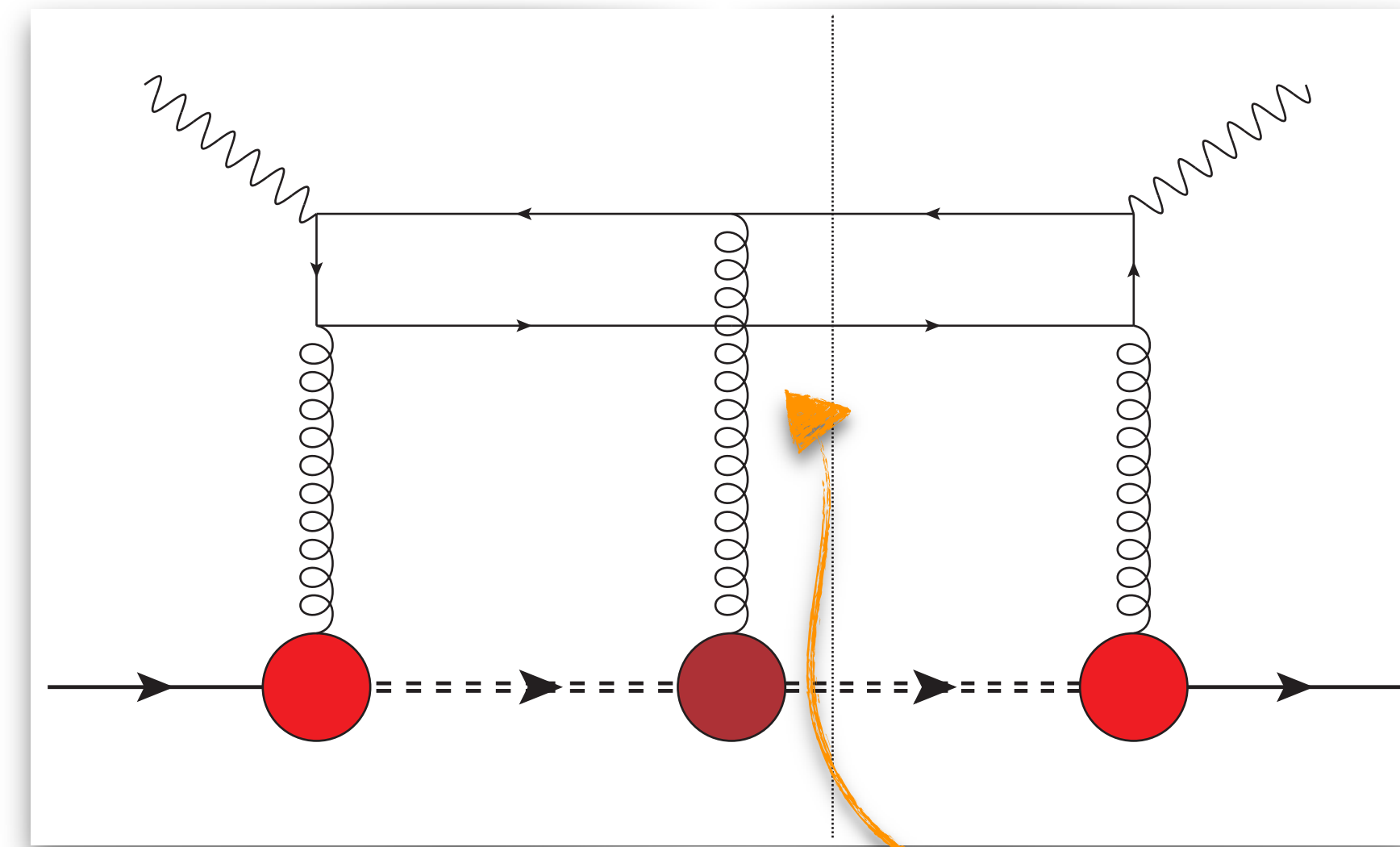
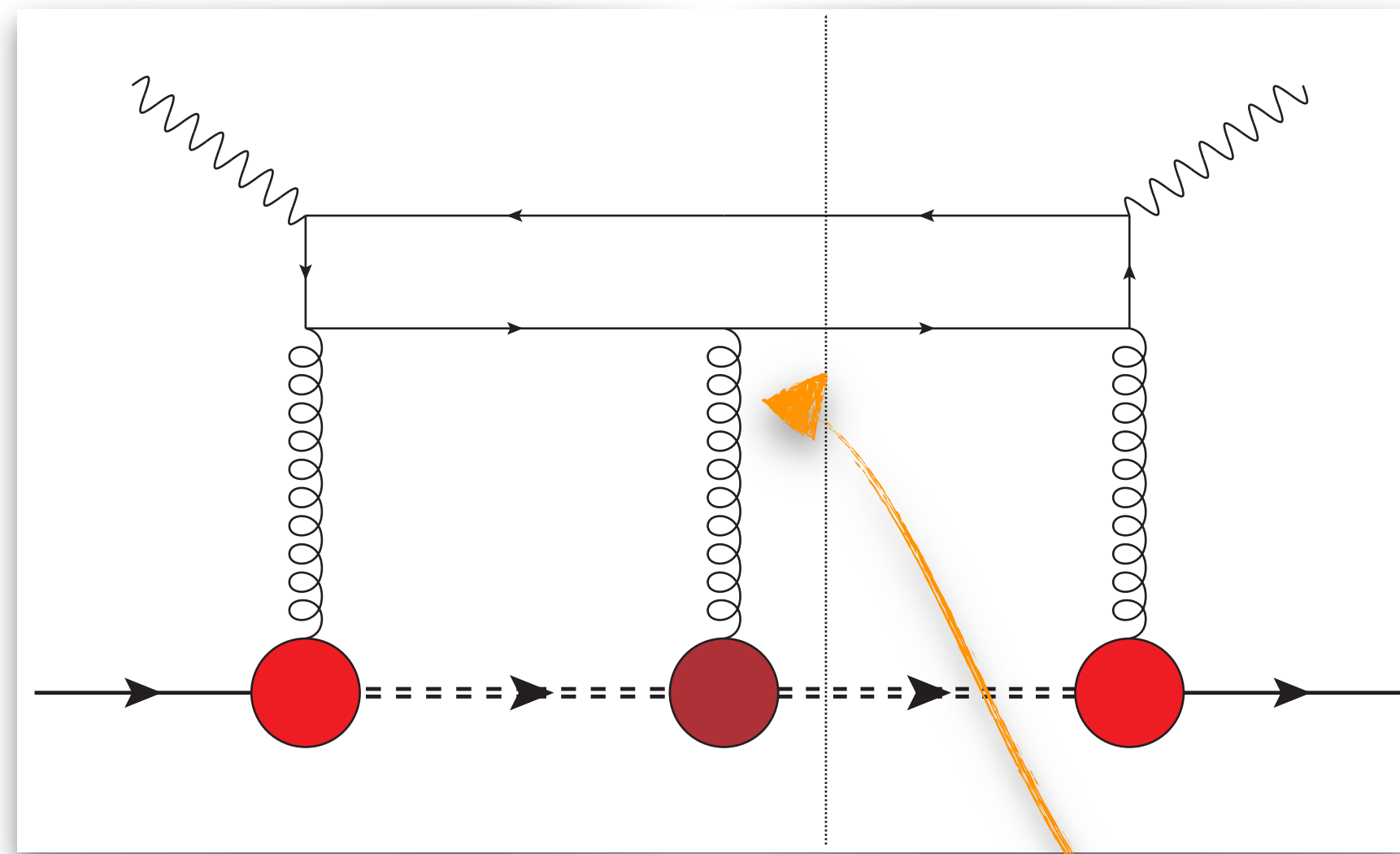
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Analytic structure of T-odd gluon TMDs

Two-jet SIDIS \Rightarrow f-type [+ , +]



- nucleon-gluon-spectator
- spectator-gluon-spectator

$$8 \times 8 \times 4$$

256 coeff. functions

$$F(x, \mathbf{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x, \mathbf{p}_T^2) g_i(\mathbf{p}_T^2) g_j(\mathbf{p}_T^2) g_k(\mathbf{p}_T^2)$$

$$C_{ijk}^{(F)}(x, \mathbf{p}_T^2) = \sum_{l=1}^7 C_{ijk}^{(F),l}(x, \mathbf{p}_T^2) \mathcal{D}_l(x, \mathbf{p}_T^2)$$



Nice, but I need more
than a 1-loop calculation
with no poles...



Nice, but I need more
than a 1-loop calculation
with no poles...

¿ Do you want more ?

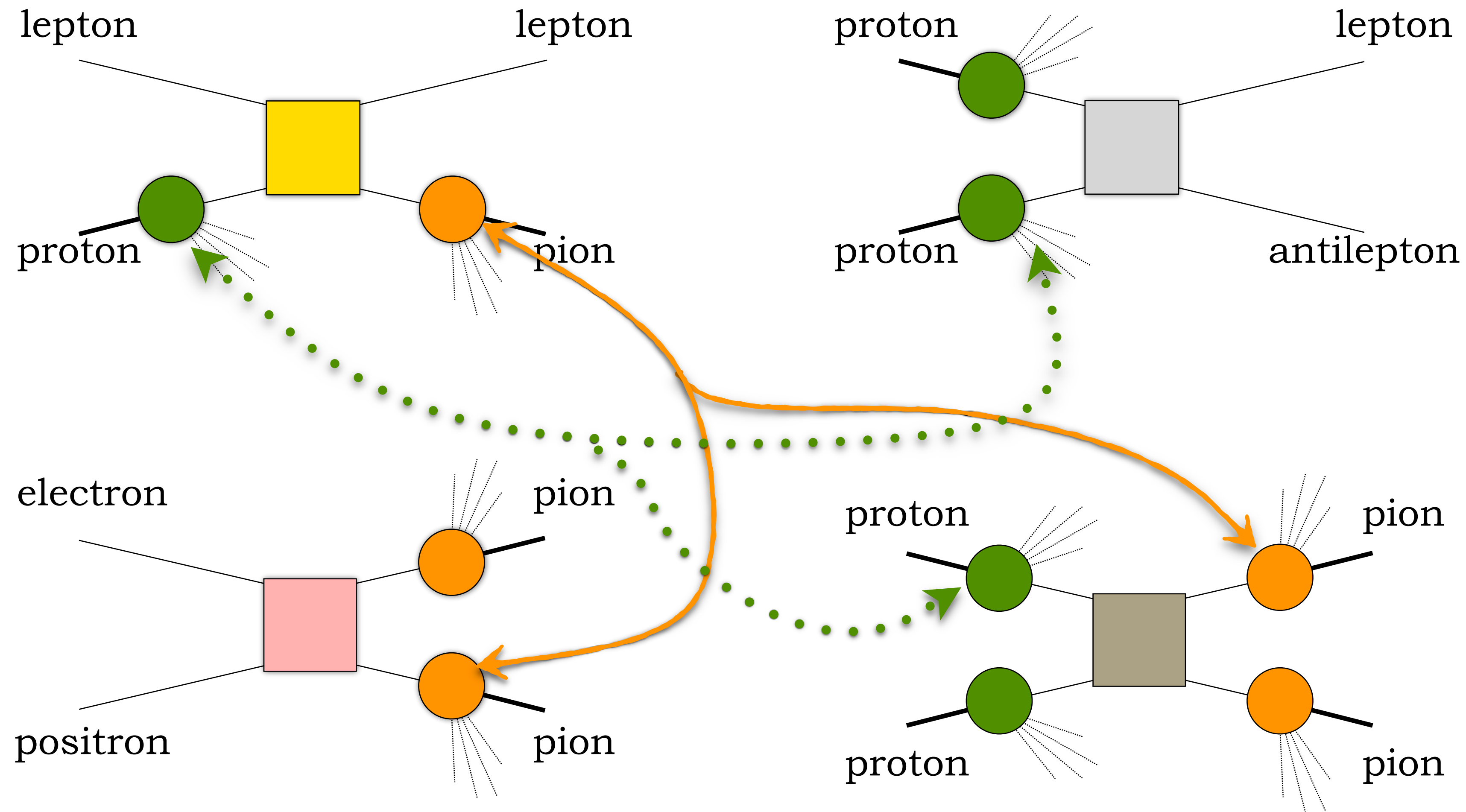
OK...



Factorization and universality

SIDIS

Drell-Yan



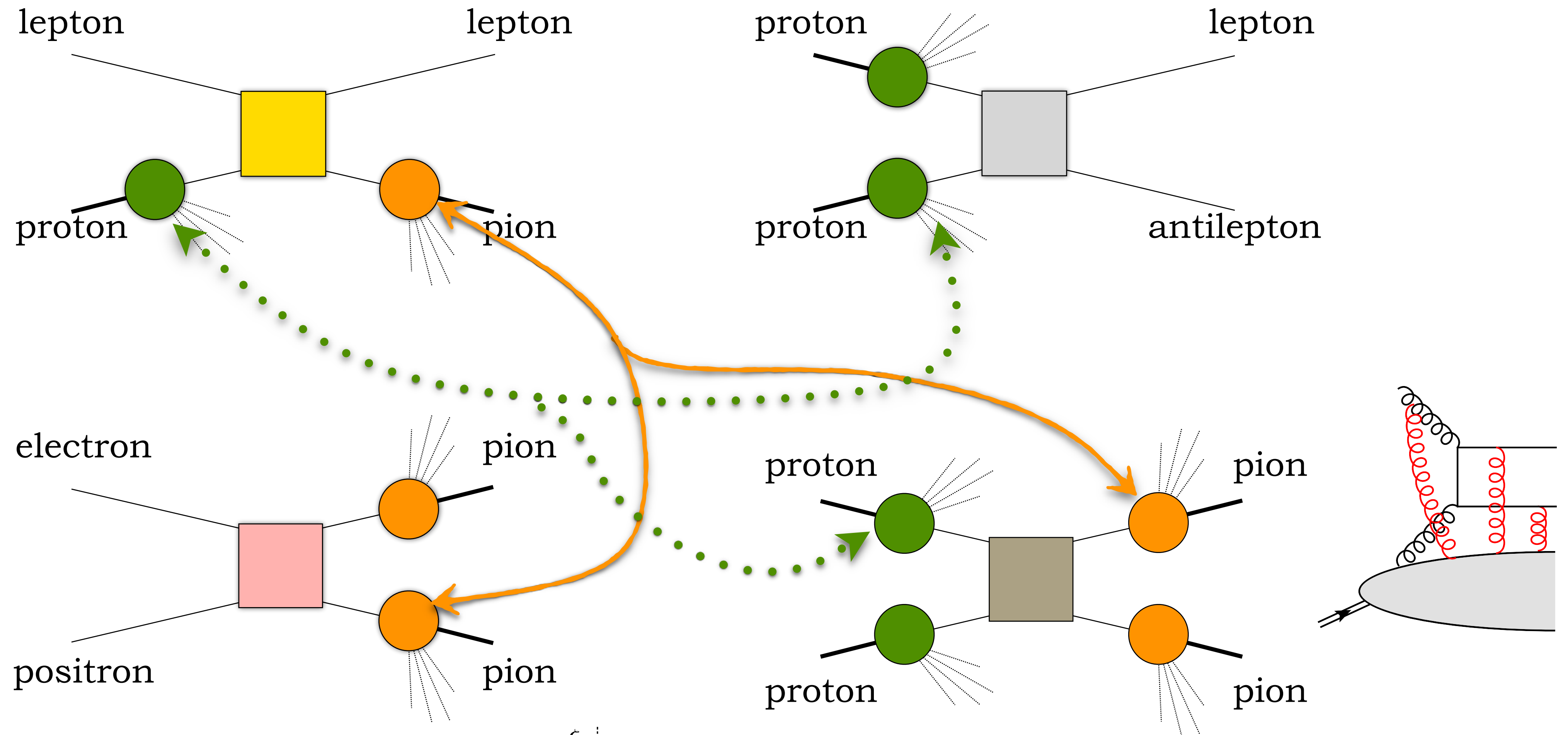
$$e^+ + e^- \rightarrow \text{hadrons}$$

$$p + p \rightarrow \text{hadrons}$$

Factorization and universality

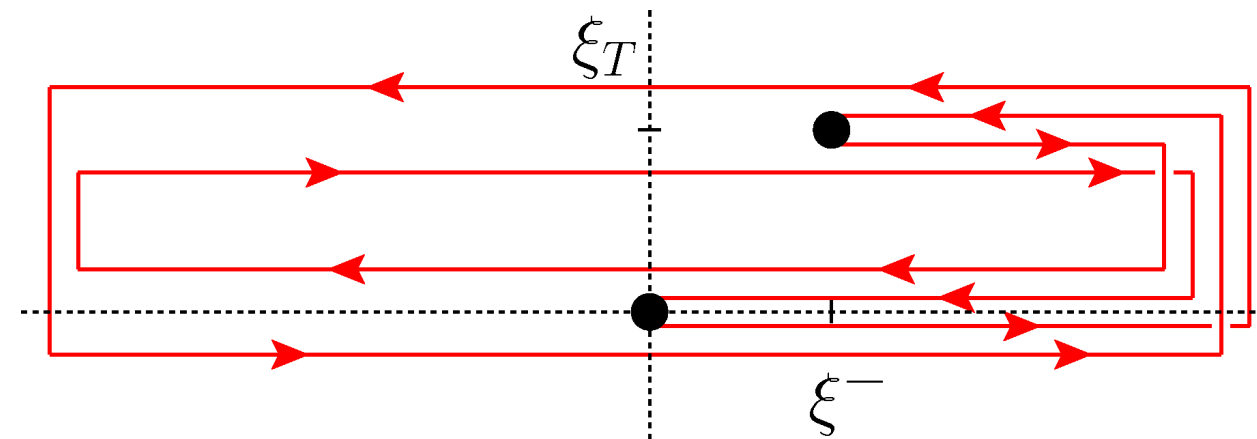
SIDIS

Drell-Yan



$$e^+ + e^- \rightarrow \text{hadrons}$$

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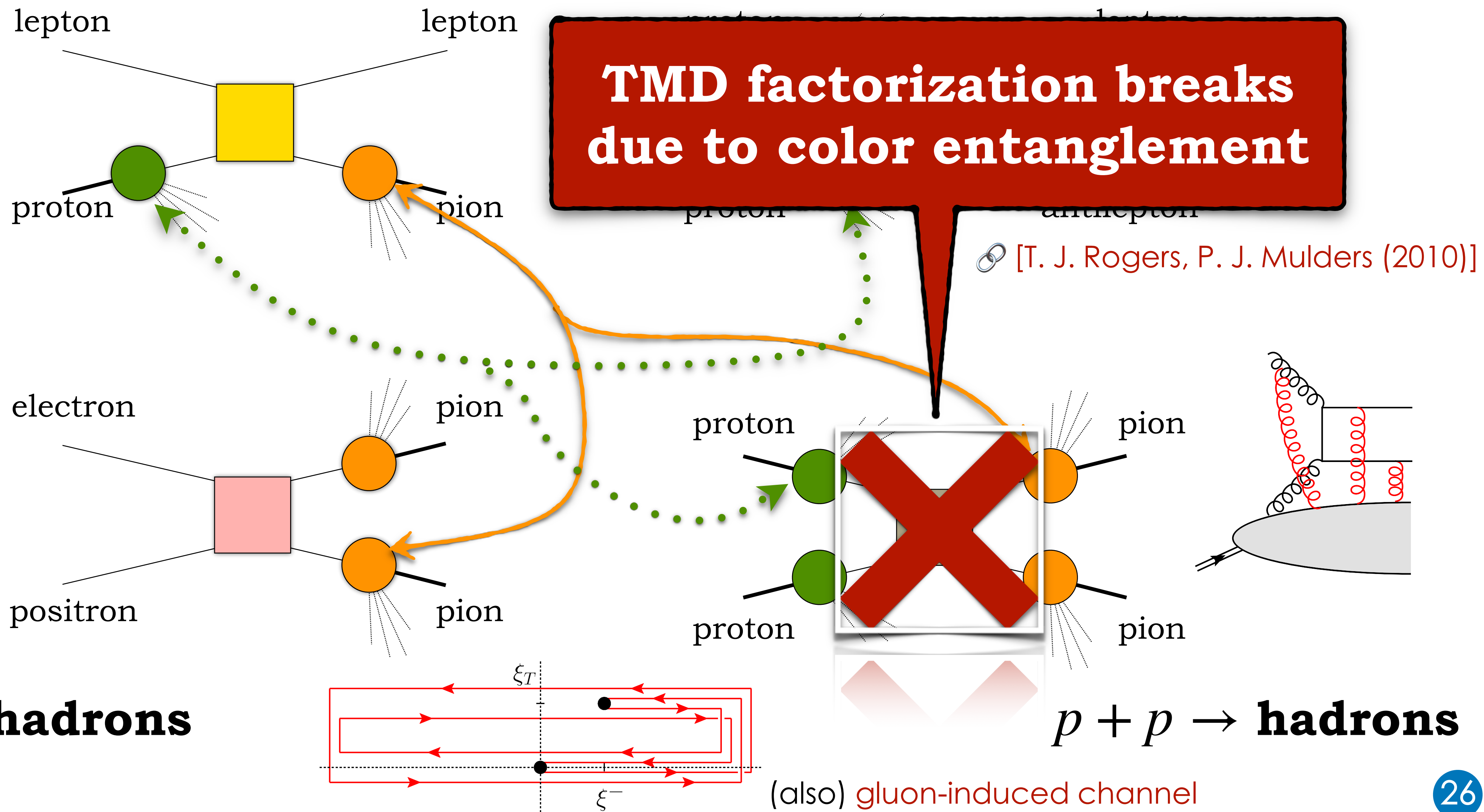


(also) gluon-induced channel

Factorization and universality

SIDIS

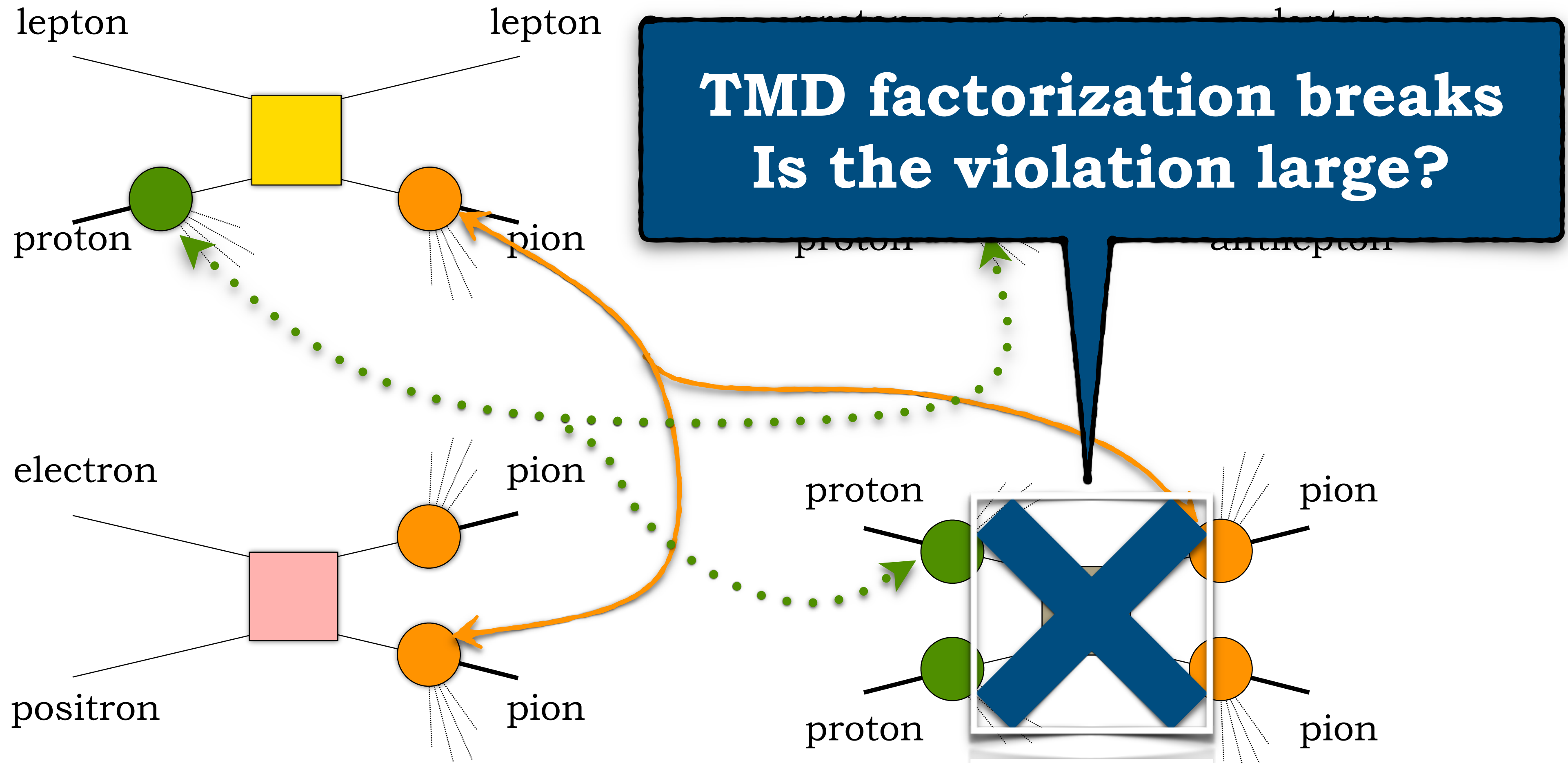
Drell-Yan



Factorization and universality

SIDIS

Drell-Yan



$$e^+ + e^- \rightarrow \mathbf{hadrons}$$

$$p + p \rightarrow \mathbf{hadrons}$$

**Proving TMD factorization
is highly nontrivial...**

**Proving TMD factorization
is highly nontrivial...**

***“...as complex as proving
remormalization”***

John C. Collins

An Early Career Path

GLUON TMDS

PHENO

Transverse momentum & polarization
3D imaging of the proton

CALCULUS

Higher-order calculations
TMD factorization properties

EXPERIMENT

Design of new-generation detectors
Synergies between EIC and LHC

COMPUTING

Proton reconstruction from EIC data
Artificial intelligence

An Early Career Path

GLUON TMDs

PHENO

Transverse momentum & polarization
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Higher-order calculations
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EXPERIMENT

Design of new-generation detectors
Synergies between EIC and LHC

COMPUTING

Proton reconstruction from EIC data
Artificial intelligence

Gluson TMDs & hadronic structure...

...a fascinating path for realizing yourself as a scientist !



Extras

A map of my journey



QCD @colliders

MN jets

di-hadron



A map of my journey



UNIVERSITÀ DELLA CALABRIA
DIPARTIMENTO DI
FISICA

PhD Studies
2014-2017

QCD @colliders



MN jets

multi-jet



Instituto de
Física
Teórica
UAM-CSIC

Post-doc
2017-2018

di-hadron

rapidity spectrum

UGD

hadron-jet

ρ -meson

forward DY

A map of my journey



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ift

Instituto de
Física
Teórica
UAM-CSIC

Post-doc
2017-2018

Nanga
Parbat

JETHAD

DY+jet


forward DY

gluon TMDs

Higgs+jet



3DSPIN
MAPPING
THE PROTON IN 3D



UNIVERSITÀ
DI PAVIA

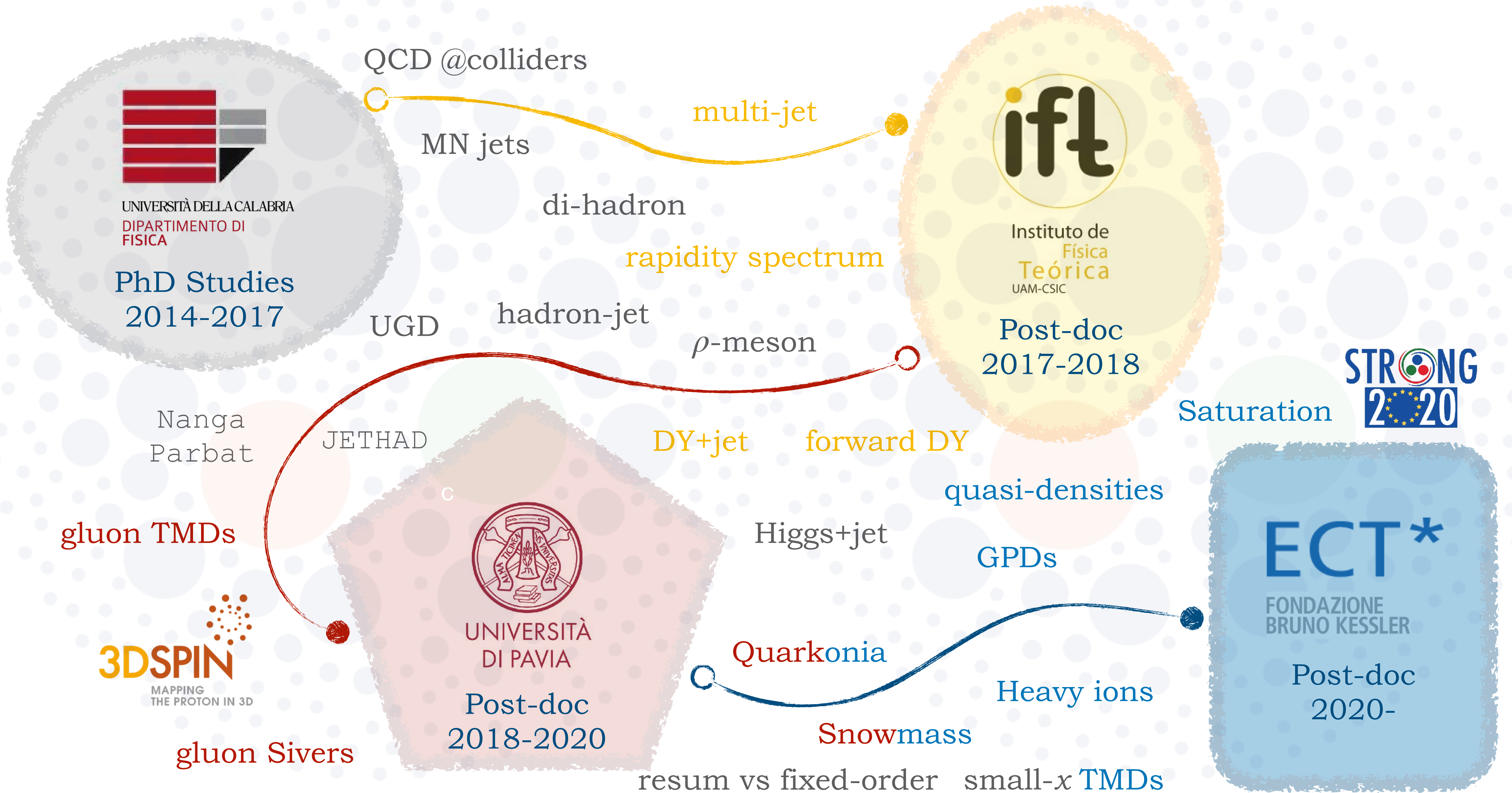
Post-doc
2018-2020

gluon Sivers

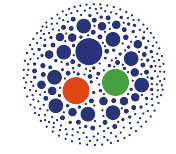
Snowmass

resum vs fixed-order

A map of my journey



Gluon TMD PDFs: A largely unexplored territory



Theory: different gauge-link structures...

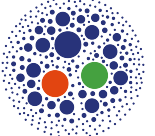

...more diversified kind of modified universality!





Pheno: golden channels for extraction

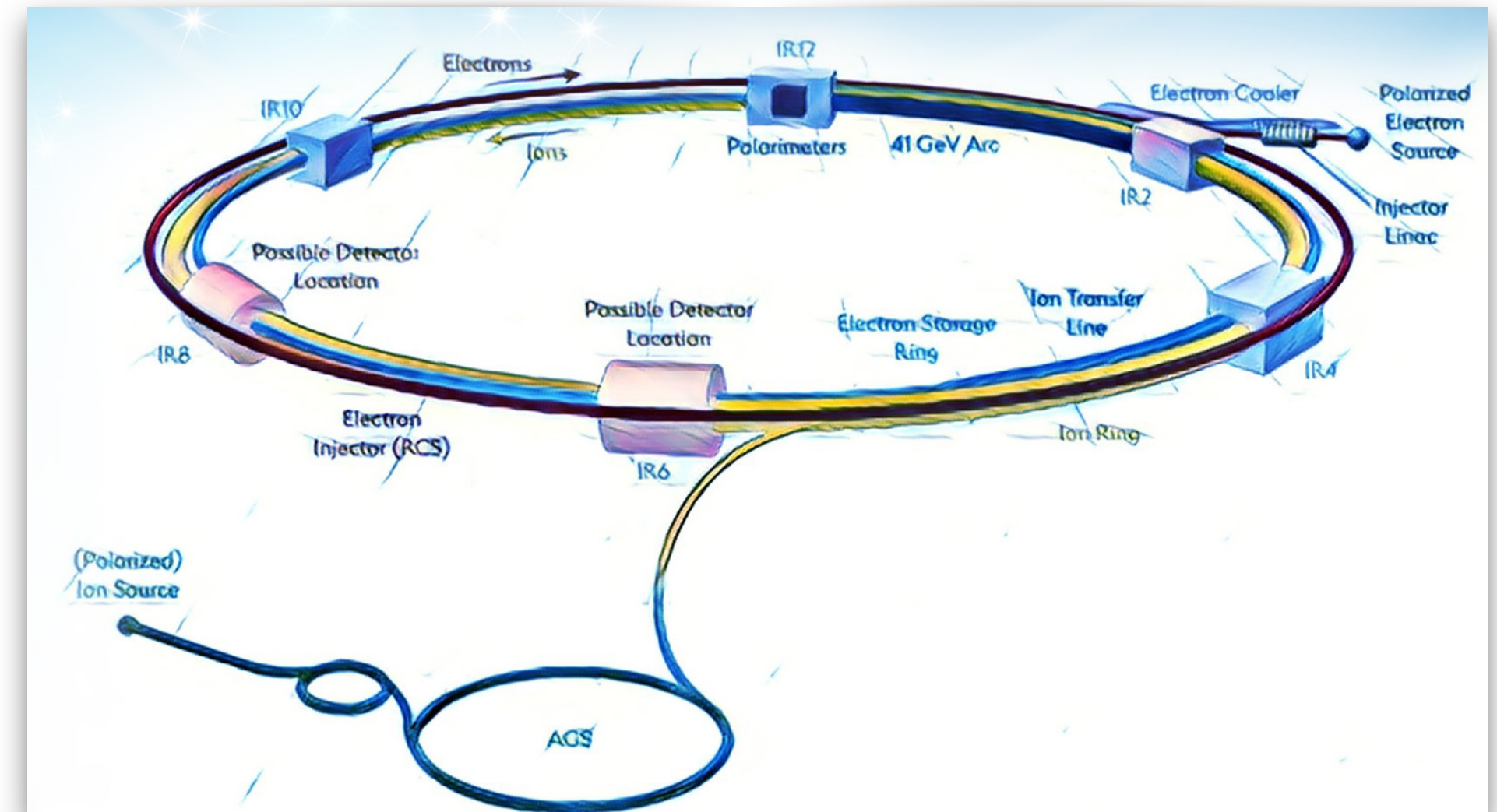
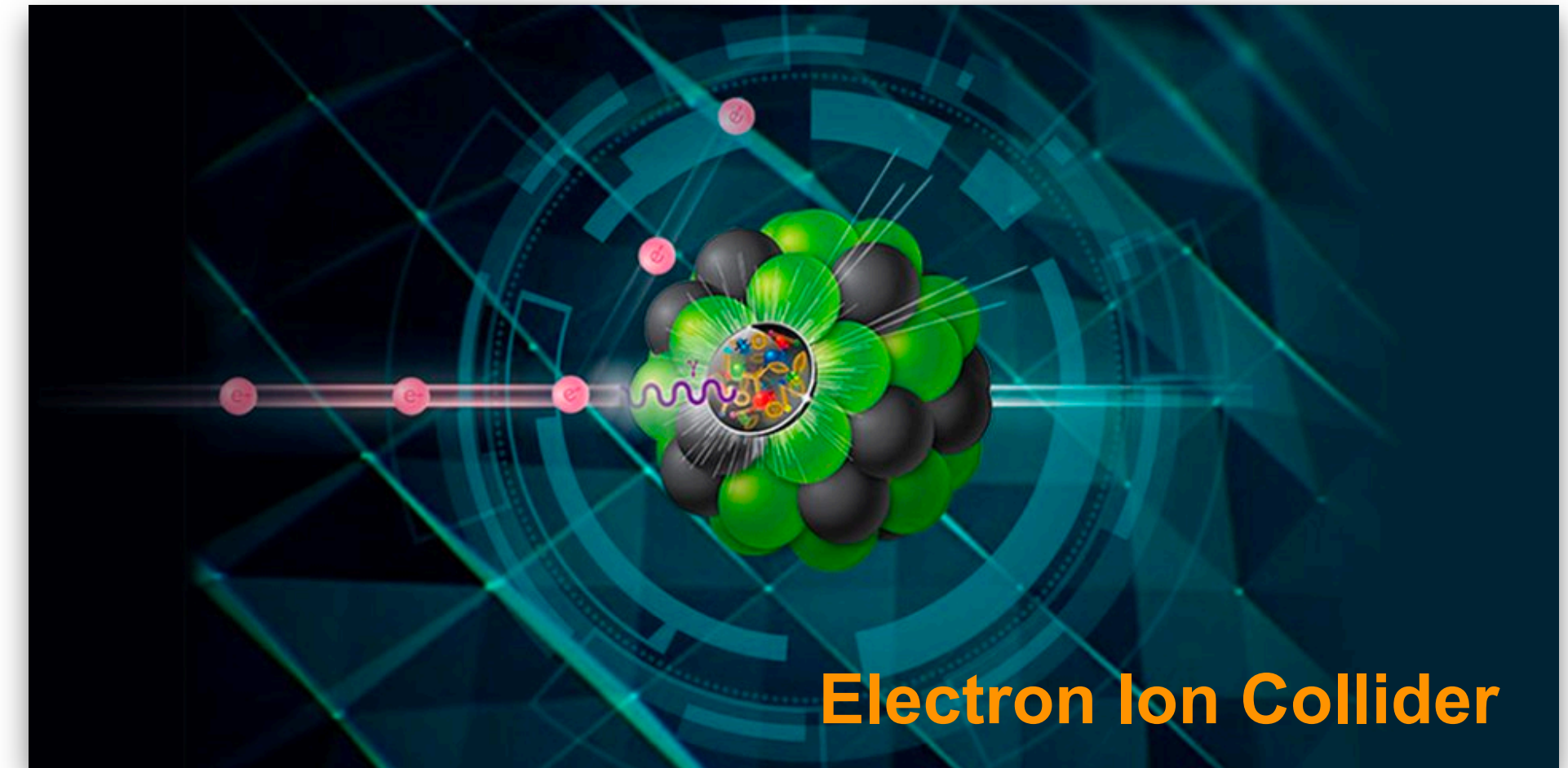
of quark TMDs are subleading for gluon TMDs

Gluon TMD PDFs: A largely unexplored territory



-  **Theory:** different gauge-link structures...
...more diversified kind of modified universality!
-  **Pheno:** golden channels for extraction
of quark TMDs are subleading for gluon TMDs

3D proton imaging

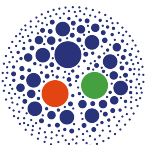
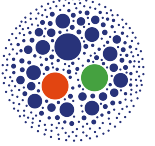
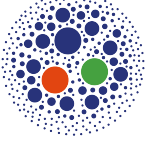
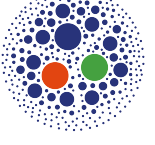
-  Gluon TMD PDFs \Rightarrow core sector of EIC studies
-  Need for a flexible model, suited to pheno

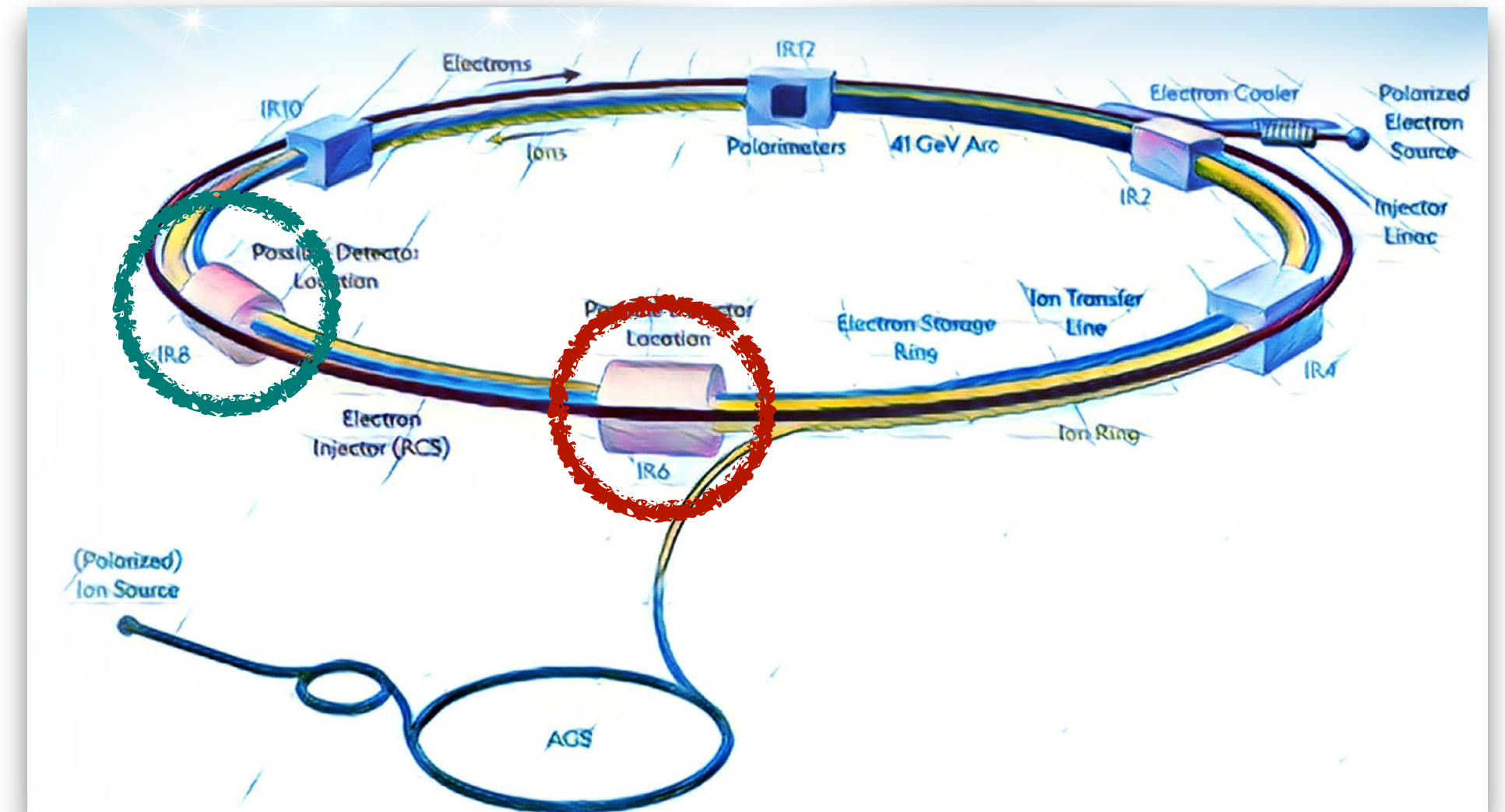
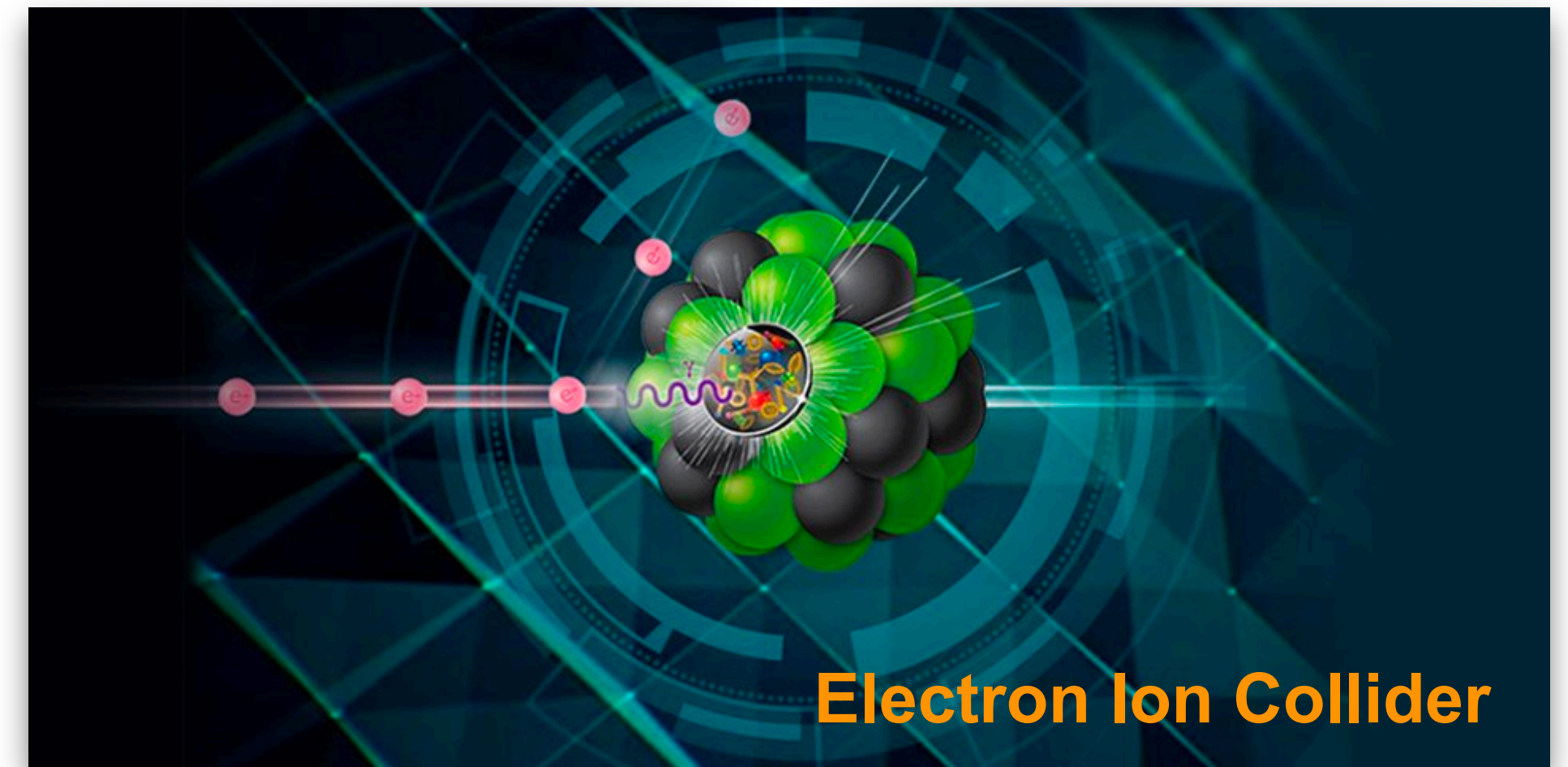


Gluon TMD PDFs: A largely unexplored territory

-  **Theory:** different gauge-link structures...
...more diversified kind of modified universality!
-  **Pheno:** golden channels for extraction
of quark TMDs are subleading for gluon TMDs

3D proton imaging

-  Gluon TMD PDFs \Rightarrow core sector of EIC studies
-  Need for a flexible model, suited to pheno
-  Gluon and nucleon polarization at twist-2
-  Window of opportunities at ePIC & 2nd detector

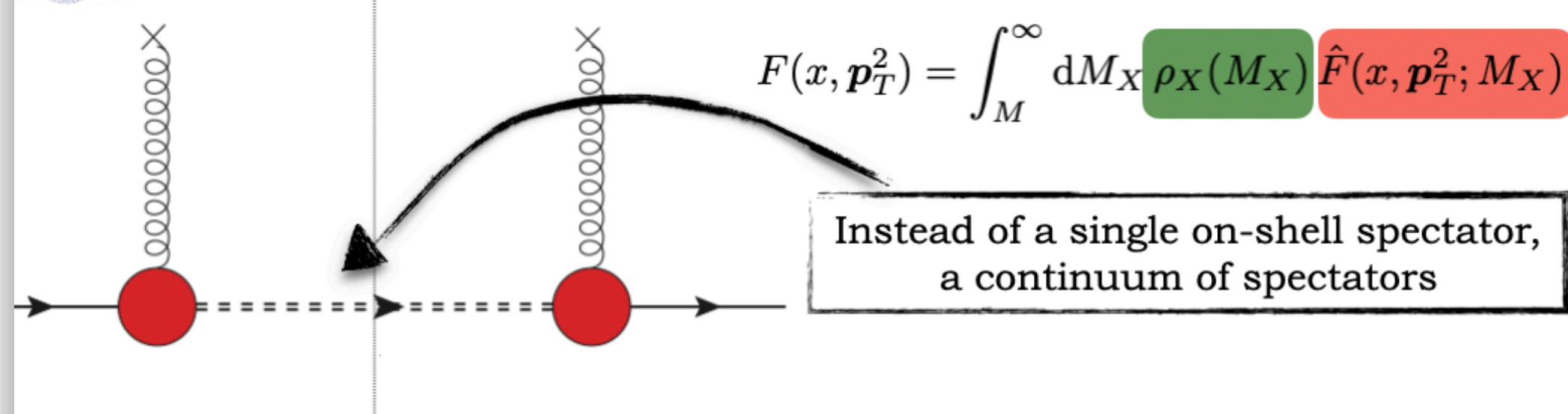


Spectator-model gluon TMD PDFs

Our model at a glance



Spectator-system spectral-mass function

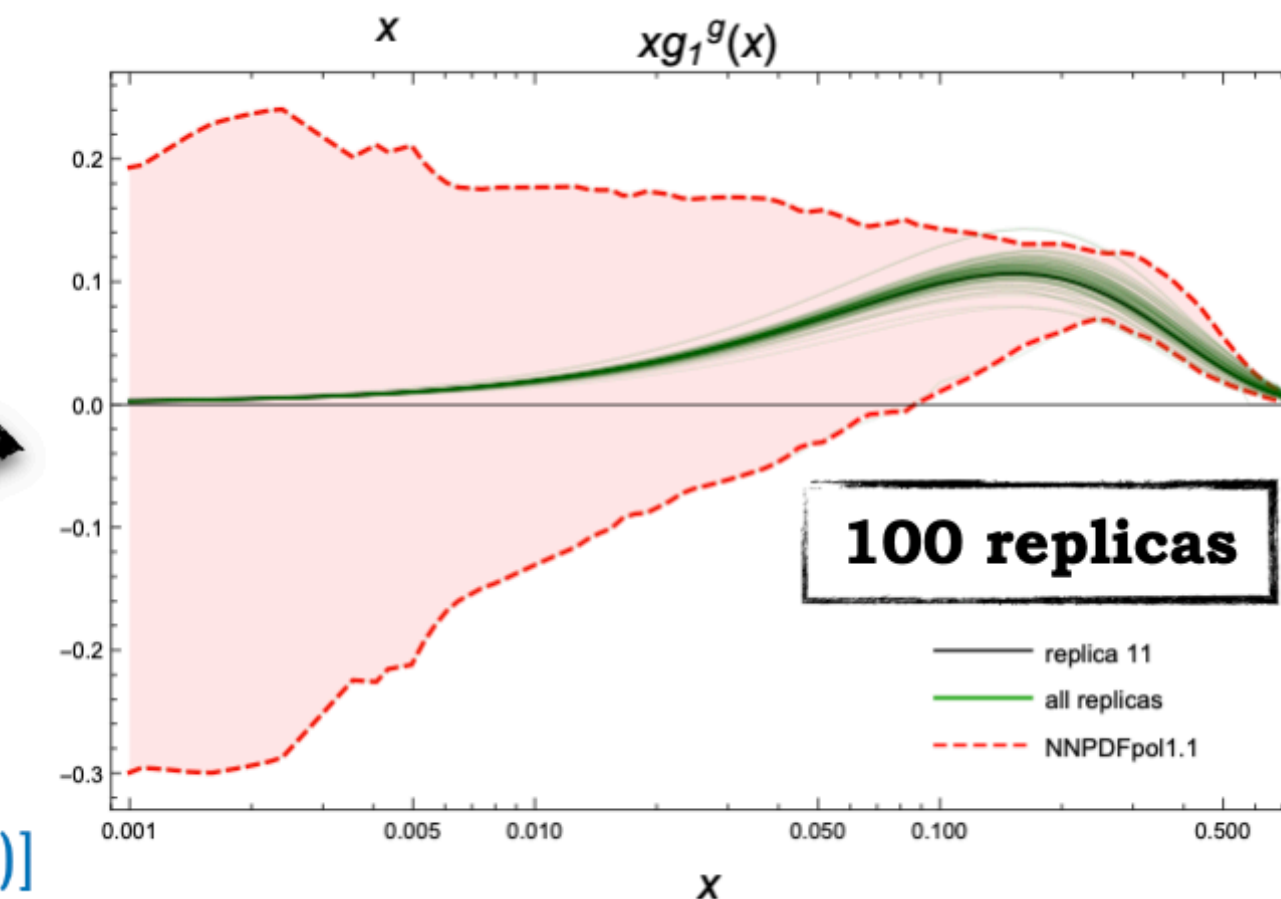
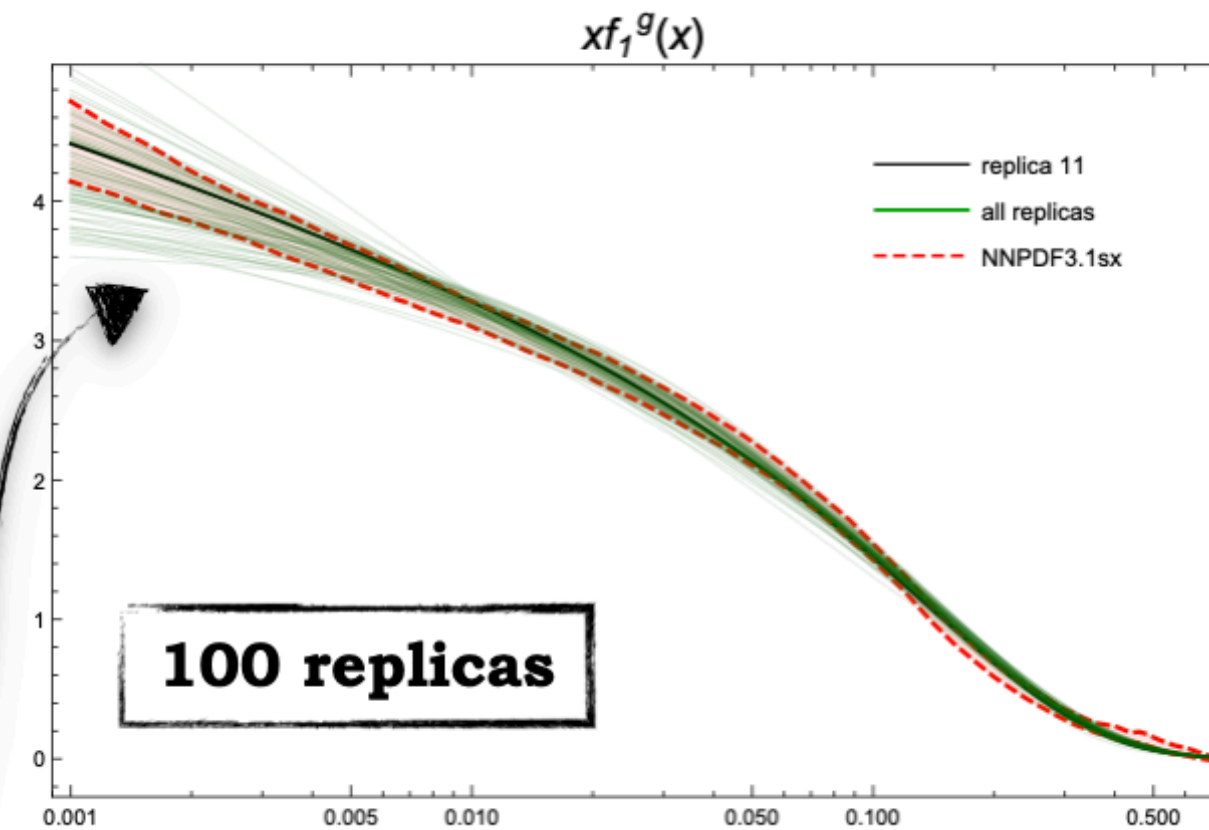


Link with collinear factorization

p_T -integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution

Spectral function **learns** small- and moderate- x info encoded in **NNPDF** collinear parametrizations (NNPDF3.1sx + NNPDFpol1.1)

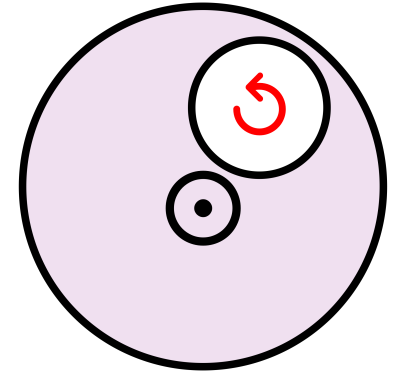
- ✓ Simultaneous fit of f_1 and g_1 PDFs
- ✓ Inclusion of small- x resummation effects (**BFKL**)
- ✓ Calculation of all leading-twist T-even gluon TMDs



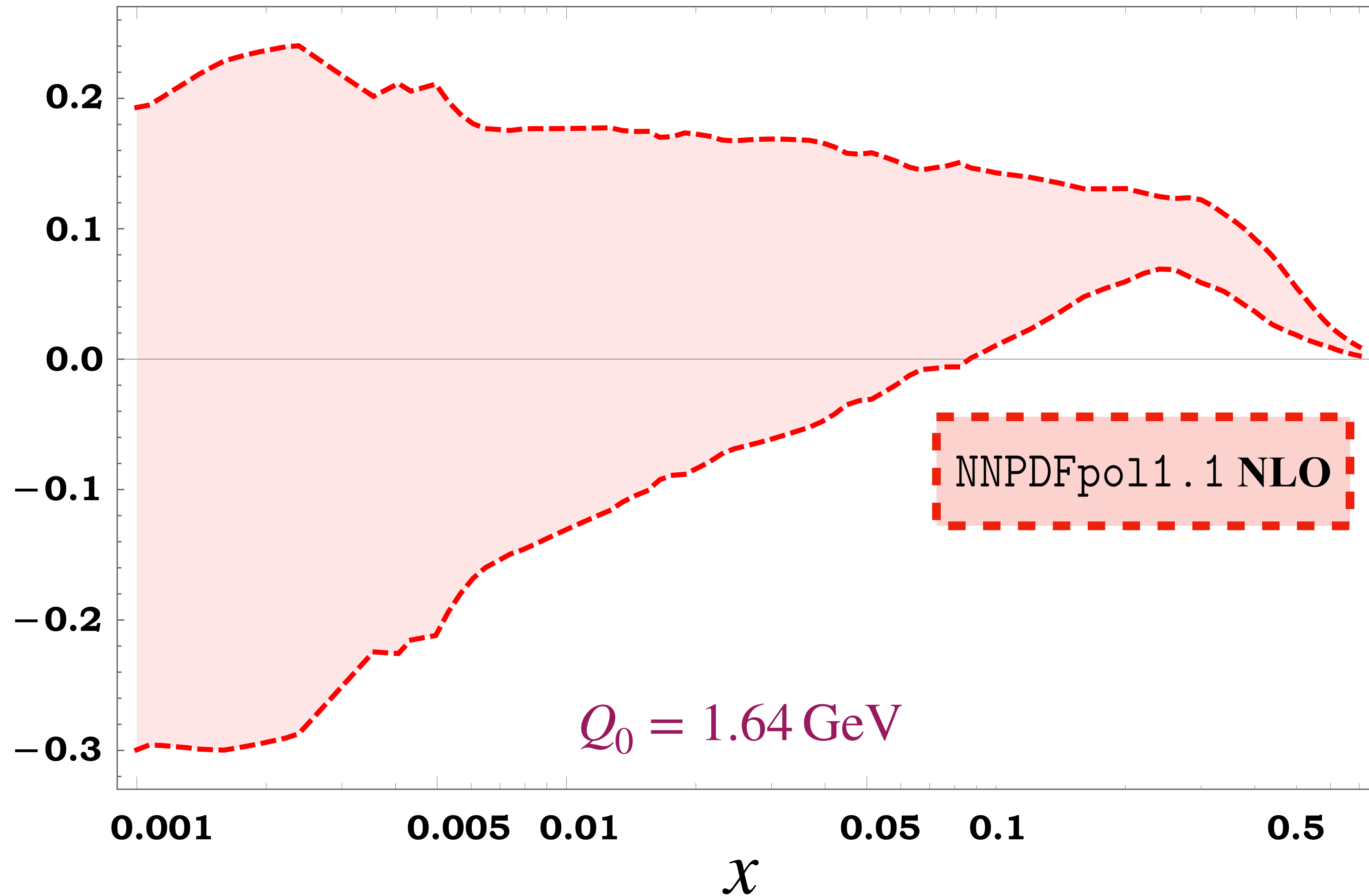
2.3 Modeling gluon TMDs [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

14

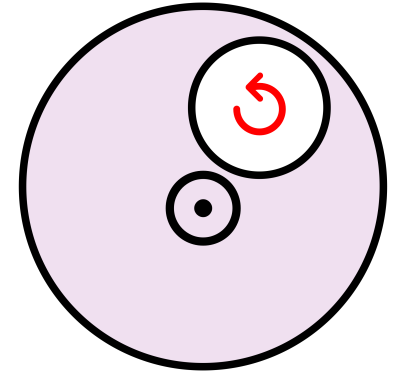
Helicity gluon collinear PDF



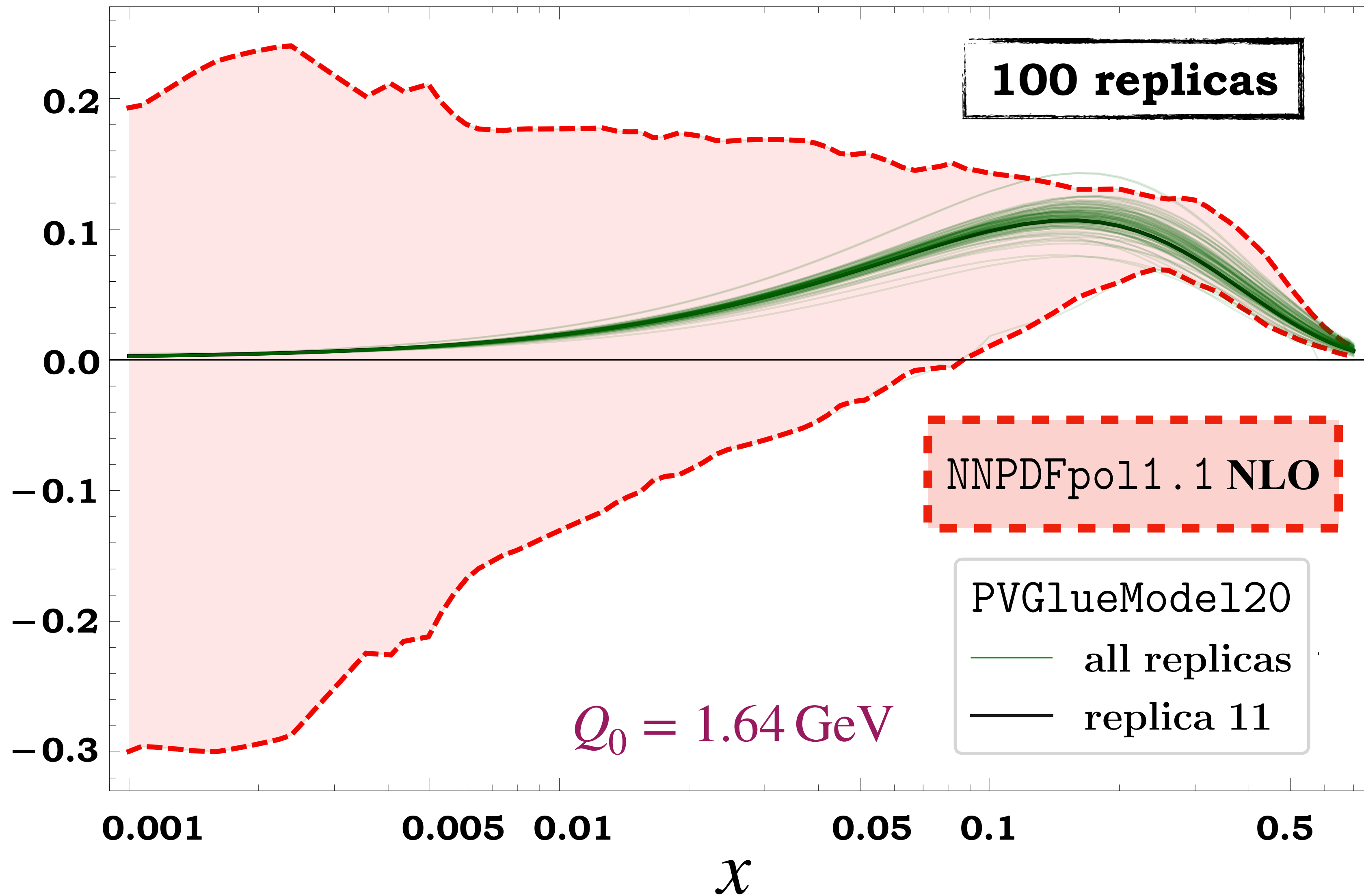
$$x g_1(x)$$



Helicity gluon collinear PDF



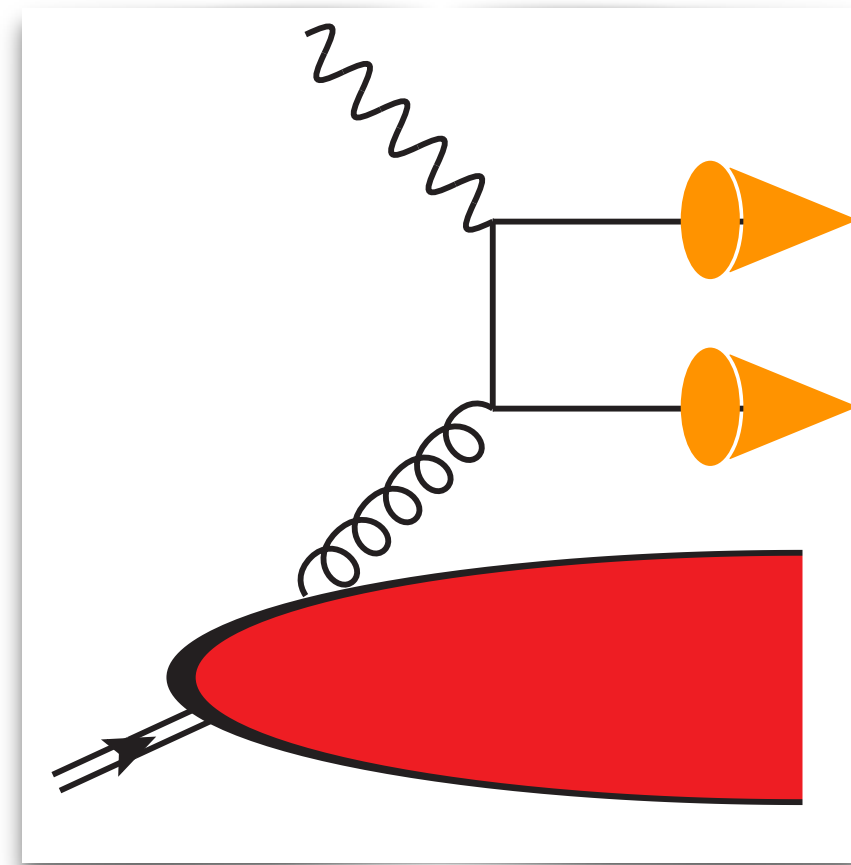
$$x g_1(x)$$



Backup

Golden channels for gluon TMD PDFs @EIC

Two-jet SIDIS



jet function

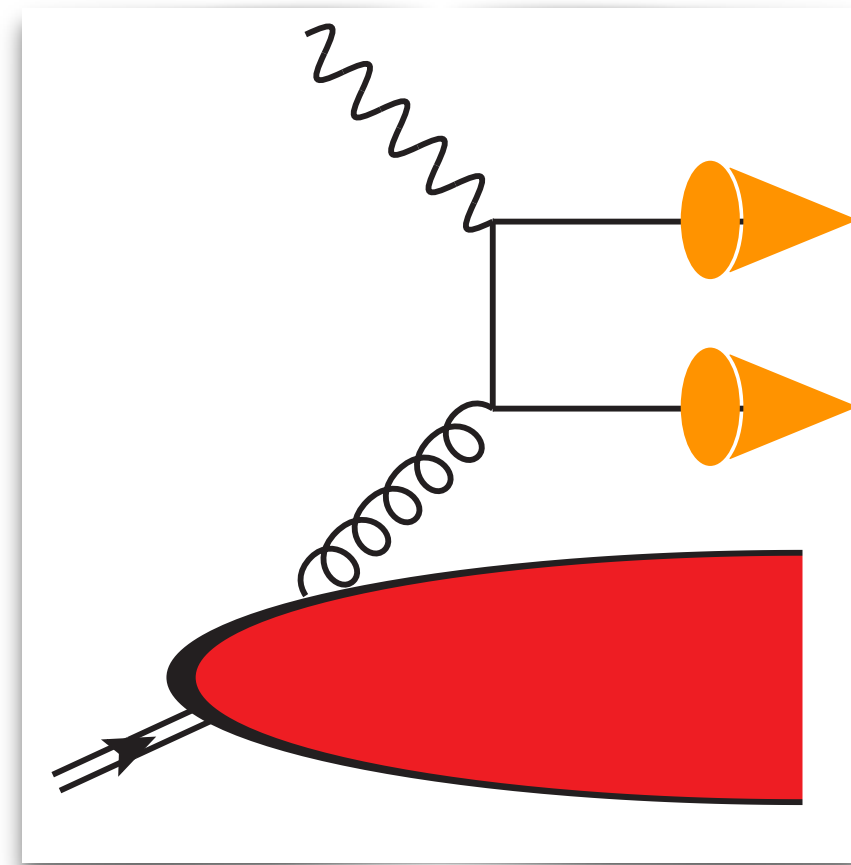
jet function

TMD PDF

Backup

Golden channels for gluon TMD PDFs @EIC

Two-jet SIDIS

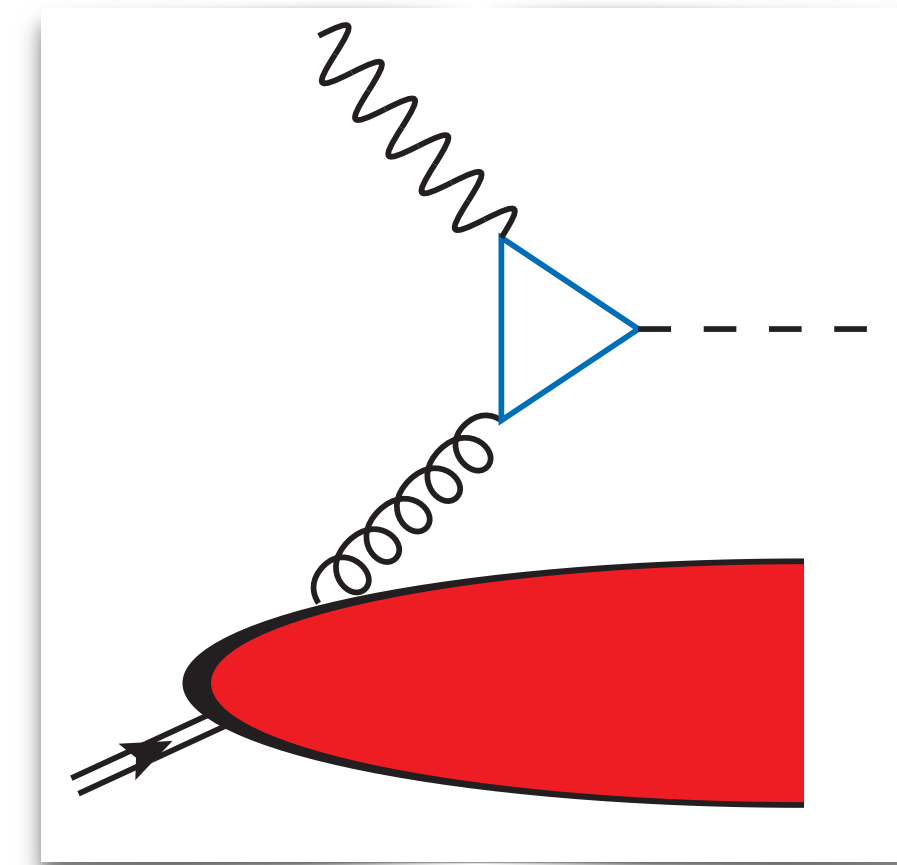


jet function

jet function

TMD PDF

Higgs in ep collisions

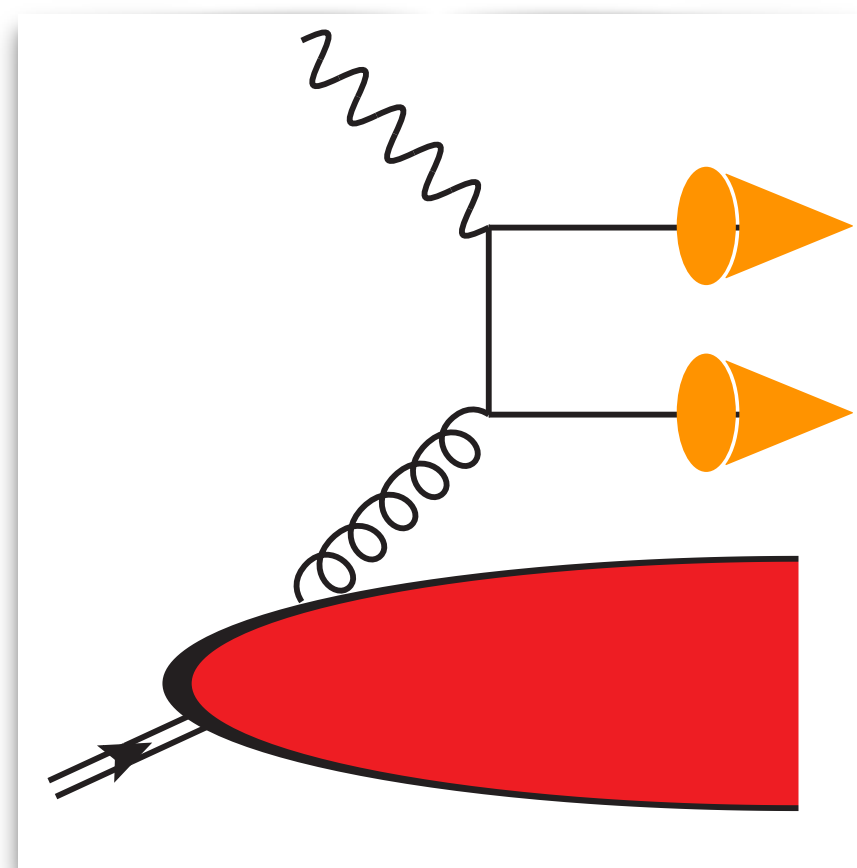


TMD PDF

Backup

Golden channels for gluon TMD PDFs @EIC

Two-jet SIDIS

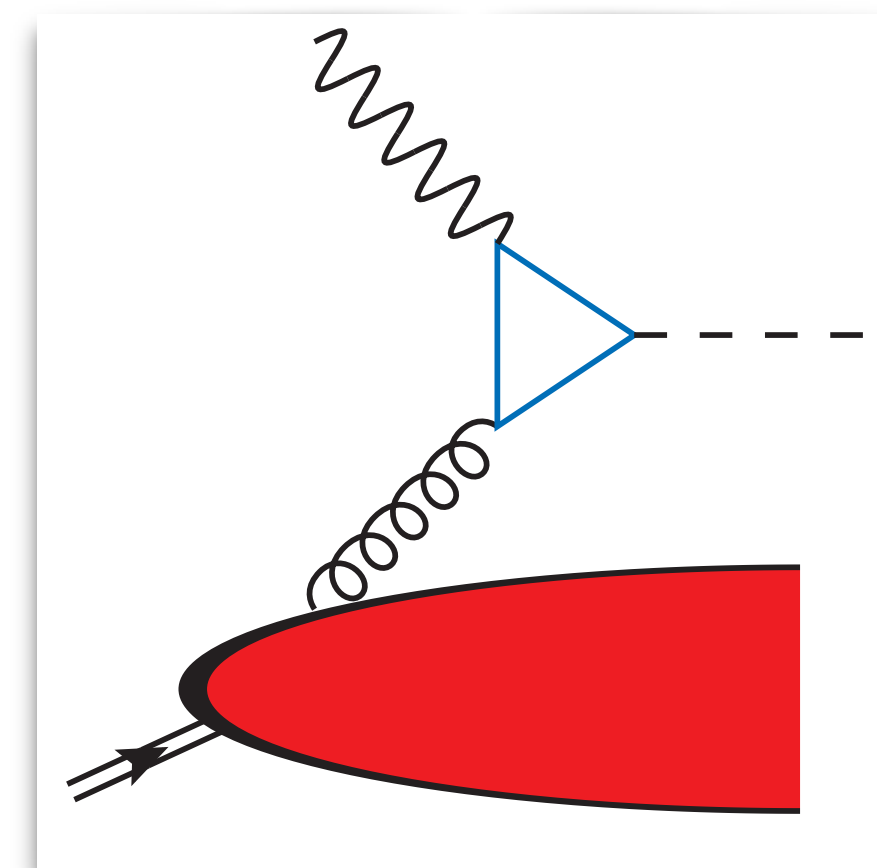


jet function

jet function

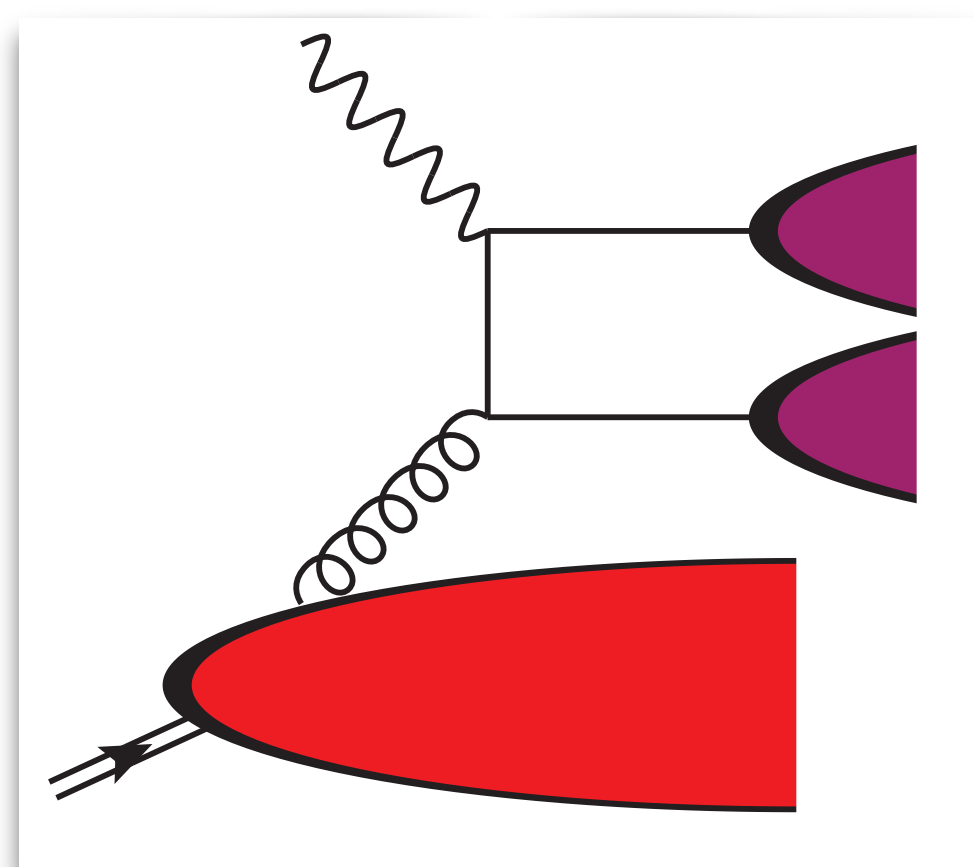
TMD PDF

Higgs in ep collisions



TMD PDF

Double D meson



TMD FF

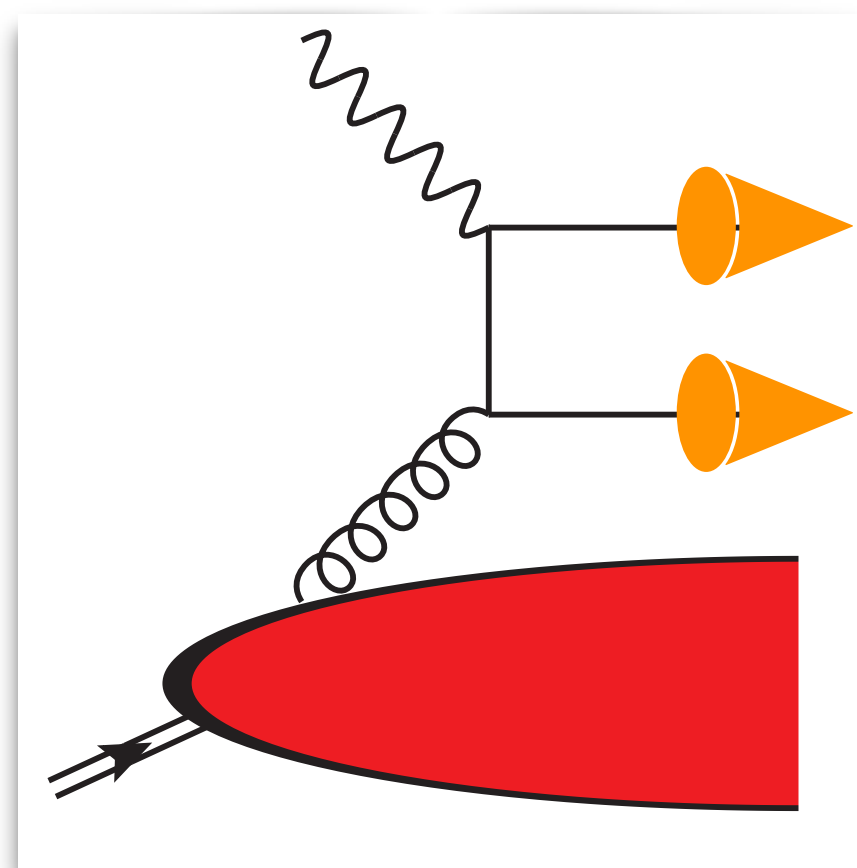
TMD FF

TMD PDF

Backup

Golden channels for gluon TMD PDFs @EIC

Two-jet SIDIS

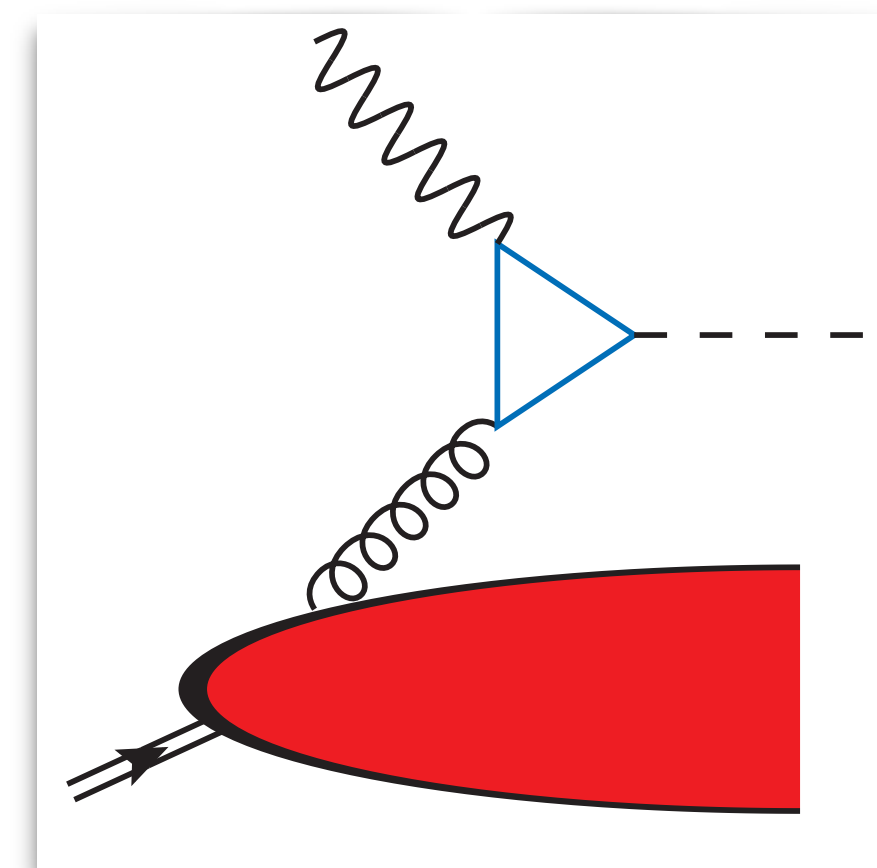


jet function

jet function

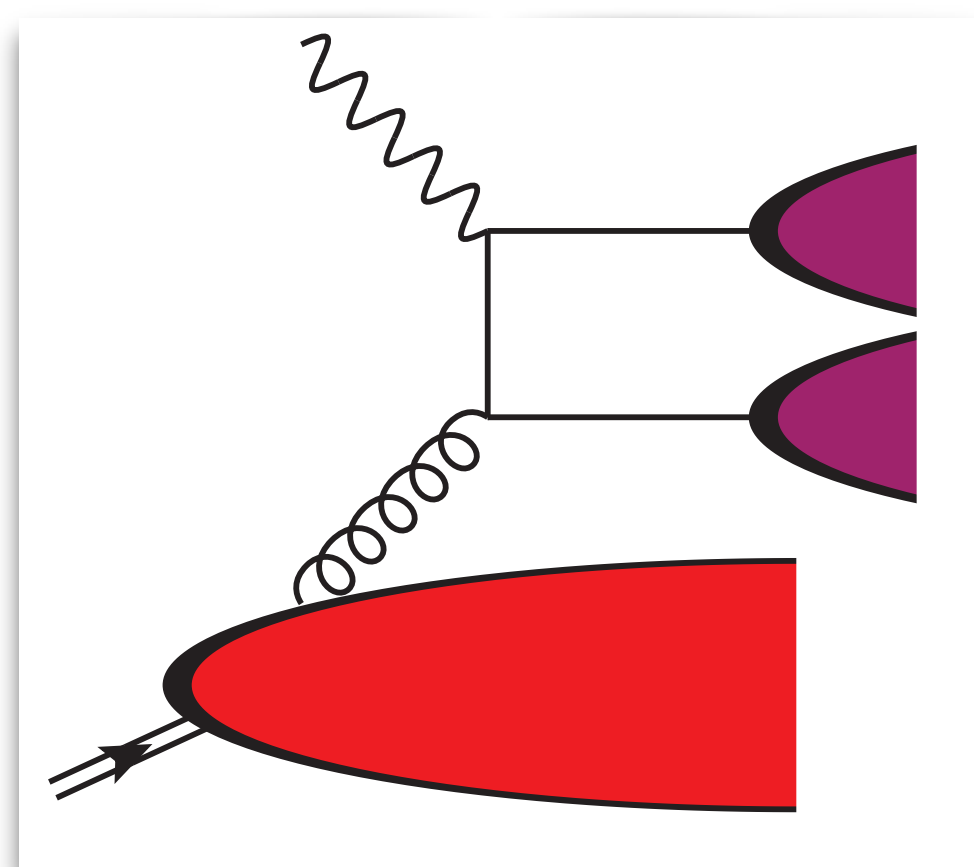
TMD PDF

Higgs in ep collisions



TMD PDF

Double D meson

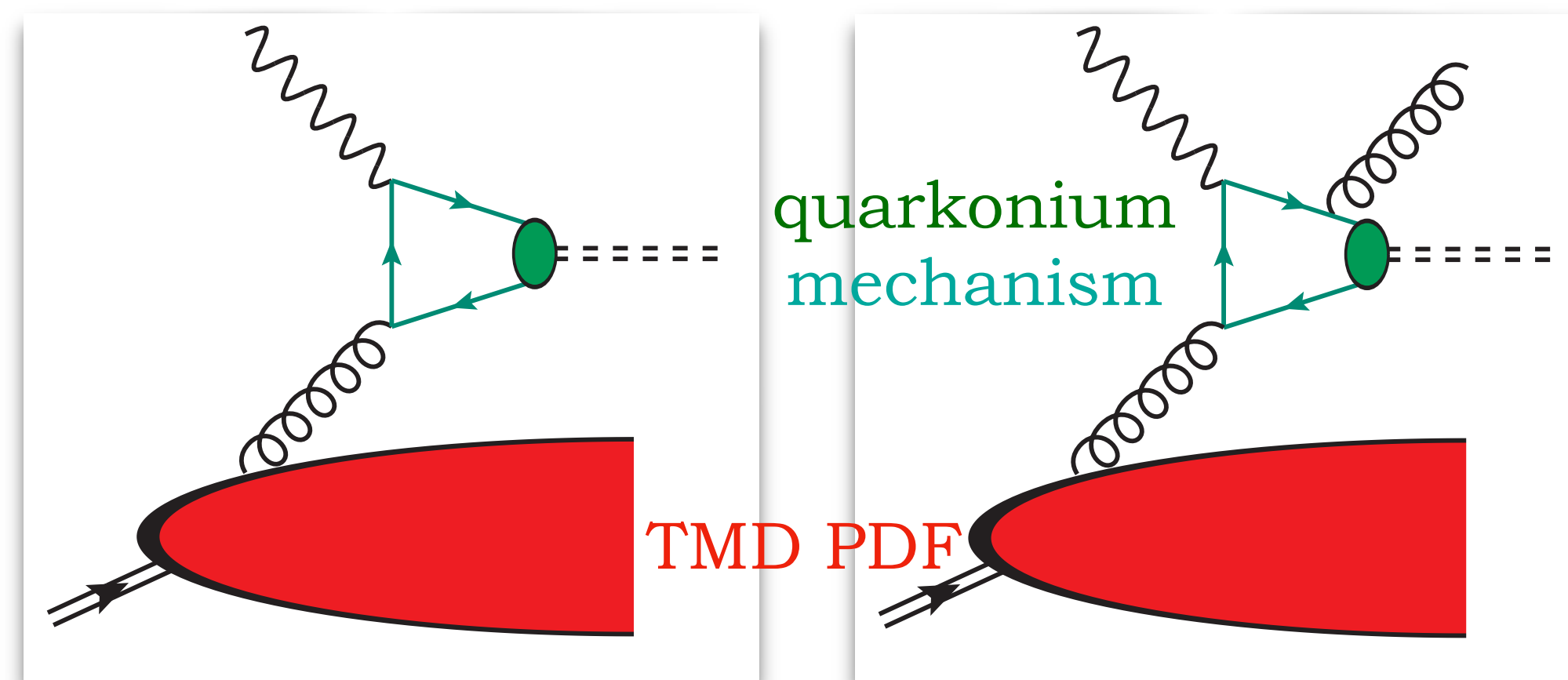


TMD FF

TMD FF

TMD PDF

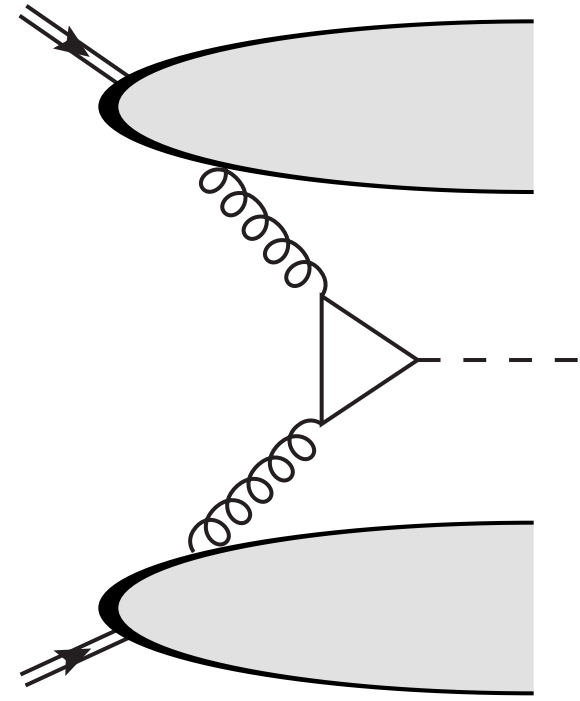
Quarkonia



TMD PDF

Backup

Higgs in gluon fusion



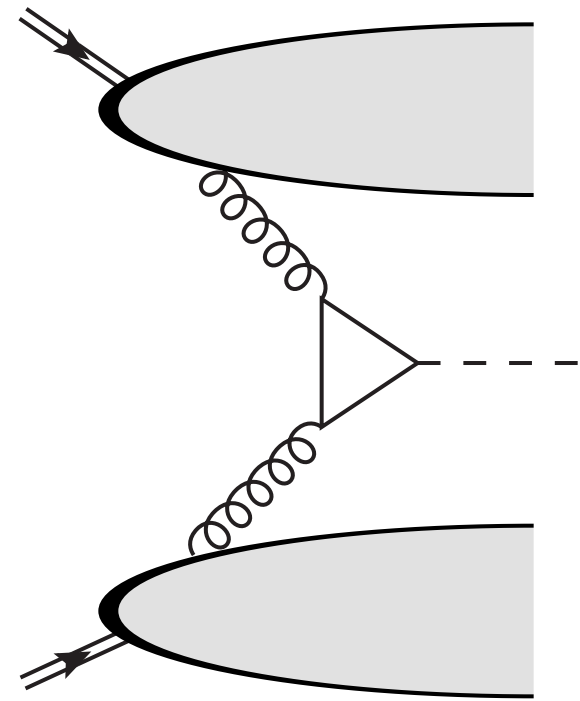
No color entanglement
TMD factorization



Large low- p_T bin @CMS
More data @HL-LHC

Golden channels for gluon TMD PDFs @LHC

Higgs in gluon fusion

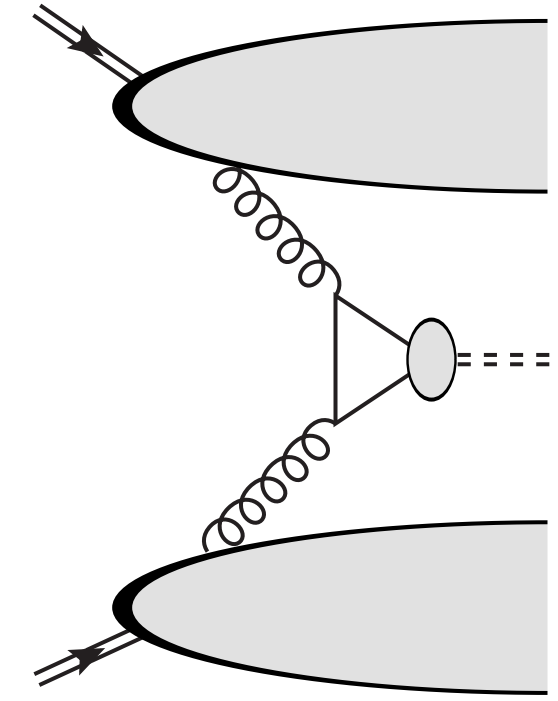


No color entanglement
TMD factorization



Large low- p_T bin @CMS
More data @HL-LHC

Single quarkonium



$\eta_{c,b}$ $J/\psi, \Upsilon$

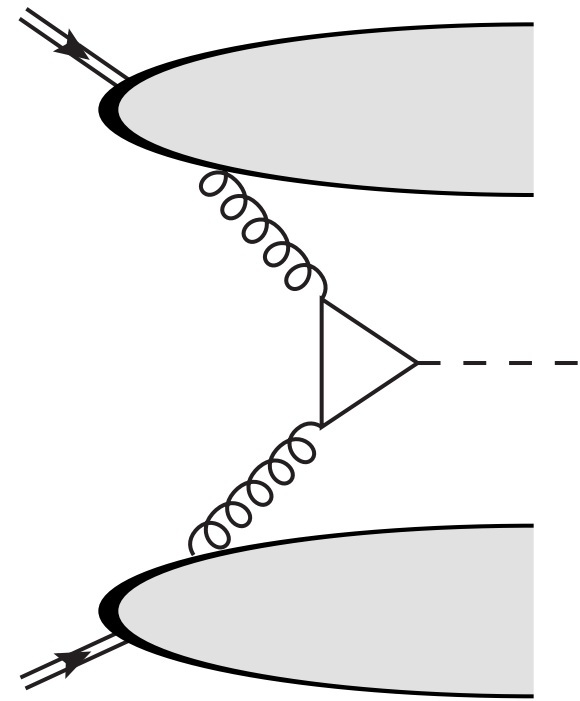
TMD factorization
C-parity selection rules



Large- p_T data @LHCb
More data @FT-LHC

Golden channels for gluon TMD PDFs @LHC

Higgs in gluon fusion

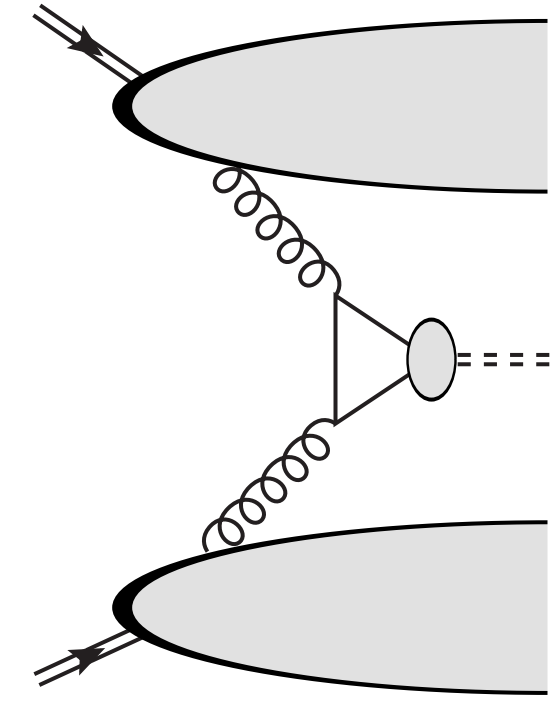


No color entanglement
TMD factorization



Large low- p_T bin @CMS
More data @HL-LHC

Single quarkonium



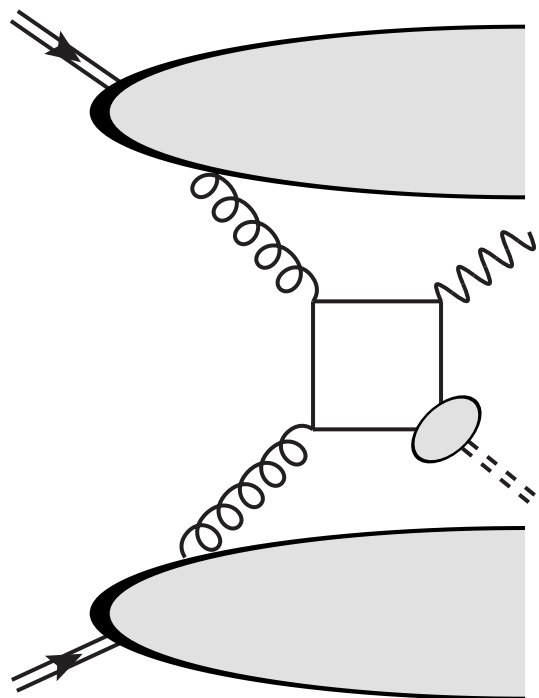
$\eta_{c,b}$ $J/\psi, \Upsilon$

TMD factorization
C-parity selection rules



Large- p_T data @LHCb
More data @FT-LHC

$J/\psi + \gamma^{(*)}$



Color entanglement
Potential TMD violation (CO)
Back-to-back suppresses CO

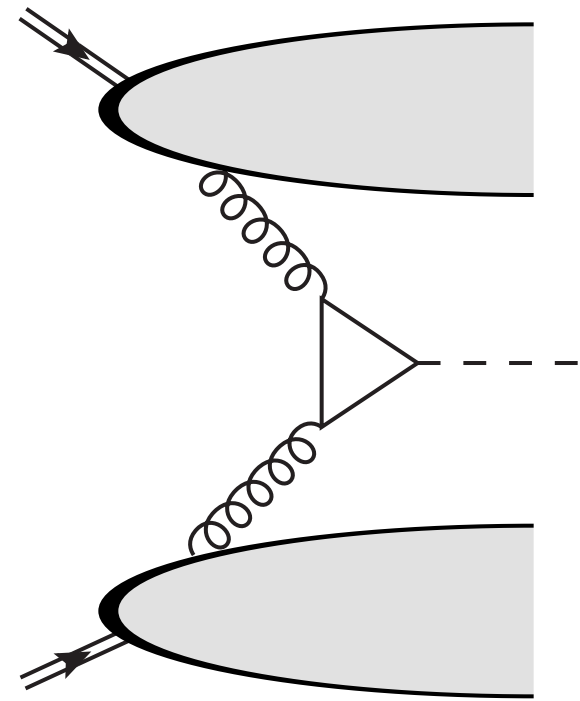


Possible studies @HL-LHC
Currently no low- p_T data

Backup

Golden channels for gluon TMD PDFs @LHC

Higgs in gluon fusion

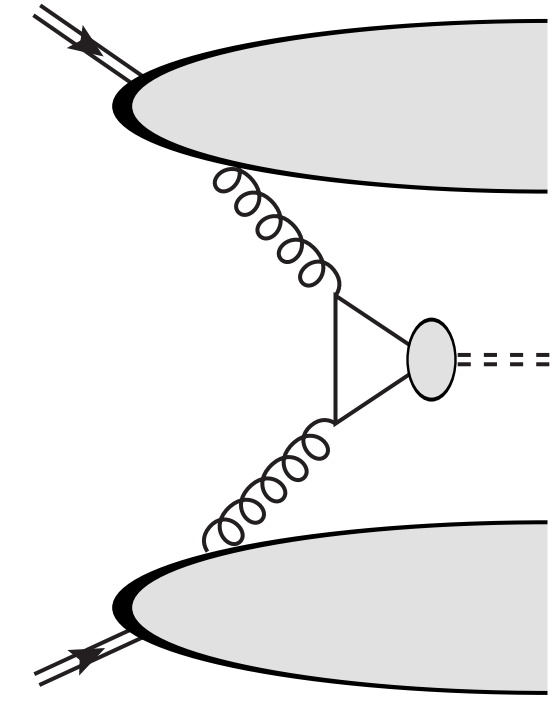


No color entanglement
TMD factorization



Large low- p_T bin @CMS
More data @HL-LHC

Single quarkonium



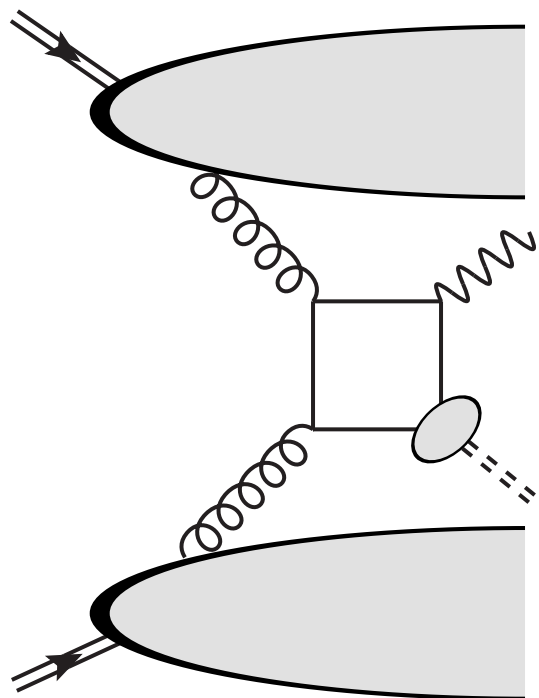
$\eta_{c,b}$ $J/\psi, \Upsilon$

TMD factorization
C-parity selection rules



Large- p_T data @LHCb
More data @FT-LHC

$J/\psi + \gamma^{(*)}$

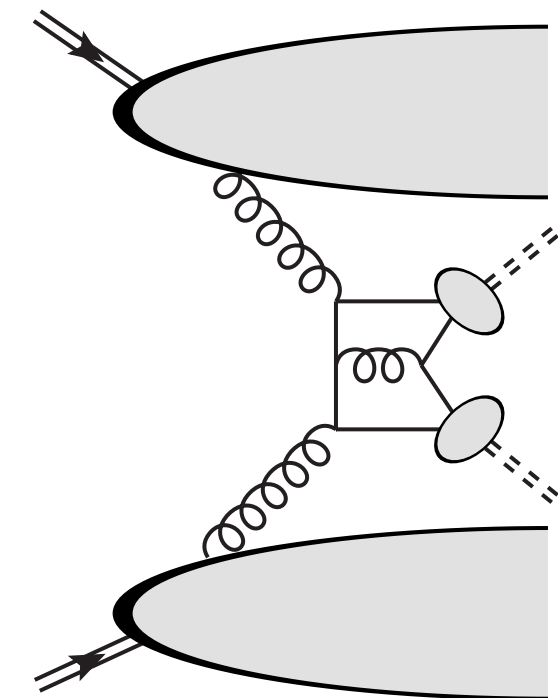


Color entanglement
Potential TMD violation (CO)
Back-to-back suppresses CO



Possible studies @HL-LHC
Currently no low- p_T data

$J/\psi + J/\psi$



No color entanglement
TMD factorization (CSM)



Low- p_T data @LHCb
Opportunities @HL- & @FT-LHC

Backup

Quarkonia: Assets & challenges

Assets

 Onia \Rightarrow clean channels of f-type gluon TMDs

Initial-state color flow \Rightarrow $[-, -]$ gauge link

(overview)  [D. Boer (2017)]

Sivers	$ep^\dagger \rightarrow e' Q \bar{Q} X$ $ep^\dagger \rightarrow e' j_1 j_2 X$
$f_{1T}^{\perp g[-,-]}$	✓
$f_{1T}^{\perp g[+,-]}$	×

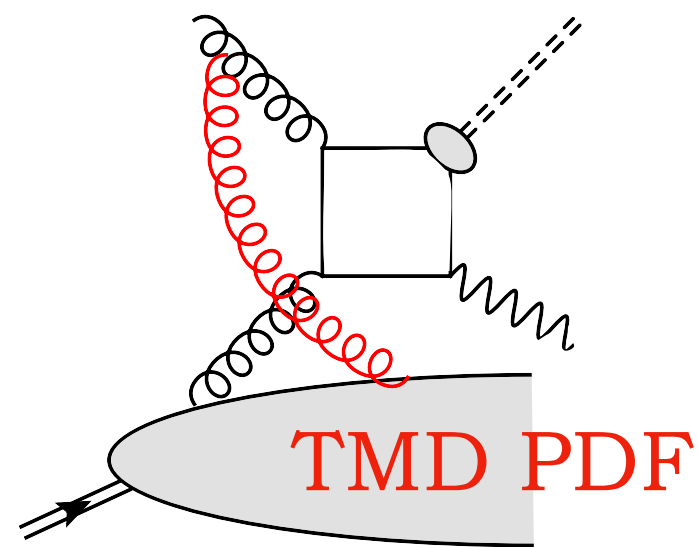
Boer-Mulders	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$
$h_1^{\perp g[-,-]}(\text{WW})$	✓
$h_1^{\perp g[+,-]}(\text{DP})$	×

Challenges

Quarkonia: Assets & challenges

Assets

Onia \Rightarrow clean channels of f-type gluon TMDs



Initial-state color flow \Rightarrow $[-, -]$ gauge link

Sivers	$ep^\dagger \rightarrow e' Q \bar{Q} X$ $ep^\dagger \rightarrow e' j_1 j_2 X$
$f_{1T}^{\perp g[-,-]}$	✓
$f_{1T}^{\perp g[+,-]}$	×

(overview) [\[D. Boer \(2017\)\]](#)

Boer-Mulders	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$
$h_1^{\perp g[-,-]}(WW)$	✓
$h_1^{\perp g[+,-]}(DP)$	×

$\eta_{c,b}$ \Rightarrow LHC complementarity, TMD factorization

$$\frac{d\sigma}{dq_T} \sim$$

at low transverse momentum
for [pseudo]scalar state

$$\sim \mathcal{C} [f_1^{g/A} f_1^{g/B}] \pm \mathcal{C} [h_1^{\perp g/A} h_1^{\perp g/B}]$$

unpolarized gluons

lin. polarized gluons

(factorization) [\[M. García Echevaría \(2019\)\]](#)

(pheno) [\[A. Bacchetta, F.G.C., J.-P. Lansberg, M. Radici, et al. \(to appear\)\]](#)

Challenges

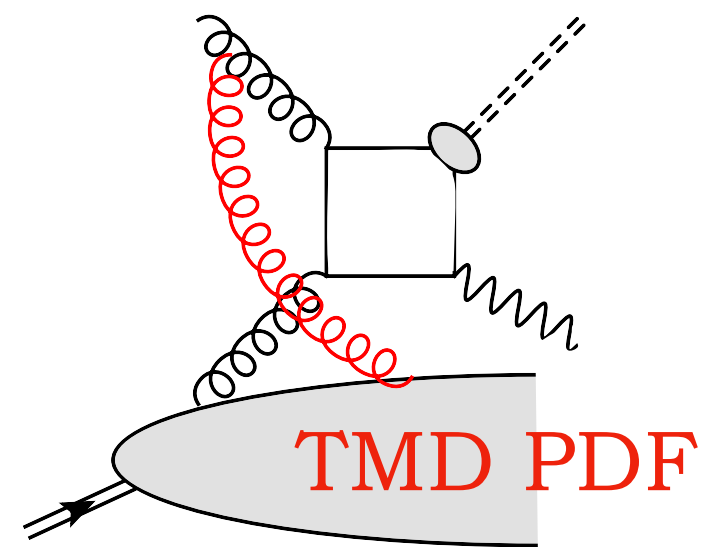
Precision TMD \Leftrightarrow production mechanism(s)

(production mechanisms, LHC pheno) [\[J.-P. Lansberg \(2020\)\]](#)

Quarkonia: Assets & challenges

Assets

Onia \Rightarrow clean channels of f-type gluon TMDs



Initial-state color flow \Rightarrow $[-, -]$ gauge link

Sivers	$ep^\dagger \rightarrow e' Q \bar{Q} X$ $ep^\dagger \rightarrow e' j_1 j_2 X$
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$$\sim \underbrace{C [f_1^{g/A} f_1^{g/B}]}_{\text{unpolarized gluons}} \pm \underbrace{C [h_1^{\perp g/A} h_1^{\perp g/B}]}_{\text{lin. polarized gluons}}$$

(factorization) [\[M. García Echevarría \(2019\)\]](#)

(pheno) [\[A. Bacchetta, F.G.C., J.-P. Lansberg, M. Radici, et al. \(to appear\)\]](#)

Challenges

Precision TMD \Leftrightarrow production mechanism(s)

(production mechanisms, LHC pheno) [\[J.-P. Lansberg \(2020\)\]](#)

Color Evaporation Model

$(Q\bar{Q})$ decorrelated from onium, semi-soft gluon emissions
Overshoots data at large p_T

Color Singlet Model

$(Q\bar{Q})$ to onium, no gluon emissions
Fails at large p_T , improves at NLO

NRQCD and Color Octet

Higher Fock states, soft gluon emissions
Problems at low p_T , fails on polarization

Quarkonia: A path toward precision

TMD PDFs & shape functions

 NRQCD \Rightarrow double expansion: $\alpha_s \oplus v$

 NRQCD \Rightarrow $d\sigma(|Q\rangle) \propto \mathcal{H} \otimes \text{LDME}$

$$|Q\rangle = \mathcal{O}(1) |Q\bar{Q}[^3S_1^{(1)}]\rangle + \mathcal{O}(v) |Q\bar{Q}[^3P_J^{(8)}g]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^1S_0^{(8)}g]\rangle \\ + \mathcal{O}(v^2) |Q\bar{Q}[^3S_1^{(1,8)}gg]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^3D_J^{(1,8)}gg]\rangle + \dots$$

S-wave quarkonium wave function

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S-wave quarkonium wave function

 TMD \Rightarrow from LDMEs to shape functions (ShFs)

 2 mechanisms: bound state + soft-gluon

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(SCET)  [S. Fleming, Y. Makris, T. Mehen (2020)]

(unpol. J/ψ)  [D. Boer, U. D'Alesio, F. Murgia, C. Pisano, P. Taelis (2020)]

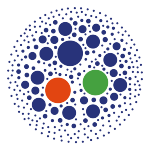
(pol. J/ψ)  [D. Boer, U. D'Alesio, L. Maxia, F. Murgia, C. Pisano, R. Sangem (2022)]

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Revised TMD shape function in SIDIS

$$\Delta^{[n]}(\kappa_T^2) \propto \frac{\alpha_s}{2\pi^2 \kappa_T^2} C_A \left(1 + \ln \frac{M_Q^2}{M_Q^2 + Q^2} \right) \langle O[n] \rangle$$

Quarkonia: A path toward precision

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 2 mechanisms: bound state + soft-gluon


 Perturbative tail \otimes LDME

 ShFs and TMD FFs exhibit different divergences

Quarkonia & Gluon TMDs: a path toward precision

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(factorization)  [M. Garcia Echevarria (2019)]

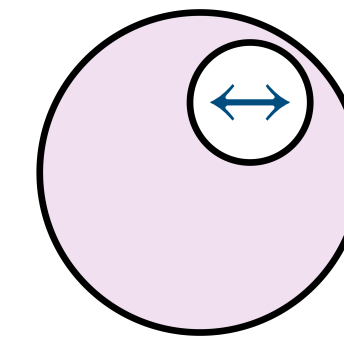
(SCET)  [S. Fleming, Y. Makris, T. Mehen (2020)]

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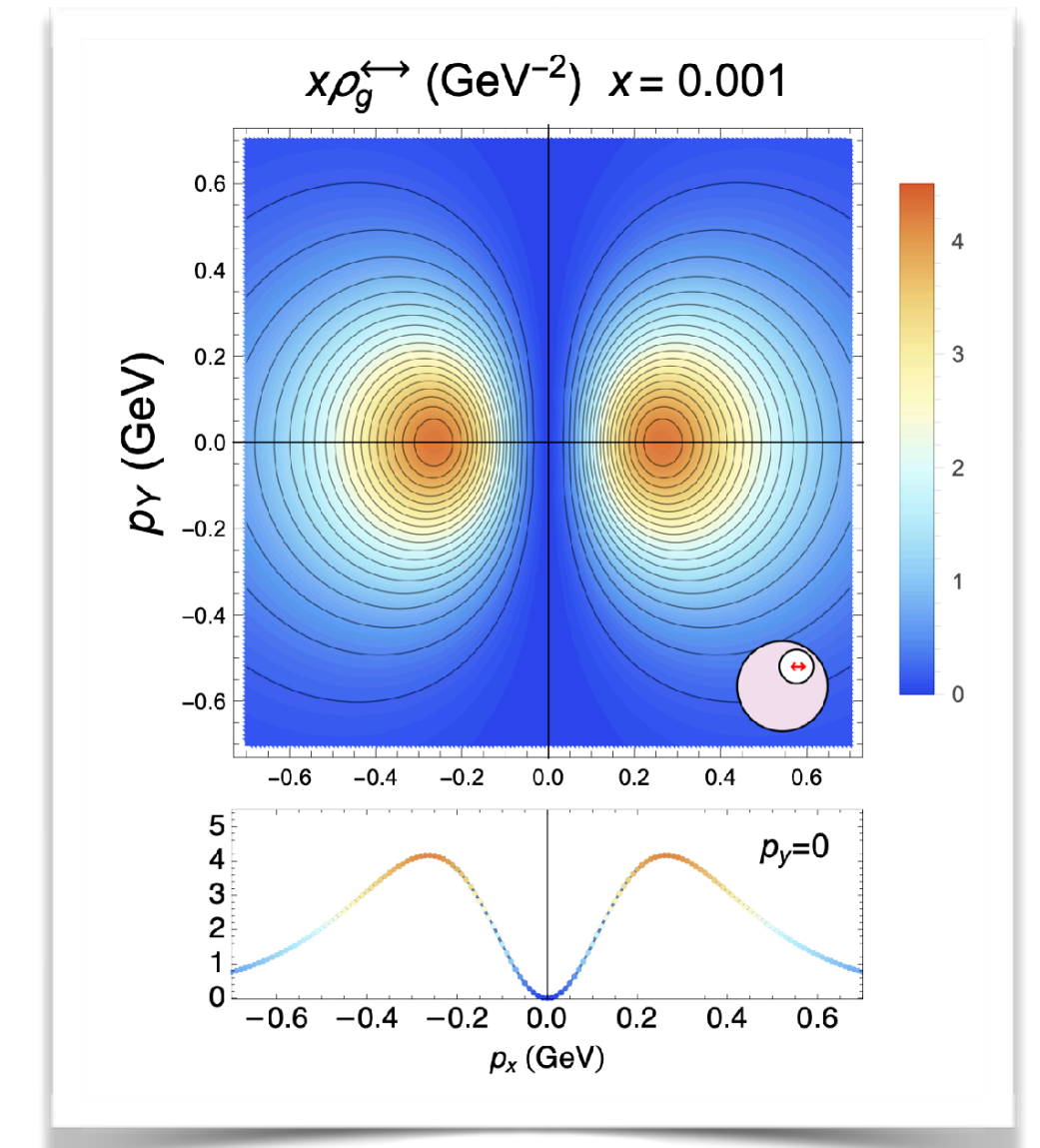
(unpol. J/ψ)  [D. Boer, J. Bor, L. Maxia, C. Pisano, F. Yuan (2023)]

3D proton imaging: LHC & EIC



EIC, LHCb, FT@LHC

Boer-Mulders

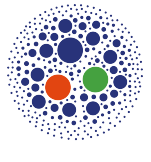


 [A. Bacchetta, F.G. C., M. Radici, P. Tael (2020)]

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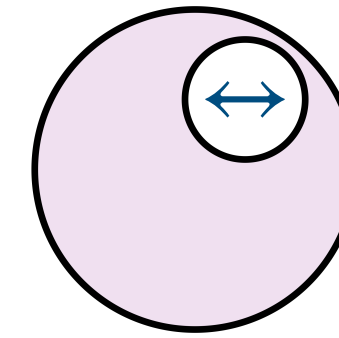
(SCET)  [S. Fleming, Y. Makris, T. Mehen (2020)]

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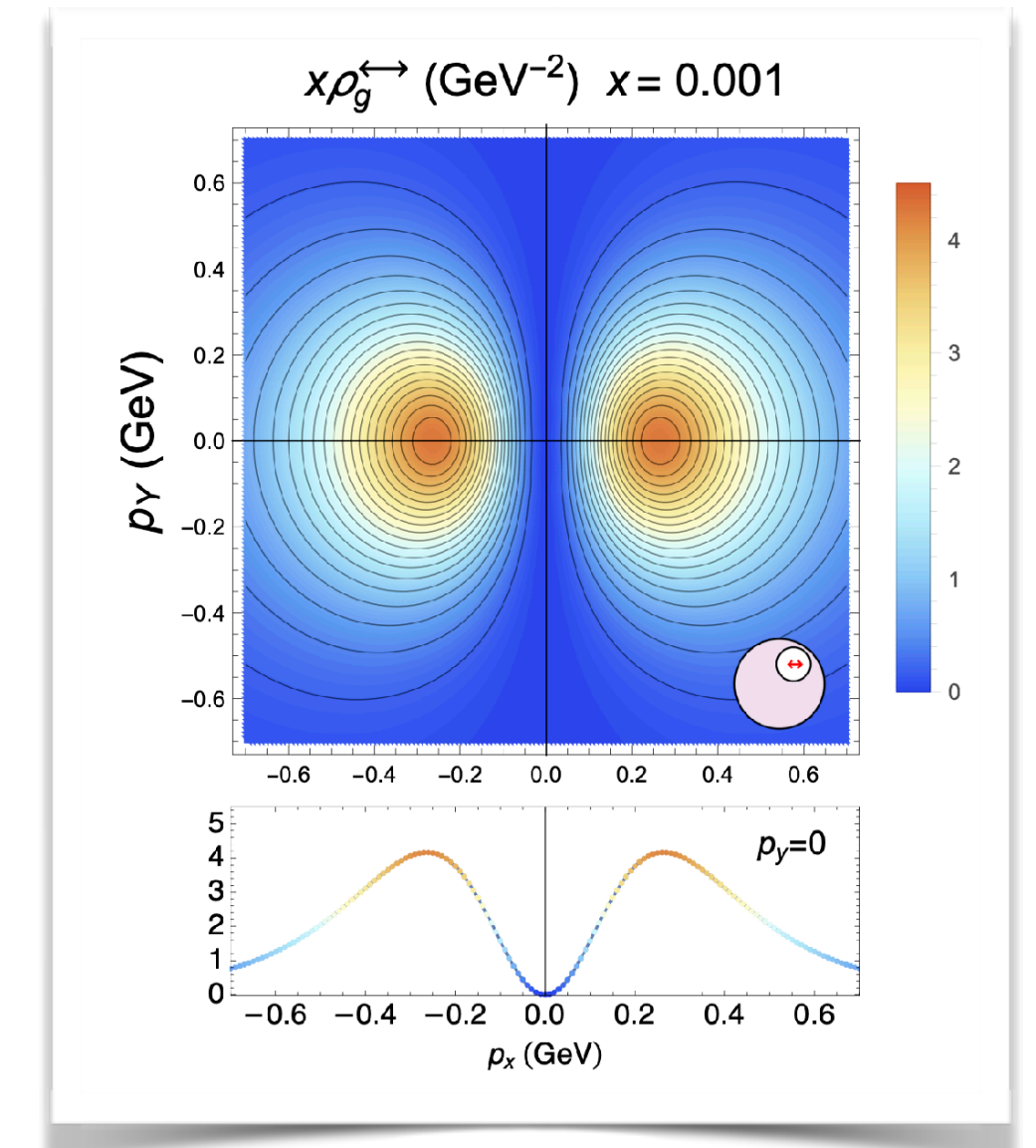
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3D proton imaging: LHC & EIC



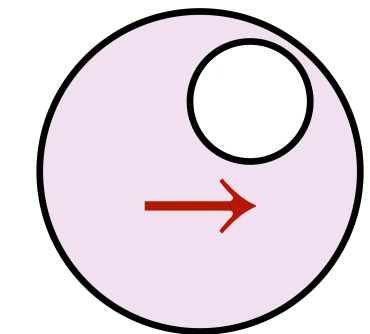
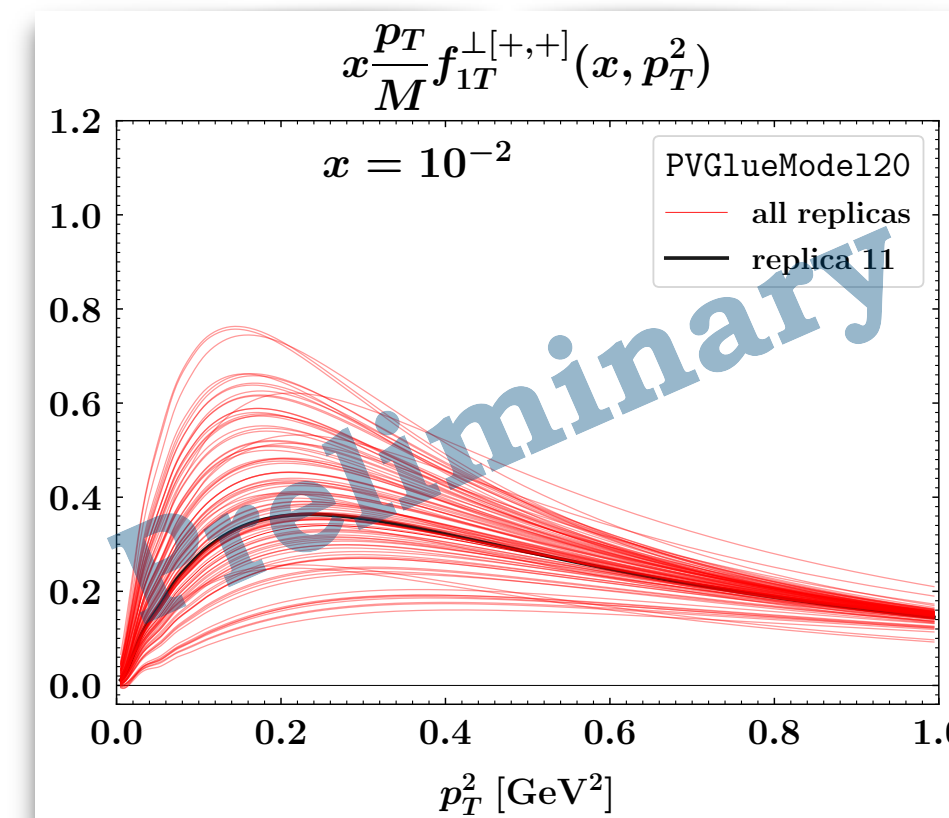
EIC, LHCb, FT@LHC

Boer-Mulders



 [A. Bacchetta, F.G. C., M. Radici, P. Tael (2020)]

[A. Bacchetta, F.G. C., M. Radici (to appear)]



EIC, LHCspin

Sivers

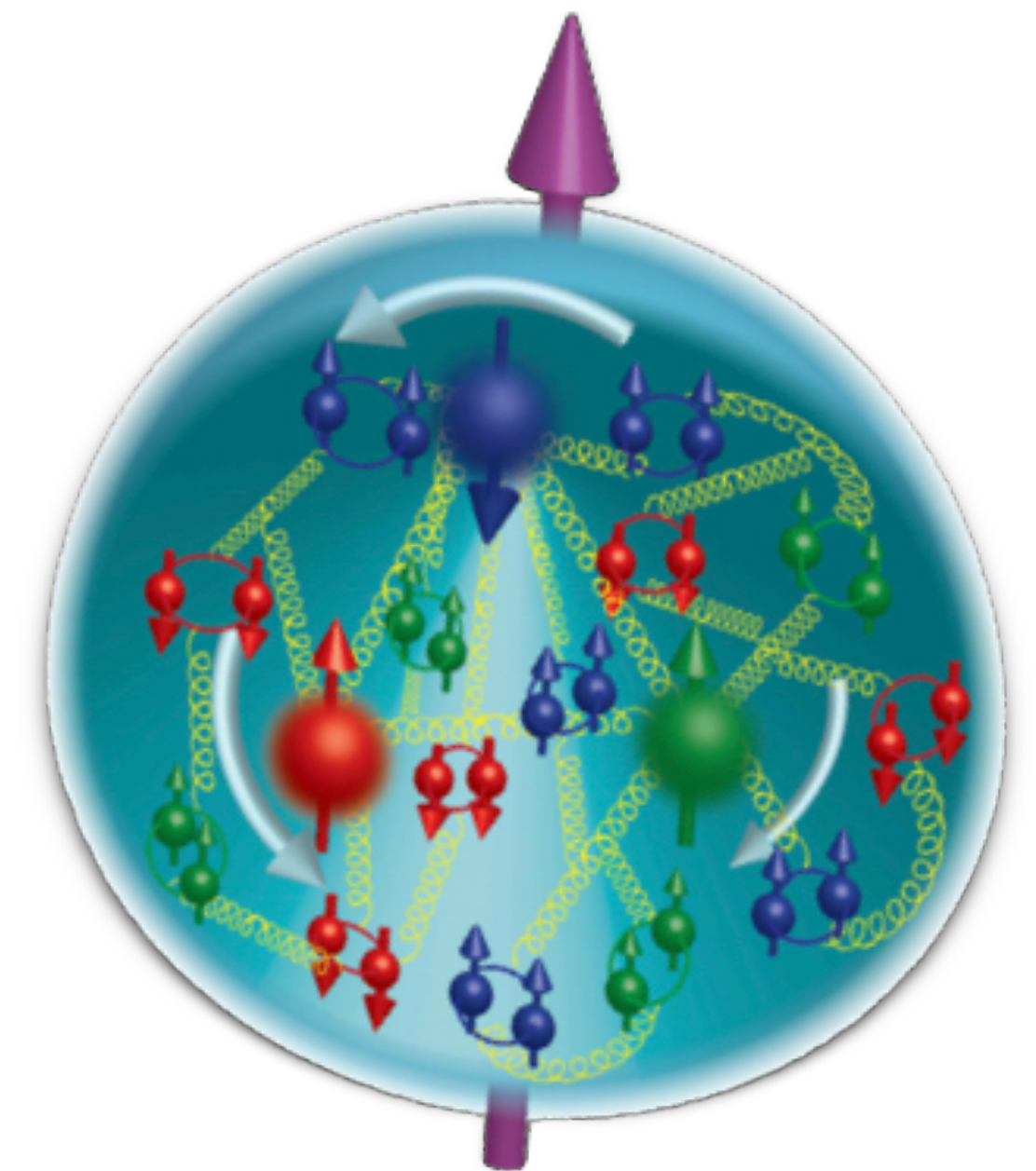
Backup



TMD factorization

Parton densities: Hors d'œuvre

- **Parton densities** → relevant for the search of **New Physics**...
 - ...crucial role in the understanding and exploration of **QCD**
- Describe the internal structure of the nucleon in terms of its elementary constituents (quarks and gluons)
- **Nonperturbative** objects that enter the expression of cross sections
- Can be *extracted* from experiments via *global fits*



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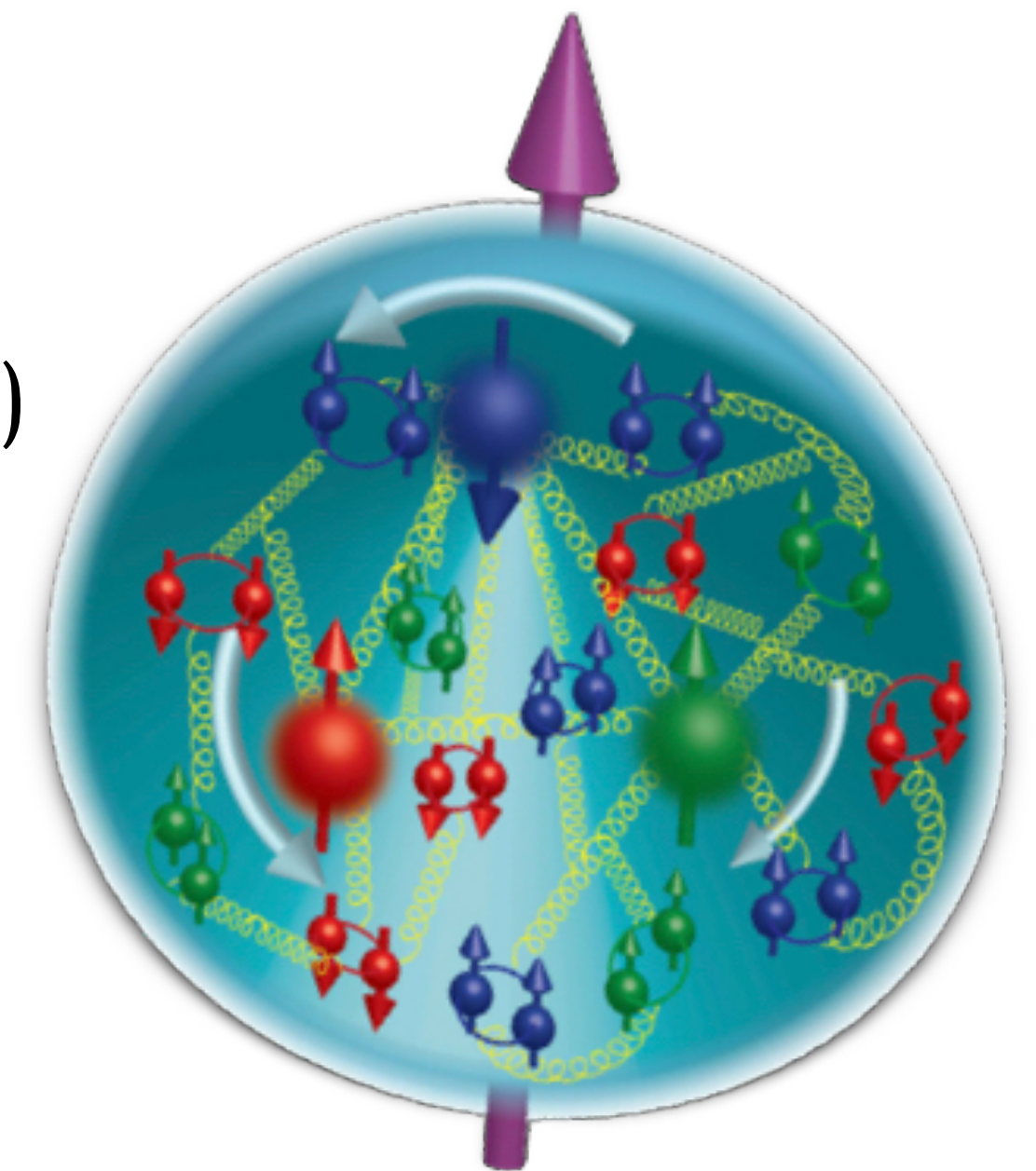
- Can be *extracted* from experiments via *global fits*

- Several types of distributions (1D collinear, **3D TMD**, **3D GPD**, ...)

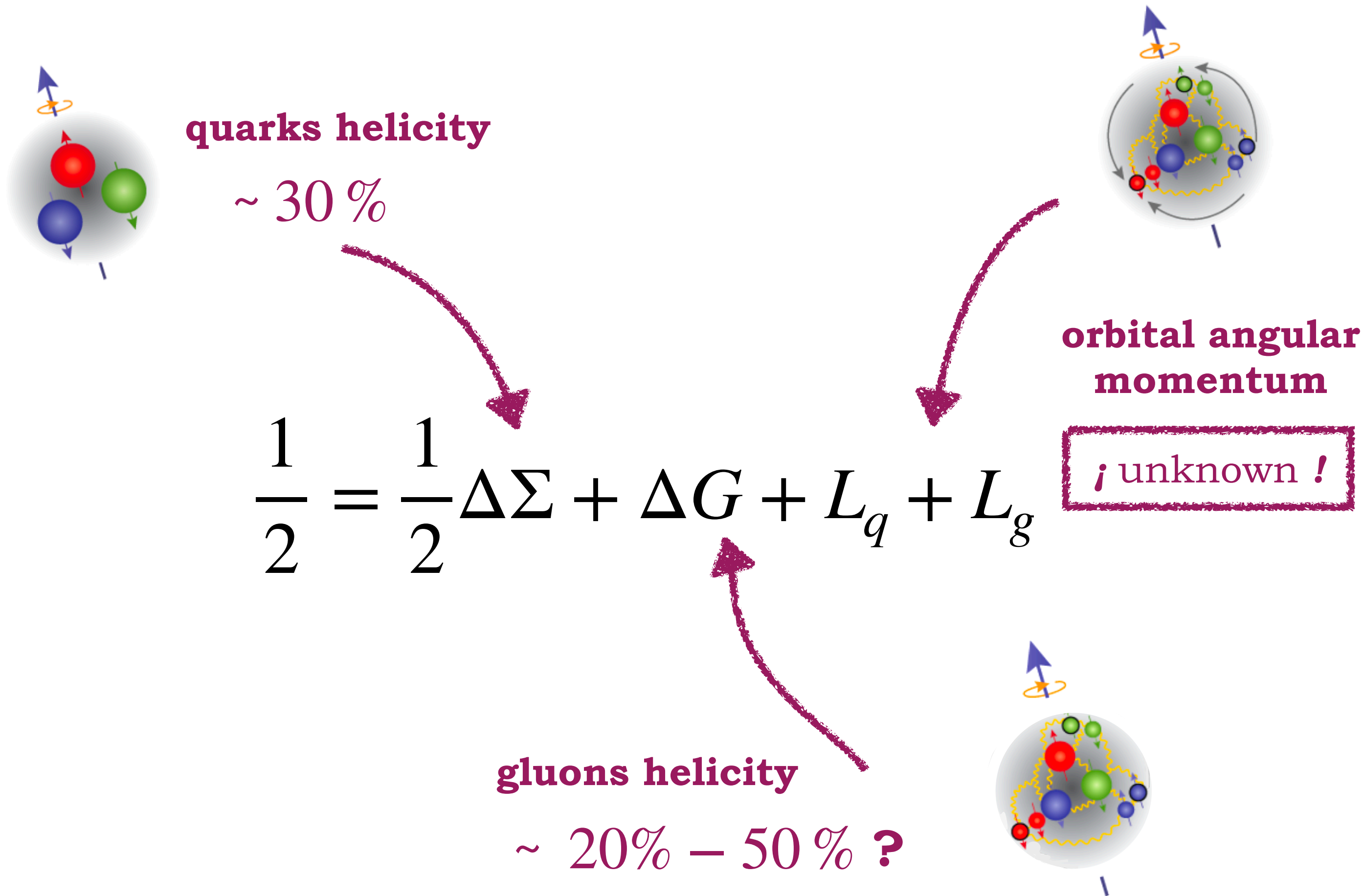
- Follow from different **factorization theorems**

- Exhibit peculiar **universality properties**

- Obey distinct **evolution equations**



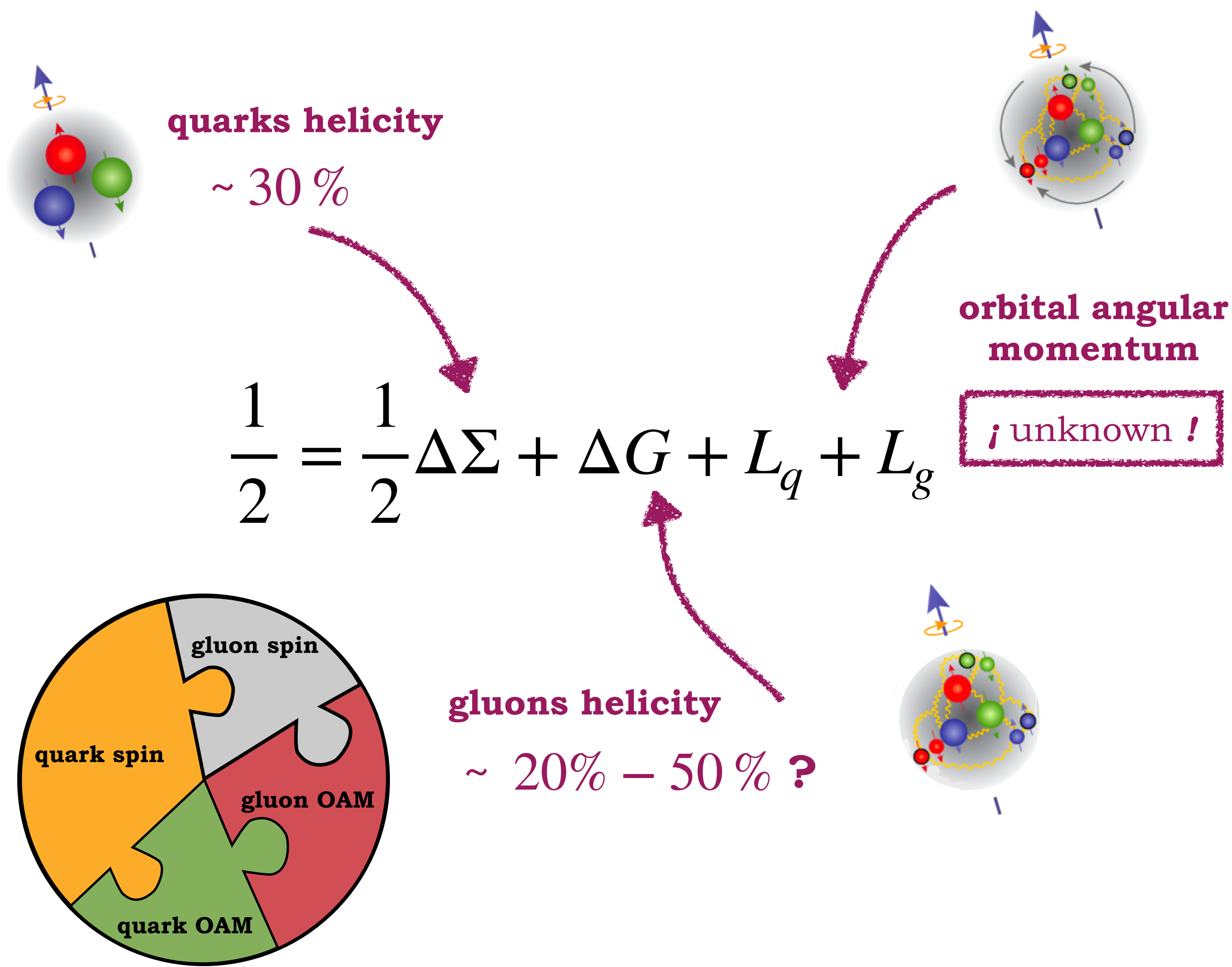
The proton spin crisis



Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...

(proton spin crisis) [EMC Collaboration, CERN (1987)]

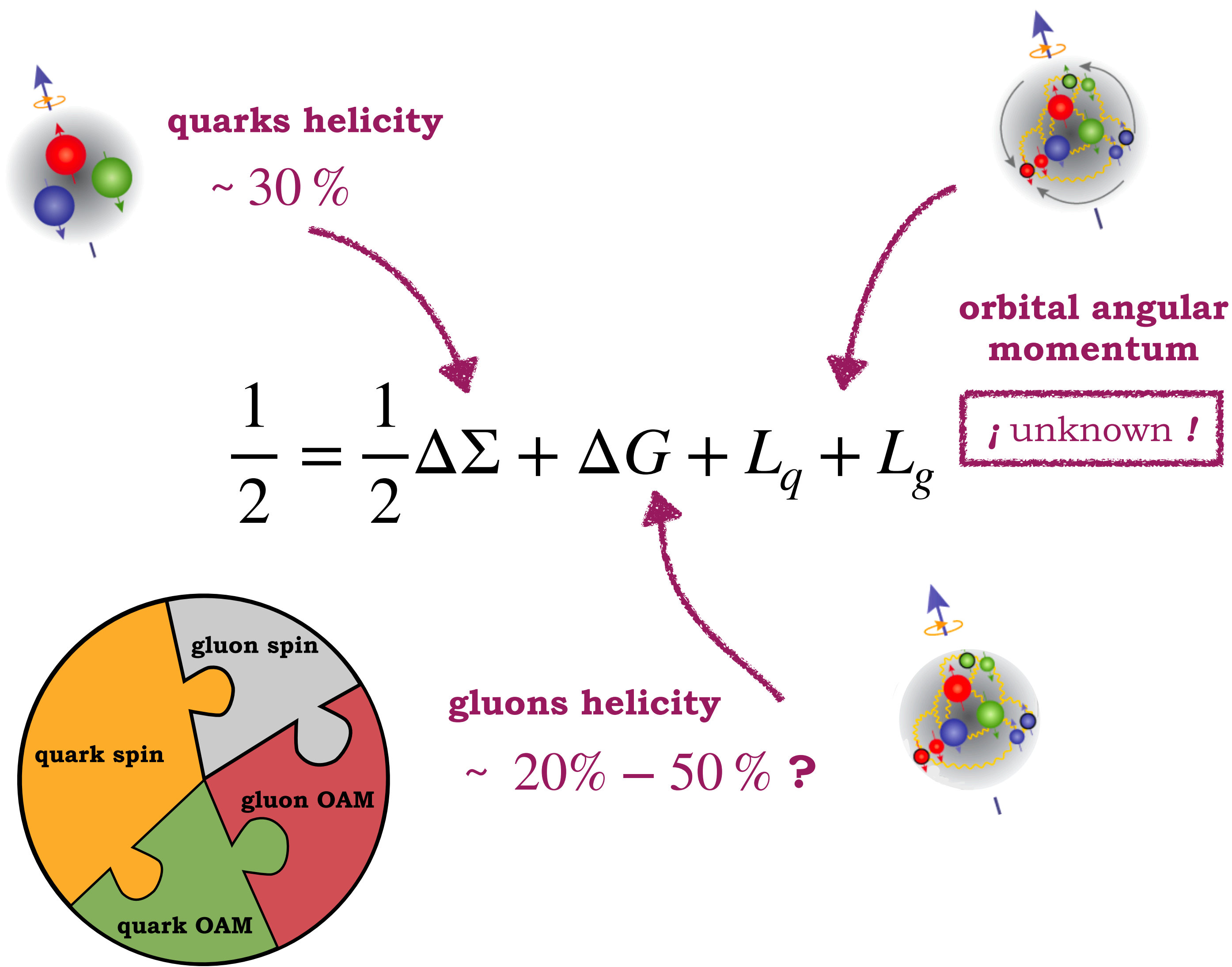
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...many other effects in hadronic interactions cannot be understood in the purely collinear approach

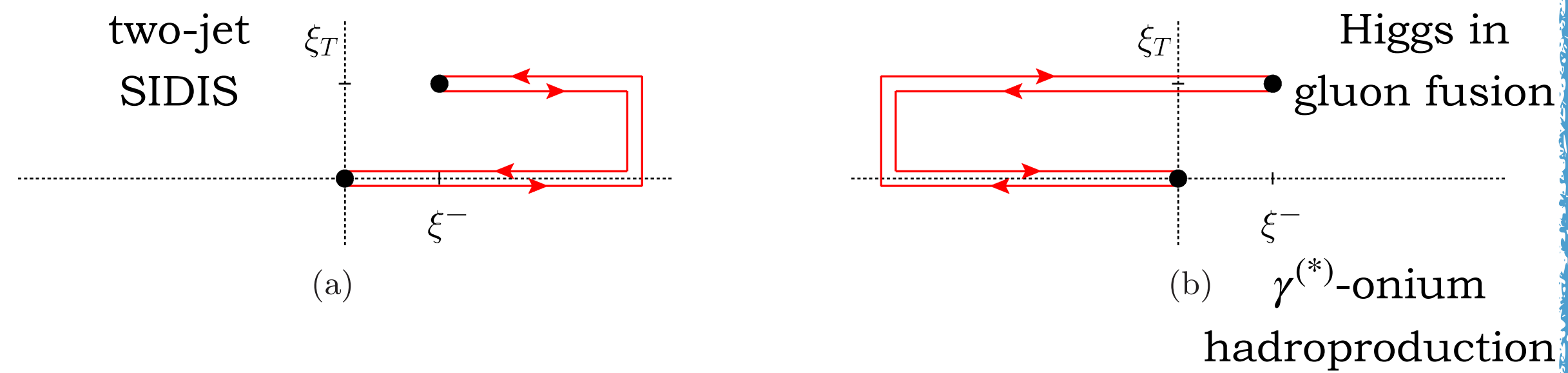
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Accessing f-type and d-type gluon TMDs

f-type (WW)

(a) [+ , +] or (b) [- , -]

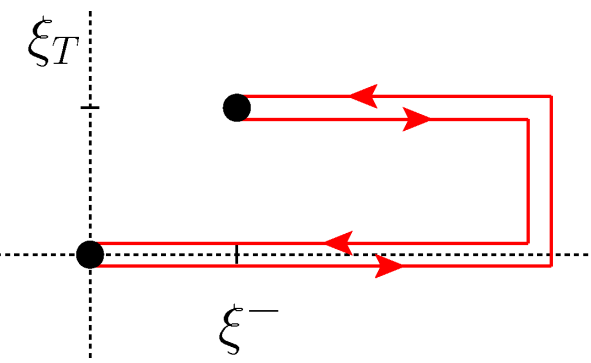


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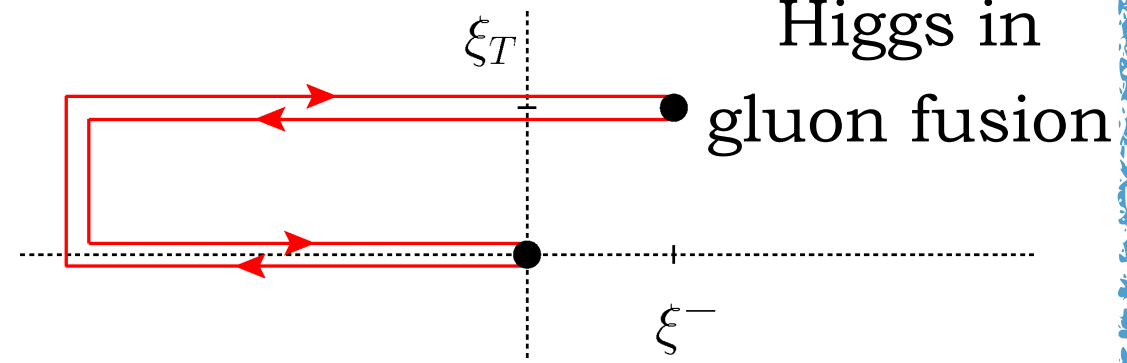
(a) [+ , +] or (b) [- , -]

two-jet
SIDIS



(a)

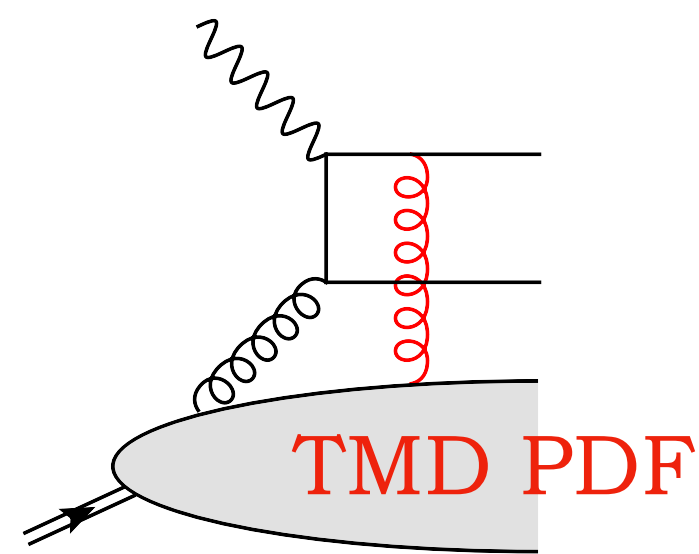
Higgs in
gluon fusion



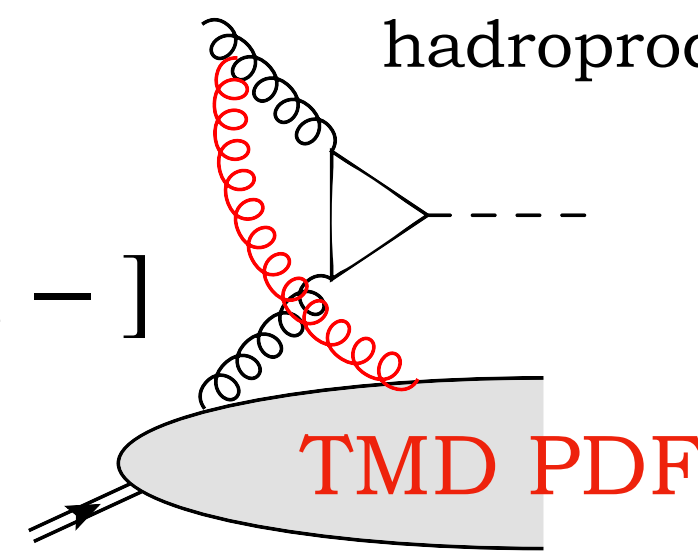
(b)

$\gamma^{(*)}$ -onium
hadroproduction

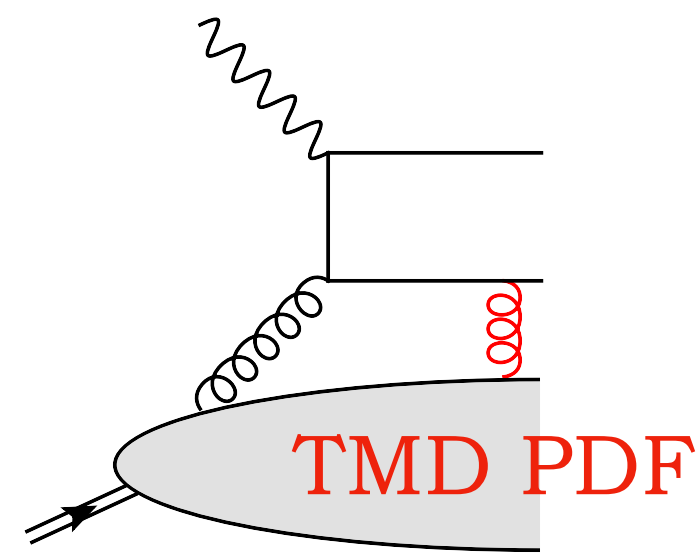
[+



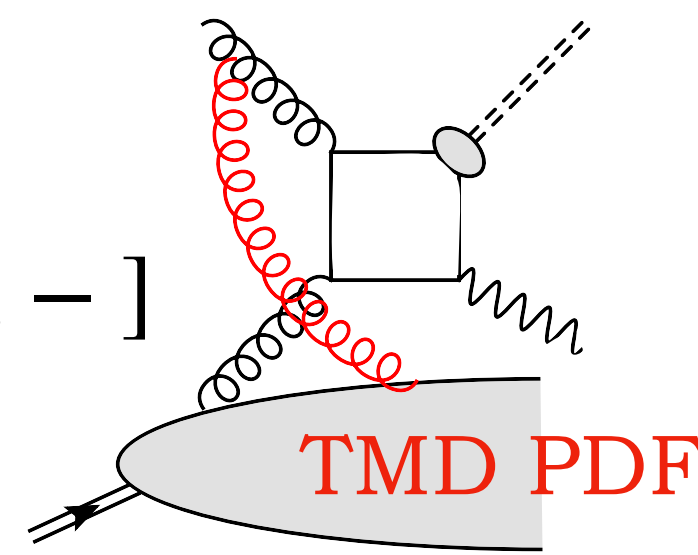
[- , -]



+]



[- , -]

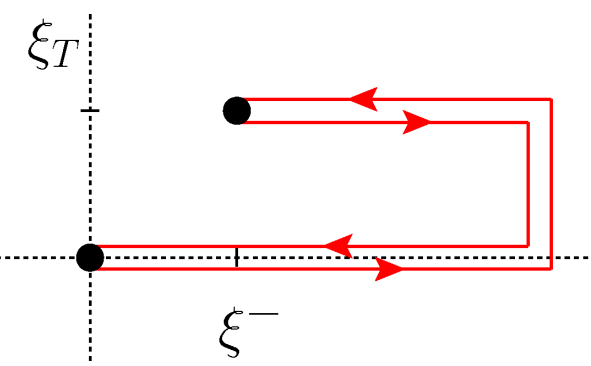


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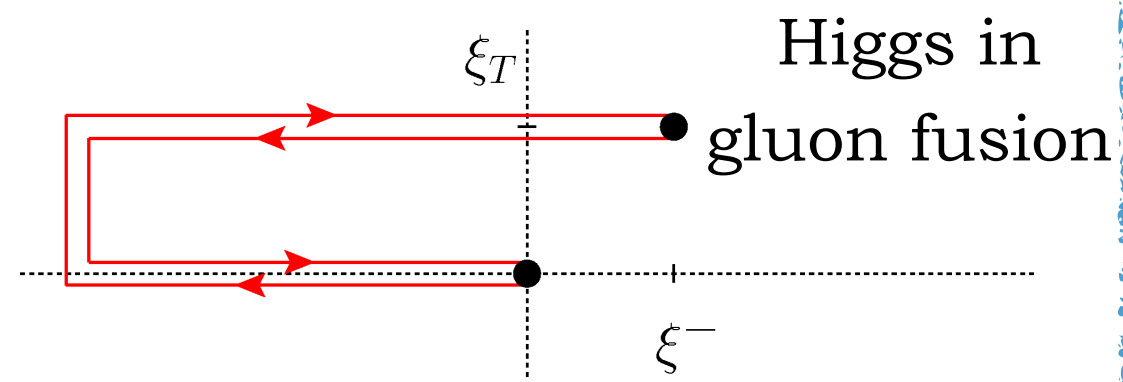
f-type (WW)

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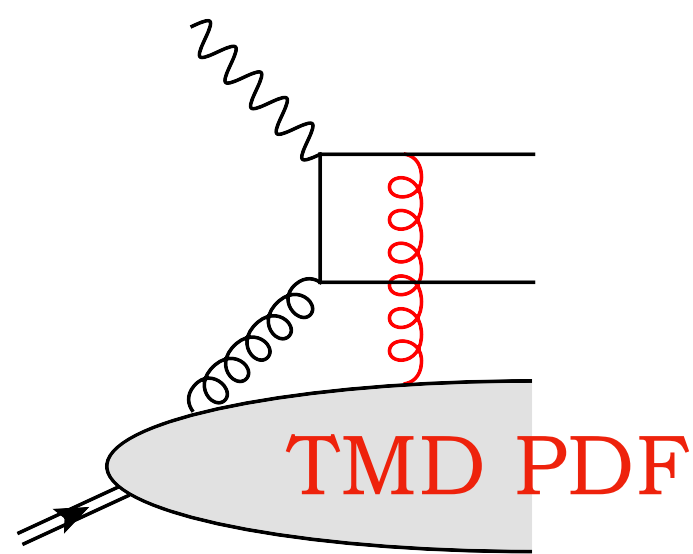


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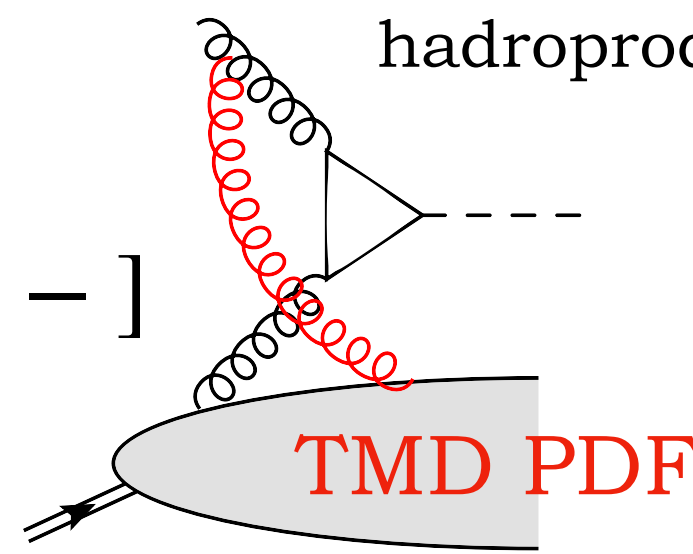
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[+]



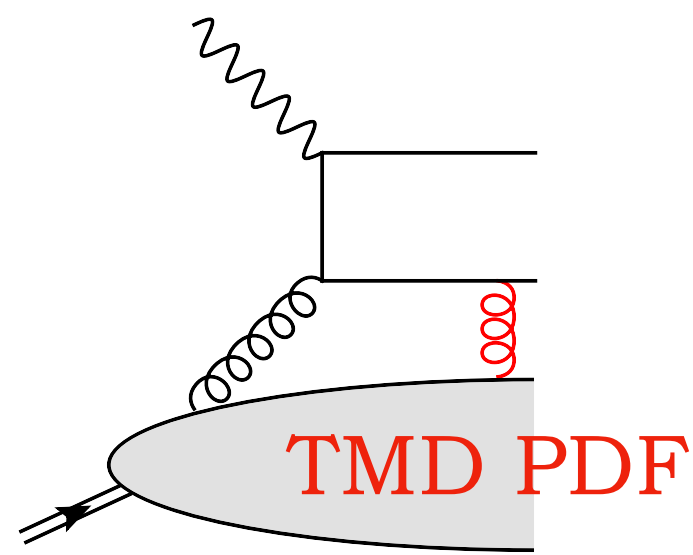
TMD PDF

[-, -]



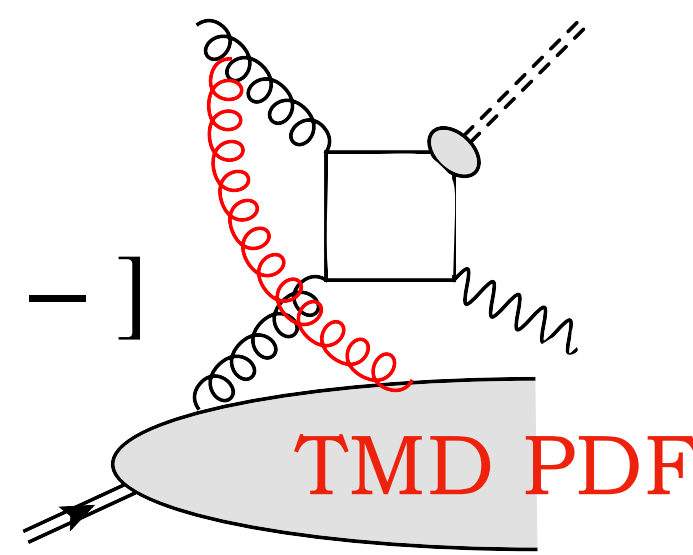
TMD PDF

[+]



TMD PDF

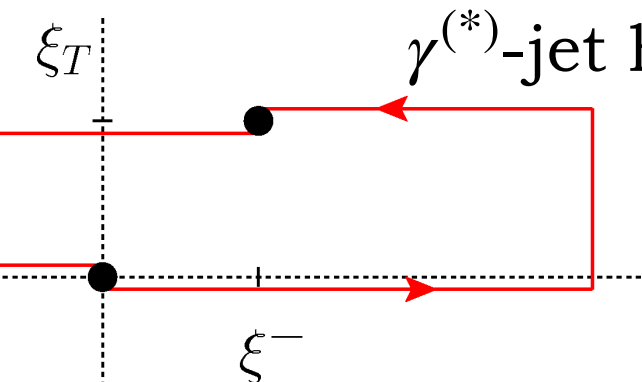
[-, -]



TMD PDF

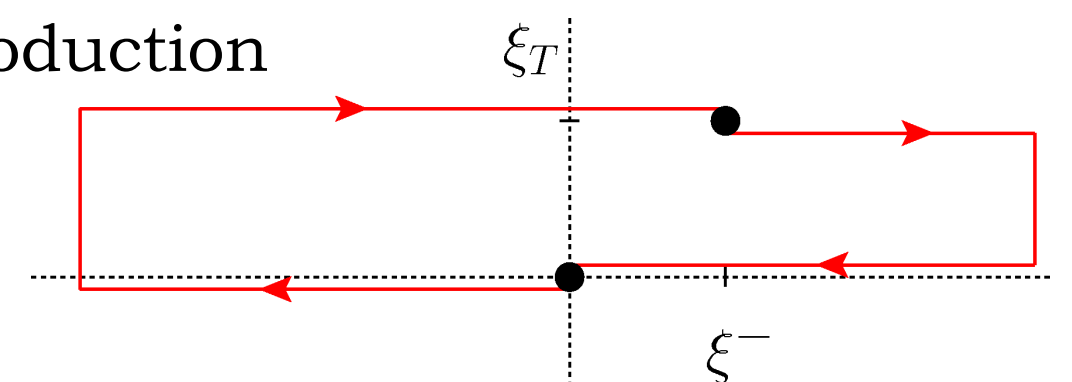
d-type (DP)

(c) $[+, -]$ or (d) $[-, +]$



(c)

$\gamma^{(*)}$ -jet hadroproduction



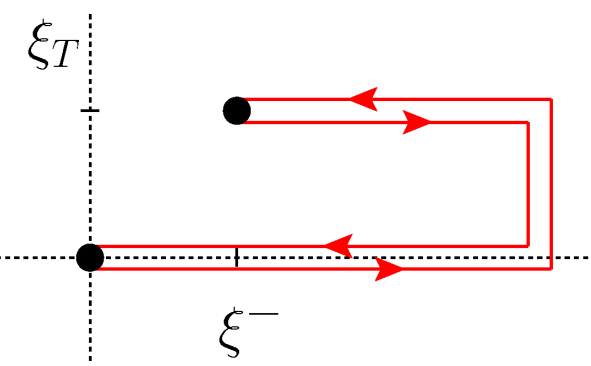
(d)

Accessing f-type and d-type gluon TMDs

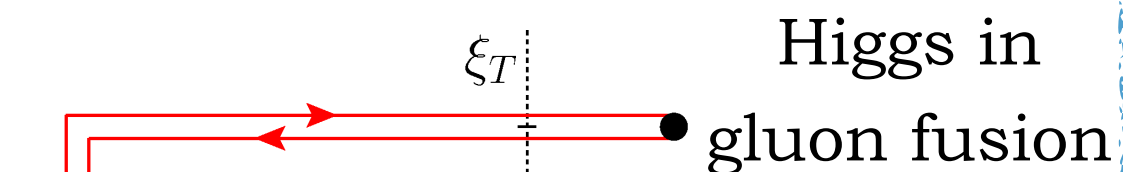
f-type (WW)

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two-jet
SIDIS



(a)

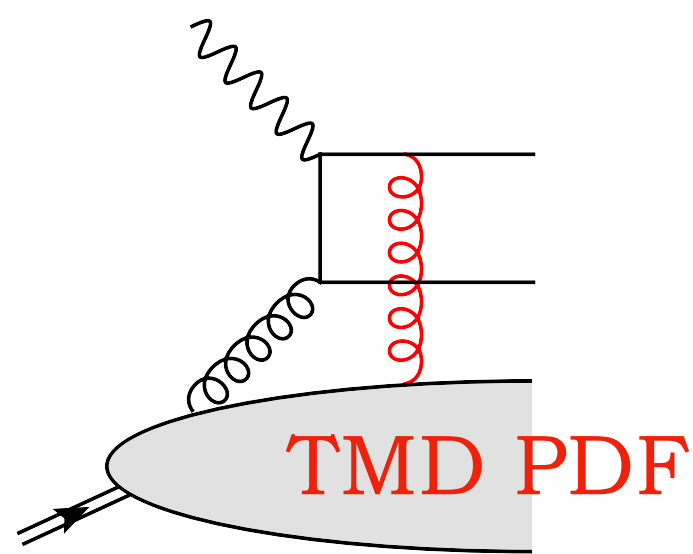


(b)

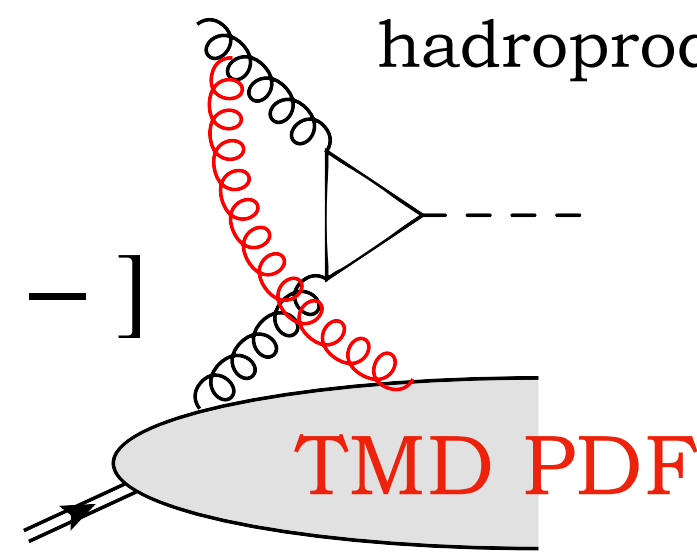
Higgs in
gluon fusion

$\gamma^{(*)}$ -onium
hadroproduction

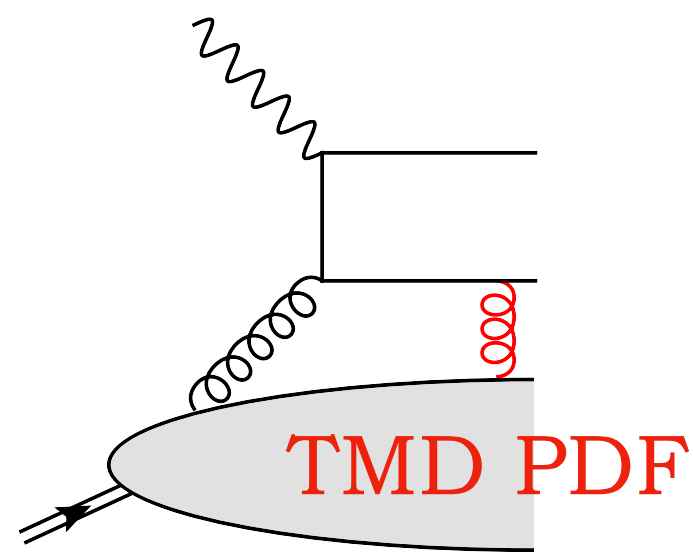
[+]



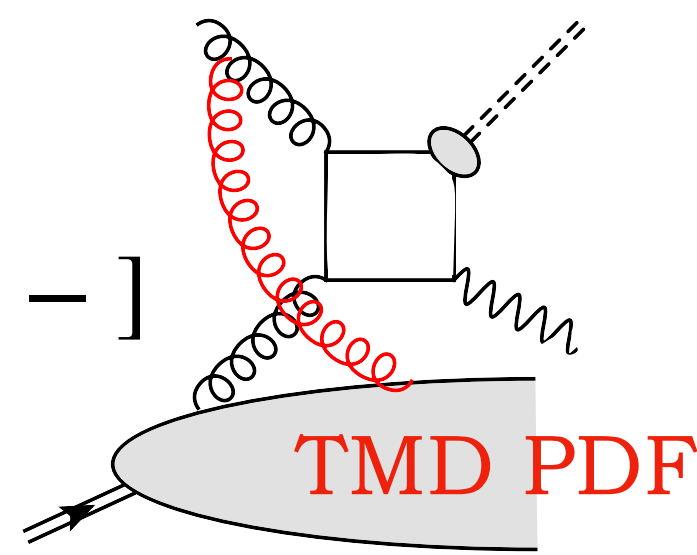
[-, -]



[+]



[-, -]

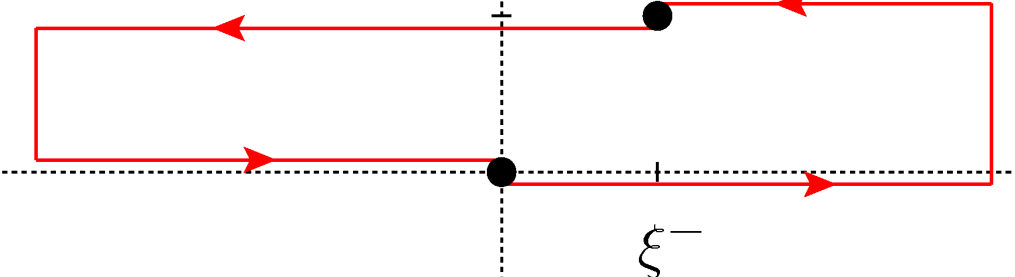


d-type (DP)

(c) $[+, -]$ or (d) $[-, +]$

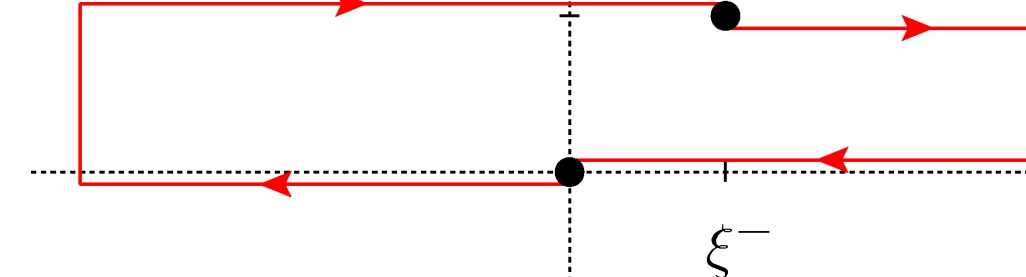
ξ_T

$\gamma^{(*)}$ -jet hadroproduction



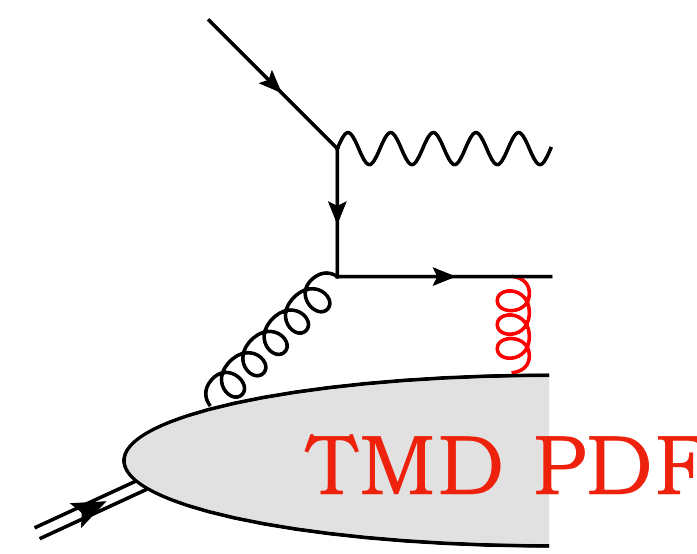
(c)

ξ_T

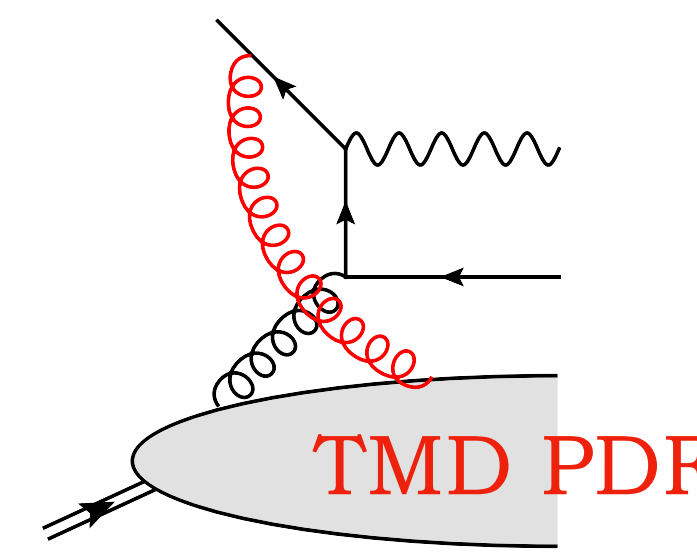


(d)

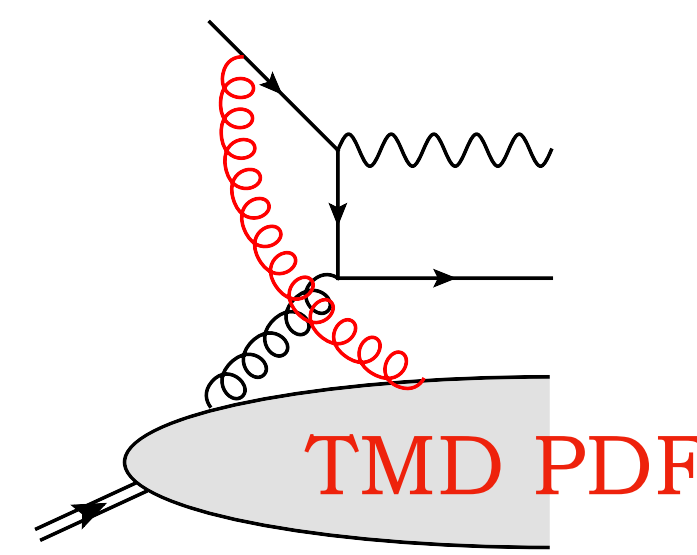
[+]



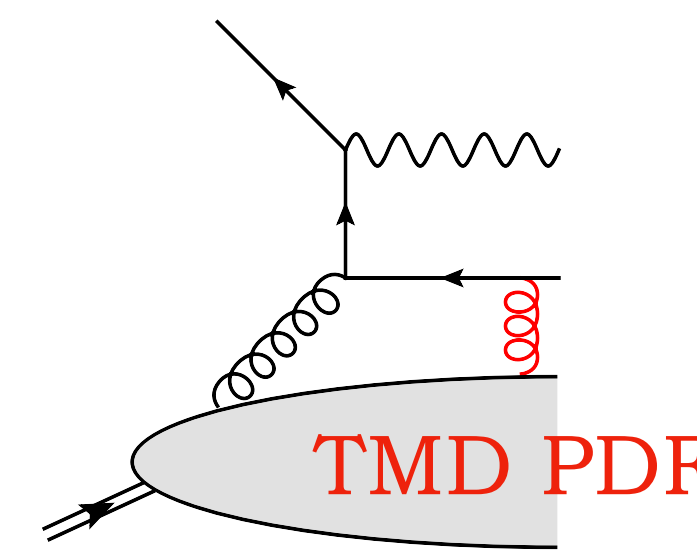
[-]



-]



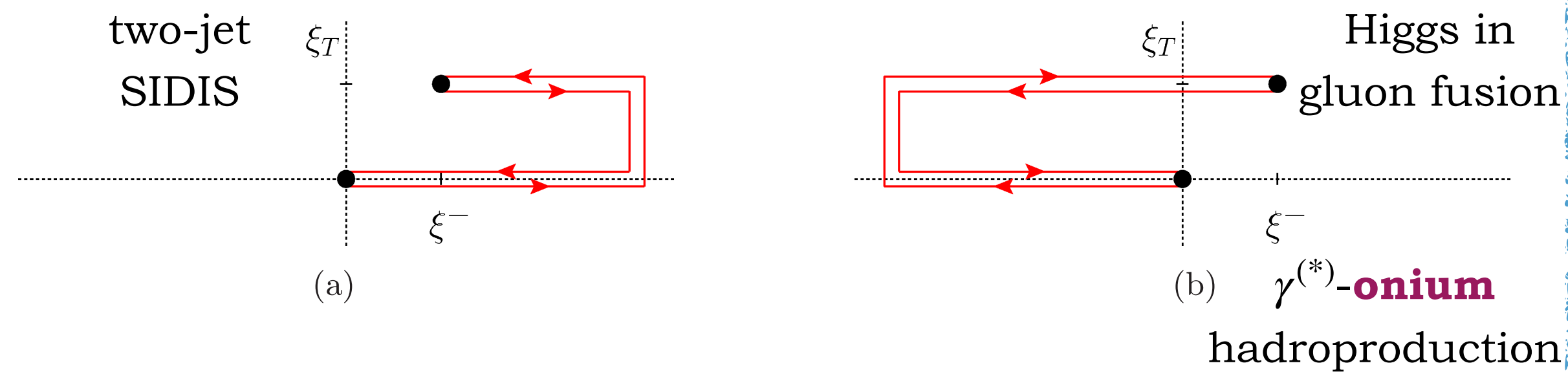
[+]



Accessing f-type and d-type gluon TMDs

f-type (WW)

(a) [+ , +] or (b) [- , -]



* Color flow annihilated within final/initial state

* *f*-type gluon TMDs $\rightarrow f^{abc}$ color structure

* Modified universality:

$$f_1^{[+,+]} = f_1^{[-,-]},$$

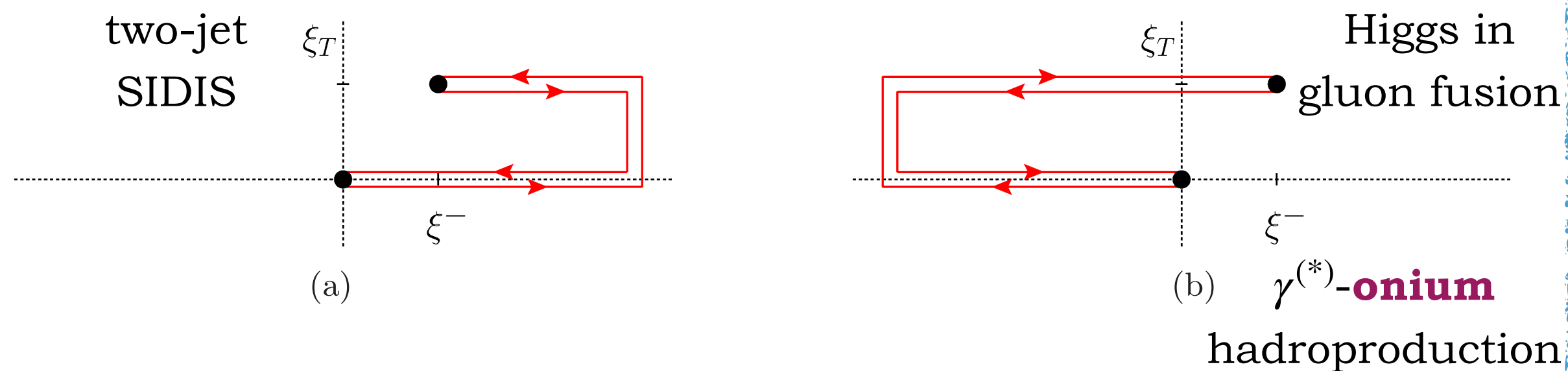
$$f_{1T}^\perp[+,+] = -f_{1T}^\perp[-,-]$$

* Phenomenology: Higgs, **quarkonia** or $\gamma\gamma$ in pp , two-jet SIDIS, heavy-quark pair SIDIS

Accessing f-type and d-type gluon TMDs

f-type (WW)

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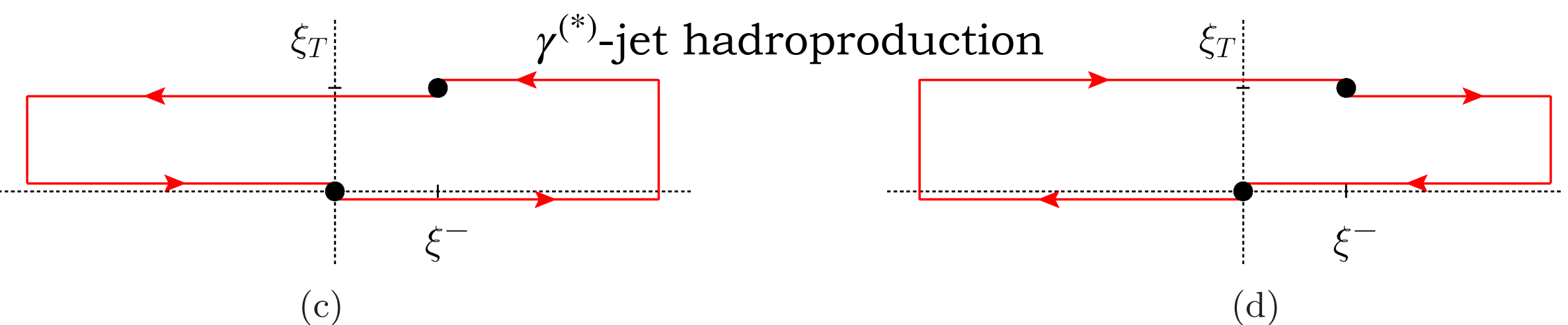
$$f_1^{[+,+]} = f_1^{[-,-]},$$

$$f_{1T}^\perp[+,+] = -f_{1T}^\perp[-,-]$$

- * Phenomenology: Higgs, **quarkonia** or $\gamma\gamma$ in pp , two-jet SIDIS, heavy-quark pair SIDIS

d-type (DP)

(c) [+ , -] or (d) [- , +]



- * Color flow involving both initial and final states

- * d-type gluon TMDs $\rightarrow d^{abc}$ color structure

- * Modified universality:

$$f_1^{[+,-]} = f_1^{[-,+]},$$

$$f_{1T}^\perp[+,-] = -f_{1T}^\perp[-,+]$$

- * Phenomenology: single hadron or $\gamma^{(*)}$ -jet hadroproduction, SIDIS or Drell-Yan (subleading)

Gauge link \rightarrow two main independent sets of TMDs, **not related** to each other

Dihadron hadroproduction and factorization breaking

* Proof of factorization violation [\[T. J. Rogers, P. J. Mulders \(2010\)\]](#)

* Assumed factorization in SCET and CGC

* Significance of low- x studies

* Size of factorization-breaking effects small?

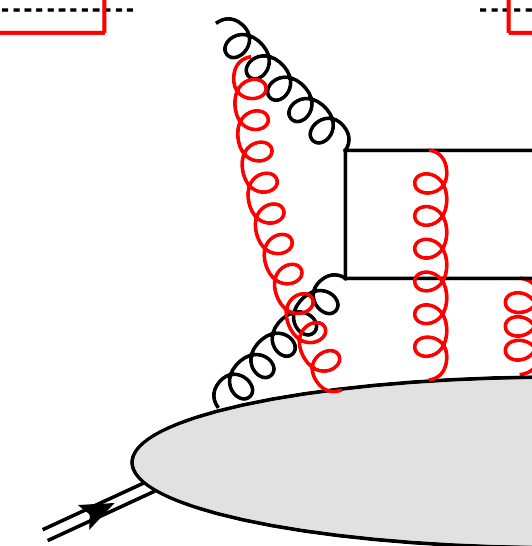
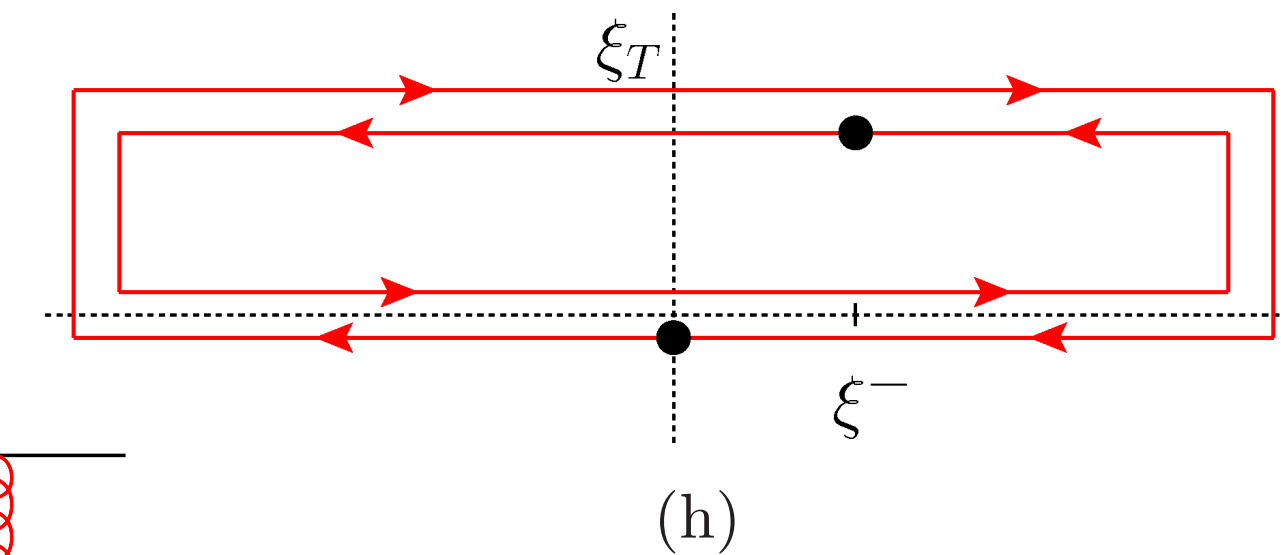
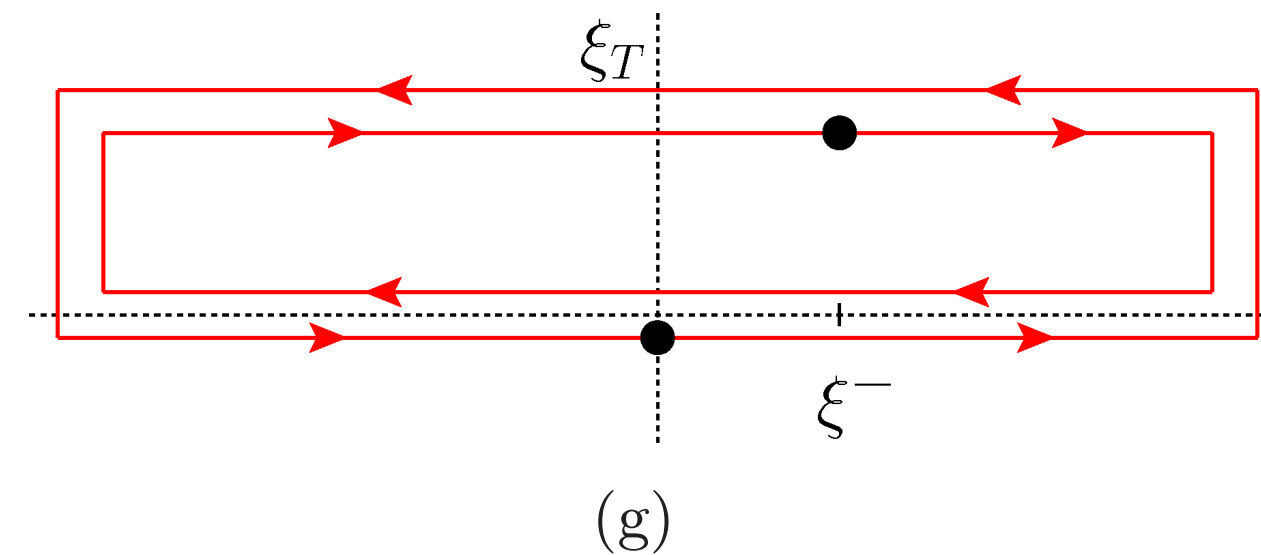
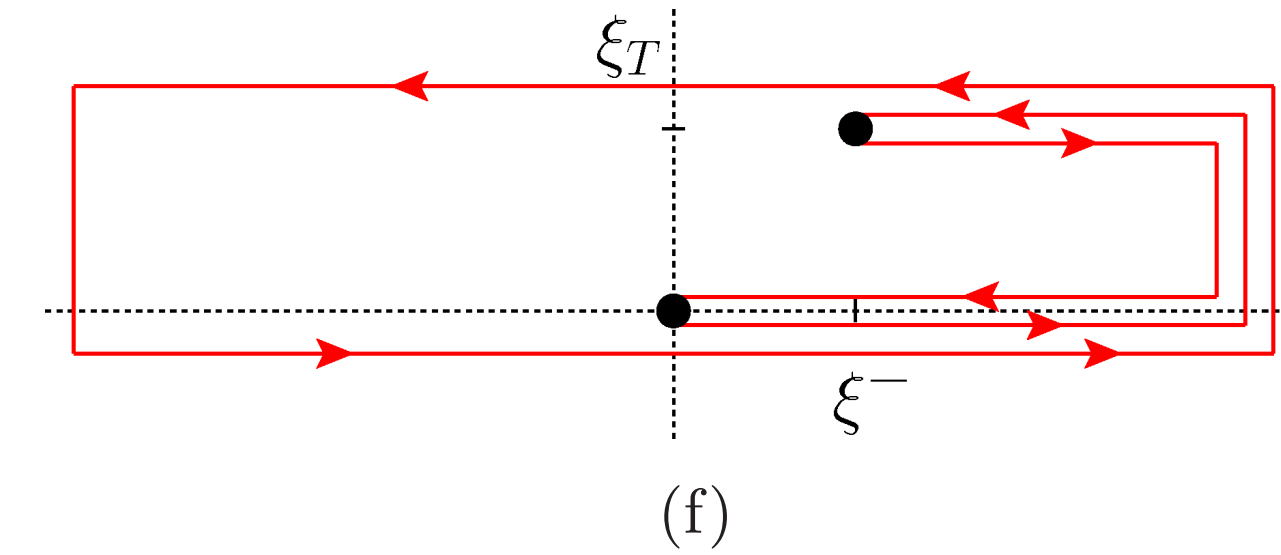
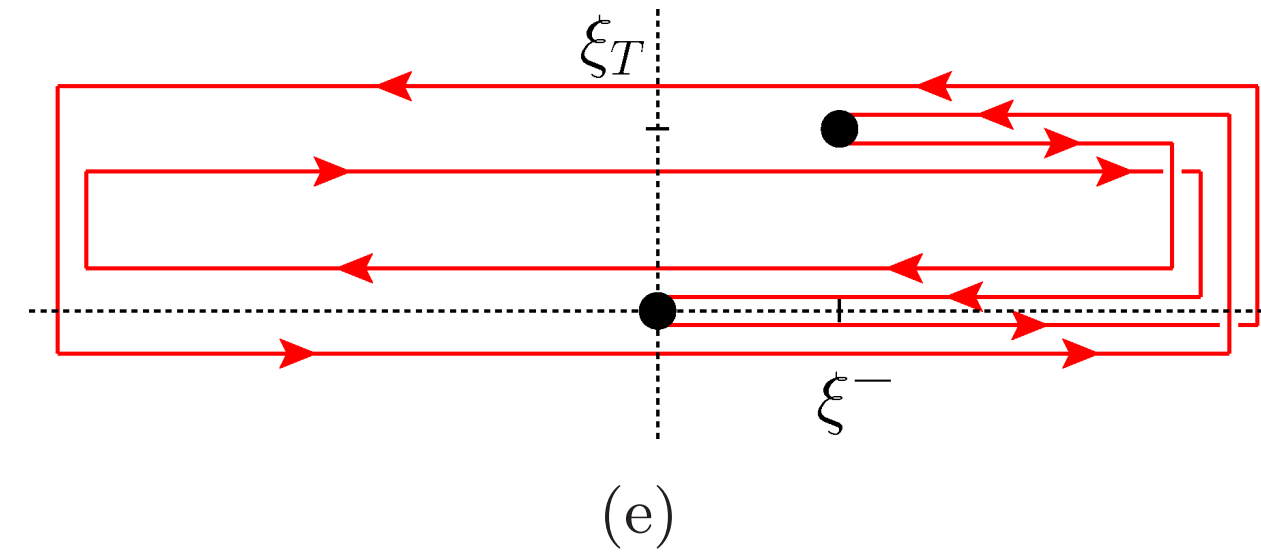
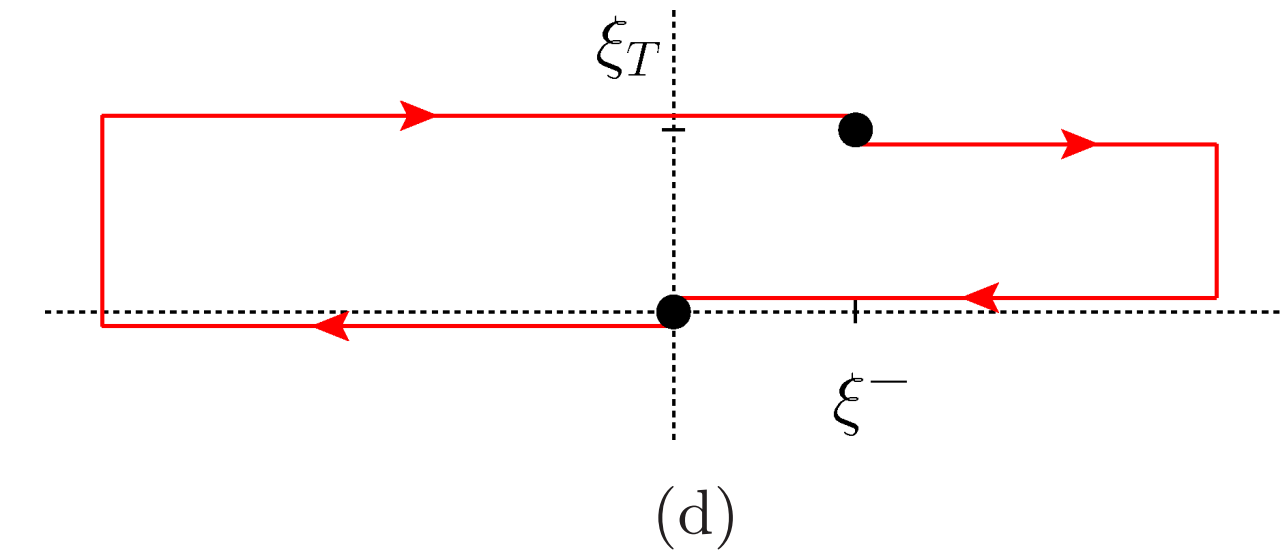
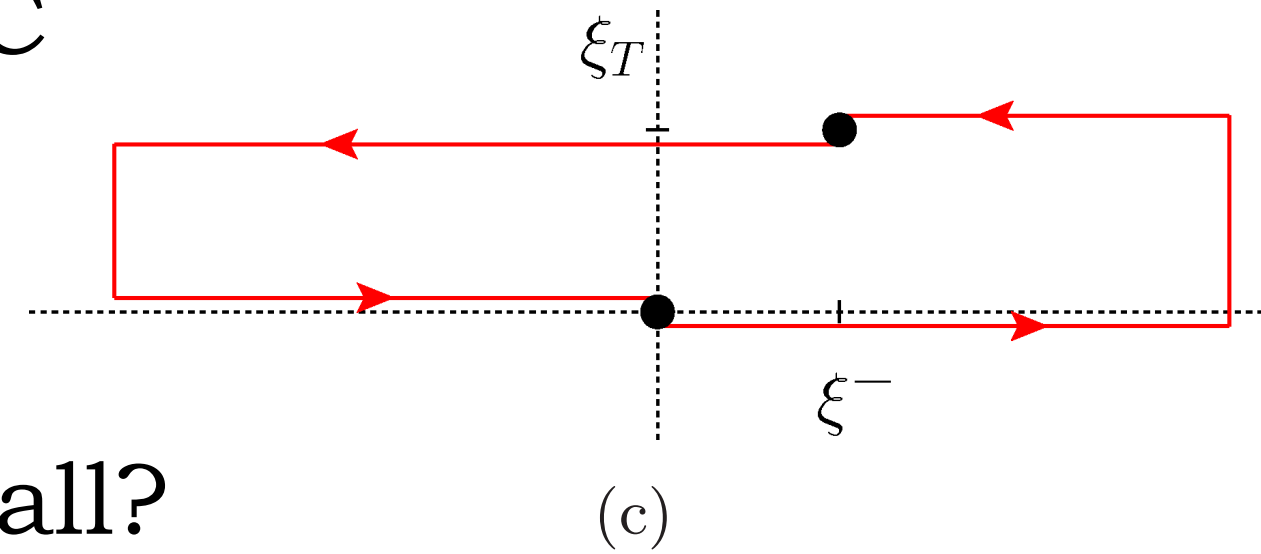
* DP TMDs:

(c) $[+, -]$ and (d) $[-, +]$

* Appearance of new gauge **loop links**:

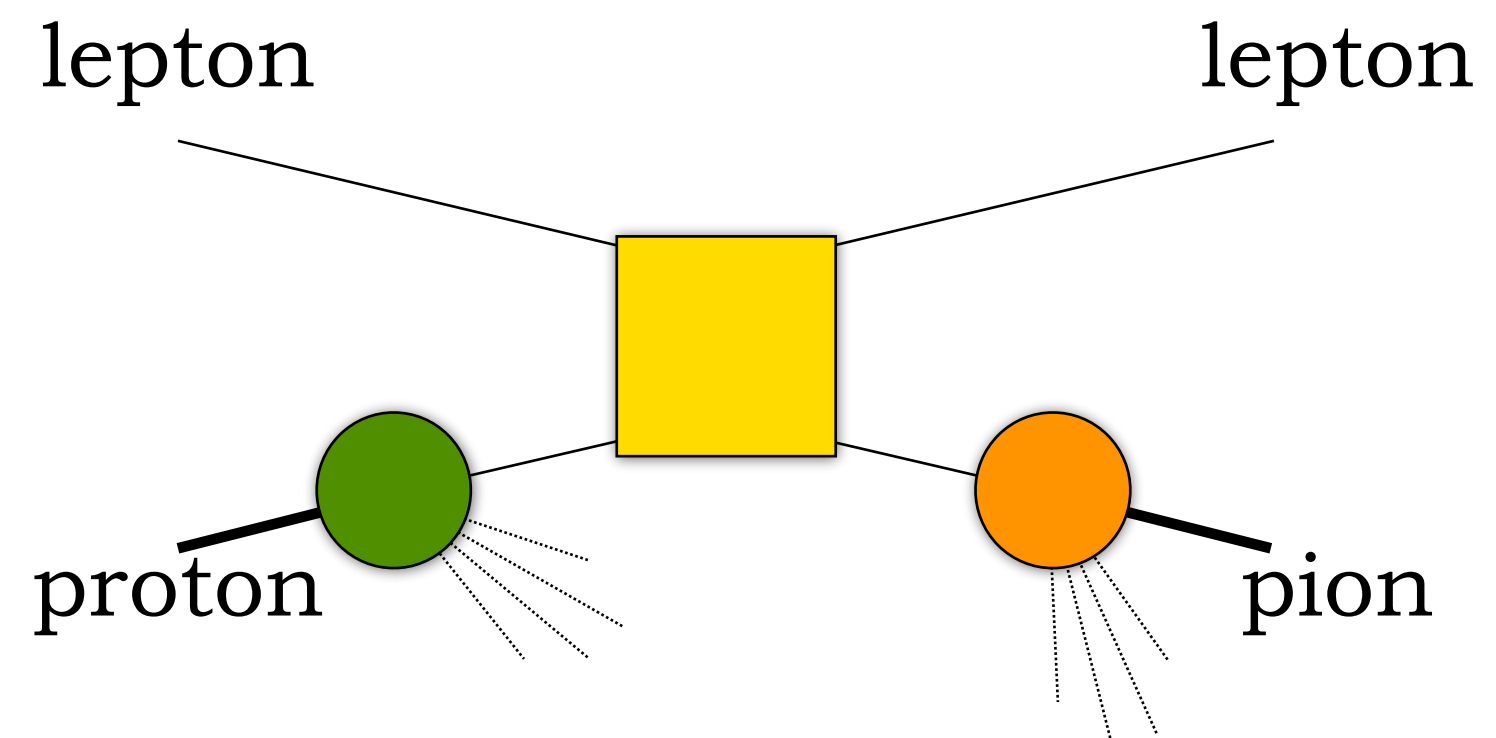
(e) $[+\square, +\square]$, (f) $[+, +\square]$,

(g) $[\square, \square]$, and (h) $[\square, \square]$



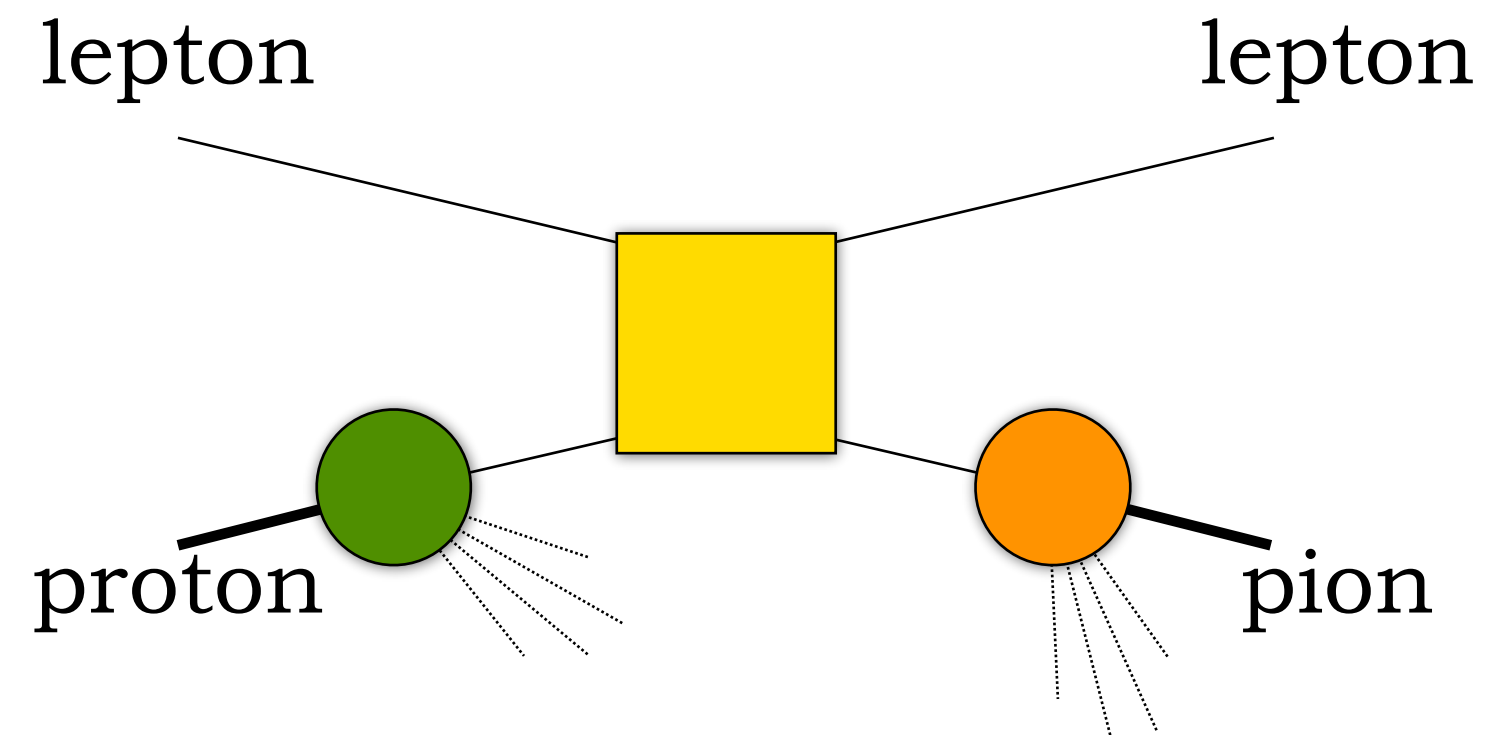
Factorization and universality

SIDIS

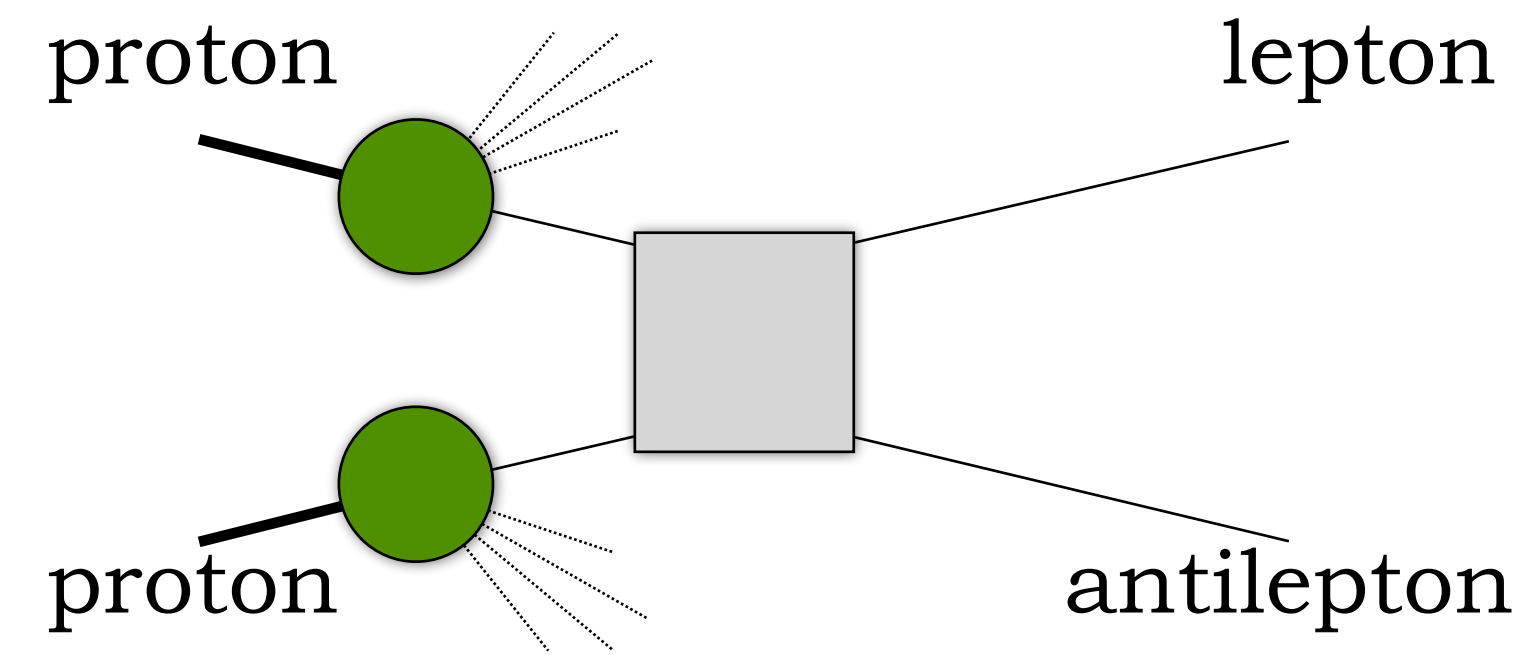


Factorization and universality

SIDIS

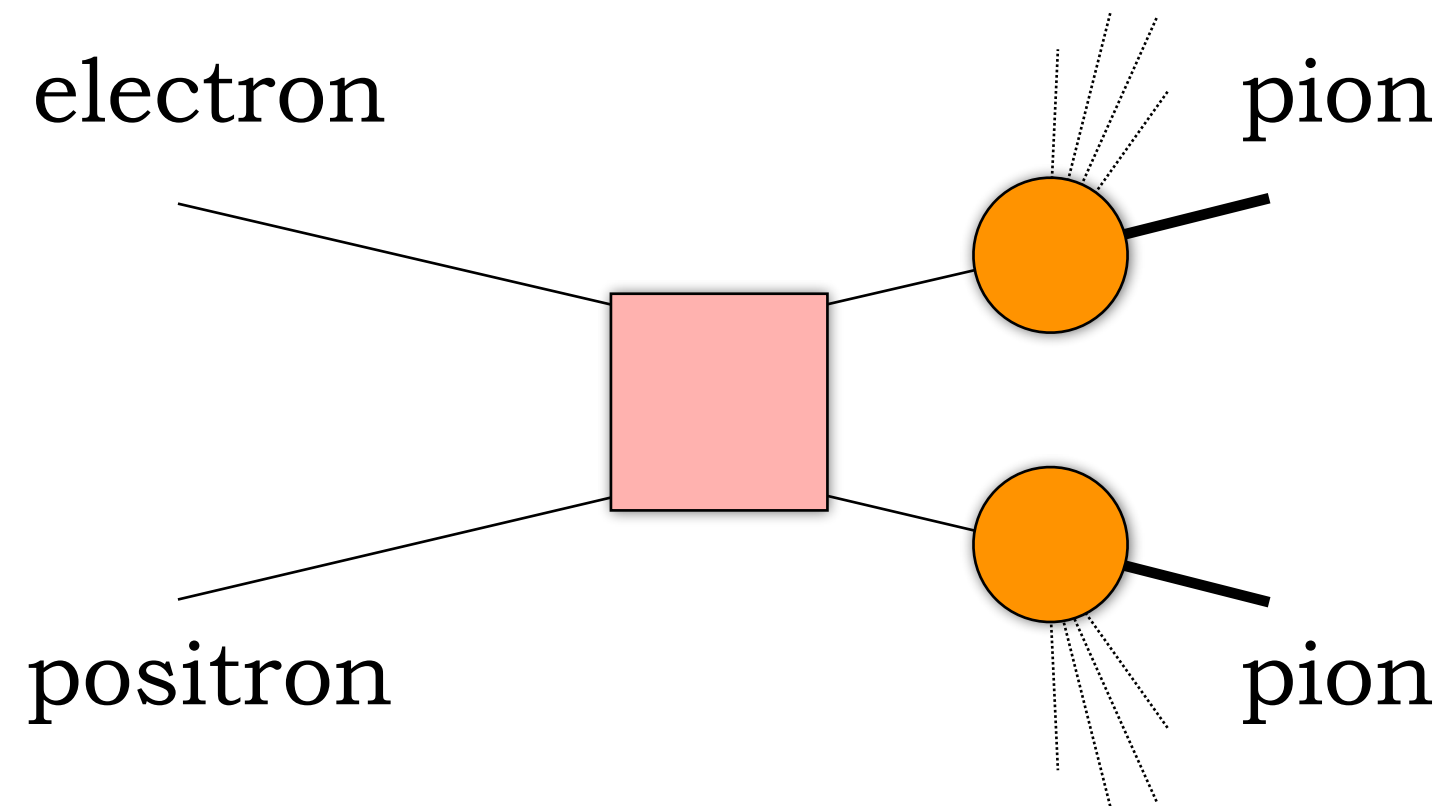
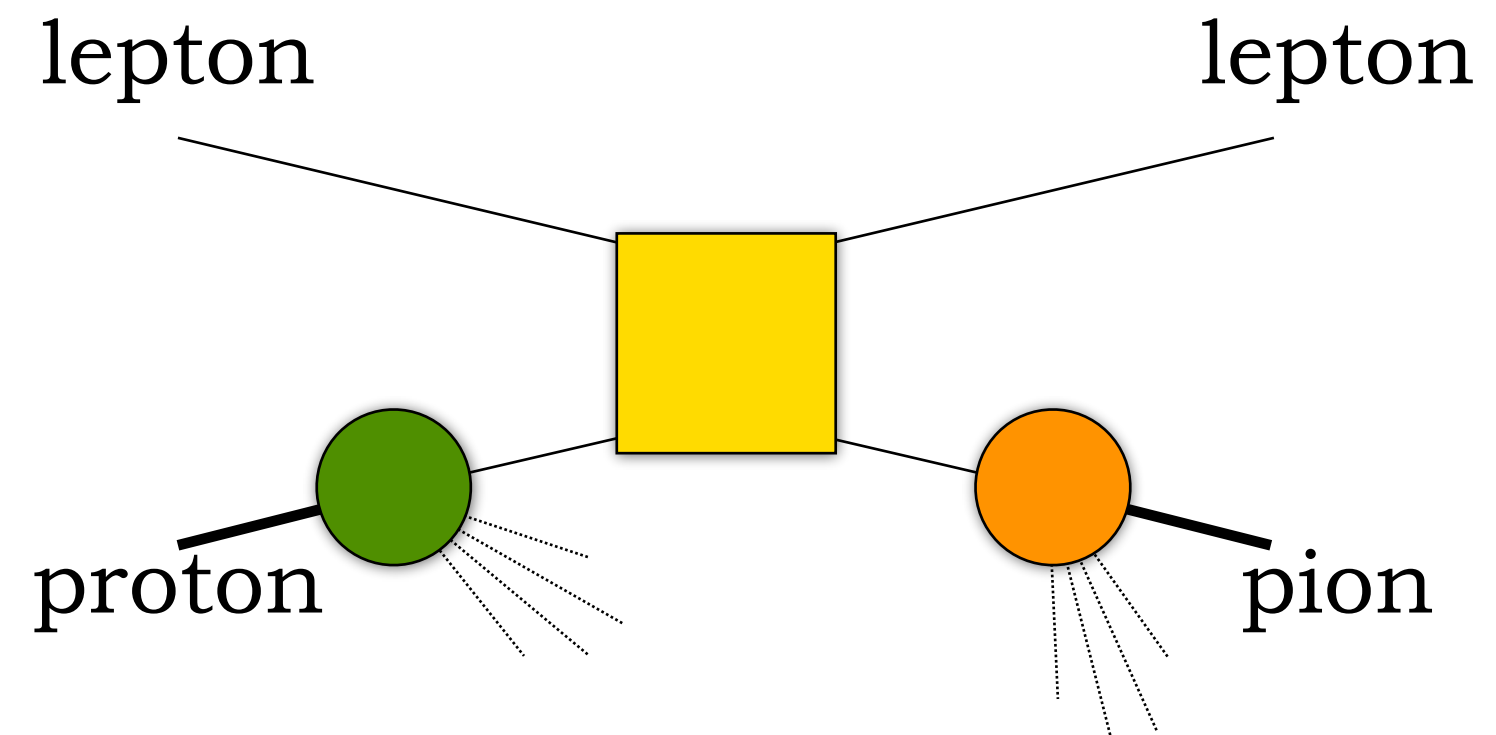


Drell-Yan



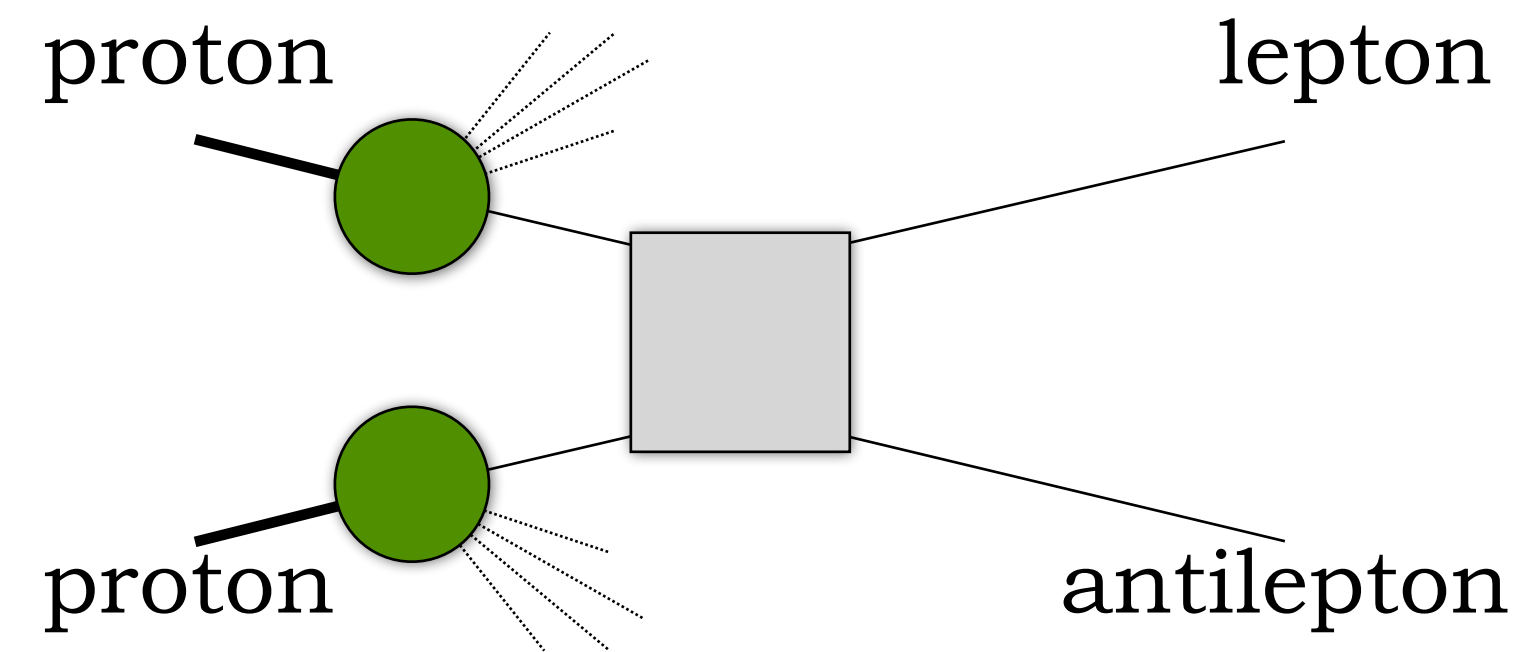
Factorization and universality

SIDIS



$$e^+ + e^- \rightarrow \mathbf{hadrons}$$

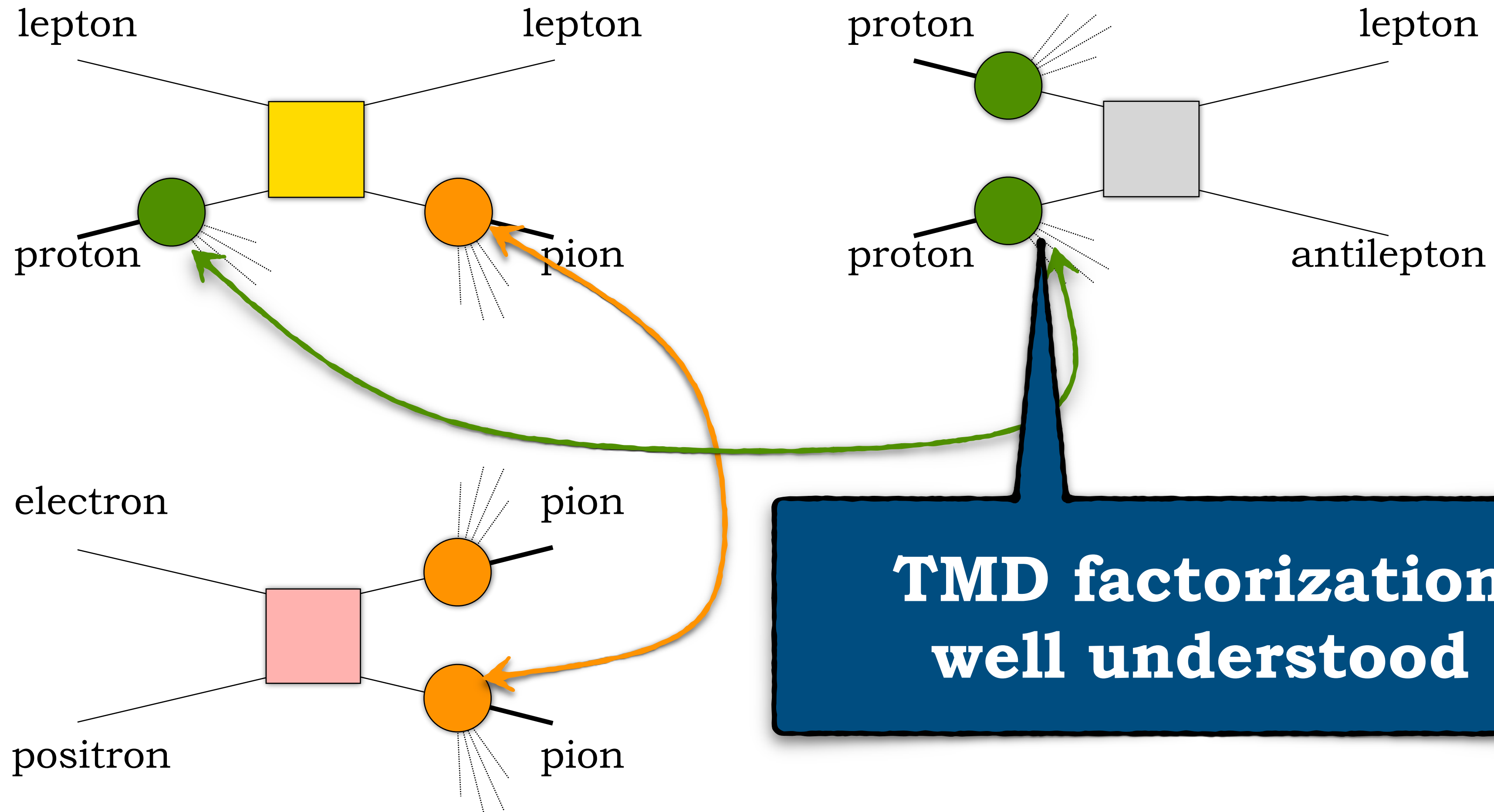
Drell-Yan



Factorization and universality

SIDIS

Drell-Yan



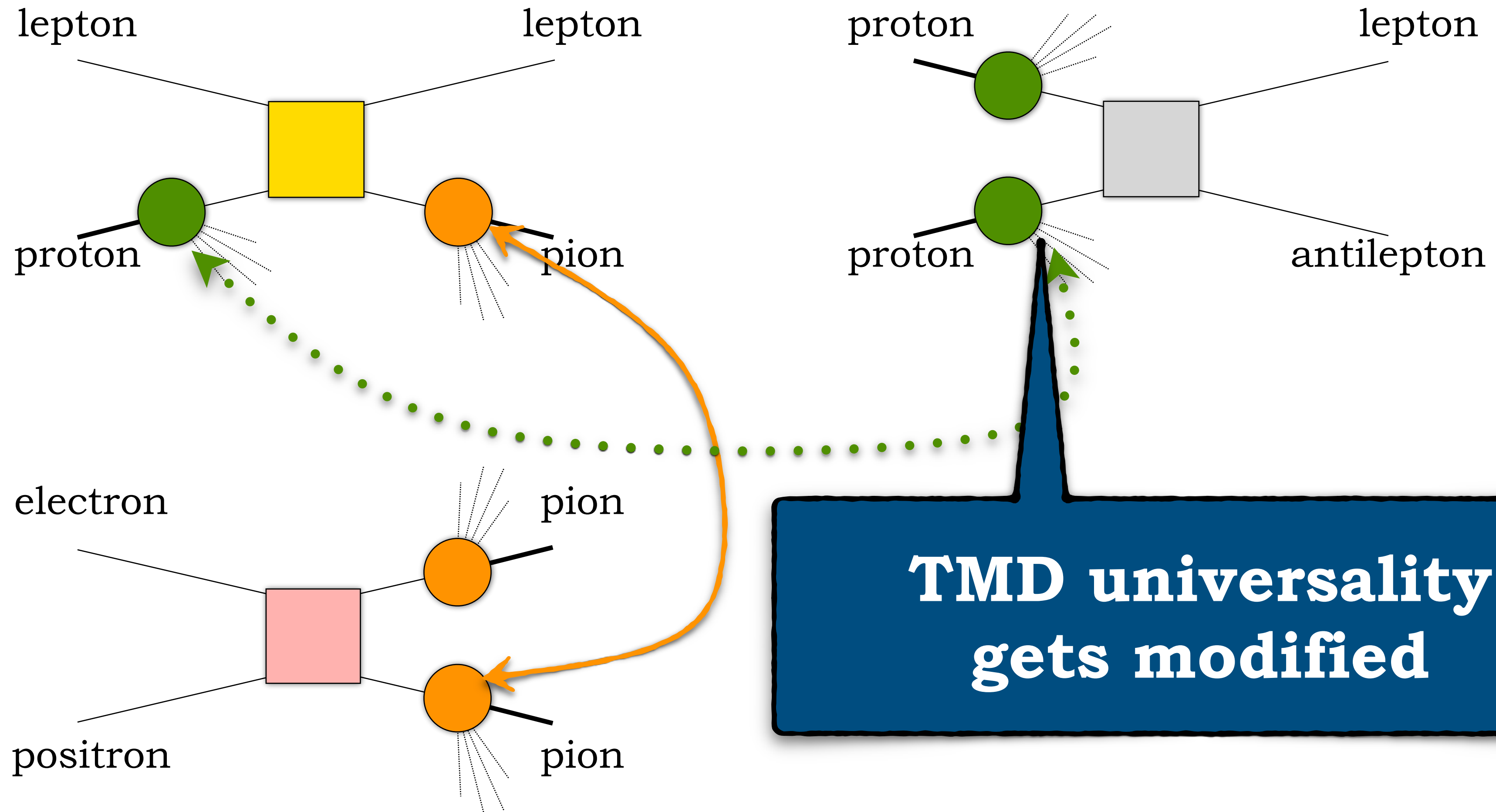
**TMD factorization
well understood**

$$e^+ + e^- \rightarrow \mathbf{hadrons}$$

Factorization and universality

SIDIS

Drell-Yan



$$e^+ + e^- \rightarrow \mathbf{hadrons}$$

Spectator-model gluon TMDs

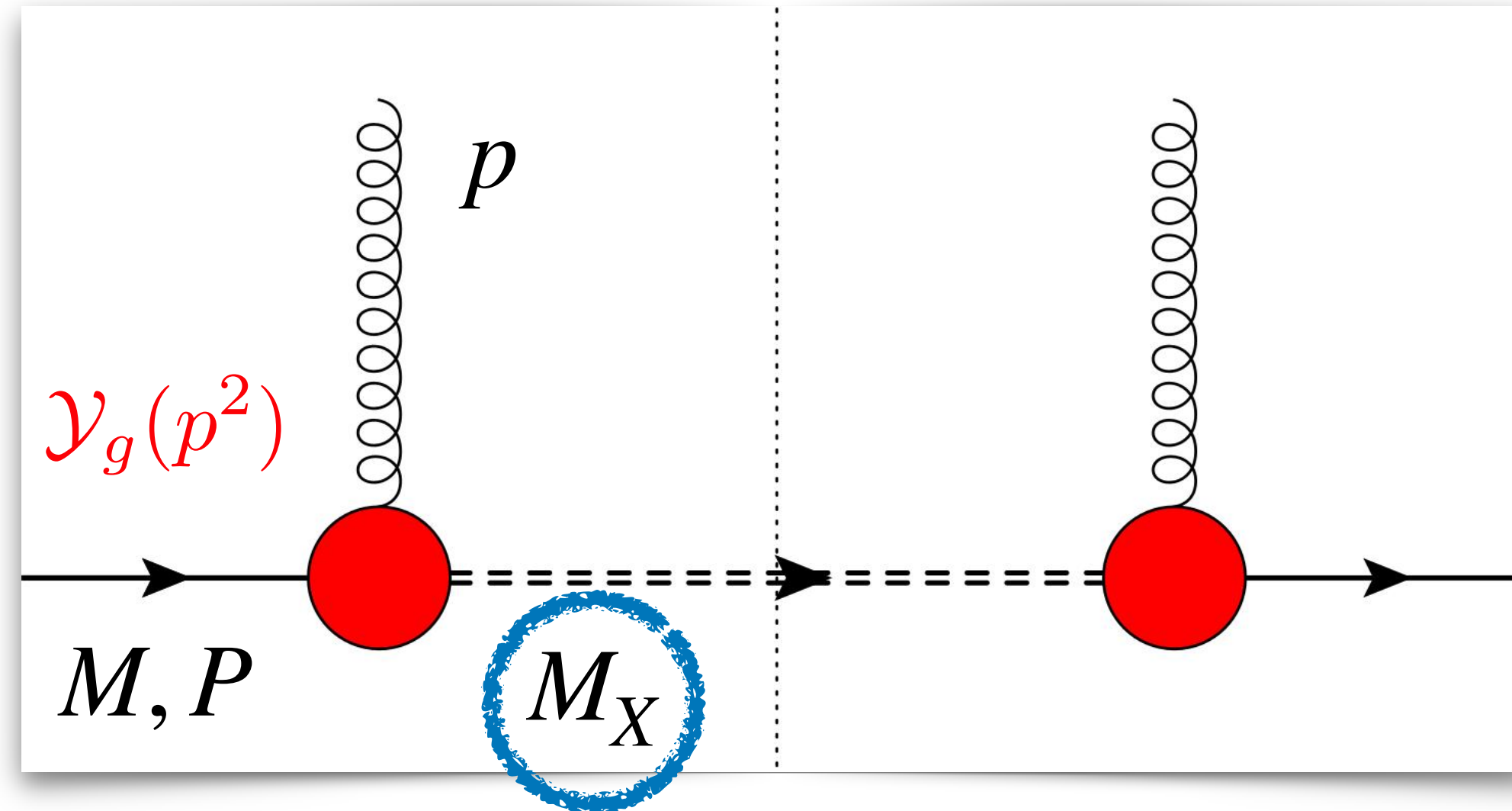
The background features a repeating pattern of circular diagrams illustrating the spectator model for gluon Transverse Momentum Distributions (TMDs). Each diagram shows a central gluon (represented by a red sphere with a red arrow) interacting with a quark (represented by a blue sphere with a blue arrow) and a spectator quark (represented by a green sphere with a green arrow). The gluon is shown as a yellow wavy line. The diagrams are arranged in a grid-like pattern, with some overlapping, and are set against a light blue and green background with a subtle grid and starburst effects.

Spectator-model gluon TMD PDFs



Spin-1/2 spectator

Lowest Fock state:
tri-quark spectator
on-shell and
with mass M_X

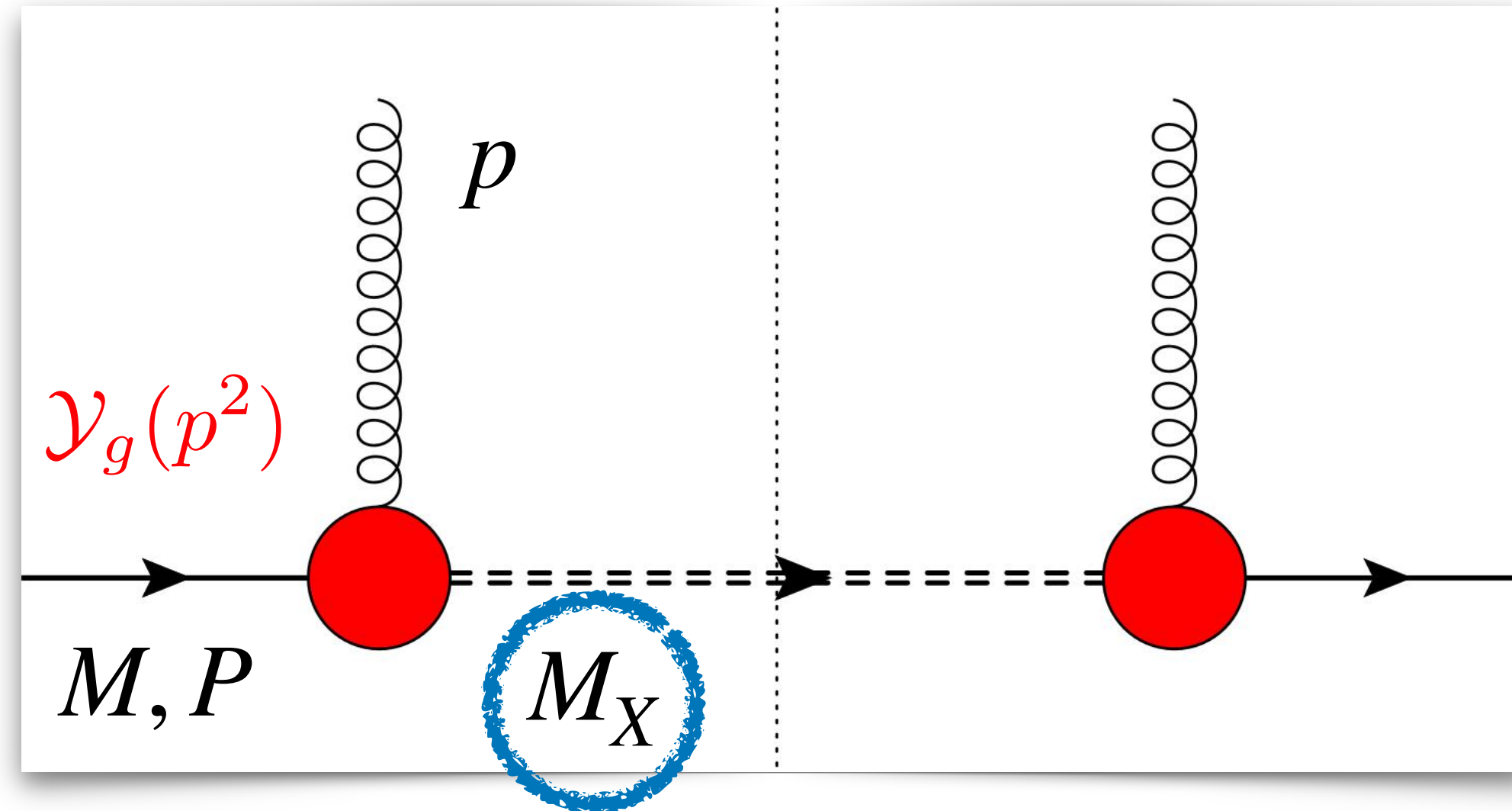


Spectator-model gluon TMD PDFs



Spin-1/2 spectator

Lowest Fock state:
tri-quark spectator
 on-shell and
 with mass M_X



Nucleon-gluon-spectator vertex

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} \text{Tr} \left[(\not{P} + M) \frac{1 + \gamma^5 \not{\xi}}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \mathcal{Y}_g^{\rho*} \mathcal{Y}_{g\sigma} (\not{P} - \not{p} + M) \right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M} \sigma^{\mu\nu} p_\nu$$

mimics proton form factors
 (conserved EM current
 of a free nucleon)



Link with collinear factorization

1. p_T -integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution
2. TMDs and PDFs *decouple* due to evolution



Link with collinear factorization

1. p_T -integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution
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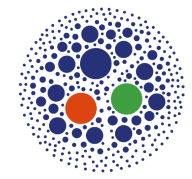
$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$



Dipolar form factor(s)

1. Cancels singularity of gluon propagator
2. Suppresses effects of high p_T
3. Compensates log divergences arising from p_T -integration
4. Adds three more parameters: $\kappa_{1,2}$ and Λ_X

Assumptions of the model



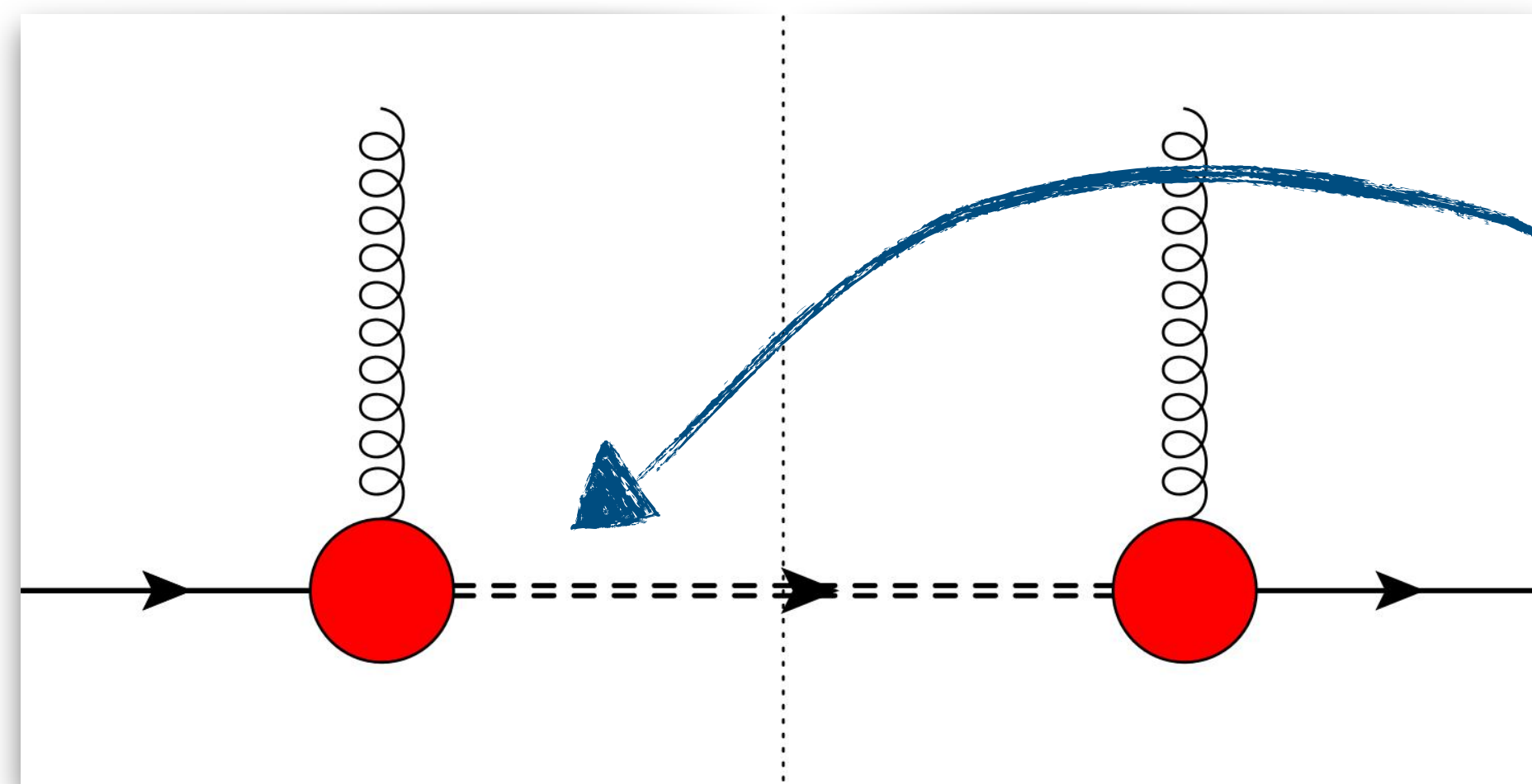
Spectator-system spectral-mass function

spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



Instead of a single on-shell spectator, a continuum of spectators

$\mathcal{V}_g(p^2)$

Assumptions of the model



Spectator-system spectral-mass function

spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X \left(M_X; \{X^{(\text{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\} \right) = \mu^{2a} \left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

low- x (high- μ^2) tail $\propto (a - b)$



$q\bar{q}$ contributions energetically available at large M_X

$$\mu^2 = M_X^2 - M^2$$

moderate- x trend

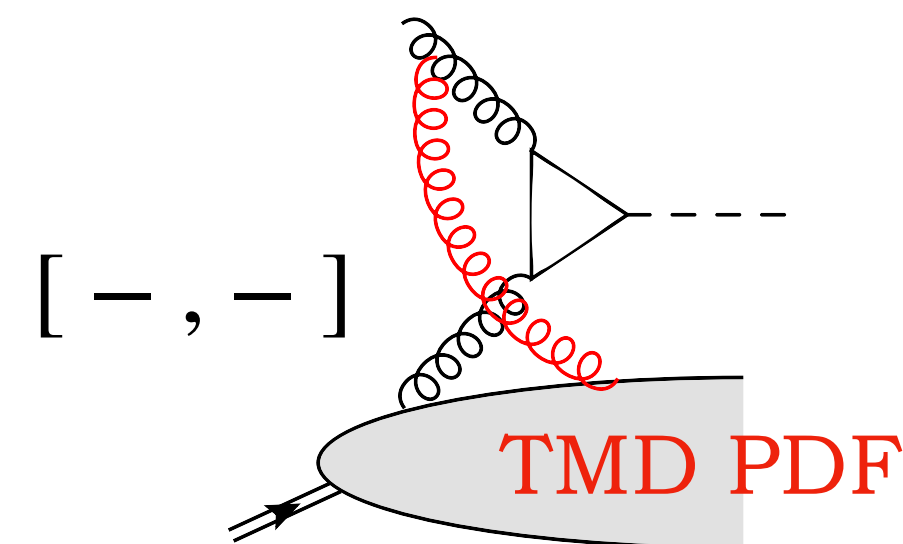
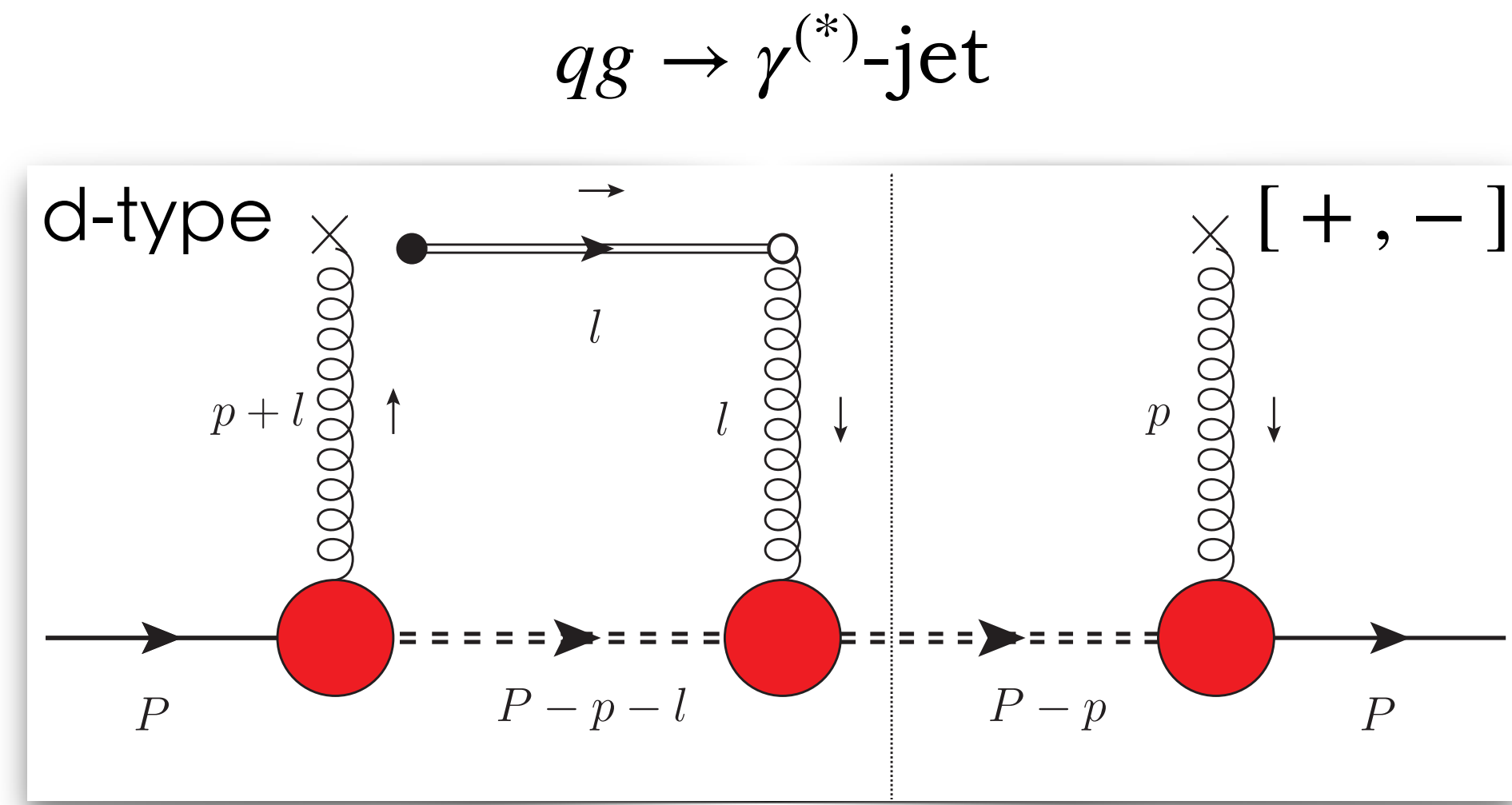
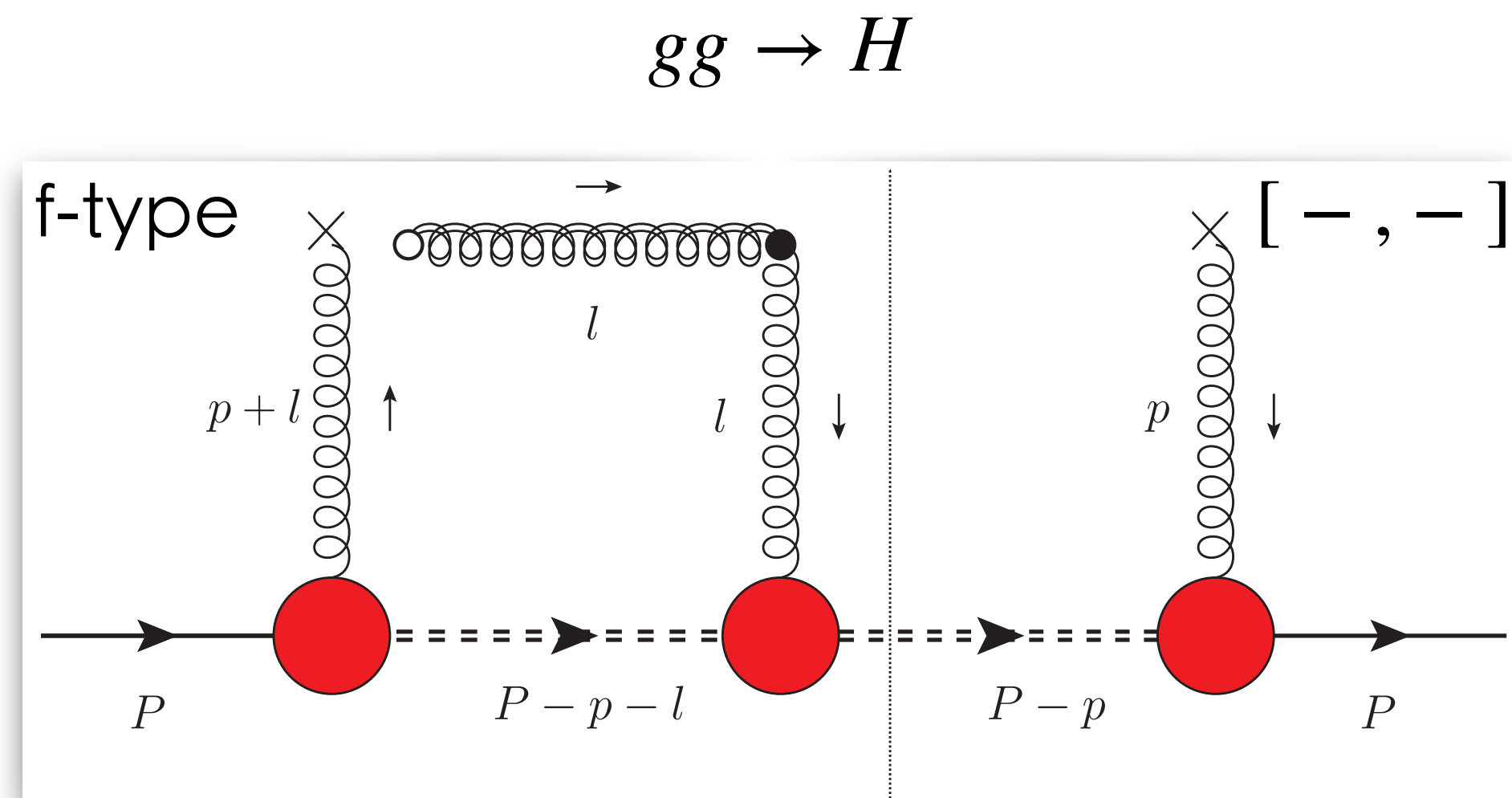
pure tri-quark contribution at low M_X

T-odd gluon TMDs in a spectator model

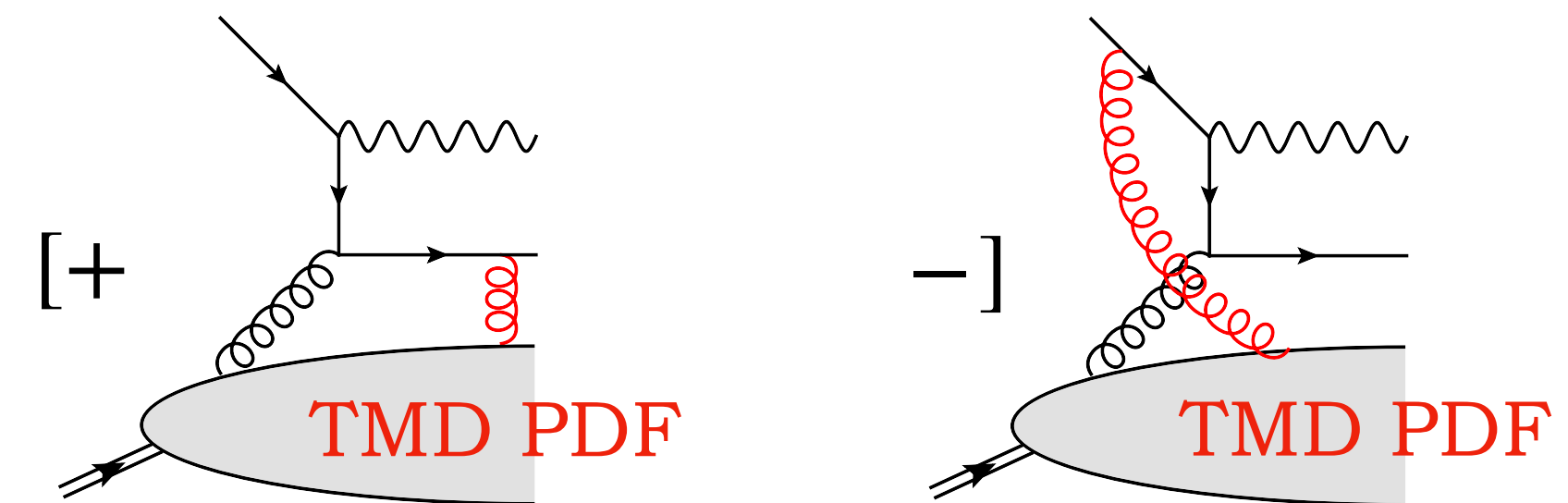
-  No residual gluon-spectator interaction at tree level
-  Interference with one-gluon exchange (eikonal)

T-odd gluon TMDs in a spectator model

- No residual gluon-spectator interaction at tree level
- Interference with one-gluon exchange (eikonal)



f-type (WW) structure

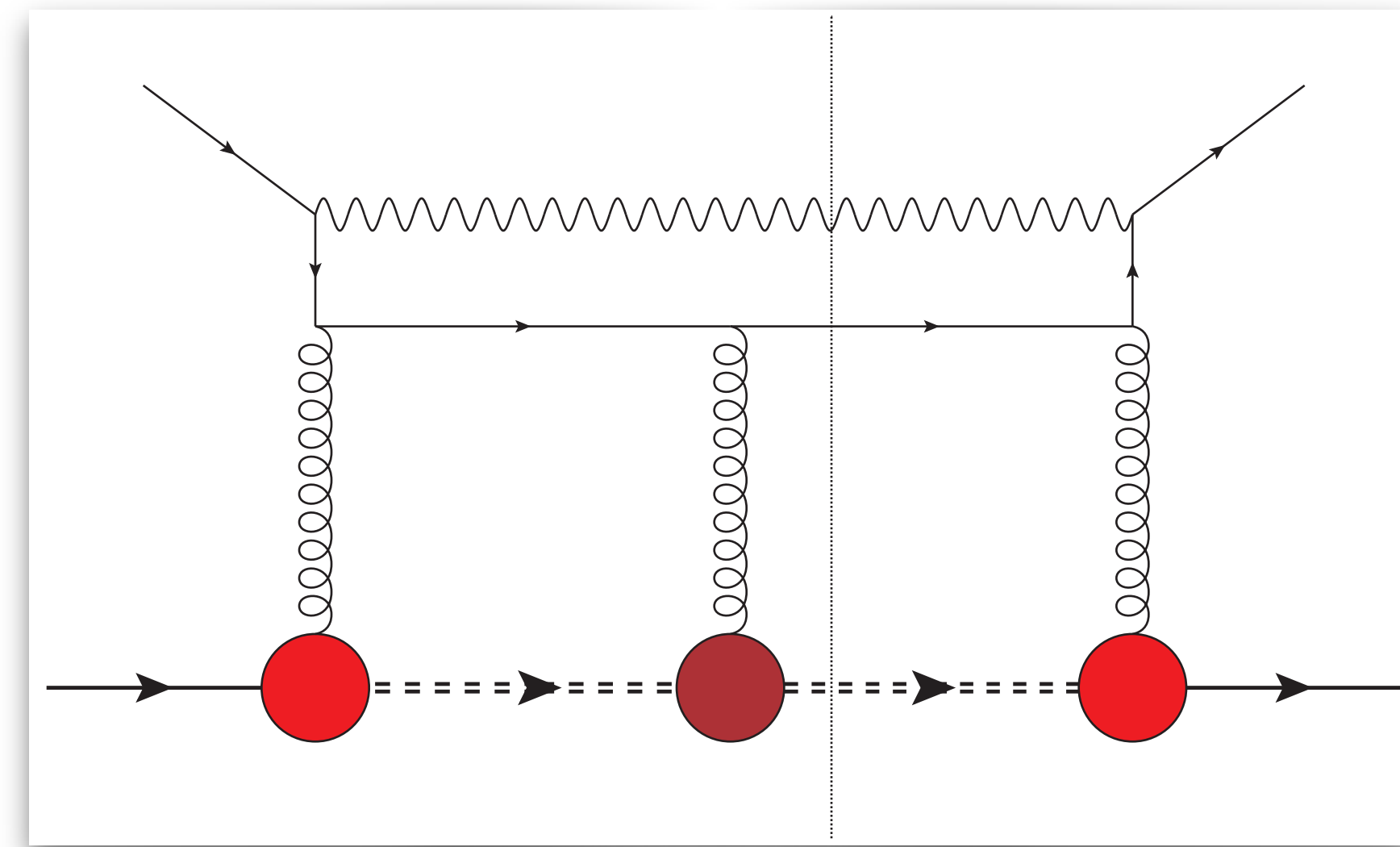
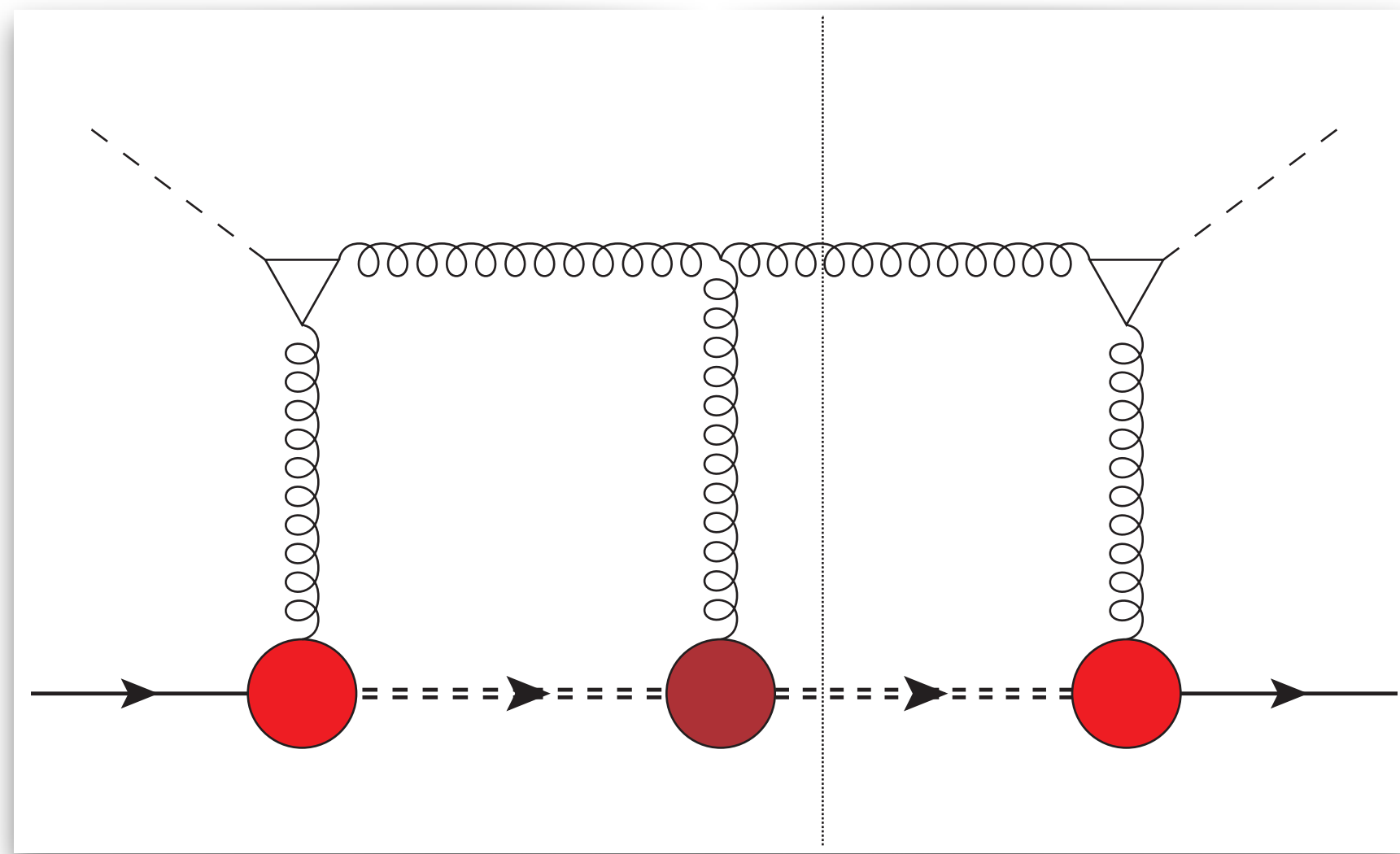


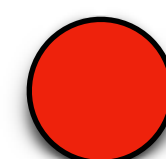
d-type (dipole) structure

T-odd gluon TMDs in a spectator model

Higgs-gluon fusion \Rightarrow f-type [+ , +]

Photon-jet emission \Rightarrow d-type [+ , -]



 nucleon-gluon-spectator

 spectator-gluon-spectator

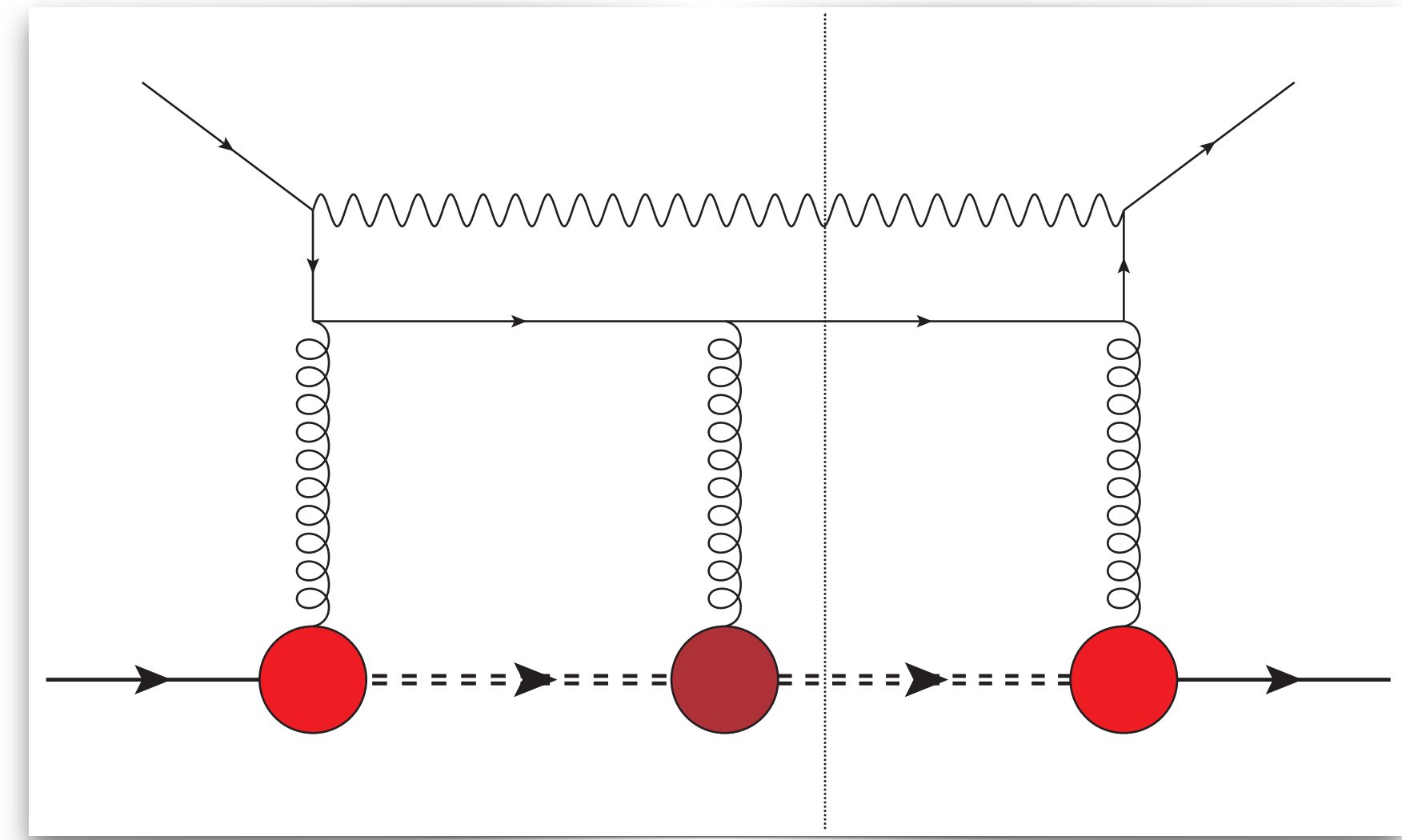
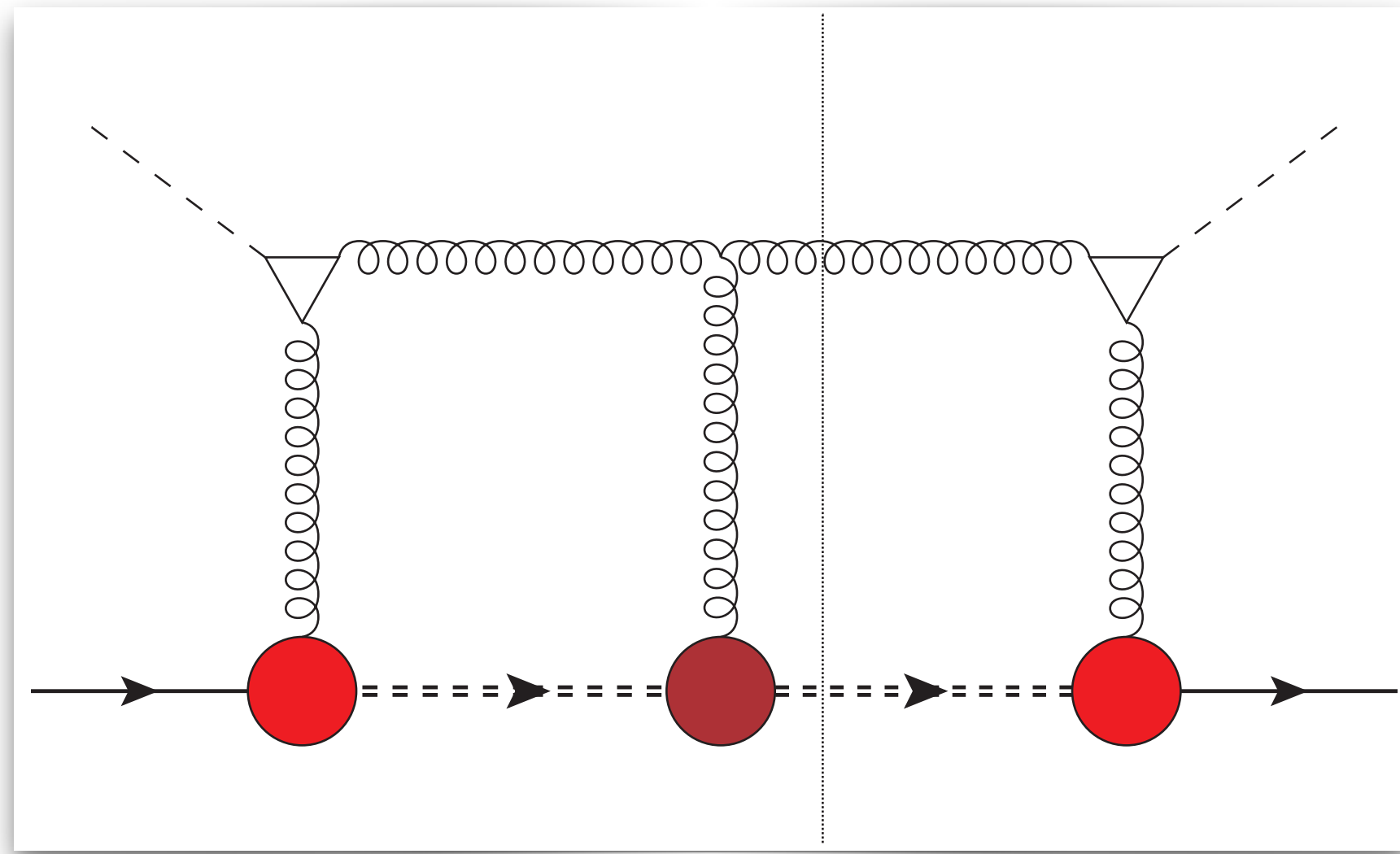
$$\mathcal{Y}_{bc}^{\mu}(p^2) = \delta_{bc} \left[g_1(p^2) \gamma_{\mu} + g_2(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_{\nu} \right]$$

$$\mathcal{X}_{abc}^{\mu}(p^2) = f^{abc} \left[g_1^f(p^2) \gamma^{\mu} + g_2^f(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_{\nu} \right] - i d^{abc} \left[g_1^d(p^2) \gamma^{\mu} + g_2^d(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_{\nu} \right]$$

T-odd gluon TMDs in a spectator model

Higgs-gluon fusion \Rightarrow f-type [+ , +]

Photon-jet emission \Rightarrow d-type [+ , -]



 nucleon-gluon-spectator

 spectator-gluon-spectator

$$\mathcal{Y}_{bc}^{\mu}(p^2) = \delta_{bc} \left[g_1(p^2) \gamma_{\mu} + g_2(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_{\nu} \right]$$

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Assumption: $g_{1,2}^d(p^2) = g_{1,2}^f(p^2) \equiv g_{1,2}(p^2)$

\Leftrightarrow

$$f_{1T}^{\perp[+,-]} = \frac{c_{[+,-]}}{c_{[+,+]}} f_{1T}^{\perp[+,+]} \equiv \frac{10}{18} f_{1T}^{\perp[+,+]}$$