Propagators in background field at Next to Eikonal order

Swaleha Mulani
(NCBJ, Poland)

In Collaboration with:
Tolga Altinoluk and Guillaume Beuf
(NCBJ, Poland)
Ongoing Work

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- Introduction
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    - Eikonal order
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- Applications of Propagators
  - SIDIS
  - Single Inclusive Production

Introduction

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  - Semi-classical approximation
  - Eikonal approximation

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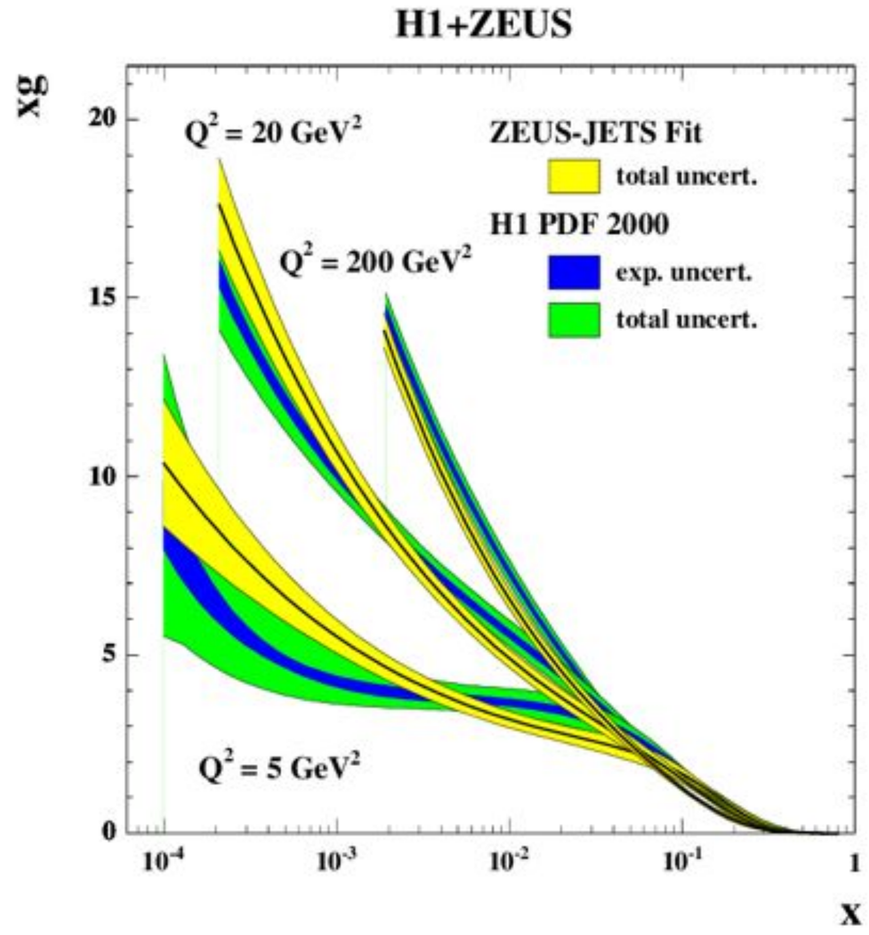
  - Semi-classical approximation
    - Dense target represented by Strong semi-classical gluon field $A^{\mu}(x)$
  - Eikonal approximation



Credit: arXiv:0901.0986

Introduction

- Generally in saturation physics in color glass condensate (CGC) framework two approximations are considered:
  - Semi-classical approximation
    - Dense target represented by Strong semi-classical gluon field $A^{\mu}(x)$
  - Eikonal approximation
    - Limit of infinite boost of $A^{\mu}(x)$
    - There is hierarchy between components of $A^{\mu}(x)$ with respect to Lorentz boost factor γ^t of the target:

$$A^- = \mathcal{O}(\gamma_t) >> A^j = \mathcal{O}(1) >> A^+ = \mathcal{O}(1/\gamma_t)$$

    - Only leading order energy term (leading term in γ^t) considered.

Eikonal Order

1. **Zero Width**

   Highly boosted background field (target) is localised in the longitudinal direction $x^+ = 0$ (zero width).

2. **Leading Component**

   Only leading component of target (-component) is considered and subleading components are neglected (suppressed by Lorentz boost factor).

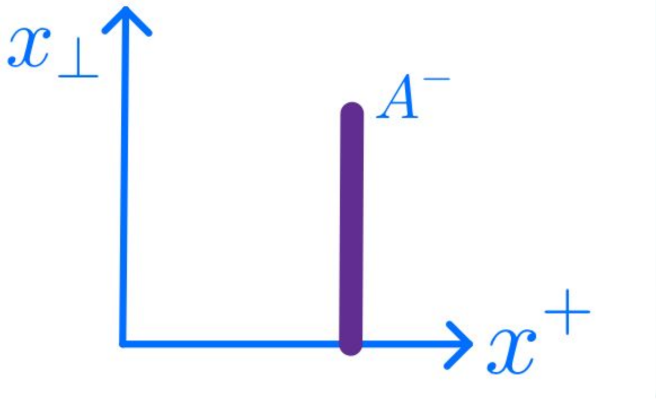
3. **x^- independence**

   Dynamics of the target are neglected (x^- dependence of target neglected).

Eikonal Order

Background field of target is :

$$A^{\mu}\left(x^{-},x^{+},\boldsymbol{x}\right)\approx\delta^{\mu-}\delta\left(x^{+}\right)A^{-}\left(\boldsymbol{x}\right)$$



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Eikonal order and Beyond it

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Beyond Eikonal:

- In very high energy accelerators (γ^t~1000 order), next-to-eikonal (NEik) order terms (power suppressed terms in high energies) are negligible while calculating observables.

- But to analyze the data from RHIC and future electron ion collider (**EIC**) (γ^t~ 10-100 order), NEik order terms might be sizable!

Eikonal order and Beyond it

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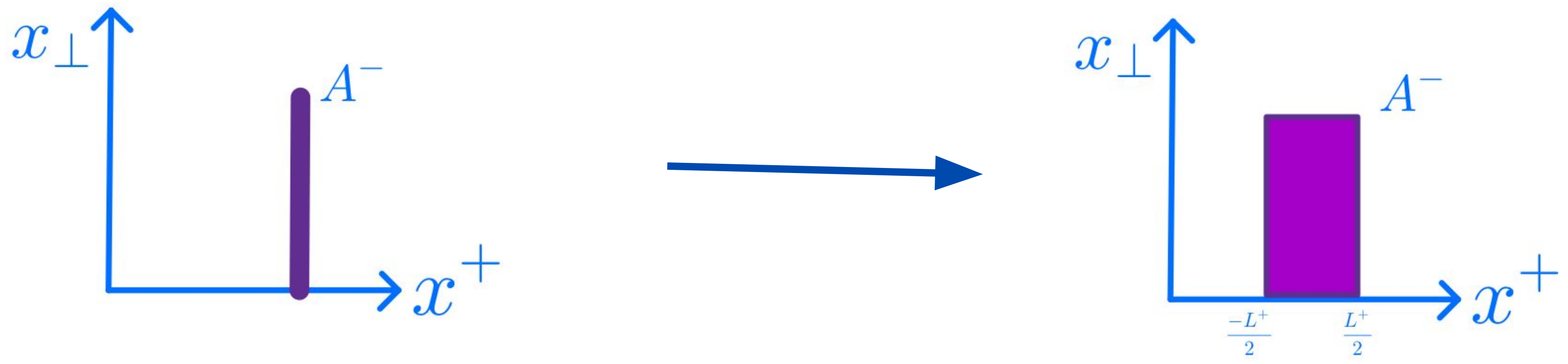
To go Beyond Eikonal:

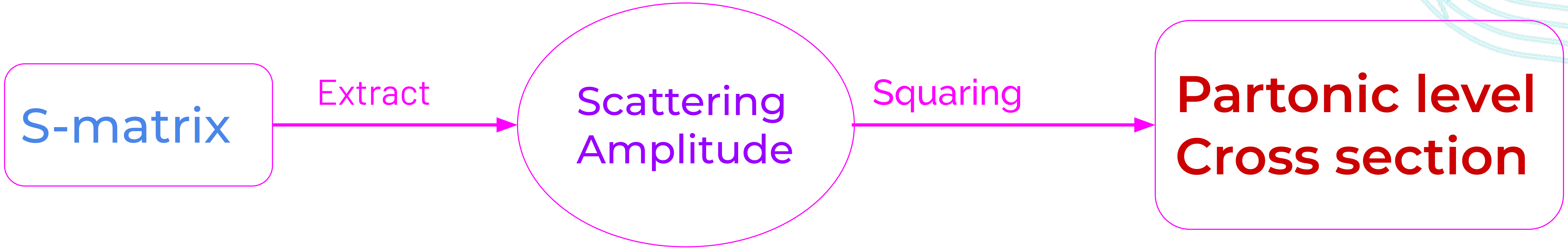
Finite Width
1.Instead of infinitely thin shockwave as a target, we consider finite width of a target.

Transverse Component
2.Instead of neglecting sub-leading components, we include transverse component of background field.

x^- dependence
3.We take into account corrections coming due to the x^- dependence of a target (consider background field is x^- dependent).



Cross-section in CGC

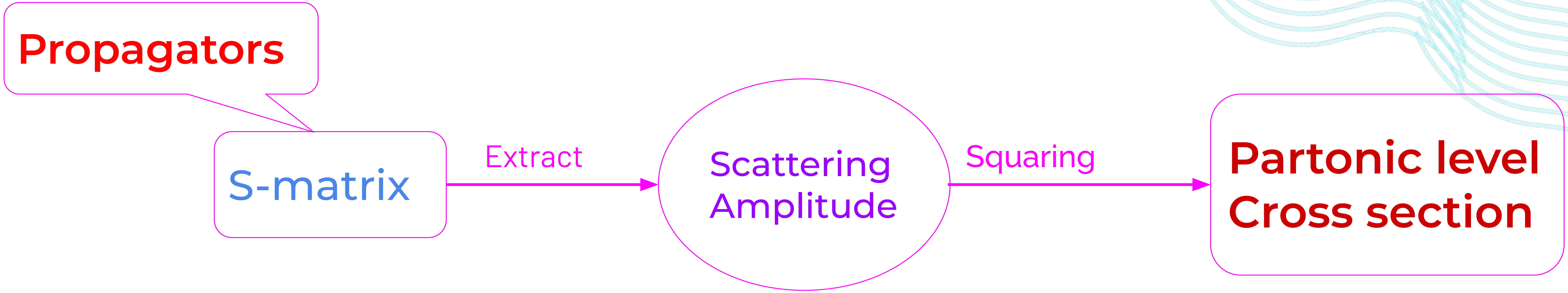


- In CGC framework for particle production cross section in pA collision is given as:

$$\sigma \propto \mathrm{PDF}_{\mathrm{proton}} \otimes \text{Partonic level cross-section} \otimes \text{fragmentation function}$$

- Our goal: To compute partonic level cross section at NEik order for different process.

Cross-section in CGC



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How to obtain these Propagators???

(in finite width medium at next-to-eikonal)

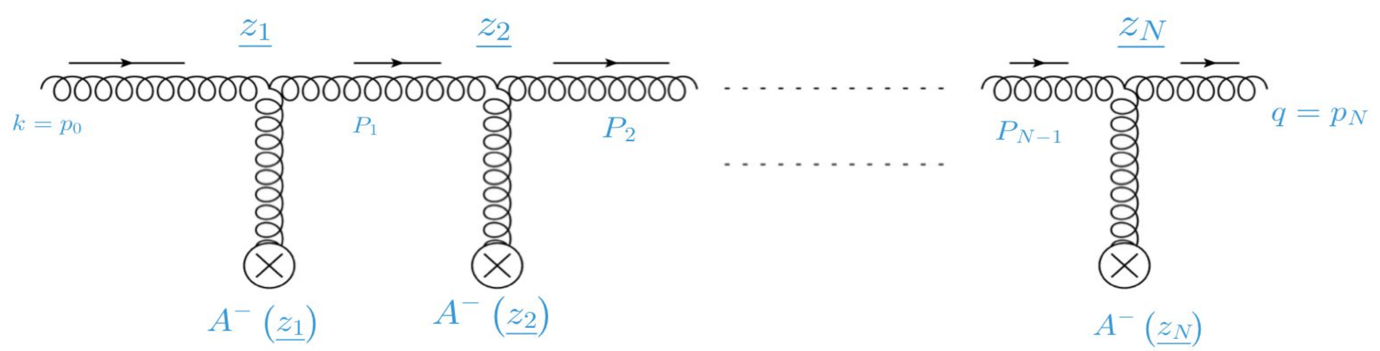
Propagators : Gluon Propagator

- **Recipe**: For gluon propagator at next-to-eikonal order travelling through entire medium
  a. First compute Eikonal order Gluon Propagator in gluon background field
  b. Use this computed gluon propagator to obtain next-to-eikonal (NEik) contributions
     i. Due to considering finite width of the target
     ii. Due to interaction with transverse components of medium
     iii. Due to dynamics of target (including x^- dependence)

Gluon Propagator at Eikonal order

- To calculate it, we re-sum multiple interaction diagrams of Gluon background field as shown in figure below.

- Only "-" component (leading component) of classical gluon background field is considered.



Similar for quarks in Altinoluk,Beuf, Czajka, Tymowska [2012.03886] , Altinoluk, Beuf [arXiv:2109.01620]

Gluon Propagator at Eikonal order

$$G_F^{\mu\nu}(x,y)\big|_{Eik} = i\delta^2(x_\perp - y_\perp)\ \delta(x^+ - y^+)\eta^\mu\eta^\nu\left[\int\frac{dk^+}{2\pi}\frac{e^{-i(x^- - y^-)k^+}}{k^+k^+}\right]$$
$$+\Bigg\{\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{e^{-ix\cdot\check{q}}\ e^{iy\cdot\check{k}}}{2k^+}\Big[2\pi\ \delta(k^+ - q^+)\Big]$$
$$\times\left[-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right]\left[\int d^2z_\perp\ e^{-i(q_\perp - k_\perp)z_\perp}\right]\Bigg\}$$
$$\times\Big[\theta(x^+ - y^+)\ \theta(k^+)\ \mathcal{U}_A(x^+, y^+; z_\perp) - \theta(y^+ - x^+)\ \theta(-k^+)\ \mathcal{U}_A^\dagger(x^+, y^+; z_\perp)\Big]$$

General expression in pure A^- background field at eikonal order for any x^+ and y^+.

(........and of course... we are going to use it in almost every calculation!)

Gluon Propagator at eikonal order:

$$G_F^{\mu\nu}(x,y)|_{Eik} = i\delta^2(x_\perp - y_\perp)\ \delta(x^+ - y^+)\eta^\mu\eta^\nu \left[\int \frac{dk^+}{2\pi} \frac{e^{-i(x^- - y^-)k^+}}{k^+k^+}\right]$$

$$+ \Bigg\{ \int \frac{d^3q}{(2\pi)^3} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-ix\cdot\check{q}}\ e^{iy\cdot\check{k}}}{2k^+} \left[2\pi\ \delta(k^+ - q^+)\right]$$

$$\times \left[-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right] \left[\int d^2z_\perp\ e^{-} \cdots\right.$$

$$\times \left[\theta(x^+ - y^+)\ \theta(k^+)\ \mathcal{U}_A(x^+, y^+; z_\perp) - \theta(y^+ - x^+)\ \theta(-k^+)\ \mathcal{U}_A^\dagger(x^+, y^+; z_\perp)\right]$$

Summations are included in Wilson lines,
Not dependent on z⁻

Where, $\mathcal{U}_A(x^+, y^+; z_\perp) = 1 + \sum_{N=1}^{\infty} \frac{1}{N!} \mathcal{P}_+ \left[-ig \int_{y^+}^{x^+} dz^+ A^-(z^+, z_\perp)\cdot T\right]^N$

Total Gluon propagator at NEik order

Total gluon propagator at NEik order travelling through the entire medium (dynamic gluon background field) for the case $x^+ > L^+/2$ and

$y^+ < -L^+/2$ with $x^+ > y^+$ is:

$$
\begin{aligned}
G_F^{\mu\nu}(x,y) = & \int \frac{d^3\underline{q}}{(2\pi)^3}\; e^{-ix\cdot\check{q}}\;\theta(q^+)\; \int \frac{d^3\underline{k}}{(2\pi)^3}\; e^{iy\cdot\check{k}}\;\theta(k^+)\;\frac{1}{q^+ + k^+} \\
& \times \left[-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right] \int d^2z_\perp e^{-i(q_\perp - k_\perp)z_\perp} \\
& \times \int dz^-\; e^{i(q^+-k^+)z^-}\;\mathcal{U}_A\Big(\frac{L^+}{2},\frac{-L^+}{2},z_\perp,z^-\Big) \\
& + \int \frac{d^3\underline{q}}{(2\pi)^3}\frac{e^{-ix\cdot\check{q}}}{2q^+} \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{e^{iy\cdot\check{k}}}{2k^+}\;\theta(k^+)\;2\pi\delta(q^+-k^+) \\
& \times \int d^2z_\perp\; e^{-iz_\perp(q_\perp-k_\perp)} \Bigg\{\Bigg(-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\Bigg) \\
& \times \Bigg(-\frac{q^j+k^j}{2}\int_{\frac{-L^+}{2}}^{\frac{L^+}{2}} dz^+ \Bigg[\mathcal{U}_A\Big(\frac{L^+}{2},z^+;z_\perp\Big)\Big(\overrightarrow{D}_{z^j} - \overleftarrow{D}_{z^j}\Big) \\
& \times \mathcal{U}_A\Big(z^+,-\frac{L^+}{2};z_\perp\Big)\Bigg] - i\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\Bigg[\mathcal{U}_A\Big(\frac{L^+}{2},z^+;z_\perp\Big)\Big(\overleftarrow{D}_{z^j}\overrightarrow{D}_{z^j}\Big)\;\mathcal{U}_A\Big(z^+,-\frac{L^+}{2};z_\perp\Big)\Bigg]\Bigg) \\
& + \Bigg(g^{\mu j}g^{\nu i} - \frac{\eta^\mu g^{\nu i}q^j}{q^+} - \frac{g^{\mu j}k^i\eta^\nu}{q^+} + \frac{\eta^\mu\eta^\nu k^i q^j}{q^+q^+}\Bigg) \\
& \times \Bigg(\int dz^+\;\mathcal{U}_A\Big(\frac{L^+}{2},z^+;z_\perp\Big)\; gT\cdot F_{ij}\;\mathcal{U}_A\Big(z^+,-\frac{L^+}{2};z_\perp\Big)\Bigg)\Bigg)\Bigg\}
\end{aligned}
$$

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&\times \Big[-g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k}) \Big] \int d^2 z_\perp e^{-i(q_\perp - k_\perp)z_\perp} \\
&\times \int dz^-\; e^{i(q^+-k^+)z^-}\; \mathcal{U}_A\Big(\frac{L^+}{2}, \frac{-L^+}{2}, z_\perp, z^-\Big) \\
&+ \int \frac{d^3\underline{q}}{(2\pi)^3}\frac{e^{-ix\cdot\check{q}}}{2q^+} \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{e^{iy\cdot\check{k}}}{2k^+}\;\theta(k^+)\; 2\pi\delta(q^+ - k^+) \\
&\times \int d^2 z_\perp\; e^{-iz_\perp(q_\perp - k_\perp)} \Bigg\{ \Big(-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k}) \Big) \\
&\times \Bigg(-\frac{q^j + k^j}{2} \int_{\frac{-L^+}{2}}^{\frac{L^+}{2}} dz^+ \Big[\mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big)\Big(\overrightarrow{D}_{z^j} - \overleftarrow{D}_{z^j}\Big) \\
&\times \mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big)\Big] - i \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \Big[\mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big)\Big(\overleftarrow{D}_{z^j}\overrightarrow{D}_{z^j}\Big)\, \mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big)\Big]\Bigg) \\
&+ \Big(g^{\mu j} g^{\nu i} - \frac{\eta^\mu g^{\nu i} q^j}{q^+} - \frac{g^{\mu j} k^i \eta^\nu}{q^+} + \frac{\eta^\mu\eta^\nu k^i q^j}{q^+q^+} \Big) \\
&\times \Big(\int dz^+\, \mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big)\; gT\cdot F_{ij}\, \mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big)\Big)\Bigg\}
\end{aligned}
$$

Including Quark Background Field

We also have to take into account quark background while computing cross-sections.

Kovchegov et al. [arXiv:1511.06737], G. V. Chirilli [arXiv:1807.11435] have included this effect.

Quark Background Field

- Due to large boost of the target along x^-: its localized in longitudinal x^+ direction around small support.
- Components of quark background field in terms of bilinear currents scale as:

$$\Psi(z)\gamma^-\Psi(z) = \mathcal{O}(\gamma_t) \text{ and } \Psi(z)\gamma^+\Psi(z) = \mathcal{O}(1/\gamma_t)$$

- If we consider projections on quark background field then,

$$\Psi(z) = \frac{\gamma^+\gamma^-}{2}\Psi(z) + \frac{\gamma^-\gamma^+}{2}\Psi(z) = \Psi^-(z) + \Psi^+(z)$$

$\mathcal{O}(\sqrt{\gamma_t})$ $\mathcal{O}(1/\sqrt{\gamma_t})$

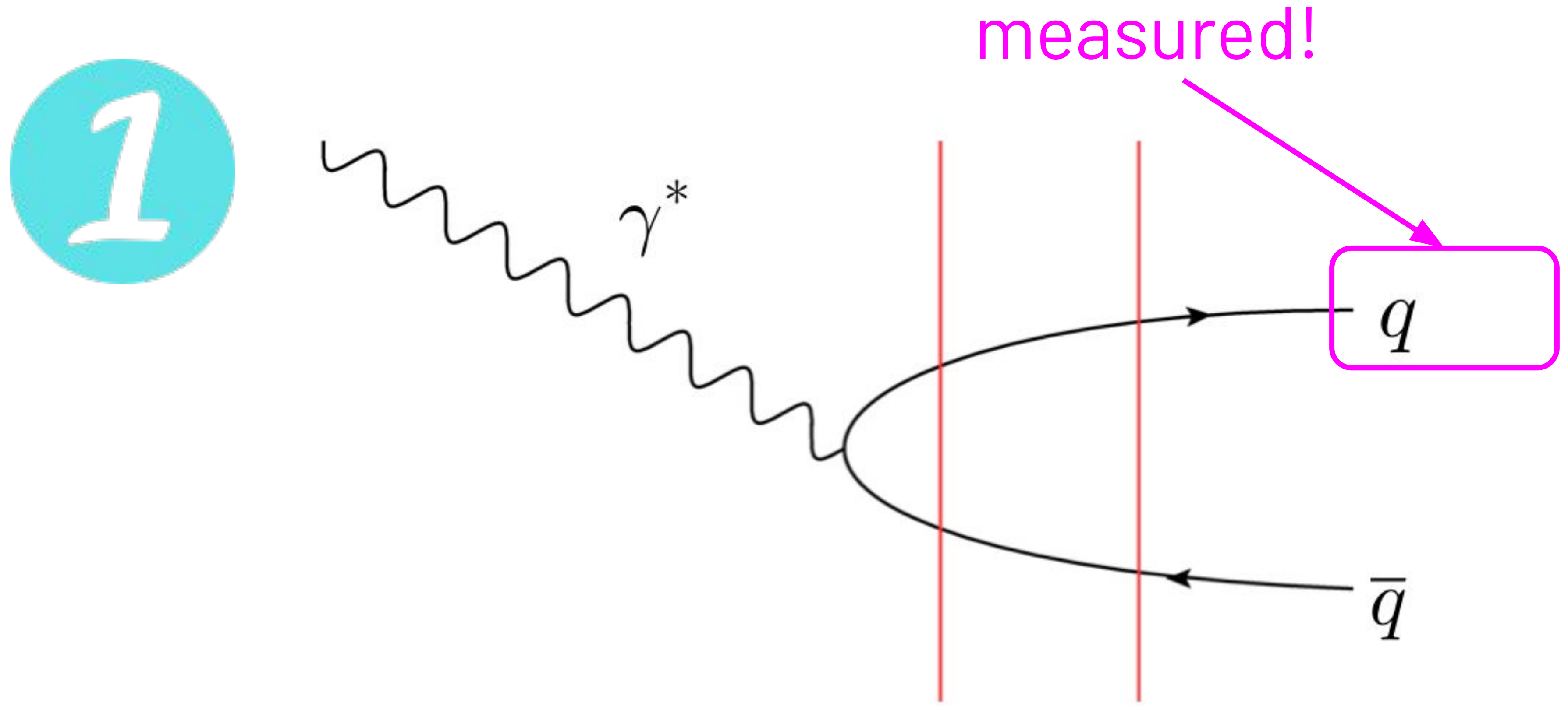
- For NEik corrections, only - component considered and + component is neglected (contribute at NNEik only).

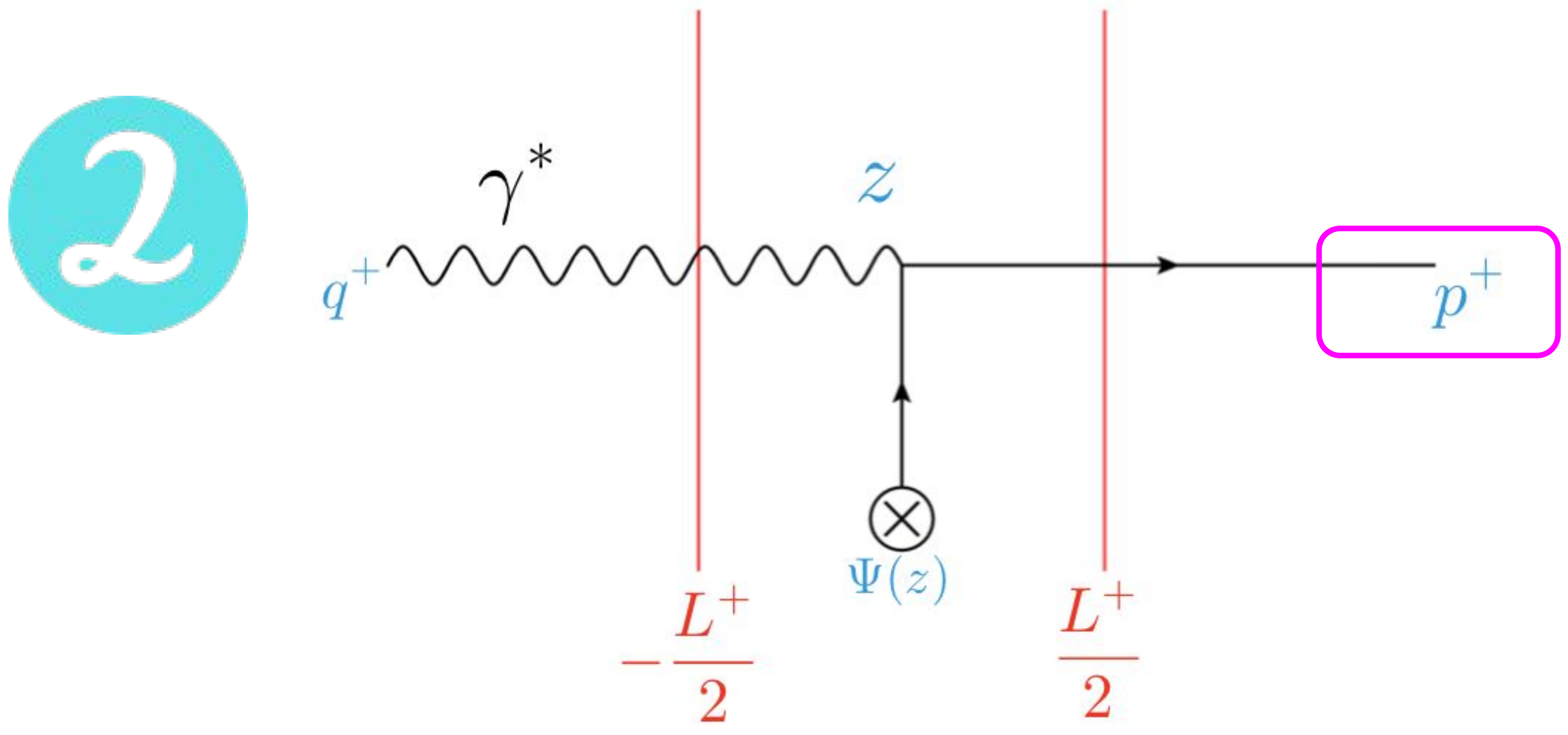
Applications

- Semi Inclusive Deep Inelastic Scattering (SIDIS)
- Single Inclusive Production

Semi Inclusive Deep Inelastic Scattering (SIDIS):

- In CGC, for this process: two kinds of contributions!
- Each of them are expected to be dominant in different kinematic regions.





- Contribution (1) is studied by Marquet, Xiao, Yuan [arXiv:0906.1454]. There is contribution at eikonal order.
- In this talk contribution coming due to (2) is discussed. No contribution at eikonal order.

SIDIS: S-matrix computation

- S-matrix at NEik order calculated : only $\Psi^-(z)$ of component considered $$S_{\gamma*\to q} = \lim_{x^+\to\infty} \int d^2x_\perp \int dx^- \, e^{i\check{p}\cdot x} \int d^4z \, \epsilon_\mu^\lambda(q) \, e^{-iq\cdot z} \, \overline{u}(p,h) \, \gamma^+ \, S_F(x,z)|^{IA}_{Eik}(-iee_f\gamma^\mu)\Psi^-(z)$$
- Two polarizations of photons are considered:
  - Longitudinal Polarization: no contribution at NEik order
  - Transverse Polarization: Contribution at NEik order

Finally, S-matrix for SIDIS process:

$\mathcal{O}(1/\gamma_t)$

$$S_{\gamma_T^*\to q} = 2\pi\delta(q^+ - p^+) \int dz^+ \int d^2z_\perp \, e^{i(q_\perp - p_\perp)z_\perp} \, \overline{u}(p,h)$$
$$\times \, \epsilon_\lambda^j(iee_f)U_F(\infty, z^+, z_\perp) \, (\frac{\gamma^j\gamma^+\gamma^-}{2})\psi(z)$$

$\mathcal{O}(\sqrt{\gamma_t})$

Similar calculations in case of q-g dijets are done in Altinoluk et al. (arXiv:2303.12691)

SIDIS: Cross-Section

Squaring amplitudes, we get cross-section for SIDIS process, in terms of Wilson lines:

$$
\frac{d^2\sigma^{\gamma_T^*\to q}}{d^2p_\perp} = \frac{e^2e_f^2}{(2\pi)^2}\frac{1}{2}\frac{1}{2q^+}\int d^2z'_\perp \int d^2z_\perp\, e^{i(q_\perp - p_\perp)(z_\perp - z'_\perp)} \int dz'^+ \int dz^+
$$
$$
\times \left\langle \overline{\psi}(z')\gamma^- \,\mathcal{U}_F^\dagger(\infty, z'^+, z'_\perp)\;\mathcal{U}_F(\infty, z^+, z_\perp)\;\psi(z)\right\rangle
$$

Over all suppression of $\mathcal{O}(1/\gamma_t)$: NEik order

SIDIS: Relation at small-x between CGC and TMD calculations

- Any color operator $\mathcal{O}$, the CGC-like target average $\langle \mathcal{O} \rangle$ is proportional to the quantum expectation value in the momentum state of target.

$$\langle \mathcal{O} \rangle = \lim_{P'_{tar} \to P_{tar}} \frac{\langle P'_{tar} | \mathcal{O} | P_{tar} \rangle}{\langle P'_{tar} | P_{tar} \rangle}$$

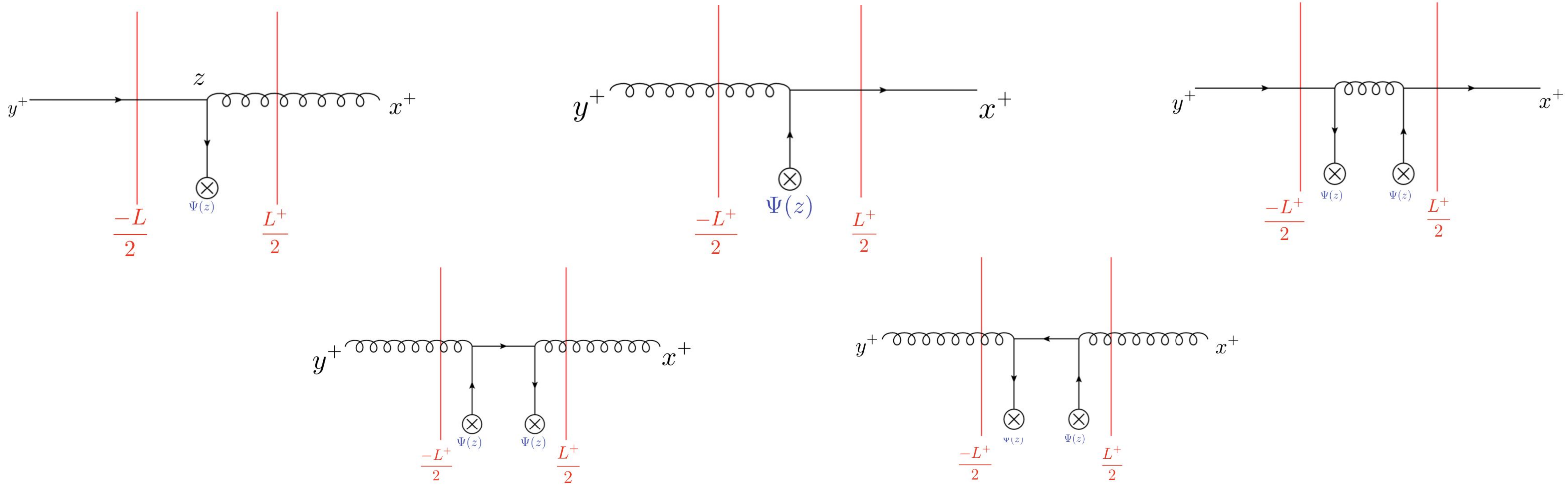
- Using this relation, we can relate obtained cross-section with unpolarized transverse momentum dependent (TMD) quark distribution.
- By comparing with quark TMD function, we get cross section:

$$\frac{d^2\sigma^{\gamma^*_T \to q}}{d^2 p_\perp} = \frac{\pi e^2 e_f^2}{W^2} f_1^q(x=0, p_\perp - q_\perp)$$

Suppression by centre of mass energy $1/W^2$ characterizes NEik contribution in terms of exchange t channel quark!

Single Inclusive Production

- First gluon propagator at NEik order in gluon background field is used to compute single inclusive gluon production cross section in gluon background field for forward pA collision.
- Contributions coming due to quark background field are taken into account.



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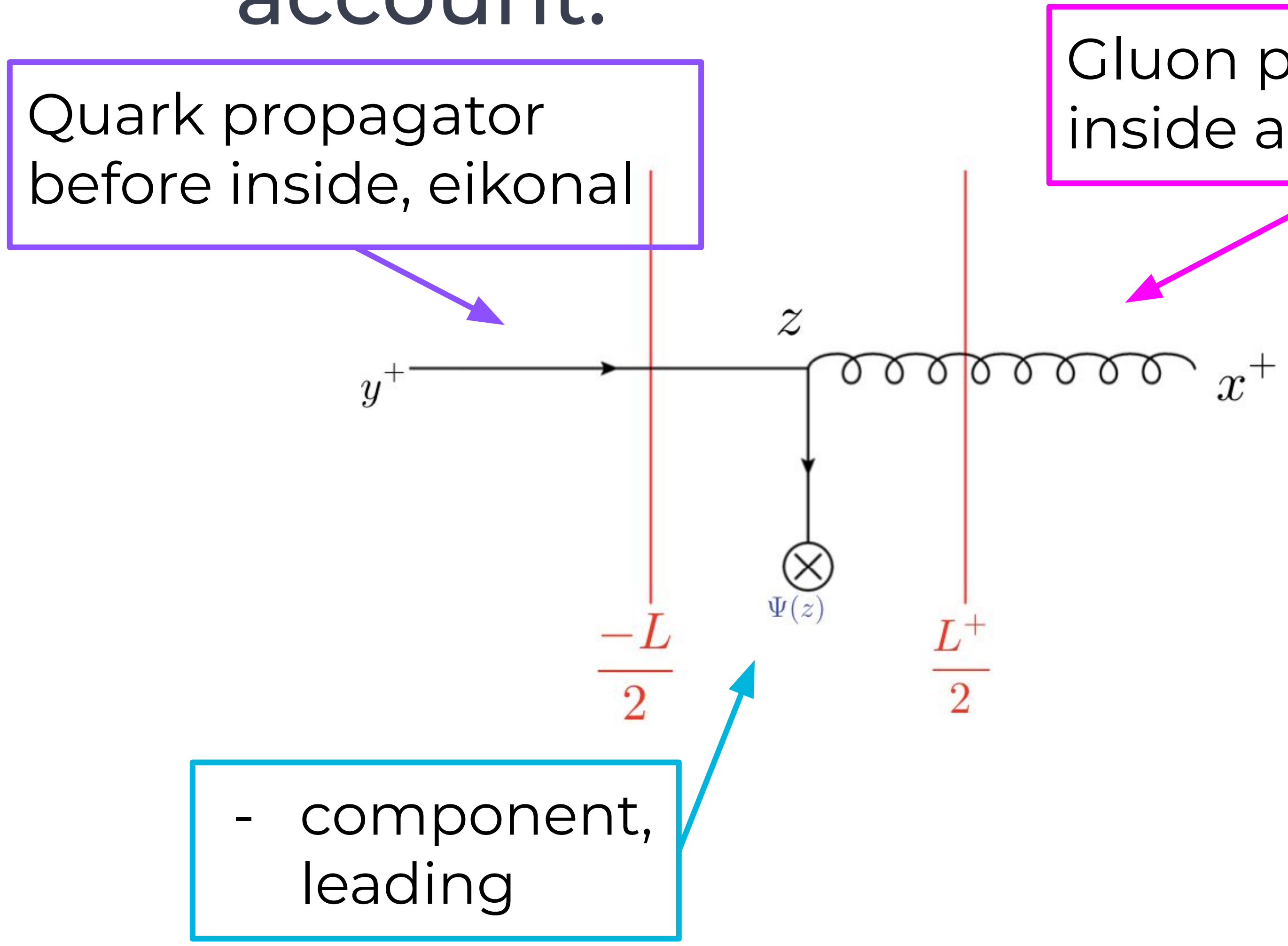

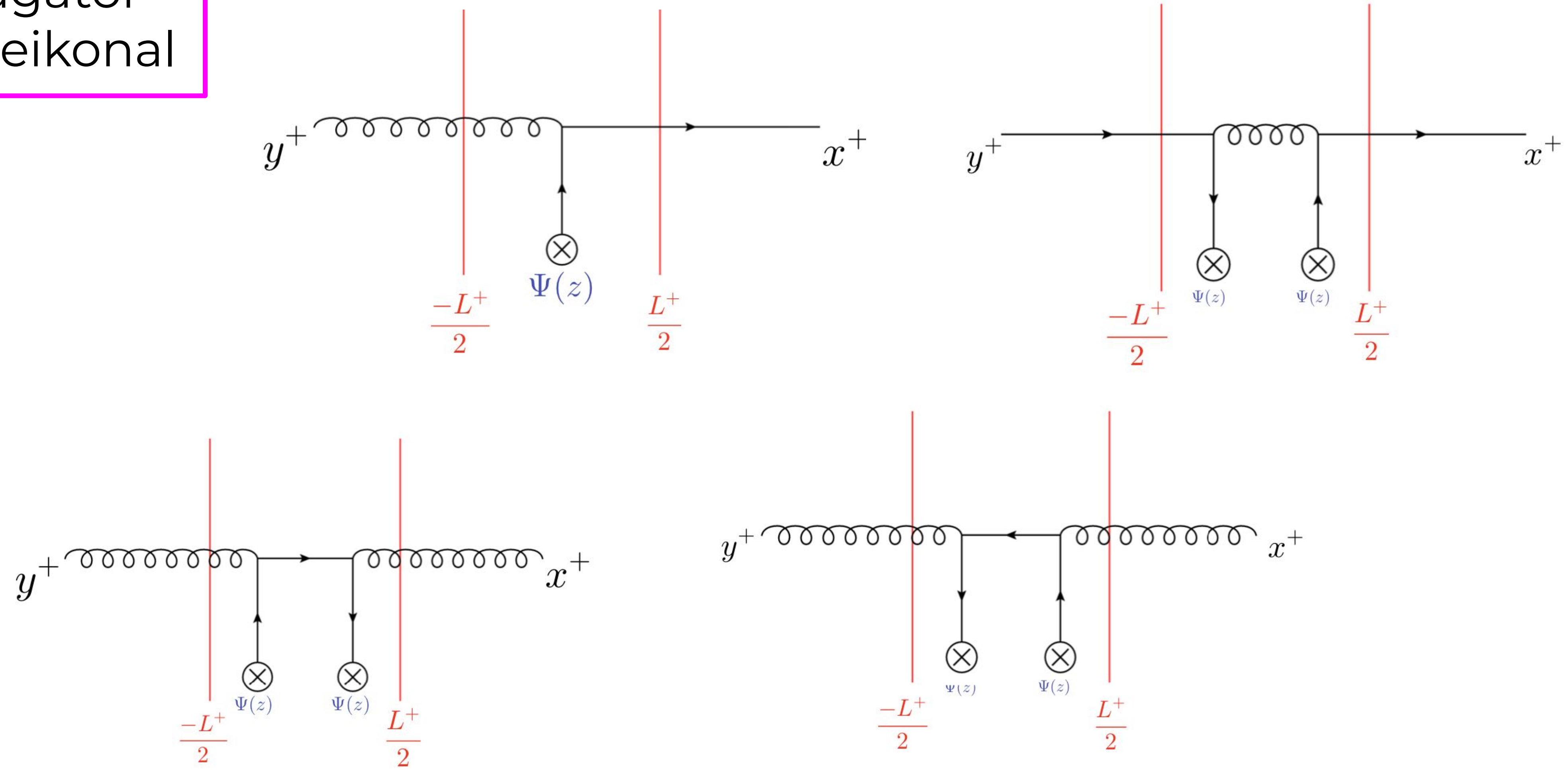

Summary

- Gluon propagator at Next-to-eikonal order in dynamic gluon background field is calculated.
  - Correction due to finite width, transverse component of target and x^- dependence of target field included.
- Obtained expression of gluon propagator is of general form therefore of general use: can be used for different scattering processes.
- SIDIS cross-section is calculated at NEik order by including quark background field.
  - expressed in terms of TMD quark distribution function.
- Quark background field contributions at NEik order will be included to obtain full single inclusive particle production at NEik accuracy in pA.

Thank you!