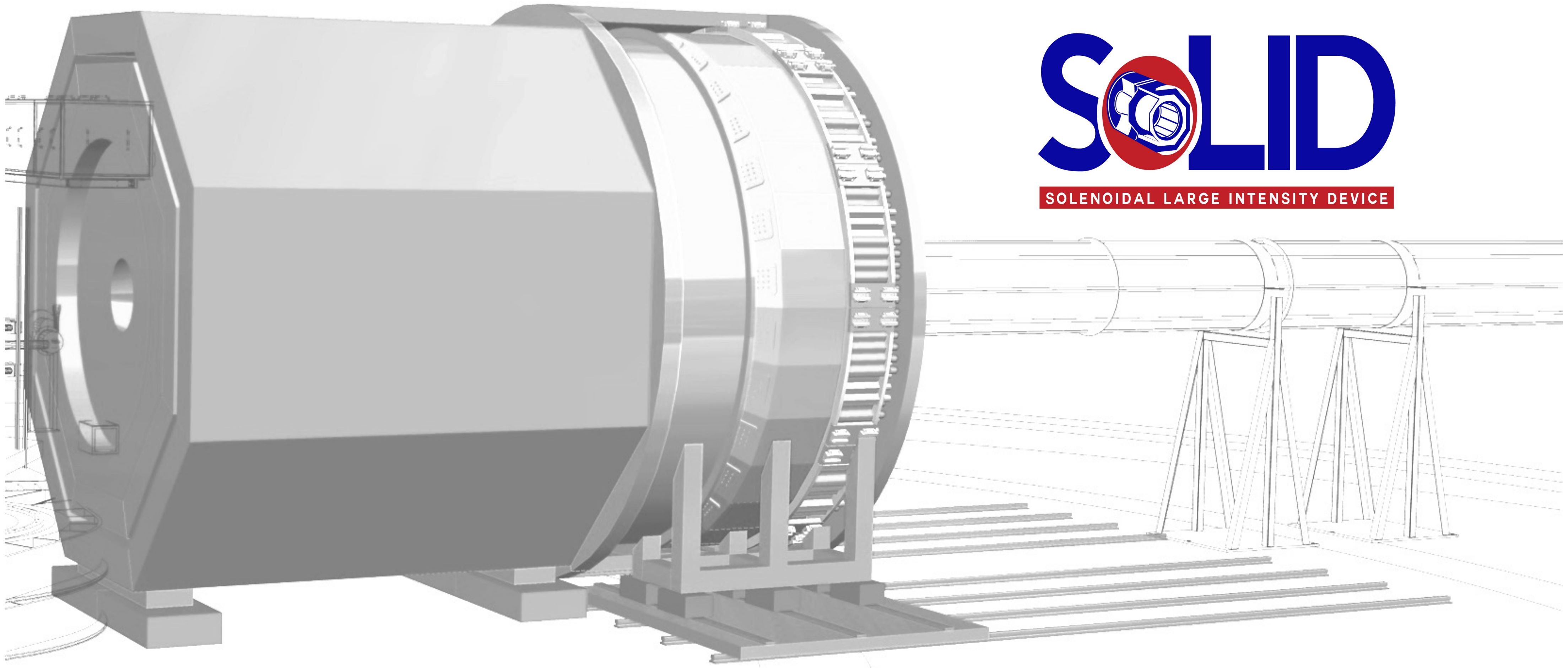
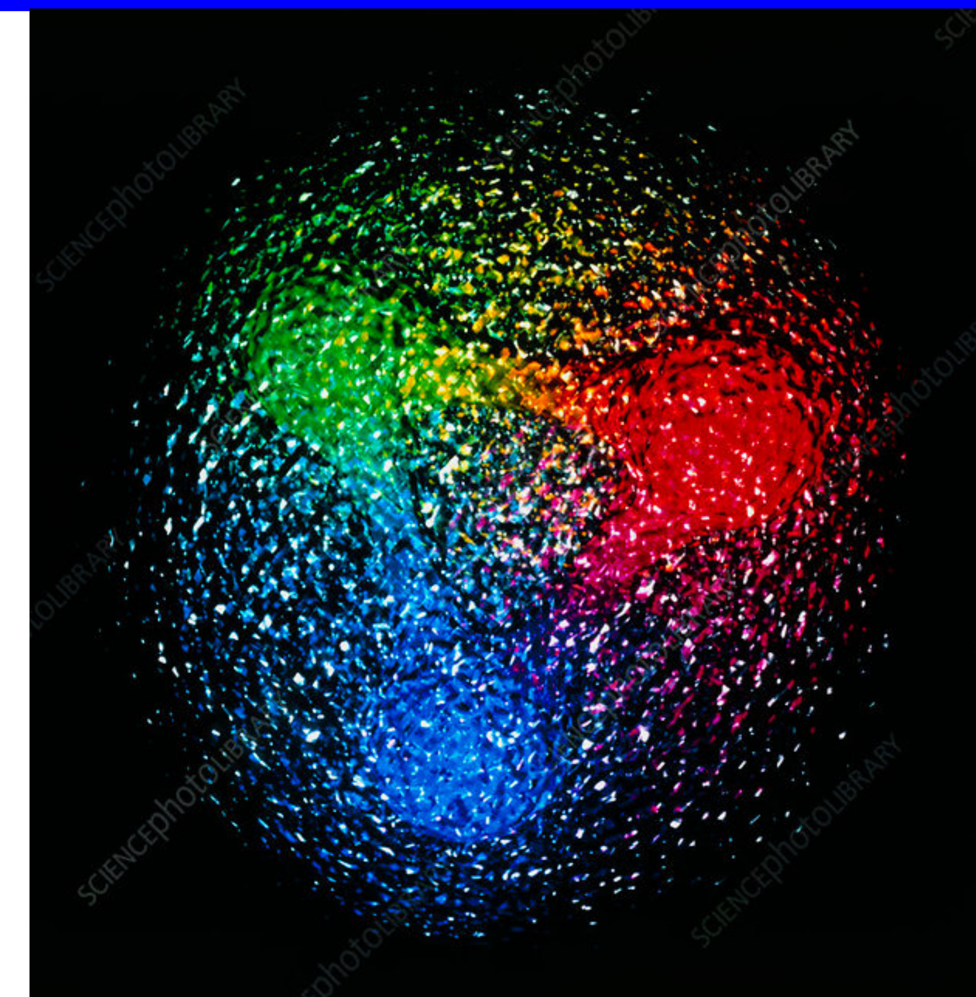
Studies of the Unpolarized SIDIS Cross Section in SoLID with Transversely/Longitudinally Polarized ^{3}He Targets at 11/8.8 GeV Beam Energies





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SoLID Collaboration Meeting, Jefferson Lab, Newport News, VA
May 8-9, 2023

Outline

- SoLID SIDIS setup with transversely and longitudinally polarized ^{3}He targets
  - *The setup and experimental details*
- Unpolarized cross-section framework
  - *Cross section without and with azimuthal modulations*
  - *Three models under considerations*
- Unpolarized cross-section SoLID projections
  - Kinematic correlations
  - Cross-section results at 11 GeV and 8.8 GeV electron beam energies
- Estimated systematic uncertainties
- Summary and outlook

Our run group experiment parasitic to the SoLID SIDIS experiments of

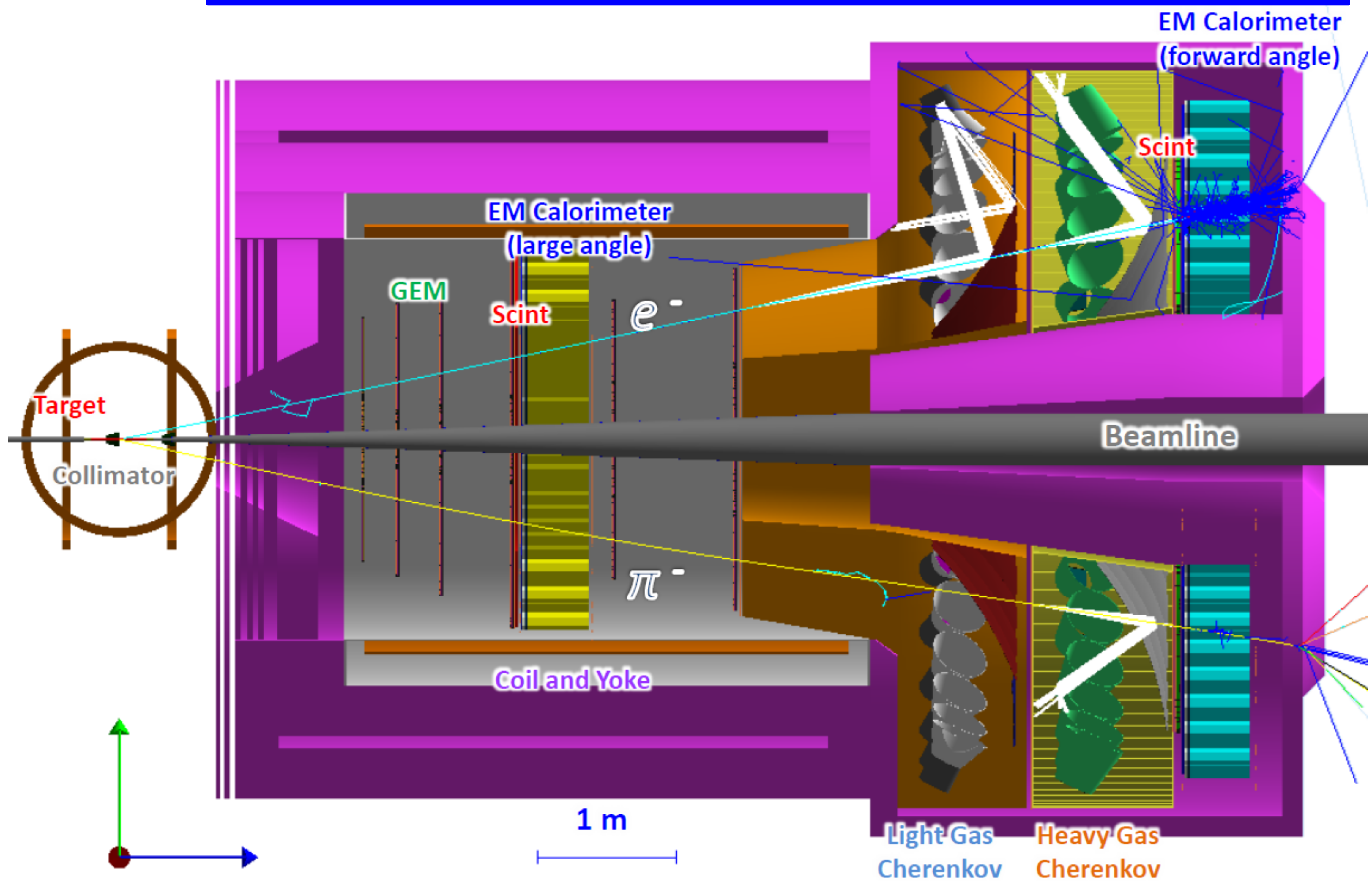
E12-10-006: Single Spin Asymmetries on Transversely Polarized ^{3}He (neutron):
Rating A Spokespersons: J.-P. Chen, H. Gao (contact), X. D. Jiang, J. Peng, X. Qian

E12-11-007: Single and Double Spin Asymmetries on Longitudinally Polarized ^{3}He (neutron):
Rating A Spokespersons: J.-P. Chen (contact), J. Huang, C. Peng, Y. Qiang, W. Yan

SIDIS: $e + p \rightarrow e' + \pi^{\pm} + X$

- Target:
  - *Length*: *40 cm*
  - *Polarization*: *~ 60%*
  - *Spin flip*: *≤ 20 mins*
  - *Polarimetry*: *~ 3%*
- GEM: six tracking chambers
- EM Calorimeter:
  Forward and Large angle
- SPD: Forward and Large angle
- LGC and HGC
- CLEO-II magnet

SoLID (SIDIS ^{3}He): 11 GeV & 8.8 GeV beam energies



Transversely polarized ^{3}He (E12-10-006)

- Approved number of days:
  - *48 days* (*11 GeV*) & *21 day* (*8.8 GeV*)
- 69 days requested for the beam on target
- 10 days requested for a study of the x-z factorization with Hydrogen and Deuterium gas using a reference target cell
- 3 days requested with a longitudinal target polarization to study the systematics of potential A_{UL} contamination
- 8 days of total overhead time requested:
  - *unpol. target runs* (*optics and detector check*)
  - *target spin flip and polarization measurements*

Longitudinally polarized ^{3}He (E12-11-007)

- Approved number of days:
  - *22.5 days* (*11 GeV*) & *9.5 day* (*8.8 GeV*)
- 32 days requested for the beam on target
- 35 days of total beam time requested to match approximately 50% statistics of the experiment E12-10-006
- When combined with E12-10-006:
  - *no beam time required for data calibration*
  - *no target runs and detector calibrations required*
- 3 days of total overhead time requested:
  - *target spin flip and polarization measurements*

Major requirements: Radiation hardness, detector resolution, kaon contamination, DAQ

Expected DAQ rates: $<$ 100 kHz

- Momentum coverage: 1.0 - 7.0 GeV/c; Polar angular coverage: 8.0° - 14.8°
  (*for hadron & electron ID*)

- Momentum coverage: 3.5 - 6.0 GeV/c; Polar angular coverage: 15.7° - 24.0°
  (*for electron ID*)

- Momentum resolution: ~ 2%; Polar angular resolution: 2 mrad

- Azimuthal angular coverage: 2π; Azimuthal angular resolution: 6 mrad

- PID (electron): detection efficiency ≥ 90%; pion contamination < 1%

- PID (pion): detection efficiency ≥ 90%; kaon contamination < 1%

- Total luminosity: $3.74 \cdot 10^{36}$ cm^{-2} sec^{-1}

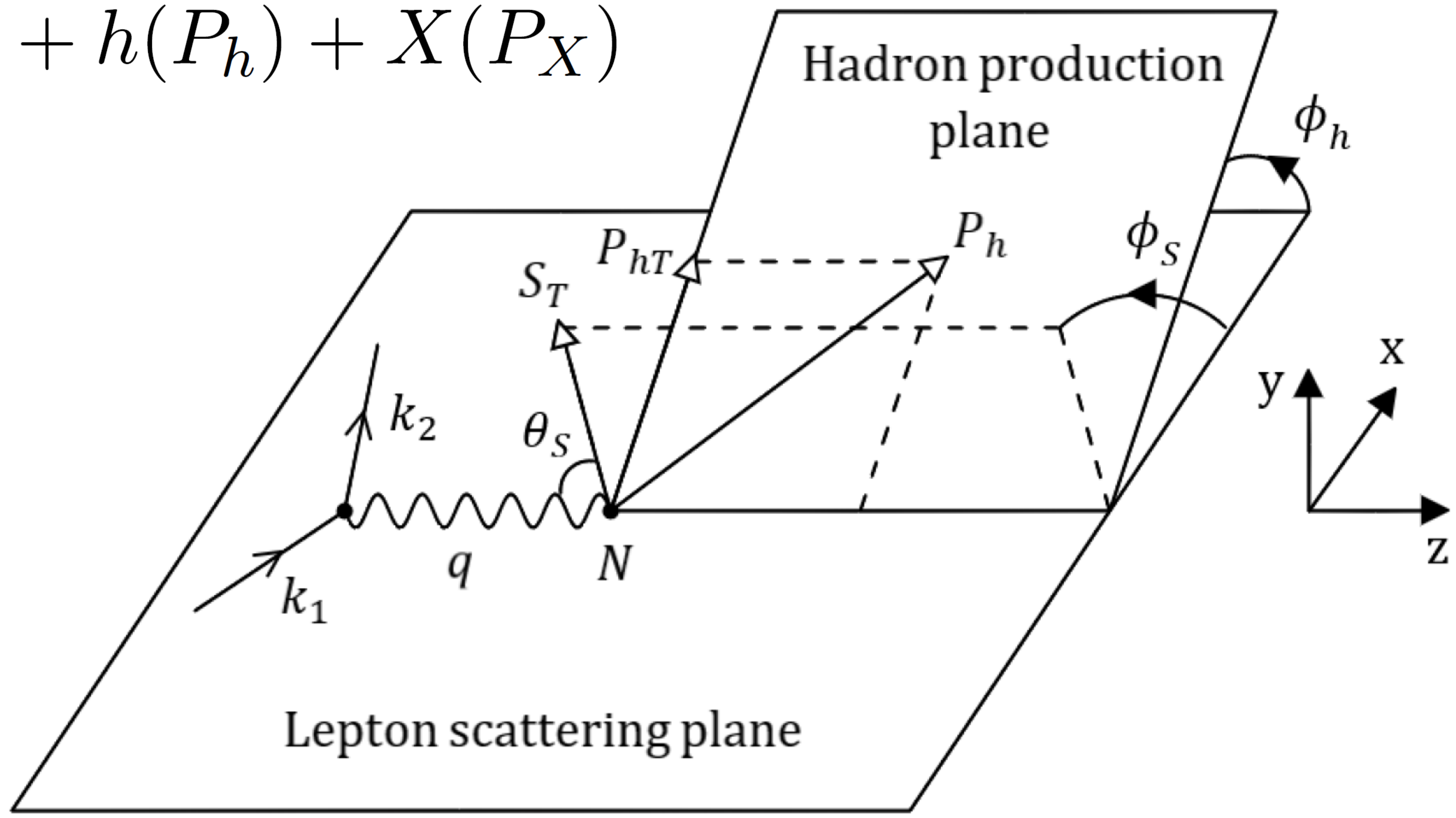
- Beam polarimetry: < 3%; Beam current: 15 μA

- Many other details in ***SoLID (Solenoidal Large Intensity Device) Updated Preliminary Conceptual Design Report***, [***https://solid.jlab.org/***](https://solid.jlab.org/)

- The SIDIS process represented as (four-momenta given in parentheses)

$$l(k_1) + N(P) \to l'(k_2) + h(P_h) + X(P_X)$$

- l - lepton beam
- N - nucleon target
- h - produced hadron
- X - undetected hadron
- q – virtual photon momentum



Azimuthal angle between hadron production and lepton scattering planes designated as ϕ_h

Kinematics of the SIDIS process:
assume one-photon exchange approximation

- Express the process cross section in terms of the following kinematic variables

$$x_{bj} = \frac{Q^2}{2P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot k_1}, \qquad z_h = \frac{P\cdot P_h}{P\cdot q}, \qquad \gamma = \frac{2M_N x_{bj}}{Q}$$

with q and Q^2 defined as $q \equiv l - l'$ and $Q^2 \equiv -q^2$

- Unpolarized SIDIS differential cross section given by

Phys. Rev. D 91, no.7, 074019 (2015)

$$\frac{d\sigma}{dx_{bj}dydz_h dP_{hT}^2 d\phi_h} \equiv \mathcal{F}_{\mathcal{UU}} = \mathcal{F}_{\mathcal{UU},\mathcal{A}} \cos 0 + \mathcal{F}_{\mathcal{UU},\mathcal{B}} \cos(\phi_h) + \mathcal{F}_{\mathcal{UU},\mathcal{C}} \cos(2\phi_h)$$

$$\mathcal{F}_{\mathcal{UU},\mathcal{A}} = 2\pi \frac{\alpha^2}{x_{bj} y Q^2}\left(1 + \frac{\gamma^2}{2x_{bj}}\right) c_1 F_{UU},$$

Unpolarized TMD

$$\mathcal{F}_{\mathcal{UU},\mathcal{B}} = 2\pi \frac{\alpha^2}{x_{bj} y Q^2}\left(1 + \frac{\gamma^2}{2x_{bj}}\right) c_2 F_{UU}^{\cos(\phi_h)},$$

$$\mathcal{F}_{\mathcal{UU},\mathcal{C}} = 2\pi \frac{\alpha^2}{x_{bj} y Q^2}\left(1 + \frac{\gamma^2}{2x_{bj}}\right) c_3 F_{UU}^{\cos(2\phi_h)}$$

ϕ_h - dependent unpolarized azimuthal modulations

with the coefficients

$$c_1 = \frac{y^2}{2(1-\varepsilon)}, \quad c_2 = \frac{y^2}{2(1-\varepsilon)}\sqrt{2\varepsilon(1+\varepsilon)}, \quad c_3 = \frac{y^2}{2(1-\varepsilon)}\varepsilon$$

Ratio of longitudinal and transverse photon fluxes

$$\varepsilon = \frac{1 - y - (\gamma^2 y^2/4)}{1 - y + (y^2/2) + (\gamma^2 y^2/4)}$$

- When $P_{hT} \sim k_\perp \ll Q$, TMD factorization known to be valid at leading twist
  - $k_\perp$ to be quark transverse momentum within the nucleon
- TMD factorization used by most phenomenological analyses
- In this scheme, the unpolarized structure function F_{UU} given by

$$F_{UU} = \sum_q e_q^2 \, x \int d^2\boldsymbol{k}_\perp \, f_q(x, k_\perp) D_q(z, p_\perp)$$

as convolution of unpolarized **TMD PDF** and **TMD FF**

- Convenient to work with PDFs and FFs in light-cone coordinates
  - $x = k^+/P^+$ and $z = P_h^-/\kappa^-$ (κ to be fragmentation quark momentum)
- Up to the order of $k_\perp/Q$, variables x and z identified with x_{bj} and z_h
- Condition of momentum conservation reads as $P_{hT} = z\, k_\perp + p_\perp$
  - $p_\perp$ to be transverse momentum of fragmentation hadron with respect to the direction of fragmentation quark

- Use the following Gaussian parameterizations for the TMD PDF and TMD FF:

$$f_q(x, k_\perp) = f_q^c(x)\, \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle} \qquad D_q(z, p_\perp) = D_q^c(z)\, \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

where $f_q^c(x)$ is the collinear PDF, and $D_q^c(z)$ is the collinear FF

- Analytical form of F_{UU} given by

$$F_{UU} = \sum_q e_q^2\, x_{bj}\, f_q^c(x_{bj})\, D_q^c(z_h)\, \frac{e^{-P_{hT}^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

where $\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$

- Gaussian widths $\langle k_\perp^2 \rangle$ and $\langle p_\perp^2 \rangle$ may have different forms of kinematical dependence

- Use the set of LHAPDF CJ15lo for the PDF, and the set of DSSFFlo for the FF

https://github.com/TianboLiu/LiuSIDIS

Phys. Rev. D 93, no.11, 114017 (2016)

Phys. Rev. D 91, 014035 (2015)

- The second structure function $F_{UU}^{cos(\phi_h)}$, associated to the $cos(\phi_h)$ modulation of the cross section, is a twist-3 quantity of the order of 1/Q

$$F_{UU}^{\cos(\phi_h)} = F_{UU}^{\cos(\phi_h)}\big|_{\text{Cahn}} + F_{UU}^{\cos(\phi_h)}\big|_{\text{BM}}$$

where

$$F_{UU}^{\cos(\phi_h)}\big|_{\text{Cahn}} = -2\sum_q e_q^2\, x \int d^2\boldsymbol{k}_\perp \frac{(\boldsymbol{k}_\perp \cdot \boldsymbol{h})}{Q} f_q(x, k_\perp) D_q(z, p_\perp)$$

as the Cahn convolution of unpolarized **TMD PDF** and **TMD FF**

$$F_{UU}^{\cos(\phi_h)}\big|_{\text{BM}} = \sum_q e_q^2\, x \int d^2\boldsymbol{k}_\perp \frac{k_\perp}{Q} \frac{P_{hT} - z\,(\boldsymbol{k}_\perp \cdot \boldsymbol{h})}{k_\perp} \Delta f_{q^\uparrow/p}(x, k_\perp)\, \Delta D_{h/q^\uparrow}(z, p_\perp)$$

$$-\frac{k_\perp}{M_p} h_1^\perp(x, k_\perp) \qquad \frac{2p_\perp}{zM_h} H_1^\perp(z, p_\perp)$$

as the Boer-Mulders convolution of **Boer-Mulders TMD PDF** and **Collins TMD FF**

- The third structure function $F_{UU}^{\cos(2\phi_h)}$, associated to the $\cos(2\phi_h)$ modulation of the cross section, consists of a twist-4 Cahn and a twist-2 Boer-Mulders contributions

$$F_{UU}^{\cos(2\phi_h)} \approx F_{UU}^{\cos(2\phi_h)}\Big|_{\rm Cahn} + F_{UU}^{\cos(2\phi_h)}\Big|_{\rm BM}$$

where

$$F_{UU}^{\cos(2\phi_h)}\Big|_{\rm Cahn} = 2\sum_q e_q^2\, x \int d^2\boldsymbol{k}_\perp \frac{2(\boldsymbol{k}_\perp\cdot\boldsymbol{h})^2 - k_\perp^2}{Q^2} f_q(x,k_\perp) D_q(z,p_\perp)$$

$$F_{UU}^{\cos(2\phi_h)}\Big|_{\rm BM} = -\sum_q e_q^2\, x \int d^2\boldsymbol{k}_\perp \frac{P_{hT}(\boldsymbol{k}_\perp\cdot\boldsymbol{h}) + z\left[k_\perp^2 - 2(\boldsymbol{k}_\perp\cdot\boldsymbol{h})^2\right]}{2k_\perp p_\perp} \times$$
$$\times\, \Delta f_{q^\uparrow/p}(x,k_\perp)\, \Delta D_{h/q^\uparrow}(z,p_\perp)$$

- Here we have the same

  i) unpolarized **TMD PDF** and **TMD FF**

  ii) **Boer-Mulders TMD PDF** and **Collins TMD FF**

- Analytical forms of the Cahn and Boer-Mulders azimuthal modulation given by

$$F_{UU}^{\cos(\phi_h)}\Big|_{\text{Cahn}} = -2\,\frac{P_T}{Q}\sum_q e_q^2\, x_{bj}\, f_q^c(x_{bj})\, D_q^c(z_h)\,\frac{z_h\langle k_\perp^2\rangle}{\langle P_T^2\rangle}\,\frac{e^{-P_{hT}^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle},$$

$$F_{UU}^{\cos(\phi_h)}\Big|_{\text{BM}} = 2e\,\frac{P_T}{Q}\sum_q e_q^2\, x_{bj}\,\frac{\Delta f_{q^\uparrow/p}(x_{bj})}{M_{\text{BM}}}\,\frac{\Delta D_{h/q^\uparrow}(z_h)}{M_{\text{C}}}\,\frac{e^{-P_{hT}^2/\langle P_T^2\rangle_{\text{BM}}}}{\pi\langle P_T^2\rangle_{\text{BM}}^4}\times$$
$$\times\frac{\langle k_\perp^2\rangle_{\text{BM}}^2\,\langle p_\perp^2\rangle_{\text{C}}^2}{\langle k_\perp^2\rangle\langle p_\perp^2\rangle}\left[z_h^2\langle k_\perp^2\rangle_{\text{BM}}\left(P_{hT}^2-\langle P_T^2\rangle_{\text{BM}}\right)+\langle p_\perp^2\rangle_{\text{C}}\langle P_T^2\rangle_{\text{BM}}\right],$$

$$F_{UU}^{\cos(2\phi_h)}\Big|_{\text{Cahn}} = 2\,\frac{P_T^2}{Q^2}\sum_q e_q^2\, x_{bj}\, f_q^c(x_{bj})\, D_q^c(z_h)\,\frac{z_h^2\langle k_\perp^2\rangle^2}{\langle P_T^2\rangle^2}\,\frac{e^{-P_{hT}^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle},$$

$$F_{UU}^{\cos(2\phi_h)}\Big|_{\text{BM}} = -eP_T^2\sum_q e_q^2\, x_{bj}\,\frac{\Delta f_{q^\uparrow/p}(x_{bj})}{M_{\text{BM}}}\,\frac{\Delta D_{h/q^\uparrow}(z_h)}{M_{\text{C}}}\,\frac{e^{-P_{hT}^2/\langle P_T^2\rangle_{\text{BM}}}}{\pi\langle P_T^2\rangle_{\text{BM}}^3}\times$$
$$\times\frac{z_h\langle k_\perp^2\rangle_{\text{BM}}^2\,\langle p_\perp^2\rangle_{\text{C}}^2}{\langle k_\perp^2\rangle\langle p_\perp^2\rangle},$$

where

$$\langle P_T^2\rangle_{\text{BM}} = \langle p_\perp^2\rangle_{\text{C}} + z_h^2\langle k_\perp^2\rangle_{\text{BM}}$$

$$\langle p_\perp^2\rangle_{\text{C}} = \frac{\langle p_\perp^2\rangle\, M_{\text{C}}^2}{\langle p_\perp^2\rangle + M_{\text{C}}^2}$$

$$\langle k_\perp^2\rangle_{\text{BM}} = \frac{\langle k_\perp^2\rangle\, M_{\text{BM}}^2}{\langle k_\perp^2\rangle + M_{\text{BM}}^2}$$

M_{C}^2 and M_{BM}^2 and all the other functional forms to be found in

JHEP 06, 007 (2019)

and

https://github.com/TianboLiu/LiuSIDIS

- In our analysis **three models** employed

  - **Default model** with Gaussian width parameters: $k_\perp^2 = 0.604\ (\mathrm{GeV/c})^2, \quad p_\perp^2 = 0.114\ (\mathrm{GeV/c})^2$
    - used in https://github.com/TianboLiu/LiuSIDIS for SoLID SIDIS asymmetry studies
    - see, e.g., preCDR https://solid.jlab.org/ and Jeopardy Updates of E12-11-007, E12-10-006, E12-11-108 at JPAC 50, https://indico.jlab.org/event/545/

  - **Barone2015 model** with $k_\perp^2 = 0.037\ (\mathrm{GeV/c})^2, \quad p_\perp^2 = 0.126 + 0.506\, z_h^2\ (\mathrm{GeV/c})^2$
    - good description of unpolarized cross-section measured by Xuefei Yan, Rev. C 95, no.3, 035209 (2017)
    - good description of HERMES data on multiplicities, Phys. Rev. D 91, no.7, 074019 (2015)

  - **Bacchetta2011 model** with $k_\perp^2 = 0.14\ (\mathrm{GeV/c})^2, \quad p_\perp^2 = 0.42\, z_h^{0.54}\,(1 - z_h)^{0.37}\ (\mathrm{GeV/c})^2$
    - also known to give good descriptions of unpolarized SIDIS cross section and HERMES multiplicity data

- **Default model** used to make unpolarized cross-section figures in our proposal

- **Three models** used in the systematic uncertainty studies

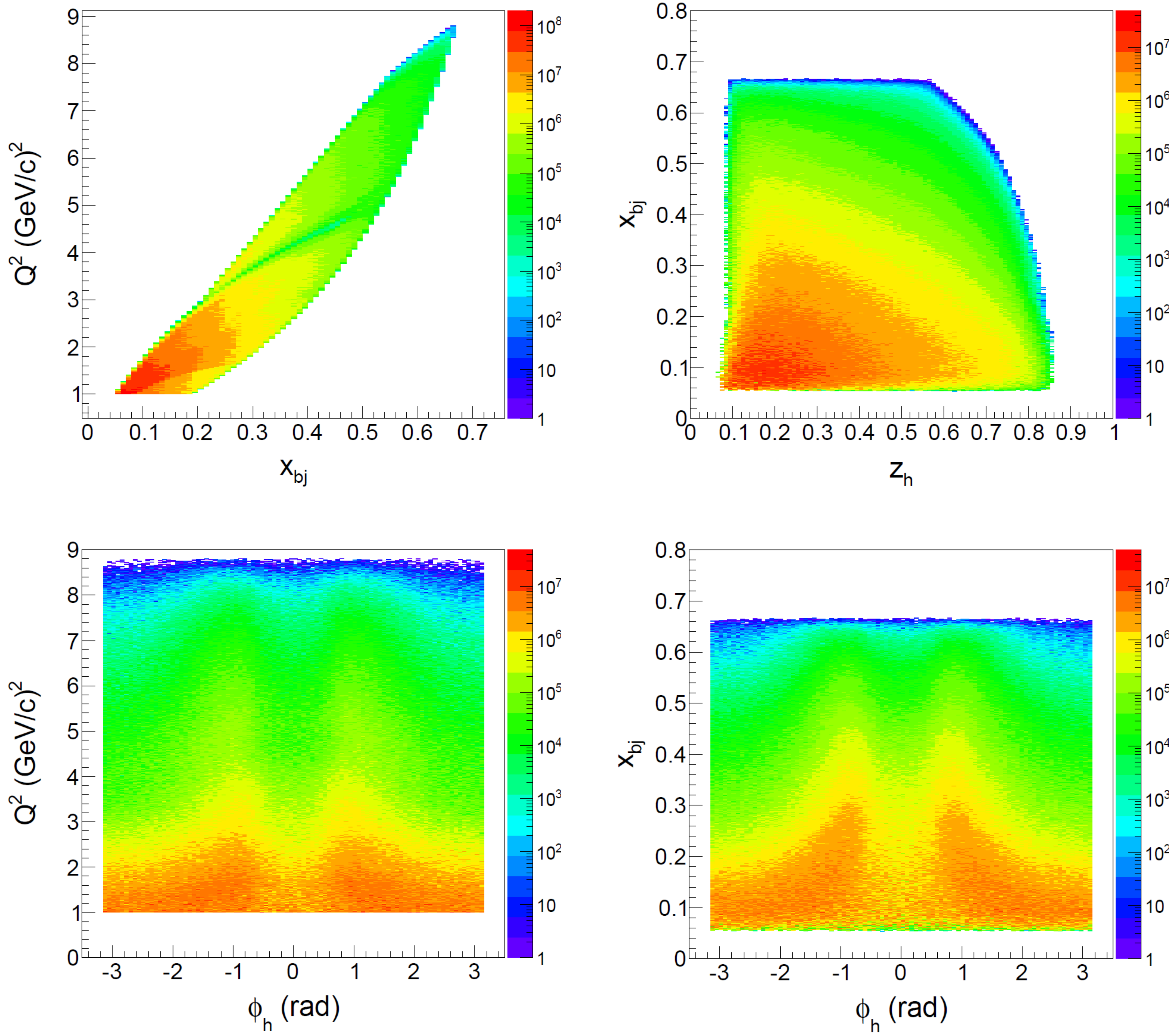
- Kinematic coverage examples of produced π^+ particles
  - *11 GeV and 8.8 GeV combined*
  - obtained after the SoLID acceptance
  - *no z_h cut implemented*

- *Phase-space correlation between* Q^2 and x_{bj} (*top-left*)

- *Phase-space correlation between* x_{bj} and z_h (*top-right*)

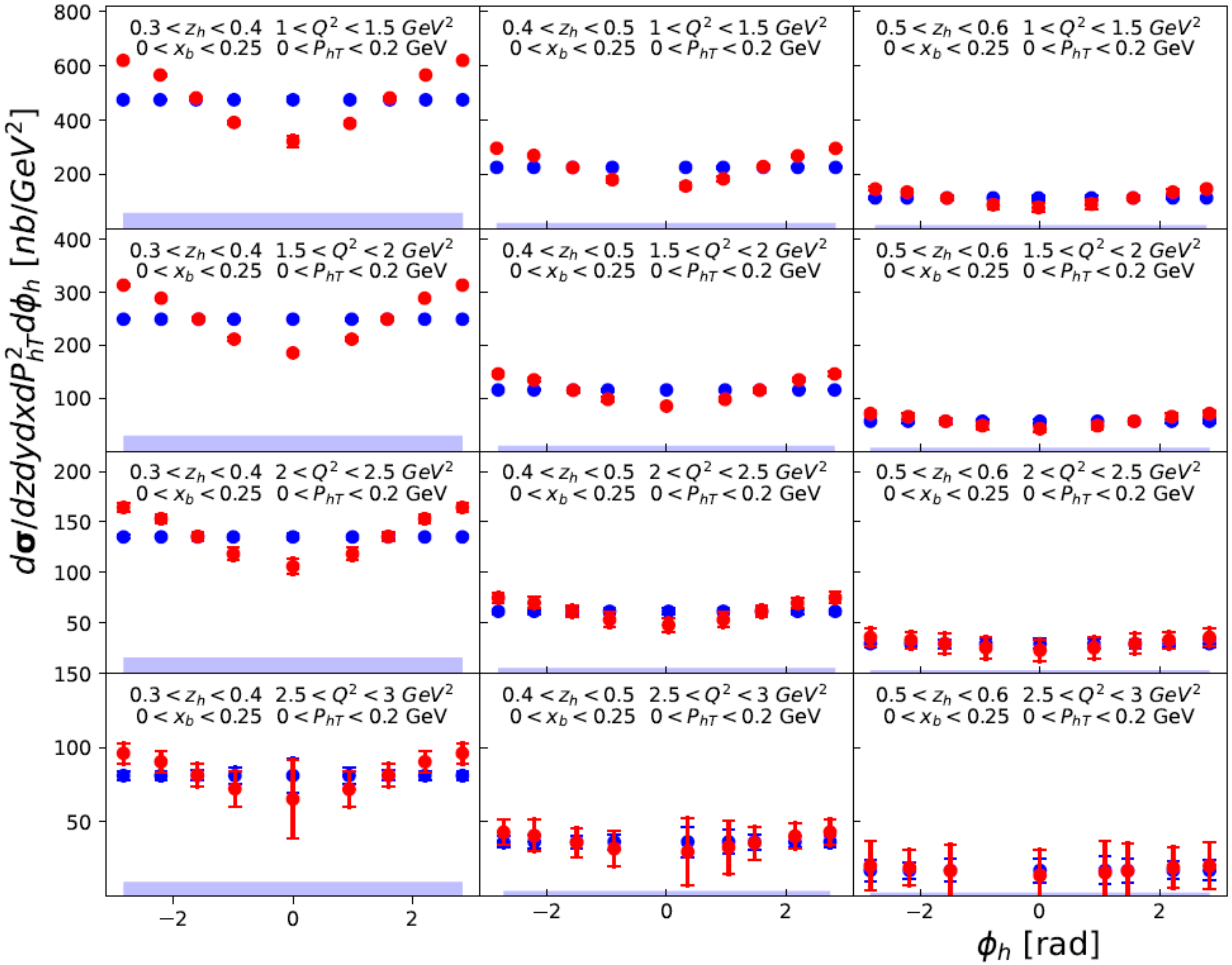
- *Phase-space correlation between* Q^2 and f_h (*bottom-left*)

- *Phase-space correlation between* x_{bj} and f_h (*bottom-right*)



First x_{bj} and first P_{hT} bin ranges

- Produced **π⁺** unpolarized cross section at **11 GeV** beam energy



Blue pseudo-data points: *cross section without azimuthal modulations*

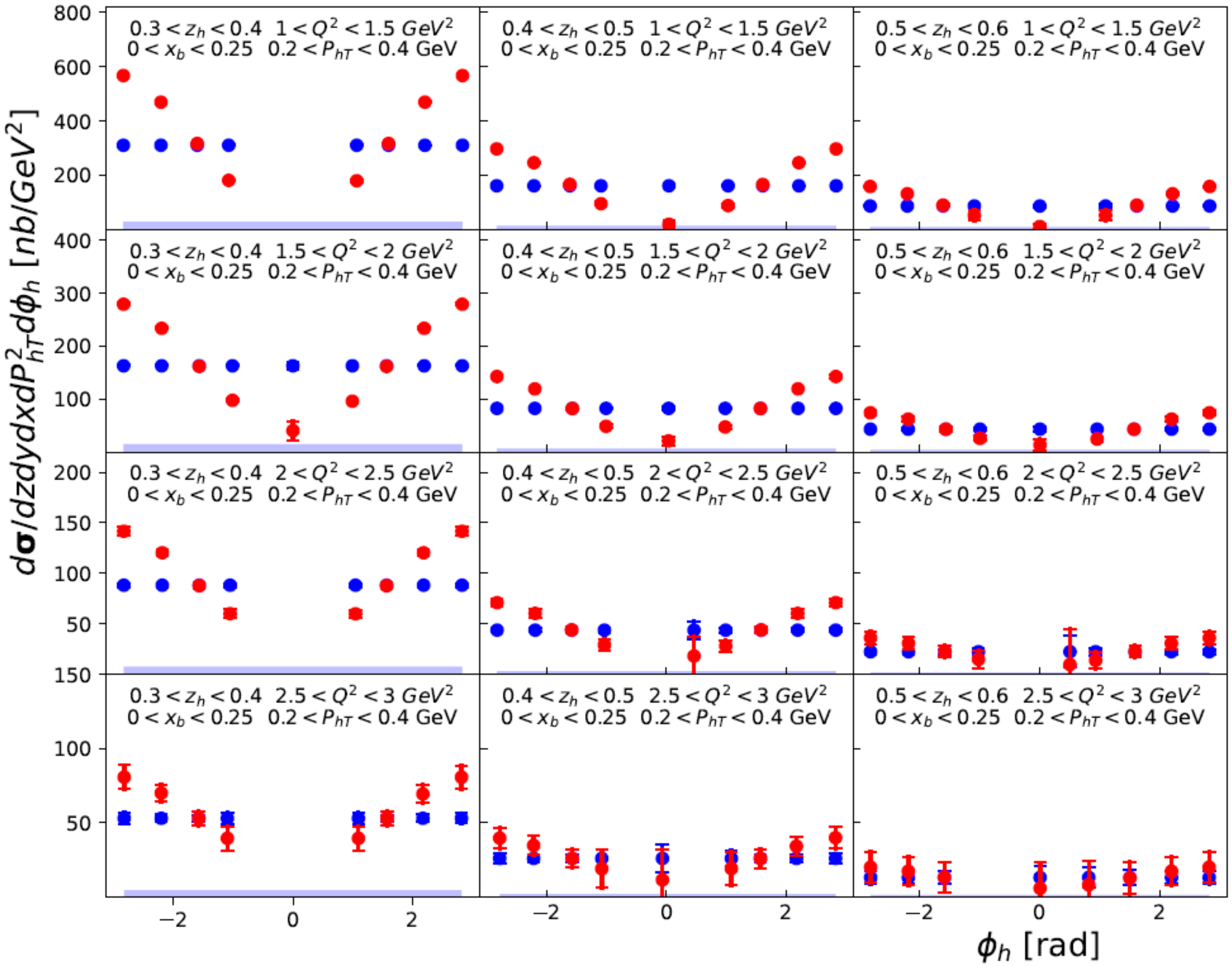
Red pseudo-data points: *cross section including azimuthal modulations*

Vertical error bars: *SoLID statistical uncertainties*

Bottom band in each plot: *SoLID total systematic uncertainties*

- Produced **π^+** unpolarized cross section at **11 GeV** beam energy

First x_{bj} and second P_{hT} bin ranges



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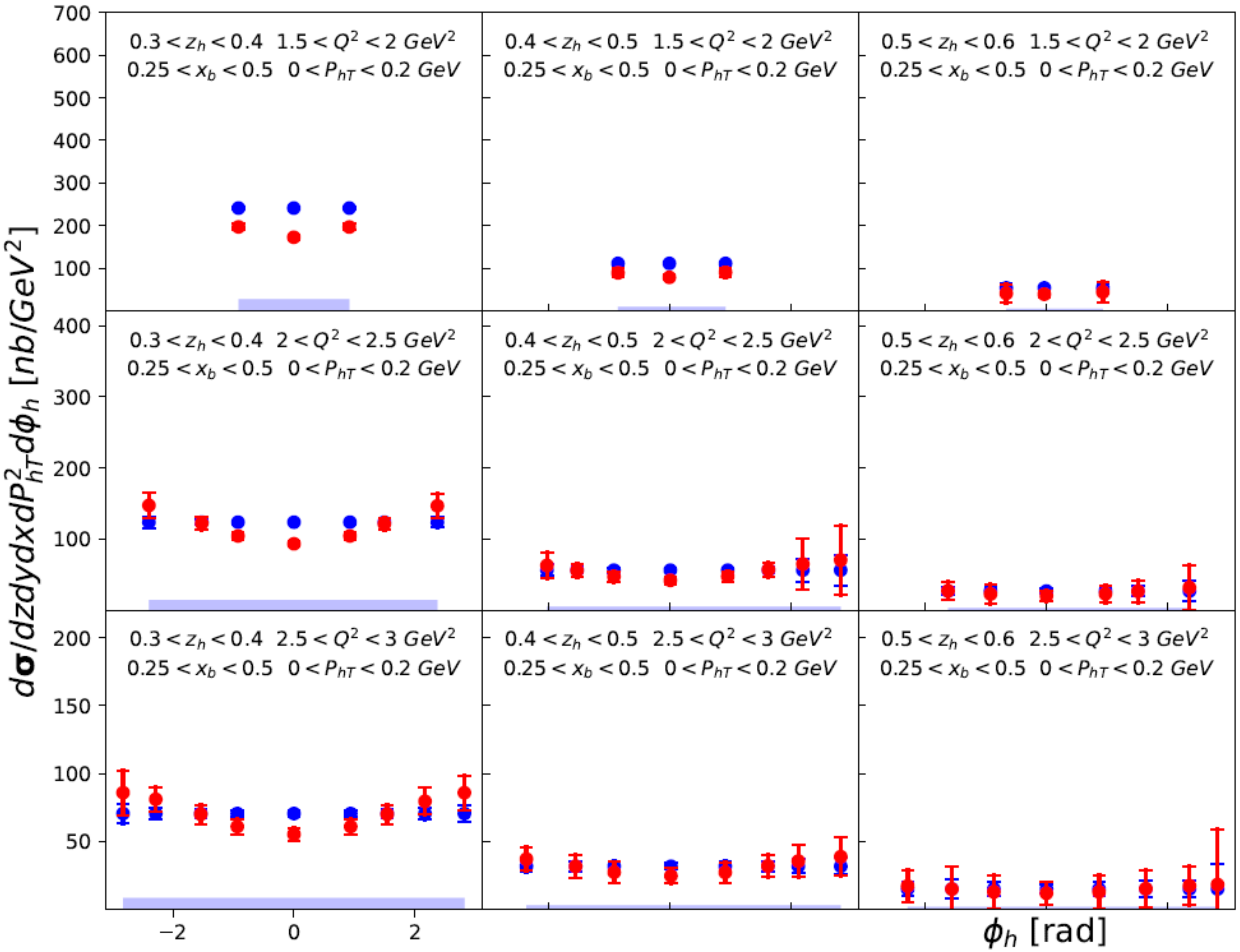

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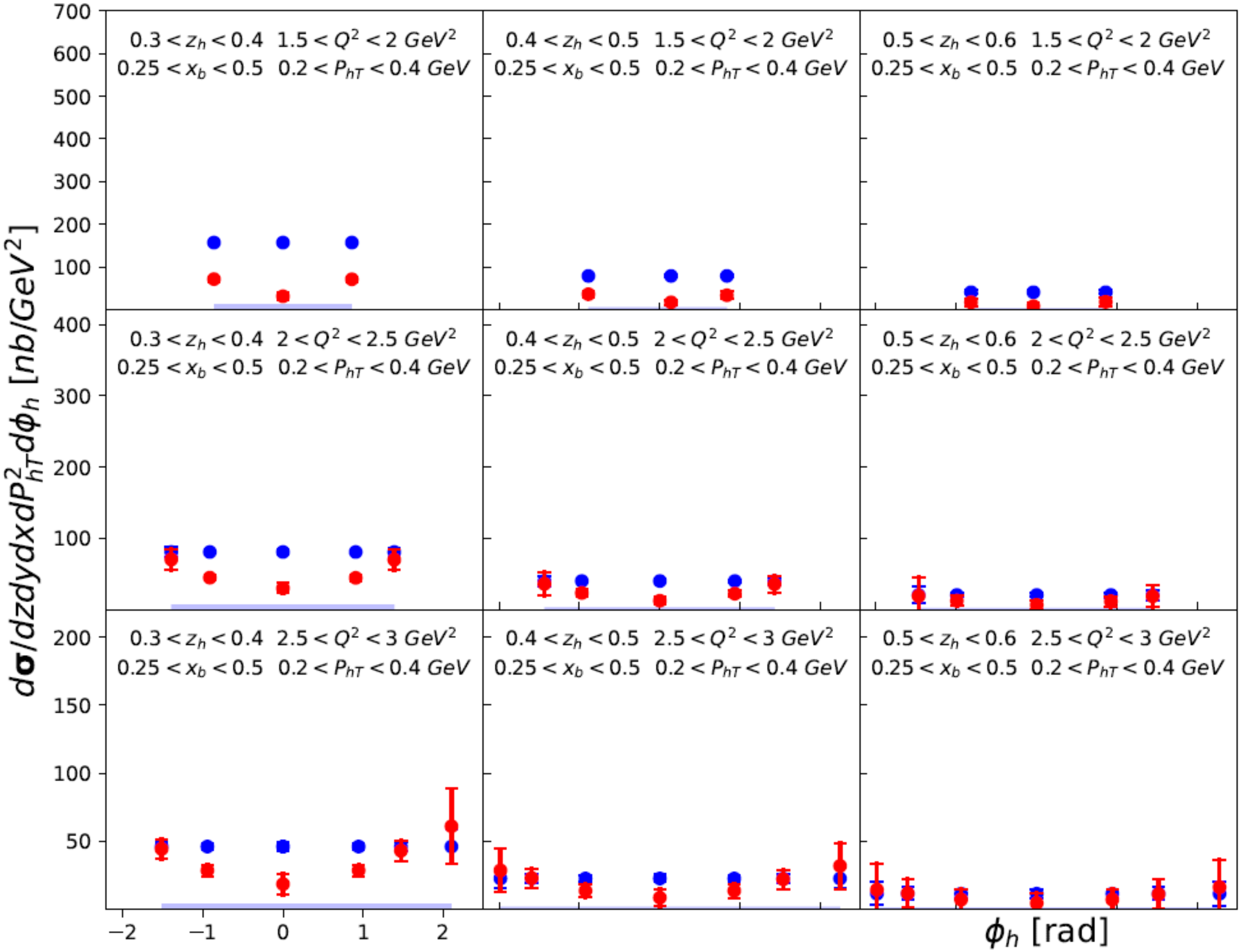

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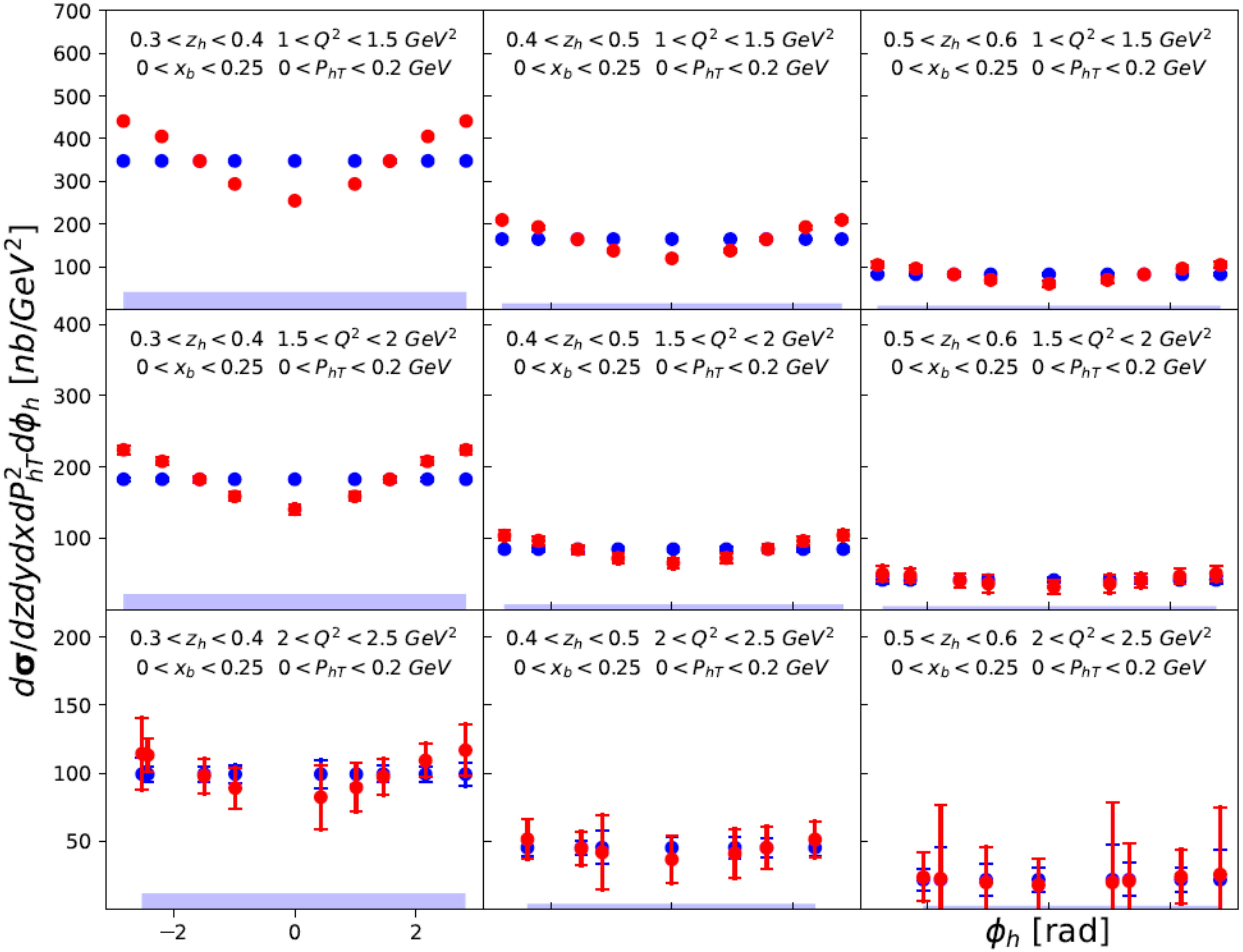
- Produced **π⁺** unpolarized cross section at **8.8 GeV** beam energy

First x_{bj} and first P_{hT} bin ranges



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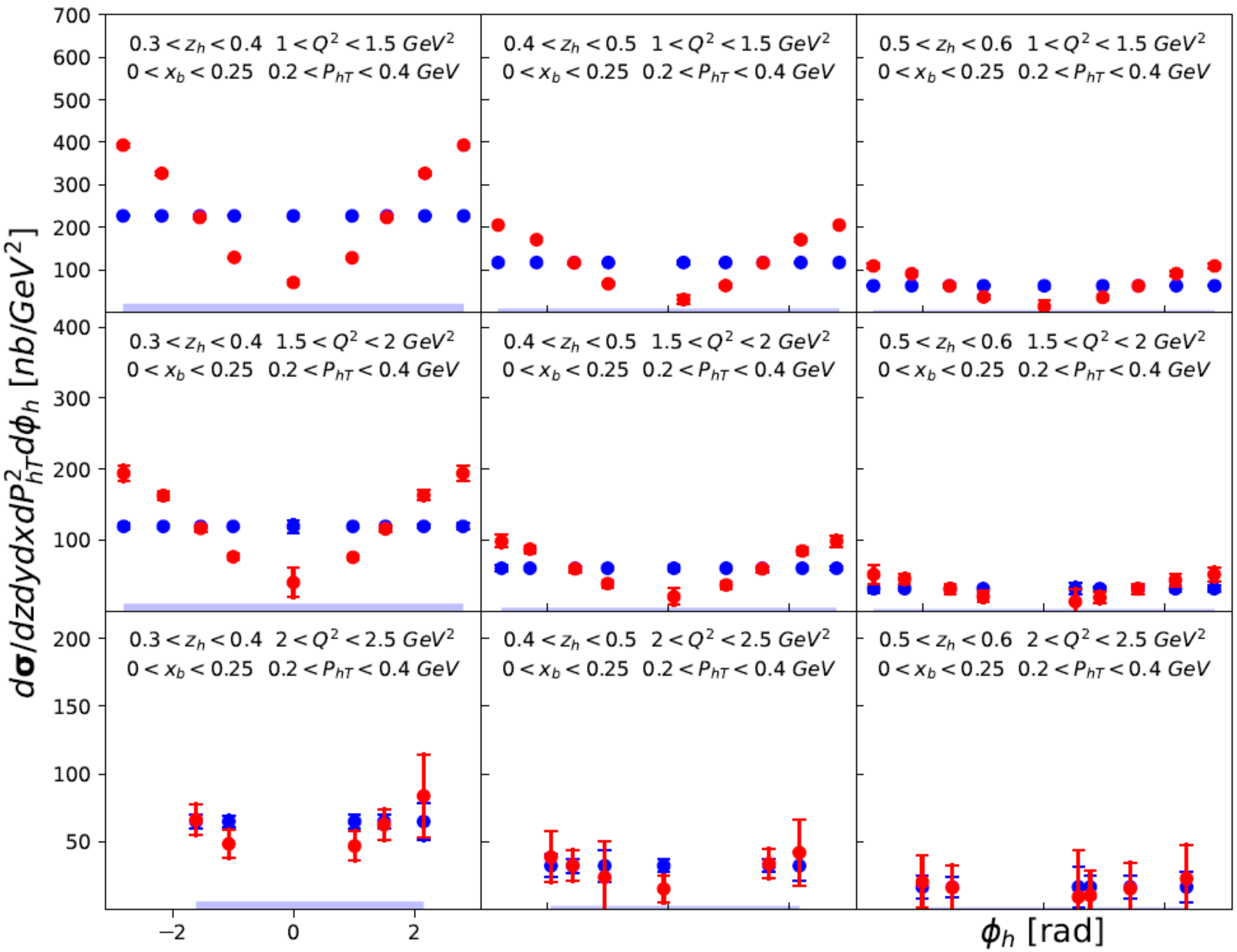
Vertical error bars: *SoLID statistical uncertainties*

Bottom band in each plot: *SoLID total systematic uncertainties*

- Produced π^+ unpolarized cross section at **8.8 GeV** beam energy

First x_{bj} and second P_{hT} bin ranges



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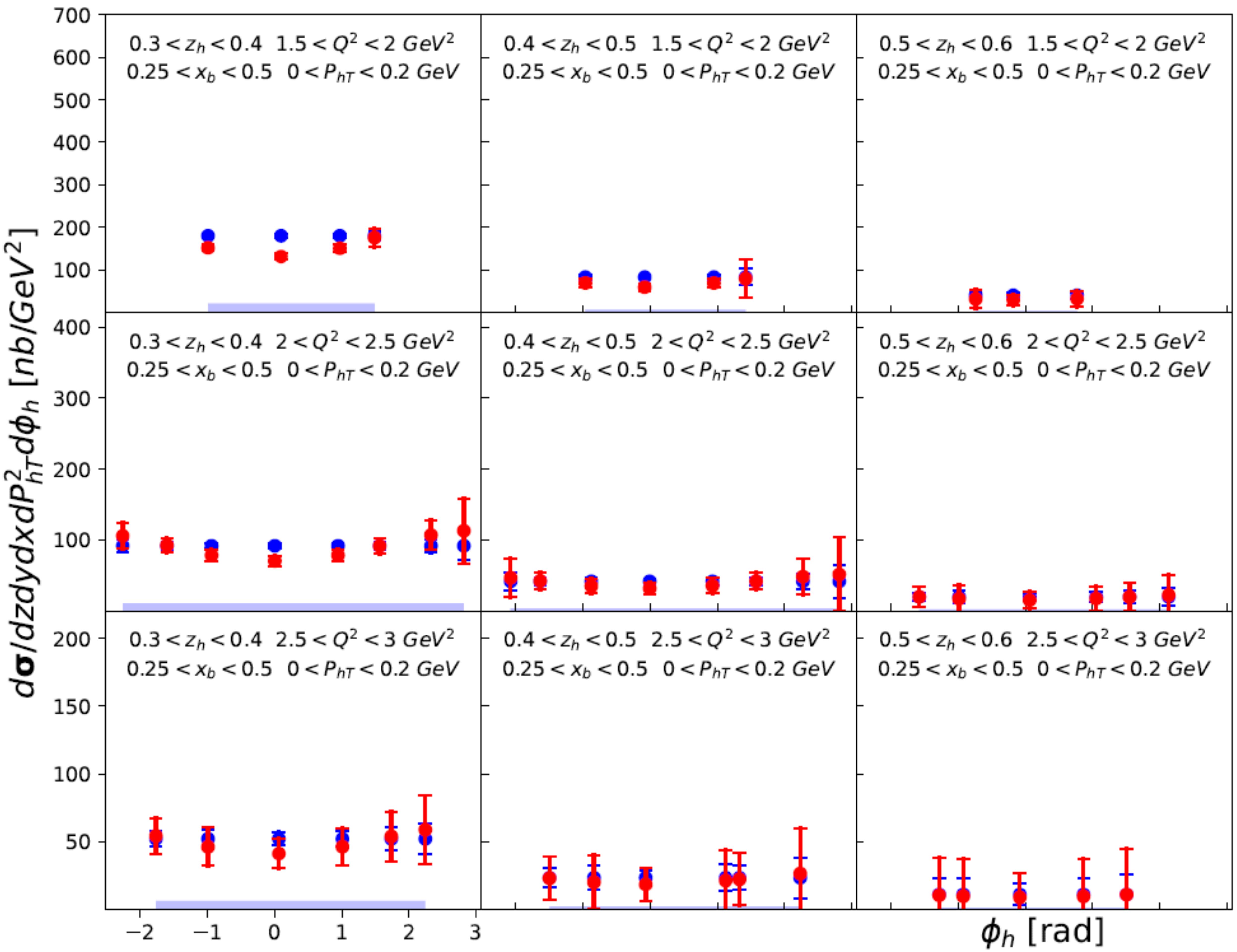
Red pseudo-data points: *cross section including azimuthal modulations*

Vertical error bars: *SoLID statistical uncertainties*

Bottom band in each plot: *SoLID total systematic uncertainties*

- Produced π^+ unpolarized cross section at **8.8 GeV** beam energy

Second x_{bj} and first P_{hT} bin ranges



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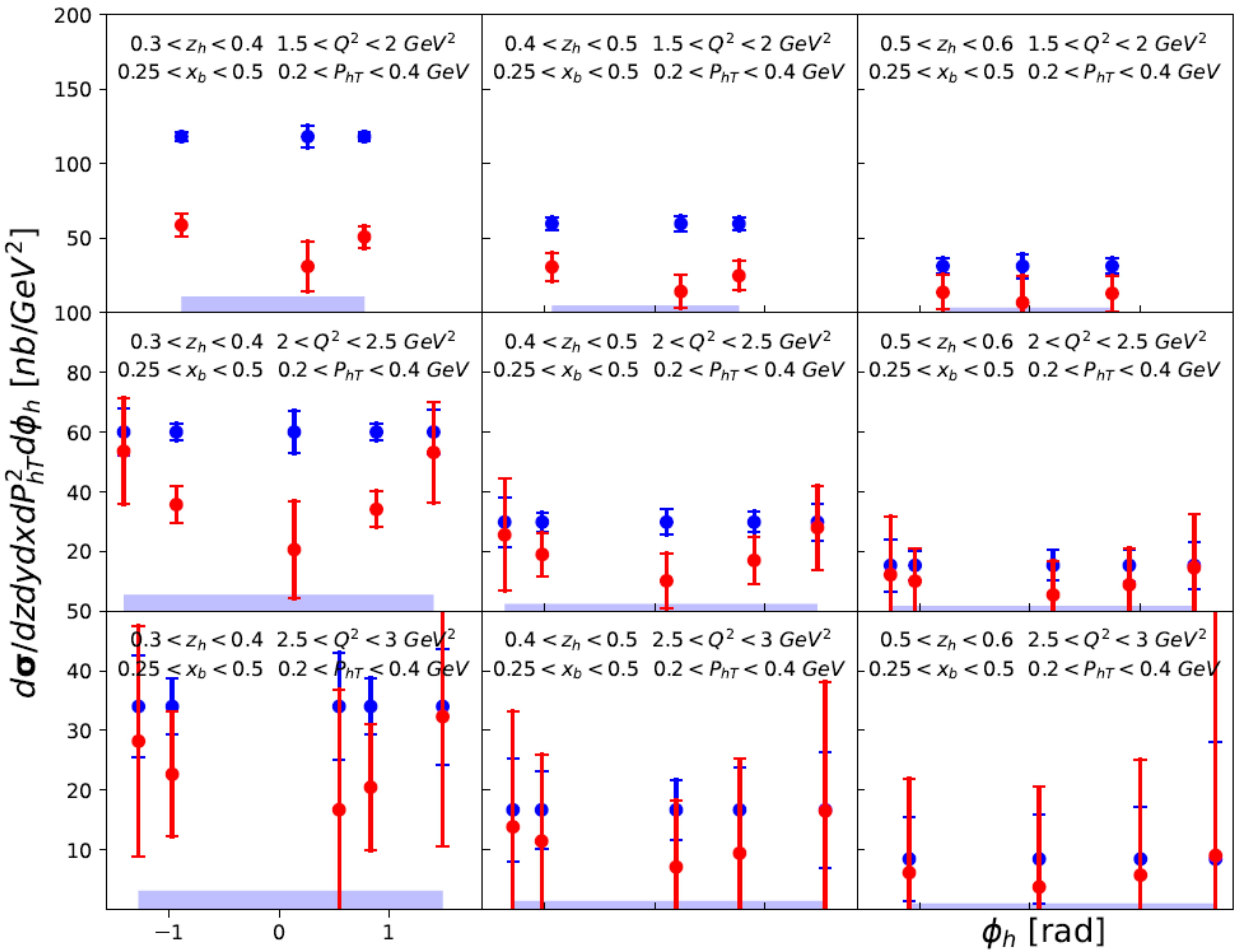
- Produced π^+ unpolarized cross section at **8.8 GeV** beam energy

Second x_{bj} and second P_{hT} bin ranges



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Vertical error bars: *SoLID statistical uncertainties*

Bottom band in each plot: *SoLID total systematic uncertainties*

- Systematic uncertainties of the $\mathcal{F}_{UU,A}$ cross section estimated from

  - *SIDIS model dependence*: *using the models discussed on slide 13th*

  - *Pion and electron identification*:
    - *using the SoLID pion detection efficiency and kaon rejection factor*
    - *using the CLAS12 inclusive electron cross-section data and similarity of the SoLD and CLASS12 electron detection numbers*

  - *Radiative corrections*: *using the recently developed MC event generator from* *Phys. Commun. 287, 108702 (2023), [arXiv:2210.03785 [hep-ph]]*

  - *Acceptance correction*: *using variations among the statistical uncertainties from the models discussed on slide 13th (obtained before and after SoLID acceptance)*

  - *Luminosity determination*: *using the beam current, target size and density*

  - *Other sources*: *using the lowest polar angle value (for acceptance) as well as W and W ' cut variations*

Systematic uncertainty budget for the unpolarized cross section

Without the azimuthal modulations included in the cross section

SIDIS model dependence

Pion and electron identification

Radiative corrections

Acceptance correction

Luminosity determination

Other sources

Total systematic uncertainty:

$0 < x_{bj} < 0.25$ and $0 < P_{hT} < 0.2$ GeV/c: ~ 9 - 13%

$0 < x_{bj} < 0.25$ and $0.2 < P_{hT} < 0.4$ GeV/c: ~ 8 - 12%

$0.25 < x_{bj} < 0.5$ and $0 < P_{hT} < 0.2$ GeV/c: ~ 9 - 13%

$0.25 < x_{bj} < 0.5$ and $0.2 < P_{hT} < 0.4$ GeV/c: ~ 8 - 12%

- In our run group proposal, we show unpolarized SoLID SIDIS cross-section results
  - for π^+ particles at 11/8.8 GeV as well as π^- particles at 11 GeV beam energies
  - based on transversely/longitudinally polarized SoLID ^{3}He targets
- Cross-section pseudo-data obtained in 5-dimensional binning of $(x_{bj}, z_h, Q^2, P_{hT}, \phi_h)$
- Cross-section pseudo-data include
  - central points from theory calculations, plus SoLID statistical and systematic uncertainties
- Systematic uncertainties estimated for π^+ particles
  - applied to both π^+/π^- cross sections
- Cross-section without azimuthal modulations include stat. and syst. uncertainties
- Cross-section with azimuthal modulations include only stat. uncertainties
  - more studies needed here for estimating syst. uncertainties
  - perhaps additional asymmetry studies needed to see if $\cos(\phi_h)$ and $\cos(2\phi_h)$ terms are separable within all experimental uncertainties

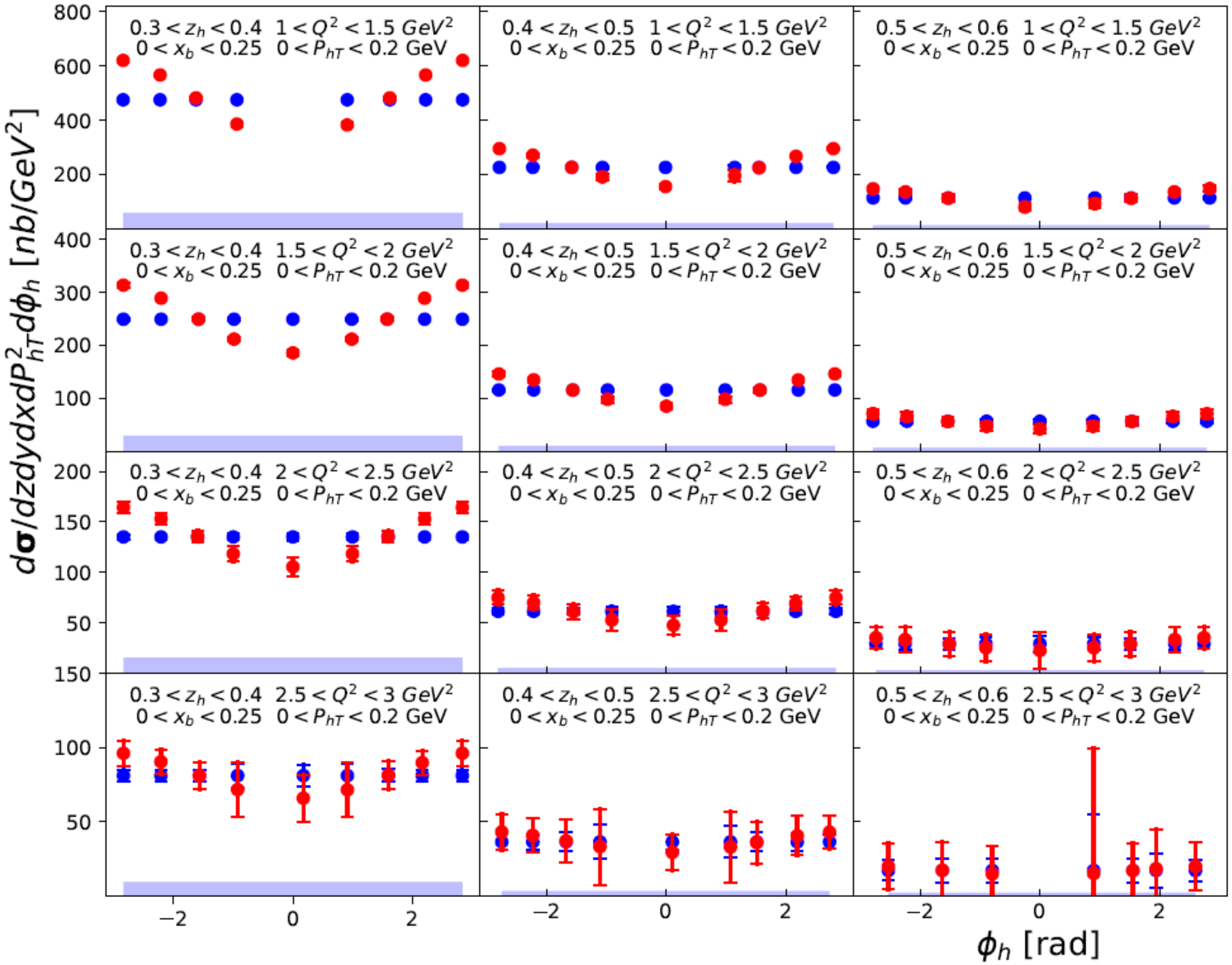
Thank You !

Acknowledgements: Haiyan Gao, Zhiwen Zhao, Jian-Ping Chen, Tianbo Liu, and the entire SoLID collaboration

Backups

- Produced π^- unpolarized cross section at **11 GeV** beam energy

First x_{bj} and first P_{hT} bin ranges



Blue pseudo-data points: *cross section without azimuthal modulations*

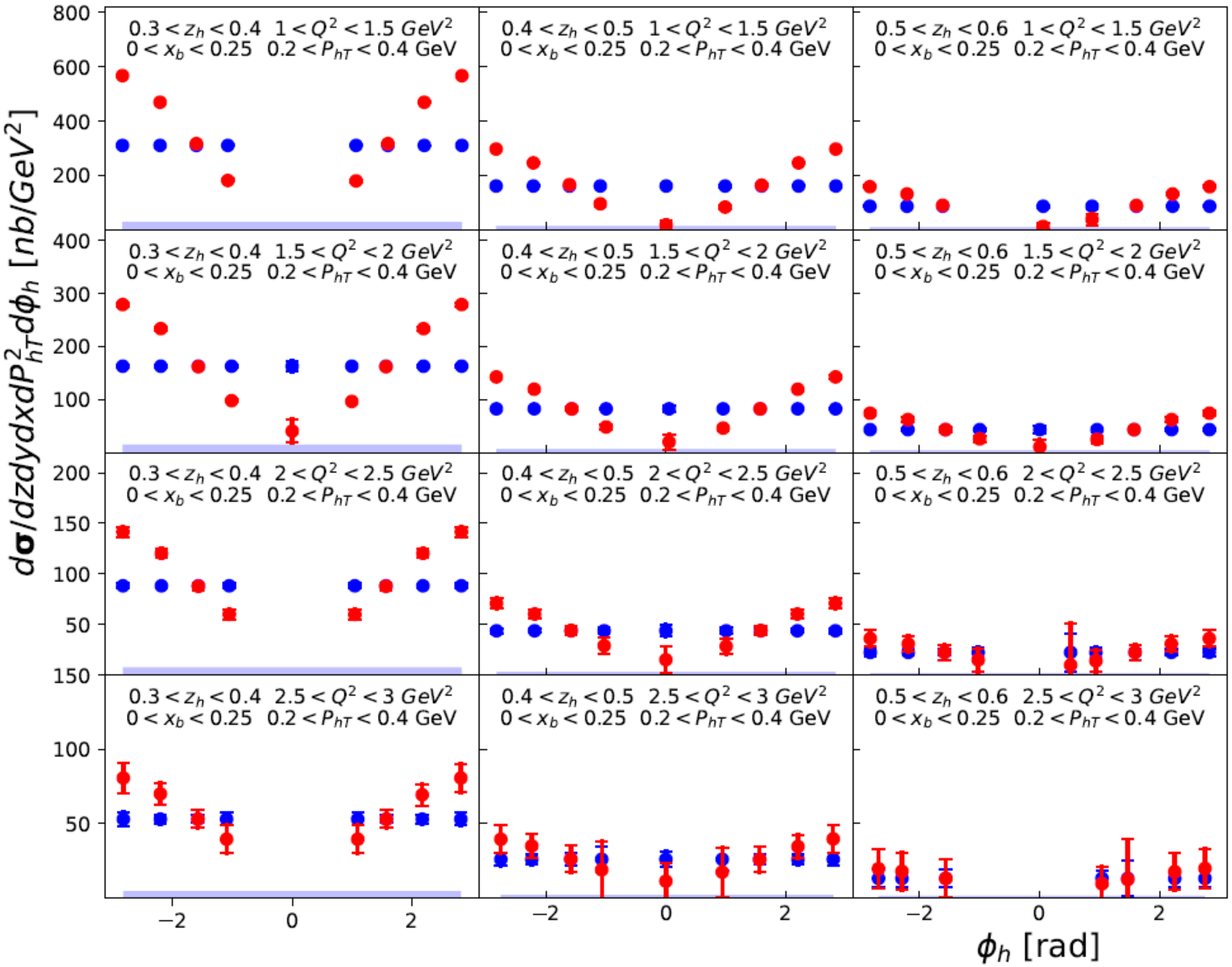
Red pseudo-data points: *cross section including azimuthal modulations*

Vertical error bars: *SoLID statistical uncertainties*

Bottom band in each plot: *SoLID total systematic uncertainties*

- Produced π^- unpolarized cross section at **11 GeV** beam energy

First x_{bj} and second P_{hT} bin ranges



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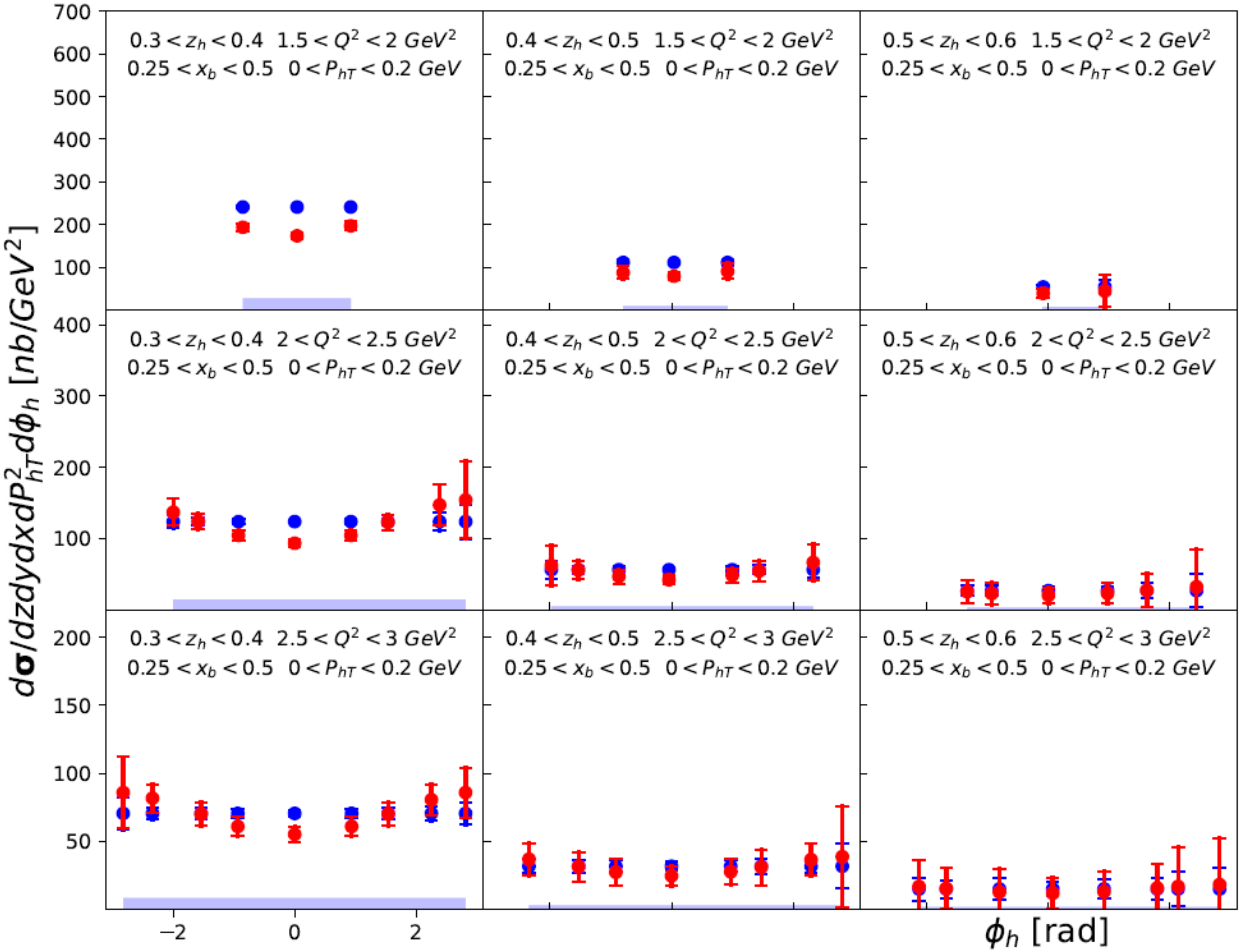
Vertical error bars: *SoLID statistical uncertainties*

Bottom band in each plot: *SoLID total systematic uncertainties*

- Produced π^- unpolarized cross section at **11 GeV** beam energy

Second x_{bj} and first P_{hT} bin ranges



Blue pseudo-data points: *cross section without azimuthal modulations*

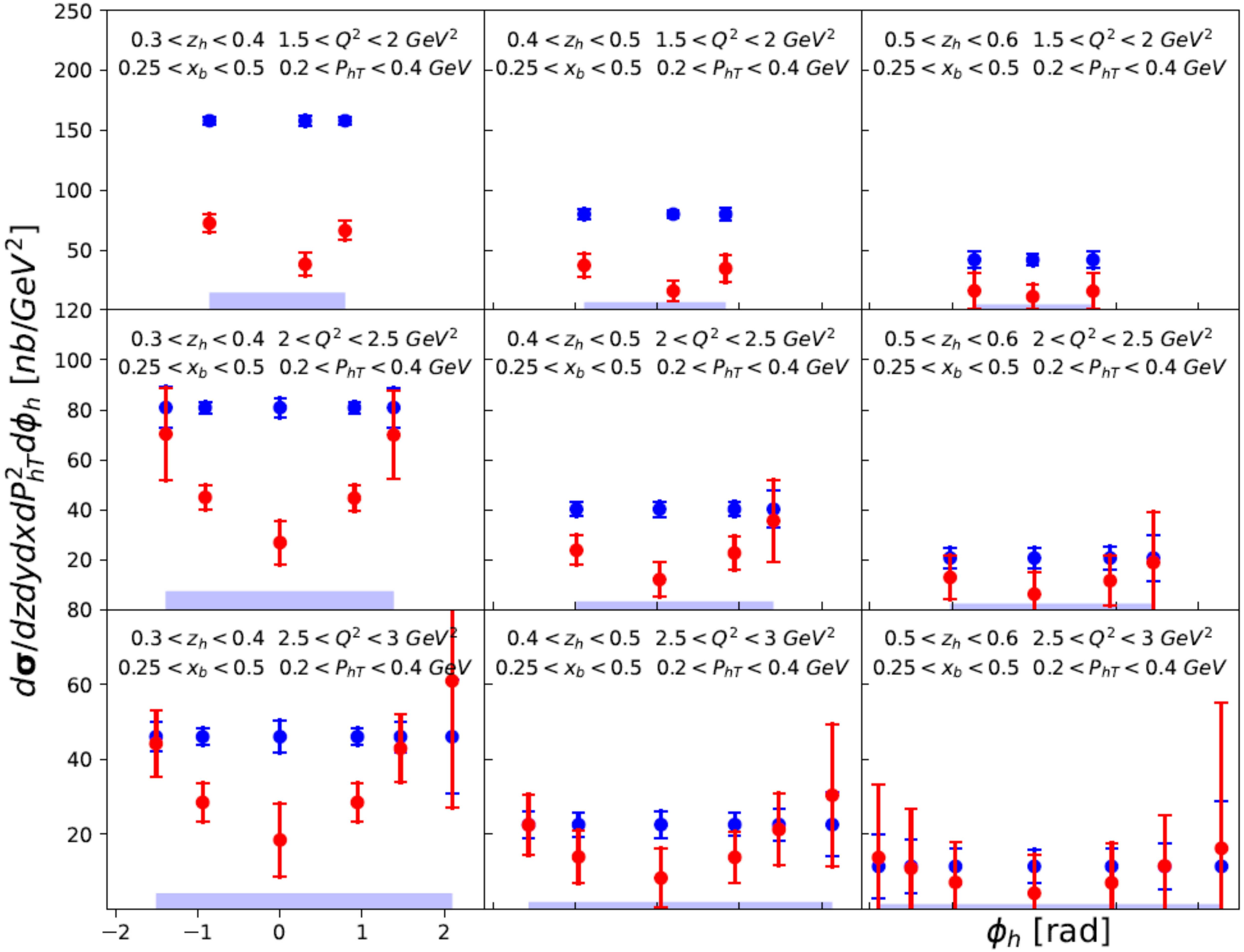
Red pseudo-data points: *cross section including azimuthal modulations*

Vertical error bars: *SoLID statistical uncertainties*

Bottom band in each plot: *SoLID total systematic uncertainties*

➢ Produced **π⁻** unpolarized cross section at **11 GeV** beam energy

Second x_{bj} and second P_{hT} bin ranges



Blue pseudo-data points: *cross section without azimuthal modulations*

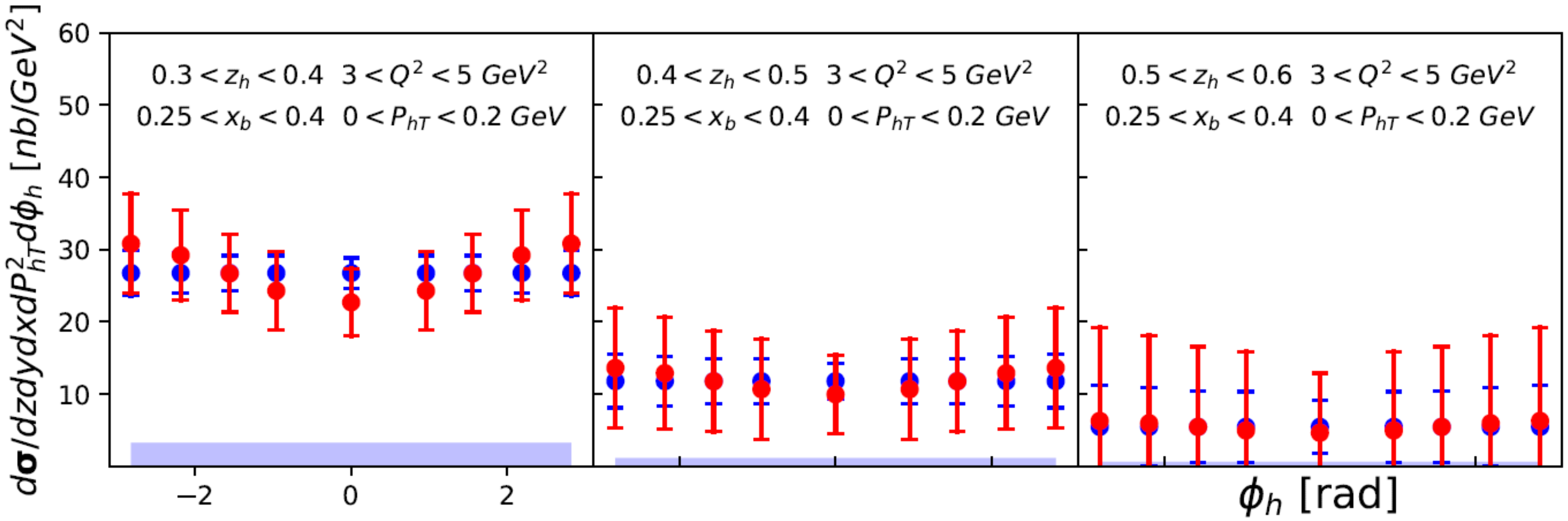
Red pseudo-data points: *cross section including azimuthal modulations*

Vertical error bars: *SoLID statistical uncertainties*

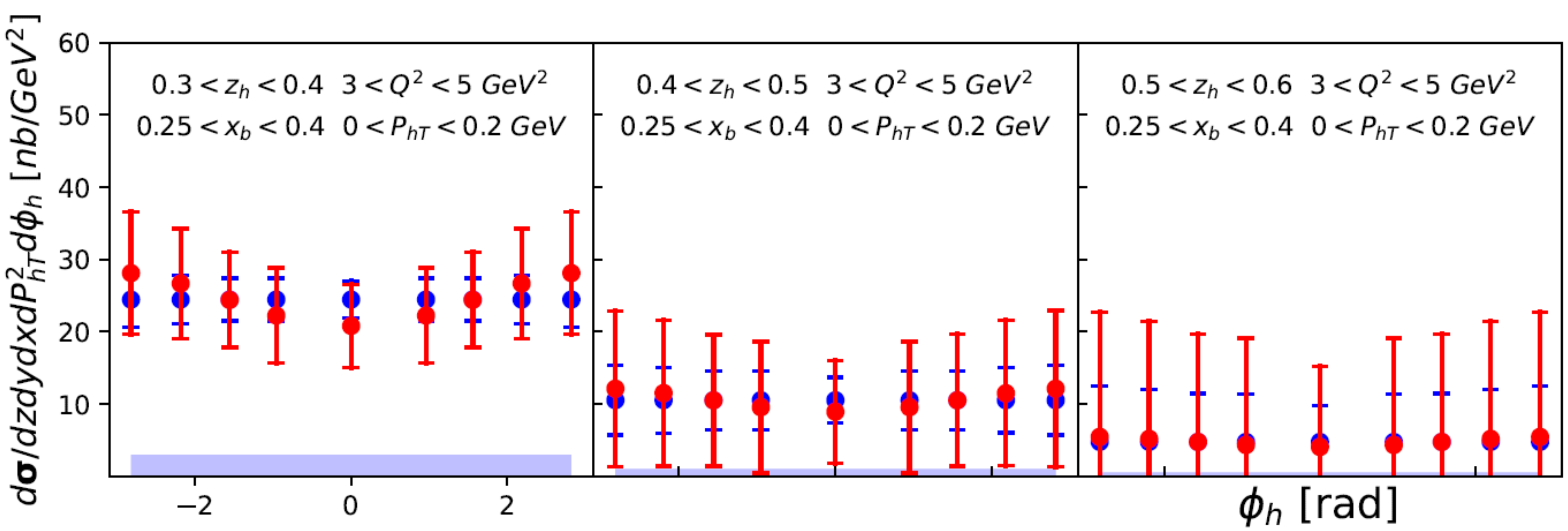
Bottom band in each plot: *SoLID total systematic uncertainties*

Third x_{bj}, first P_{hT} and higher Q^2 bin ranges

- Produced **π⁺** unpolarized cross section at **11 GeV** beam energy



- Produced **π⁻** unpolarized cross section at **11 GeV** beam energy



Blue pseudo-data points: *cross section without azimuthal modulations*

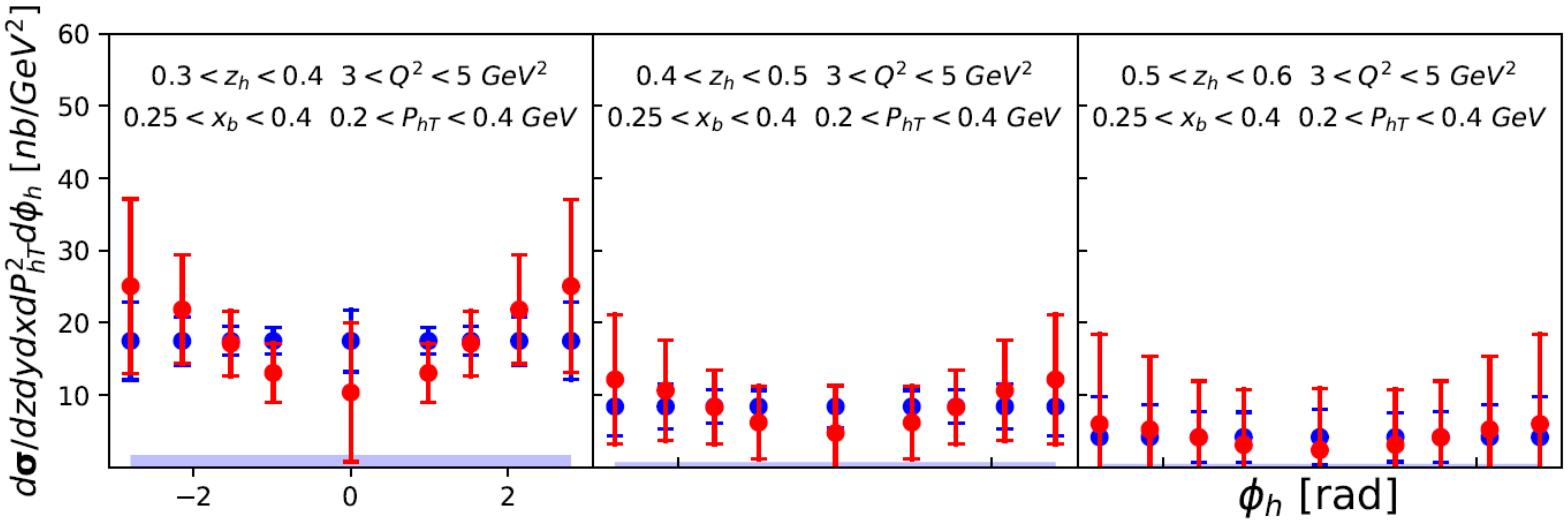
Red pseudo-data points: *cross section including azimuthal modulations*

Vertical error bars: *SoLID statistical uncertainties*

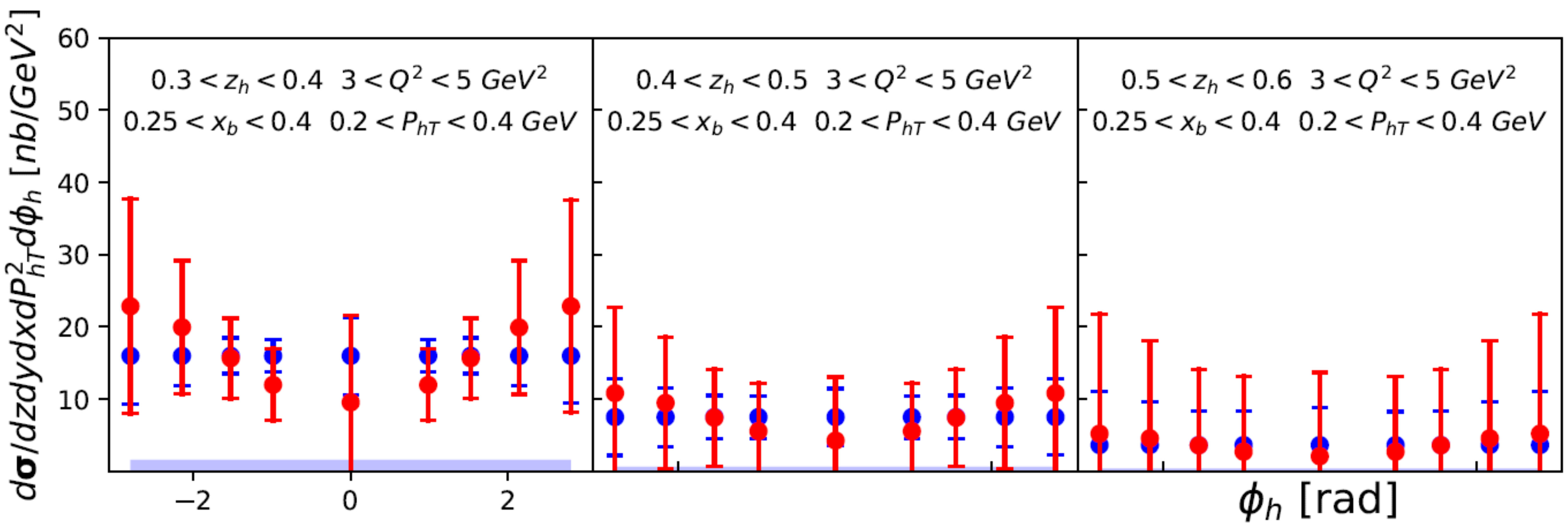
Bottom band in each plot: *SoLID total systematic uncertainties*

Third x_{bj}, second P_{hT} and higher Q^2 bin ranges

- Produced **π⁺** unpolarized cross section at **11 GeV** beam energy



- Produced **π⁻** unpolarized cross section at **11 GeV** beam energy



Blue pseudo-data points: *cross section without azimuthal modulations*

Red pseudo-data points: *cross section including azimuthal modulations*

Vertical error bars: *SoLID statistical uncertainties*

Bottom band in each plot: *SoLID total systematic uncertainties*