Tests of low-energy QCD via light pseudoscalar mesons

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Science at the Luminosity Frontier:
Jefferson Lab at 22 GeV Workshop

Jefferson Laboratory, Virginia, January 25, 2023

Based on *Phys. Rept. 945 (2022), 1-105,*
L. Gan, B. Kubis, E.P., S. Tulin

Outline

1. Introduction and Motivation

2. $\eta \rightarrow 3\pi$: light quark mass extraction

3. Theory Inputs for hadronic light-by-light for g-2 of the muon

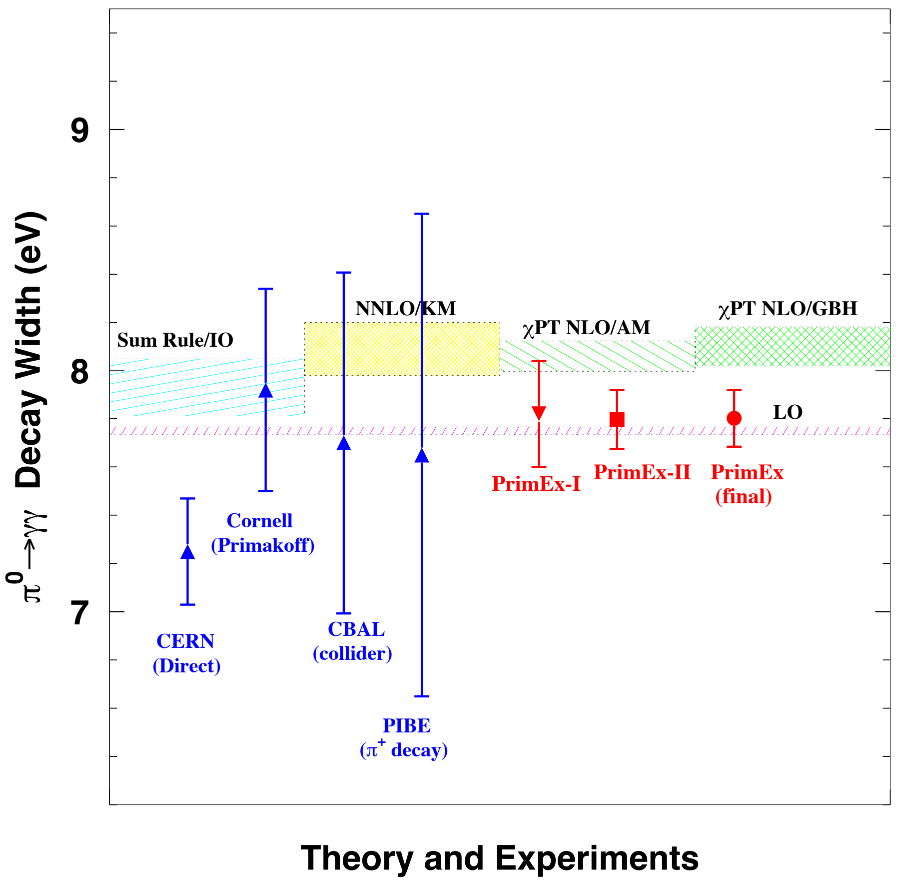
4. Conclusion and Outlook

1. Introduction and Motivation

1.1 Why is it interesting to study π^0, η and η' physics?

- π^0 is the pseudo-Goldstone boson of chiral perturbation theory
- It is one of the most fundamental degree of freedom
- There are still some puzzles about this particle:

Gan, Kubis, E. P., Tulin'22



From L.Gan

1.1 Why is it interesting to study η and η' physics?

- Quantum numbers $I^G J^{PC} = 0^+ 0^{-+}$
  - *C*, *P* eigenstates, all additive quantum numbers are zero
  - flavour-conserving laboratory for symmetry tests

- η: pseudo-Goldstone boson, $\boxed{M_\eta = 547.862(17)\ \mathrm{MeV}}$, Γ_η = 1.31 keV

  All decay modes forbidden at leading order by *symmetries* (C, P, angular momentum, isospin/G-parity. . .)

- η': not a Goldstone boson due to $U(1)_A$ anomaly $\boxed{M_{\eta'} = 957.78(6)\ \mathrm{MeV}}$
  $\Gamma_{\eta'}$ = 196 keV

- Theoretical methods:
  - (large-N_c) chiral perturbation theory, RChPT
  - dispersion relations to resum final state interactions
  - Vector-meson dominance

1.1 Why is it interesting to study π^0, η and η' physics?

- In the study of π^0, η and η' physics, large amount of data have been collected:

  ⇨ *CBall, WASA, KLOE & KLOEII, BESIII, A2@MAMI, CLAS, GlueX*

  ⇨ More to come: *JEF, REDTOP (Elam et al'22), LHCb?*

  22 GeV JLab upgrade: very high precision could be achieved : radiative decay width and the transition form factor of π^0 (at 0.95% precision), η(2%) and η' (3.4%) via Primakoff effect.

- Unique opportunity:
  - Test chiral dynamics at low energy
  - Extract fundamental parameters of the Standard Model: ex: light quark masses
  - Study of fundamental symmetries: P & CP and C & CP violation
  - Looking for beyond Standard Model Physics ⇨ Dark Sector

  ⇨ See talk by *M. Pospelov*

Rich physics program at η,η' factories

From S.Tulin

Standard Model highlights
- Theory input for light-by-light scattering for $(g\text{-}2)_\mu$
- Extraction of light quark masses
- QCD scalar dynamics

Fundamental symmetry tests
- P,CP violation
- C,CP violation

[Kobzarev & Okun (1964), Prentki & Veltman (1965), Lee (1965), Lee & Wolfenstein (1965), Bernstein et al (1965)]

Dark sectors (MeV—GeV)
- Vector bosons
- Scalars
- Pseudoscalars (ALPs)

(Plus other channels that have not been searched for to date)

Channel	Expt. branching ratio	Discussion
$\eta \to 2\gamma$	39.41(20)%	chiral anomaly, η–η' mixing
$\eta \to 3\pi^0$	32.68(23)%	$m_u - m_d$
$\eta \to \pi^0\gamma\gamma$	$2.56(22) \times 10^{-4}$	χPT at $\mathcal{O}(p^6)$, leptophobic B boson, light Higgs scalars
$\eta \to \pi^0\pi^0\gamma\gamma$	$< 1.2 \times 10^{-3}$	χPT, axion-like particles (ALPs)
$\eta \to 4\gamma$	$< 2.8 \times 10^{-4}$	$< 10^{-11}$[52]
$\eta \to \pi^+\pi^-\pi^0$	22.92(28)%	$m_u - m_d$, C/CP violation, light Higgs scalars
$\eta \to \pi^+\pi^-\gamma$	4.22(8)%	chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta \to \pi^+\pi^-\gamma\gamma$	$< 2.1 \times 10^{-3}$	χPT, ALPs
$\eta \to e^+e^-\gamma$	$6.9(4) \times 10^{-3}$	theory input for $(g-2)_\mu$, dark photon, protophobic X boson
$\eta \to \mu^+\mu^-\gamma$	$3.1(4) \times 10^{-4}$	theory input for $(g-2)_\mu$, dark photon
$\eta \to e^+e^-$	$< 7 \times 10^{-7}$	theory input for $(g-2)_\mu$, BSM weak decays
$\eta \to \mu^+\mu^-$	$5.8(8) \times 10^{-6}$	theory input for $(g-2)_\mu$, BSM weak decays, P/CP violation
$\eta \to \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
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$\eta \to \pi^+\pi^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \to e^+e^-e^+e^-$	$2.40(22) \times 10^{-5}$	theory input for $(g-2)_\mu$
$\eta \to e^+e^-\mu^+\mu^-$	$< 1.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \to \mu^+\mu^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \to \pi^+\pi^-\pi^0\gamma$	$< 5 \times 10^{-4}$	direct emission only
$\eta \to \pi^\pm e^\mp \nu_e$	$< 1.7 \times 10^{-4}$	second-class current
$\eta \to \pi^+\pi^-$	$< 4.4 \times 10^{-6}$	P/CP violation
$\eta \to 2\pi^0$	$< 3.5 \times 10^{-4}$	P/CP violation
$\eta \to 4\pi^0$	$< 6.9 \times 10^{-7}$	P/CP violation

Gan, Kubis, E. P., Tulin'22

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Gan, Kubis, E. P., Tulin’22

2. $\eta \to 3\pi$: light quark mass extraction

In collaboration with G. Colangelo, S. Lanz and H. Leutwyler (ITP-Bern)

Phys. Rev. Lett. 118 (2017) no.2, 022001

Eur.Phys.J. C78 (2018) no.11, 947

2.1 Decays of η

- η decay from PDG:

$$M_\eta = 547.862(17)\ \text{MeV}$$

η DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
	Neutral modes		
Γ_1	neutral modes	(72.12 ± 0.34) %	S=1.2
Γ_2	2γ	(39.41 ± 0.20) %	S=1.1
Γ_3	$3\pi^0$	(32.68 ± 0.23) %	S=1.1
	Charged modes		
Γ_8	charged modes	(28.10 ± 0.34) %	S=1.2
Γ_9	$\pi^+\pi^-\pi^0$	(22.92 ± 0.28) %	S=1.2
Γ_{10}	$\pi^+\pi^-\gamma$	$(\ 4.22 \pm 0.08)$ %	S=1.1

2.1 Why is it interesting to study $\eta \to 3\pi$?

- Decay forbidden by isospin symmetry $\eta(I^G = 0^+) \to 3\pi(I^G = 1^-)$

$$\boxed{A = (m_u - m_d)\, A_1 + \alpha_{em} A_2}$$

- α_{em} effects are small — *Sutherland'66, Bell & Sutherland'68; Baur, Kambor, Wyler'96, Ditsche, Kubis, Meissner'09*

- Decay rate measures the size of isospin breaking ($m_u - m_d$) in the SM:

$$L_{QCD} \to \boxed{L_{IB} = -\frac{m_u - m_d}{2}\left(\bar{u}u - \bar{d}d\right)}$$

Unique access to ($m_u - m_d$)

2.2 Quark mass ratio

- In the following, extraction of Q from $\eta \to \pi^+ \pi^- \pi^0$

$$\Gamma_{\eta\to\pi^+\pi^-\pi^0} = \frac{1}{Q^4}\frac{M_K^4}{M_\pi^4}\frac{\left(M_K^2 - M_\pi^2\right)^2}{6912\pi^3 F_\pi^4 M_\eta^3}\int_{s_{\min}}^{s_{\max}} ds \int_{u_-(s)}^{u_+(s)} du \, \left|M(s,t,u)\right|^2$$

Determined from experiment

Determined from:
- Dispersive calculation
- ChPT

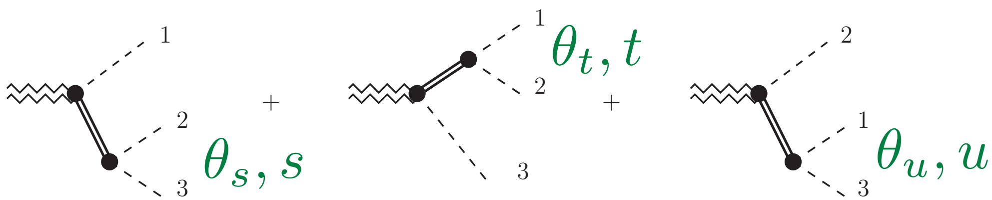
Fit to Dalitz distr.

$$\boxed{\left[Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}\right]} \qquad \left[\hat{m} \equiv \frac{m_d + m_u}{2}\right]$$

- Aim: Compute M(s,t,u) with the *best accuracy*

2.3 Method

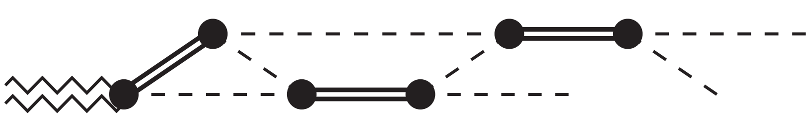
- Decompose amplitide in partial waves:



- Usual assumption: 3 BWs (ρ^+, ρ^-, ρ^0) + background term

⇒ Improve to include final states interactions + restore unitarity

- Use a *Khuri-Treiman* approach or *dispersive* approach
  Restore 3 body unitarity and take into account the final state interactions in a systematic way



2.3 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$\boxed{M(s,t,u) = M_0(s) + \left(s-u\right) M_1(t) + \left(s-t\right) M_1(u) + M_2(t) + M_2(u) - \frac{2}{3} M_2(s)}$$

Fuchs, Sazdjian & Stern'93

Anisovich & Leutwyler'96

- M_I isospin *I* rescattering in two particles
- Amplitude in terms of S and P waves ⇨ exact up to NNLO ($\mathcal{O}(p^6)$)
- Main two body rescattering corrections inside M_I

2.3 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$\boxed{M(s,t,u) = M_0^0(s) + (s-u)M_1^1(t) + (s-t)M_1^1(u) + M_0^2(t) + M_0^2(u) - \frac{2}{3}M_0^2(s)}$$

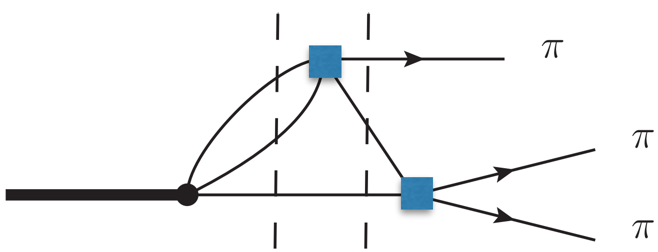
- Unitarity relation:

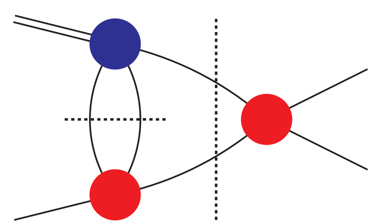
$\pi\pi \to \pi\pi$

$$\boxed{disc\left[M_\ell^I(s)\right] = \rho(s)t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)}$$

right-hand cut

left-hand cut

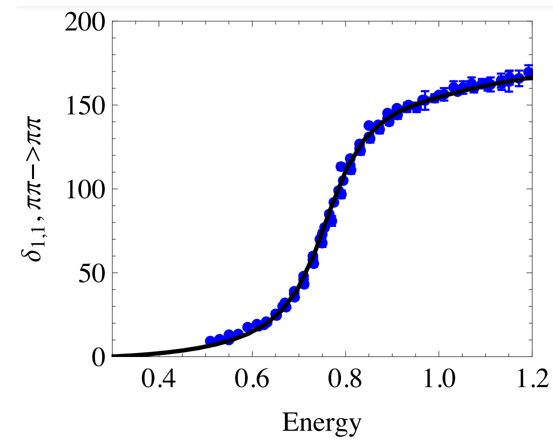




input

Roy analysis

Colangelo et al.'01



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- Unitarity relation:

$$\boxed{disc\left[M_\ell^I(s)\right] = \rho(s)t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)}$$

- Relation of dispersion to reconstruct the amplitude everywhere:

$$\boxed{M_I(s) = \Omega_I(s)\left(P_I(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^n}\frac{\sin\delta_I(s')\,\hat{M}_I(s')}{|\Omega_I(s')|\left(s'-s-i\varepsilon\right)}\right)}$$

$$\left[\Omega_I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}ds'\frac{\delta_I(s')}{s'(s'-s-i\varepsilon)}\right)\right]$$

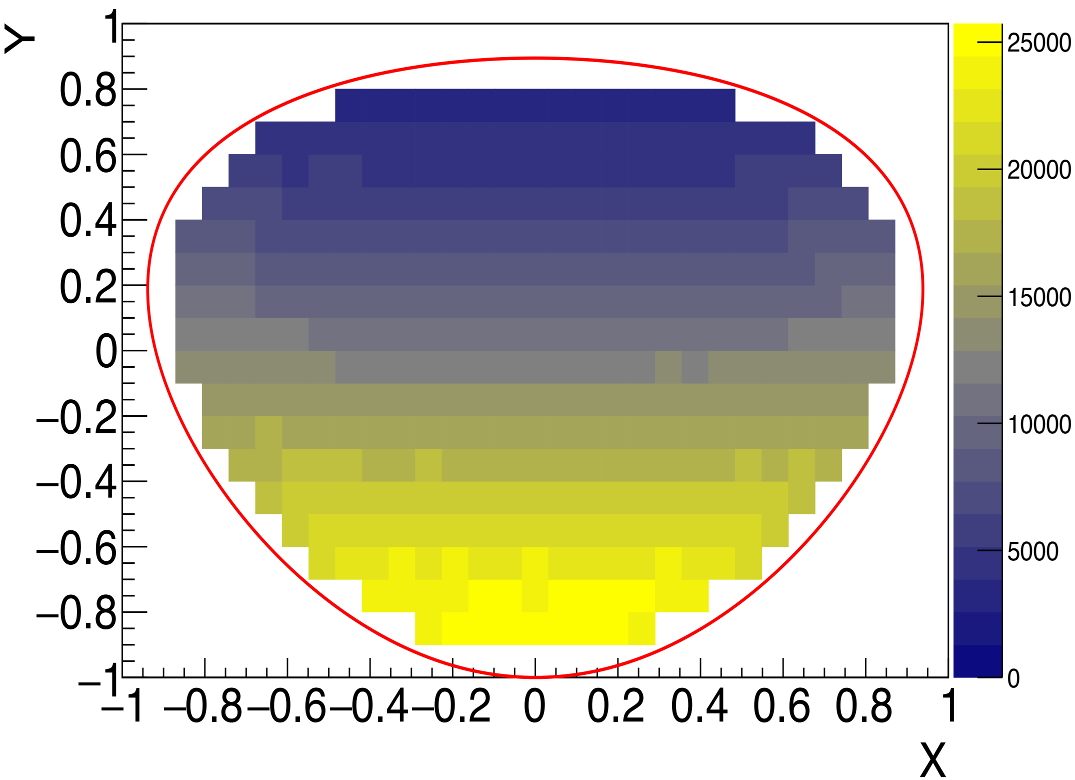
Omnès function

Gasser & Rusetsky'18

- $P_I(s)$ determined from a fit to NLO ChPT + experimental Dalitz plot

2.4 $\eta \to 3\pi$ Dalitz plot

- In the charged channel: experimental data from *WASA, KLOE, BESIII*



KLOE'16

$$\left|A(s,t,u)\right|^2 = N\begin{pmatrix} 1 + aY + bY^2 \\ +dX^2 + fY^3 + \ldots \end{pmatrix}$$

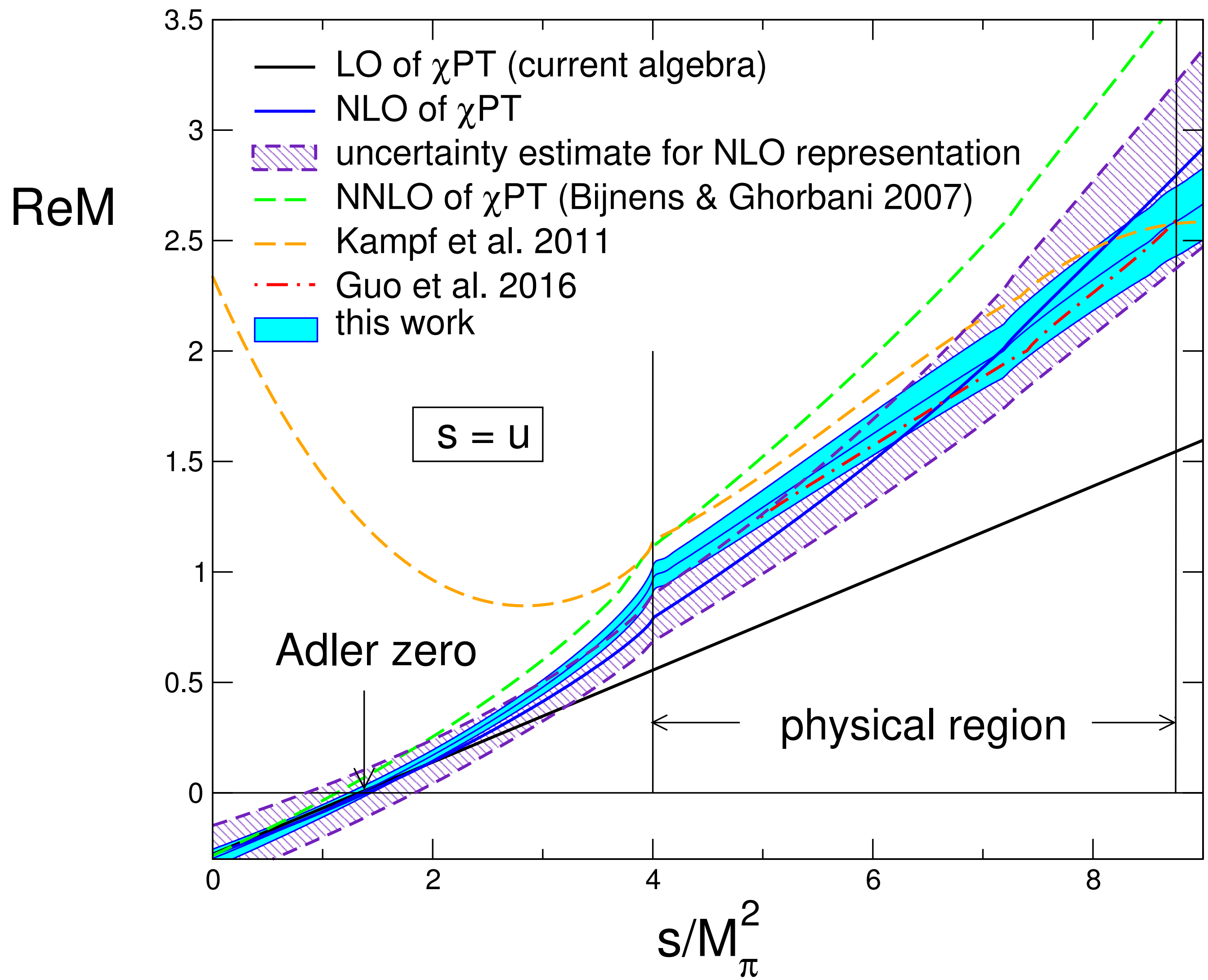
$$X = \sqrt{3}\,\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

- New data expected from *CLAS* and *GlueX* with very different systematics

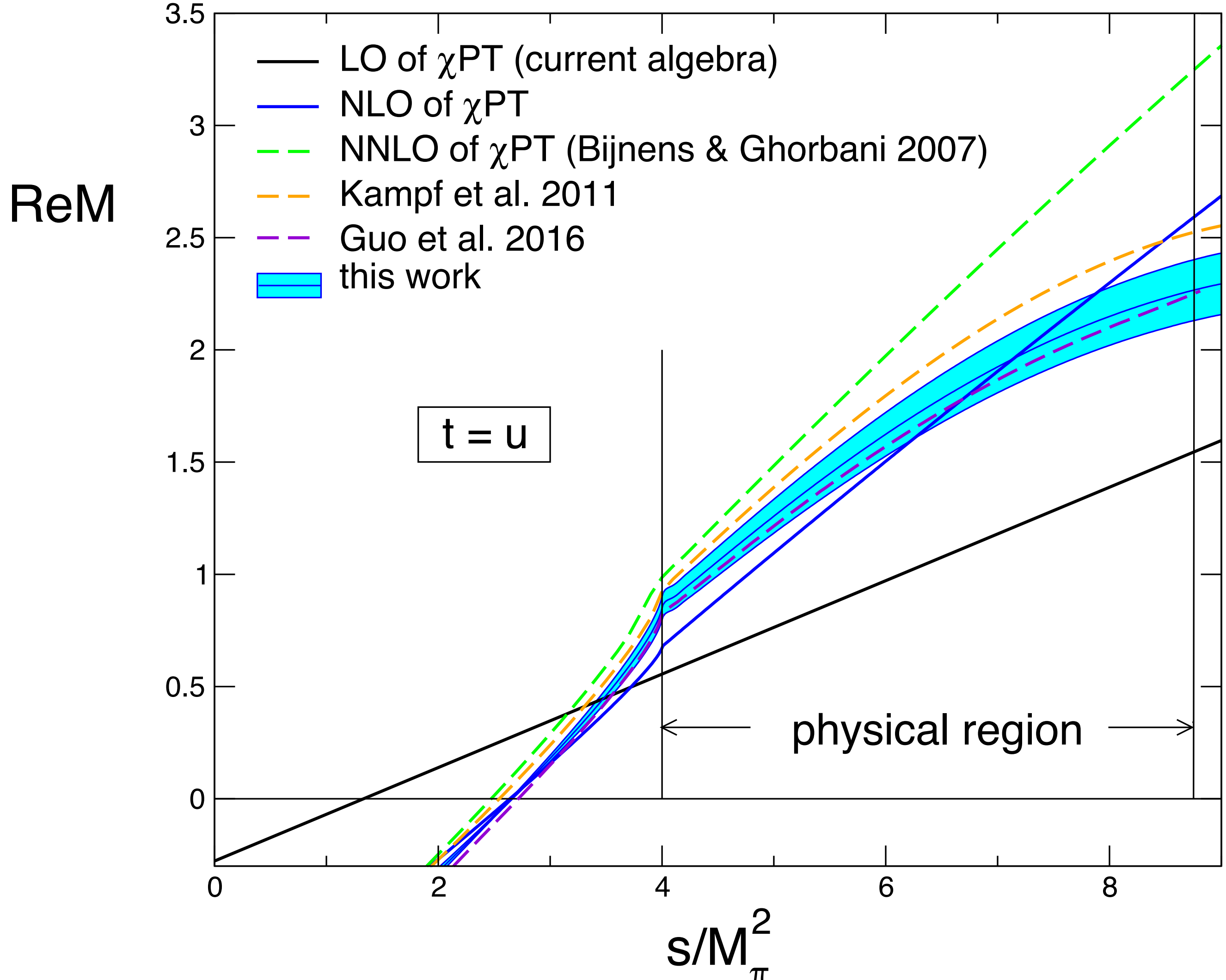
2.5 Results: Amplitude for $\eta \to \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line s = u :

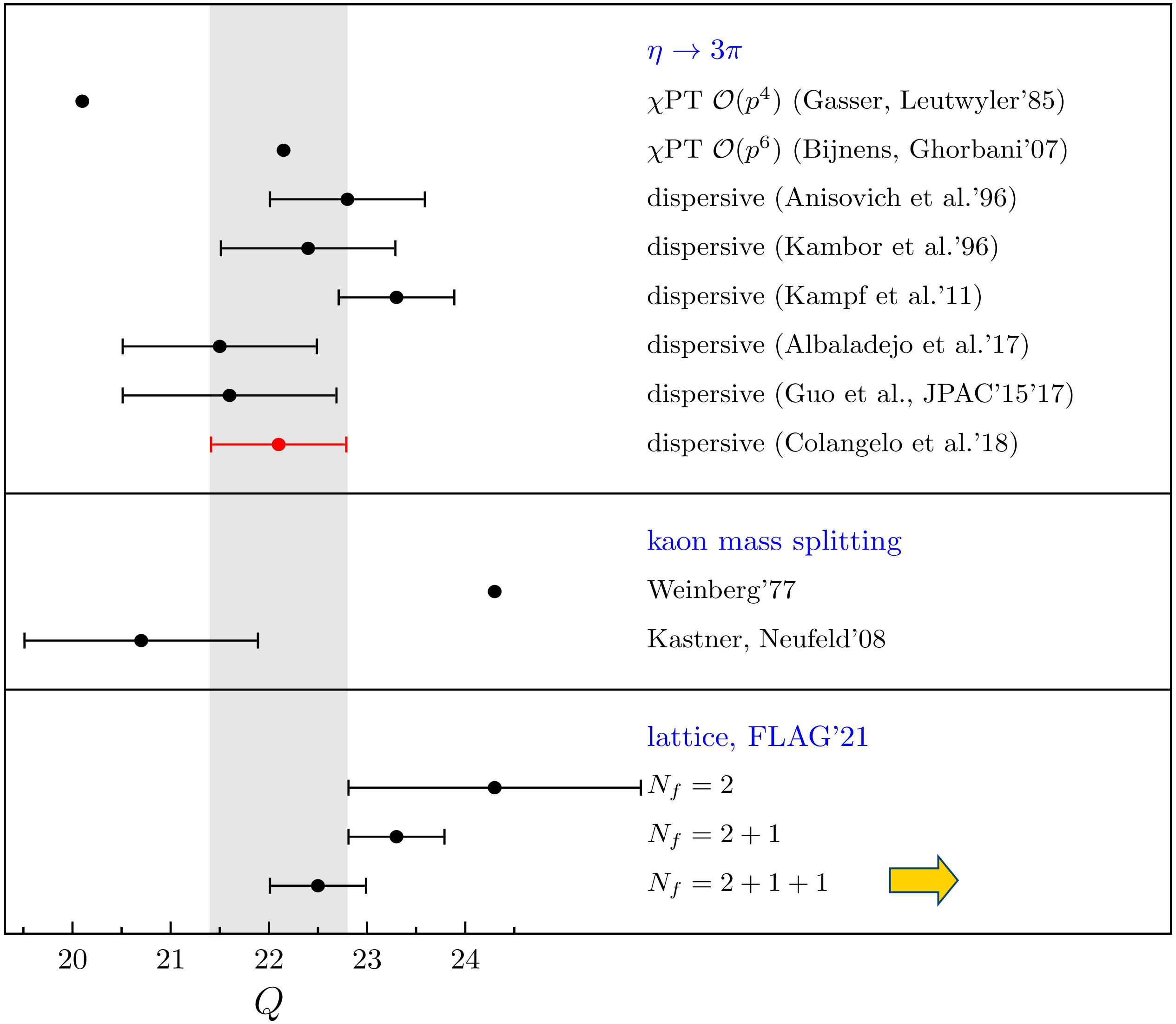


2.5 Results: Amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line t = u :



Quark mass ratio



$Q = 22.1 \pm 0.7$

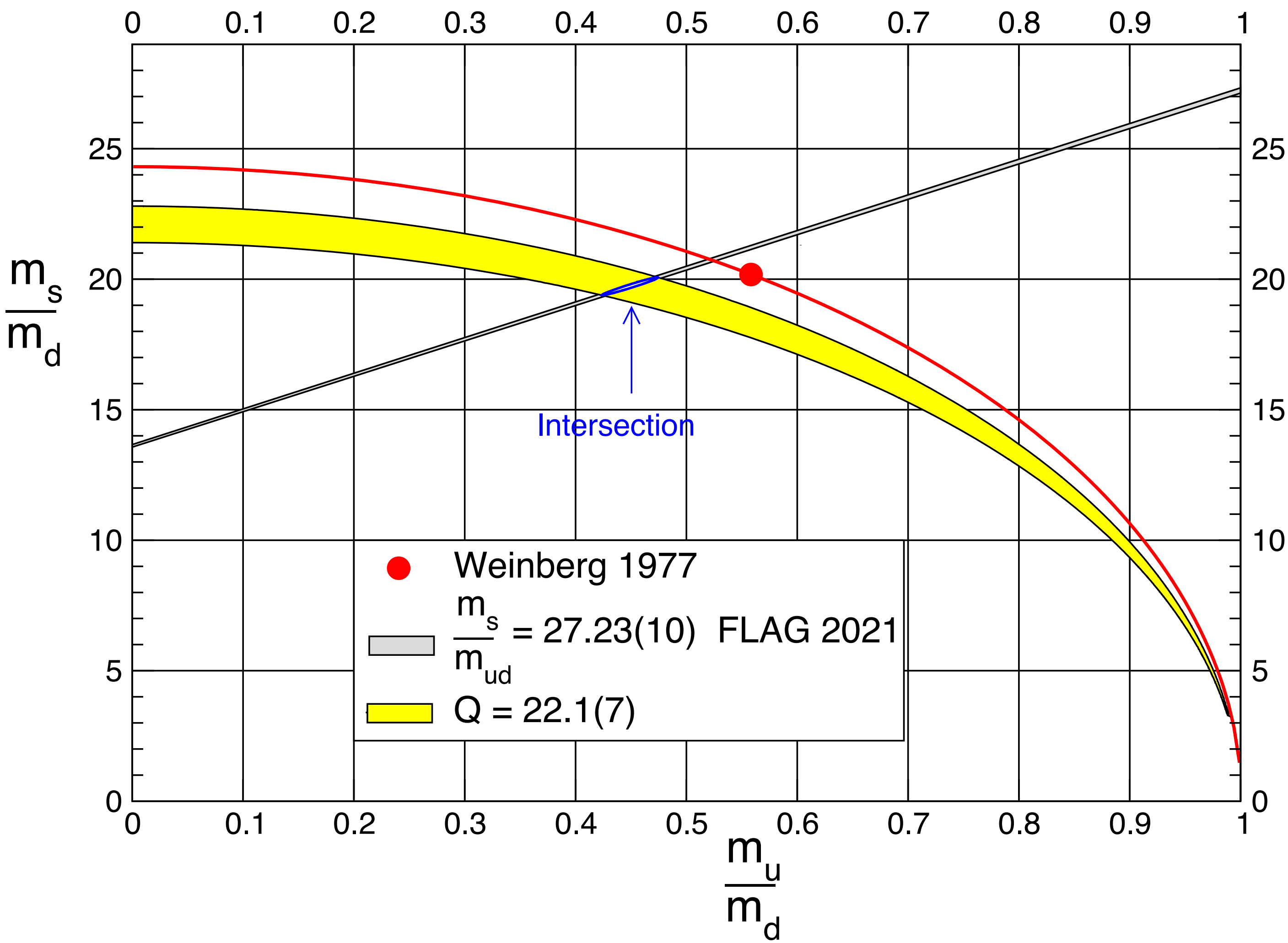
New lattice results

Shift of Q towards smaller values

Better *agreement* with $\eta \to 3\pi$ result

- Experimental systematics needs to be taken into account

Light quark masses



$\boldsymbol{Q = 22.1 \pm 0.7}$

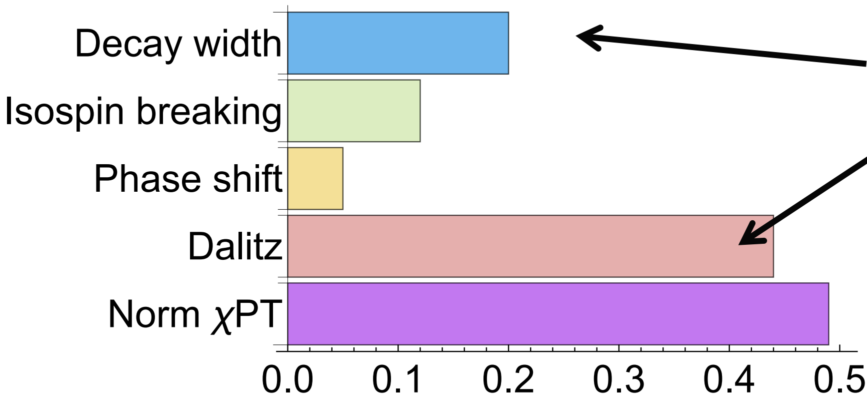
$\boldsymbol{\frac{m_u}{m_d} = 0.44 \pm 0.03}$

- Smaller values for Q ⇨ smaller values for m_s/m_d and m_u/m_d than LO ChPT

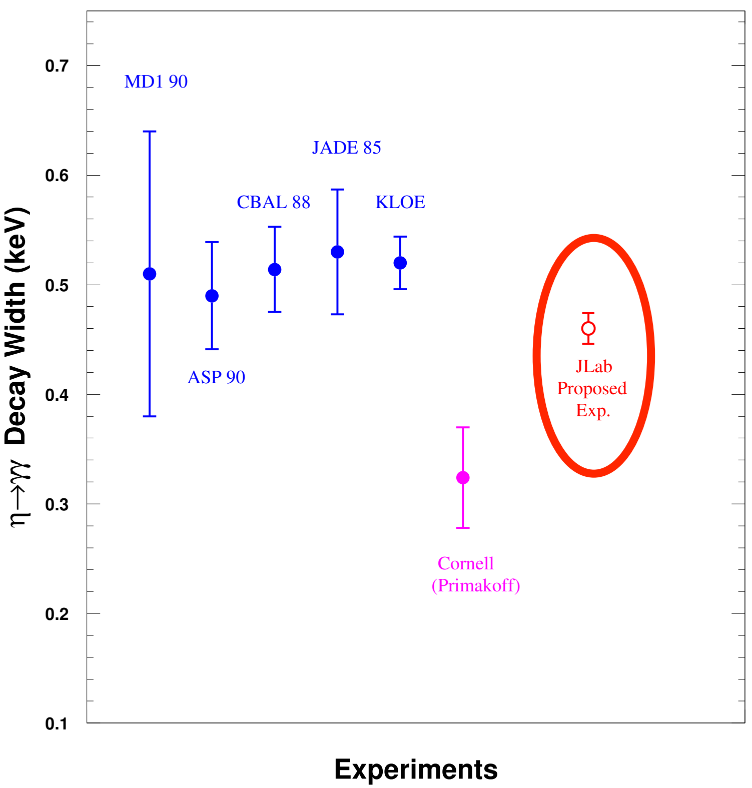
2.6 Prospects

Gan, Kubis, E. P., Tulin'22

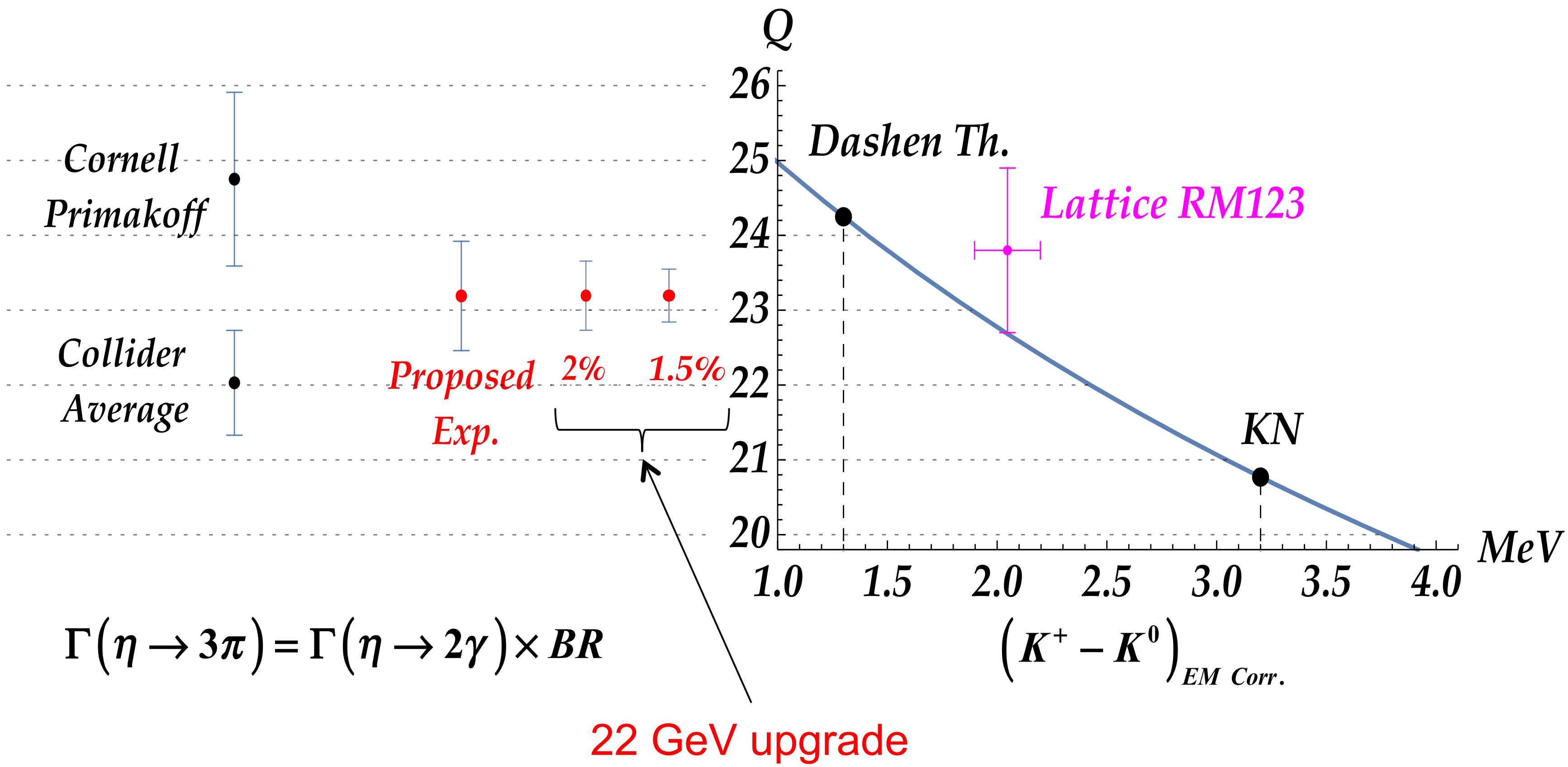
- Uncertainties in the quark mass ratio



Can be investigated and reduced at *future facilities*



2.7 Expected Impact of JLab 22 GeV program

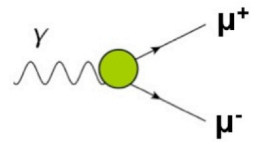


$$\Gamma\left(\eta \to 3\pi\right) = \Gamma\left(\eta \to 2\gamma\right) \times BR$$

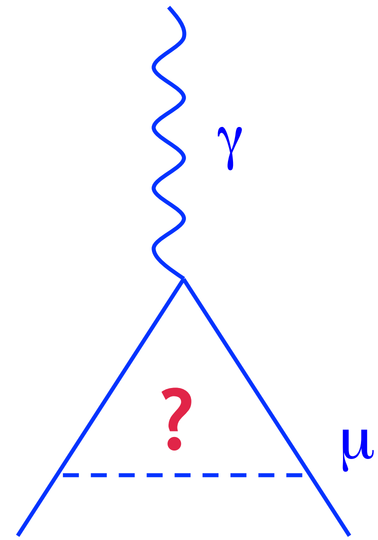
3. Theory Inputs for hadronic light-by-light for g-2 of the muon

3.1 Introduction

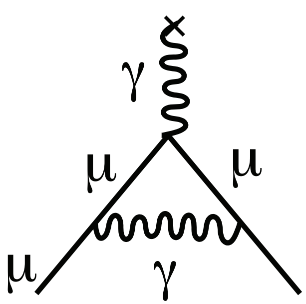
$$a_\mu = \frac{(g-2)_\mu}{2}$$ **Anomalous magnetic moment**

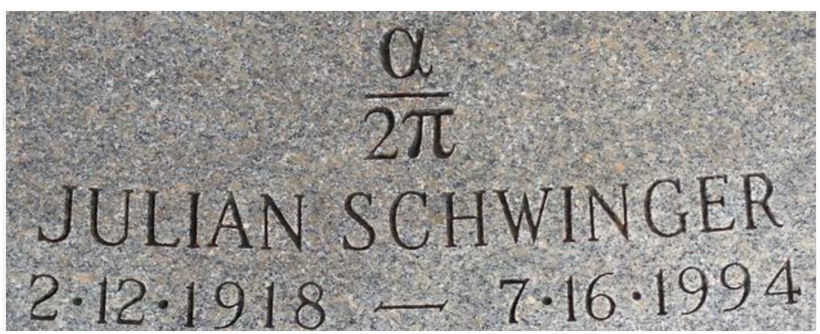


- The gyromagnetic factor of the muon is modified by loop contribution
- Predicted by Dirac to be 2
- Schwinger computed the first order correction



QED





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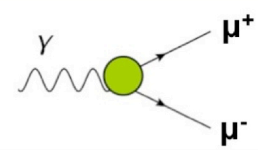
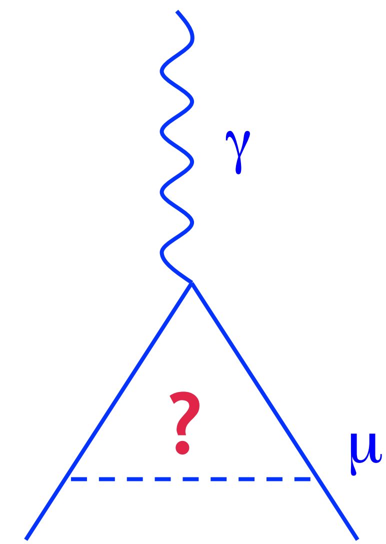


- The gyromagnetic factor of the muon is modified by loop contribution

- We can also study a_e with better experimental precision but if new physics heavy then more sensitivity in a_μ

$$\boxed{a_\ell^{NP}(\Lambda_{NP}) \propto \mathcal{O}\left(\frac{m_\ell^2}{\Lambda_{NP}^2}\right)} \Rightarrow \boxed{\frac{a_\mu^{NP}}{a_e^{NP}} \propto \mathcal{O}\left(\frac{m_\mu^2}{m_e^2}\right) \approx 43{,}000}$$

  a_τ even more sensitive but insufficient experimental accuracy *Eidelman, Giacomini, Ignatov, Passera'07*



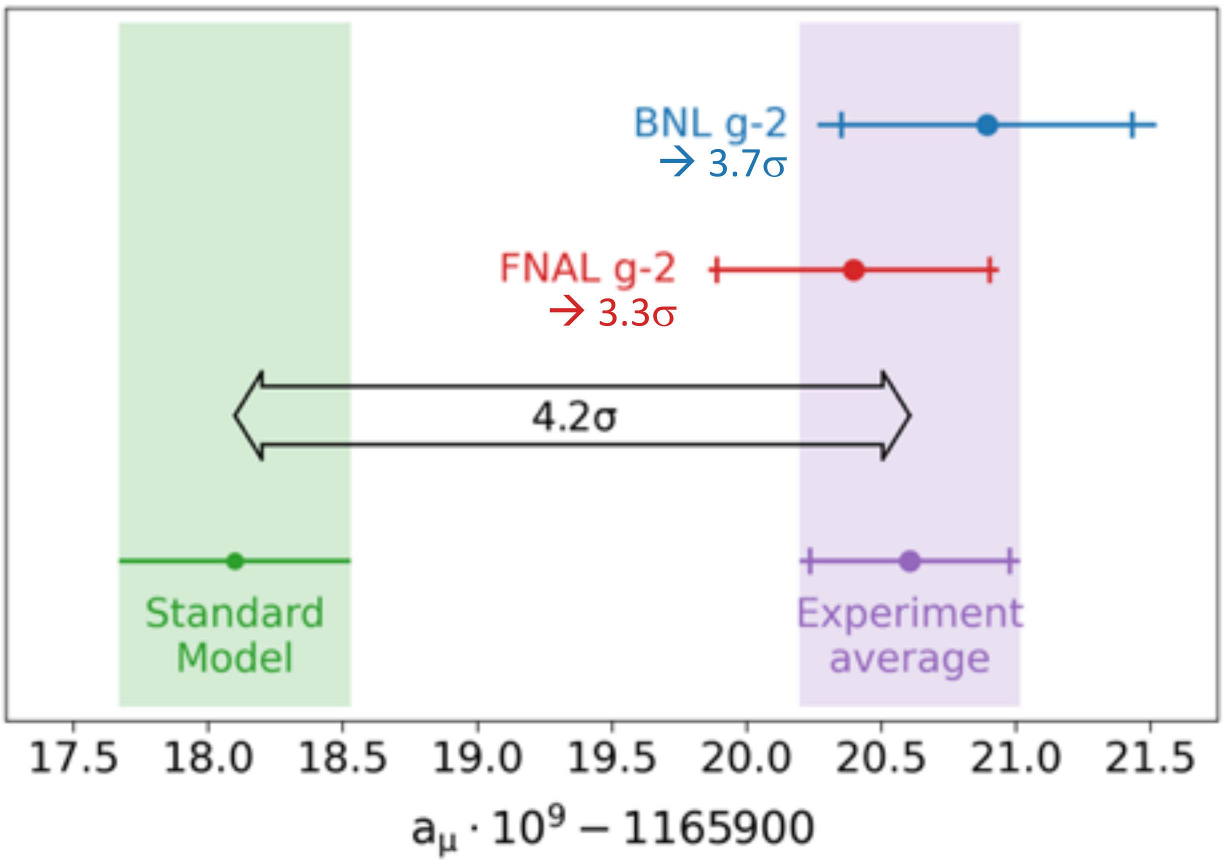
- But a_e important if NP is light
  ⇒ Important constraints on NP scenarios

Giudice, Paradisi, Passera'12

3.2 Experimental result

FNAL g-2
Chris Polly

$a_\mu(SM) = 0.00116591810(43) \rightarrow 368$ ppb



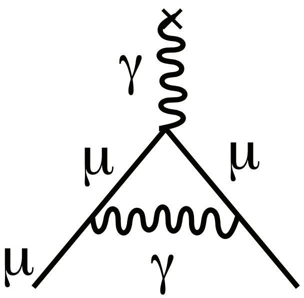
- Individual tension with SM
  - BNL: 3.7σ
  - FNAL: 3.3σ

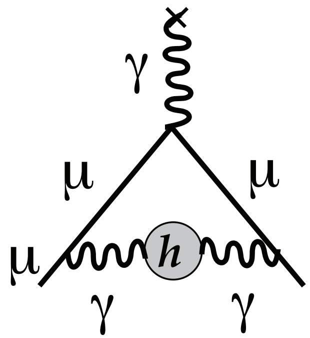
$a_\mu(Exp) - a_\mu(SM) = 0.00000000251(59) \rightarrow 4.2\sigma$

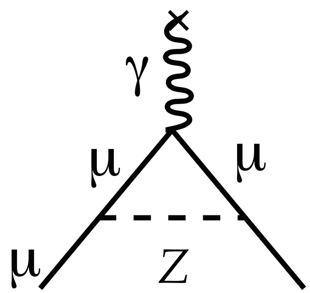
3.3 Contribution to $(g\text{-}2)_\mu$

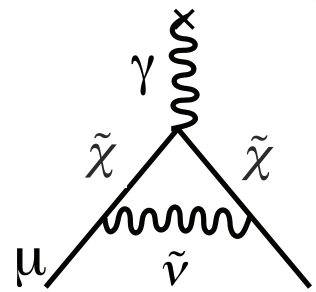
Hoecker'11

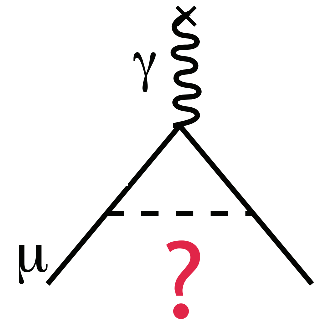
QED | Hadronic | Weak | SUSY… ? | … or some unknown type of new physics ?



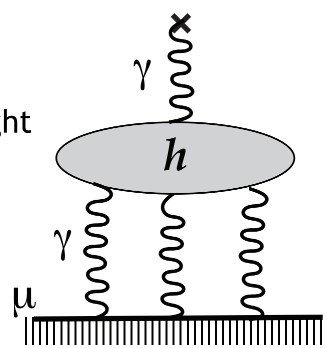


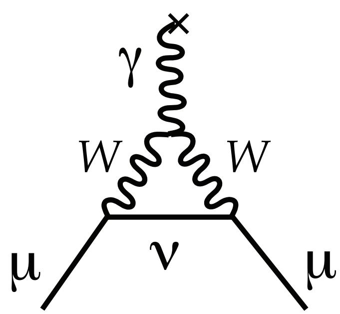


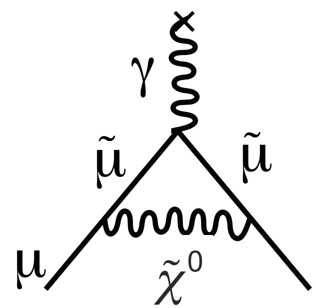




"Light-by-light scattering"







… or no effect on a_μ, but new physics at the LHC? That would be interesting as well !!

Need to compute the SM prediction with high precision! ⇒ *Not so easy!*

3.4 Confronting measurement and prediction

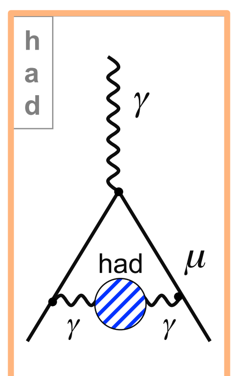
- Theoretical Prediction:

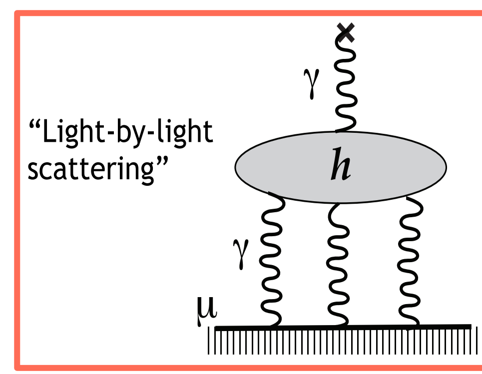
Colangelo et al. Snowmass 2022

Contribution	Result in 10^{-10} units
QED(leptons)	11658471.885 ± 0.004
HVP(leading order)	693.1 ± 4.0
HVP(higher order) (*)	-8.59 ± 0.07
HLBL	9.2 ± 1.8
EW	15.4 ± 0.1
Total	11659181.0 ± 4.3

- Important contribution comes from virtual hadrons in the loop!

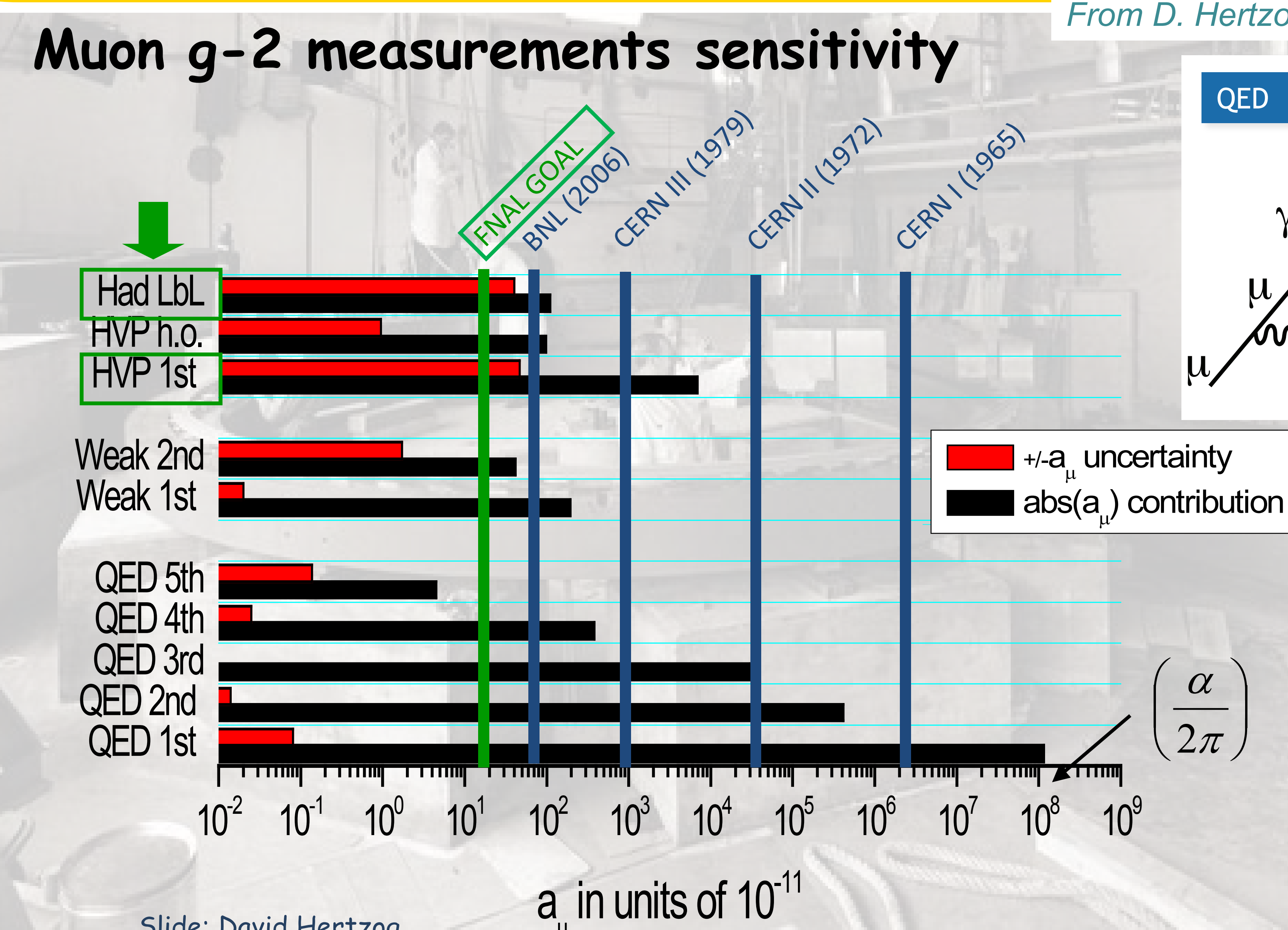
- Tackled using :
  - Models
  - Dispersion Relations
  - Lattice QCD

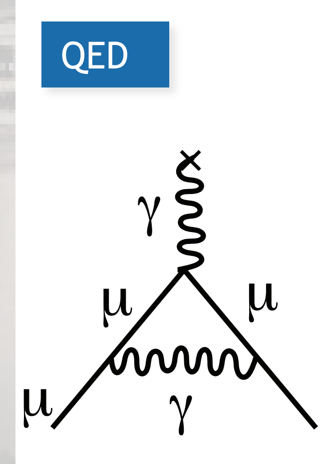




3.5 On the importance of hadronic contributions

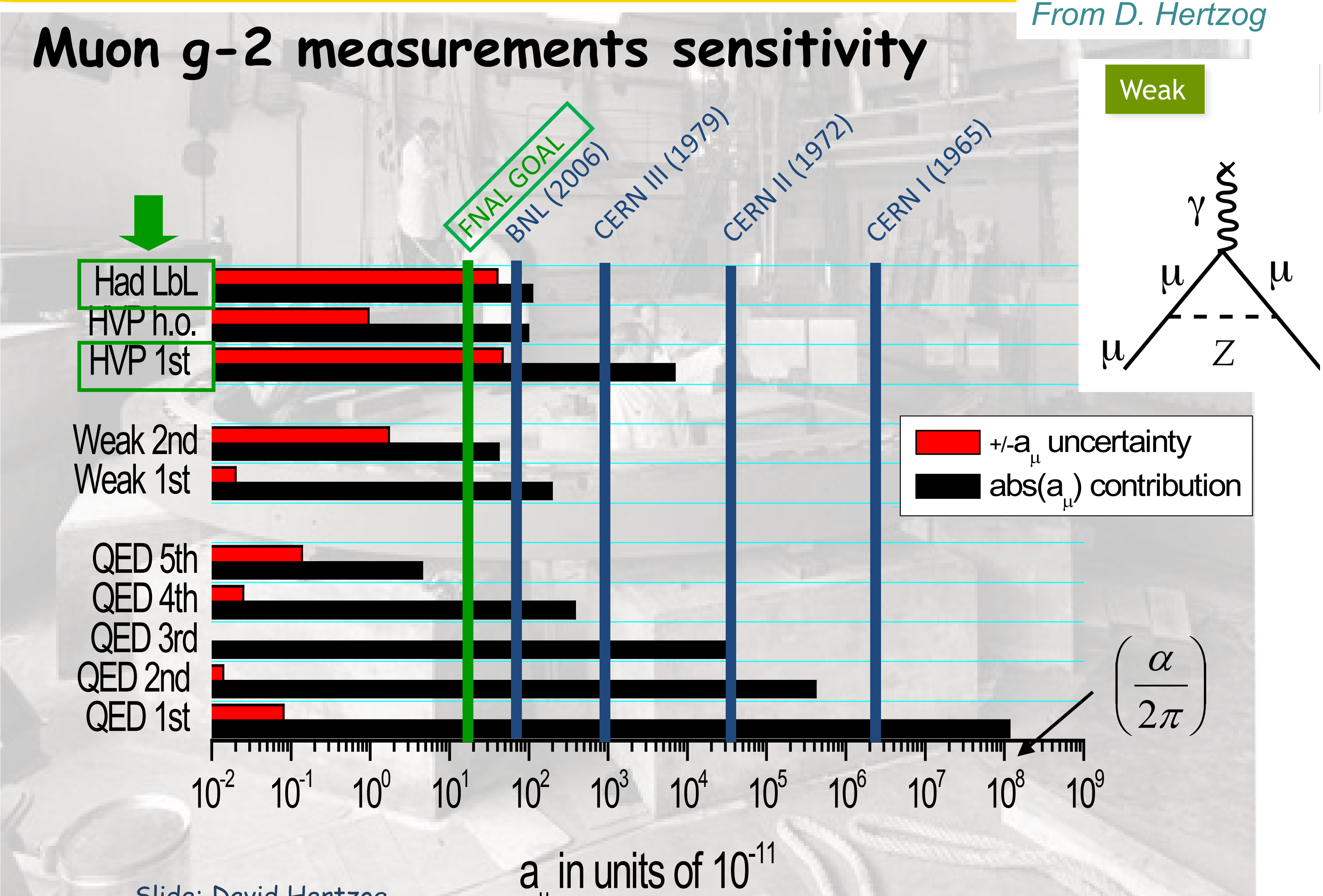
From D. Hertzog





3.5 On the importance of hadronic contributions

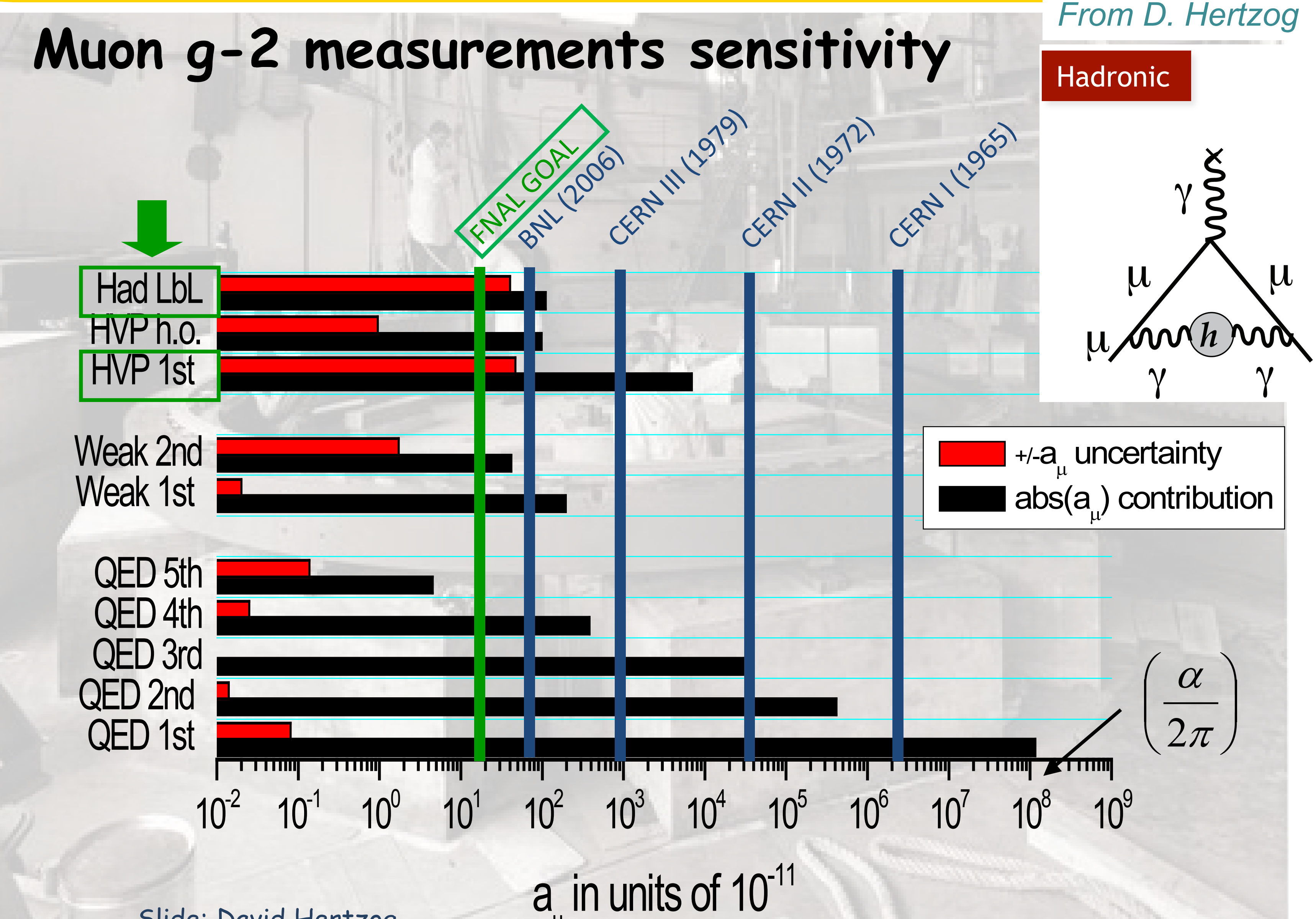
From D. Hertzog

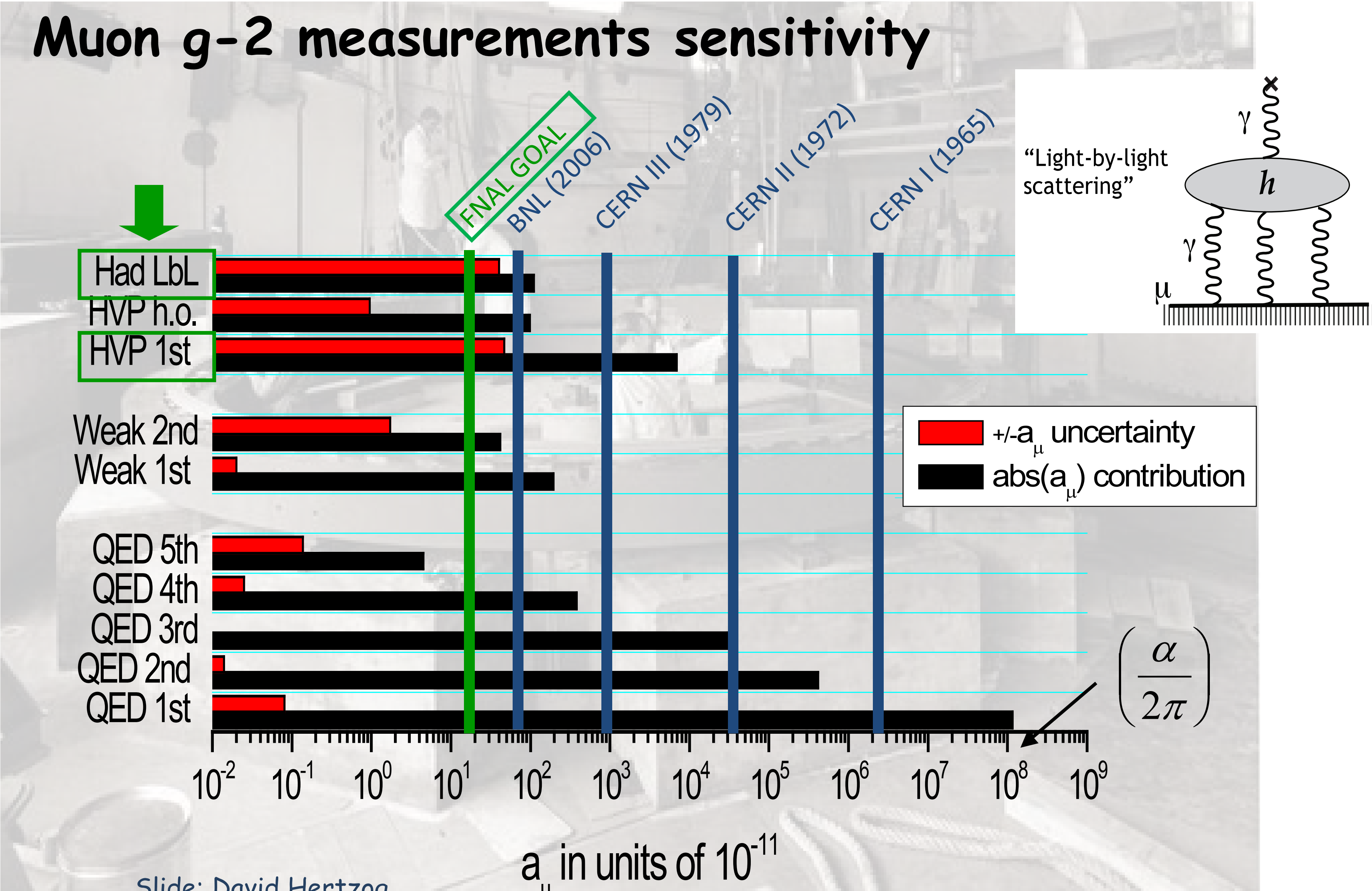


3.5 On the importance of hadronic contributions

From D. Hertzog

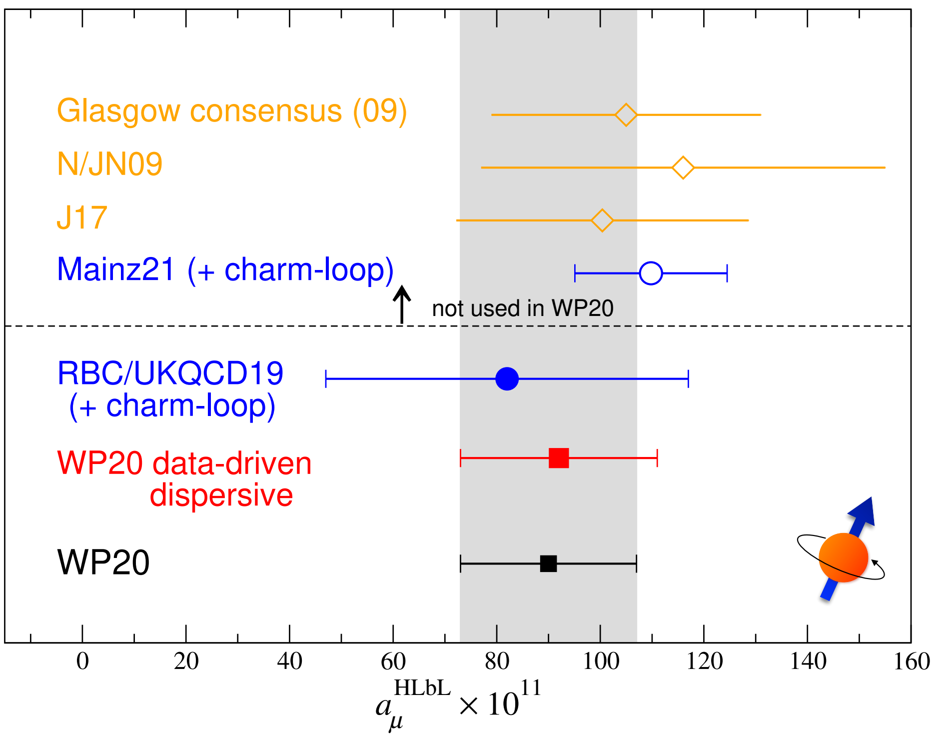
Hadronic

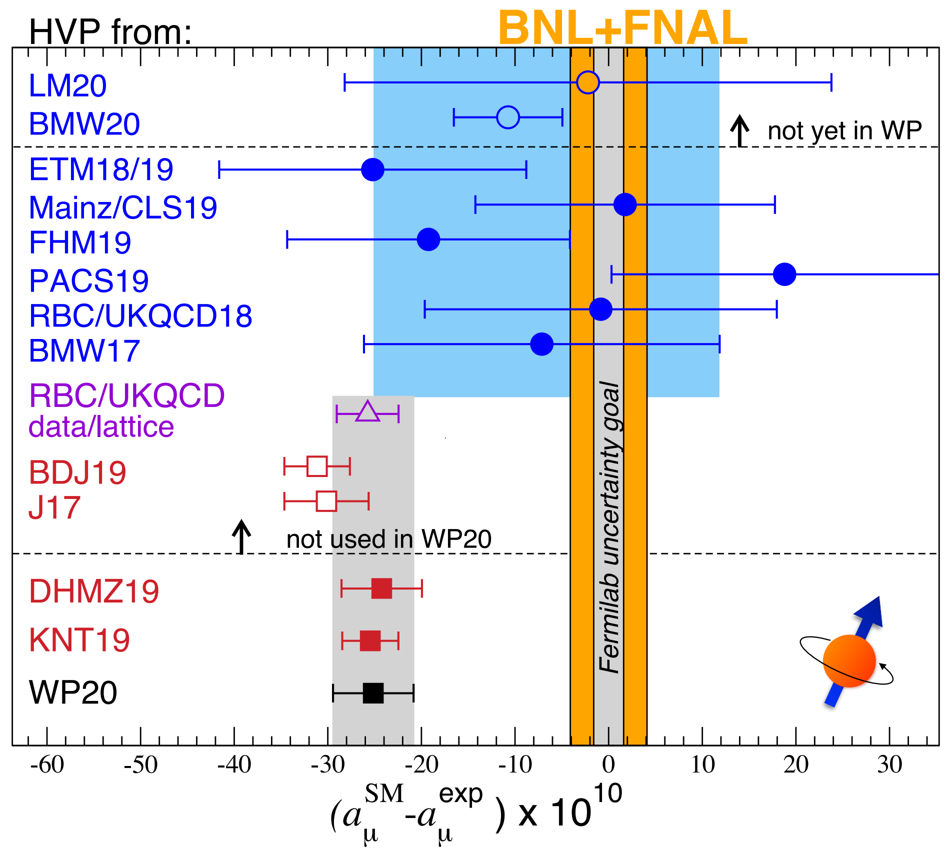



Muon g-2 measurements sensitivity
FNAL GOAL
BNL (2006)
CERN III (1979)
CERN II (1972)
CERN I (1965)
"Light-by-light scattering"
γ
h
γ
μ
Had LbL
HVP h.o.
HVP 1st
Weak 2nd
Weak 1st
QED 5th
QED 4th
QED 3rd
QED 2nd
QED 1st
+/-aμ uncertainty
abs(aμ) contribution
α/2π
10^-2 10^-1 10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9
aμ in units of 10^-11
Slide: David Hertzog

3.6 On the importance of experimental inputs for HLbL

Colangelo et al.
Snowmass 2022



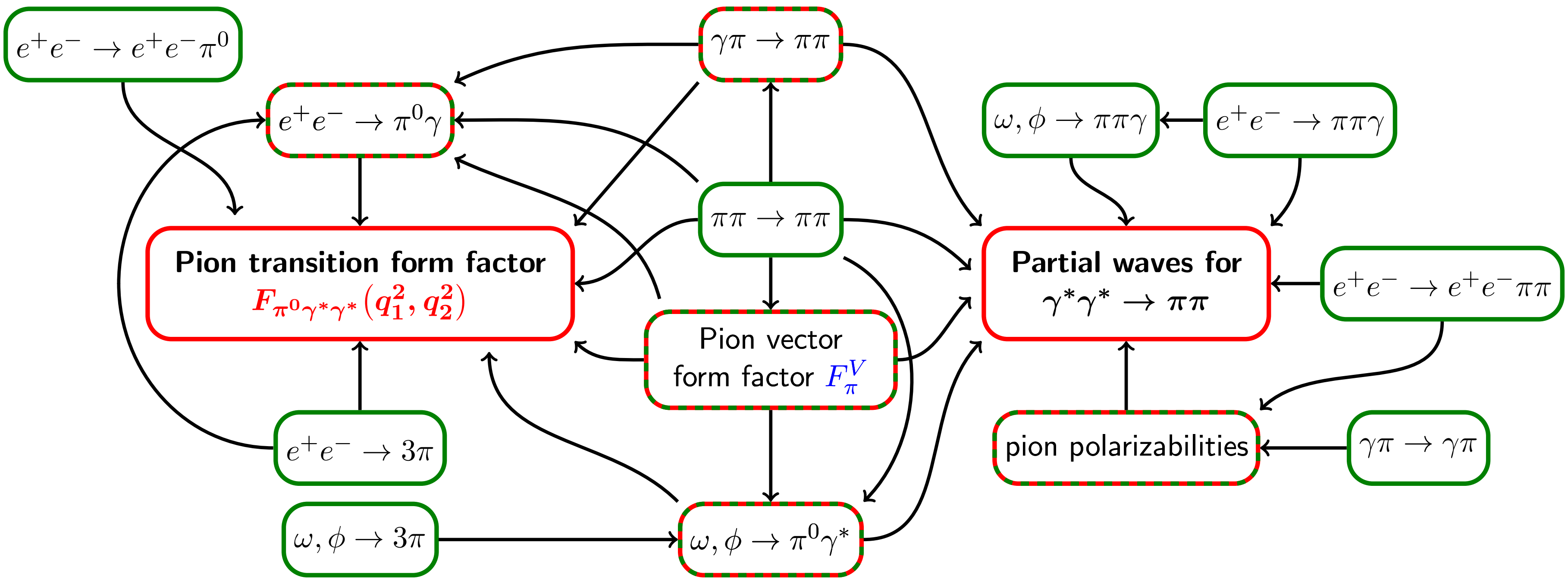


While several lattice collaborations have results for HVP, the LbL lattice calculations are much more challenging

⇨ the data driven dispersive approach dominates

3.6 Contribution of π^0, η and η' physics

Colangelo, Hoferichter, Kubis, Procura & Stoffer'14

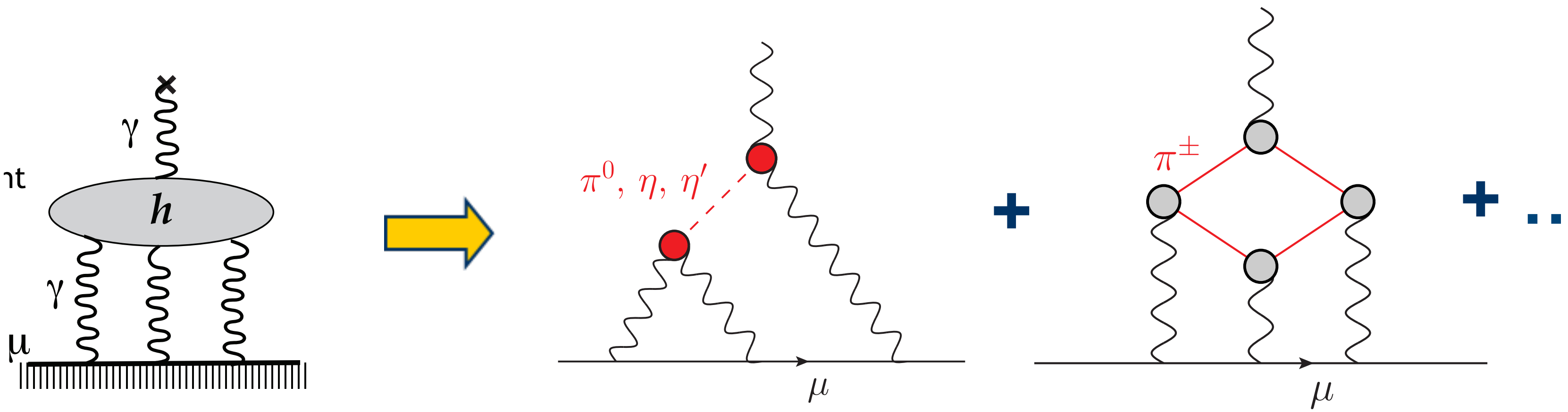


- Reconstruction of $\gamma^*\gamma^* \to \pi\pi, \pi^0$: combine experiment and theory constraints
- Need input on $\gamma^*\gamma^*$ matrix elements for as many states as possible

3.6 Contribution of π^0, η and η' physics

g-2 Theory Initiative White Paper'20

- π^0, η and η' physics enter in one meson contribution



$$a_\mu^{P\text{-pole}} = \left(\frac{\alpha}{\pi}\right)^3 \int dQ_1 dQ_2 d\tau \Big[w_1(Q_1, Q_2, \tau) F_{P\gamma^*\gamma^*}(-Q_1^2, -Q_3^2) F_{P\gamma^*\gamma^*}(-Q_2^2, 0)$$
$$+ w_2(Q_1, Q_2, \tau) F_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2) F_{P\gamma^*\gamma^*}(-Q_3^2, 0) \Big]$$

Weight functions

Meson Transitions form factors

3.6 Contribution of π^0, η and η' physics

- Experimentally, measuring the doubly-virtual TFFs $F_{P\gamma*\gamma*}$ is very challenging.

- In time-like region ($q^2_{1,2} > 0$), the TFF can be measured in principle through the double-Dalitz decay, $P \to e^+e^-e^+e^-$ but very difficult because of small partial width ⇒ only upper limits for π^0 and η

- Other possibility: use vector meson decays $V \to Pl^+l^-$: Recent results
  - $\omega \to \pi^0\mu^+\mu^-$ from NA60 and $\omega \to \pi^0 e^+e^-$ from A_2
  - $\varphi \to \pi^0 e^+e^-$ from KLOE-II
  - $J/\psi \to Pe^+e^-$ ($P = \pi^0, \eta, \eta'$) from BESIII
  - $\eta' \to \omega e^+e^-$ only BR reported by BESIII
  - $e^+e^- \to \omega\pi^0$ from various collaborations
  - $e^+e^- \to \phi\eta'$ and $e^+e^- \to J/\psi\eta'$ from BESIII

- In space-like region ($q^2_{1,2} = -Q^2_{1,2} < 0$), two-photon fusion $e^+e^- \to \gamma^*\gamma^* e^+e^- \to Pe^+e^-$, where the virtualities of the photons are measured by detecting the outgoing leptons. In double tag ⇒ double virtual TFF but cross sections are very small, only one result from BaBar in η' .
  Possibility via virtual Primakoff effect, $e^-A \to \gamma^*\gamma^* e^-A \to Pe^-A$
  ⇒ *under investigation at JLab*

3.6 Contribution of π^0, η and η' physics

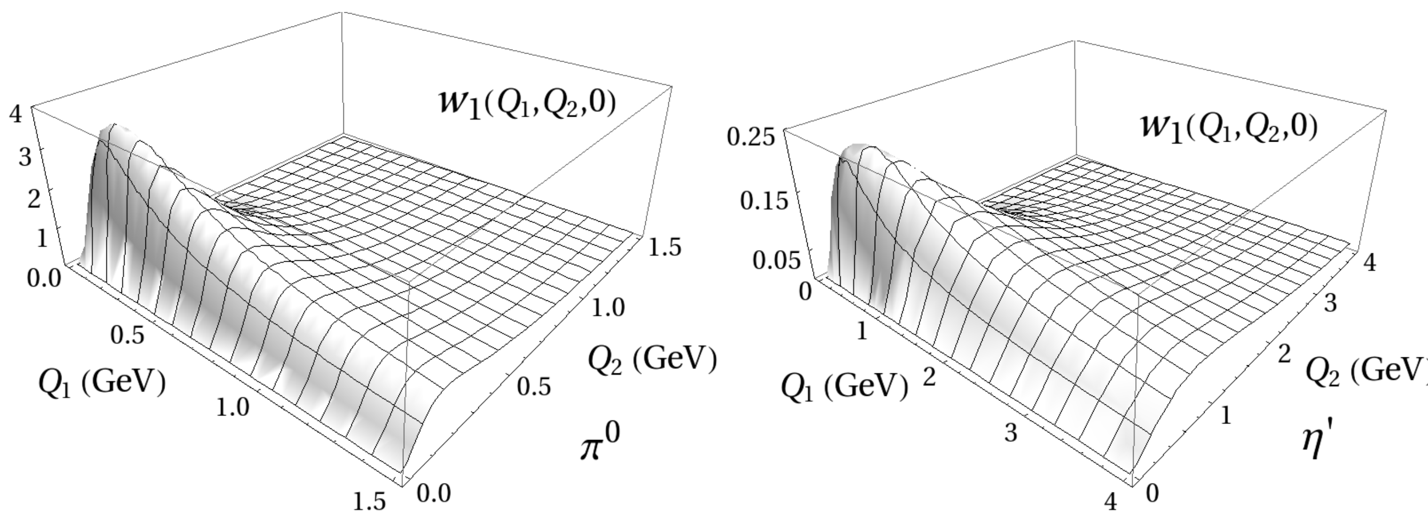
- Many more results from single-virtual TFFs $F_{P\gamma\gamma*}$:

- In time-like region:
  - $\pi^0 \to l^+l^-\gamma$ from NA62 and A_2 experiments
  - $\eta \to \mu^+ \mu^- \gamma$ from NA60
  - $\eta \to e^+ e^- \gamma$ from A_2 at MAMI and WASA-at-COSY
  - $\eta' \to e^+ e^- \gamma$ from BESIII
  - for larger momenta: $e+e- \to \gamma* \to P\gamma$ at collider facilities: SND and CMD2 at Novosibirsk VEPP-2000, BaBar and CLEO

- In space-like region: $e^+e^- \to \gamma^*\gamma^*e^+e^- \to Pe^+e^-$ at the e^+e^- collider facilities. Only one outgoing lepton is detected (single-tag mode). The other untagged photon is almost real (where the associated lepton is not detected).
  Measurements by CLEO, CELLO, BaBar and Belle.
  Low Q^2 region data are missing ⇨ on-going activities at BESIII and KLOE-II

3.6 Contribution of π^0, η and η' physics

g-2 Theory Initiative White Paper



- Weight contribution ⇒ low energy dominates

$$F_{\pi^0\gamma^*\gamma^*} = F^{\rm disp}_{\pi^0\gamma^*\gamma^*} + F^{\rm eff}_{\pi^0\gamma^*\gamma^*} + F^{\rm asym}_{\pi^0\gamma^*\gamma^*}$$

Use experimental data with dispersive analysis to reconstruct from dominant low-energy singularities (2/3 pions intermediate states)

$$F^{\rm disp}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = F^{\rm disp}_{vs}(q_1^2, q_2^2) + F^{\rm disp}_{vs}(q_2^2, q_1^2)$$

3.6 Contribution of π^0, η and η' physics

g-2 Theory Initiative White Paper

issue	experimental input [I] or cross-checks [C]
axials, tensors, higher pseudoscalars	$\gamma^{(*)}\gamma^* \to 3\pi,\ 4\pi,\ K\bar{K}\pi,\ \eta\pi\pi,\ \eta'\pi\pi$ [I]
missing states	inclusive $\gamma^{(*)}\gamma^* \to$ hadrons at 1–3 GeV [I]
dispersive analysis of $\eta^{(\prime)}$ TFFs	$e^+e^- \to \eta\pi^+\pi^-$ [I]
	$\eta' \to \pi^+\pi^-\pi^+\pi^-$ [I]
	$\eta' \to \pi^+\pi^-e^+e^-$ [I]
	$\gamma\pi^- \to \pi^-\eta$ [C]
dispersive analysis of π^0 TFF	$\gamma\pi \to \pi\pi$ [I]
	high accuracy Dalitz plot $\omega \to \pi^+\pi^-\pi^0$ [C]
	$e^+e^- \to \pi^+\pi^-\pi^0$ [C]
	$\omega, \phi \to \pi^0 l^+ l^-$ [C]
pseudoscalar TFF	$\gamma^{(*)}\gamma^* \to \pi^0, \eta, \eta'$ at arbitrary virtualities [I,C]
pion, kaon, $\pi\eta$ loops (including scalars and tensors)	$\gamma^{(*)}\gamma^* \to \pi\pi,\ K\bar{K},\ \pi\eta$ at arbitrary virtualities, partial waves [I,C]

$$F_{vs}^{\text{disp}}(-Q_1^2, -Q_2^2) = \frac{1}{\pi^2}\int_{4M_\pi^2}^{s_{\text{iv}}} dx \int_{s_{\text{thr}}}^{s_{\text{is}}} \frac{dy\,\rho(x,y)}{(x+Q_1^2)(y+Q_2^2)},$$

Isovector/scalar

$$\rho(x,y) = \frac{q_\pi^3(x)}{12\pi\sqrt{x}}\,\text{Im}\left[(F_\pi^V(x))^* f_1(x,y)\right],$$

3.6 Contribution of π^0, η and η' physics

g-2 Theory Initiative White Paper

- Dispersively to reconstruct the TFFs $F_{\eta\gamma*\gamma*}$ from singly-virtual input only is more challenging because of different Isospin decomposition

$$F_{\eta^{(\prime)}\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{vv^{(\prime)}}(q_1^2, q_2^2) + F_{ss^{(\prime)}}(q_1^2, q_2^2)$$

Rely on $\eta' \to \pi\pi\gamma$ as input

3.6 Contribution of π^0, η and η' physics

g-2 Theory Initiative White Paper

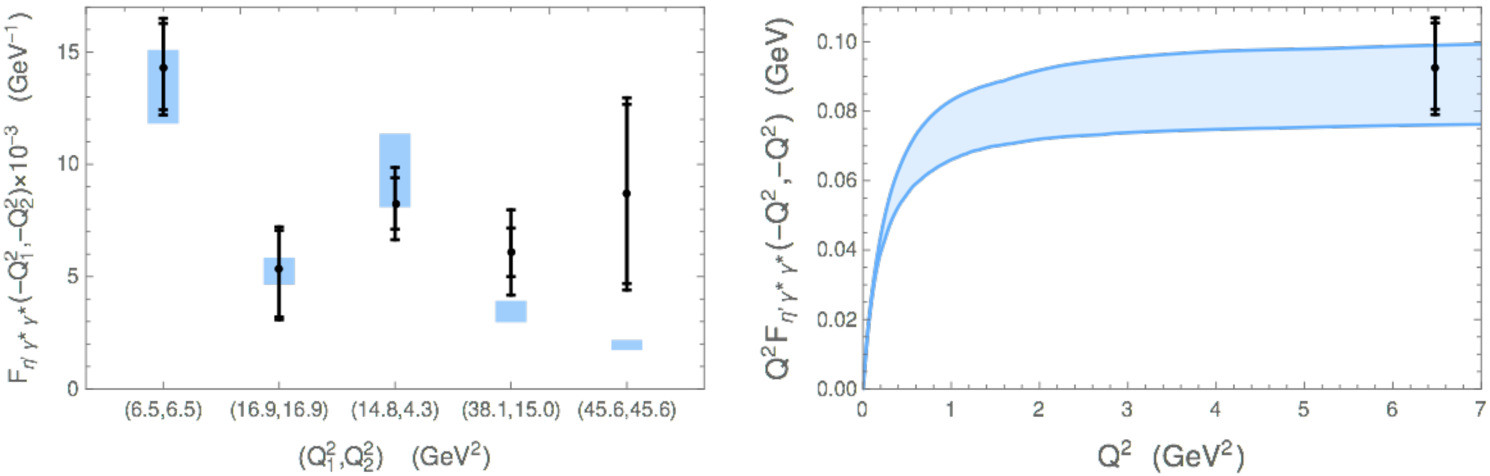
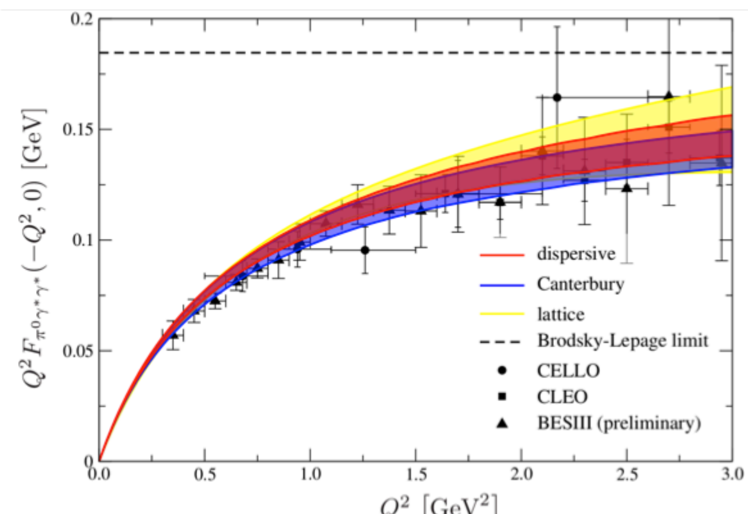
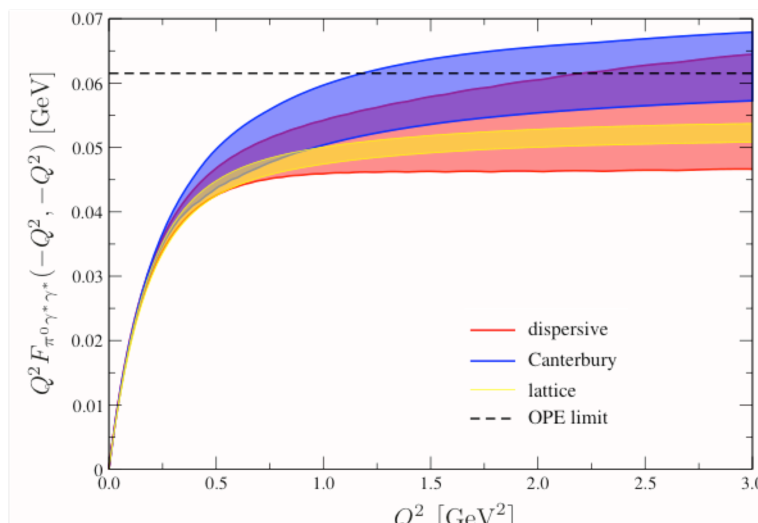


Figure 59: Left: BABAR data points [108] with statistical errors (inner bars) and statistical and systematic combined (outer bars) in black, together with the CA prediction including errors (blue bands). Right: The analogous plot for the diagonal $Q^2 F_{\eta'\gamma^*\gamma^*}(-Q^2,-Q^2)$ TFF.





3.6 Contribution of π^0, η and η' physics

g-2 Theory Initiative White Paper

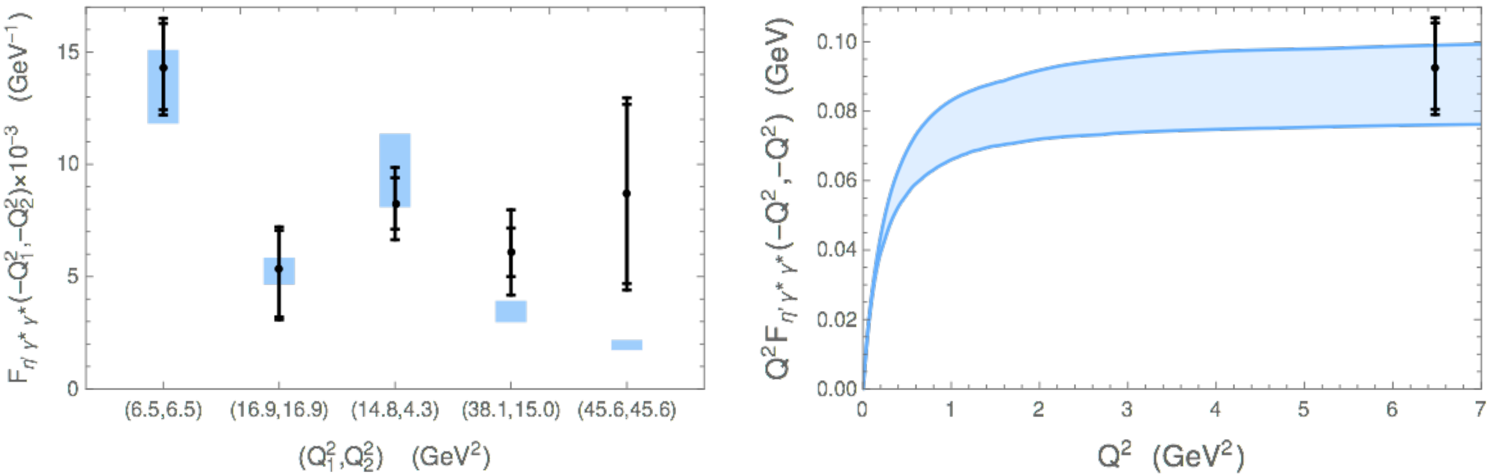
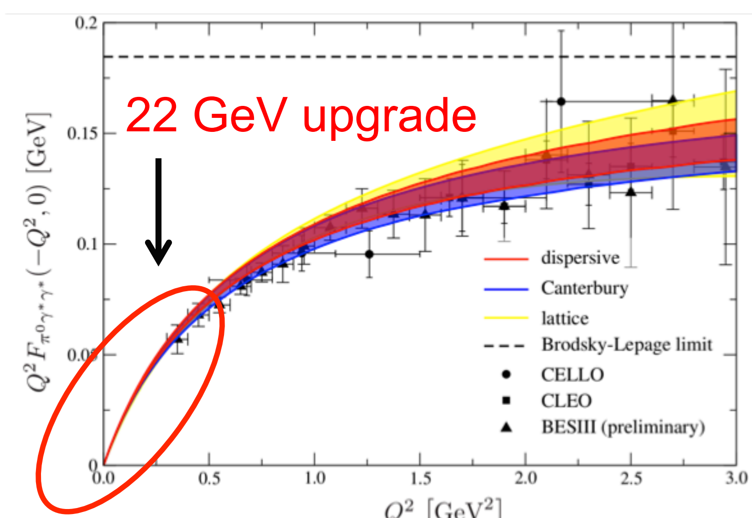
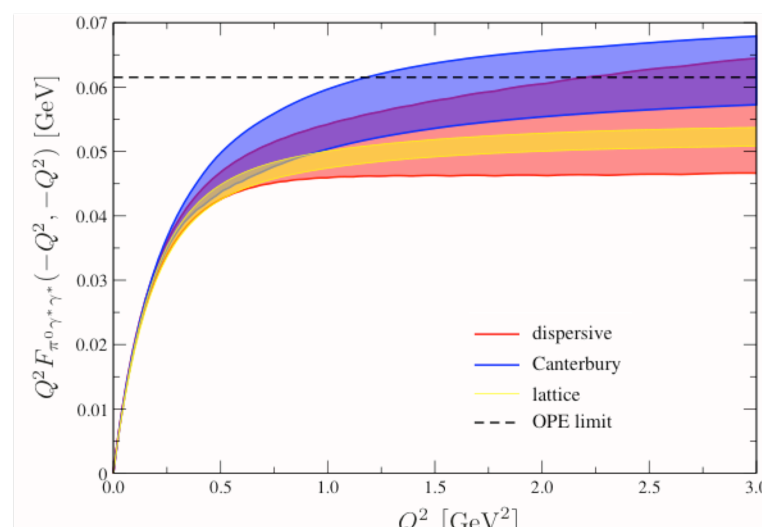


Figure 59: Left: BABAR data points [108] with statistical errors (inner bars) and statistical and systematic combined (outer bars) in black, together with the CA prediction including errors (blue bands). Right: The analogous plot for the diagonal $Q^2F_{\eta'\gamma^*\gamma^*}(-Q^2,-Q^2)$ TFF.





3.6 Contribution of π^0, η and η' physics

g-2 Theory Initiative White Paper

- η and η' physics enter in one meson contribution

$$a_\mu^{\pi^0\text{-pole}}(\text{disp}) = 63.0^{+2.7}_{-2.1} \times 10^{-11} ,$$

$$a_\mu^{\eta/\eta'\text{-pole}} = [14.7(1.9)/13.6(0.8)] \times 10^{-11}$$

3.6 Contribution of π^0, η and η' physics

g-2 Theory Initiative White Paper

Contribution	PdRV(09) [475]	N/JN(09) [476, 596]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	−	−	−	− 1(3)
tensors	−	−	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	−	21(3)	20(4)	15(10)
c-loop	2.3	−	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

Table 15: Comparison of two frequently used compilations for HLbL in units of 10^{-11} from 2009 and a recent update with our estimate. Legend: PdRV = Prades, de Rafael, Vainshtein ("Glasgow consensus"); N/JN = Nyffeler / Jegerlehner, Nyffeler; J = Jegerlehner.

Work remains to be done in the theory and experimental sides on η and η' TFFs

4. Conclusion and Outlook

Conclusion and Outlook

- π^0, η and η' allows to study the fundamental properties of QCD and test the SM
  - Extraction of fundamental parameters of the SM
    ⇨ e.g. light quark masses
  - Study of chiral dynamics
  - Fundamental inputs for calculating LbL of the anomalous magnetic moment of the muon

- To studies η and η' with the best precision: Development of amplitude analysis techniques consistent with analyticity, unitarity, crossing symmetry dispersion relations allow to take into account *all rescattering effects* being as model independent as possible combined with ChPT ⇨ Provide parametrization for experimental studies

- Several improvements and puzzle could be investigated with the JLab 22 GeV upgrade ⇨ work remains to be done between theorists and experimentalists