Science at the Luminosity Frontier: Jefferson Lab at 22 GeV Workshop

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QCD energy-momentum tensor and the mechanical properties of hadrons

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Outline

- Introduction: Meaning of Energy-Momentum Tensor (EMT) in QCD
- Gravitational Form Factors (GFFs) and the processes to measure them
- D-term form factor $D(t)$ and the "least known" global property of nucleon
- Interpretation, Visualization of Forces, the Mechanical Properties of Hadrons
- Conclusions

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Energy momentum tensor (EMT) in general

- **coupling to gravitational field**

$$\hat{T}_{\mu\nu}(x) = \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{grav}}}{\delta g^{\mu\nu}(x)}$$

- **EMT conservation:**

in classical and quantum case: $\boldsymbol{\partial^\mu \hat{T}_{\mu\nu} = 0}$

- **Poincaré group generators:**

$\hat{P}^\mu = \int \text{d}^3x \; \hat{T}^{0\mu}$, $\hat{M}^{\kappa\nu} = \int \text{d}^3x \; (x^\kappa \hat{T}^{0\nu} - x^\nu \hat{T}^{0\kappa})$,

algebra: $[\hat{P}^\mu, \hat{P}^\nu] = 0$, $[\hat{M}^{\mu\nu}, \hat{P}^\kappa] = i(g^{\mu\kappa}\hat{P}^\nu - g^{\nu\kappa}\hat{P}^\mu)$, $[\hat{M}^{\mu\nu}, \hat{M}^{\kappa\sigma}] = i(g^{\mu\kappa}\hat{M}^{\nu\sigma} - g^{\nu\kappa}\hat{M}^{\mu\sigma} - g^{\mu\sigma}\hat{M}^{\nu\kappa} + g^{\nu\sigma}\hat{M}^{\mu\kappa})$

- **Casimir operators:**

$\hat{P}^\mu \hat{P}_\mu \to m^2$, $\hat{W}^\mu \hat{W}_\mu \to m^2 \, J(J+1)$ where $\hat{W}^\kappa = -\frac{1}{2}\varepsilon^{\kappa\mu\nu\sigma}\hat{M}_{\mu\nu}\hat{P}_\sigma$

- **classify particles:**

mass & spin

Energy momentum tensor (EMT) in QCD

- $\hat{T}_{\mu\nu} = \sum_q T^q_{\mu\nu} + \hat{T}^g_{\mu\nu}$

  quark, gluon $\hat{T}^q_{\mu\nu}$, $\hat{T}^g_{\mu\nu}$ each gauge-invariant, but not conserved separately

  $$T^{\mu\nu}_q = \frac{1}{4}\overline{\psi}_q\Big(-i\overleftarrow{\mathcal{D}}^\mu\gamma^\nu - i\overleftarrow{\mathcal{D}}^\nu\gamma^\mu + i\overrightarrow{\mathcal{D}}^\mu\gamma^\nu + i\overrightarrow{\mathcal{D}}^\nu\gamma^\mu\Big)\psi_q - g^{\mu\nu}\overline{\psi}_q\Big(-\frac{i}{2}\overleftarrow{\not{\mathcal{D}}} + \frac{i}{2}\overrightarrow{\not{\mathcal{D}}} - m_q\Big)\psi_q$$

  $$T^{\mu\nu}_g = F^{a,\mu\eta}\,F^{a,}{}_\eta{}^\nu + \frac{1}{4}\,g^{\mu\nu}F^{a,\kappa\eta}\,F^{a,}{}_{\kappa\eta}$$

- classical Lagrangian symmetric under scale transformations for $m_q \to 0$

  $\hookrightarrow j^\mu = x_\nu \hat{T}^{\mu\nu}$ is conserved $\partial_\mu j^\mu = \hat{T}_\mu^{\ \mu} = \sum_q m_q\bar{\psi}_q\psi_q$ when $m_q \to 0$

- quantum corrections
  break conformal symmetry
  in QED, QCD $\to$ trace anomaly

  $$\hat{T}_\mu^{\ \mu} \equiv \frac{\beta(g)}{2g}\,F^{a,\mu\nu}F^{a,}{}_{\mu\nu} + (1+\gamma_m)\sum_q m_q\bar{\psi}_q\psi_q$$

S.L.Adler, J.C.Collins and A.Duncan, PRD **15**, 1712 (1977).
N.K.Nielsen, NPB **120**, 212 (1977).
J.C.Collins, A.Duncan and S.D.Joglekar, PRD **16**, 438 (1977).

Definition Gravitational Form Factors (GFFs) for nucleon

in the $A - B - \bar{c} - D$ notation

$$\langle p'|\hat{T}^a_{\mu\nu}|p\rangle = \bar{u}(p')\Bigg[\, A^a(t,\mu^2)\,\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} + B^a(t,\mu^2)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M} + \bar{c}^a(t,\mu^2)g_{\mu\nu} + D^a(t,\mu^2)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M}\Bigg]u(p)$$

EMT conservation: $A(t) = \sum_a A^a(t,\mu^2)$, $B(t)$, $D(t)$ scale invariant, $\sum_a \bar{c}^a(t,\mu^2) = 0$

constraints: **mass** $\Leftrightarrow$ $A(0) = 1$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon momentum

spin $\Leftrightarrow$ $B(0) = 0$ $\Leftrightarrow$ total anomalous gravitomagnetic moment vanishes *

D-term $\Leftrightarrow$ $D(0) \equiv D$ $\rightarrow$ unconstrained! **Least known global property!**

$$2P = (p' + p)$$
$$\Delta = (p' - p)$$
$$t = \Delta^2$$

notation: $A^q(t) + B^q(t) = 2\,J^q(t)$

$D^q(t) = \frac{4}{5}\,d_1^q(t) = \frac{1}{4}\,C^q(t)$ or $C^q(t)$

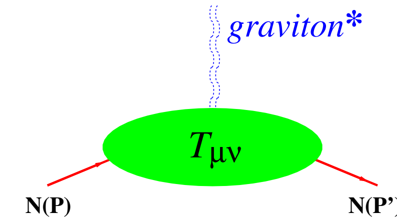
$A^q(t) = M_2^q(t)$

* equivalent to: total nucleon spin $J^q + J^g = \frac{1}{2}$ is due to quarks + gluons (via Gordon identity)

Brief history of GFFs (many notations!)

- 1962 Kobzarev & Okun: $f_1(t)$, $f_2(t)$, $f_3(t)$, $f_4(t)$, $f_5(t)$, $f_6(t)$ (including parity violation)
- 1966 Pagels: spin-$\frac{1}{2}$ $\theta_1(t)$, $\theta_2(t)$, $\theta_3(t)$
- 1980 Novikov, Shifman, Voloshin, Zakharov $\psi' \to J/\psi\,\pi\pi$ $\rightsquigarrow$ triggered modest interest
- 1990 Donoghue, Gasser, Leutwyler "light Higgs" $\to \pi\pi$ $\rightsquigarrow$ some more modest interest
- 1995 Ji: mass decomposition $\rightsquigarrow$ a little more interest (at that time, now much activity!)
- 1997 Ji: nucleon spin decomposition & GPDs $\boldsymbol{A^q(t)}$, $\boldsymbol{B^q(t)}$, $\underbrace{C^q(t)}$, $\boldsymbol{\bar{c}^a(t)}$ $\rightsquigarrow$ boom!
- 1999 Polyakov, Weiss: "double distributions" and D-term $\rightsquigarrow$ $\boldsymbol{D^a(t)}$
- 2003 Polyakov: stress tensor and 3D interpretation in Breit frame $\rightsquigarrow$ "Druck-term"
- models: 1997 Ji et al (bag), 2007 Goeke et al (chiral quark soliton), ...
- lattice: 2002 Gadiyak et al, 2003 Hägler et al, ... 2019 Shanahan, Detmold
- 2019 Lorcé, Moutarde, Trawiński: elastic frames, 2D interpretation
- 2021 Freese, Miller: more on forces in hadrons on the light front
- 2021 Panteleeva, Polyakov: Abel transforms between 3D $\leftrightarrow$ 2D
- far from complete, tip of iceberg

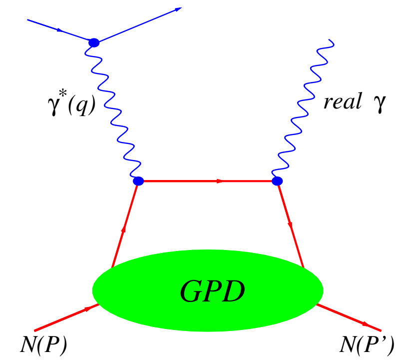
Measuring in experiment



- **gravitational interaction** in principle, but 10^{39} weaker than em

- **hard-exclusive em reactions & GPDs**

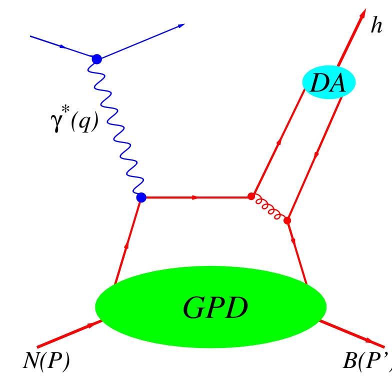
  Müller et al Fortsch. Phys. **42**, 101 (1994)
  Ji, PRL **78**, 610 (1997); PRD **55**, 7114 (1997)
  Radyushkin, PLB **380**, 417, PLB **385**, 333 (1996)
  Collins, Frankfurt, Strikman, PRD **56**, 2982 (1997)



DVCS

$$\int \frac{\mathrm{d}\lambda}{2\pi} e^{i\lambda x} \langle N'(p')|\overline{\psi}_q(-\tfrac{\lambda n}{2})\, n_\mu \gamma^\mu\, \mathcal{W}(-\tfrac{\lambda n}{2}, \tfrac{\lambda n}{2})\, \psi_q(\tfrac{\lambda n}{2})\, |N(p)\rangle$$

$$= \overline{u}(p')\left[n_\mu \gamma^\mu\, \boldsymbol{H^q(x,\xi,t)} + \overline{u}(p')\, \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M}\, \boldsymbol{E^q(x,\xi,t)}\right] u(p)$$



HMP

- **GPDs** $\rightarrow$ **polynomiality** $\rightarrow$ **GFFs** (Ji 1997)

$$\int \mathrm{d}x\; x\, H^q(x,\xi,t) = \boldsymbol{A^q(t)} + \xi^2 \boldsymbol{D^q(t)}$$
$$\int \mathrm{d}x\; x\, E^q(x,\xi,t) = \boldsymbol{B^q(t)} - \xi^2 \boldsymbol{D^q(t)}$$

$$\mathrm{Re}\mathcal{H}(\xi,t) + i\,\mathrm{Im}\mathcal{H}(\xi,t) = \sum_q e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H_q(x,\xi,t).$$

CFF

$\rightarrow$ talks by Simonetta Liuti, Kyugseon Joo, Volker Burkert

Applications of GFFs

- proton mass decomposition $\rightsquigarrow$ $A^a(0)$, $\bar{c}^a(0)$ $\rightsquigarrow$ entire workshops
  Ji 1995, Hatta et al 2018, Tanaka 2019, Metz et al, Rodini et al 2020, Lorcé et al 2021, Ji 2021

- proton spin decomposition $\rightsquigarrow$ $J^a(t)$ $\rightsquigarrow$ even more workshops
  Ji 1996, many works, Leader & Lorcé 2014, many works, Ji, Yuan, Zhao 2021

- D-term $\rightsquigarrow$ $D(0)$ $\rightsquigarrow$ "last unknown global property" of proton
  slowly becoming "least known" global property $\rightsquigarrow$ this talk

- mechanical properties of the nucleon $\rightsquigarrow$ $D(t)$
  very first insights $\rightsquigarrow$ this talk

What means "D-term is least known global property"?

$|N\rangle$ = **strong**–interaction particle. Use other forces to probe it! Simplest observables:

em:	$\partial_\mu J^\mu_{\rm em} = 0$	$\langle N'\vert J^\mu_{\rm em}\vert N\rangle$	$\longrightarrow$	$G_E(t),\ G_M(t)$	$\longrightarrow$	$Q,\ \mu,\ \ \dots$
weak:	PCAC	$\langle N'\vert J^\mu_{\rm weak}\vert N\rangle$	$\longrightarrow$	$G_A(t),\ G_P(t)$	$\longrightarrow$	$g_A,\ g_p,\ \ \dots$
gravity:	$\partial_\mu T^{\mu\nu}_{\rm grav} = 0$	$\langle N'\vert T^{\mu\nu}_{\rm grav}\vert N\rangle$	$\longrightarrow$	$A(t),\ B(t),\ D(t)$	$\longrightarrow$	$M,\ J,\ D,\ \ \dots$

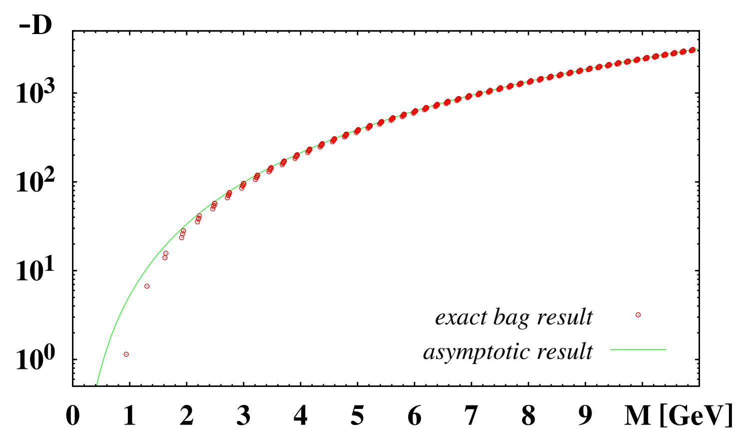
global properties:

$$
\begin{aligned}
Q_p &= 1.602176487(40) \times 10^{-19}\text{C} \\
\mu_p &= 2.792847356(23)\mu_N \\
g_A &= 1.2694(28) \\
g_P &= 8.06(0.55) \\
M_p &= 938.272013(23)\,\text{MeV} \\
J_p &= \tfrac{1}{2} \\
\boldsymbol{D} &= \textbf{?}
\end{aligned}
$$

first insights on D of π^0 and proton
Kumano, Song, Teryaev, PRD97, 014020 (2018)
Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)

Selected theory results on the D-term

- **free spin-$\frac{1}{2}$ theory** $D = 0$ Donoghue et al (2002), Hudson, PS (2018)
- **Goldstone bosons** chiral symmetry breaking $D = -1$
  Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)
- **nuclei** (liquid drop model, Walecka model) $D \propto A^{7/3}$ $\rightarrow$ DVCS with nuclei!
  Polyakov (2002), Guzey, Siddikov (2006); Liuti, Taneja (2005)
- **Q-balls** N^{th} excited Q-ball state: $M \propto N^3$ but $D \propto N^8$ Mai, PS (2012)
- **nucleon, bag model** $D = -1.15 < 0$ Ji, Melnitchouk, Song (1997)
- **chiral quark soliton** Goeke et al, PRD75 (2007) (see next slides)
- **χPT** Belitsky, Ji (2002), Alharazin, Djukanovic, Gegelia, Polyakov PRD102 (2020) 7, 076023
- **lattice QCD** Göckeler et al, PRL92 (2004), ...
  Shanahan, Detmold (2019)
- **dispersion relations**
  Pasquini, Polyakov, Vanderhaeghen (2014)
- **excited states**
  in bag model Neubelt et al (2019)
- **review** Polyakov and PS, (2018)



of all properties, D-term most sensitive
details of interaction $\Rightarrow$ dynamics!

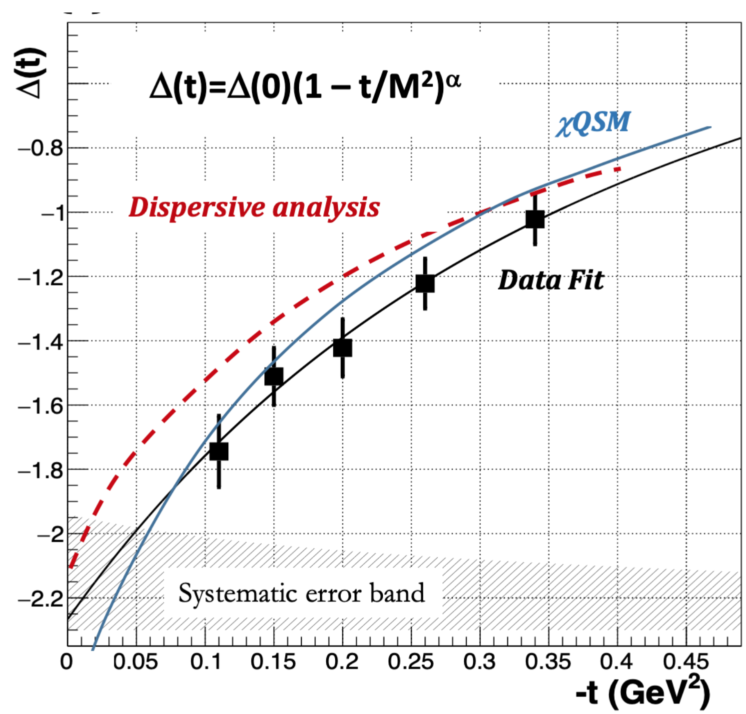
Experimental results on GFFs

- determining $A^a(t)$, $B^a(t)$ of nucleon from data difficult $\rightsquigarrow$ not possible to "deconvolute" $\text{CFF} = \text{Re}\mathcal{H}(\xi,t) + i\,\text{Im}\mathcal{H}(\xi,t) = \sum_q e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H_q(x,\xi,t)$ to get GPDs

- for $D^a(t)$ situation somewhat better $\rightsquigarrow$ fixed-t dispersion relation
  $\text{Re}\mathcal{H}(\xi,t) = \Delta(t,\mu^2) + \frac{1}{\pi}\,\text{P.V.} \int_0^1 \text{d}\xi' \left[\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right] \text{Im}\mathcal{H}(\xi',t)$ and $\Delta(t,\mu^2) \xrightarrow{\mu^2\to\infty} 5\sum_q e_q^2\, \boldsymbol{D^q(t,\mu^2)} + \ldots$
  Anikin & Teryaev 2008, Diehl & Ivanov 2007

- proton $D^q(t)$ from JLab DVCS data
  Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)
  assume corrections small (NLO, $\mathcal{E}(\xi,t)$, d_3^q, d_5^q, $\ldots$)
  $\mathcal{Im}\,\mathcal{H}$ $\rightsquigarrow$ BSA Girod et al PRL 100 (2008) 162002
  $\mathcal{Re}\,\mathcal{H}$ $\rightsquigarrow$ σ_{unp} Jo et al PRL 115 (2015) 212003

  fit: $\Delta(0, 1.5\,\text{GeV}^2) = -2.27 \pm 0.16 \pm 0.36$
  $M^2 = 1.02 \pm 0.13 \pm 0.21\text{GeV}^2$
  $\alpha = 2.76 \pm 0.23 \pm 0.48$
  at $\langle Q^2\rangle = 1.5\,\text{GeV}^2$



$D_{u+d}(0) = -1.63 \pm 0.11 \pm 0.26$ + syst using large N_c
cf. chiral quark soliton model (χQSM) Goeke et al 2008, dispersion relations Pasquini et al 2014
lattice (gluon Shanahan, Detmold 2019); fits: KM 2015, Kumerički Nature 2019 $\rightsquigarrow$ JLab 22 GeV

- D-term of π^0

  EMT form factors of unstable particles

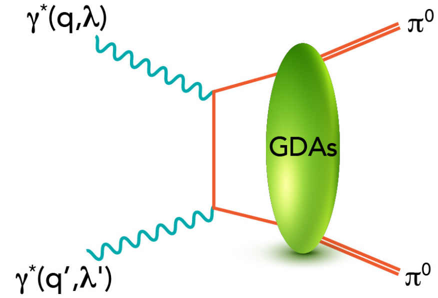
  generalized distribution amplitudes (analyt. cont. of GPDs)

  in $\gamma\gamma^* \to \pi^0\pi^0$ in e^+e^- Belle, PRD 93, 032003 (2016)

  $D^Q_{\pi^0} \approx -0.7$ at $\langle Q^2 \rangle = 16.6$ GeV^2

  Kumano, Song, Teryaev, PRD97, 014020 (2018)

  compatible with soft pion theorem: total $D_{\pi^0} = -1$ for $m_\pi \to 0$



- gluon GFFs of proton

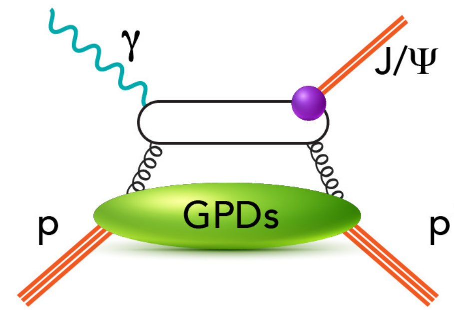
  $\rightsquigarrow$ threshold J/ψ photo-production

  Kharzeev 1995, 2021; Guo et al 2021

  factorization for heavy vector mesons Ivanov et al, 2004,

  does it apply at threshold? Sun, Tong, Yuan 2021

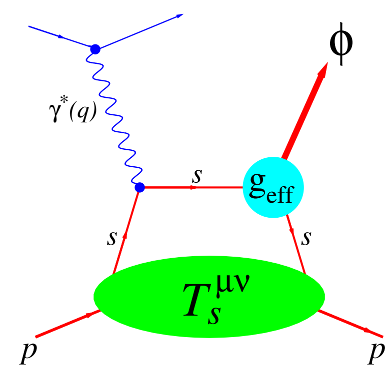
  $\to$ Monday talks by Kiminad Mamo and Lubomir Pentchev



- $D^s(t)$ of proton from $ep \to e'p'\phi$ at threshold

  effective approach (not colinear factorization)

  Hatta and Strikman 2021 $\to$ talk by Mark Strikman, Wednesday(?)



Interpretation of GFFs

- **static 3D EMT** in Breit frame $\Delta^\mu = (0, \vec{\Delta})$ $\quad T_{\mu\nu}(\vec{r}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3}\, e^{-i\vec{\Delta}\,\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle$
  M.V.Polyakov, PLB 555 (2003) 57

$$\int d^3r\, T_{00}(\vec{r}) = M \qquad \text{known}$$

$$\int d^3r\, \varepsilon^{ijk}\, s_i\, r_j\, T_{0k}(\vec{r},\,\vec{s}) = \frac{1}{2} \qquad \text{known}$$

$$-\frac{2}{5}\,M \int d^3r\, \left(r^i r^j - \frac{r^2}{3}\,\delta^{ij}\right) T_{ij}(\vec{r}) \equiv \boldsymbol{D} \qquad \textbf{new!}$$

with: $\quad \boldsymbol{T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\,\delta_{ij}}$ $\quad$ **stress tensor**

$\boldsymbol{s(r)}$ related to distribution of *shear forces*
$\boldsymbol{p(r)}$ distribution of *pressure* inside hadron
$\Big\} \longrightarrow$ **"mechanical properties"**

- **relation to stability**: EMT conservation $\Leftrightarrow$ $\partial^\mu \hat{T}_{\mu\nu} = 0$ $\Leftrightarrow$ $\nabla^i T_{ij}(\vec{r}) = 0$

  $\hookrightarrow$ necessary condition for stability $\int_0^\infty dr\, \boldsymbol{r^2\, p(r)} = 0$ (von Laue, 1911)

  $$\boldsymbol{D} = -\frac{16\pi}{15}\,M \int_0^\infty dr\; r^4 s(r) = 4\pi M \int_0^\infty dr\; \boldsymbol{r^4\, p(r)} \rightarrow \text{balance of internal forces}$$

- 2D interpretation Lorcé et al (2019); Freese, Miller (2021)

  Abel transformations Panteleeva, Polyakov (2021)

mechanical radius

- $T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(r)\,\delta_{ij} =$ symmetric 3×3 matrix $\rightarrow$ diagonalize:

$$\frac{2}{3}s(r) + p(r) = \text{ normal force (eigenvector } \vec{e}_r) \rightsquigarrow \textbf{positive definite!}$$

$$-\frac{1}{3}s(r) + p(r) = \text{ tangential force } (\vec{e}_\theta,\ \vec{e}_\phi, \text{ degenerate for spin 0 and } \tfrac{1}{2})$$

- **mechanical stability** $\Leftrightarrow$ normal force directed towards outside

$$\Leftrightarrow T^{ij}e_r^j\,dA = \underbrace{[\tfrac{2}{3}s(r) + p(r)]}_{>0}e_r^i\,dA \quad \Rightarrow \quad \boldsymbol{D < 0}$$

Perevalova et al (2016)

- $$\langle r^2\rangle_{\rm mech} = \frac{\int d^3r\ r^2[\frac{2}{3}s(r) + p(r)]}{\int d^3r\ [\frac{2}{3}s(r) + p(r)]} = \frac{6D(0)}{\int_{-\infty}^{0}{\rm d}t\,D(t)} \quad \text{vs} \quad \langle r^2_{\rm ch}\rangle = \frac{6\,G'_{E,p}(0)}{G_{E,p}(0)}$$

"anti-derivative"

- proton: $\langle r^2\rangle_{\rm mech} \approx 0.75\,\langle r^2_{\rm ch}\rangle$ from chiral quark soliton model, Goeke et al 2007 ($\rightarrow$ V. Burkert)

- neutron: same $\langle r^2\rangle_{\rm mech}$ as proton while $\langle r^2_{\rm ch}\rangle_{\rm neut} = -(0.11\,{\rm fm})^2$ insightful, but not particle size!

- in chiral limit $\langle r^2\rangle_{\rm mech}$ finite (!) vs $\langle r^2_{\rm ch}\rangle$ divergent

$\Rightarrow$ **mechanical radius better concept for particle size than electric charge radius**

2D vs 3D interpretation

- **3D density** not exact, "relativistic corrections" for $r \lesssim \lambda_{\text{Compt}} = \frac{\hbar}{mc}$
  2D densities exact partonic probability densities

  known since earliest days:
  - Yennie, Levy, Ravenhall, Rev. Mod. Phys. 29 (1957) 144
  - Sachs, Physical Review 126 (1962) 2256
  - Belitsky, Radyushkin, Phys. Rept. 418, 1 (2005), Sec. 2.2.2
  - X.-D. Ji, PLB254 (1991) 456 (Skyrme model, not a big effect)
  - G. Miller, PRC80 (2009) 045210 (toy model, very dramatic effect)
  - Lorcé, PRL 125 (2020) 232002 quasi-probabilistic phase-space average á la Wigner
  - Jaffe, arXiv:2010.15887 (not possible to measure spatial dependence of nucleon matrix elements)
  - Freese, Miller, 2102.01683 (expectation value of a local operator within spatially-localized state)
  - Epelbaum, Gegelia, Lange, Meißner, Polyakov, arXiv:2201.02565

- **2D densities** = **partonic probability densities** (unitarity)
  must (and better be) exact! → M. Burkardt (2000)
  apply to any particle (including the light pion)

  3D densities = **mechanical response functions**
  correlation functions (≠ probabilities!)
  if corrections "reasonably small" → we do not need to worry

  relative correction for $\langle r_E^2\rangle = \int d^3r\, r^2 T_{00}(r)/m$ is $\delta_{\text{rel}} = 1/(2m^2R^2)$ Hudson, PS PRD (2007)

  numerically $\underbrace{\text{pion}}_{220\,\%}$, $\underbrace{\text{kaon}}_{25\,\%}$, $\underbrace{\text{nucleon}}_{3\,\%}$, $\underbrace{\text{deuterium}}_{1\times10^{-3}}$, $\underbrace{{}^{4}\text{He}}_{5\times10^{-4}}$, $\underbrace{{}^{12}\text{C}}_{3\times10^{-5}}$, $\underbrace{{}^{20}\text{Ne}}_{6\times10^{-6}}$, $\underbrace{{}^{56}\text{Fe}}_{5\times10^{-7}}$, $\underbrace{{}^{132}\text{Xe}}_{6\times10^{-8}}$, $\underbrace{{}^{208}\text{Pb}}_{2\times10^{-8}}$

- for nucleon 3D description strictly justified in large-N_c limit
  S. Coleman: "$1/N_c$ only small parameter in QCD at all energies" (in Aspects of Symmetry)

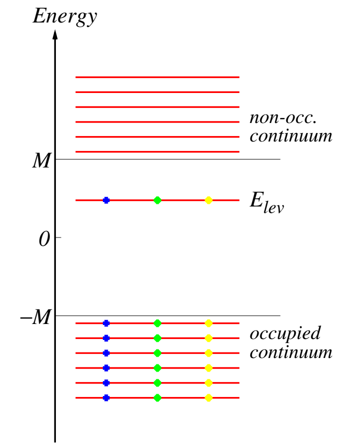
- **important: nucleon mass → heavy, quark mass → anything**

illustration in chiral quark-soliton model

- $\mathcal{L}_{\rm eff} = \overline{\Psi}\,(i\,\partial\!\!\!/ - M\,U^{\gamma_5})\Psi,\quad U = \exp(i\tau^a\pi^a/f_\pi)$
  Diakonov, Petrov, Pobylitsa, NPB 306, 809 (1988)

  solve in large-N_c limit, where $U(x) \to U(\vec{x})$ static mean field
  Witten NPB 223 (1983) 433

  Hamiltonian $H = -i\gamma^0\gamma^i\nabla^i + \gamma^0 M\,U^{\gamma_5}$ with $H\Phi_n(\vec{x}) = E_n\Phi_n(\vec{x})$
  spectrum discrete level and continua



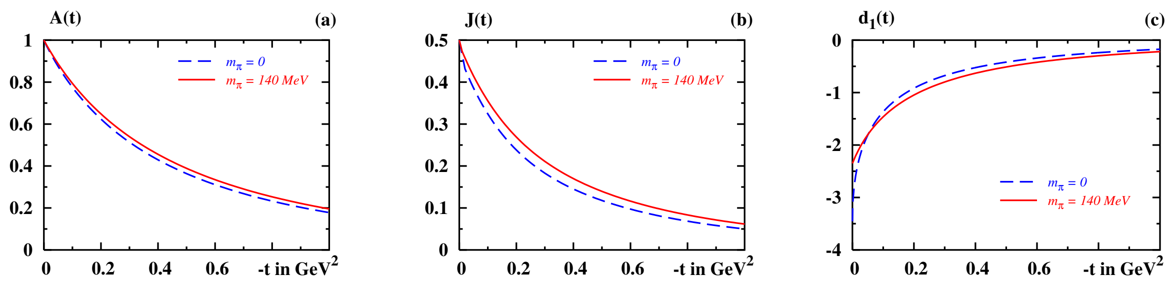
$$\langle N'|\hat{T}_{\mu\nu}|N\rangle = \bar{u}(\vec{p}\,')\left[\frac{P_\mu P_\nu}{M_N}A(t) + \frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N}J(t) + \frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N}D(t)\right]u(\vec{p})$$

$$= \lim_{T\to\infty}\frac{\int\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}\,\mathcal{D}U\;J_N(\tfrac{T}{2})\;\hat{T}_{\mu\nu}\;J_N^\dagger(-\tfrac{T}{2})e^{-\int d^4x_E\,\mathcal{L}_{\rm eff}}}{\int\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}\,\mathcal{D}U\;J_N(\tfrac{T}{2})\;J_N^\dagger(-\tfrac{T}{2})e^{-\int d^4x_E\,\mathcal{L}_{\rm eff}}}$$

$$= 2M_N\int d^3x\;e^{i(\vec{p}\,'-\vec{p})\vec{x}}\,N_c\sum_n\overline{\Phi}_n(\vec{x})(i\gamma^\mu\partial^\nu + i\gamma^\nu\partial^\mu)\Phi_n(\vec{x}) \;+\; \mathcal{O}(1/N_c)$$

$$= 2M_N\int d^3r\;e^{i\vec{\Delta}\,\vec{r}}\,T_{\mu\nu}(\vec{r}) \;+\; \mathcal{O}(1/N_c^2)$$

Goeke et al, PRD75 (2007) 094021

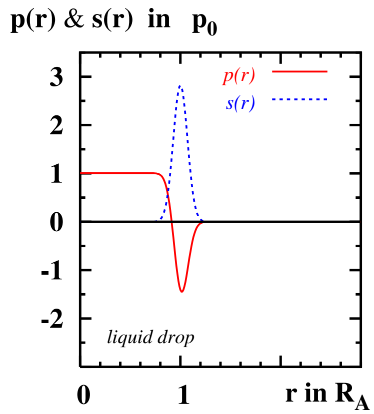


recall $2J(t) = A(t)+B(t)$, $d_1(t) = \frac{4}{5}D(t)$

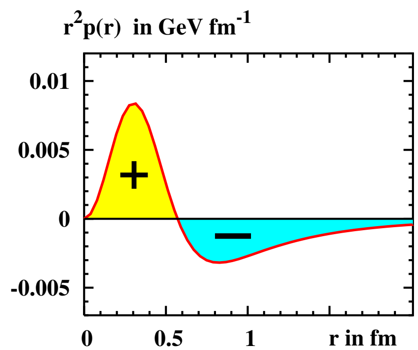
visualization of forces

- **liquid drop model for large nuclei**

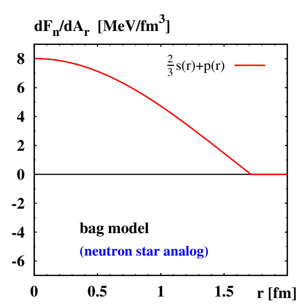
  $p(r) = p_0\,\Theta(R_A - r) - \frac{1}{3}p_0 R_A\,\delta(r - R_A)$, $\;s(r) = \gamma\,\delta(r - R_A)$
  $R_A = R_0\,A^{1/3}$, $m_A = m_0\,A$, surface tension $\gamma = \frac{1}{2}p_0 R_A$
  D-term $\boldsymbol{D = -\frac{4\pi}{3}\,m_A\,\gamma\,R_A^4 \approx -0.2\,A^{7/3}}$
  M.V.Polyakov PLB555 (2003)
  Guzey, Siddikov (2006); Liuti, Taneja (2005)



- **chiral quark soliton model**
  Goeke et al, PRD75 (2007) 094021
  $p(r) > 0$ for $r > r_0 = 0.57$,
  $p(r) < 0$ for $r < r_0$



- **bag model**
  Neubelt et al PRD101 (2020) 034013
  normal force directed towards outside
  $dF_n^i = T^{ij} e_r^j\,dA_r = \underbrace{[\tfrac{2}{3}\,s(r) + p(r)]}_{>0} e_r^i\,dA_r$



- first visualization based on data → talk by Volker Burkert
  role of gluons → lattice studies → Shanahan, Detmold 2019
  many model interesting studies; intersting include Coulomb forces, D-term divergent
  Kubis & Meissner (2000), Donoghue et al (2002), Varma & PS (2020), Metz et al (2021)

Conclusions

- **EMT** crucial operator $\rightsquigarrow$ gravitational form factors (GFFs)
  important applications: mass & spin decompositions + more!
- **measurability** $A(t)$ and $B(t)$ difficult (cannot "invert CFF")
  $\boldsymbol{D(t)}$ from fixed-t dispersion relation more directly and less model-dependently
- **crucial: scale dependence + wide range of $\boldsymbol{\xi}$** $\rightsquigarrow$ measurement at different energies
  EIC good energies but ξ-coverage? Crucial compare JLab 6, **JLab 12**, **JLab 22**
- **$\boldsymbol{D}$-term least known** global property, for fermions generated dynamically
  negative: Goldstone bosons, models, dispersion relations, lattice QCD, experiment
- **3D interpretation** strictly correct in large N_c and intuitive (pressure is 3D!)
  2D formalism can be given a meaning, mathematically equivalent (Abel transform)
- **mechanical stability** $\rightsquigarrow$ normal force $\frac{2}{3}s(r)+p(r)$ positive definite
  more than analogy & fully consistent
- **visualization of internal forces**
  appealing and insightful application
- **mechanical radius** $\langle r_p^2\rangle_{\text{mech}} = \langle r_n^2\rangle_{\text{mech}}$
  smaller than $\langle r_p^2\rangle_{\text{el}}$ in physical situation, finite in chiral limit
- **mechanical properties** $\rightsquigarrow$ many fascinating lessons to learn about hadrons

Thank you!

Support slides

D-term in theory

- **free spin-0 particle** $D = -1$
  Pagels 1966; Hudson, PS 2017
- **free spin** $\frac{1}{2}$ **particle** $D = 0$
  Donoghue et al, (2002), Hudson, PS PRD97 (2018) 056003
- **Goldstone bosons** chiral symmetry breaking $D = -1$
  Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)

$$\begin{aligned}
D_\pi &= -1 + 16\,a\,\frac{m_\pi^2}{F^2} + \frac{m_\pi^2}{F^2}\,I_\pi - \frac{m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\
D_K &= -1 + 16\,a\,\frac{m_K^2}{F^2} + \frac{2m_K^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\
D_\eta &= -1 + 16\,a\,\frac{m_\eta^2}{F^2} - \frac{m_\pi^2}{F^2}\,I_\pi + \frac{8m_K^2}{3F^2}\,I_K + \frac{4m_\eta^2 - m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4)
\end{aligned}$$

$a = L_{11}(\mu) - L_{13}(\mu)$
$I_i = \frac{1}{48\pi^2}\,(\log\frac{\mu^2}{m_i^2} - 1)$
$i = \pi,\ K,\ \eta.$

$D_\pi = -0.97 \pm 0.01$
$D_K = -0.77 \pm 0.15$
$D_\eta = -0.69 \pm 0.19$
Donoghue, Leutwyler (1991)
estimates: Hudson, PS (2017)

- **nuclei** (liquid drop model, Walecka model) $\boldsymbol{D \approx -0.2 \times A^{7/3}}$ $\rightarrow$ DVCS with nuclei!
  Polyakov (2002),
  Guzey, Siddikov (2006);
  Liuti, Taneja (2005)

$$\begin{aligned} {}^{12}\text{C}: \quad D &= -6.2 \\ {}^{16}\text{O}: \quad D &= -115 \\ {}^{40}\text{Ca}: \quad D &= -1220 \\ {}^{90}\text{Zr}: \quad D &= -6600 \\ {}^{208}\text{Pb}: \quad D &= -39000 \end{aligned}$$

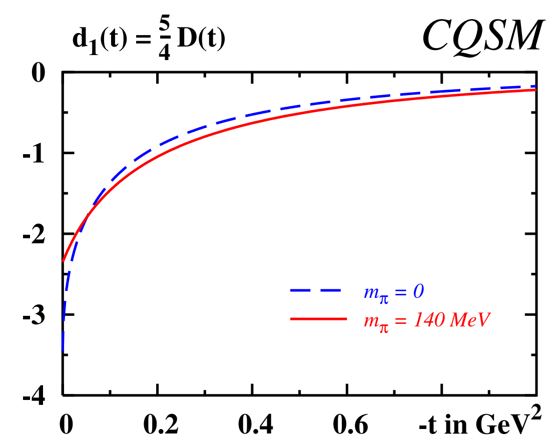
- **Q-balls** N^{th} excited Q-ball state: mass $M \propto N^3$ but $\boldsymbol{D \propto N^8}$
  Mai, PS PRD**86**, 096002 (2012)

- **nucleon, bag model** $\boldsymbol{D = -1.15 < 0}$
  Ji, Melnitchouk, Song (1997)

- **chiral quark soliton**
  Goeke et al, PRD75 (2007)

$$d_1(m_\pi) = \overset{\circ}{d}_1 + \frac{5\,k\,g_A^2 M}{64\,\pi\,f_\pi^2}\,m_\pi + \dots$$

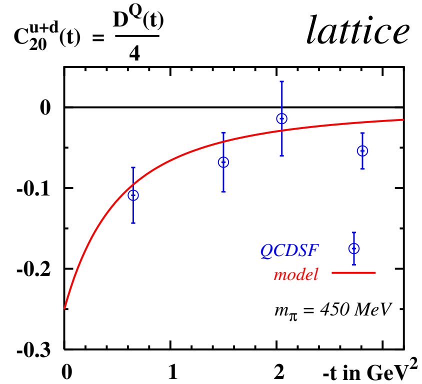
$$\overset{\circ}{d}{}_1'(0) = -\frac{k\,g_A^2\,M}{32\,\pi f_\pi^2\,m_\pi} + \dots \quad k = \begin{cases} 1, & N_c \text{ finite} \\ 3, & N_c \to \infty \end{cases}$$



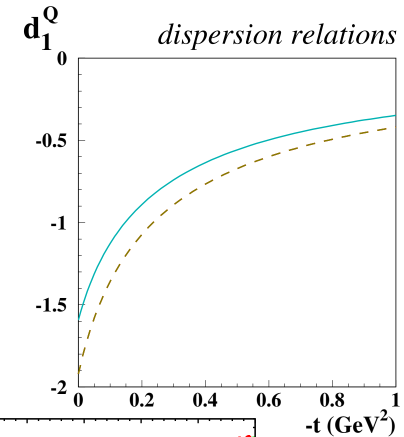
- **χPT**
  Belitsky, Ji (2002), Diehl et al (2006),
  Alharazin, Djukanovic, Gegelia, Polyakov PRD102 (2020) 7, 076023

- **non-relativistic limit** $\boldsymbol{D = -\,N_c^2\,\frac{4\pi^2 - 15}{45}} = -\,4.89$
  Neubelt et al (2019) (in bag)

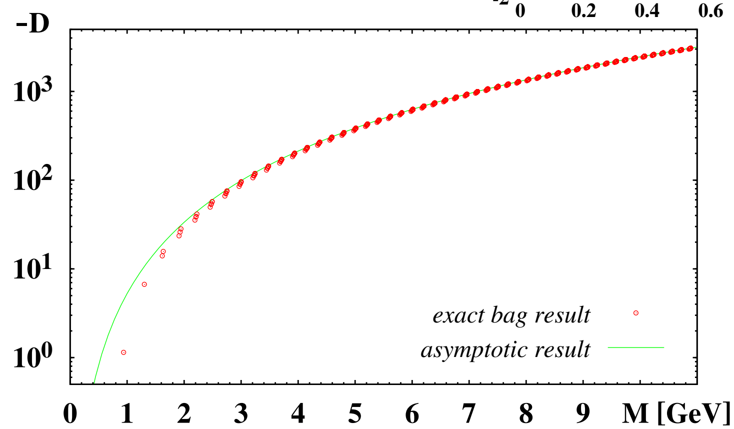
- **lattice:** QCDSF
  Göckeler et al, PRL92 (2004)
  $\mu = 2\,\text{GeV}$, $m_\pi = 450\,\text{MeV}$
  disconnected diagrams neglected
  recently:
  $D^g(t) < 0$ with $|D^g(t)| > |D^Q(t)|$
  Shanahan, Detmold, PRD99 (2019)



- **dispersion relations** $d_1^Q(t) = \frac{5}{4}\,D^Q(t)$
  Pasquini, Polyakov, Vanderhaeghen (2014)
  pion PDFs are input, scale $\mu^2 = 4\,\text{GeV}^2$



- **excitated stats**
  in bag model Neubelt et al (2019)
  M over 1 order of magnitude
  D over 3 orders of magnitude



of all properties, D-term most sensitive (parameters, excitations)

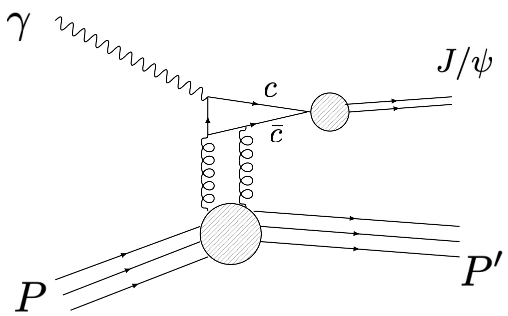
$\Rightarrow$ dynamics!

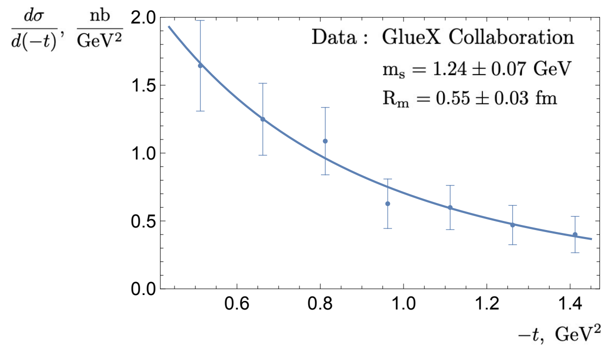
keep in mind: free spin $\frac{1}{2}$ theory $\to D = 0$;

i.e. D-term of nucleon due to dynamics!

- **form factor of** $\hat{T}_\mu^{\ \mu} = \frac{\beta(g)}{2g} F^2 + \mathcal{O}(m_q)$ from J/ψ photoproduction at threshold

Hatta 2019, Kharzeev 2021





Gluex PRL 123, 072001 (2019).

$\sqrt{\langle r^2_{\text{trace}} \rangle} = 0.55 \pm 0.03$ fm $<$ charge radius ~ 0.84 fm

$\sqrt{\langle r^2_{\text{traceless}} \rangle_g} \sim (0.3\text{–}0.35)$ fm of $A^g(t) = A^g(0) + \frac{1}{6}\, t\, \langle r^2_{traceless} \rangle_g + \ldots$ from QCD sum rules

Braun, Górnicki, Mankiewicz, Schäfer, PLB 302, 291 (1993)

explanation:

$\langle r^2_{\text{trace}} \rangle_g$ due to one-instanton contributions, vs $\langle r^2_{\text{traceless}} \rangle_g$ from instanton-anti-instanton
i.e. suppressed by instanton packing fraction Diakonov, Polyakov, Weiss (1996)

relation to other EMT form factors:

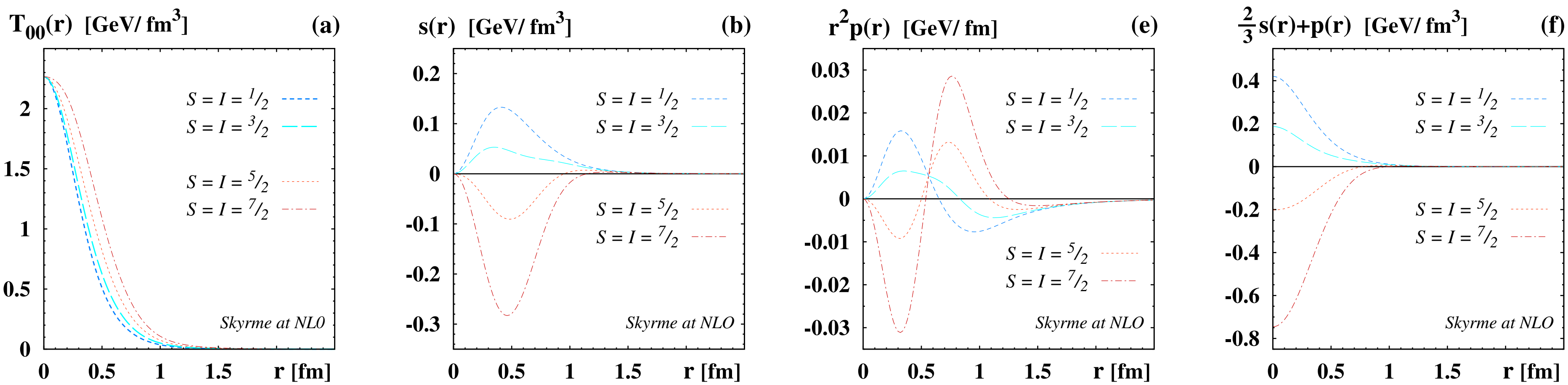
form factor $\langle p' | \hat{T}^\mu_{\ \mu} | p \rangle = \bar{u}(p')u(p)\, F_{\text{tr}}(t)$ where $F_{\text{tr}}(t) = 1 + \frac{1}{6}\, t\, \langle r^2_{\text{tr}} \rangle + \mathcal{O}(t^2)$

$$F_{\text{tr}}(t) = A(t) + \frac{t}{4M^2}\, B(t) - \frac{3t}{4M^2}\, D(t) = 1 + t \left(\frac{dA(0)}{dt} - \frac{3D}{4M^2} \right) + \mathcal{O}(t^2)$$

$\langle r^2_{\text{trace}} \rangle = 6\, A'(0) - \frac{9D}{2M^2}$ "mass radius"

Skyrme model nucleon, Δ vs large-N_c artifacts Witten 1979

- in large N_c baryons = rotational excitations of soliton with $S = I = \underbrace{\frac{1}{2}, \frac{3}{2}}_{\text{observed}}, \underbrace{\frac{5}{2}, \ldots}_{\text{artifacts}}$



$$M_B = M_{\text{sol}} + \frac{S(S+1)}{2\Theta}$$

nucleon $s(r) \neq \gamma\delta(r-R)$
Δ much more diffuse

$$\int_0^\infty \mathrm{d}r\, r^2 p(r) = 0$$

stability requires:
$p(r) > 0$ in center,
negative outside
okay for nucleon, Δ
$\Longrightarrow$ implies $D < 0$

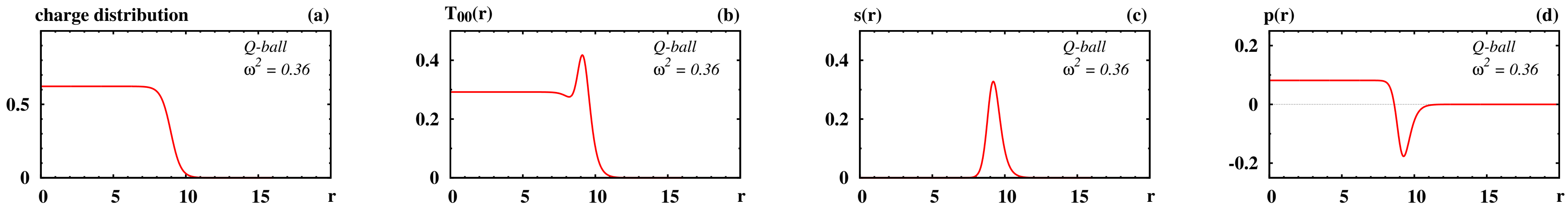
mechanical stability
$T^{ij}\, da^j \geq 0$
$\Leftrightarrow \ \frac{2}{3}s(r) + p(r) \geq 0$
artifacts do not satisfy!
$\Rightarrow$ have positive D-term!
So do not exist!
dynamical understanding
Perevalova et al (2016)

$\Rightarrow$ particles with positive D unphysical!!!

Q-balls $\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V$, $V = A(\Phi^*\Phi) - B(\Phi^*\Phi)^2 + C(\Phi^*\Phi)^3$

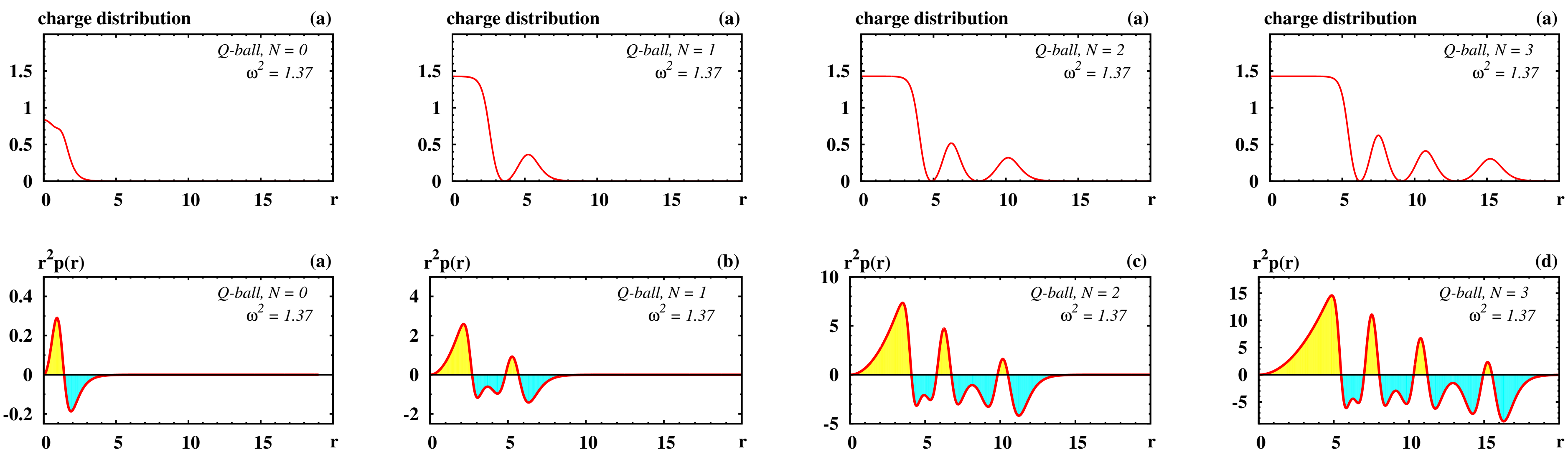
global U(1) symmetry, solution $\Phi(t,\vec{r}) = e^{i\omega t}\,\phi(r)$

- ground state properties for large Q-ball



- excitations: $N = 0$ ground state, $N = 1$ first excited state, etc Volkov, Wohnert 2002; Mai, PS 2012

charge density exhibits N shells, $p(r)$ exhibits $(2N+1)$ zeros

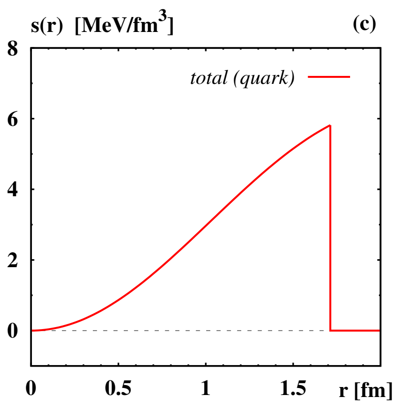


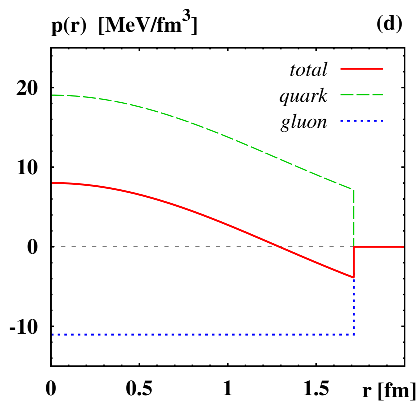
excited states unstable, but $\int_0^\infty \mathrm{d}r\, r^2 p(r) = 0$ always valid, and D-term always negative!

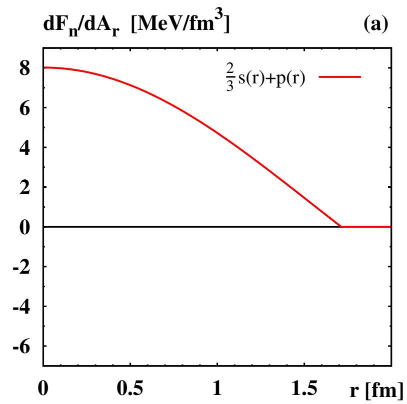
bag model Neubelt, Sampino, Hudson, Tezgin, PS, PRD101 (2020) 034013

- free quarks + boundary condition, formulated in large-N_c
- $T^{\mu\nu}(r) = T^{\mu\nu}_{\text{quarks}}(r) + T^{\mu\nu}_{\text{bag}}(r)$

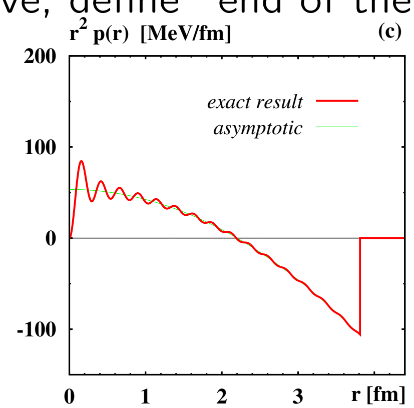
  $T^{\mu\nu}_{\text{bag}}(r) = B\,\Theta(R-r)\,g^{\mu\nu}$ binding effect ("mimics gluons" Jaffe & Ji 1991)
- all densities defined with Θ-functions, assume non-zero values at $r = R$







- only exception:
  the normal force = $\frac{2}{3}\,s(r) + p(r) > 0$ for $r < R$, becomes exactly zero at $r = R$
- this is how one determines the radius of a neutron star:
  solve Tolman-Oppenheimer-Volkoff equations with an "equation of state"
  where "radial pressure" $\frac{2}{3}\,s(r) + p(r)$ turns negative, define "end of the system"
- excitated states different pattern than Q-balls:
  $p(r)$ has one node (here 3163th excited state)
  but $D \sim \text{const} \times M^{8/3}$ bag & Q-balls
  deeper reason?



D-term in the presence of long-range forces

Simple relativistic classical model of a finite size particle Białynicki-Birula, Phys. Lett. A 182 (1993) 346

non-interacting "dust particles" within R described by phase-space distribution $\Gamma(\vec{r},\vec{p},t)$ feel 3 forces:

- massive scalar field force (attractive, mass m_S, short range $\sim \frac{1}{r}\,e^{-m_s r}$)
- massive vector field force (repulsive, mass $m_V > m_S$, even shorter range $\sim \frac{1}{r}\,e^{-m_V r}$)
- massless vector field force (repulsive, Coulomb force, infinite range $\sim \frac{1}{r}$)

$$\begin{aligned}\Big[(m-g_S\phi)(\partial_t+\vec{v}\cdot\vec{\nabla}_r)+m\,\vec{F}\cdot\vec{\nabla}_p\Big]\,\Gamma(\vec{r},\vec{p},t) &= 0\,,\\ \partial_\alpha G^{\alpha\beta}+m_V^2V^\beta &= g_V\,j^\beta,\\ (\Box+m_S^2)\phi &= g_S\,\rho\,,\\ \partial_\alpha F^{\alpha\beta} &= e\,j^\beta,\end{aligned}$$

with $j^\alpha(\vec{r},t)=\displaystyle\int\frac{d^3p}{E_p}\,p^\alpha\,\Gamma(\vec{r},\vec{p},t),\quad \rho(\vec{r},t)=\int\frac{d^3p}{E_p}\,m\,\Gamma(\vec{r},\vec{p},t).$ relativistically invariant.

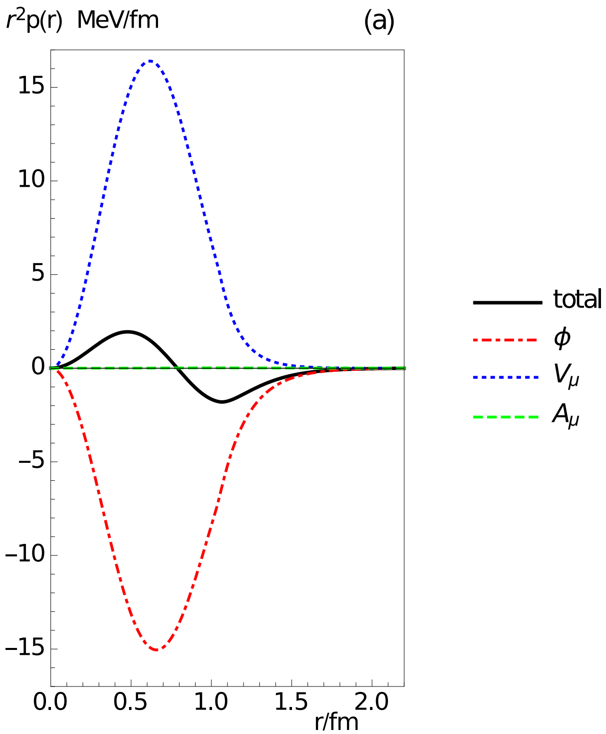
parameters from model QHD-I of the mean field theory of nuclear matter Serot, Walecka (1986)

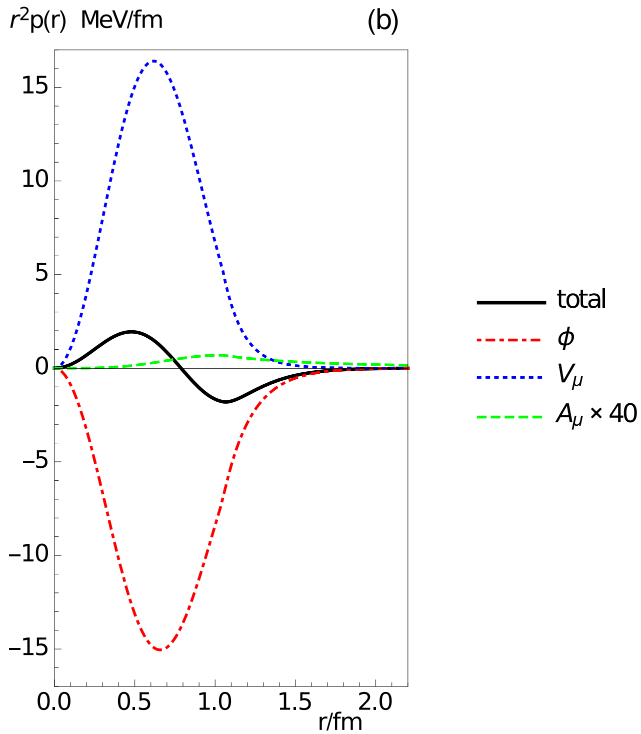
$$m_S=550\,\text{MeV},\quad m_V=783\,\text{MeV},\quad \frac{g_S^2}{4\pi\hbar c}=7.29\,,\quad \frac{g_V^2}{4\pi\hbar c}=10.84\,,\quad \alpha=\frac{e^2}{4\pi\hbar c}=\frac{1}{137},$$

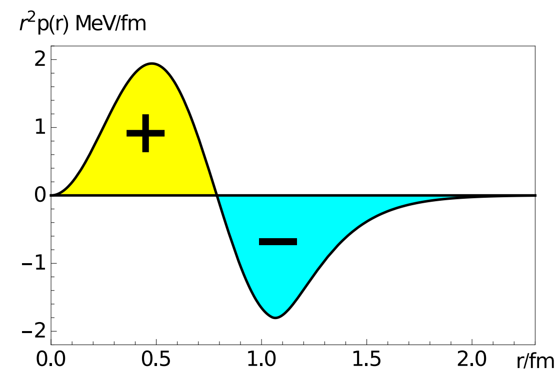
Can be solved analytically, describes particle of charge radius 0.71 fm ("proton") Białynicki-Birula (1993)

We use it to investigate in consistent framework effects of long-range forces Varma, PS (2020)

• **usual features in inner region $r < 2\,\mathrm{fm}$**







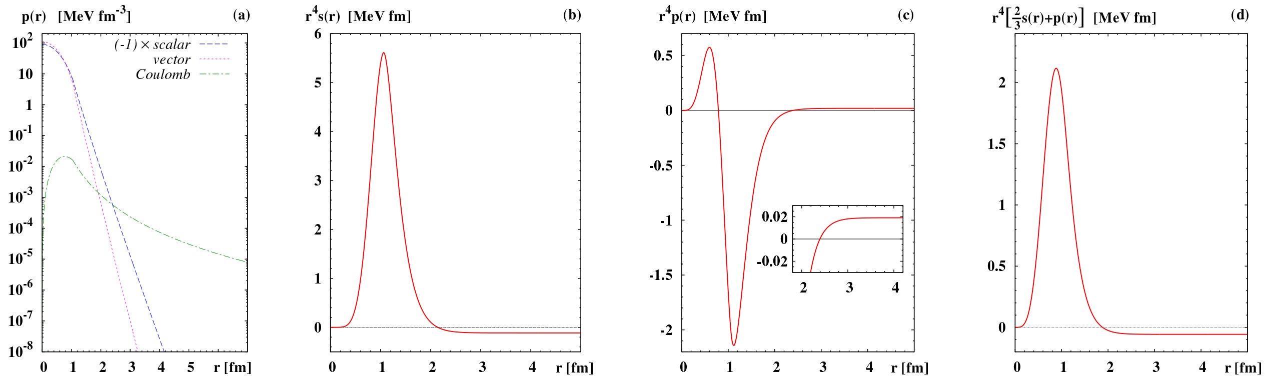
strong forces (scalar and vector fields ϕ and V^μ) make large contributions
about $10\times$ smaller than in chiral quark soliton ("residual nuclear forces")
Coulomb field minuscule contribution, hardly visible

$p(r)$ exhibits node at $r = 0.788\,\mathrm{fm}$, balance of forces:

$$\int dr\; r^2 p_i(r) = \begin{cases} -10.916\,\mathrm{MeV} & \text{for} \quad i = \text{scalar}, \\ +10.891\,\mathrm{MeV} & \text{for} \quad i = \text{vector}, \\ +\;\;0.025\,\mathrm{MeV} & \text{for} \quad i = \text{Coulomb}. \end{cases}$$

So far, same picture as in systems with short-range forces.
But we are looking at the region of $r < 2\,\mathrm{fm}$. Let's look more closely at larger r ...

- **unusual features in outer region $r > 2\,\mathrm{fm}$**



- at large $r > 2\,\mathrm{fm}$, Coulomb contribution takes over! Consequences!!

- shear forces $s(r)$ exhibit a node (in short-range systems $s(r) > 0$)
  $p(r)$ has 2nd node at 2.4 fm (short-range systems one node)
  normal force turns negative (in short range systems $>$ 0)

- model is still mechanically stable:
  dust particles within $R = 1.05\,\mathrm{fm}$
  where features "as usual"

- D-term is affected by that ...
  (most sensitive to dynamics!!)

- **consequences for D-term**

$$D(t) = \text{(regular strong part)} + \frac{\alpha}{\pi}\Big(-\frac{11}{18} + \frac{\pi^2 M}{4\sqrt{-t}} + \frac{2}{3}\log\frac{(-t)}{M^2}\Big)$$ QED part model-independent!

- from QED diagrams Donoghue, Holstein, Garbrecht, Konstandin,

- long-range tail of densities ⇔ small-t behavior of $D(t)$
  due to exchange of massless photons (also the "classical Coulomb potential")

- model independent features, seen in
  Kubis, Meissner, Nucl. Phys. A **671**, 332 (2000)
  Metz, Pasquini, Rodini, PLB **820**, 136501 (2021)
  X. Ji and Y. Liu, arXiv:2110.14781 [hep-ph]

Deeper reason:
$T^{ij}(r) = -E^i E^j + \frac{1}{2}\delta^{ij}\vec{E}^{\,2} = -\sigma^{ij}$
(σ^{ij} Maxwell stress tensor, with $\vec{E} \sim \frac{1}{r^2}$ for $r > R$)

$\rightarrow$

$$T_{00}(r)_{\text{QED}} = \frac{1}{2}\,\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4}$$
$$s(r)_{\text{QED}} = -\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4}$$
$$p(r)_{\text{QED}} = \frac{1}{6}\,\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4}$$

Important: in classical model **consistently** incorporated!
balance of forces: von Laue condition $\boldsymbol{\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0}$
consistent nonperturbative solution, proton stable!