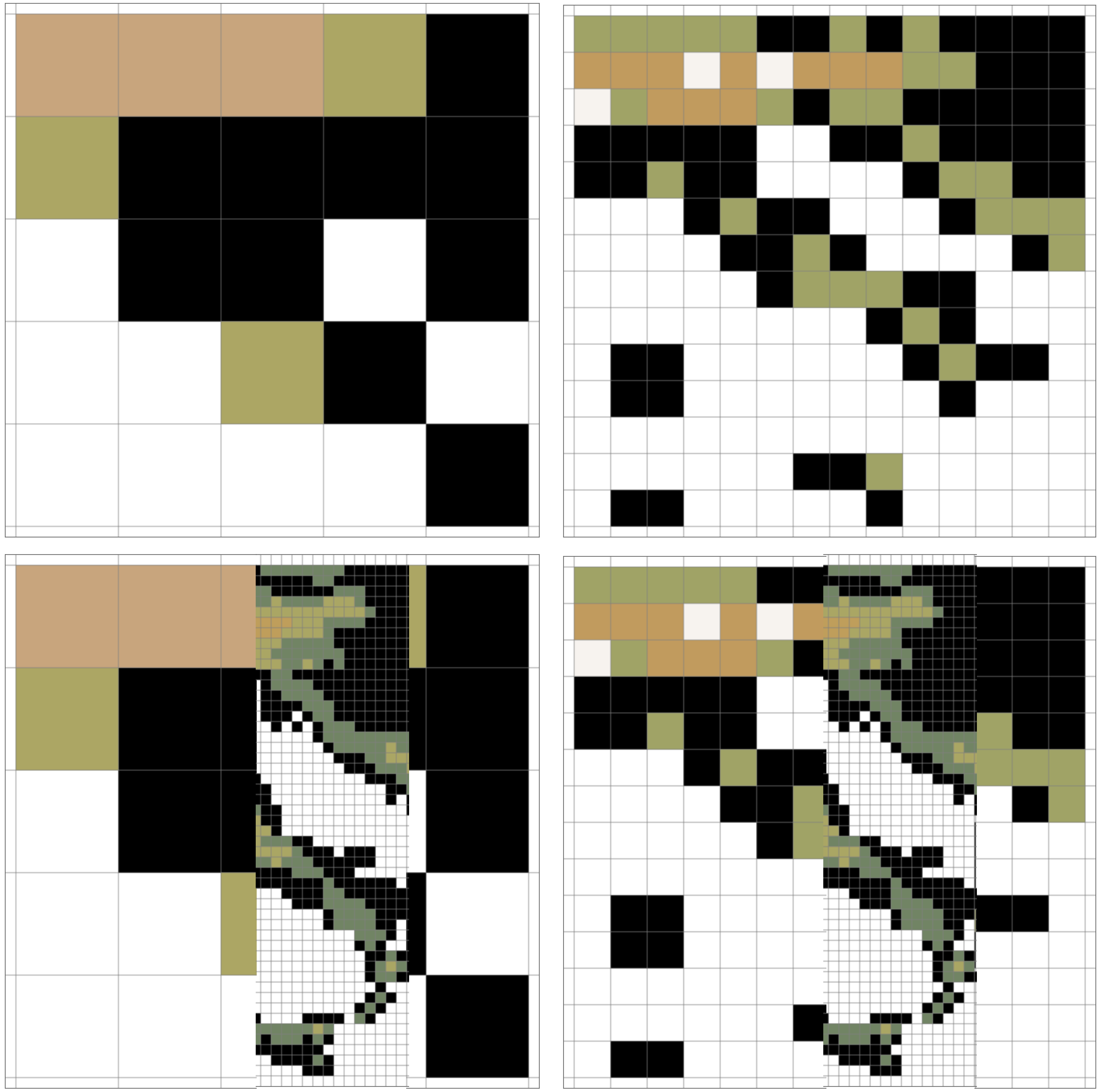
ALESSANDRO BACCHETTA

STUDIES OF STRUCTURE FUNCTIONS IN SIDIS – THEORY

IMPROVING OUR 3D MAPS



Present data

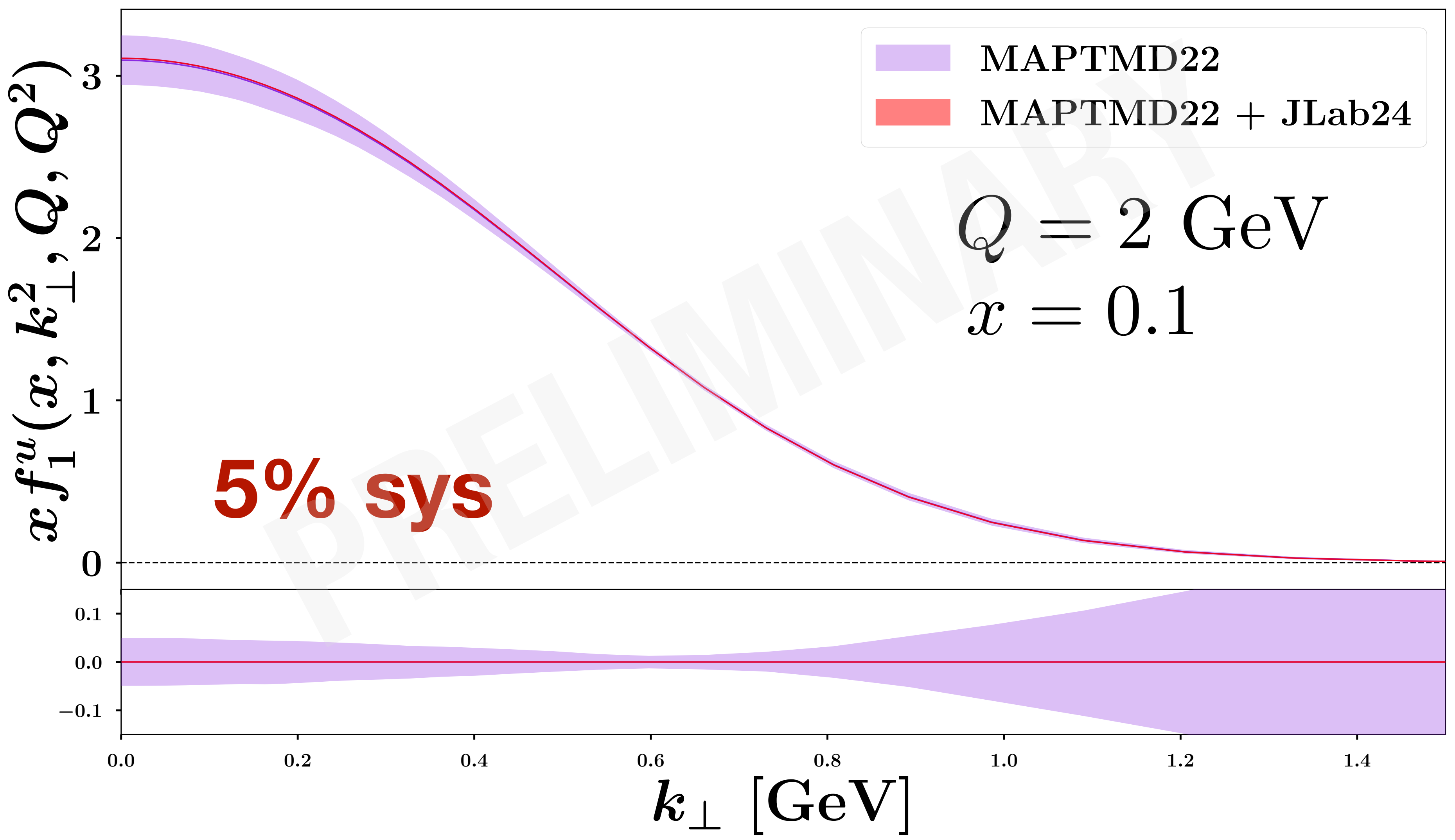
+ EIC

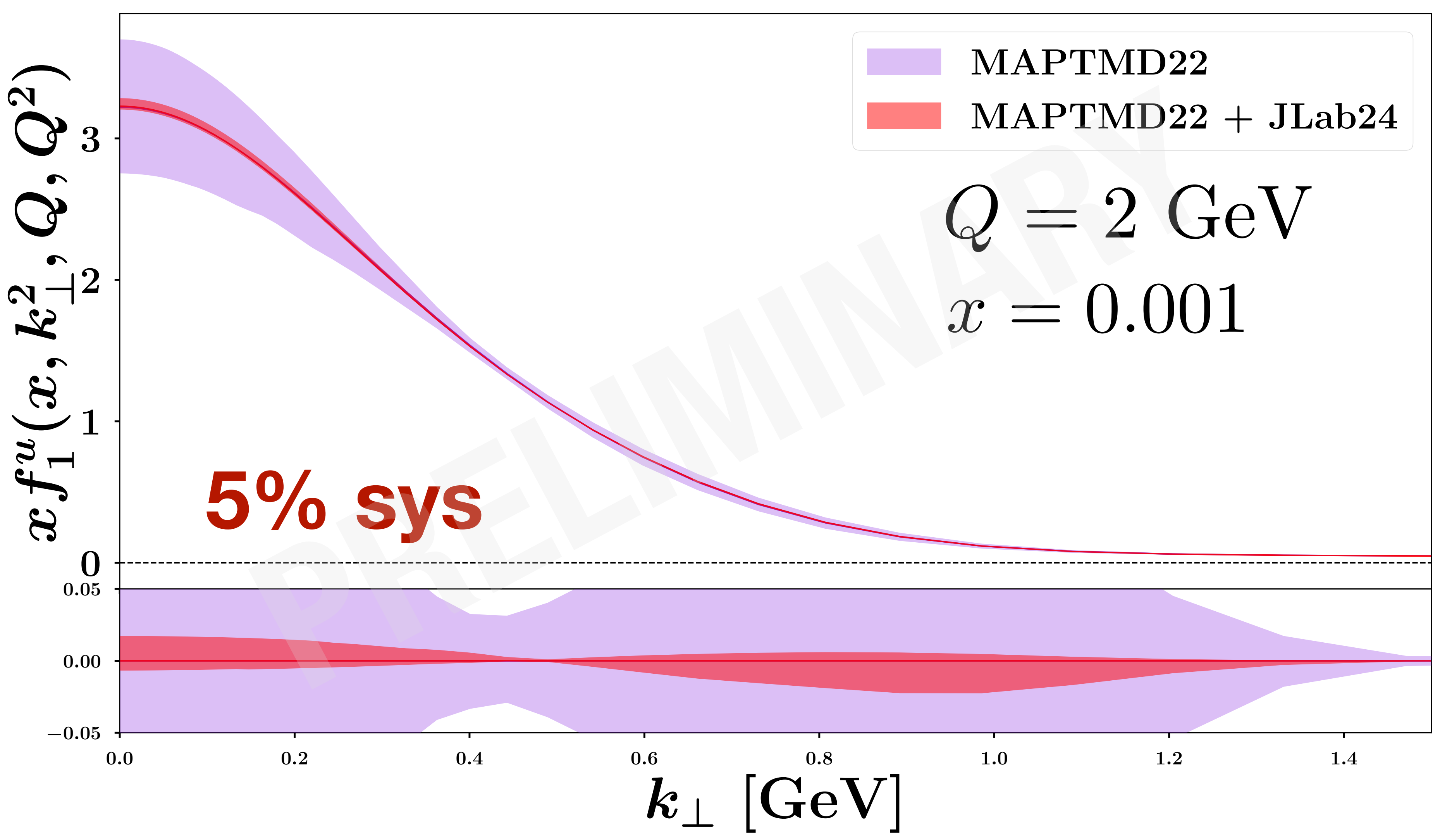
+ JLab 24

+ EIC
+ JLab 24

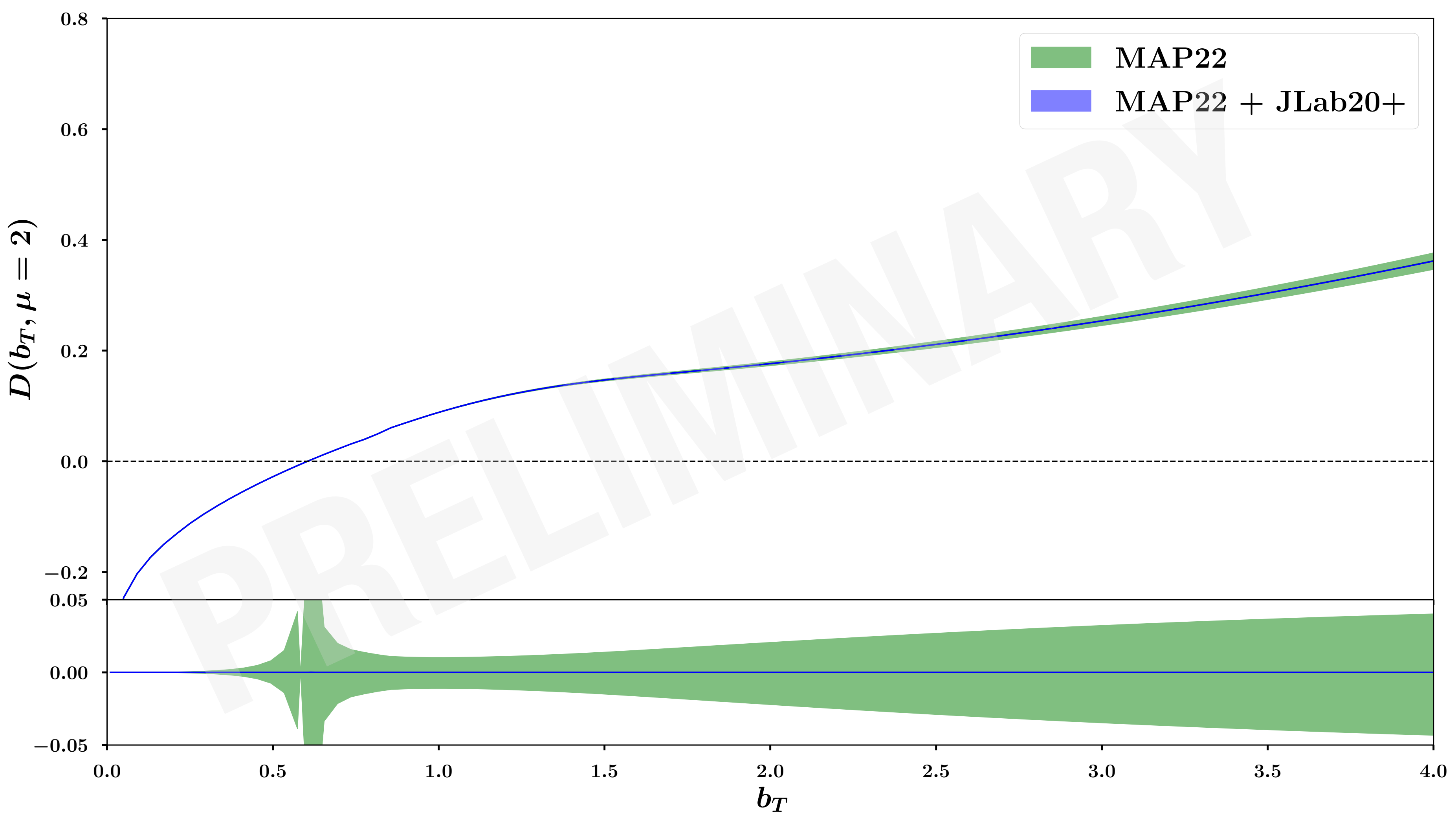
JLAB 24 IMPACT STUDIES ON TMDS

M. Cerutti, talk at Trento workshop Sep 2022





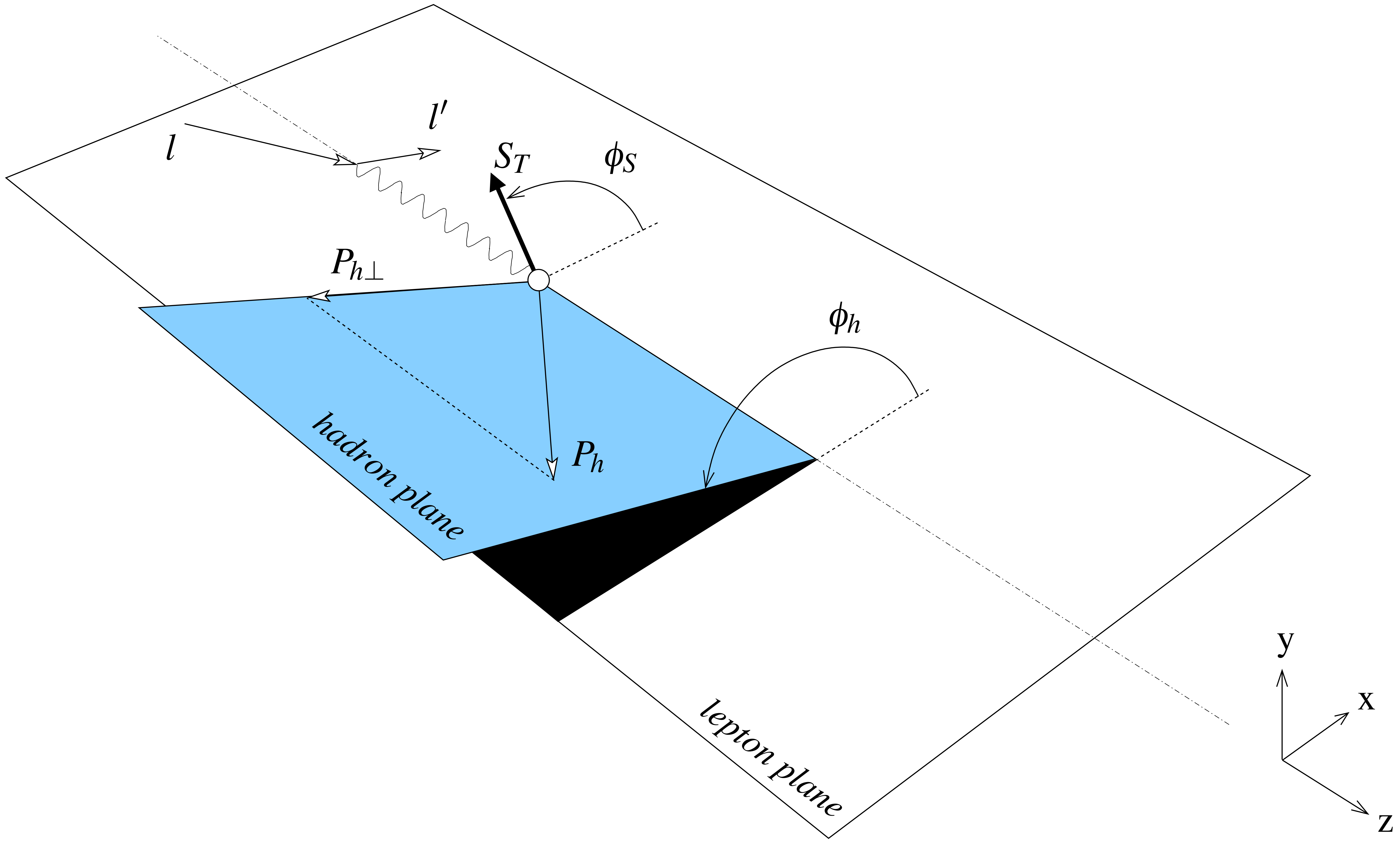
Collins-Soper kernel
(driving TMD evolution)



JLab 24 can have a very significant impact in reducing the errors on TMDs and their evolution

SIDIS KINEMATICS

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, hep-ph/0611265



Q = photon virtuality

M = hadron mass

$P_{h\perp}$ = hadron transverse momentum = P_T $\qquad q_T^2 \approx P_{h\perp}^2/z^2$

SIDIS STRUCTURE FUNCTIONS

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, hep-ph/0611265

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\,\Bigg\{ F_{UU,T} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\, F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\; F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\, F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\left[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)}\right. \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\, F_{UT}^{\sin\phi_S} \\
&\qquad \left. + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)}\right] + S_T\lambda_e\left[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)}\right. \\
&\qquad \left. + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\, F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)}\right]\Bigg\}
\end{aligned}
$$

LIST OF STRUCTURE FUNCTIONS

Bacchetta, Boer, Diehl, Mulders, [arXiv:0803.0227](https://arxiv.org/abs/0803.0227)

	observable	twist
"SIDIS F_T"	$F_{UU,T}$	2
"SIDIS F_L"	$F_{UU,L}$	4
"Cahn" - $f^{\perp}$	$F_{UU}^{\cos\phi_h}$	3
"Boer-Mulders"	$F_{UU}^{\cos 2\phi_h}$	2
e, $g^{\perp}$ and friends	$F_{LU}^{\sin\phi_h}$	3
	$F_{UL}^{\sin\phi_h}$	3
"Kotzinian-Mulders"	$F_{UL}^{\sin 2\phi_h}$	2
"SIDIS g_1"	F_{LL}	2
	$F_{LL}^{\cos\phi_h}$	3
"Sivers"	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2
	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4
"Collins"	$F_{UT}^{\sin(\phi_h+\phi_S)}$	2
"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2
f_T and friends	$F_{UT}^{\sin\phi_S}$	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3

Not all of them are easy to access at EIC due to: x-range, twist, evolution, prefactors

Examples:

probably high-x effect, probably suppressed by evolution, small prefactor (→ $F_{LT}^{\cos(\phi_h-\phi_S)}$)

probably high-x effect, twist-3, small prefactor (→ $F_{LT}^{\cos\phi_S}$)

LOW AND HIGH P_T

	observable	twist (low P_T)	twist (high P_T)
"SIDIS F_T"	$F_{UU,T}$	2	2
"SIDIS F_L"	$F_{UU,L}$	4	2
"Cahn" - $f^\perp$	$F_{UU}^{\cos\phi_h}$	3	2
"Boer-Mulders"	$F_{UU}^{\cos 2\phi_h}$	2	2
e, $g^\perp$ and friends	$F_{LU}^{\sin\phi_h}$	3	2
	$F_{UL}^{\sin\phi_h}$	3	2
"Kotzinian-Mulders"	$F_{UL}^{\sin 2\phi_h}$	2	
"SIDIS g_1"	F_{LL}	2	2
	$F_{LL}^{\cos\phi_h}$	3	2
"Sivers"	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	3
	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4	3
"Collins"	$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	3
"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	3
f_T and friends	$F_{UT}^{\sin\phi_S}$	3	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	3
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3	
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

There are several possibilities:

- Twist 2 TMD matching twist 2 PDF
- Twist 3 TMD matching twist 2 PDF
- Twist 2 TMD matching twist 3 PDF
- Expected mismatch
- Twist 4 TMD matching twist 2 PDF?

UNPOLARIZED AND AZIMUTHALLY INDEPENDENT PART

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, [hep-ph/0611265]

Integrated over ϕ_h

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{h\perp}^2} = \frac{2\pi\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{2\pi F_{UU,T}(x,z,P_{h\perp}^2,Q^2) + \varepsilon\,2\pi F_{UU,L}(x,z,P_{h\perp}^2,Q^2)\right\}$$

Integrated over $P_{h\perp}$

$$\frac{d\sigma}{dx\,dy\,dz} = \frac{4\pi\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_{UU,T}(x,z,Q^2) + \varepsilon\,F_{UU,L}(x,z,Q^2)\right\}$$

Inclusive DIS

$$\frac{d\sigma}{dx\,dy} = \frac{4\pi\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_T(x,Q^2) + \varepsilon\,F_L(x,Q^2)\right\}$$

(alternatively)

$$\frac{d\sigma}{dx\,dQ^2} \approx \frac{2\pi\alpha^2}{x\,Q^2}\left[1+(1-y)^2\right]F_2(x,Q^2)\left\{1 - \frac{y^2}{1+(1-y)^2}\,\frac{R}{1+R}\right\}$$

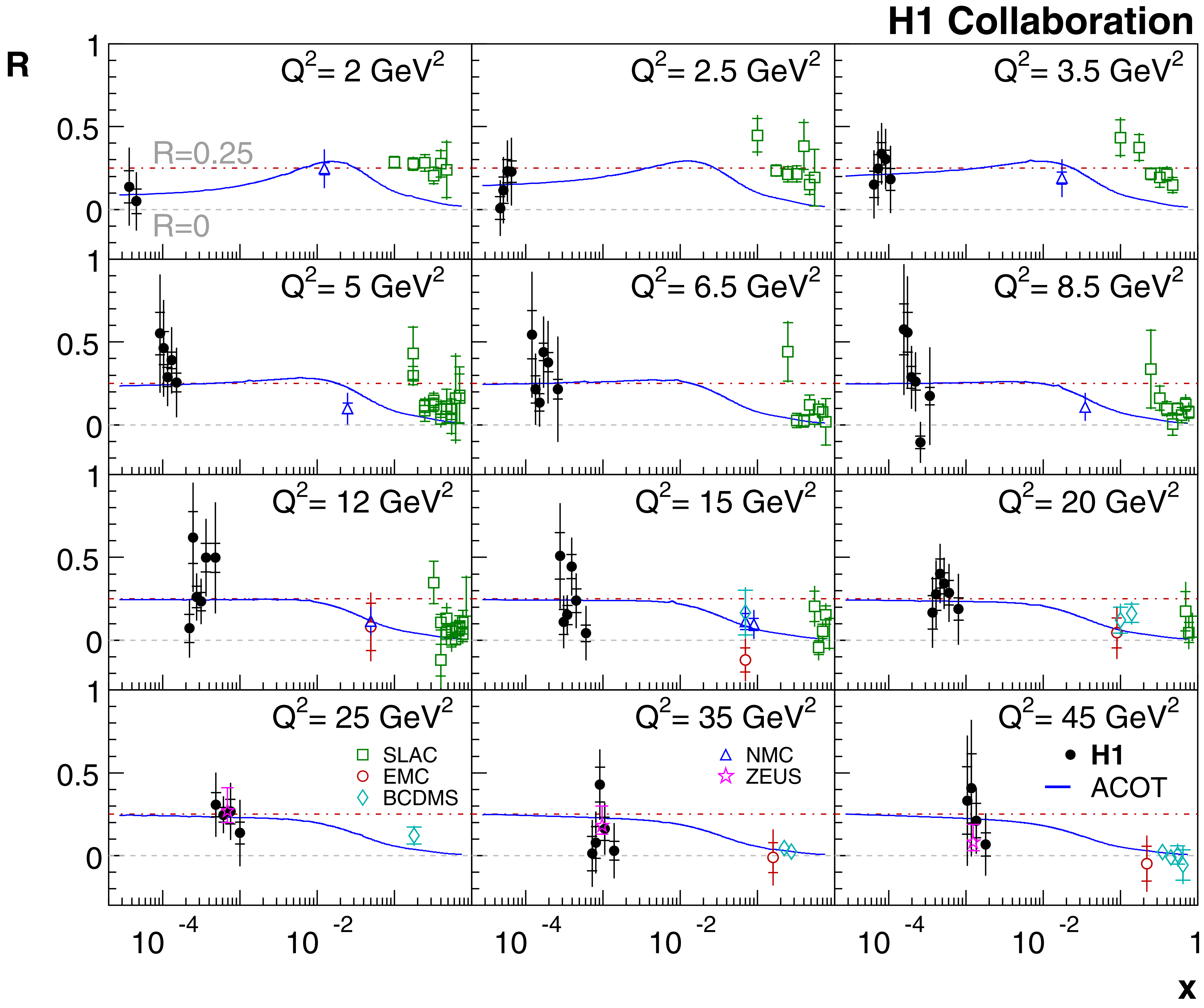
$$F_2 \approx F_T + F_L$$

In all cases, we can define

$$R = \frac{F_{UU,L}}{F_{UU,T}}$$

HOW LARGE IS R? INCLUSIVE DIS

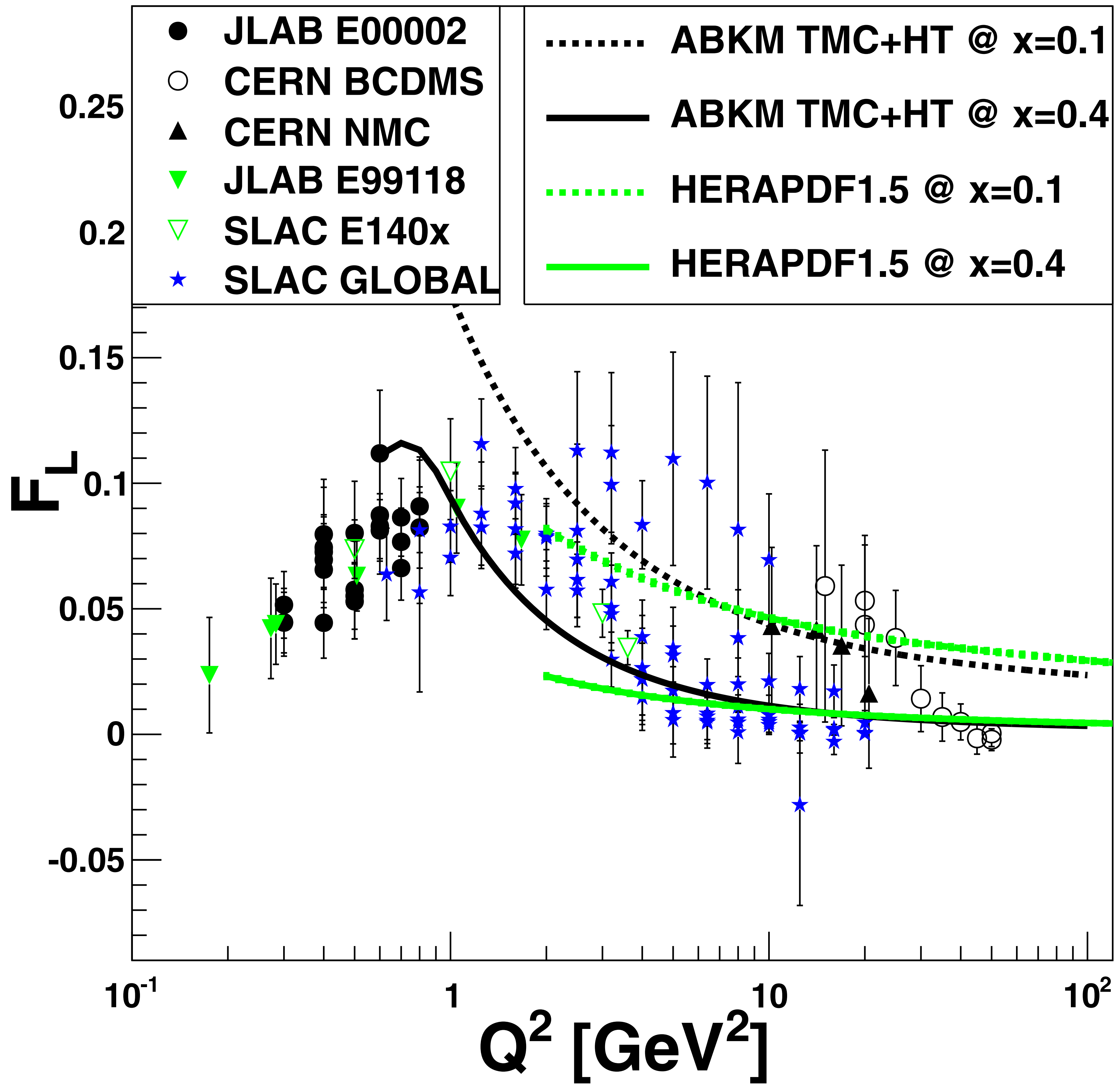
H1 collaboration, [arxiv:1012.4355]



$$R \approx 25\,\%$$

HOW LARGE IS F_L? INCLUSIVE DIS

Tvaskis et al., Hall C, arxiv:1606.02614



Does not behave clearly as $1/Q^2$

Higher twist and mass corrections are relevant

WHAT ABOUT SIDIS?

(schematically only)

Bacchetta, Boer, Diehl, Mulders, arXiv:0803.0227

Low transverse momentum

$$F_{UU,T} = \mathcal{C}\Big[f_1 D_1\Big]$$

Twist 2

$$F_{UU,L} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{P_{h\perp}^2}{Q^2}\right)$$

Twist 4

High transverse momentum

$$F_{UU,T} = \frac{\alpha_s}{P_{h\perp}^2}\Big[f_1 \otimes D_1\Big]_A$$

$$F_{UU,L} = \frac{\alpha_s}{P_{h\perp}^2}\frac{P_{h\perp}^2}{Q^2}\Big[f_1 \otimes D_1\Big]_B = 2\, F_{UU}^{\cos 2\phi_h}$$

Integrated

$$F_{UU,T} = x f_1 D_1 + \alpha_s \Big[f_1 \otimes D_1\Big]_C$$

$$F_{UU,L} = \alpha_s \Big[f_1 \otimes D_1\Big]_D$$

All of them can contain corrections $\mathcal{O}\left(\frac{M^2}{Q^2}, \frac{P_{h\perp}^2}{Q^2}\right)$

Nice about FUUL:
at low transverse momentum
is the cleanest twist-4 signal

POSSIBLE TERMS AT LOW TRANSVERSE MOMENTUM

see, e.g., Wei, Song, Chen, Liang, arxiv:1611.08688

$$F_{UU,L} = \frac{M^2}{Q^2}\mathcal{C}\Big[\frac{4k_T^2}{M^2} f_1 D_1 + \frac{m^2}{M^2} f_1 D_1 + \tilde{f}_2 D_1 + f_1 \tilde{D}_2 + \dots\Big]$$

kinematic twist 4
(à la Wandzura-Wilczek)

mass corrections

dynamic twist 4

sometimes denoted with

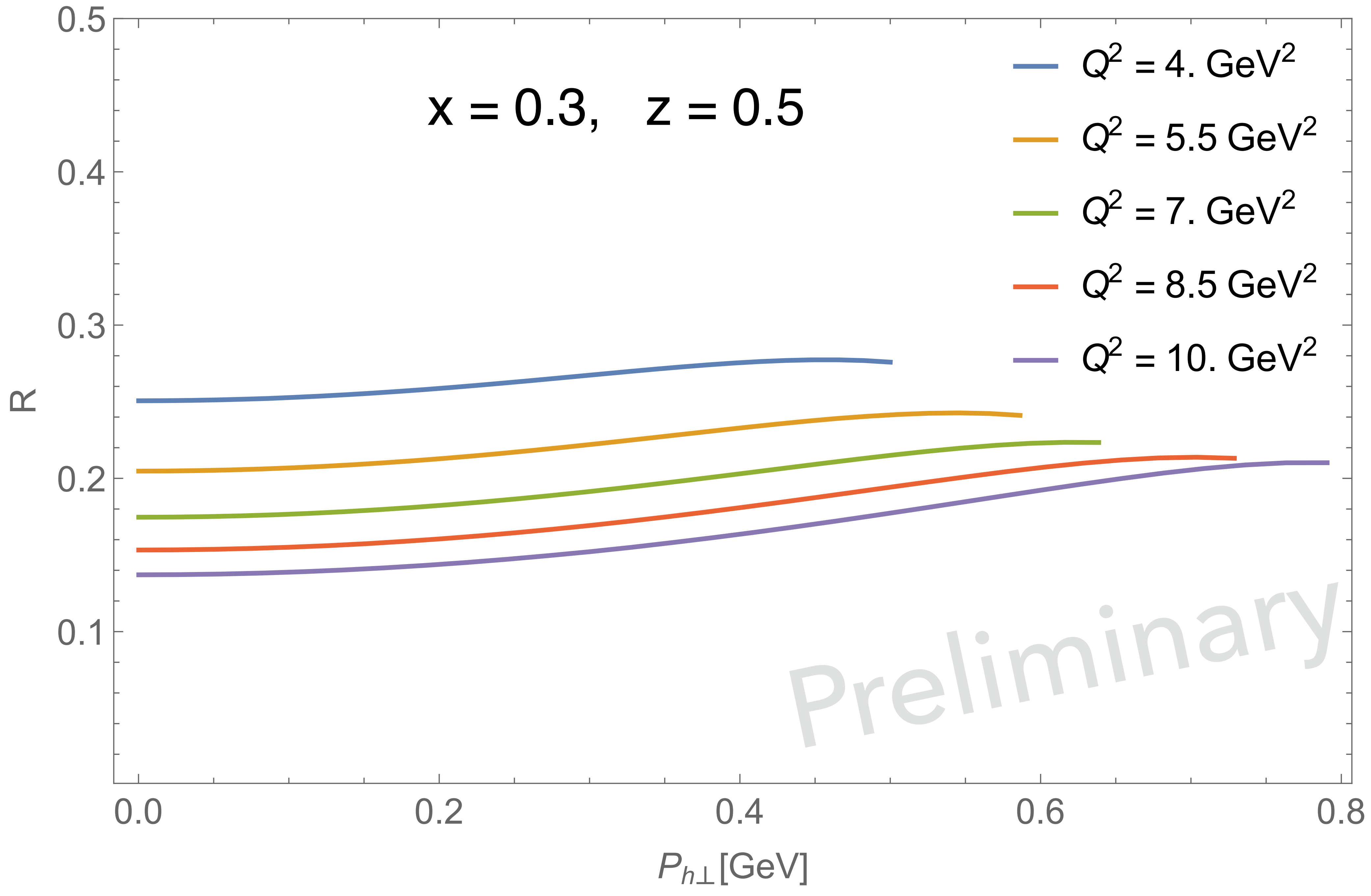
f_3

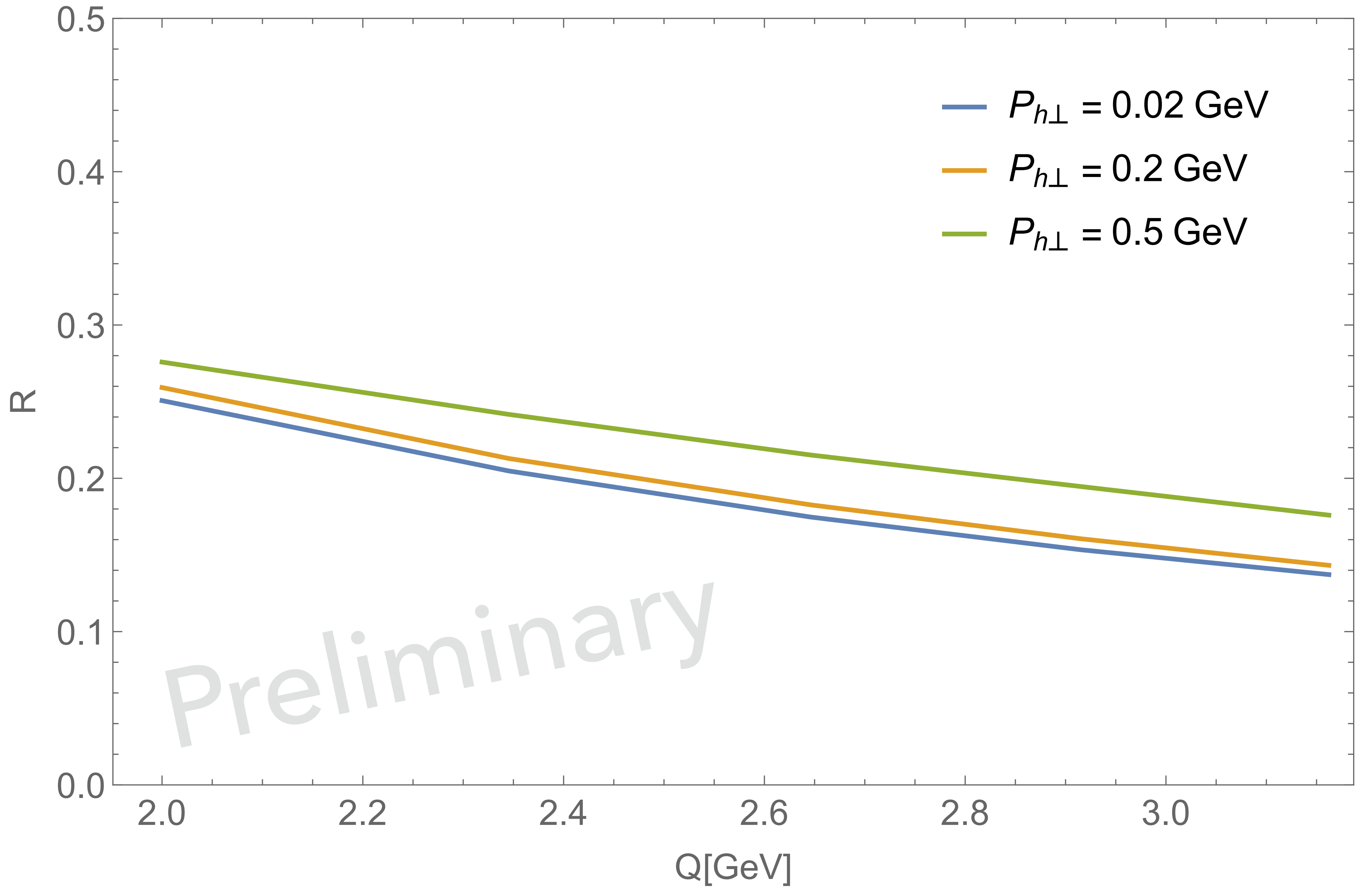
factorization breaking terms?

HOW LARGE COULD IT BE @JLAB24?

similar approach as Anselmino et al., hep-ph/0501196

$$F_{UU,L} \stackrel{?}{=} \frac{M^2}{Q^2}\mathcal{C}\left[\frac{4k_T^2}{M^2} f_1 D_1\right]$$



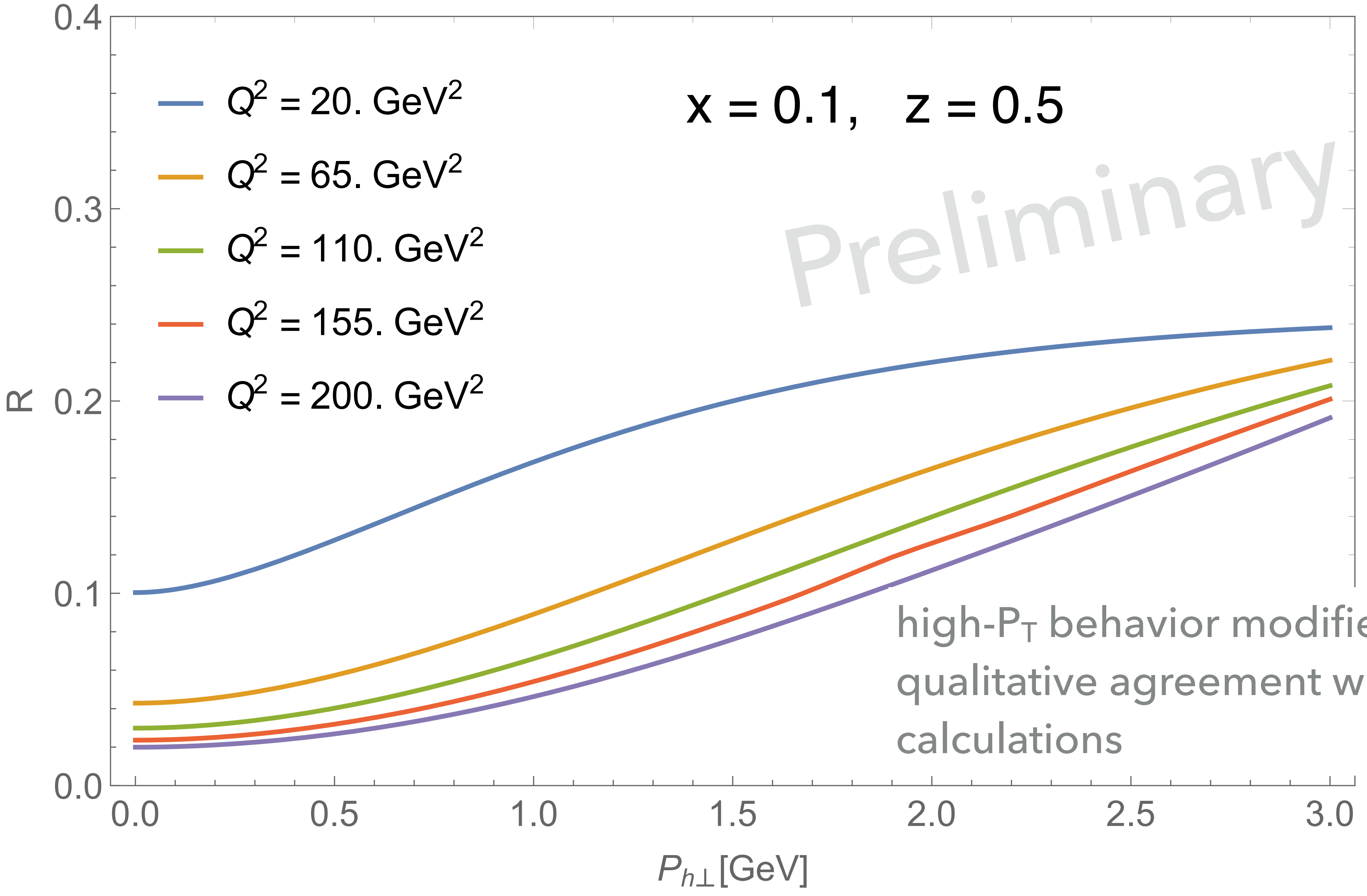


Large size, even at small transverse momentum. Decreases less than $1/Q^2$

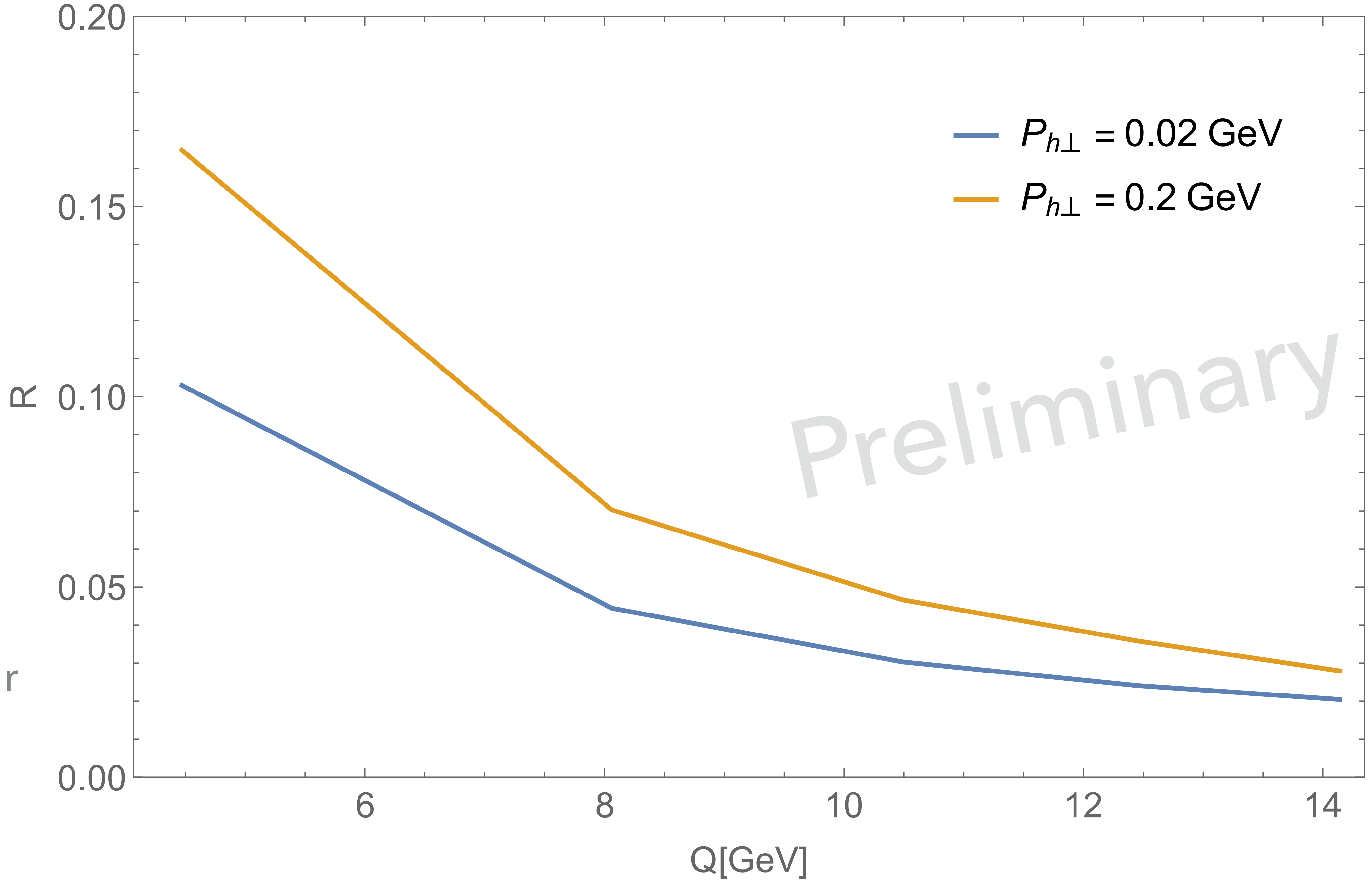
HOW LARGE COULD IT BE @EIC?

similar approach as Anselmino et al., [hep-ph/0501196](hep-ph/0501196)

$$F_{UU,L} \stackrel{?}{=} \frac{M^2}{Q^2}\mathcal{C}\left[\frac{4k_T^2}{M^2} f_1 D_1\right]$$



high-P_T behavior modified to be in qualitative agreement with collinear calculations

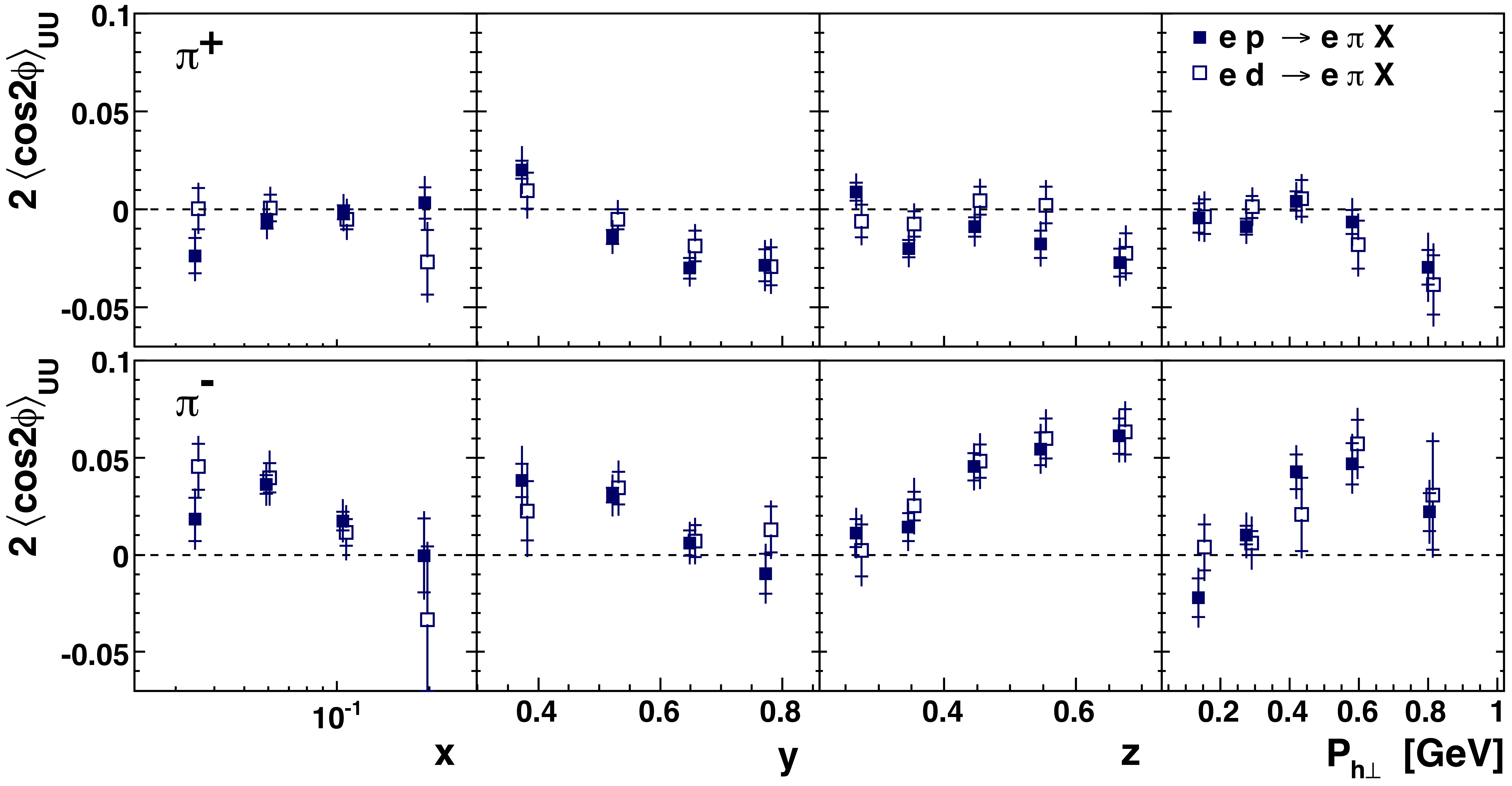


Smaller at small transverse momentum. Decreases $1/Q^2$ more evident

WHAT ABOUT THE CONNECTION TO COS2Φ?

$$F_{UU,L} = 2\, F_{UU}^{\cos 2\phi_h}$$

only valid at high transverse momentum and leading order



At HERMES, the cos2φ structure function is below 5%

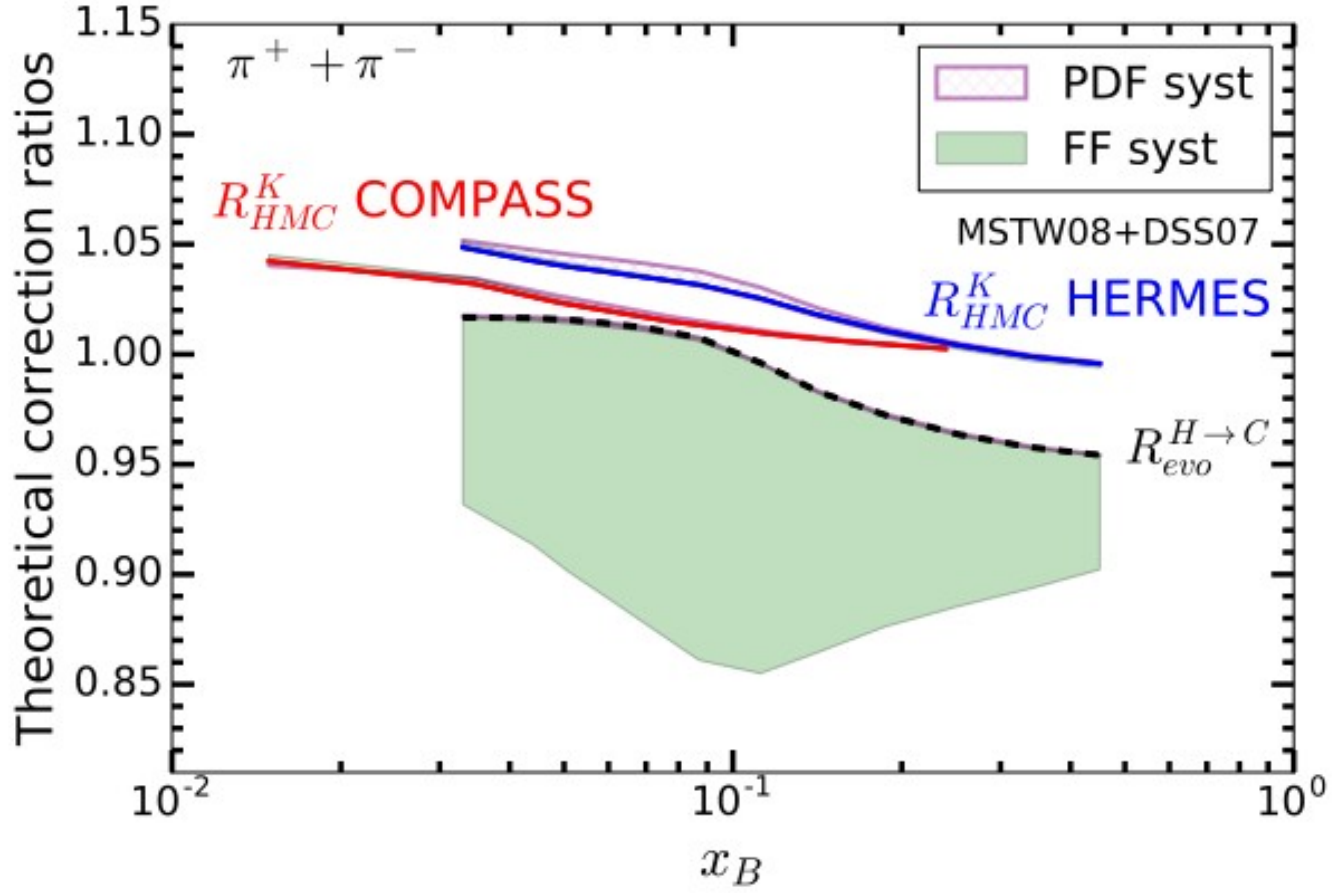
WHAT ABOUT HADRON MASS EFFECTS?

courtesy of A. Accardi

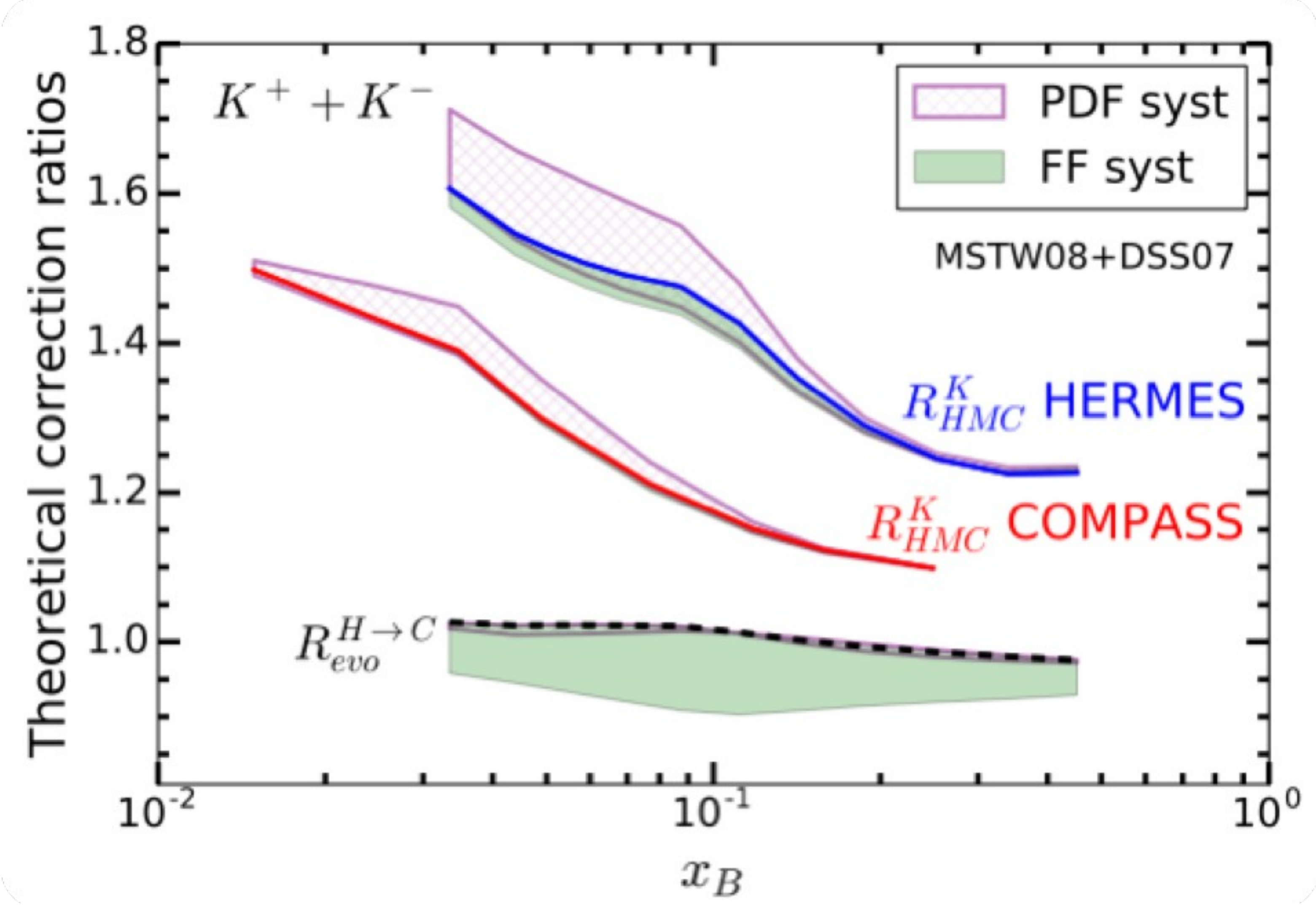
No specific study on F_L, but studies on unpolarized multiplicities

- **Corrections:** $R^h_{HMC} = M^{h(0)}/M^h$

Pions



Kaons



For pions, 5% effects

For kaons, 50% effects!

- **TMD fits with HMCs** *Scimemi, Vladimirov, JHEP 06, 137 (2020)*

  “visible effect on fit quality especially for HERMES”

CONCLUSIONS

- The F_{UUL} structure function is not small at JLab kinematics (~20%)
- The function is smaller at high Q and low P_T
- There are no good estimates of this function in most of the phase space
- In general, the world of “higher twist effects” (kinematic, dynamic, mass…) needs to be seriously quantified