Measurement of the Bjorken sum rule and α_s with JLab@22 GeV

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Importance of measuring α_s

The QCD coupling is (by far) the least known fundamental coupling:

- QED: $\Delta\alpha/\alpha = 1.5 \times 10^{-10}$
- Weak: $\Delta G_F/G_F = 5.1 \times 10^{-7}$
- Gravity: $\Delta G_N/G_N = 2.2 \times 10^{-5}$
- QCD: $\Delta\alpha_s/\alpha_s = 7.6 \times 10^{-3}$ (with Lattice QCD determinations) or 8.5×10^{-3} without.

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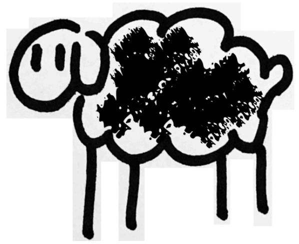
- QCD: $\Delta\alpha_s/\alpha_s \simeq \%$ 

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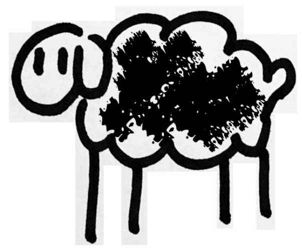
- QCD: $\Delta\alpha_s/\alpha_s \simeq \%$ 

Precise knowledge of α_s is needed for:

- Understanding QCD;
- High precision SM and beyond SM studies at high-energy facilities.

Ex. beyond SM physics can be revealed by checking the universality of α_s: different processes yield slightly different α_s.

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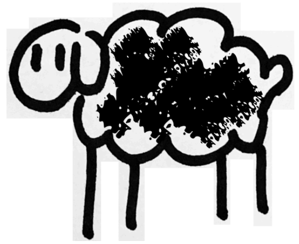

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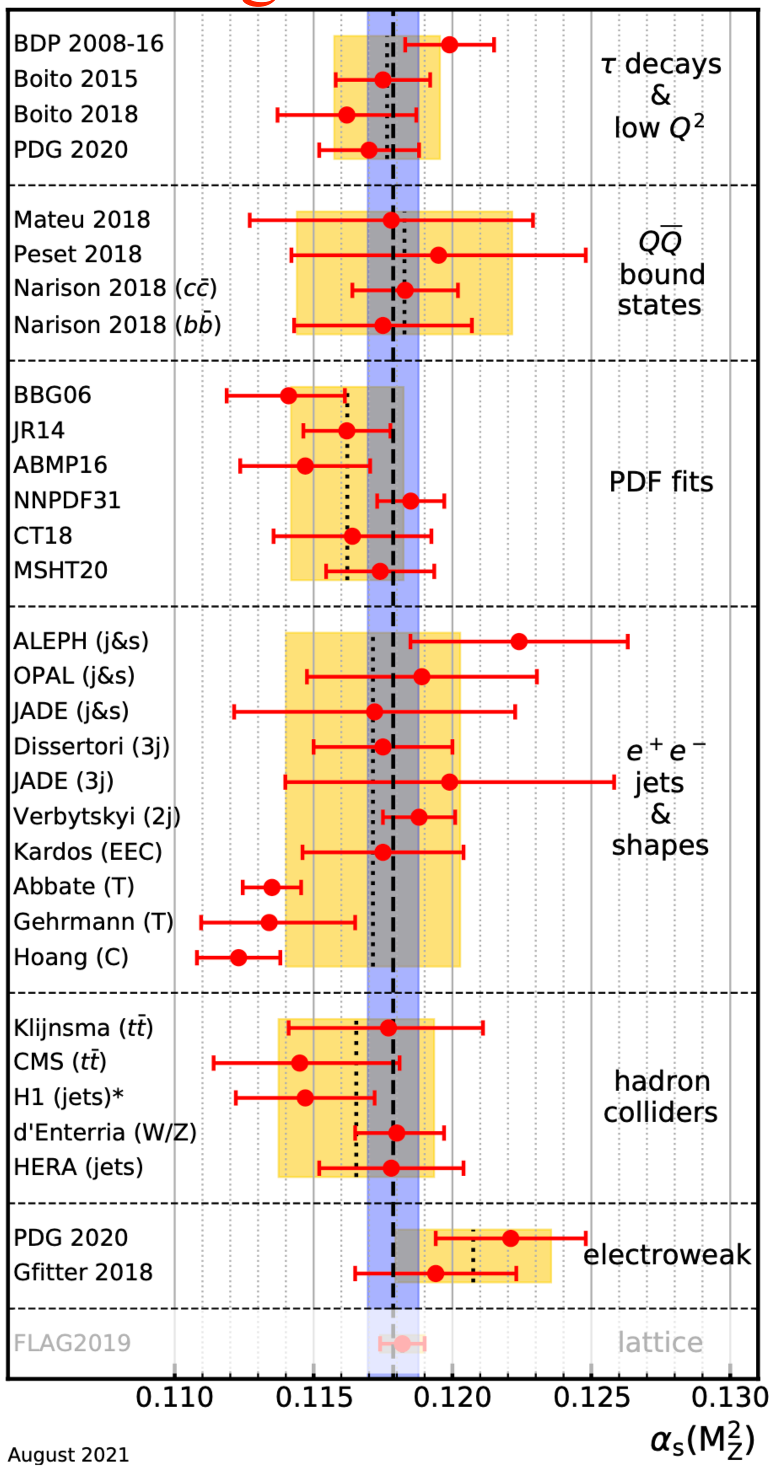
Most precise determinations of α_s are from lattice QCD. But it is not directly applicable for the above as it accounts only for QCD, no beyond SM physics or other interactions (there is recent work toward including QED).

Importance of measuring α_s

Presently, 8.5×10^{-3} is achieved by combining many independent measurements with larger uncertainties.



⇒High precision experimental determinations are necessary.

Importance of measuring α_s

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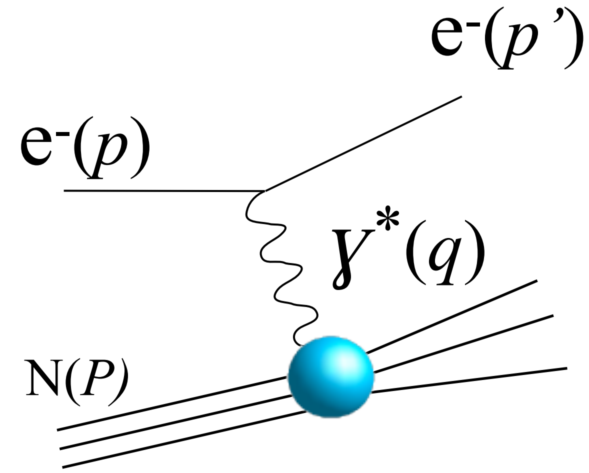
High precision experimental determinations are necessary.

We will discuss here the prospect of measuring α_s at JLab@22 GeV with the Bjorken Sum Rule.

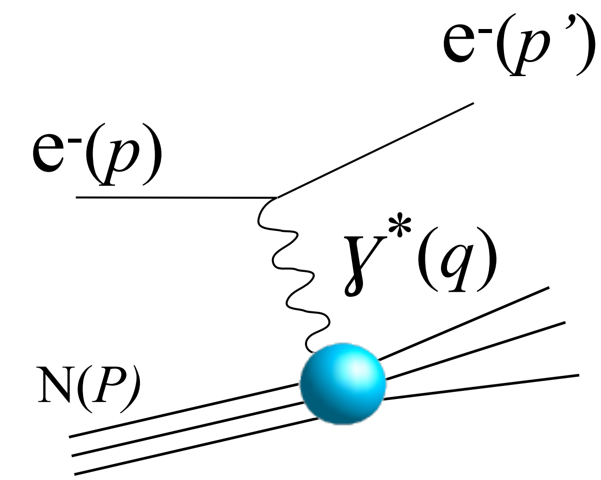
Although the Bjorken Sum Rule is important and interesting on its own, but here, it is only a mean to our end.

Definition of kinematic variables in inclusive eN scattering



- $p=(E,\mathbf{p})$, $p'=(E-\nu,\mathbf{p}-\mathbf{q})$, $q=(\nu,\mathbf{q})$
- γ^* virtual photons: q^2
- $q^2<0$; $Q^2=-q^2$.
- At high energy, Bjorken scaling variable $x=Q^2/2M\nu$ is more convenient than ν.

Structure functions



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Cross section: $\sigma=\sigma_{\mathrm{Mott}}[\alpha F_1(x,Q^2)+\beta F_2(x,Q^2)+\gamma g_1(x,Q^2)+\varpi g_2(x,Q^2)]$

spin independent   spin dependent

Bjorken sum rule: $\Gamma_1^{p-n}=\int_0^1 g_1^p(x,Q^2)-g_1^n(x,Q^2)dx=\frac{g_A}{6}$ +pqcd corrections

g_A: Nucleon axial charge

Considering the nucleon inclusive spin structure, α_s can be extracted from:

- Q^2-evolution of $g_1(x, Q^2)$. Complex: involves DGLAP global fit, non-perturbative inputs: quark and gluon distributions, possibly higher-twists for low-Q^2 / large-x data.

- Q^2-evolution of moment $\int_0^1 g_1(x, Q^2)dx$. Simpler: no x-dependence, non-perturbative inputs: more-or-less well measured axial charges a_0, a_3 and a_8 (+ possibly higher-twists for low-Q^2 data). Issues: unmeasurable low-x contribution, a_0 is Q^2-dependent and may have contribution from gluon ΔG pdf (but not the case in $\overline{MS}$).

- Q^2-evolution of isovector moment $\int_0^1 g_1^{p-n}(x, Q^2)dx$, i.e Bjorken sum. Simplest. Axial charge $a_3 = g_A$ precisely measured ($g_A = 1.2762 \pm 0.0005$). DGLAP-evolution known to higher order than single nucleon case (nowadays, this is often the limitation in extracting α_s). No gluon contribution. But low-x issue and demands measurement on polarized p and n.

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$$+ \frac{M^2}{Q^2} \left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s) \right] + \ldots$$

Nucleon's First spin structure function

Nucleon axial charge. (Value of $\Gamma_1^{p-n}(Q^2)$ in the $Q^2 \to \infty$ limit)

pQCD radiative corrections ($\overline{MS}$ Scheme.)

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$\Rightarrow$ Two possibilities to extract $\alpha_s(M_Z)$:

- Do an absolute measurement of $\Gamma_1^{p-n}(Q^2)$ and solve the Bj SR for $\alpha_s(Q^2)$.
  - One α_s per Γ_1^{p-n} experimental data point.

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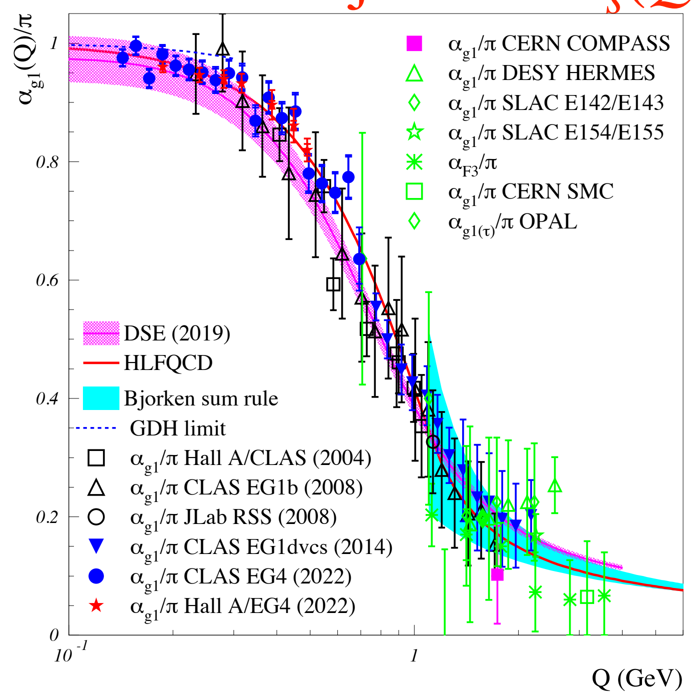

The Bj SR allows to extract $\alpha_s(Q^2)$ at any Q^2 value!

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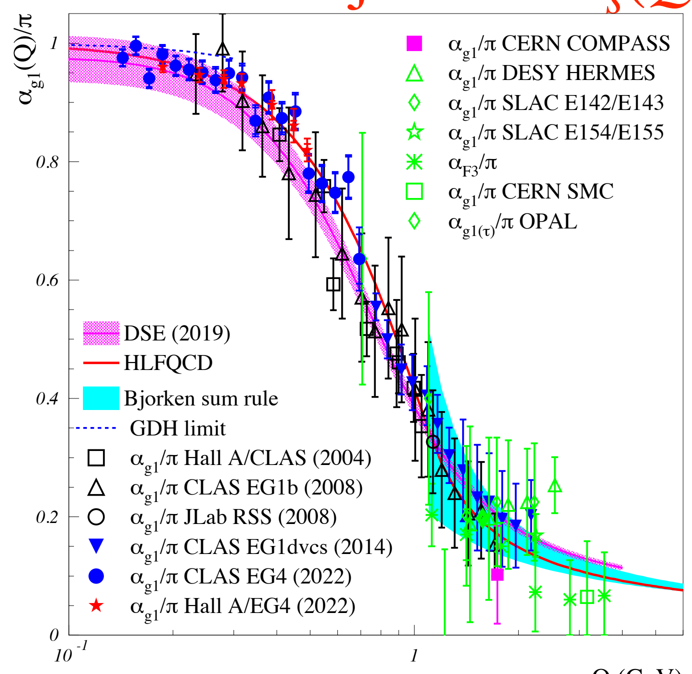

The Bj SR allows to extract $\alpha_s(Q^2)$ at all scale!

- Poor systematic accuracy, typically $\Delta\alpha_s/\alpha_s \sim 10\%$ at high energy $\Rightarrow$ Not competitive.

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- Measurement of Q^2**-dependence** of $\Gamma_1^{p-n}(Q^2)$.
  - Need Γ_1^{p-n} at several Q^2 points. Only one (or a few) value of α_s.
  - Good accuracy: 1990's CERN/SLAC data yielded: $\alpha_s(M_Z)$=0.120±0.009

Altarelli, Ball, Forte, Ridolfi, Nucl.Phys. B496 337 (1997)

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- Statistical uncertainties are expected to be negligible:
  - JLab is a high-luminosity facility;
  - A JLab@22 GeV program would include polarized DVCS and TMD experiments. Those imply long running times compared to those needed for inclusive data gathering;
  - High precision data already available from 6 GeV and 12 GeV for the lower Q^2 bins and moderate x.

- Looking at the 6 GeV CLAS EG1dvcs data, required statistics for DVCS and TMD experiments imply statistical uncertainties < 0.1% on the Bjorken sum. For the present exercise we will use 0.1% on all Q^2-points with Q^2-bin sizes increasing exponentially with Q^2.

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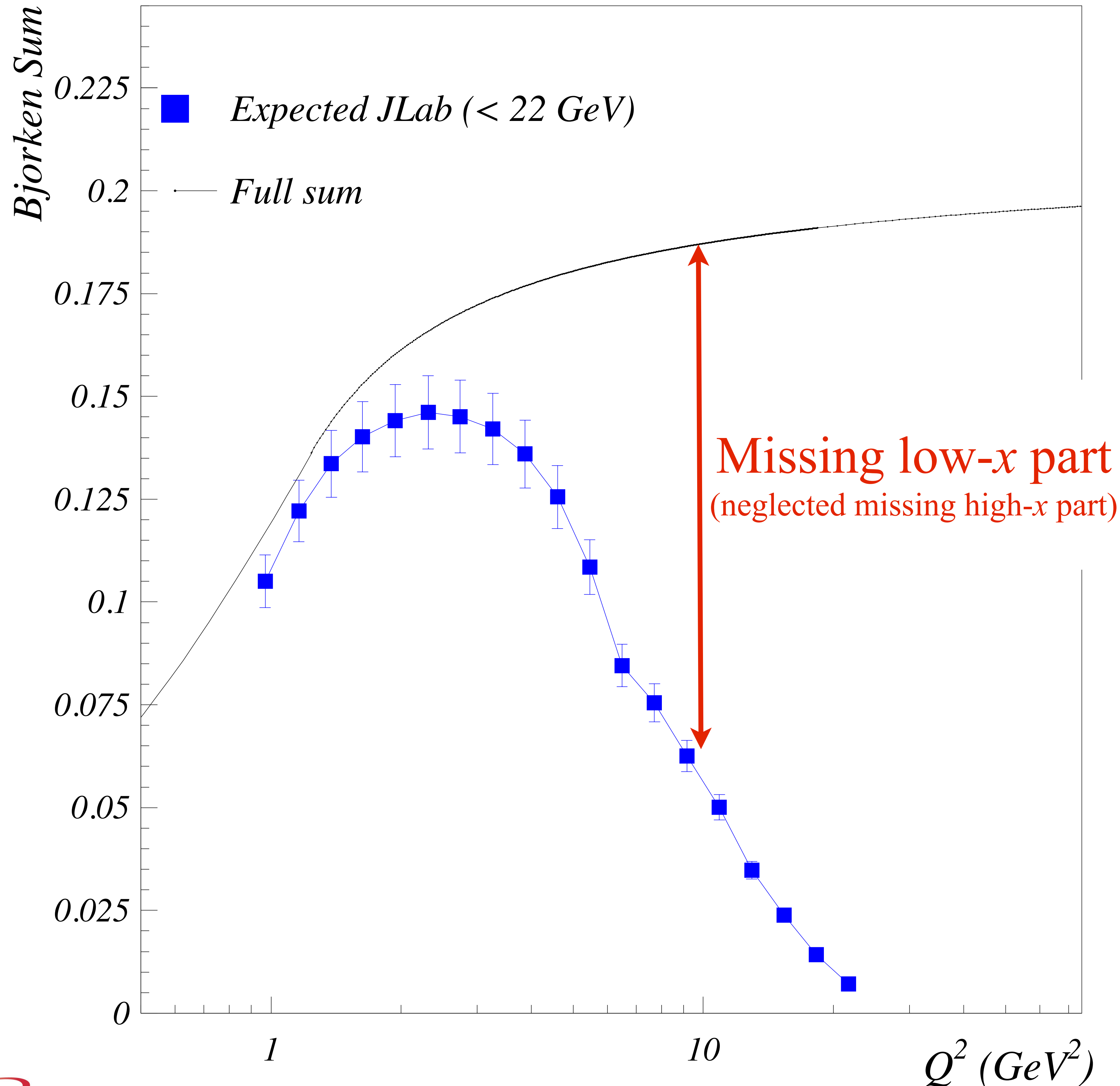
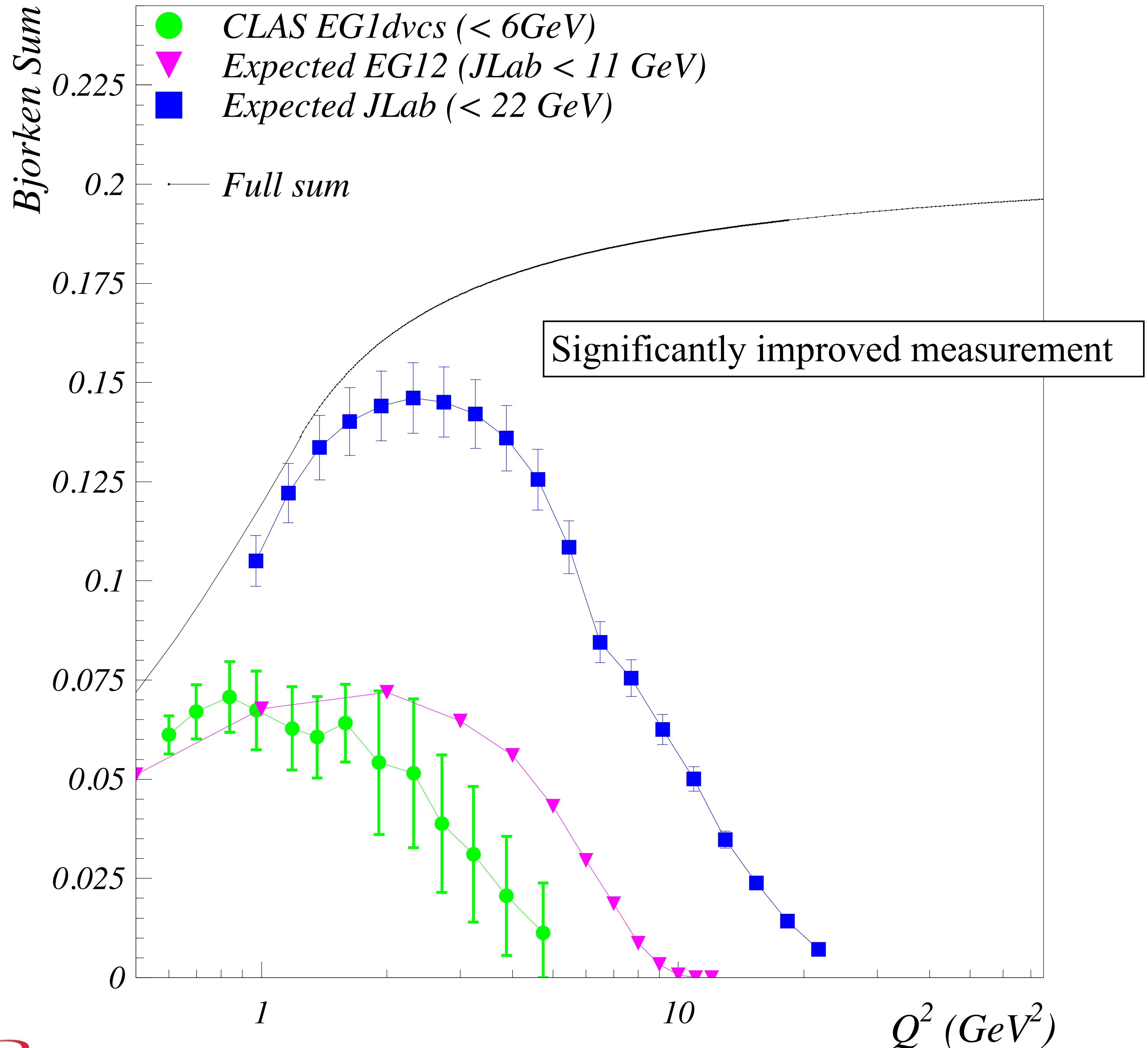
- Use 6% for experimental systematics (i.e. not including the uncertainty on unmeasured low-x).
  - Nuclear corrections:
    - D: negligible assuming we can tag the ~spectator proton
    - ^{3}He: 2% (5% on n, which contribute to 1/3 to the Bjorken sum: 5%/3≃2%)
  - Polarimetries: Assume ΔP_e-ΔP_N = 3%.
  - Radiative corrections: 1%
  - F_1 to form g_1 from A_1: 2%
  - g_2 contribution to longitudinal asym: Negligible, assuming it will be measured.
  - Dilution/purity:
    - Bjorken sum from P & D: 4%
    - Bjorken sum from P & ^{3}He: 3%
  - Contamination from particle miss-identification: Assumed negligible.
  - Detector/trigger efficiencies, acceptance, beam currents: Neglected (asym).

Adding in quadrature: ~5%
We will use 6%.

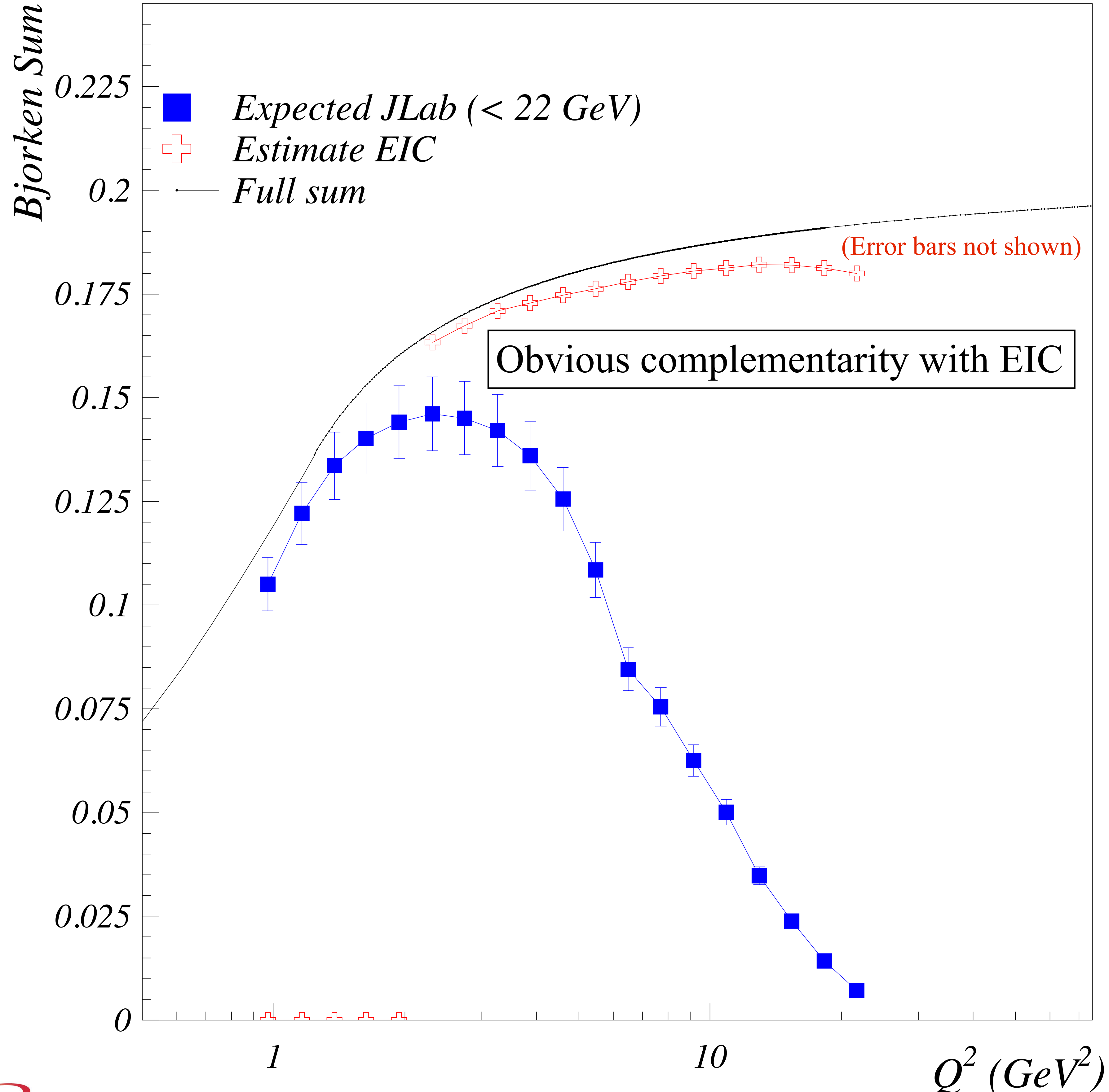
Under these assumptions:



Comparison with JLab at 6 and 11 GeV



Comparison with EIC



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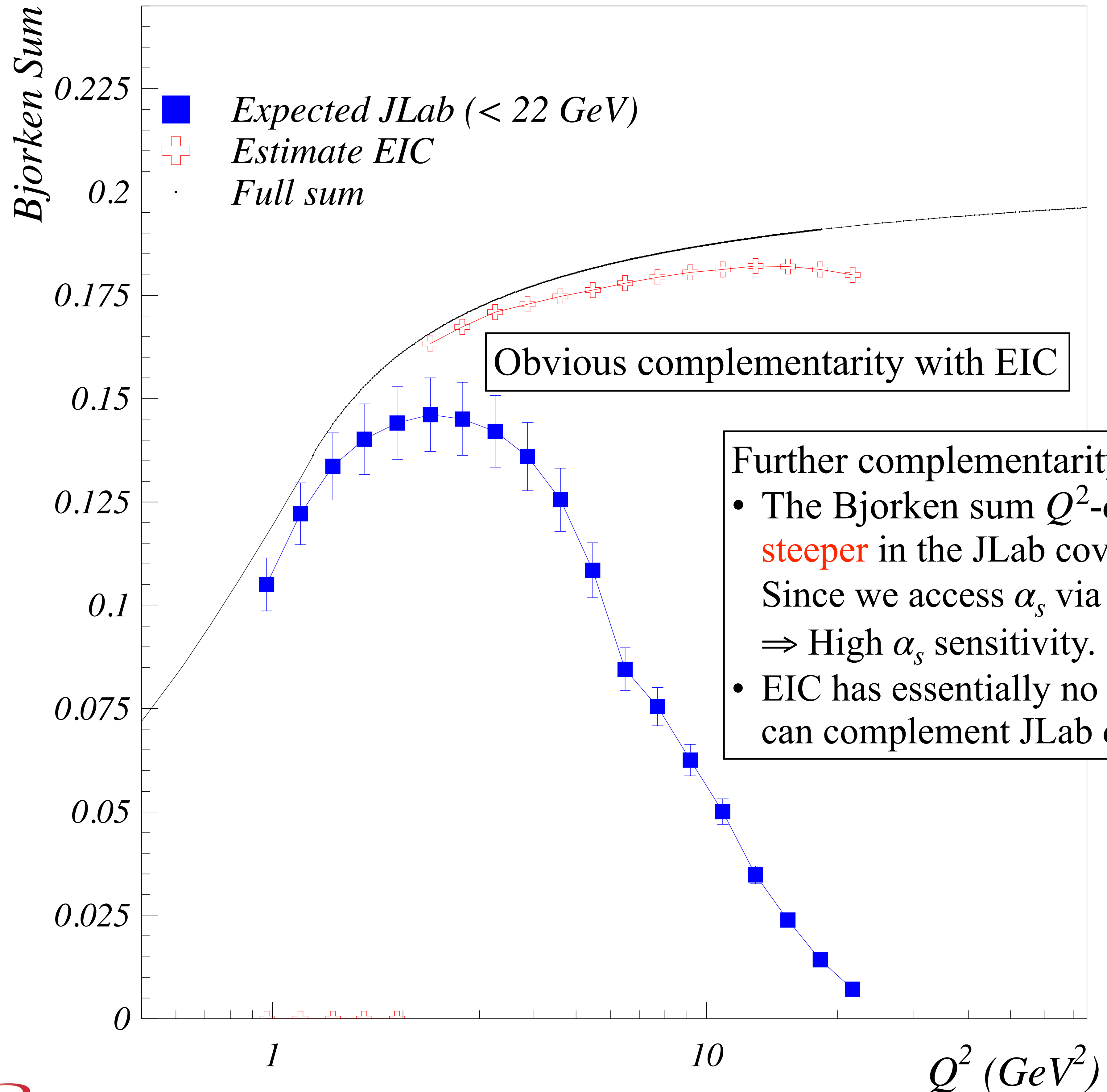


Obvious complementarity with EIC

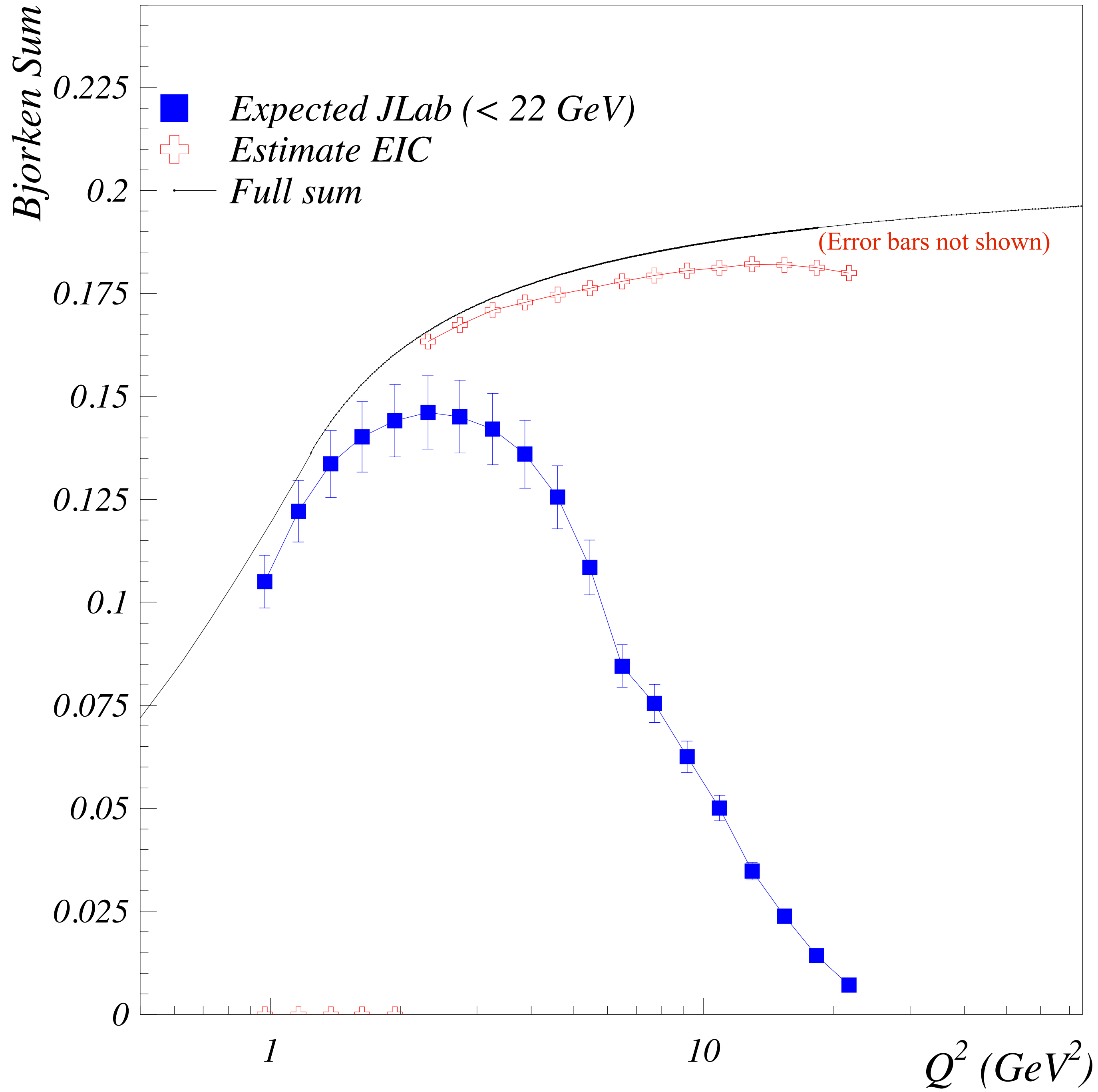
Further complementarity:

- The Bjorken sum Q^2-dependence is up to 50 times steeper in the JLab covered range than that of EIC. Since we access α_s via relative Q^2-dependence $\Rightarrow$ High α_s sensitivity.
- EIC has essentially no unmeasured low-x issue and can complement JLab data.

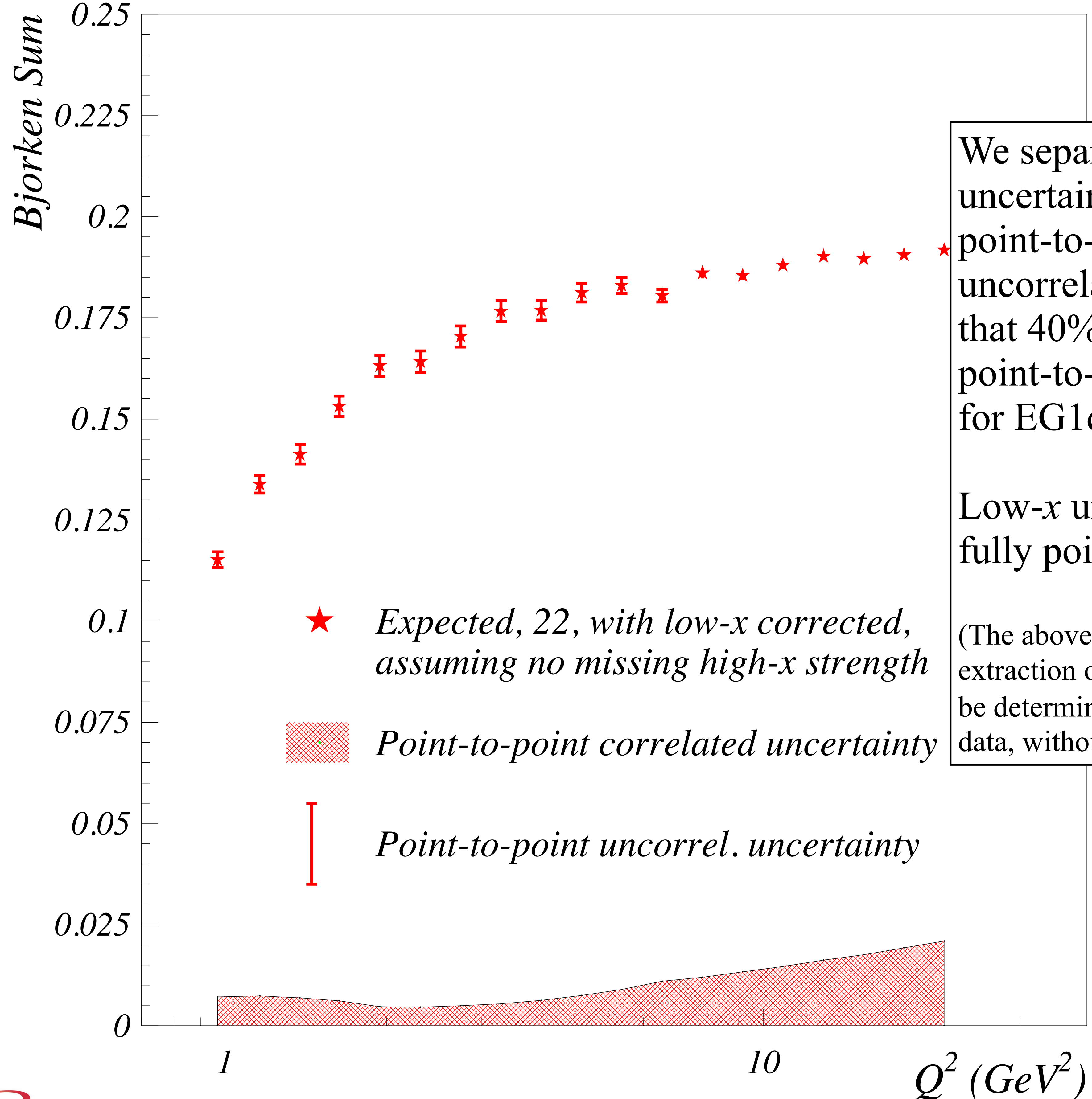
Low-x uncertainty

- For the Q^2 bins covered by EIC, global fits will be available up to the lowest x covered by EIC.
  $\Rightarrow$ assume 10% uncertainty on that missing (for the JLab measurement) low-x part.
  Assume 100% for the very small-x contribution not covered by EIC.

- For the 5 lowest Q^2 bins not covered by EIC:
  - Bin #5 close to the EIC coverage $\Rightarrow$ Constrained extrapolation, assume 20% uncertainty on missing low-x part.
  - Bin #4, assume 40% uncertainty, Bin #3, assume 60%, Bin #2, assume 80%, Bin #1, assume 100%.



Bjorken sum rule at JLab@22 GeV (meas.+low-x)

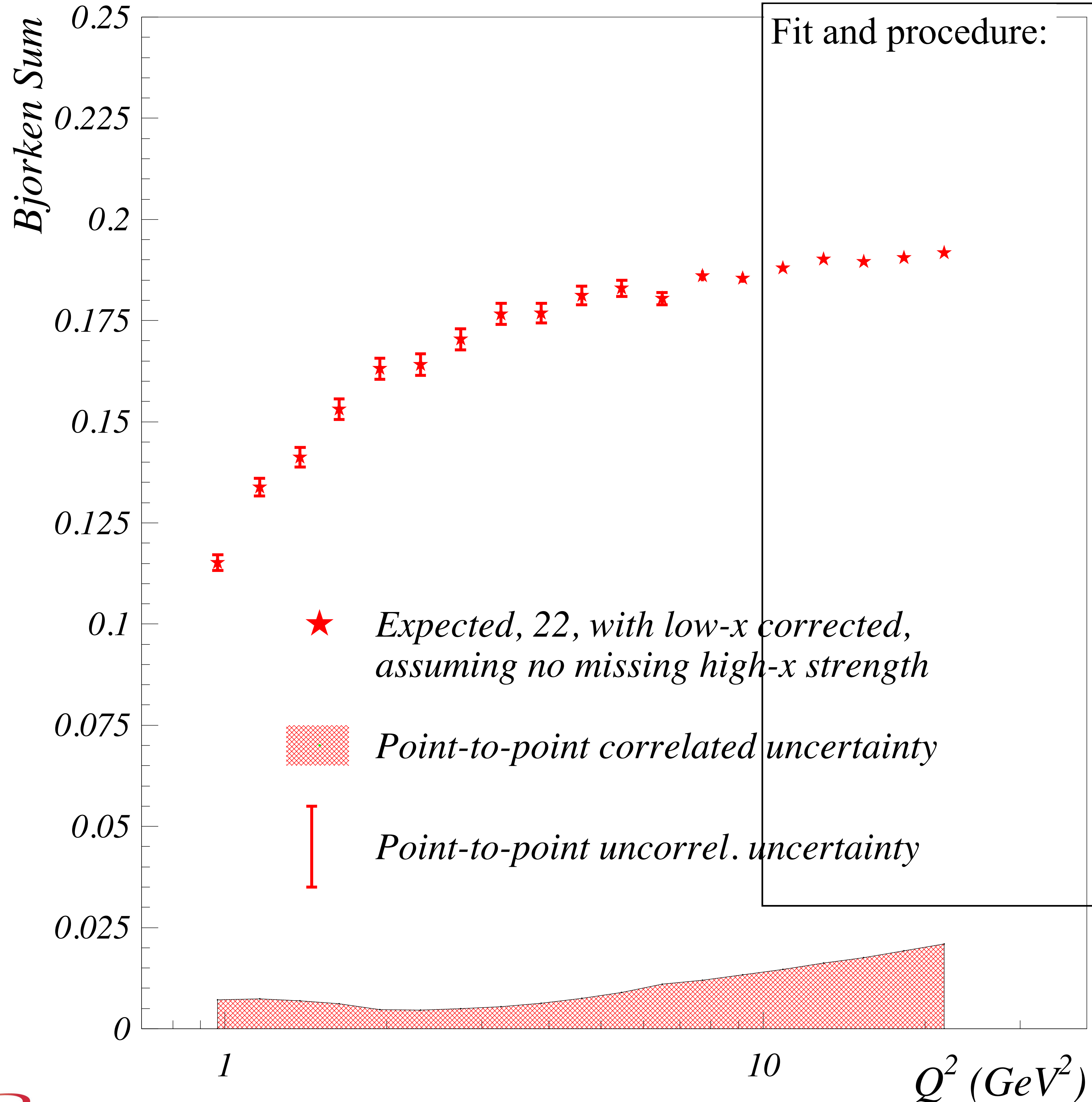


We separate the total <u>experimental</u> uncertainty (i.e. excluding the low-x error) in point-to-point correlated and uncorrelated contributions, assuming that 40% of the total uncertainty is point-to-point correlated (as obtained for EG1dvcs Bjorken sum analysis).

Low-x uncertainty is assumed to be fully point-to-point correlated.

(The above assumptions are not crucial for the extraction of α_s. Also, the proper separation would be determined from analysis of the actual 22 GeV data, without assumption.)

Extraction of $\alpha_s(M_Z)$



Fit and procedure:

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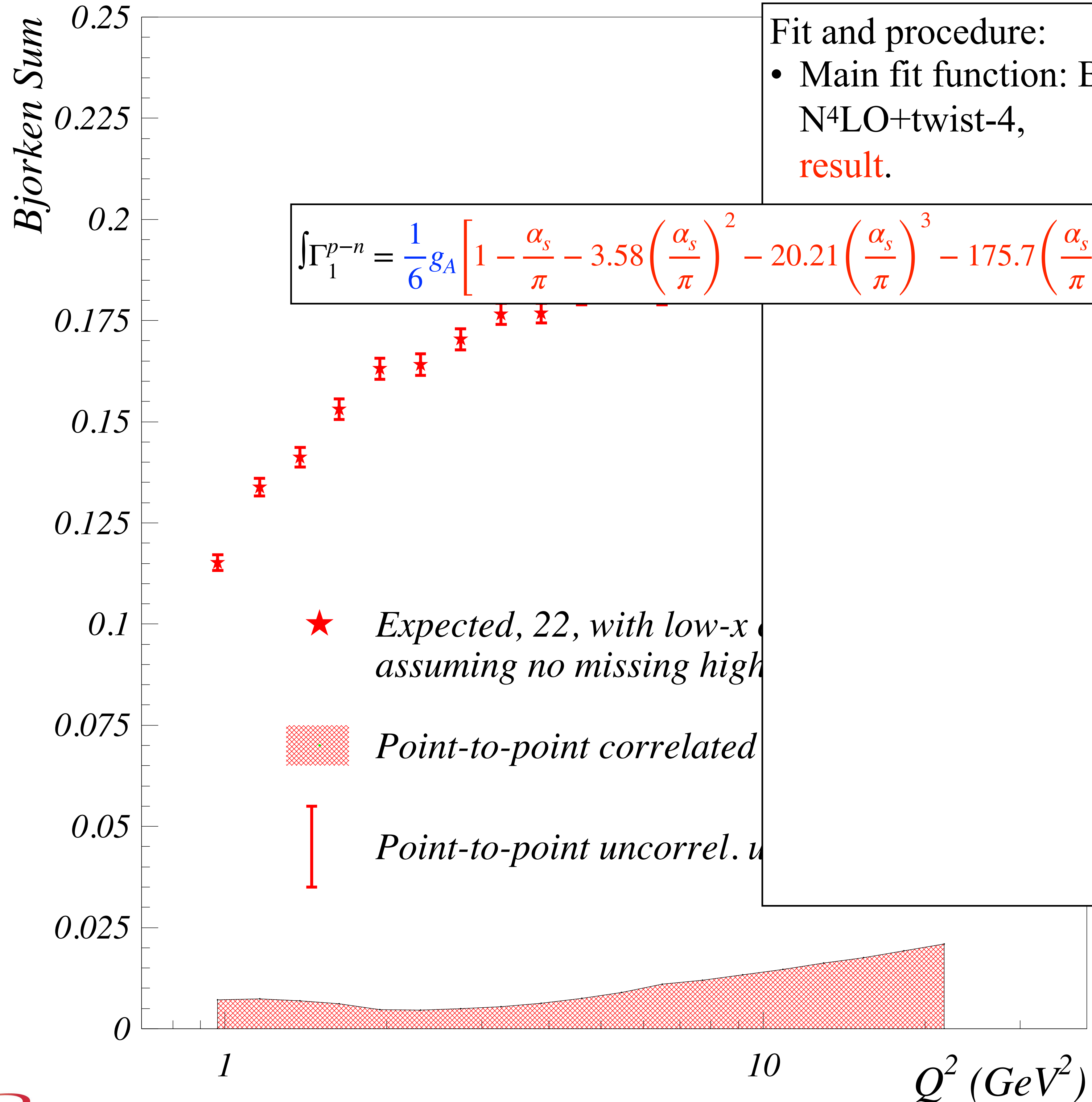

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- Main fit function: Bjorken sum approximant at N[4]LO+twist-4, for main result.

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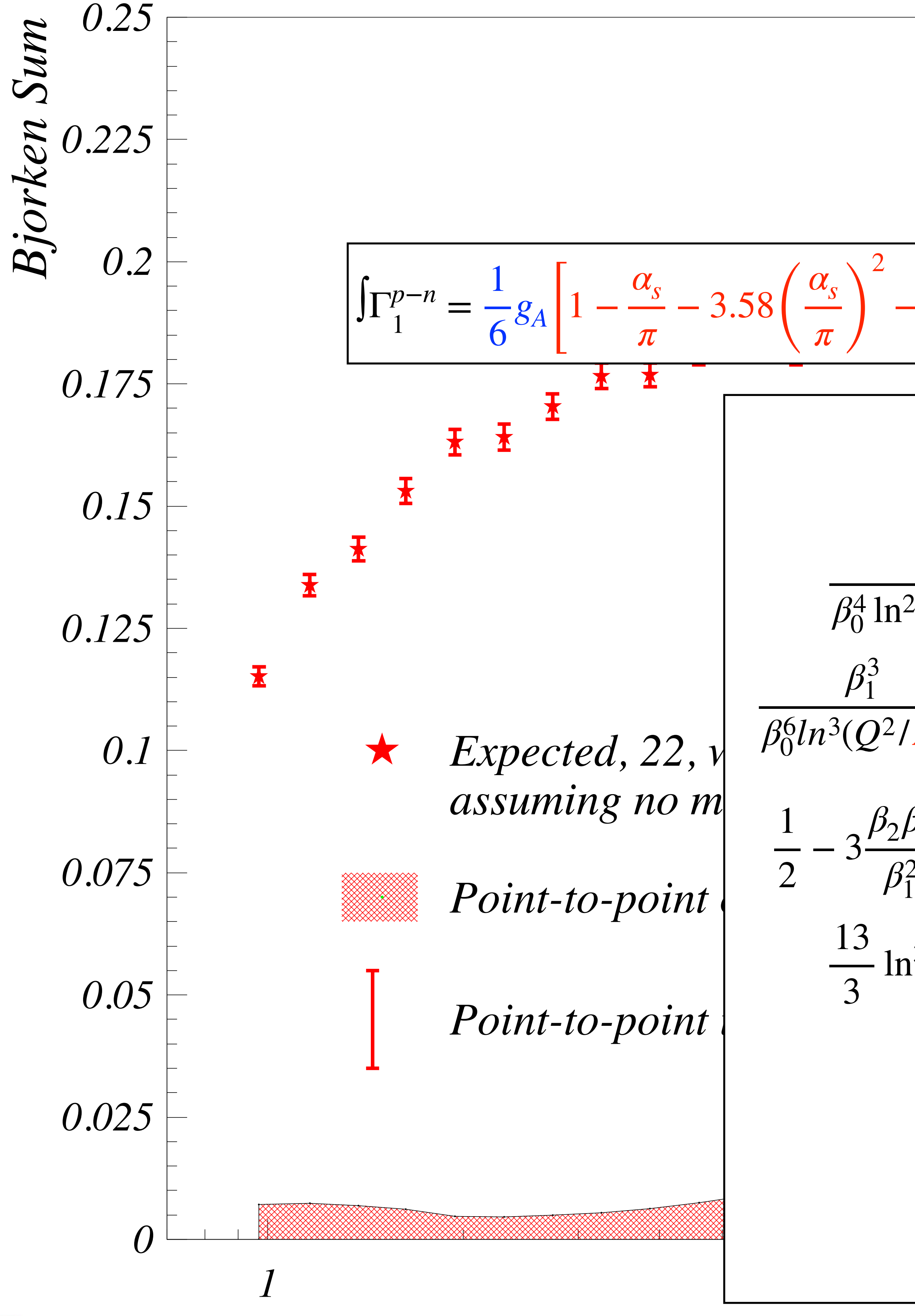

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$$\begin{aligned}
\alpha_s^{\overline{\rm MS}}(Q) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_s^2)}\Bigg[1 &- \frac{\beta_1}{\beta_0^2}\frac{\ln(\ln(Q^2/\Lambda_s^2))}{\ln(Q^2/\Lambda_s^2)} + \\
&\frac{\beta_1^2}{\beta_0^4 \ln^2(Q^2/\Lambda_s^2)}\left(\ln^2(\ln(Q^2/\Lambda_s^2)) - \ln(\ln(Q^2/\Lambda_s^2)) - 1 + \frac{\beta_2\beta_0}{\beta_1^2}\right) + \\
&\frac{\beta_1^3}{\beta_0^6 ln^3(Q^2/\Lambda_s^2)}\Bigg(-\ln^3(\ln(Q^2/\Lambda_s^2)) + \frac{5}{2}\ln^2(\ln(Q^2/\Lambda_s^2)) + 2\ln(\ln(Q^2/\Lambda_s^2)) - \\
&\frac{1}{2} - 3\frac{\beta_2\beta_0}{\beta_1^2}\ln(\ln(Q^2/\Lambda_s^2)) + \frac{\beta_3\beta_0^2}{2\beta_1^3}\Bigg) + \frac{\beta_1^4}{\beta_0^8 ln^4(Q^2/\Lambda_s^2)}\Bigg(\ln^4(\ln(Q^2/\Lambda_s^2)) - \\
&\frac{13}{3}\ln^3(\ln(Q^2/\Lambda_s^2)) - \frac{3}{2}\ln^2(\ln(Q^2/\Lambda_s^2)) + 4\ln(\ln(Q^2/\Lambda_s^2)) + \frac{7}{6} + \\
&\frac{7}{6} + \frac{3\beta_2\beta_0}{\beta_1^2}\left(2\ln^2(\ln(Q^2/\Lambda_s^2)) - \ln(\ln(Q^2/\Lambda_s^2)) - 1\right) - \\
&\frac{\beta_3\beta_0^2}{\beta_1^3}\left(2\ln(\ln(Q^2/\Lambda_s^2)) + \frac{1}{6}\right)\Bigg]
\end{aligned}$$

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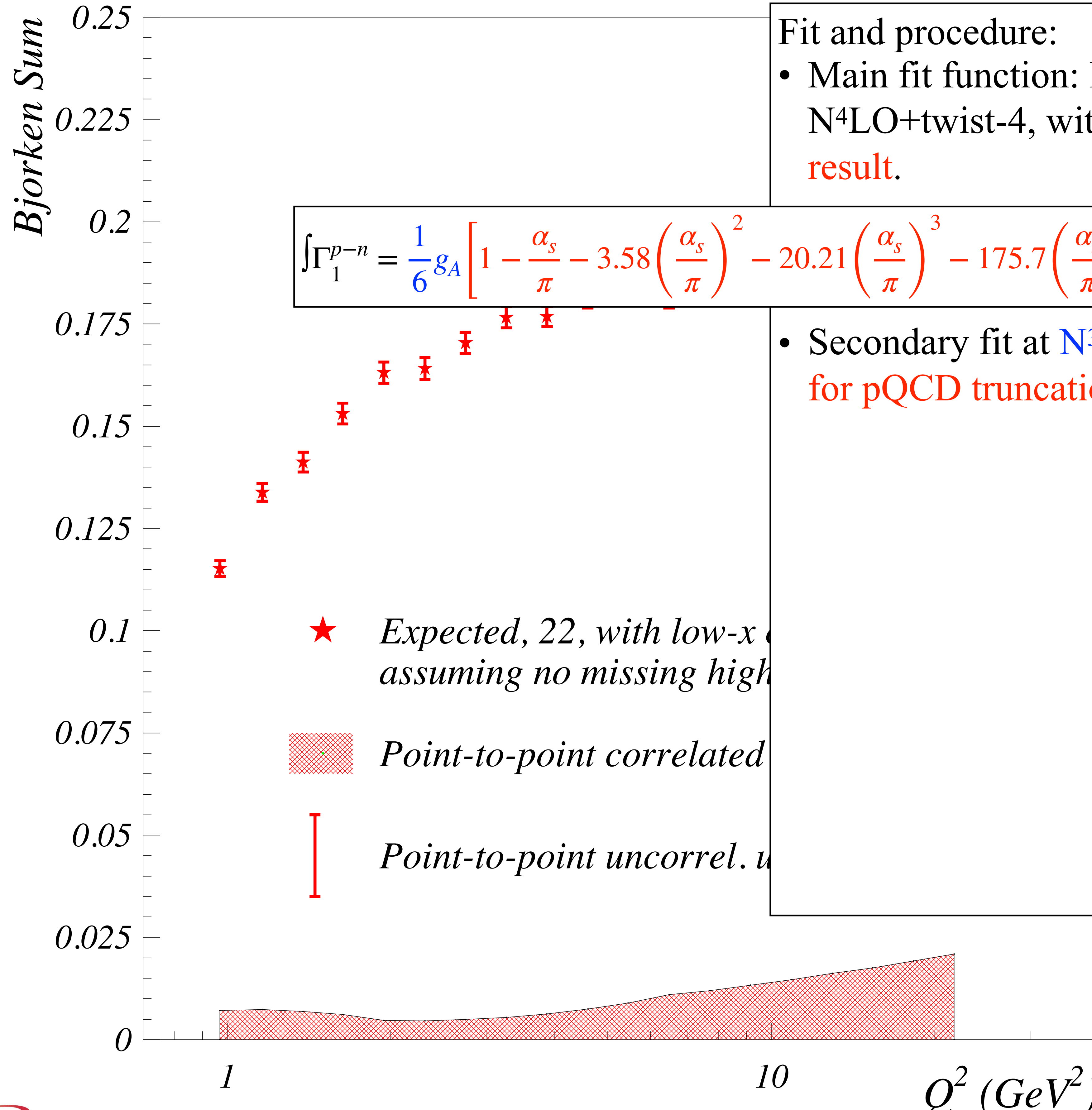

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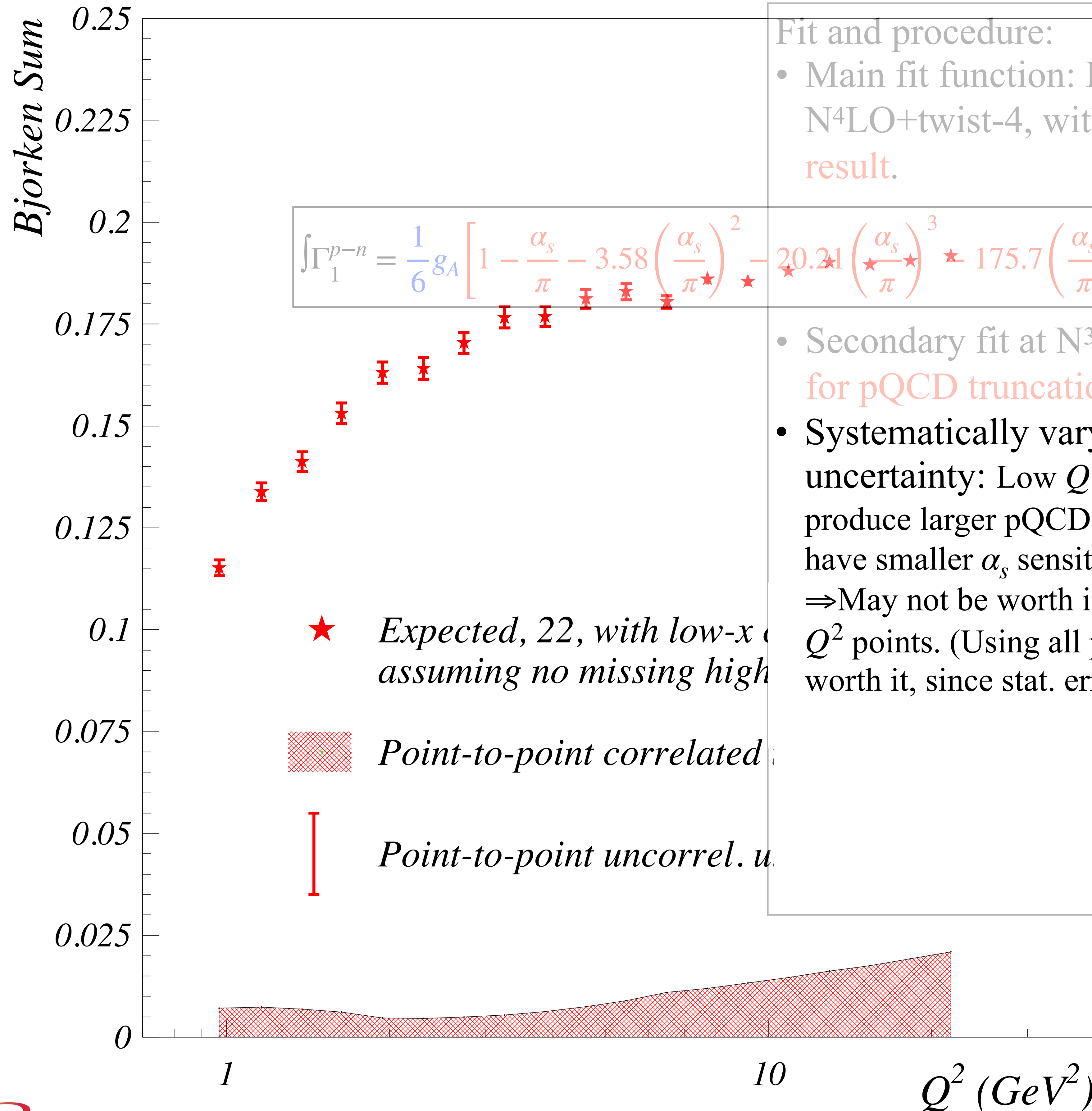
Extraction of $\alpha_s(M_Z)$



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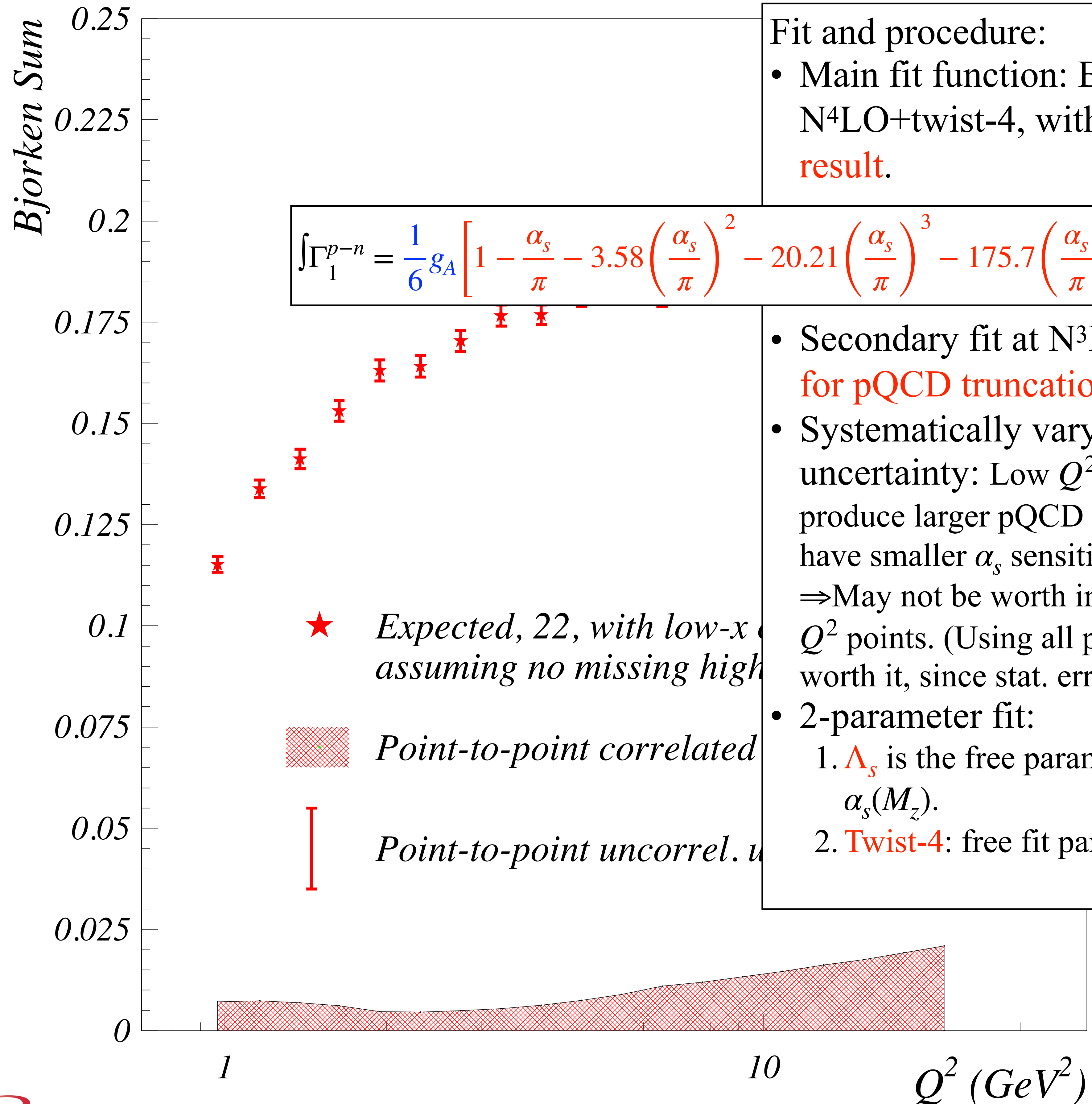

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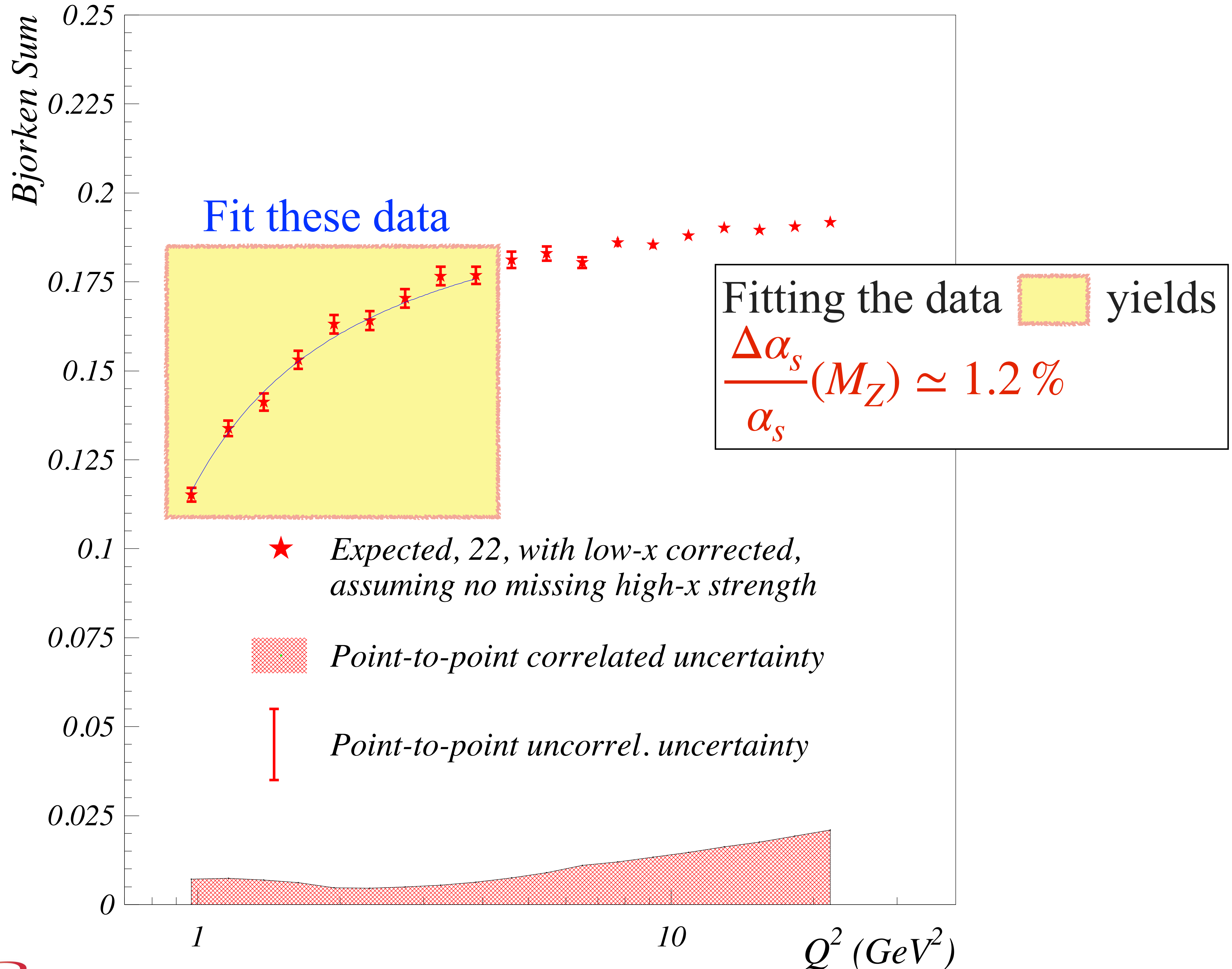
- 2-parameter fit:
  1. Λ_s is the free parameter of interest. From it, we obtain $\alpha_s(M_z)$.
  2. Twist-4: free fit parameter.

Extraction of $\alpha_s(M_Z)$



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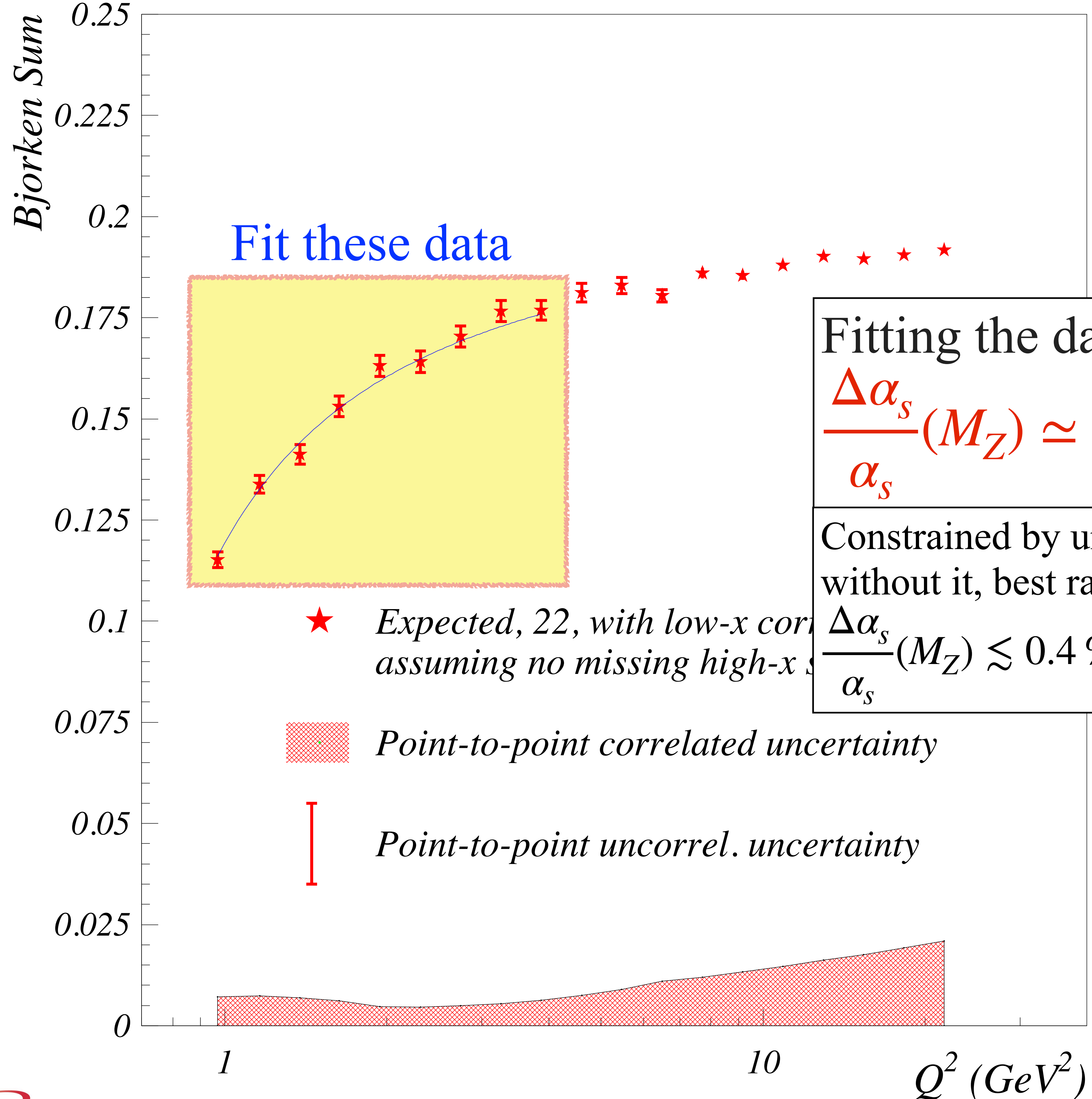

Fitting the data yields

$$\frac{\Delta\alpha_s}{\alpha_s}(M_Z) \simeq 1.2\,\%$$

Constrained by uncertainty on pQCD series: without it, best range sits at low Q^2 and yields

$$\frac{\Delta\alpha_s}{\alpha_s}(M_Z) \lesssim 0.4\,\%$$

Comparison JLab@22 GeV and EIC

JLab@22 GeV

- Covers region with strong Q^2-dependence: best sensitivity to α_s.
- Small Higher-Twist uncertainties.
- Finer Q^2 binning (19 bins (JLab) vs 7 bins (EIC)).

EIC

- Best low-x coverage.
- No Higher-Twist uncertainties
- Smaller pQCD uncertainties.

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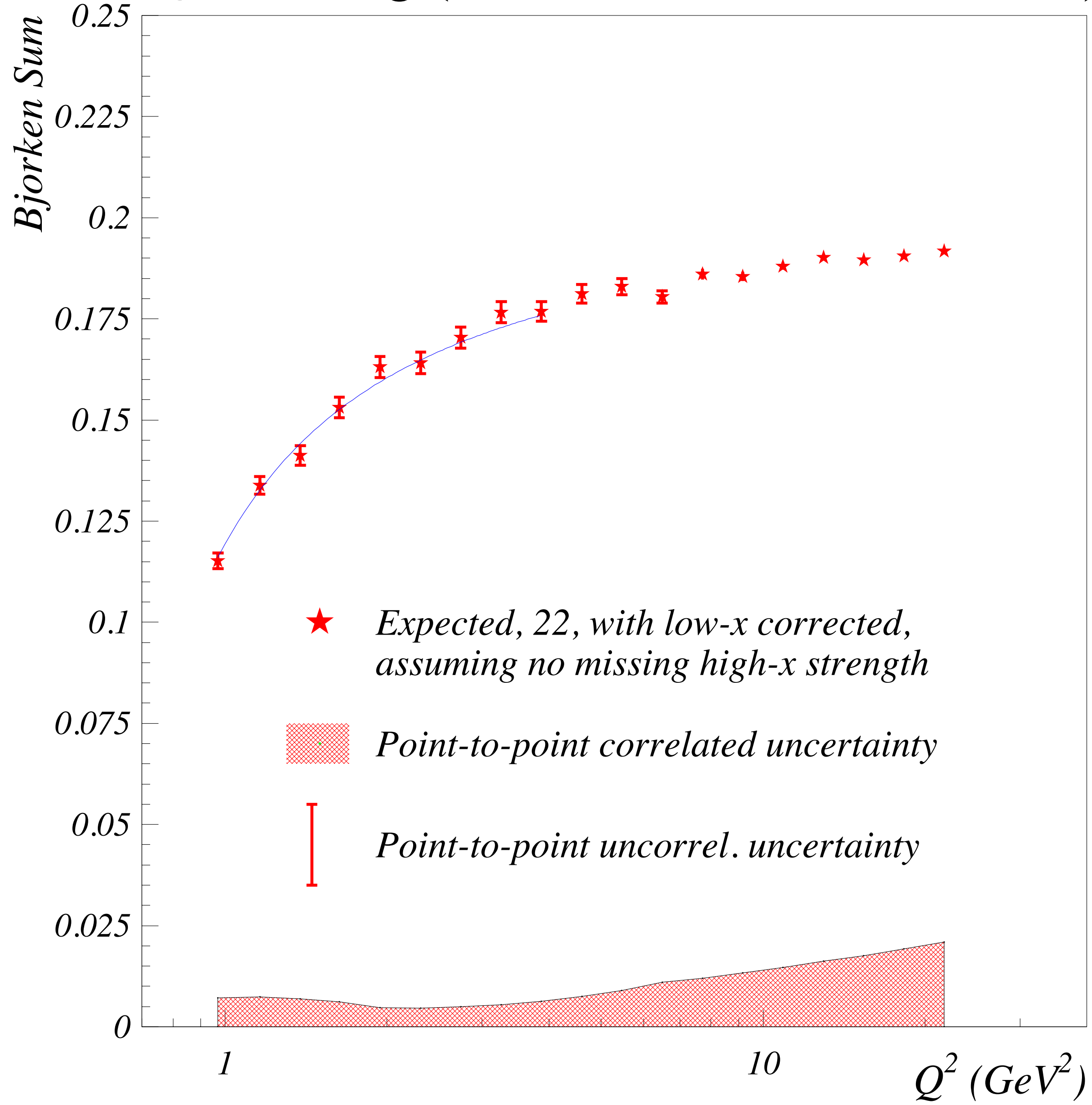

$\frac{\Delta\alpha_s}{\alpha_s} \simeq 1.2\,\%$. Take advantage of EIC low-x data

EIC

- Best low-x coverage.
- No Higher-Twist uncertainties
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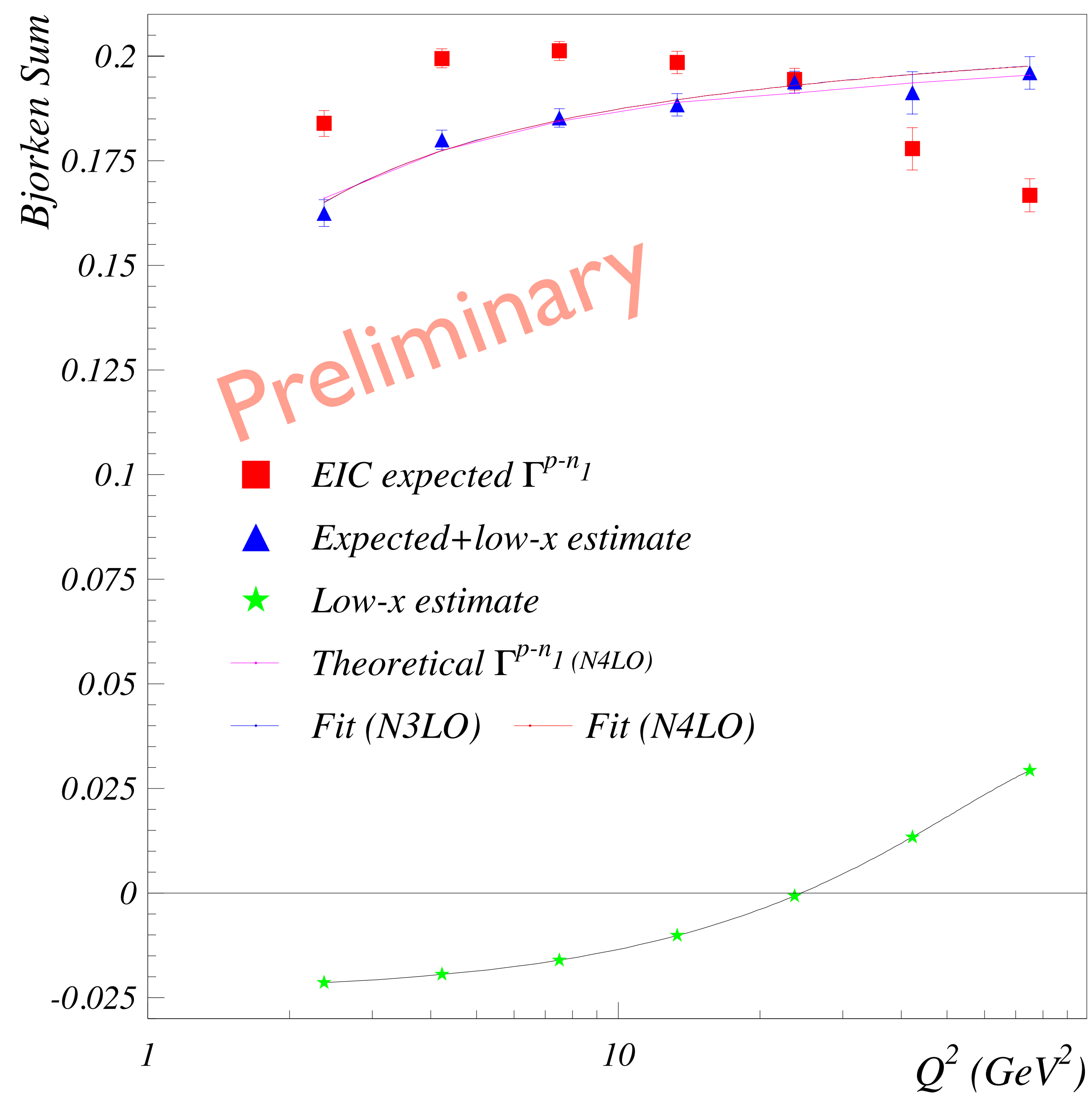
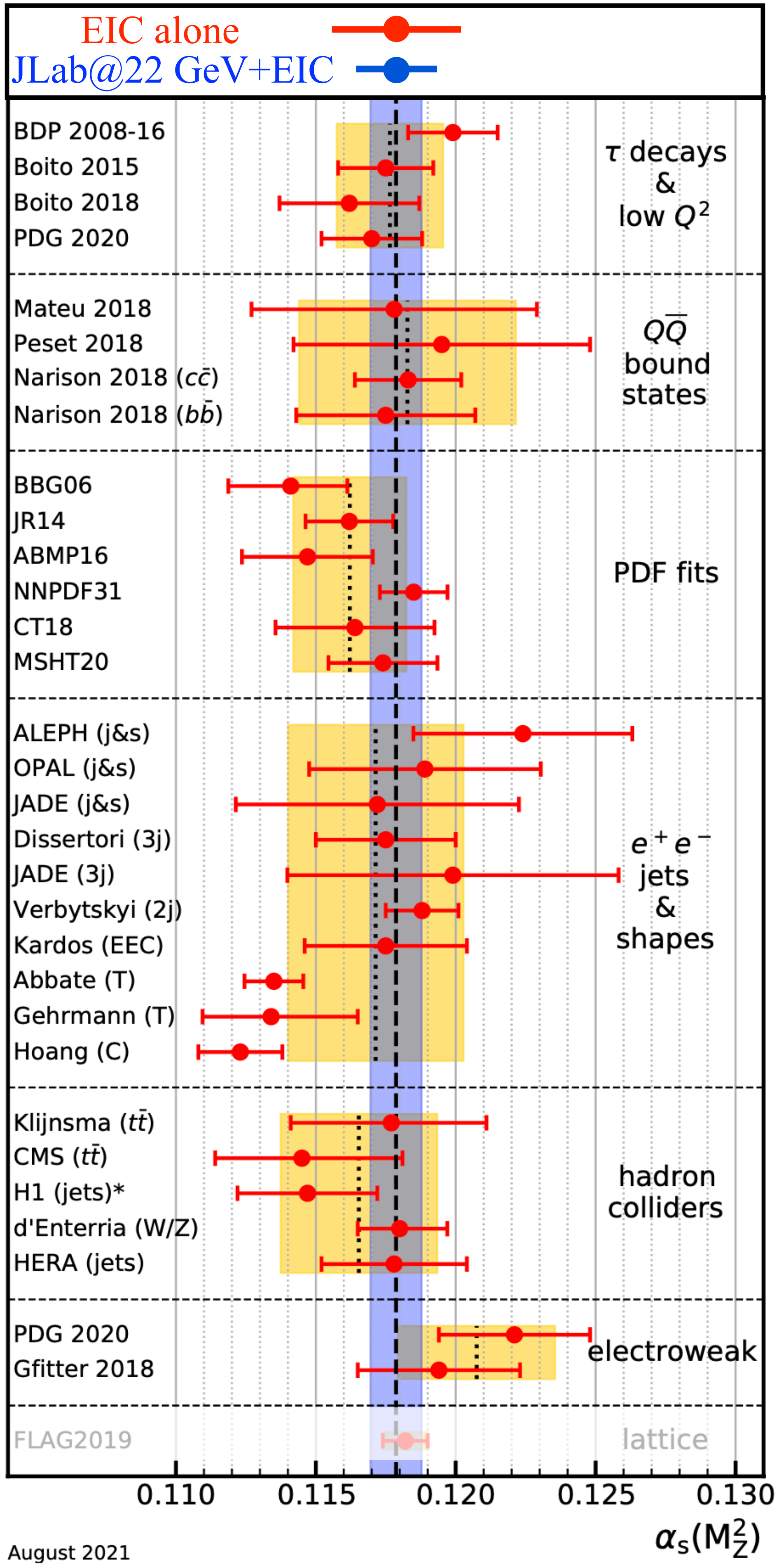


Preliminary study from EIC ECCE consortium: $\frac{\Delta\alpha_s}{\alpha_s} \simeq 1.9\,\%$ with EIC alone. Error ~evenly split between stats and syst.

Comparison with current best world experimental data (PDG 2021):



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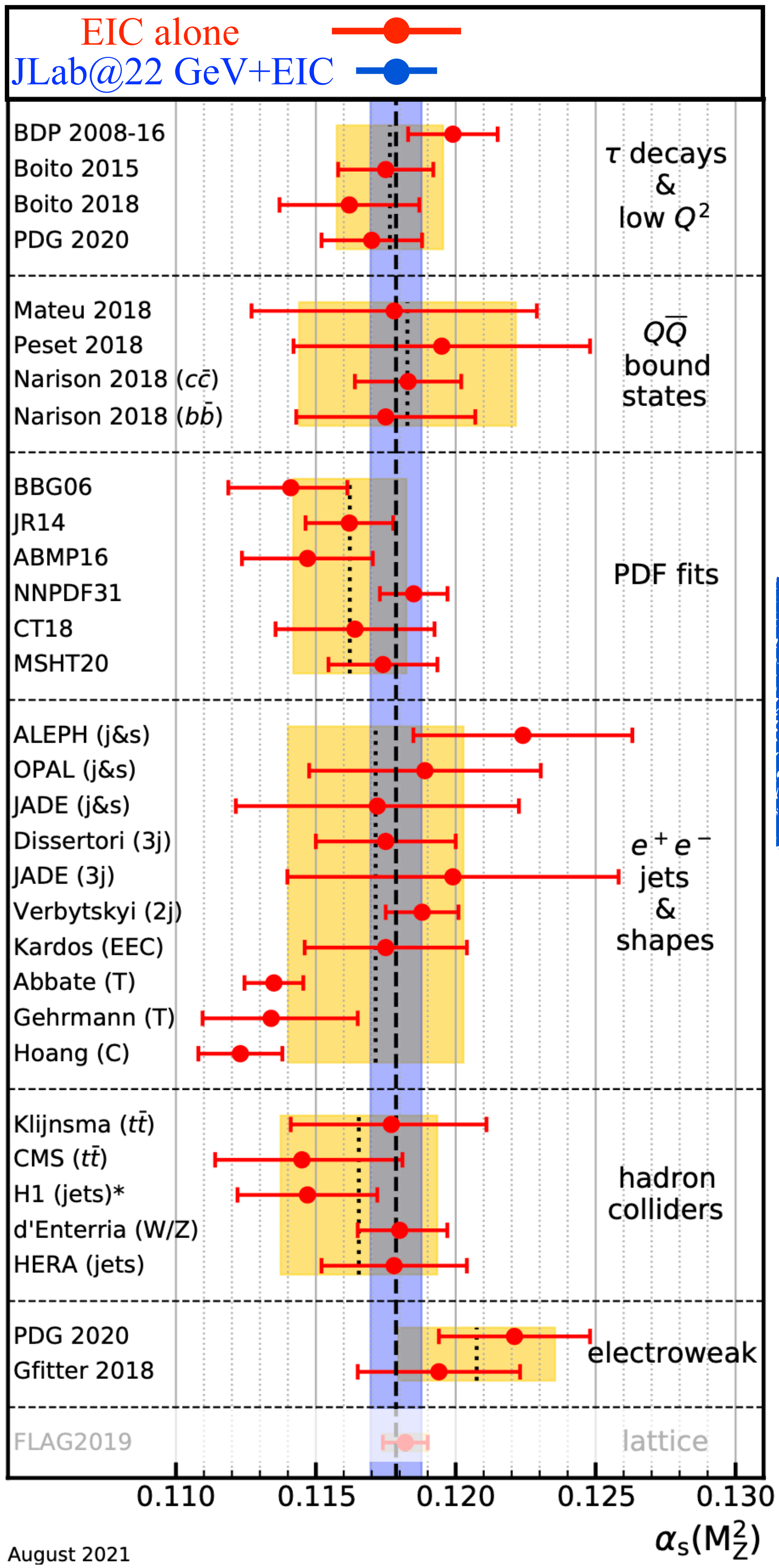
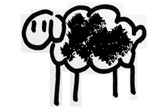


Conclusion

Under reasonable assumptions, JLab@22 GeV can yield a compelling 1.2% measurement of $\alpha_s(M_Z)$. Input from EIC data is important (low-x needed for moments).

Summary

- Of the 4 fundamental couplings, α_s has by far the lowest accuracy: 
- Accurate experimental determinations of $\alpha_s(M_Z)$ are crucial for QCD, SM and beyond SM studies.
- The Bjorken sum $\Gamma_1^{p-n}(Q^2)=\int g_1^{p-n}(x,Q^2)dx$ offers a simple and competitive method to determine α_s.
- This preliminary study indicates that JLab@22 GeV can provide a determination at the 1% level.
- This assumes available polarized data at low-x from EIC. On-going studies indicate that a EIC-only determination of $\alpha_s(M_Z)$ with the Bjorken sum would reach a ~2% accuracy.
- Possibilities of further improvement:
  1. Improved knowledge of pQCD series: $\alpha_s(M_Z)$ at β_4 already available. Preliminary N⁵LO results for Γ_1^{p-n} available.
  2. Improved perturbative methods, e.g., Principle of Maximum Conformality, a method minimizing pQCD truncation, has been worked out for Γ_1^{p-n}.
  3. Used conservative 6% for total systematics uncertainties. 5% or below may be reachable.
- This is but one of several ways to determine α_s with JLab@22 GeV. Others, e.g., global fits of (un)polarized PDFs may also provide competitive measurements. Put together, they have the potential to be provide a leading contribution toward a better determination of α_s: