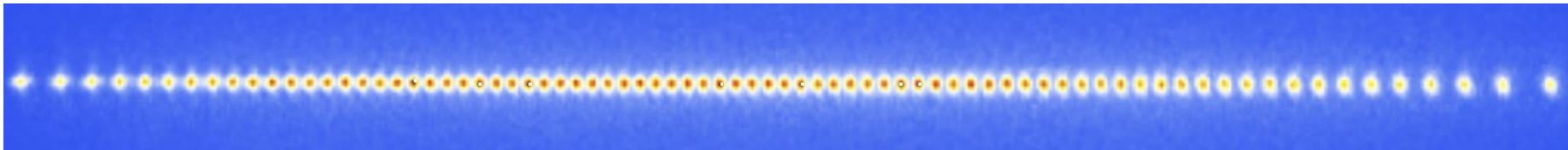


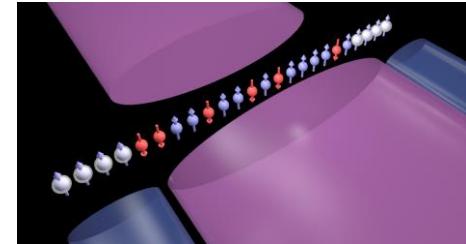
Towards Trapped-Ion Analog Simulation of Lattice Gauge Theories



Outline

Introduction:

- Trapped-Ion Quantum systems



Trapped-Ion Quantum Simulation of LGTs

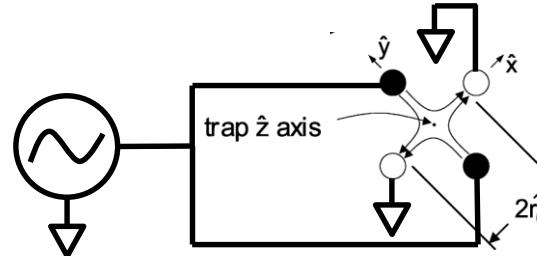
- Digital realizations of Schwinger model (2016, 2022)
- Analog route: Three-spin interactions, [B. Andrade et al., QST 7 034001, \(2022\)](#)
- Hybrid Analog-Digital simulations, [Z. Davoudi et al., PRR 7 034001, \(2022\)](#)

Outlook and Perspectives



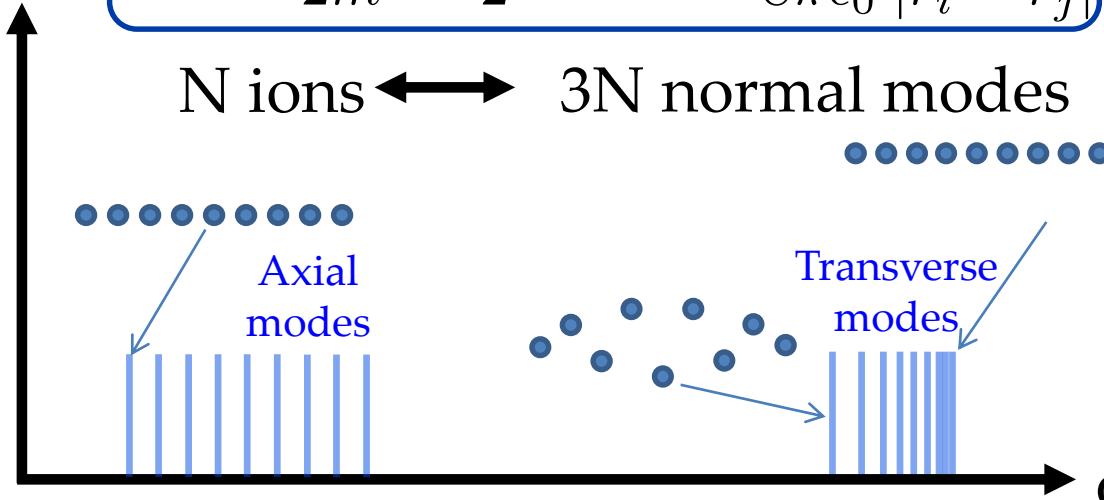
- Experimental progress at Rice University

Self-organized Ion crystals



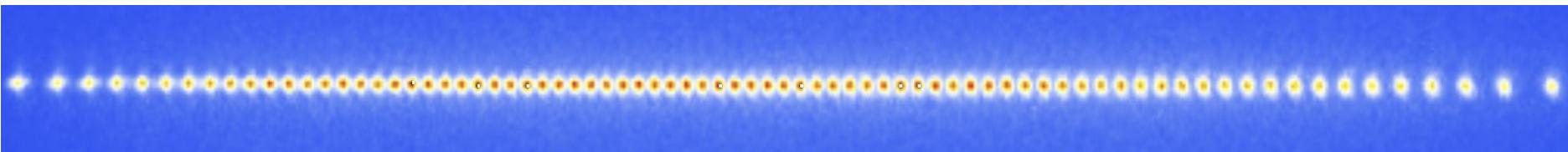
$$H_M = \frac{\hat{p}^2}{2m} + \frac{m}{2}\omega_{\alpha}^2 x_{\alpha i}^2 + \frac{q^2}{8\pi\epsilon_0} \frac{1}{|r_i - r_j|}$$

N ions \longleftrightarrow 3N normal modes



Mechanical Paul trap, https://www.youtube.com/watch?v=pG1TcnpsY_8

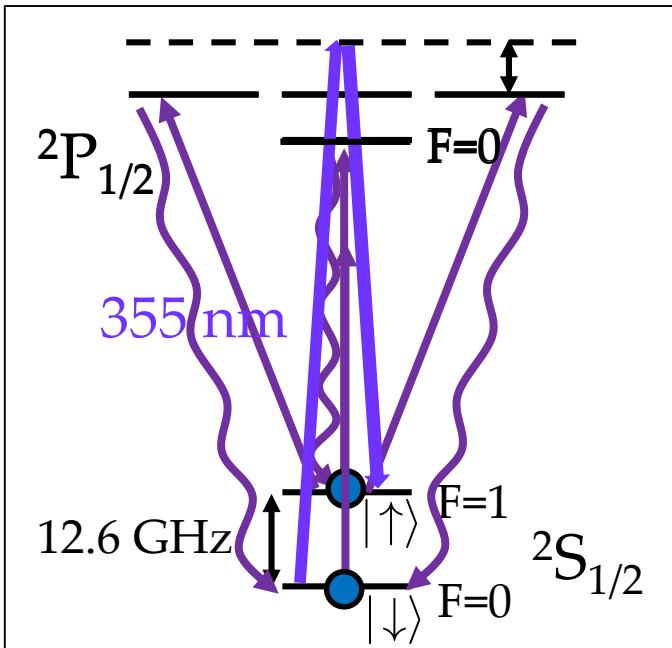
A many-body system assembled atom by atom



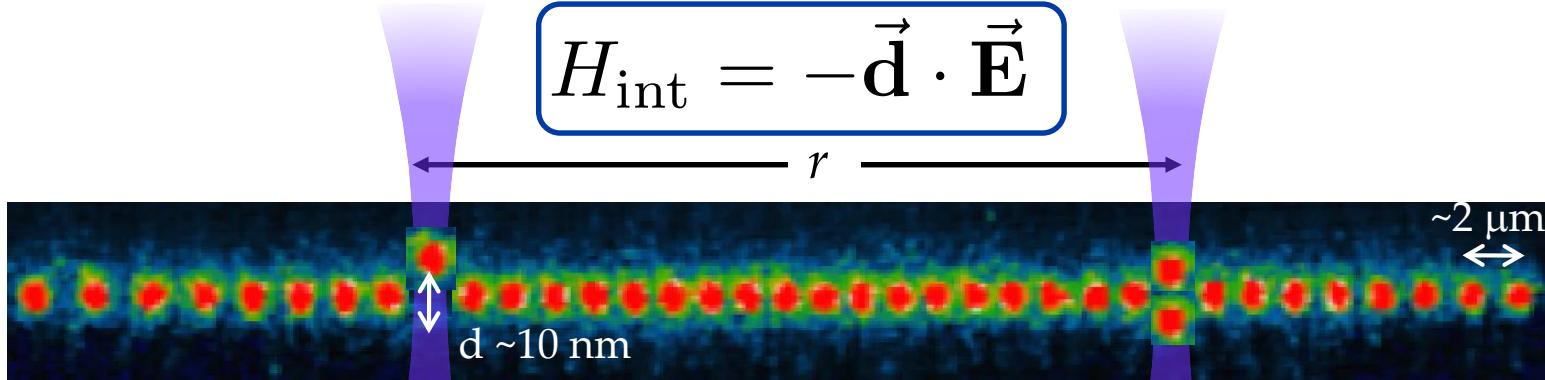
$^{171}\text{Yb}^+$ Clock states qubit

- Coherence time: $T_2 > 10$ minutes [1]
- High fidelity state preparation: > 99.9% in $\sim 10 \mu\text{s}$
- High speed readout: > 99.9% in $\sim 100 \mu\text{s}$
- High Fidelity one (>99% in 1 μs) and two qubit gates (~99% in 500 μs)

[1] P. Wang et al., Nat. Comm. **12**, 233 (2021)



Wiring Trapped-ion Qubits with Laser beams



Spin dependent force

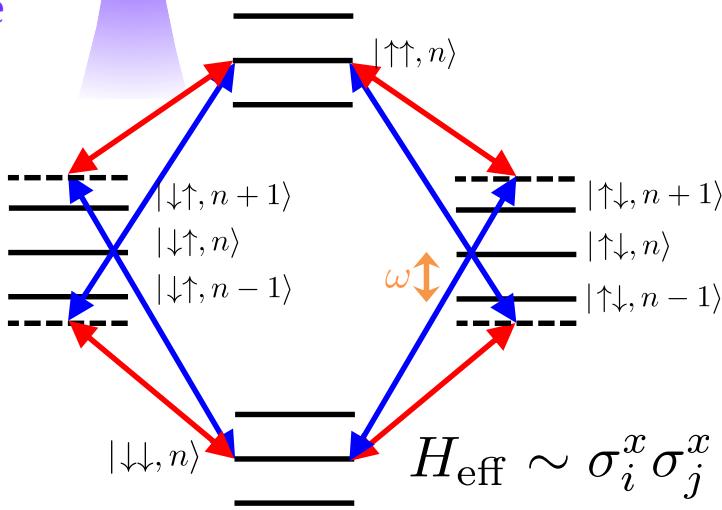
$$|\downarrow\rangle|\downarrow\rangle \rightarrow |\downarrow\rangle|\downarrow\rangle$$
$$|\uparrow\rangle|\uparrow\rangle \rightarrow |\uparrow\rangle|\uparrow\rangle$$

$$\Delta E = \frac{e^2}{\sqrt{\delta^2 + r^2}} - \frac{e^2}{r} \sim \frac{(e\delta)^2}{2r^3}$$

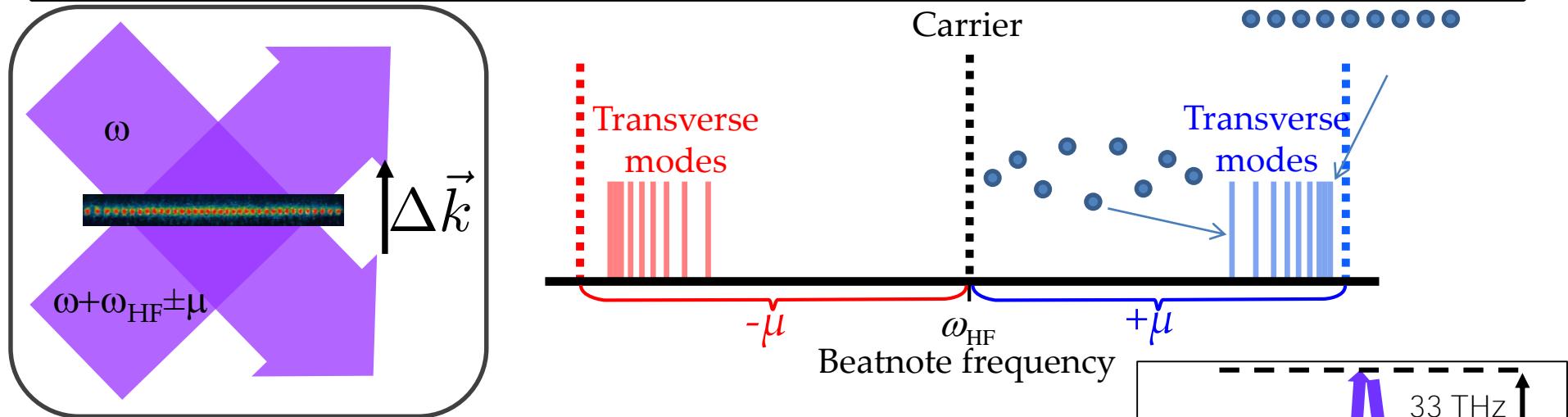
$$|\downarrow\rangle|\uparrow\rangle \rightarrow e^{i\varphi}|\downarrow\rangle|\uparrow\rangle$$
$$|\uparrow\rangle|\downarrow\rangle \rightarrow e^{i\varphi}|\uparrow\rangle|\downarrow\rangle$$

$$\varphi = \frac{\Delta E t}{\hbar}$$

Cirac & Zoller (1995)
Molmer & Sorensen (1999)
Solano, et al. (1999)
Milburn, et al. (2000)



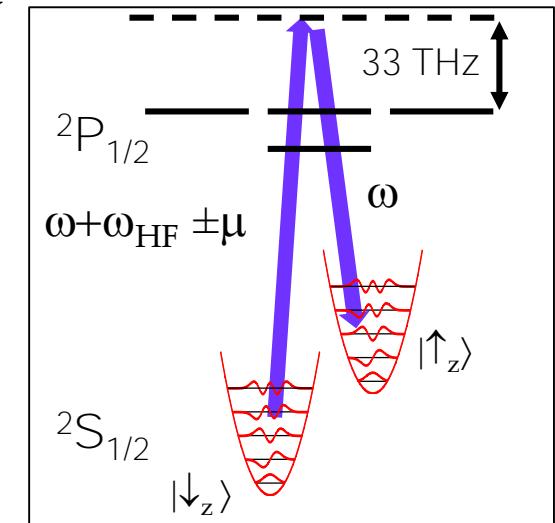
Generating Spin Hamiltonians



$$H_{eff} = \sum_{i < j} J_{ij} \sigma_i^x \sigma_j^x + \boxed{\sum_{i,\alpha} B_\alpha \sigma_i^\alpha}$$

$$\boxed{\frac{J_0}{|i - j|^\alpha}}$$

$$0.5 < \alpha < 2$$



Non-Equilibrium Studies with Trapped Ions

Long-Range Transverse Field Ising Model

- Breaks Integrability
- Theoretically Challenging
- Model for quantum systems in nature

Reviews:

- C. Monroe, *et al.*, RMP **93** 025001 (2021)
- N. Defenu, *et al.*, RMP in press (2023)

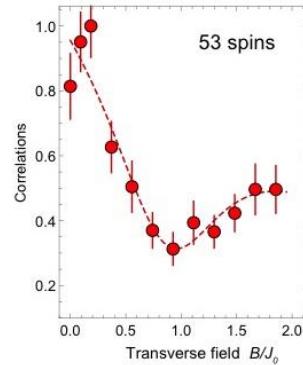
$$H_{\text{eff}} = \sum_{ij} J_{ij} \sigma_i^x \sigma_j^x + B \sum_i \sigma_i^z$$
$$J_{ij} \sim \frac{J_0}{|i - j|^\alpha}$$

Quantum quenches and Thermalization in Spin LR Systems

Many-Body Localization [1,2]

[1] Smith *et al.*, (Nature Phys. 2016)

[2] Bridges *et al.*, (Science 2019)



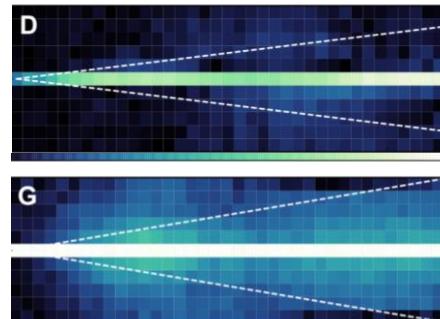
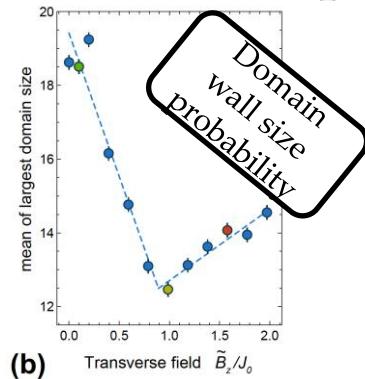
Dynamical Phase transition [3,4]

[3] Zhang, GP, *et al.*, Nature (2017)

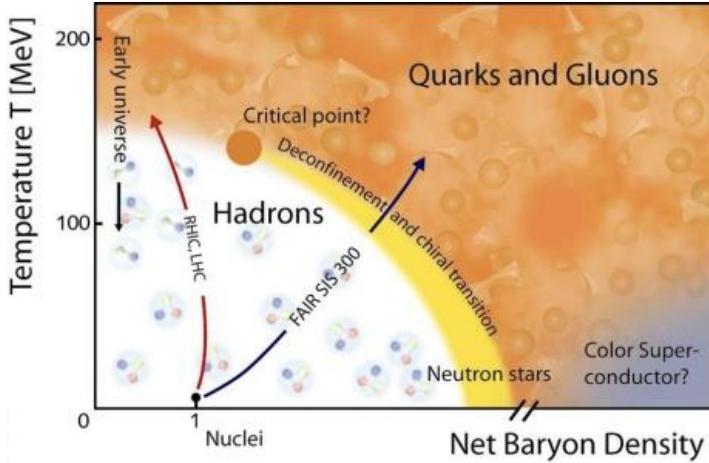
[4] Jurcevic, *et al.*, PRL (2017)

Confinement of Domain Walls [5]

[5] Tan, Becker, *et al.*, Nature Phys. (2021)



From Spin models to NP and HEP models



Solving QCD on a lattice:

- Phase diagram largely unexplored
- MonteCarlo has sign problem

Roadblock for both Real-time dynamics
and equilibrium calculations

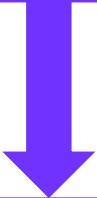
$$\mathcal{L}(\psi, F^{\mu\nu}, \dots)$$



$$H(\sigma_i^x, a^\dagger, \dots)$$

From Spin models to NP and HEP models

$$\mathcal{L} = \bar{\psi}(iD^\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



Kogut – Susskind staggered fermions
+
Jordan-Wigner Transformation

$$H = \frac{1}{2a} \sum_n [\sigma_n^+ L_n^+ \sigma_{n+1}^- + \text{h.c.}] + m \sum_n (-1)^n \sigma_n^z + \frac{g^2 a}{2} \sum_n L_n^2$$

$$L_n = \frac{E_n}{g}$$

Two Challenges:

- 1) Engineering three-body interactions
- 2) Preserve Gauss Law

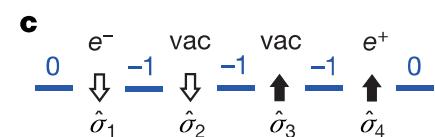
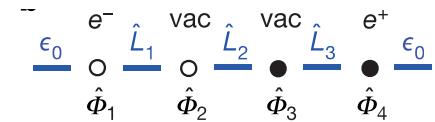
$$L_n - L_{n-1} + \frac{1}{2} [\sigma_n^z + (-1)^n] = \text{const.}$$

Odd lattice sites

$$\begin{aligned} \bullet_n &\equiv \uparrow_n \equiv \text{vac} & L_n &= L_{n-1} \\ \circ_n &\equiv \downarrow_n \equiv e^- & L_n &= L_{n-1} - 1 \end{aligned}$$

Even lattice sites

$$\begin{aligned} \bullet_n &\equiv \uparrow_n \equiv e^+ & L_n &= L_{n-1} + 1 \\ \circ_n &\equiv \downarrow_n \equiv \text{vac} & L_n &= L_{n-1} \end{aligned}$$



1+1D Schwinger model with trapped ions

$$H = w \sum_n [\sigma_n^+ e^{i\theta_n} \sigma_{n+1}^- + \text{h.c.}] + \frac{m}{2} \sum_n (-1)^n \sigma_n^z + J \sum_n L_n^2$$

Implementation with long range interactions

$$L_n - L_{n-1} = \frac{1}{2} [\sigma_n^z + (-1)^n]$$

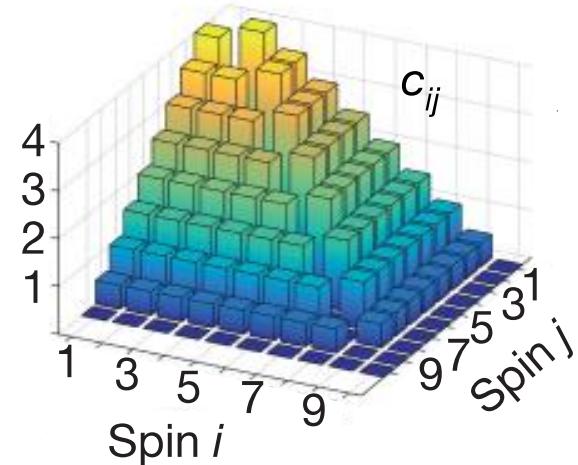
$$\sigma_n^- \rightarrow \prod_{l < n} e^{i\theta_l} \sigma_n^-$$

$$H = w \sum_{n=1}^{N-1} [\sigma_n^+ \sigma_{n+1}^- + \text{h.c.}] + \frac{m}{2} \sum_{n=1}^N c_n \sigma_n^z + \frac{J}{2} \sum_{n=1}^{N-2} \sum_{l=n+1}^{N-1} \sigma_n^z \sigma_l^z$$

Particle-
antiparticle
hopping

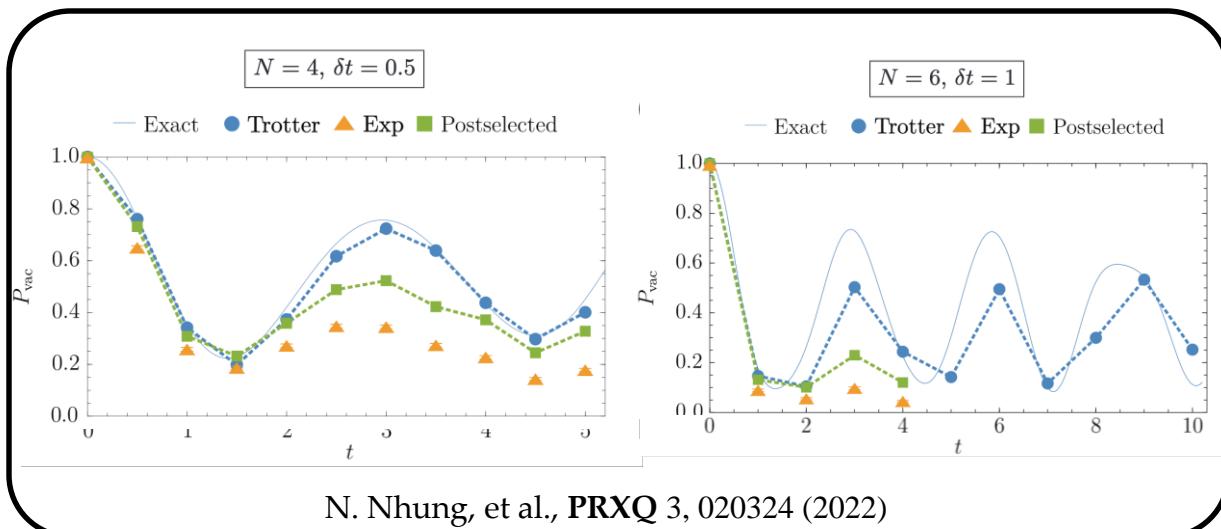
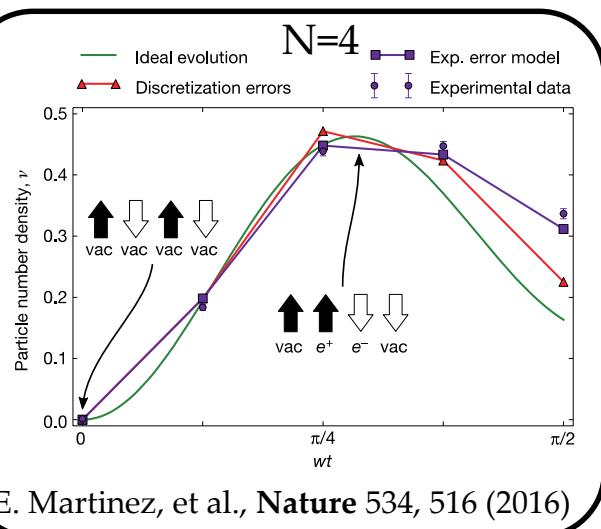
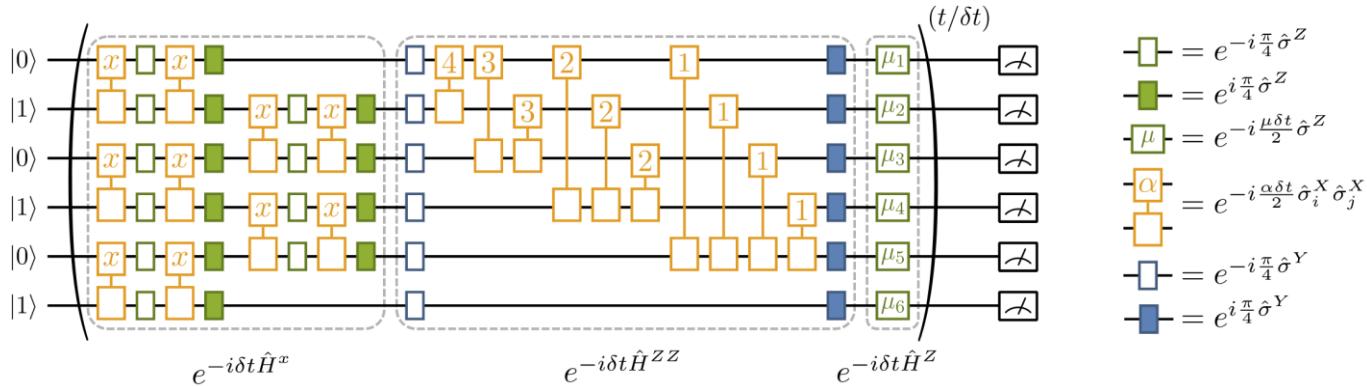
Mass
term

Gauge field



E. Martinez, et al., **Nature** 534, 516 (2016)
C. Mudrik, et al., **NJP** 19, 103020 (2017)

1+1D Schwinger model: digital approach



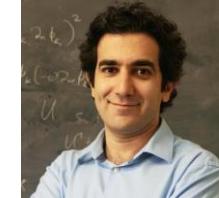
Analog proposal for the 1+1D Schwinger model

$$H = w \sum_{n=1}^{N-1} [\sigma_n^+ \sigma_{n+1}^- + \text{h.c.}] + \frac{m}{2} \sum_{n=1}^N c_n \sigma_n^z + \frac{J}{2} \sum_{n=1}^{N-2} \sum_{l=n+1}^{N-1} \sigma_n^z \sigma_l^z$$

Particle-antiparticle hopping

Mass

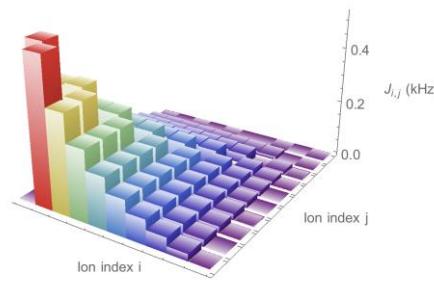
Gauge field



Z. Davoudi

A. Seif

M. Hafezi



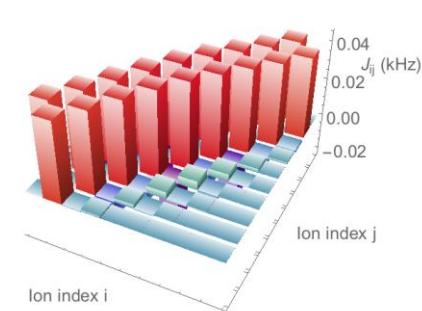
Transverse modes

Beatnote frequency



A. N. Shaw

C. Monroe

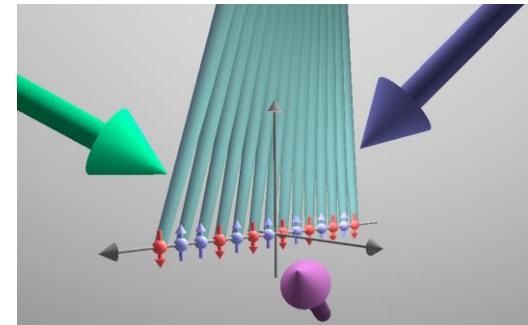


Axial modes

Axial modes

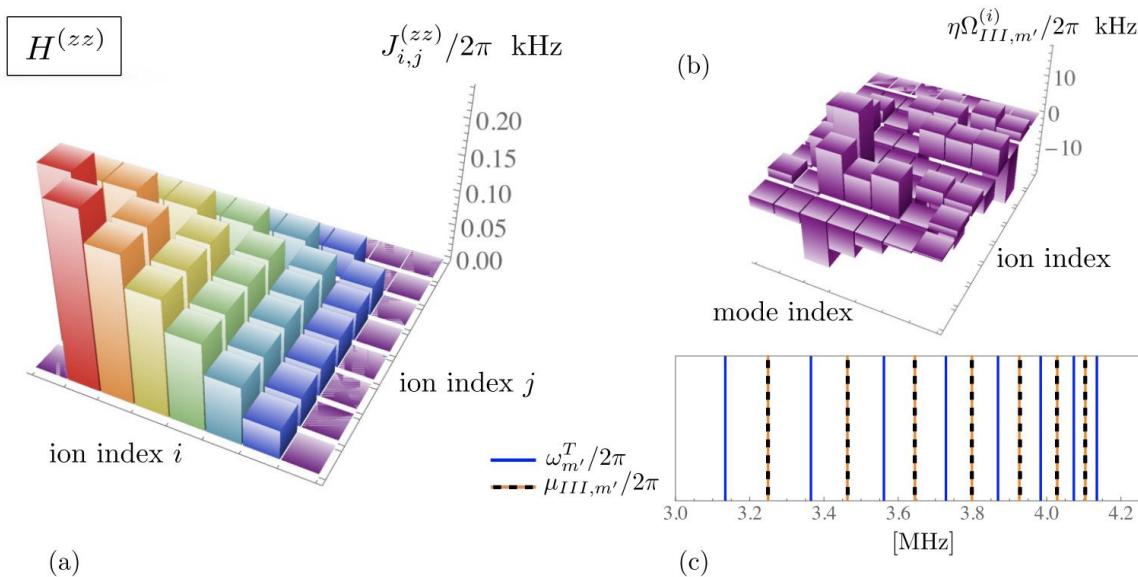
ω_{HF}

Beatnote frequency



Z. Davoudi et al., PRR 2, 023015 (2020)

Analog proposal for the 1+1D Schwinger model



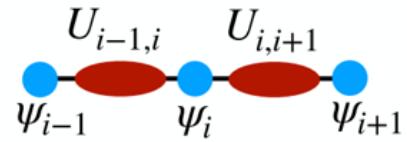
Multi-amplitude and multi-frequency Optimization with full magnus expansion:

- Residual spin-phonon coupling
- Cross-talk between orthogonal normal modes
- Higher order processes

The higher-order interaction route

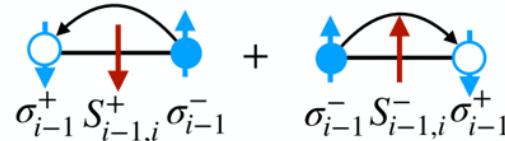
$$H_{U(1)} = x \sum_{i=1}^{N_{\text{stag}}-1} [\psi_i^\dagger U_i \psi_{i+1} + \text{h.c.}] + \sum_{i=1}^{N_{\text{stag}}-1} E_i^2 + \mu \sum_{i=1}^{N_{\text{stag}}} (-1)^i \psi_i^\dagger \psi_i$$

hopping field mass



Quantum Link formulation

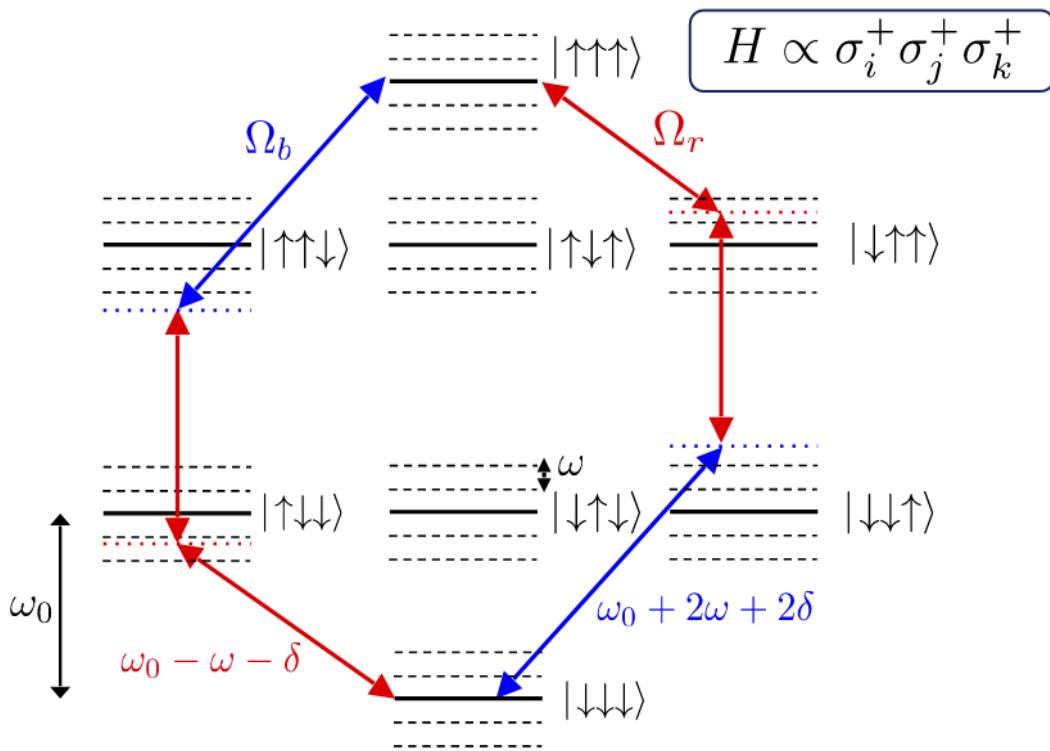
$$H_{\text{QLM}} = J \sum_{i=1}^{N_{\text{stag}}-1} [\sigma_i^+ S_i^+ \sigma_{i+1}^- + \text{h.c.}] + \sum_{i=1}^{N_{\text{stag}}-1} S_z^2 + \mu \sum_{i=1}^{N_{\text{stag}}} (-1)^i \sigma_i^z$$



$$H_{\text{Ion}} = J \sum_{i=1}^{N_{\text{stag}}-1} [\sigma_{2i-1}^+ \sigma_{2i}^+ \sigma_{2i+1}^- + \text{h.c.}] + \mu \sum_{i=1}^{N_{\text{stag}}} (-1)^i \sigma_{2i-1}^z$$

$$H \sim \sigma_{i-1}^+ \sigma_i^+ \sigma_{i+2}^+$$

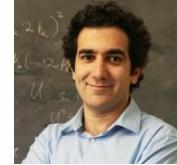
Generalized Molmer-Sorensen scheme



B. Andrade et al., QST 7 034001, (2022)



Z. Davoudi



M. Hafezi



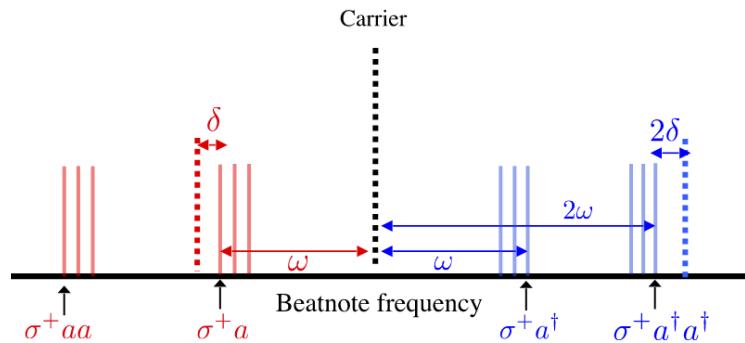
B. Andrade



A. Seif



T. Grass



Generalized Molmer-Sorensen scheme

Effective Hamiltonian in single mode approximation:

$$\mathcal{H}_{\text{eff}}^{(\sigma)} = -\frac{1}{4} \sum_i \frac{\eta_{\text{COM}}^2}{\delta} \left[\Omega_r^2 \left(n + \frac{1}{2} \right) - \frac{1}{8} \eta_{\text{COM}}^2 \Omega_b^2 \left(n^2 + n + 1 \right) \right] \sigma_i^z$$

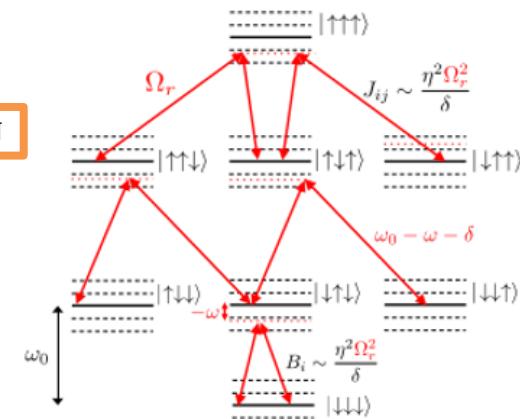
ODD

$$\mathcal{H}_{\text{eff}}^{(\sigma\sigma)} = \frac{1}{4} \sum_i \sum_{j \neq i} \frac{\eta_{\text{COM}}^2}{\delta} \left[\Omega_r^2 + \frac{1}{2} \eta_{\text{COM}}^2 \Omega_b^2 \left(n + \frac{1}{2} \right) \right] \sigma_i^+ \sigma_j^-$$

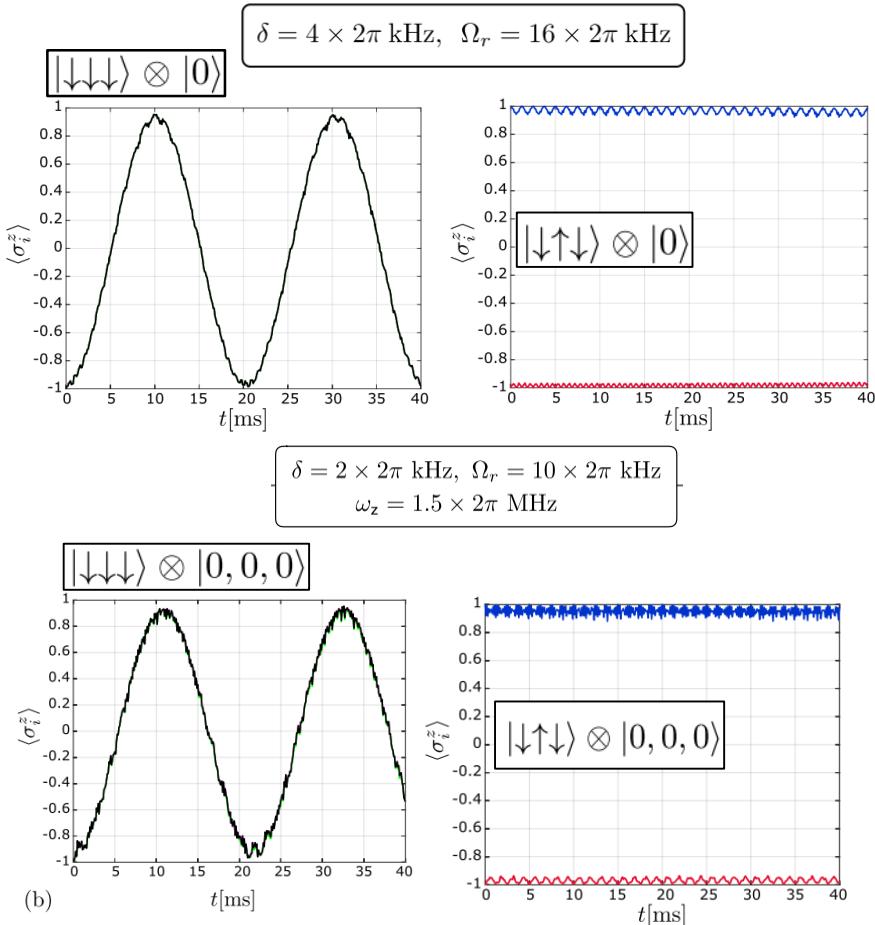
ODD

$$\mathcal{H}_{\text{eff}}^{(\sigma\sigma\sigma)} = \sum_{i,j,k} \left[\frac{\eta_{\text{COM}}^4}{16\delta^2} \Omega_r^2 \Omega_b \sigma_i^+ \sigma_j^+ \sigma_k^+ + \text{h.c.} \right]$$

EVEN

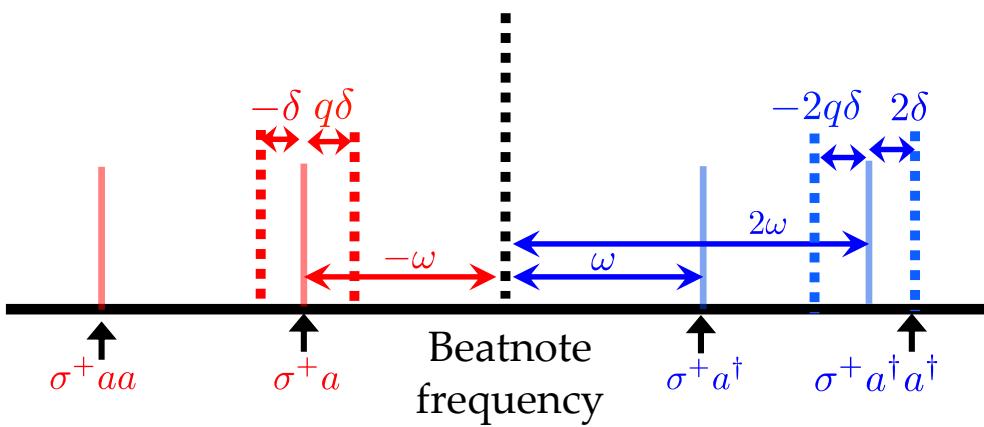


Two-body term suppression



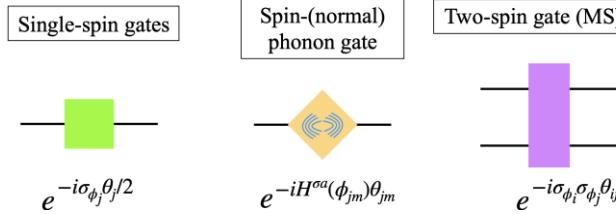
Multifrequency drive to suppress unwanted
2-body interactions

Single phonon
mode

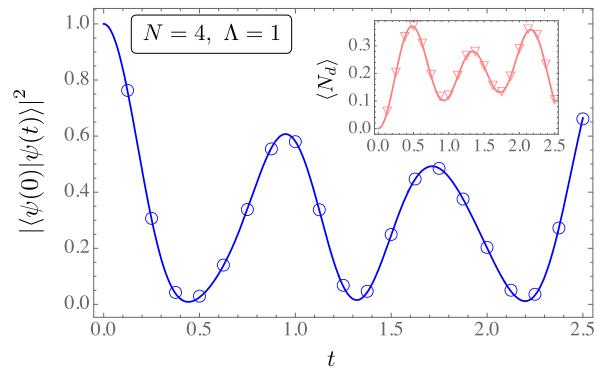
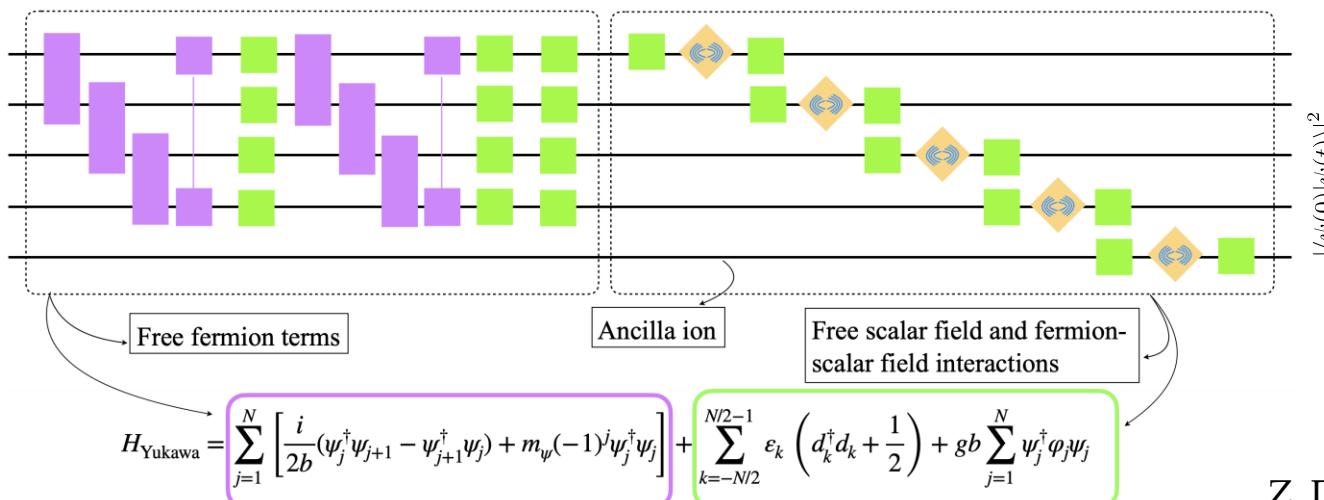


Three phonon
modes

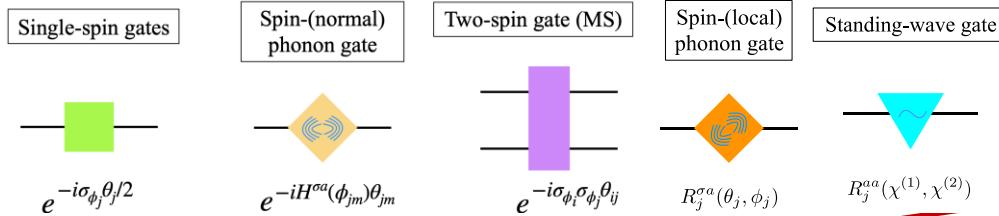
Hybrid Analog-Digital simulation of Yukawa model



Spin → Fermions $\psi_i = \prod_{l < j} (i\sigma_l^z)\sigma_j^-$ Z. Davoudi N. M. Linke
 Collective modes → scalar bosons $d_k^\dagger, d_k, \varphi_j$



Spin-phonon realization of Schwinger model



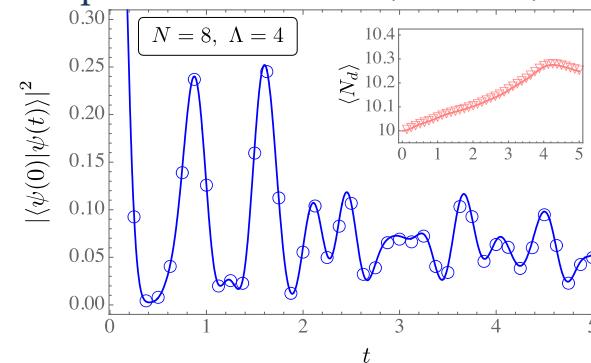
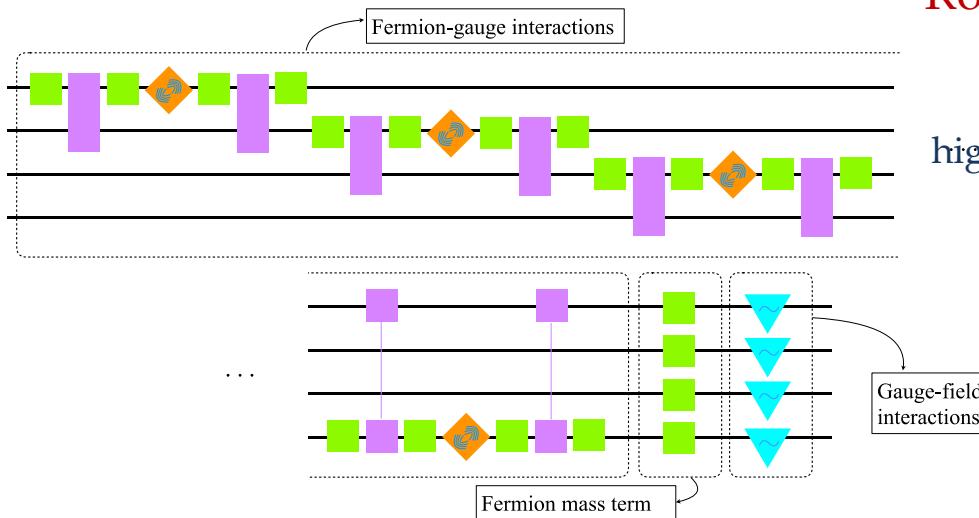
Gauge bosons local in space

$$\frac{i}{2b} \sum_{j=1}^N (\psi_j^\dagger U_j^\dagger \psi_{j+1} - \psi_j U_j \psi_{j+1}^\dagger) + m \sum_{j=1}^N (-1)^j \psi_j^\dagger \psi_j + \frac{g^2 b}{2} \sum_{j=1}^N E_j^2 \text{ with } [E_j, U_{j'}] = U_j \delta_{j,j'}$$

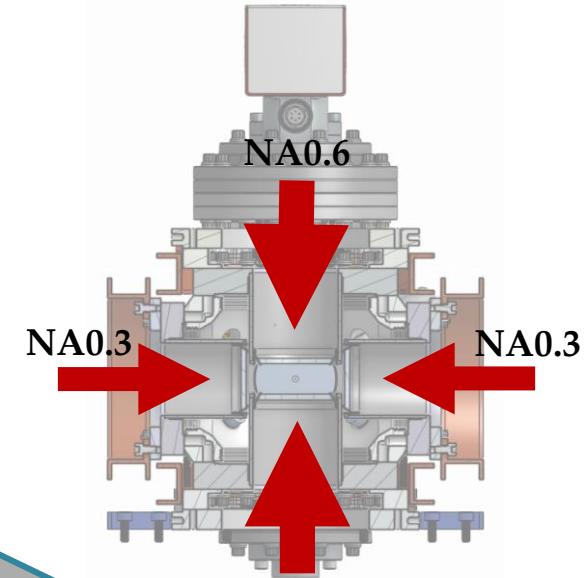
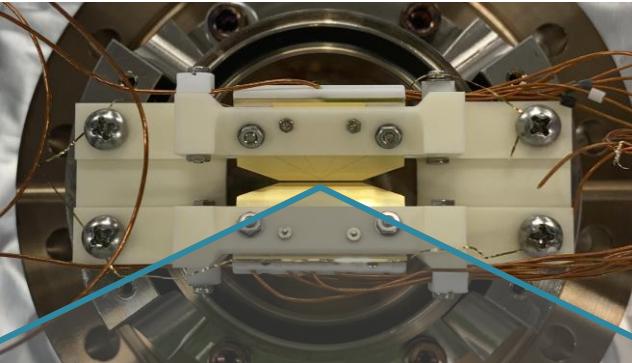
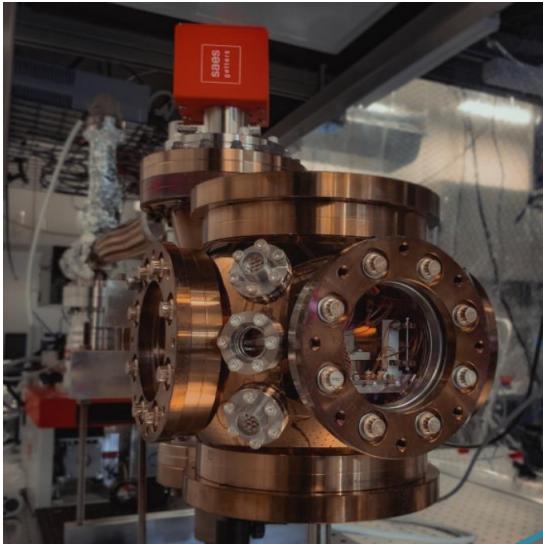
Rotor algebra \neq phonon algebra

$$\downarrow \text{HOBM } \frac{g^2 b}{2} \sum_{j=1}^N \left[-2M(d_j^\dagger d_j) + (d_j^\dagger d_j)^2 \right]$$

highly occupied bosonic mode, PRA 94,052321 (2016)

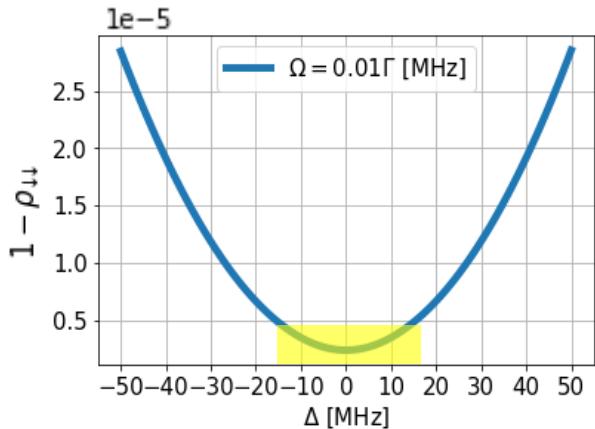


Experimental System at Rice

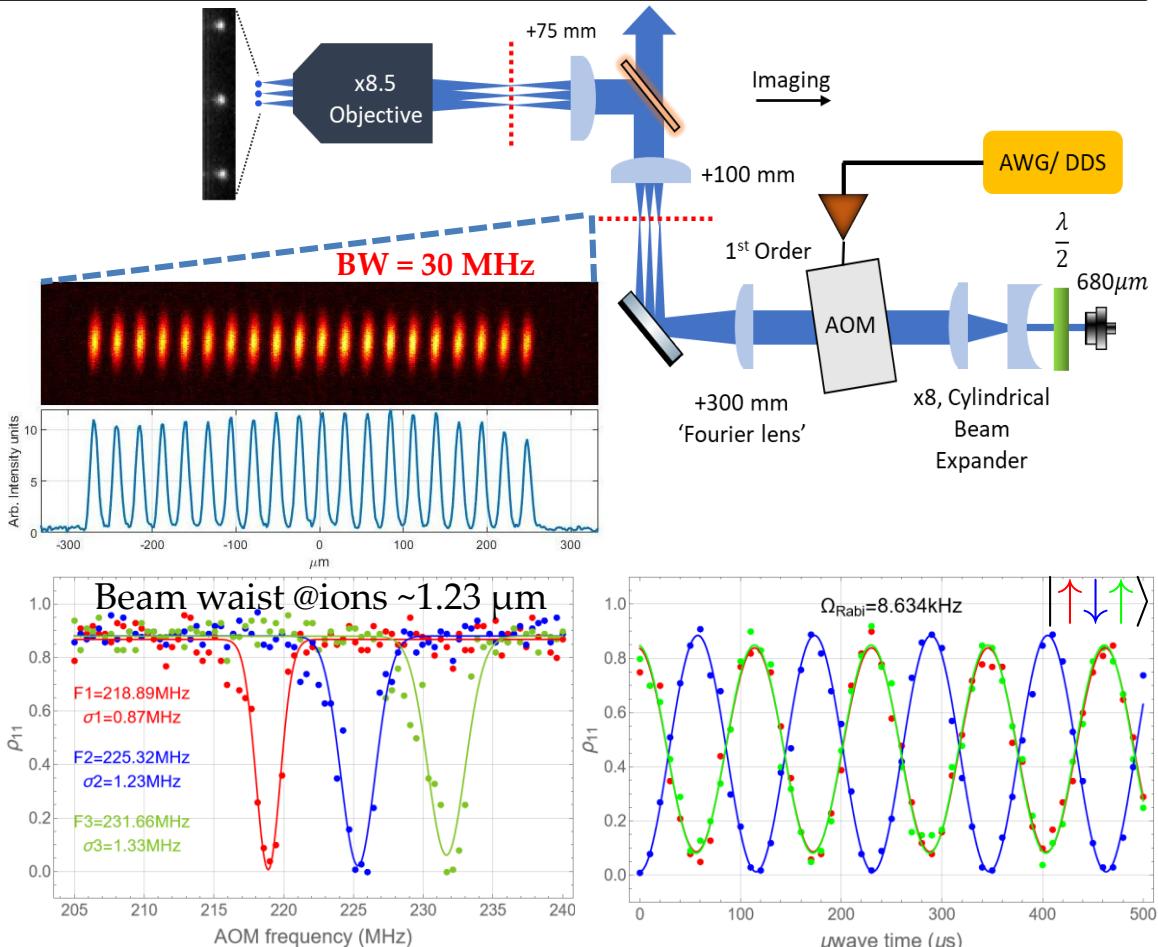


Individual Optical pumping

OP weakly dependent
on frequency

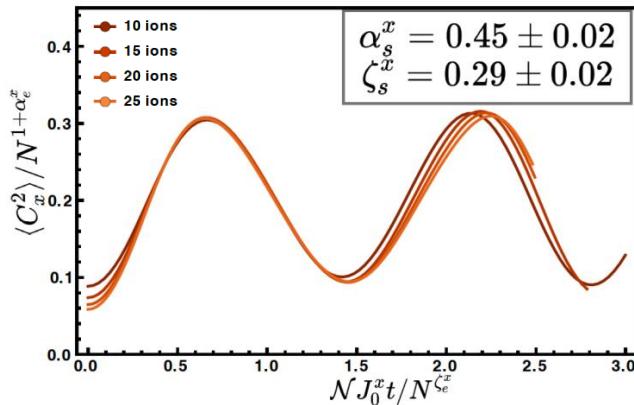
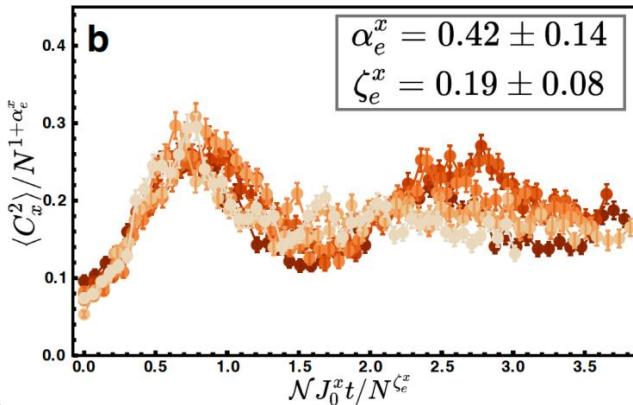


Simple Tweezer array
approach based on
Elliptical beams



Other Directions

Critical non-equilibrium dynamics

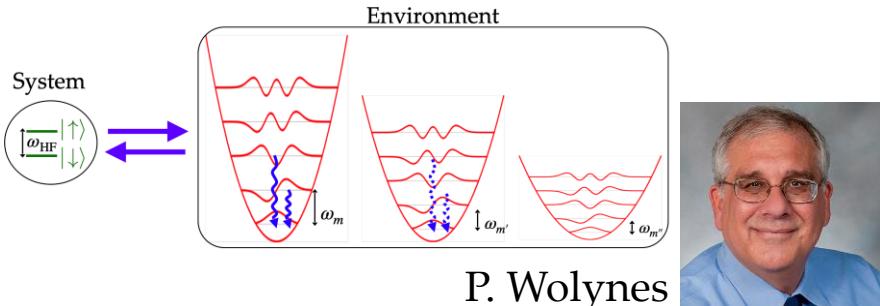


M. Maghrebi C. Monroe

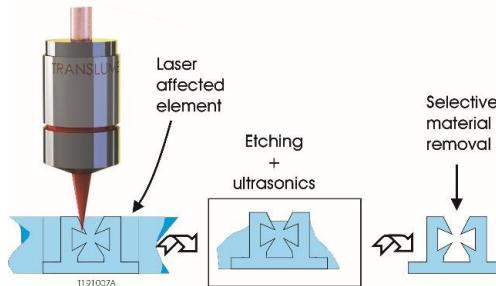


W. Morong A. De

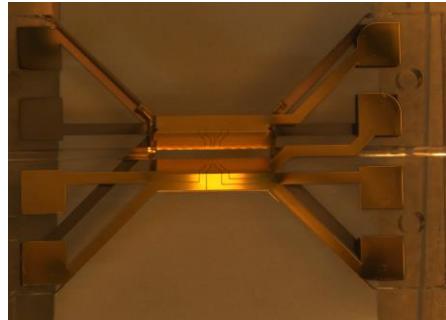
Quantum Chemistry/Electron transfer



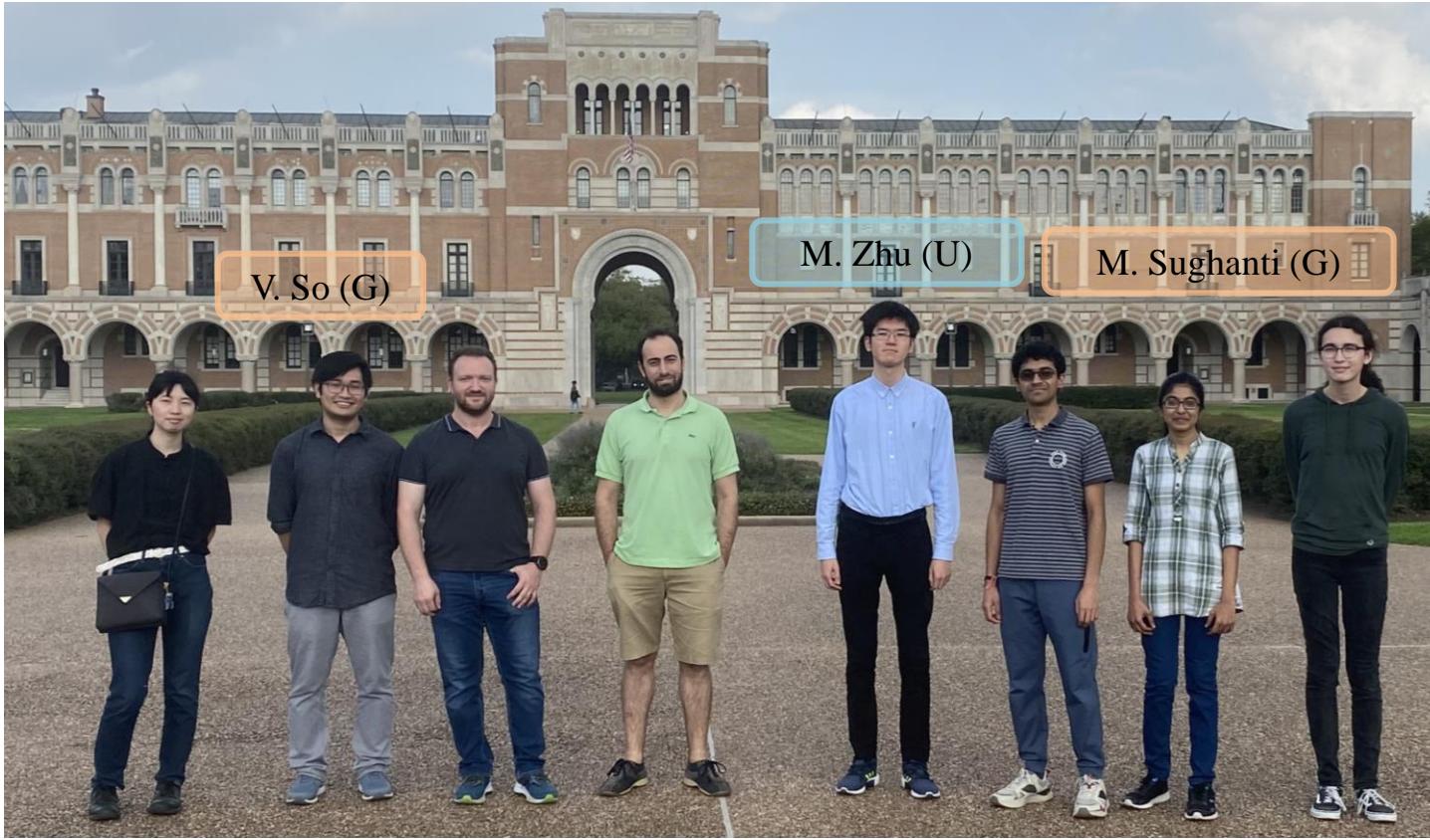
Ion Trapping made easy: Monolithic trap



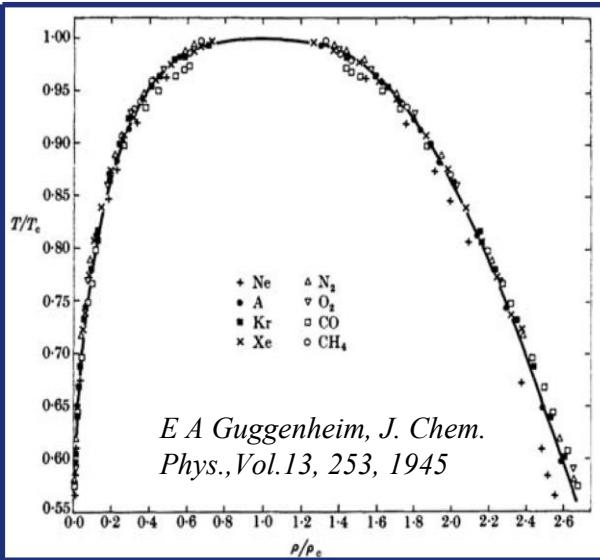
N. M. Linke + Translume



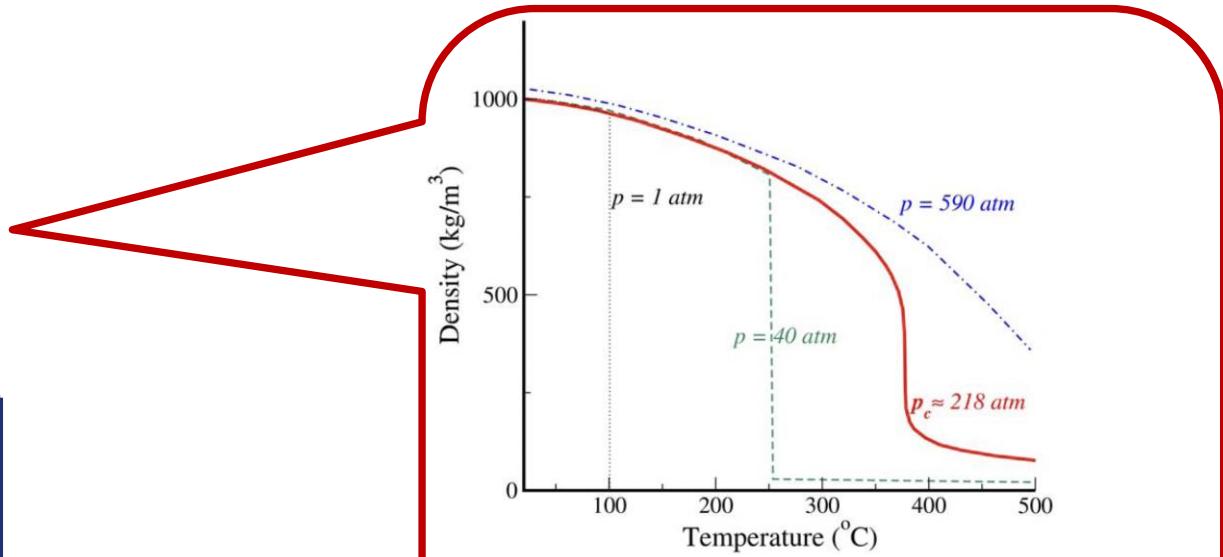
Acknowledgments



Phase transitions and criticality



Density and Temperatures for different systems scaled with a “critical exponent” to collapse → “Universal properties” of phase transition.



At 218 atm and 374°C water and vapor co-exists. Critical point for the phase transition.

Can we find universal scaling in non-equilibrium quantum systems?

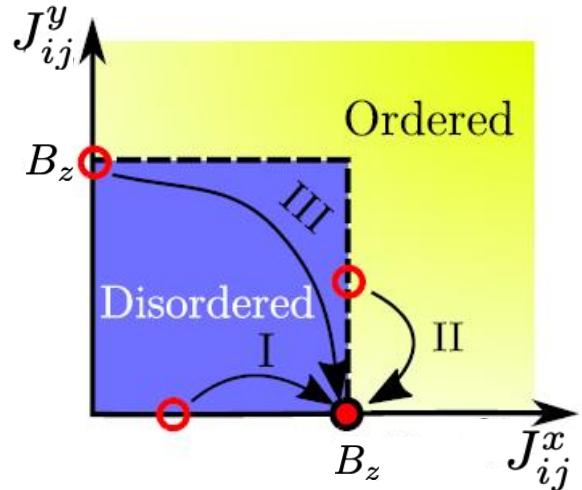
Non equilibrium criticality in quench dynamics

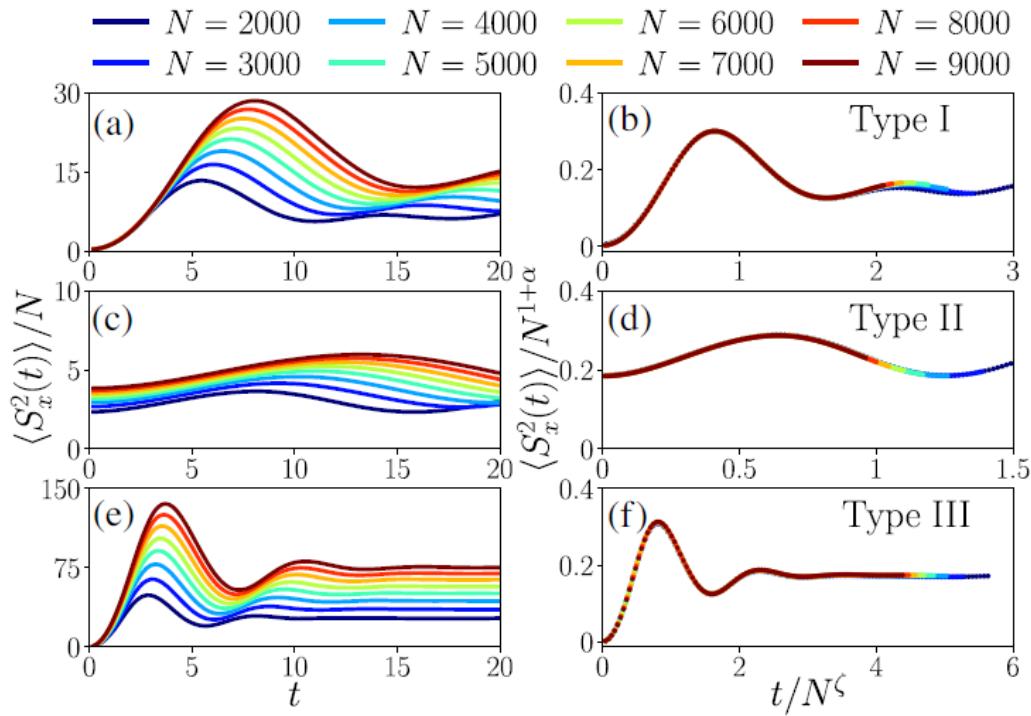
Prototype model for long-range interaction: [Lipkin-Meshkov-Glick model \(LMG\)](#)
(a.k.a. “all-to-all” interactions)

$$H = -\frac{1}{N} \sum_{i < j} J_{ij}^x \sigma_i^x \sigma_j^x + J_{ij}^y \sigma_i^y \sigma_j^y - B_z \sum_i \sigma_i^z$$

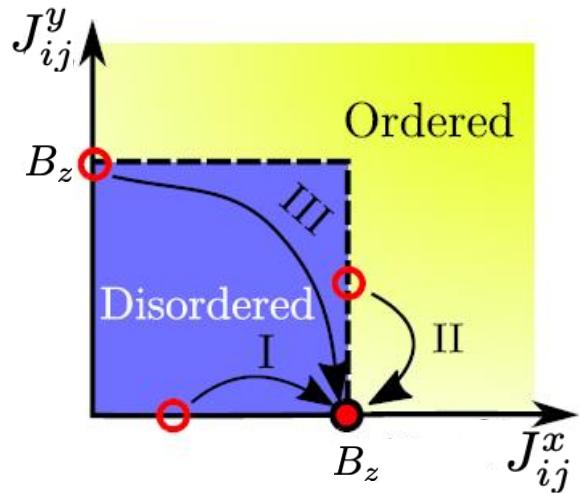
Order Parameters: $\langle S_x \rangle, \langle S_y \rangle$

Observables: $S_a = \frac{1}{2} \sum_i \sigma_i^a, \quad \frac{1}{N} \langle S_x^2(t) \rangle = N^\alpha f\left(\frac{t}{N^\xi}\right)$



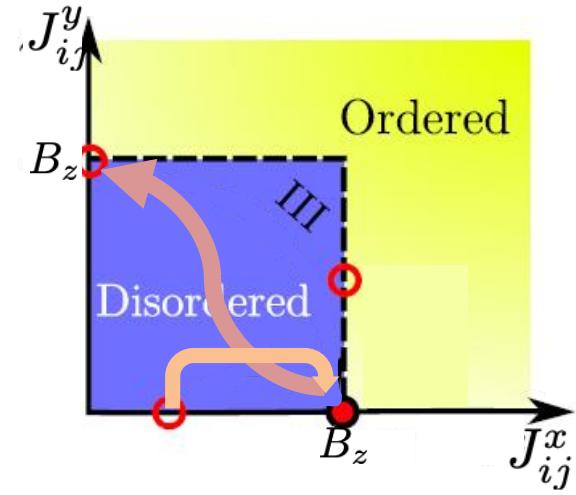
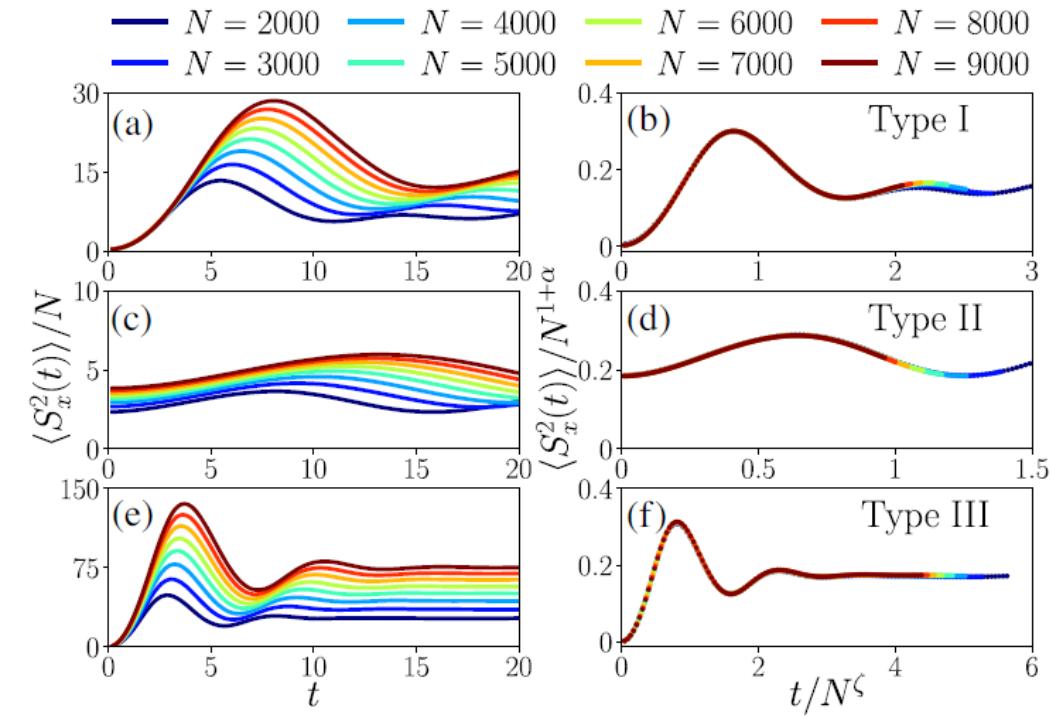


Depending on the initial state, the LMG dynamics scales with different exponents



	Type I	Type II	Type III	TCP	QCP
ζ	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$
α	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
$T_{\text{eff}}^{\text{IR}}$	Finite	0	$\sim N^{1/3}$	T_c	0

Adiabatically preparing a critical state is experimentally expensive.



	Type I	Type II	Type III	TCP	QCP
ζ	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$
α	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
$T_{\text{eff}}^{\text{IR}}$	Finite	0	$\sim N^{1/3}$	T_c	0

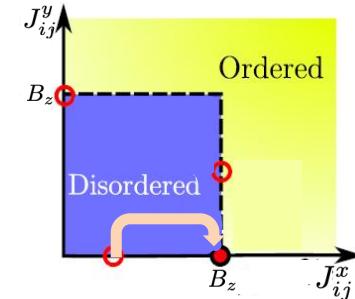
Double
Quench
Approach

Type I quench

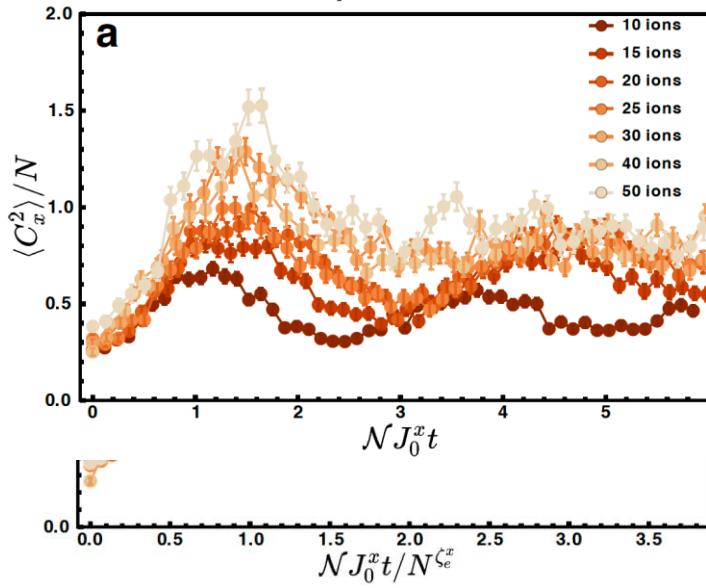
$| \downarrow z \downarrow z \downarrow z \dots \downarrow z \rangle$

$$H = \sum_{ij} J_{ij} \sigma_x^i \sigma_x^j + B_z \sum_i \sigma_z^i$$

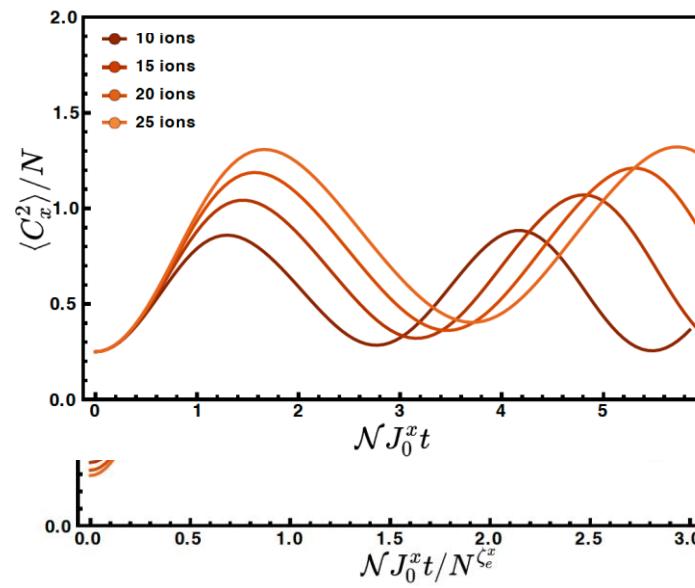
$\langle S_x^2 \rangle$



Experiment



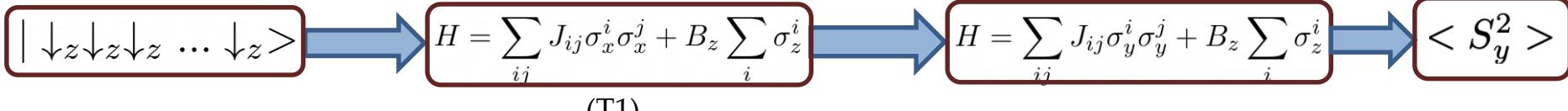
Simulation



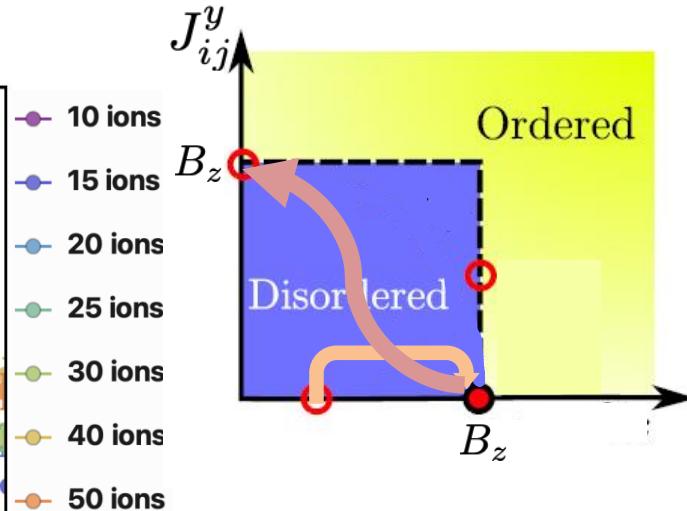
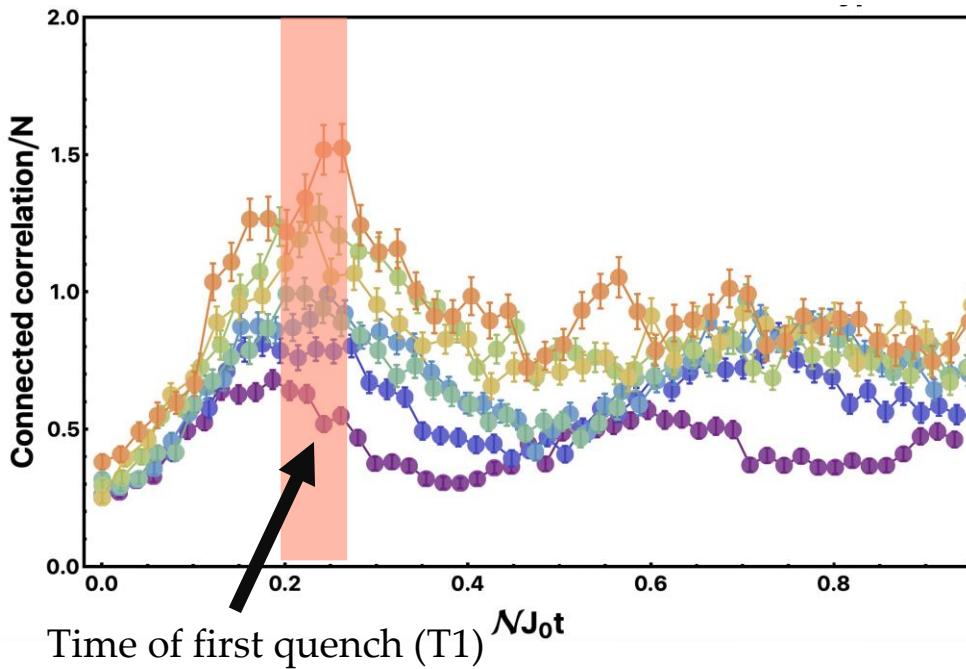
Critical exponents consistent with a thermal phase transition

$$\mathcal{N} = \frac{\sum_{ij} J_{ij}}{N - 1}$$

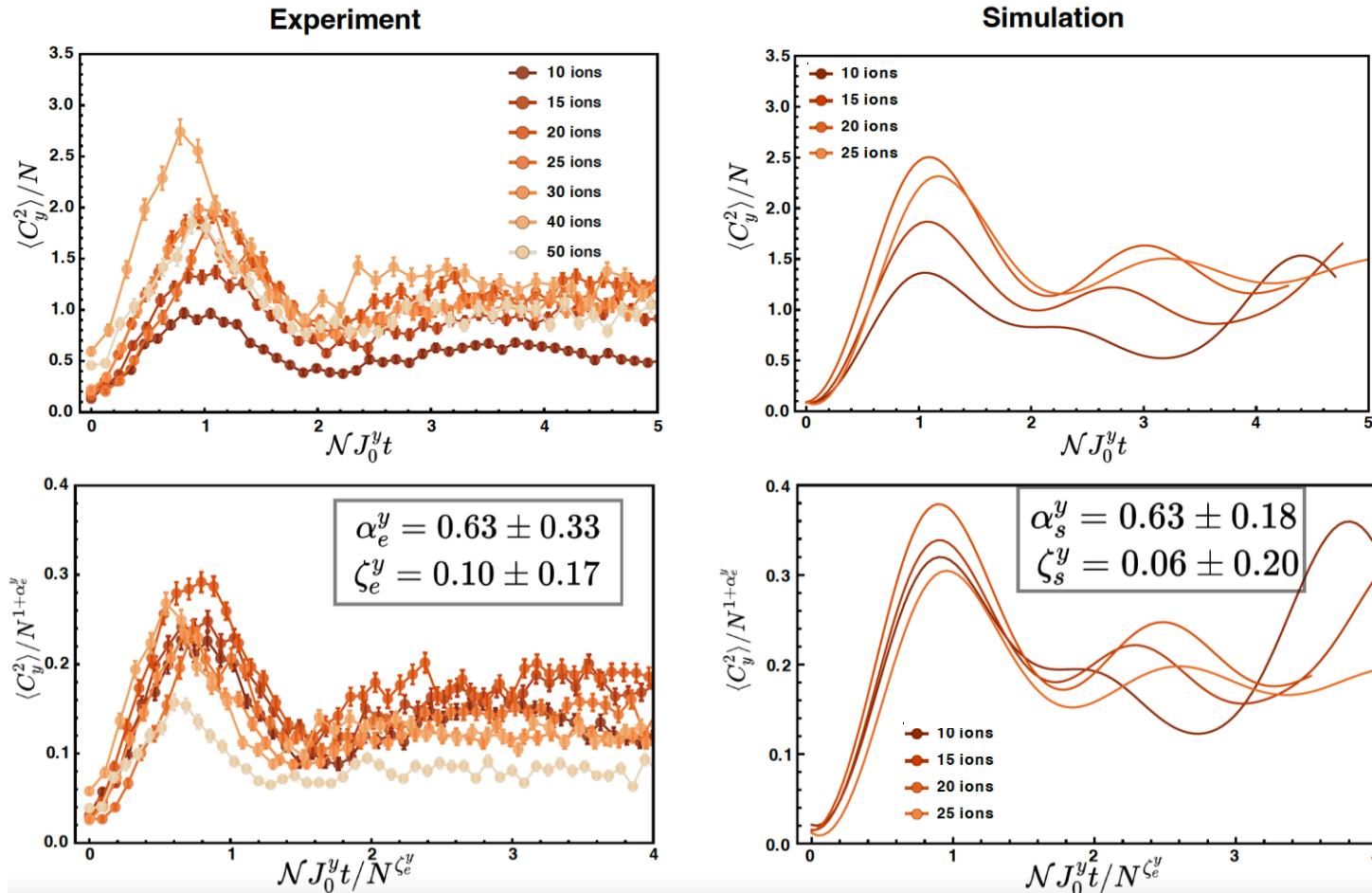
Double quench



Unscaled critical fluctuation after type 1 quench



Critical scaling behavior after double quench with 10-50 ions



Critical exponents consistent with a non-equilibrium phase transition.

Hierarchy of quenches

- One can keep doing multiple quenches.
- There are different universality classes for different sequence of quenches
- The scaling exponents are related by recursive relations

$$\alpha_D^{(n+1)} = \frac{1 + \alpha_S^{(n)}}{2},$$

$$\alpha_S^{(n+1)} = \alpha_S^{(n)},$$

$$\zeta^{(n+1)} = \frac{1 - \alpha_S^{(n)}}{4},$$

	Type-I-III-III-...			Type-II-III-III-...			Type-III-III-III-...		
k	$\alpha_D^{(k)}$	$\alpha_S^{(k)}$	$\zeta^{(k)}$	$\alpha_D^{(k)}$	$\alpha_S^{(k)}$	$\zeta^{(k)}$	$\alpha_D^{(k)}$	$\alpha_S^{(k)}$	$\zeta^{(k)}$
1	1/2	0	1/4	1/3	-1/3	1/3	1/3	-1/3	1/3
2	3/4	1/2	1/8	2/3	1/3	1/6	2/3	1/3	1/6
3	7/8	3/4	1/16	5/6	2/3	1/12	5/6	2/3	1/12
4	15/16	7/8	1/32	11/12	5/6	1/24	11/12	5/6	1/24
5	31/32	15/16	1/64	23/24	11/12	1/48	23/24	11/12	1/48
6	63/64	31/32	1/128	47/48	23/24	1/96	47/48	23/24	1/96
7	127/128	63/64	1/256	95/96	47/48	1/192	95/96	47/48	1/192
8	255/256	127/128	1/512	191/192	95/96	1/384	191/192	95/96	1/384

Acknowledgments



A. De



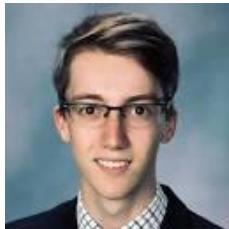
W. Morong → AWS



C. Monroe



M. Maghrebi



P. Cook



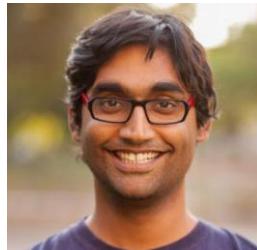
D. Paz



W. L. Tan → IonQ



A. V. Gorshkov



P. Titum



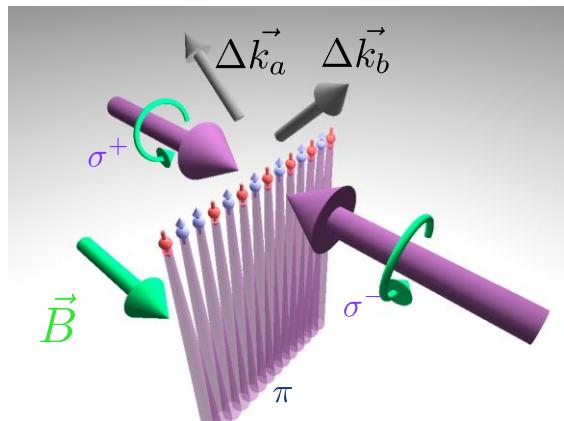
New Opportunities in Quantum Simulation

QAOA, [PNAS 117 \(41\), 25396 \(2020\)](#)

MB Dephasing, [PRL 125, 120605 \(2020\)](#)

Confinement, [Nature Phys. 17, 742 \(2021\)](#)

- Global Interaction Control
- One set of normal modes

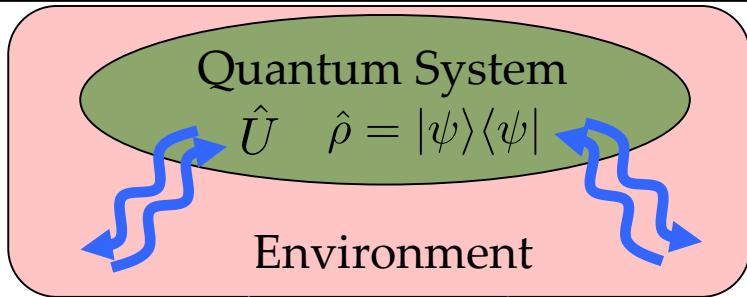


 RICE

- Quantum magnetism models
- Optimization problems
- Quantum Spin glasses
- High Energy Physics
- **Driven-Dissipative quantum systems**



Dissipative Many-body Systems



Dissipative Phase Transitions (DPT)

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar} [H, \rho] + \mathcal{L}(\rho)$$

Properties of the Average state
 $\langle O \rangle = \text{Tr}(\rho O)$

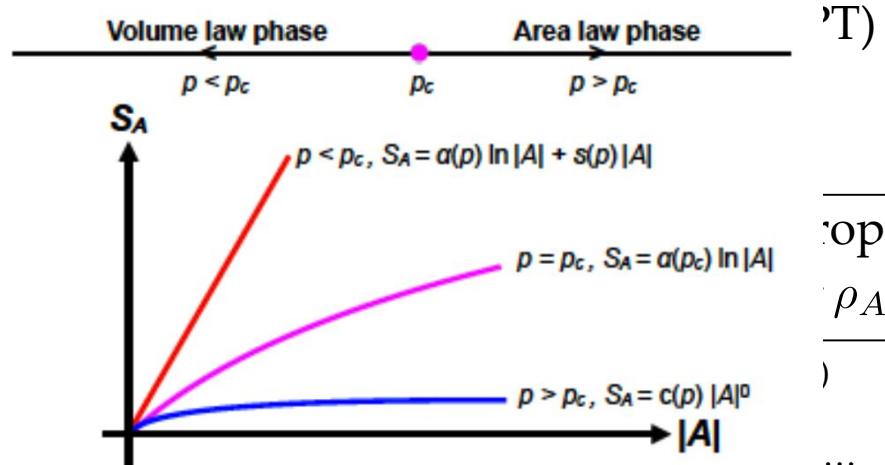
L. M. Sieberer, et al., Rep. Prog. in Phys. 79, 096001 (2016).

T. E. Lee, et al., Phys. Rev. Lett. 110, 257204 (2013).

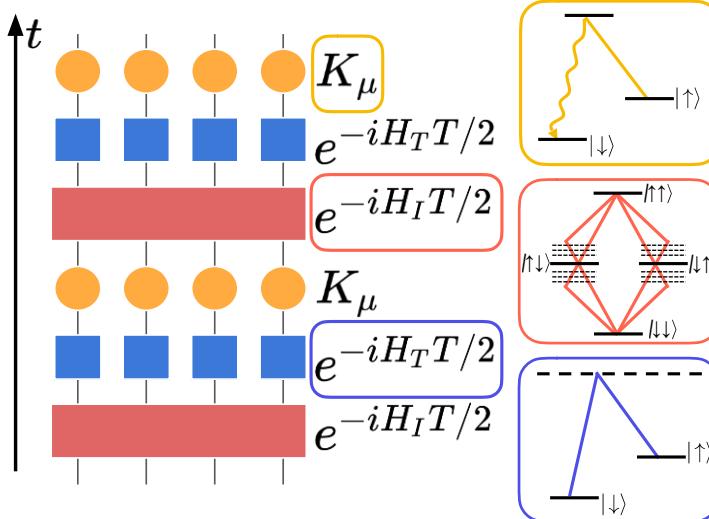
J. Jin, et al., Phys. Rev. X 6, 031011 (2016).

M. F. Maghrebi, et al., Phys. Rev. B 93, 014307 (2016)

Measurement-induced phase transitions



Dissipative Floquet Systems



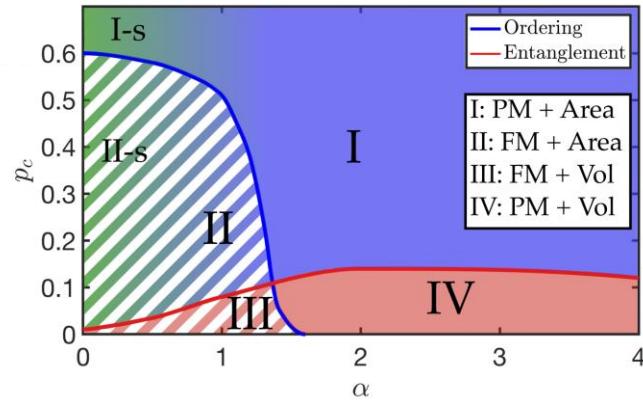
$$K_{i,0} = \sqrt{p} |\downarrow\rangle\langle\downarrow|_i, \quad K_{i,1} = \sqrt{p} |\downarrow\rangle\langle\uparrow|_i, \quad K_{i,2} = \sqrt{1-p} \mathbf{1}_i$$

$$H_I = \sum_{i < j} J_{ij} \sigma_i^x \sigma_j^x, \quad J_{ij} \sim \frac{J_0}{|i - j|^\alpha}$$

$$H_T = B_z \sum_i \sigma_i^z$$

DPT
Order-Disorder ?

$$X^2 = \frac{1}{L} \text{Tr}(\rho_{ss} \sum_{i \neq j} \sigma_i^x \sigma_j^x)$$



MIPT
Volume-Area law ?

$$S_A = -\text{Tr}(\rho_A \log \rho_A)$$

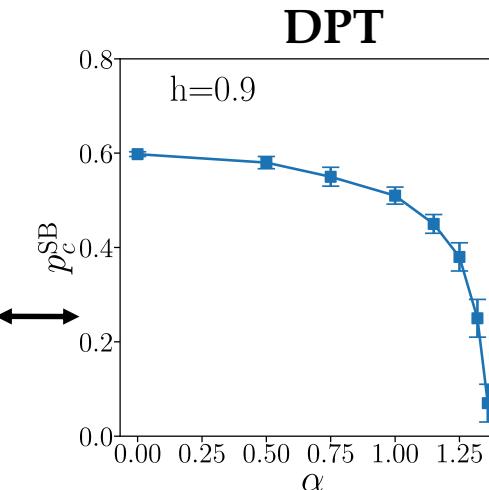
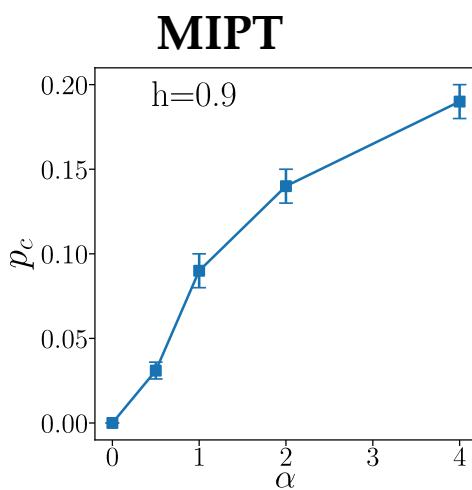
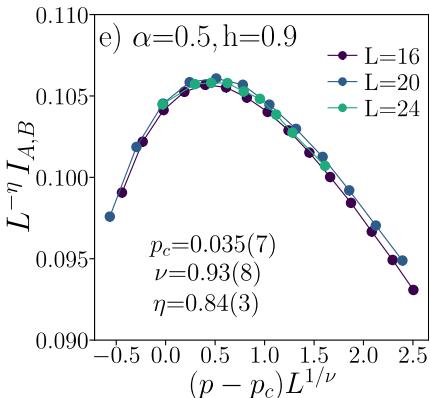
Dissipative and Measurement-induced PTs

Questions:

- Are these transitions present in a Hamiltonian system with long-range interactions?
- Are they connected or exclusive?

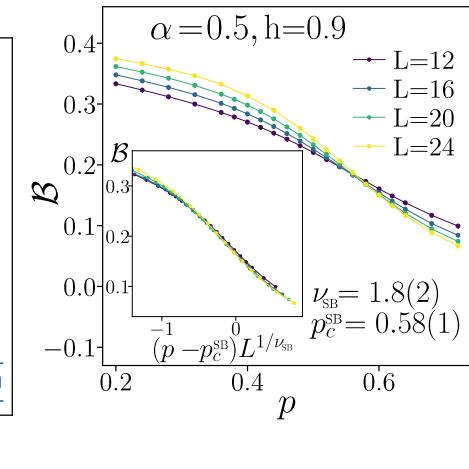
Mutual information

$$I_{A,B} = S(A) + S(B) - S(A \cup B)$$



Binder Cumulant

$$\mathcal{B} = 1 - \frac{\langle S_x^4 \rangle}{\langle S_x^2 \rangle^2}$$



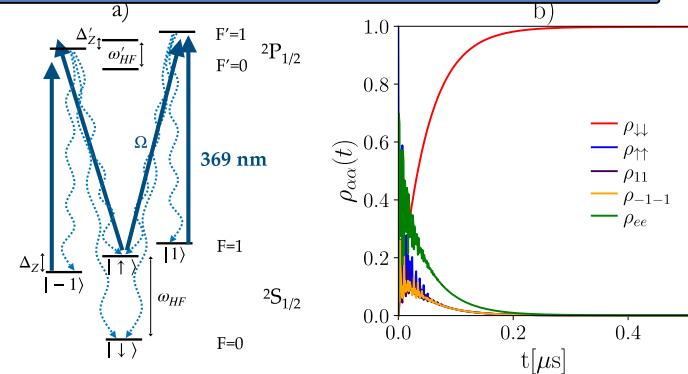
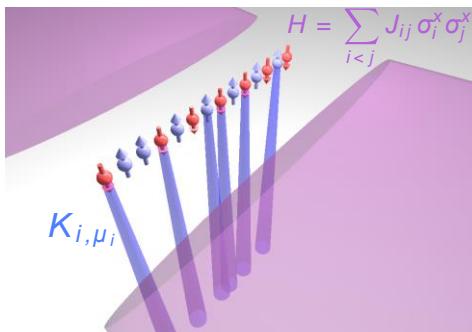
Experimental Considerations

DPT:

Global Long-range Interactions

+

Local Optical Pumping

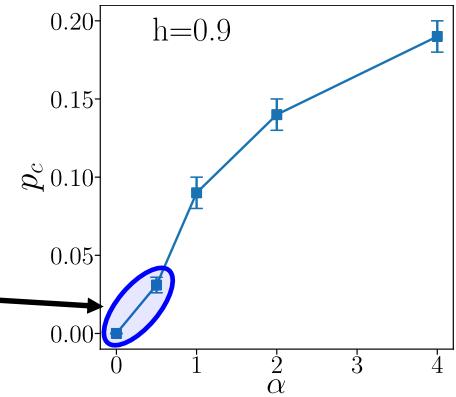


MIPT:

1) Detecting $S(\rho_A)$

2) Postselection $\rightarrow 2^{pLT}$ measurements

3) Detect individual qubits with negligible crosstalks



Acknowledgments



Roman Zhuravel
(Postdoc)

Visal So
(Grad Student)

Abhishek Menon
(Grad Student)

Midhuna D. Sughani
(Grad Student)

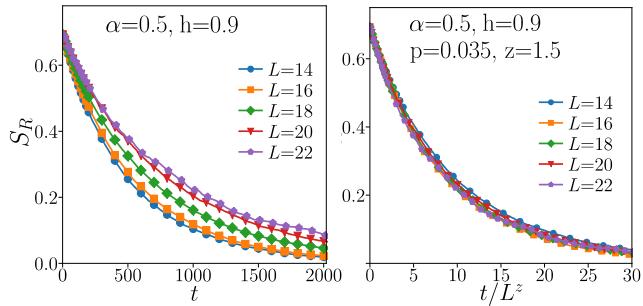
April Sheffield
(Undergraduate)



Collaborators :

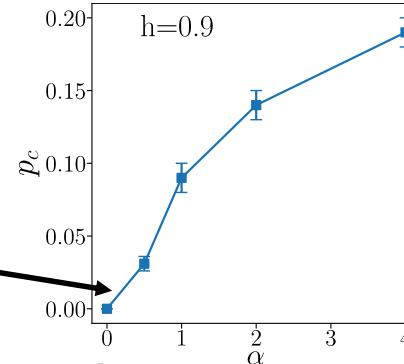
- Marcello Dalmonte
- Rosario Fazio
- Piotr Sierant
- Giuliano Chiriacò
- Federica Surace
- Xhek Turkeshi
- Zohreh Davoudi
- Mohammad Hafezi
- Tobias Grass
- Barbara Andrade
- Alireza Seif
- Norbert Linke

Experimental Considerations

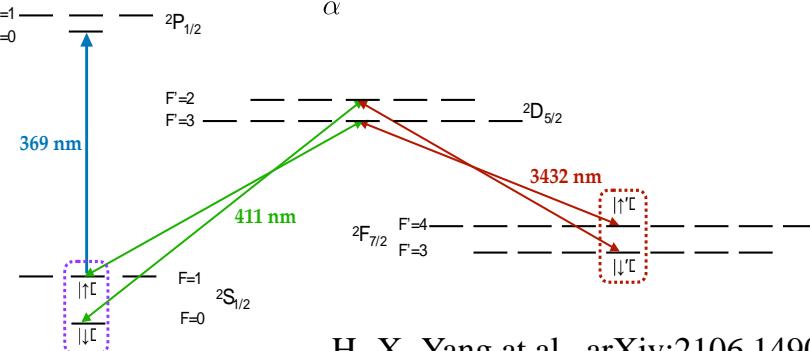


- 1) Detecting $S(\rho_A)$ of an entangled Ancilla
M. J. Gullans and D. A. Huse, PRL 125 (2020)
C. Noel et al., arXiv:2106.05881 (2021)

2) 2^{pLT} overhead manageable if α and, therefore, p_c is decreased



3) Qubit hiding to avoid crosstalks during measurements

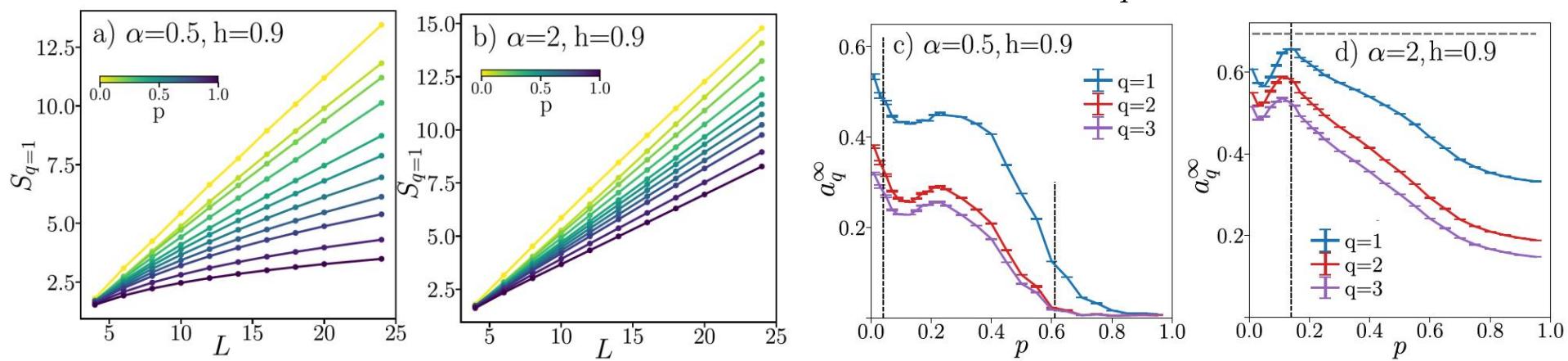


Participation Entropies

$$S_q = \frac{1}{1-q} \ln \left(\sum_{\beta=1}^{2^L} |\psi_{\beta}|^{2q} \right)$$

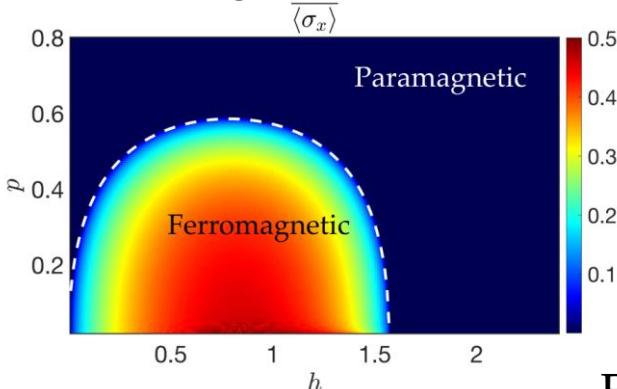
The scaling with L of S_q distinguishes if wave functions that are delocalized, multifractal and localized.

$$a_q(L) \sim a_q^{\infty} + b_1/L + b_2/L^2$$



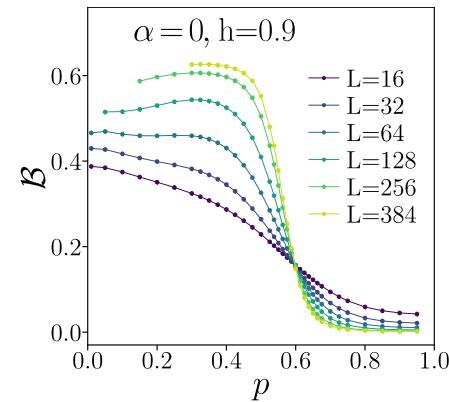
Dissipative Symmetry-Breaking Phase transition

Magnetization

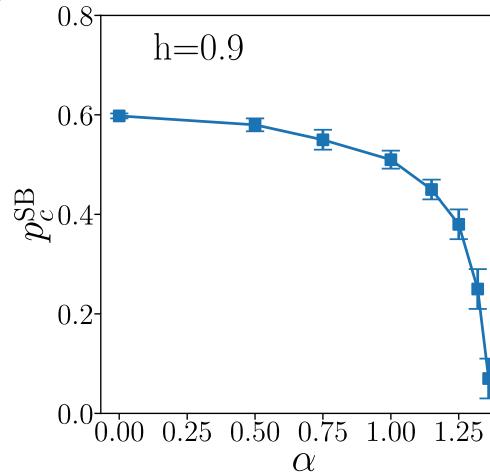
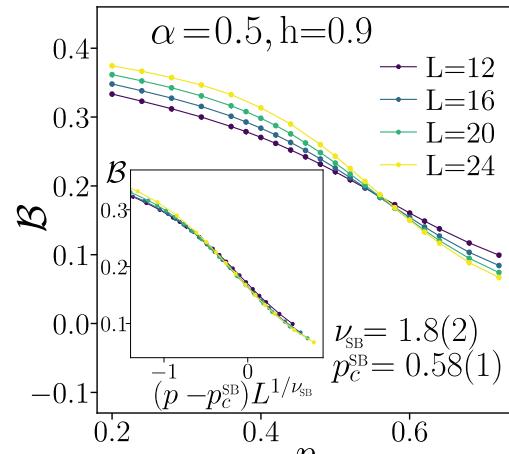


Cluster mean field
for $\alpha = 0$

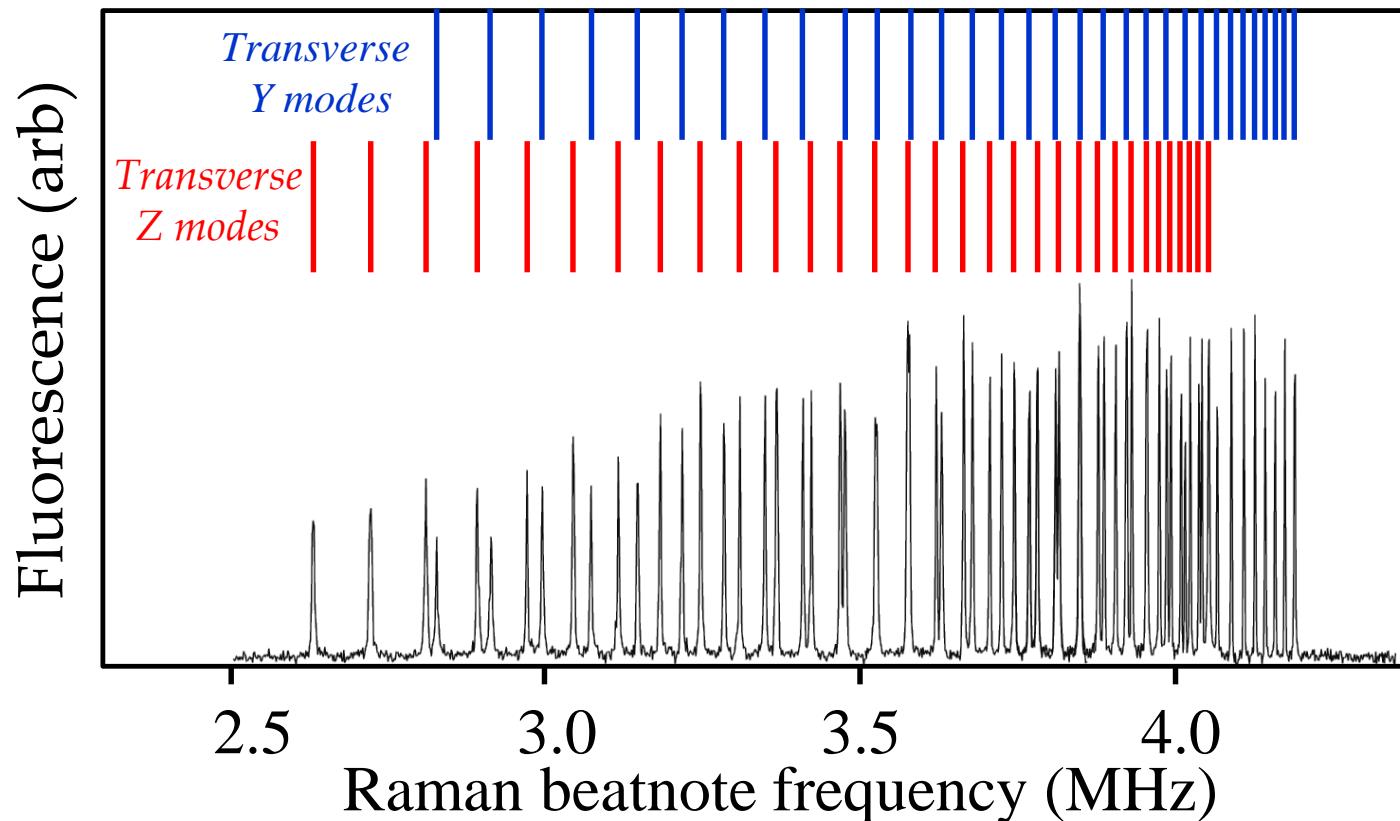
Binder cumulant



Exact Diagonalization $\alpha > 0$



Self-organized Ion crystals



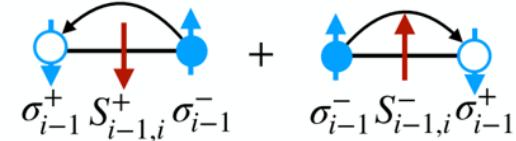
From Spin models to HEP models



$$H_{\text{Ion}} = J \sum_{i=1}^{N_{\text{stag}}-1} [\sigma_{2i-1}^+ \sigma_{2i}^+ \sigma_{2i+1}^- + \text{h.c.}] + \mu \sum_{i=1}^{N_{\text{stag}}} (-1)^i \sigma_{2i-1}^z$$



$$H_{\text{QLM}} = J \sum_{i=1}^{N_{\text{stag}}-1} [\sigma_i^+ S_i^+ \sigma_{i+1}^- + \text{h.c.}] + \sum_{i=1}^{N_{\text{stag}}-1} S_i^2 + \mu \sum_{i=1}^{N_{\text{stag}}} (-1)^i \sigma_i^z$$



Laser-ion interactions

