Evolution and the modelling of GPDs

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PARTONS

- **PARTONS** (PARton Tomography Of Nucleon Software – https://partons.cea.fr/) software framework for the **phenomenology of 3D hadron structure** (GPDs and TMDs) **[Berthou et al, 2018]**
- Developed since **2012**, **open source** and readily available in a virtual machine environment. Users can run **XML scenarios** to compute directly observables from already implemented GPD models, or develop their own modules in C++. **Plenty of examples and documentation! GPD models** (GK, VGG, Vinnikov, ...), **evolution**, **observables** (DVCS, TCS, DVMP, ...), **neural network fits of CFFs**, **event generator EpIC**, ...
- See also **Gepard** (https://gepard.phy.hr/))

Outlook

- GPDs at small ξ
    - t dependence
    - ξ dependence
- Deconvolution problem for $x < \xi$
- Deconvolution problem at moderate x and ξ

GPDs at small ξ – t dependence

- When $x \gg \xi$, negligible asymmetry between incoming $(x-\xi)$ and outgoing $(x+\xi)$ parton longitudinal momentum fraction $\rightarrow$ **smooth limit of GPDs**

$$H(x,\xi,t,\mu^2) \approx H(x,0,t,\mu^2) \ \text{ for } x \gg \xi \,. \tag{1}$$

- Extraction of the t-dependent PDF $H(x,0,t,\mu^2)$?
  - Forward limit gives ordinary PDFs

$$H(x,0,t=0,\mu^2) = f(x,\mu^2) \,. \tag{2}$$

  - First Mellin moment gives elastic form factors

$$\int \mathrm{d}x \, H(x,0,t) = F_1(t) \,. \tag{3}$$

  - Simplistic model of t-dependent PDF:

$$H(x,0,t,\mu^2) \propto f(x,\mu^2) F_1(t) \,. \tag{4}$$

GPDs at small ξ – t dependence

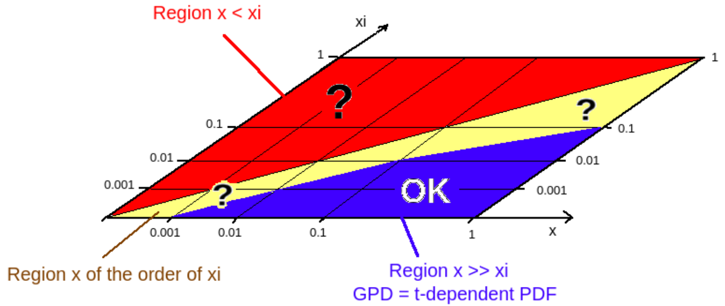
- Better modelling of the t-dependent PDF requires more data, more difficult to obtain with larger systematic uncertainty
  - x-dependence at $\xi = 0$ computed on the lattice from the **non-local euclidean matrix elements** (LaMET **[Ji, 2013]**, short-distance factorization **[Radyushkin, 2017]**, ...)
  - Higher order Mellin moments of GPDs (**generalized form factors**) computed on the lattice with local operators (limited by operator mixing to the first 3)
  - Experimental data from exclusive processes: **most of these data have a particular sensitivity to the region $x \approx \xi$, so precisely not $x \gg \xi$!**
- How does one leverage the experimental data in the small x and ξ domain to constrain t-dependent PDFs?

- Why don't we just assume

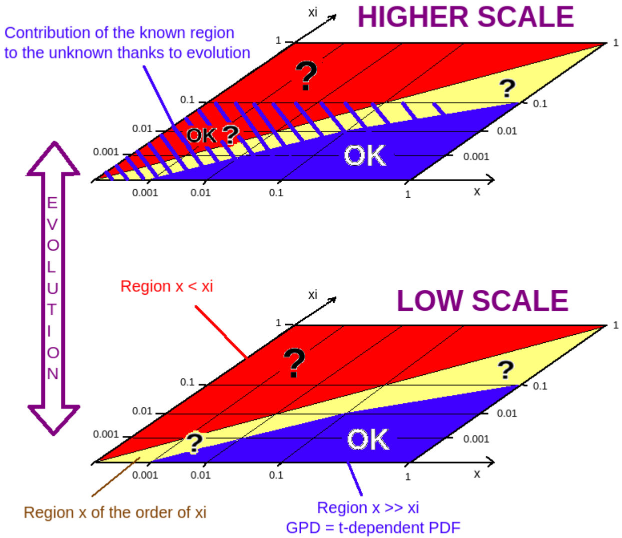
$$H(x,\xi,t,\mu^2) \approx H(x,0,t,\mu^2) \quad \text{for } \xi \ll 1 \text{ even if } x \approx \xi? \tag{5}$$

Because significant asymmetry between incoming and outgoing ($x+\xi \gg x-\xi$) parton momentum means very different dynamics, materialized *e.g.* by a very different behavior under evolution.

No reason for the ξ dependence to be negligible even at very small ξ. Skewness ratios $\frac{H(x,x)}{H(x,0)}$ as large as 1.6 have been advocated at small x. [Frankfurt et al, 1998] [Shuvaev et al, 1999]



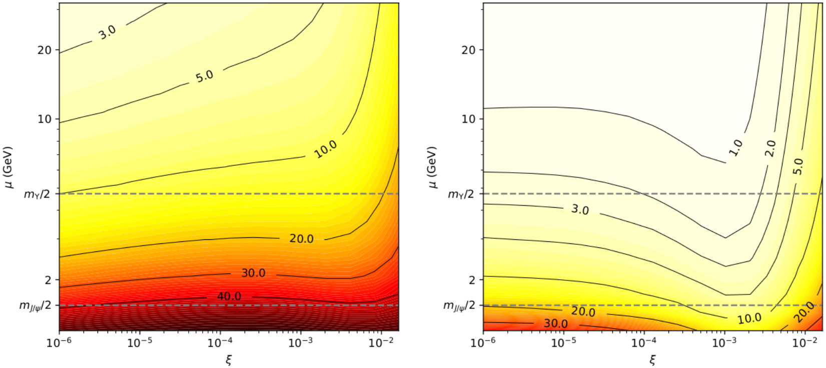
GPDs at small ξ – ξ dependence



- Evolution displaces the GPD from the large x to the small x region
- Significant ξ dependence arises perturbatively in the small x and ξ region
- But how does it compare to the unknown ξ dependence at initial scale?

Obviously depends on the range of evolution, value of x and ξ, and profile of the known t-dependent PDF.

Example: working at $t = 0$, with the MMHT2014 PDF **[Harland-Lang et al, 2015]** at 1 GeV (**prior knowledge of t-dependent PDF**). We want to assess the dominance of the region $x \gg \xi$ at initial scale in the value of the GPD on the diagonal as scale increases.
Pessimistic assumption on unknown ξ dependence at $x = \xi$ for 1 GeV: 60%.



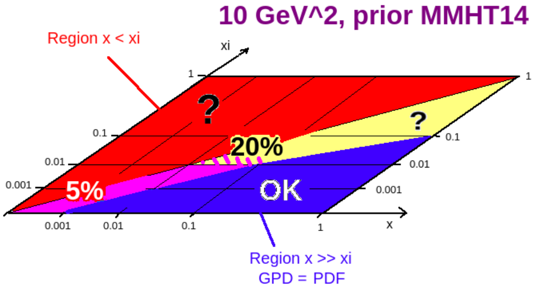
Uncertainty on the diagonal of the light sea quarks (left) and gluons (right) depending on $x = \xi$ and μ.
Stronger μ effect for gluons, divergence of PDFs at small x visible.

[HD, Winn, Bertone, hep-ph:2302.07861 (2023)]

GPDs at small ξ – ξ dependence

Generating perturbatively the ξ dependence offers a well defined functional space for GPDs at small ξ which verifies the main theoretical constraints (polynomiality of Mellin moments, positivity, limits, ...)
By subtracting the degree of freedom of the ξ dependence, we have regularized the deconvolution problem, and we have an evaluation of the uncertainty associated to this regularization.
Better modelling: include missing higher order corrections by varying the scales $\rightarrow$ use higher order evolution

Deconvolution problem for $x < \xi$



- Summary of the situation for H^g at $t = 0$ with MMHT2014 PDFs as prior
- What is happening for $x < \xi$, and what is the **deconvolution problem**?
- GPDs satisfy a **polynomiality property** arising from Lorentz covariance: **[Ji, 1998], [Radyushkin, 1999]**

$$\int_{-1}^{1} \mathrm{d}x\, x^n H^q(x,\xi,t,\mu^2) = \sum_{k=0\ even}^{n} A^q_{n,k}(t,\mu^2)\xi^k + \mathrm{mod}(n,2)\xi^{n+1}\, C^q_n(t,\mu^2)\,. \quad (6)$$

red contribution: if a function $D^q(\alpha, t, \mu)$ is odd in α, **[Polyakov, Weiss, 1999]**

$$\int_{-1}^{1} \mathrm{d}x\, x^n\, \Theta\left(1 - \frac{|x|}{|\xi|}\right) \mathrm{sgn}(\xi) D^q\left(\frac{x}{\xi}, t, \mu^2\right) = \mathrm{mod}(n,2)\xi^{n+1} \int_{-1}^{1} \mathrm{d}\alpha\, \alpha^n D^q(\alpha, t, \mu^2)\,. \quad (7)$$

Deconvolution problem for $x < \xi$

Dispersive formalism of DVCS [Anikin, Teryaev, 2007], [Diehl, Ivanov, 2007] at LO

$$S^q(t, Q^2) = 2e_q^2 \int_{-1}^{1} \mathrm{d}\alpha \, \frac{D^q(\alpha, t, \mu^2)}{1-\alpha} \,. \tag{8}$$

Since α is integrated out, only hope comes from the knowledge of the LO scale dependence of the D-term (ERBL equation). How effective is evolution to constrain it?

Shadow distributions

Find a distribution with reasonable shape such that it gives no experimental contribution at one scale, and check how big its contribution becomes as you move from the initial scale $\rightarrow$ measures worst case uncertainty propagation from experiment to fit

Deconvolution problem for $x < \xi$

- [HD, Lorcé, Moutarde, Sznajder, Trawinski, Wagner, Eur.Phys.J.C 81 (2021) 4, 300]: a very simple shadow D-term cause causes an **inflation of uncertainty by a factor 20 with full correlation between fitted parameters** over a range of $Q^2 \in [1.5, 4]$ GeV^2
- Preliminary prediction (EIC): over range in $Q^2 \in [1.5, 50]$ GeV^2, **inflation of uncertainty reduced to a factor 7 thanks to sole increase of range in Q^2** [HD, Ph.D. thesis (2022)]

Deconvolution problem at moderate x and ξ

General deconvolution problem: Compton form factors (CFFs) given by **[Radyushkin, 1997], [Ji, Osborne, 1998], [Collins, Freund, 1999]**

$$\mathcal{H}^q(\xi, t, Q^2) = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} \, T^q \left(\frac{x}{\xi}, \alpha_s, \frac{Q^2}{\mu^2} \right) H^q(x, \xi, t, \mu^2) \, . \tag{9}$$

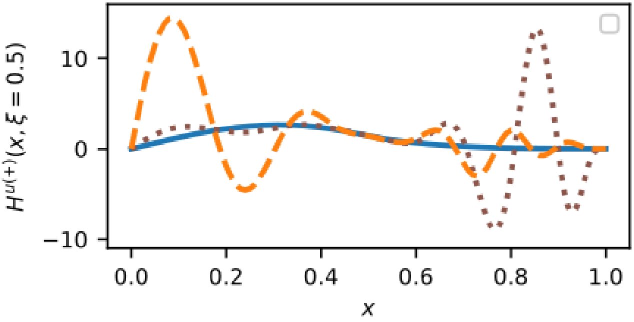
What is a reasonable shape for shadow GPD?

1. **Double distributions** **[Radyushkin, 1997]** as polynomials in their two variables (α, β)?
2. **Neural network model** of double distributions?

Deconvolution problem at moderate x and ξ

Double distributions as polynomials in their two variables (α, β) **[Bertone, HD, Mezrag, Moutarde, Sznajder, Phys.Rev.D 103 (2021) 11, 114019]**

- Enforces polynomiality by construction
- Analytical computation of the CFF $\rightarrow$ exact cancellation possible at least up to NLO
- Precise test of the accuracy of evolution: at NLO, should vary as $\mathcal{O}(\alpha_s^2)$



- **Result**: the three models give CFFs that vary by $\approx 10^{-5}$ at moderate ξ over a range of $[1, 100]$ GeV^2 $\rightarrow$ **enormous inflation of uncertainty from experimental data at moderate** ξ
- **Limitation**: large fluctuations at large x unphysical, incompatible with positivity constraints

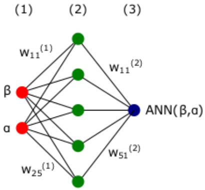
Deconvolution problem at moderate x and ξ

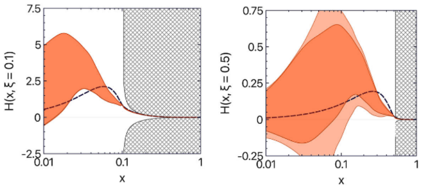
Neural network model of double distributions [HD, Grocholski, Moutarde, Sznajder, Eur.Phys.J.C 82 (2022) 3, 252]

- Enforces polynomiality by construction
- More flexible without the need of very large polynomial powers (precision issue for floating point computation)
- More flexible framework to implement positivity constraint: mock constraint

$$|H^q(x,\xi,t)| \leq \sqrt{f^q\left(\frac{x+\xi}{1+\xi}\right) f^q\left(\frac{x-\xi}{1-\xi}\right) \frac{1}{1-\xi^2}} \qquad (10)$$

- Proof of concept – closure test :





Conclusion

- Different regions of GPDs and different sources of data imply different modelling strategies, **but can be implemented in a common framework!**
- Flexible modelling of the t-dependent PDF (neural networks)
- Generate small ξ dependence perturbatively **but with uncertainty!**
- When uncertainty too large, switch to a flexible modelling at intermediate ξ (neural network)
- **What has not been evoked**: flavor and helicity! Having u, d, g twist-two GPDs H, E, $\tilde{H}$, $\tilde{E}$, etc. $\rightarrow$ proliferation of functions to fit
- **What has not been evoked**: systematic errors in the data! We have discussed the uncertainty related to the modelling (**unknow ξ dependence at small scale**, **deconvolution of experimental data**, ...) but there is the significant systematical uncertainty of the data itself (higher twist contaminations, ...)

Thank you for your attention!

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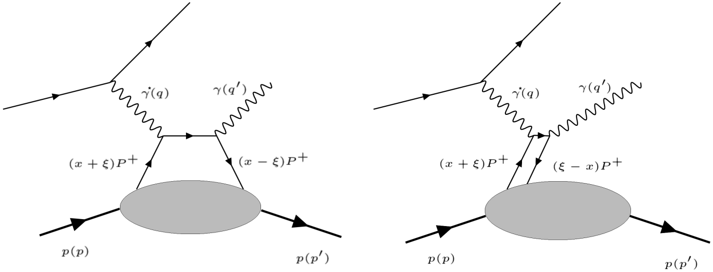
Deeply virtual Compton scattering and the structure of hadrons

DVCS is the scattering of a lepton on a hadron via a photon of large virtuality, producing a real photon in the final state. It is an **exclusive process** with an intact recoil proton.

- x is the average light-front plus-momentum (longitudinal momentum in a fast moving hadron) fraction of the struck parton
- ξ describes the light-front plus-momentum transfer, linked to Björken's variable x_B
- $t = \Delta^2$ is the total four-momentum transfer squared



GPDs were introduced more than two decades ago in **[Müller et al, 1994]**, **[Radyushkin, 1996]** and **[Ji, 1997]**.

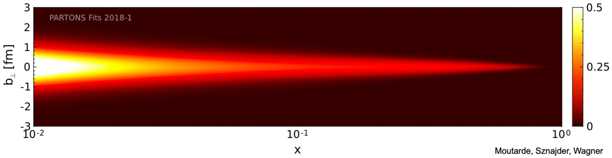
Tree-level depiction of DVCS for $x > |\xi|$ (left) and $\xi > |x|$ (right)

Deeply virtual Compton scattering and the structure of hadrons

Impact parameter distribution (IPD) **[Burkardt, 2000]**

$$I_a(x, \mathbf{b}_\perp, \mu^2) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} F^a(x, 0, t = -\Delta_\perp^2, \mu^2) \tag{11}$$

is the density of partons with plus-momentum x and transverse position $\mathbf{b}_\perp$ from the center of plus momentum in a hadron $\rightarrow$ **hadron tomography**



Density of up quarks (valence GPD) in an unpolarized proton from a parametric fit to DVCS data in the PARTONS framework **[Moutarde et al, 2018]**.

Deeply virtual Compton scattering and the structure of hadrons

- Remarkably, GPDs allow access to gravitational form factors (GFFs) of the **energy-momentum tensor (EMT)** [Ji, 1997] defined for parton of type a

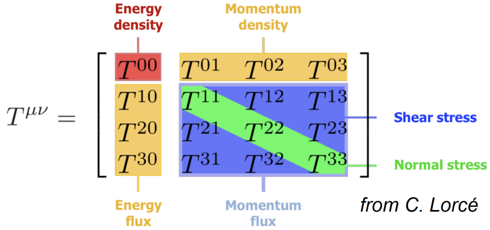
Gravitational form factors **[Lorcé et al, 2017]**

$$\langle p', s' | T_a^{\mu\nu} | p, s\rangle = \bar{u}(p', s')\Big\{ \frac{P^\mu P^\nu}{M} A_a(t, \mu^2) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C_a(t, \mu^2) + M\eta^{\mu\nu} \bar{C}_a(t, \mu^2)$$
$$+ \frac{P^{\{\mu} i\sigma^{\nu\}\rho}\Delta_\rho}{4M}\left[A_a(t, \mu^2) + B_a(t, \mu^2)\right] + \frac{P^{[\mu} i\sigma^{\nu]\rho}\Delta_\rho}{4M} D_a(t, \mu^2)\Big\} u(p, s) \quad (12)$$

where

$$\Delta = p' - p, \ \ t = \Delta^2, \ \ P = \frac{p + p'}{2} \quad (13)$$

Deeply virtual Compton scattering and the structure of hadrons



In the Breit frame ($\vec{P} = 0,\ t = -\vec{\Delta}^2$), radial distributions of energy and momentum in the proton are described by Fourier transforms of the **GFFs** w.r.t. variable $\vec{\Delta}$ **[Polyakov, 2003]**.

- Example of such distribution: radial pressure anisotropy profile

$$s_a(r,\mu^2) = -\frac{4M}{r^2}\int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}}\,\frac{t^{-1/2}}{M^2}\frac{d^2}{dt^2}\left[t^{5/2}\,C_a(t,\mu^2)\right] \tag{14}$$

- This pressure profile can be extracted from **GPDs** thanks to *e.g.* for quarks

$$\int_{-1}^{1} dx\, x\, H^q(x,\xi,t,\mu^2) = A_q(t,\mu^2) + 4\xi^2 C_q(t,\mu^2) \tag{15}$$

$$\int_{-1}^{1} dx\, x\, E^q(x,\xi,t,\mu^2) = B_q(t,\mu^2) - 4\xi^2 C_q(t,\mu^2) \tag{16}$$

Extraction of GFFs

- At this stage, we don't need to fully extract the GPDs H or E to conveniently access the GFF $C_q(t,\mu^2)$. The **polynomiality property** gives that the GFF $C_q(t,\mu^2)$ only depends on the D-term via

$$\int_{-1}^{1} \mathrm{d}z\, zD^q(z,t,\mu^2) = 4C_q(t,\mu^2) \tag{17}$$

- The experimental data is sensitive to the D-term through the **subtraction constant** defined by the **dispersion relation** (see *e.g.* [Diehl, Ivanov, 2007])

DVCS dispersion relation

$$\mathcal{C}_H(t,Q^2) = \mathrm{Re}\,\mathcal{H}(\xi,t,Q^2) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi'\,\mathrm{Im}\,\mathcal{H}(\xi',t,Q^2)\left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right) \tag{18}$$

The subtraction constant $\mathcal{C}_H(t,Q^2)$ is a function of the D-term given at LO by

$$\mathcal{C}_H(t,Q^2) = 2\sum_q e_q^2 \int_{-1}^{1} \mathrm{d}z\, \frac{D^q(z,t,Q^2)}{1-z} \tag{19}$$

Extraction of GFFs

- How do we get from

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} \quad \text{to} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) \text{ ?} \tag{20}$$

- This is a prototype of the more complicated GPD extraction problem we will face later on. The known solution is through evolution.
- Let's expand the D-term on a basis of Gegenbauer polynomials

$$D^q(z,t,\mu^2) = (1-z^2) \sum_{\text{odd } n} d_n^q(t,\mu^2) \, C_n^{3/2}(z) \tag{21}$$

Then

GFF C_a extraction

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \quad \text{and} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) = \frac{4}{5} \, d_1(t,\mu^2) \tag{22}$$

Extraction of GFFs

- Since the LO subtraction constant reads

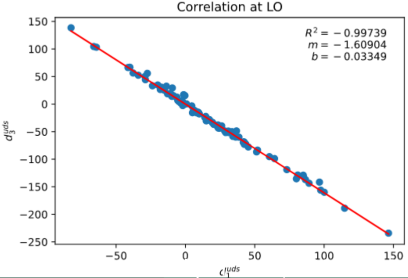
$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \tag{23}$$

if we allow d_3^q to be non-zero, at some scale μ_0^2, we can have $d_1^q(\mu_0^2) = -d_3^q(\mu_0^2)$, so a **vanishing subtraction constant, but non-zero GFF** $C_q(\mu_0^2)$. If the effect of evolution is not significant enough, these configurations are not ruled out and add a considerable uncertainty.

$d_1^{uds}(\mu_F^2)$	-0.5 ± 1.2

$\longrightarrow$

$d_1^{uds}(\mu_F^2)$	11 ± 25
$d_3^{uds}(\mu_F^2)$	-11 ± 26



Deconvoluting a Compton form factor

- Question was raised 20 years ago. Evolution was proposed as a crucial element in **[Freund, 1999]**, but the question has remained essentially open.
- We show that GPDs exist which bring contributions to the LO and NLO CFF of only subleading order even under evolution. We call them **LO and NLO shadow GPDs**.

Definition of an NLO shadow GPD

For a given scale μ_0^2,

$$\forall \xi, \forall t,\ T^q_{NLO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0 \quad \text{and} \quad H^q(x, \xi = 0, t = 0, \mu_0^2) = 0 \tag{24}$$

$$\text{so for } Q^2 \text{ and } \mu^2 \text{ close enough to } \mu_0^2,\ T^q_{NLO}(Q^2, \mu^2) \otimes H^q(\mu^2) = \mathcal{O}(\alpha_s^2(\mu^2)) \tag{25}$$

- Let H^q be an NLO shadow GPD, and G^q be any GPD. Then G^q and $G^q + H^q$ have the same forward limit, and the same NLO CFF up to a numerically small and theoretically subleading contribution.

Shadow GPDs at leading order

- Complete details in **[Bertone, HD, Mezrag, Moutarde, Sznajder, Phys.Rev.D 103 (2021) 11, 114019]**
- We search for our shadow GPDs as simple **double distributions (DD)** $F(\beta, \alpha, \mu^2)$ to respect polynomiality, with a zero D-term. Then, thanks to dispersion relations, we can restrict ourselves to the imaginary part only $\mathrm{Im}\ T^q(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0$.
- We search our DD as a polynomial of order N in (β, α), characterised by $\sim N^2$ coefficients c_{mn}:

$$F(\beta, \alpha, \mu_0^2) = \sum_{m+n \leq N} c_{mn}\, \alpha^m \beta^n \tag{26}$$

Shadow GPDs at next-to-leading order

- **First study beyond leading order:** Apart from the **LO** part, the NLO CFF is composed of a **collinear part** (compensating the α_s^1 term resulting from the convolution of the LO coefficient function and the evoluted GPD) and a genuine **1-loop NLO** part.

$$\mathcal{H}^q(\xi, Q^2) = C_0^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_1^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_{coll}^q \otimes H^{q(+)}(\mu_0^2) \log\left(\frac{\mu^2}{Q^2}\right) \tag{27}$$

An explicit calculation of each term for our polynomial double distribution gives that

$$\mathrm{Im}\ T_{coll}^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto$$

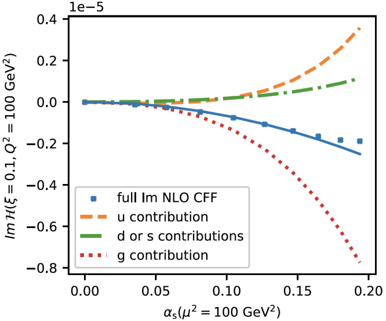
$$\alpha_s(\mu^2) \log\left(\frac{\mu^2}{Q^2}\right)\left[\left(\frac{3}{2} + \log\left(\frac{1-\xi}{2\xi}\right)\right)\mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N+1} \frac{k_w^{(coll)}}{(1+\xi)^w}\right] \tag{28}$$

and assuming $\mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) = 0$,

$$\mathrm{Im}\ T_1^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto \alpha_s(\mu^2)\left[\log\left(\frac{1-\xi}{2\xi}\right)\mathrm{Im}\ T_{coll}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N-1} \frac{k_w^{(1)}}{(1+\xi)^w}\right]$$

Shadow GPDs at next-to-leading order

- By linearity of both the CFF convolution and the evolution equation, we can evaluate separately the contribution to the CFF of a quark shadow NLO GPD under evolution.
- We probe the prediction of evolution as $\mathcal{O}(\alpha_s^2(\mu^2))$ with our previous NLO shadow GPD on a lever-arm in Q^2 of $[1, 100]$ GeV^2 (typical collider kinematics) using APFEL++ code.



- The fit by $\alpha_s^2(\mu^2)$ is very good up to values of α_s of the order of its $\overline{MS}$ values. For larger values, large logs and higher orders slightly change the picture.
- The numerical effect of evolution remains very small. For a GPD of order 1, the NLO CFF is only of order 10^{-5}.

Perspectives

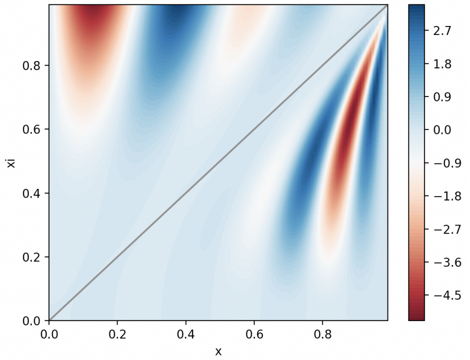
- Other exclusive processes can be expressed in terms of GPDs. Close parent to DVCS is **time-like Compton scattering** (TCS) [Berger et al, 2002]. Although its measurement will reduce the uncertainty, especially on $\mathrm{Re}\,\mathcal{H}$ [Jlab proposal PR12-12-001], and produce a valuable check of the universality of the GPD formalism, the similar nature of its convolution (see [Müller et al, 2012]) makes it subject to the same shadow GPDs.
- **Deeply virtual meson production** (DVMP) [Collins et al, 1997] is also an important source of knowledge on GPDs, with currently a larger lever arm in Q^2. The process involves form factors of the general form

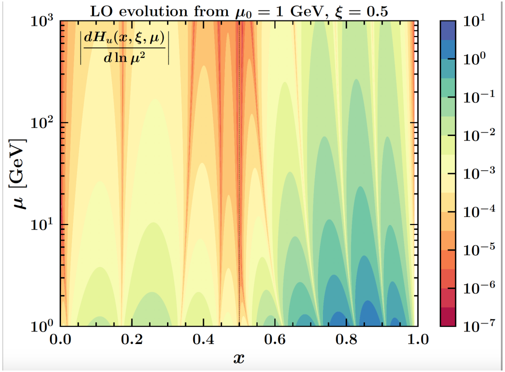
$$\mathcal{F}(\xi, t) = \int_0^1 \mathrm{d}u \int_{-1}^1 \frac{\mathrm{d}x}{\xi} \phi(u) T\left(\frac{x}{\xi}, u\right) F(x, \xi, t) \tag{30}$$

  where $\phi(u)$ is the leading-twist meson distribution amplitude (DA).
- At LO, the GPD and DA parts of the integral factorize and shadow GPDs cancel the form factor.
- Situation at NLO remains to be clarified, it is foreseeable new shadow GPDs (dependent on the DA) could be generated also for this process.

Shadow GPDs at next-to-leading order

- Cancelling both terms gives rise to two additional systems with a linear number of equations. The first NLO shadow GPD is found for $N = 21$, and adding the condition that the DD vanishes at the edges of its support gives a first solution for $N = 25$ (see below).





Color plot of an NLO shadow GPD at initial scale 1 GeV^2, and its evolution for $\xi = 0.5$ up to 10^6 GeV^2 via APFEL++ and PARTONS **[Bertone]**.

Evolution of GPDs

GPD's dependence on scale is given by **renormalization group equations**. In the limit $\xi = 0$, usual DGLAP equation:

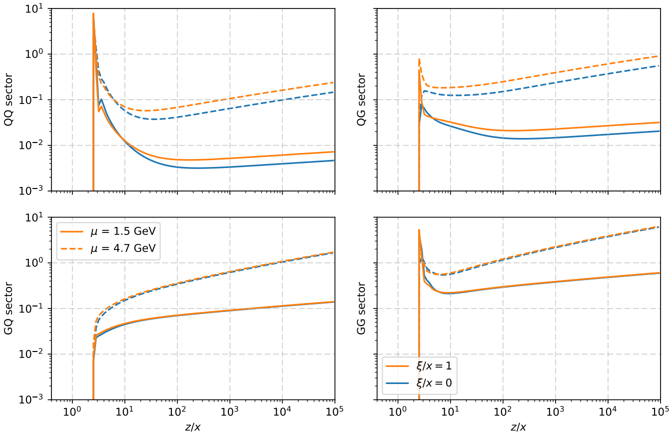
$$\frac{\mathrm{d}f^{q+}}{\mathrm{d}\mu}(x,\mu) = \frac{C_F\alpha_s(\mu)}{\pi\mu}\left\{\int_x^1 \mathrm{d}y\,\frac{f^{q+}(y,\mu)-f^{q+}(x,\mu)}{y-x}\left[1+\frac{x^2}{y^2}\right] + f^{q+}(x,\mu)\left[\frac{1}{2}+x+\log\left(\frac{(1-x)^2}{x}\right)\right]\right\} \quad (31)$$

But in the limit $x = \xi$:

$$\frac{\mathrm{d}H^{q+}}{\mathrm{d}\mu}(x,x,\mu) = \frac{C_F\alpha_s(\mu)}{\pi\mu}\left\{\int_x^1 \mathrm{d}y\,\frac{H^{q+}(y,x,\mu)-H^{q+}(x,x,\mu)}{y-x} + H^{q+}(x,x,\mu)\left[\frac{3}{2}+\log\left(\frac{1-x}{2x}\right)\right]\right\} \quad (32)$$

Assuming that GPD = t-dependent PDF at small ξ and $x \approx \xi$ is incompatible with evolution, which generates an intrinsic ξ dependence!

Evolution of GPDs



Behaviour of $\Gamma^{ab}(z/x, \xi/x; \mu_0 = 1\text{ GeV}, \mu)$ – **provided x and ξ are small enough, evolution range is large enough and PDF not too singular at small x, the large z region at initial scale with $z \gg \xi$ dominates evolution**

The Shuvaev transform

- Remarkably,

$$\Gamma^{ga}(x,\xi,z;\mu_0,\mu) \approx S^g(x,\xi,n) \star \mathcal{M}(n,y) \star \Gamma^{ga}(y,0,z;\mu_0,\mu) \quad (33)$$

The GPD evolution operator is nicely approximated by the Shuvaev transform of the Mellin transform of its limit for $\xi = 0$ (DGLAP evolution operator). The approximation is excellent as soon as $z > 4\xi$.