Measurement of antineutrino scattering on free protons in $MINER\nu A$

Tejin Cai

York University

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Nucleon and nuclear effects are inseparable





Neutrino measurements depend on models. Most factor out nucleon and nuclear effects:

- **1** Neutrino-nucleon interactions
- 2 Fermi motion, binding energy
- **3** Final state interactions
- 4 Event kinematics → combine uncertainty from previous steps!





Nucleon form factor impacts oscillation physics



 ν_{μ} event rate in DUNE near (left) and far (right) detector.

$$E_{\nu}^{\rm rec,had} = E_l + \sum_p E_{\rm kin} + \sum_{\pi^{\pm},\pi^0,\gamma} E_{\rm tot}$$

- Solid blue: GENIEv3 10a_02_11a, CCQE uses dipole F_A.
- Dashed black: replace dipole F_A with z-expansion F_A fitted to LatCat LQCD.

 $10\% \sim 20\%$ effect.

Meyer, Walker-Loud, and Wilkinson, 2022¹.

Measure the charged current elastic (CCE) scattering

 $\bar{\nu}_{\mu} + p \to \mu^+ + n,$

with the Medium Energy NuMI neutrino beam, and the scintillators in ${\rm MINER}\nu{\rm A}.$

- No nuclear effect on hydrogen.
- Large carbon background.
- POT: 1.12×10^{21}
- No. of hydrogen atoms: ~ 2.61×10^{29}

There is an equal number of carbon:

- Nuclear effects that alter the kinematics of the outgoing particles.
- Use this fact to constrain this background

 $MINER \nu A$

MINER ν A detector



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Boost hydrogen fraction with nuclear effects

 $\bar{\nu}_{\mu}H$ is a 2-body process. Final state neutron kinematics predictable from muon measurement.



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Fermi motion, nuclear effects, and final state interaction cause neutrons from carbon to deviate.



Boost hydrogen fraction with nuclear effects

 $\bar{\nu}_{\mu}H$ is a 2-body process. Final state neutron kinematics predictable from muon measurement.

Fermi motion, nuclear effects, and final state interaction cause neutrons from carbon to deviate.

Capture this deviation in the neutron direction, using $\delta \theta_{\rm P}$ and $\delta \theta_{\rm R}$



Neutrons inside the detector interact with hydrogen or carbon to produce charged secondary particles.



Most prompt neutron energy deposits due to knockout protons.

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Neutron signature



An incoming anti-neutrino scatters off a hydrogen producing neutron. The neutron undergoes secondary interactions to produce visible proton.

Blobbing algorithm



Probability for interacting neutron to have main candidate

Quasi-elastic like (QELike) event selection

Event topology:

■ 1 µ⁺ and no reconstructed hadronic track

Neutron selection:

- ≥ 1 three-dimensional neutron candidate
- Leading candidate energy deposited 10 cm away from the muon axis.

Muon acceptance due to detector shape:

- $\blacksquare 1.5 \text{ GeV} < E_{\mu} < 20 \text{ GeV}$
- $\theta_{\mu} < 20^{\circ}$ w.r.t. to neutrino beam

NuMI Antineutrino Mode: 1.12×10^{21} Protons on Target

Total re	coil ene	rgy cut:
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Condition	$E_{\rm max}^{\rm recoil} \ ({\rm GeV})$
$Q_{ m OE}^2 < 0.3 \; ({ m GeV}/c)^2$	$0.04 + 0.43 Q_{ m QE}^2 / ({ m GeV}/c)^2$
$Q_{\rm OE}^2 < 1.4 \; ({\rm GeV}/c)^2$	$0.08 + 0.3 Q_{\rm OE}^2 / ({\rm GeV}/c)^2$
$Q_{\rm QE}^2 > 1.4 \; ({\rm GeV}/c)^2$	0.50



CCE hydrogen QELike CCQE carbon 2p2h

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Created different angular regions – Hydrogen signal in the center. Outer regions are used for fit and validation – expand each region in Q^2

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Predicted hydrogen angles - concentrated in the center.

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Carbon QELike (CCQE) – more spread out due to Fermi motion and final state interactions.

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2p2h and resonant - all over the place but different.

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2p2h and resonant - all over the place but different.

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Fit using sideband region



CCQE is the dominant background. Small 2p2h, resonant, and Non-QELike contributions. The fitted model are well constrained by data.

Check fit with validation region



CCQE is the dominant background. Small 2p2h, resonant, and Non-QELike contributions. The fitted model are well constrained by data.

Apply fit to signal region



Projecting the fit into the signal region. Difference between data and background is the physics. More than 5000 hydrogen events!

Apply fit to signal region



Subtract background, and correct detector smearing using D'Agostini iterative unfolding with 4 iterations.

Efficiency

 $\epsilon = \frac{\text{signal events recorded}}{\text{total signal events produced}}$

- Peaks at $Q^2 \sim 0.3 \; (\text{GeV}/c)^2$
- Low Q²: Low acceptance because neutron needs to produce protons that span at least 2 planes.
- high Q²: Reconstruction inefficiency and larger opening angles and less detector material to contain neutrons.



Cross section prediction

- The single nucleon Llewelyn Smith equation convolutes with flux and muon acceptance.
 - Necessary for theory calculations.
- We use a standard vector form factor parameterization. (BBBA2005, Bradford et al., 2005²)
- F_A fit assumes z-expansion form³ (details in the backup).



Cross Section and Fitting $\ensuremath{\mathit{F}_{\mathrm{A}}}$

Cross section

Deuterium Fit: Meyer et al., 2016³

BBBA2007: Bodek et al. 2007⁴



Figure: Extracted cross section and ratio to a dipole form factor.

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Systematic and statistical uncertainty

- Dominated by statistical uncertainty from background subtraction, despite enhanced signal
- Systematic uncertainties from residuals of background subtraction
- Particle responses in the "other" category, dominated by neutron systematics.



LQCD: Lin 20225

$F_{\rm A}$ fit and axial radius of the nucleon

Result of F_A fit:

Deuterium fit applies z-expansion to bubble chamber data focused for $Q^2 < 1 \ (\text{GeV}/c)^2$. Hydrogen fit data up to $Q^2 = 5 \ (\text{GeV}/c)^2$. Tension between hydrogen and deuterium at larger Q^2 .



Cross Section and Fitting $\ensuremath{\mathit{F}_{\mathrm{A}}}$

LQCD: Lin 20225

$F_{\rm A}$ fit and axial radius of the nucleon

Result of F_A fit:

A fit to lattice QCD calculation by Lin⁵ is consistent with our fitted F_A up to $1 \ (\text{GeV}/c)^2$



LQCD: Lin 20225

$\ensuremath{\mathit{F}_{\mathrm{A}}}\xspace$ fit and axial radius of the nucleon

Result of F_A fit:

Calculate proton radius from
$$F_A$$
 for $Q^2 \rightarrow 0$.
 $F_A(Q^2) = F_A(0) \left(1 - \frac{\langle r_A^2 \rangle}{3!}Q^2 + \frac{\langle r_A^4 \rangle}{5!}Q^4 + \dots\right),$
 $\frac{1}{F_A(0)} \frac{dF_A}{dQ^2}\Big|_{Q^2=0} = -\frac{1}{6} \langle r_A^2 \rangle$
 $\langle r_A^2 \rangle = 0.53(25) \text{fm}^2$
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Filled circle: full error budget. Open square: incomplete. Red band: this result. Courtesy of Aaron Meyer.

Summary

Made a new F_A measurement in 30 years and the only statistically significant measurement on free nucleon.

- The result will aid in better understanding the weak nucleon structure.
- An important ingredient for oscillation physics \rightarrow impacts cross section.
- The new techniques developed are useful for current and future experiments:
 - neutron reconstruction
 - method of background constraint
- Can we reduce uncertainties in future iterations?

Cross Section and Fitting $F_{\rm A}$

Thank you! The MINERvA Collaboration



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Backup

Backup

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Backup

Mann et al., 1973⁶

First neutrino - deuterium measurement

Reported exactly 50 years ago:

- \blacksquare Flux uncertainty was 15%
- Extracting form factor needs model-dependent deuterium corrections
 - Fermi motion
 - Pauli exclusion principle



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Measurements with heavier targets



MiniBooNE QE measurement Aguilar-Arevalo et al., 2010⁷ Butkevich and Perevalov, 2013⁸ MiniBooNE – inclusive, lepton-only measurement – interpreted as F_A on carbon, insensitive to lepton-hadron correlation.

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Measurements with heavier targets

MINER ν A ν_{μ} QE measurement Fiorentini et al..2013⁹



- MiniBooNE inclusive, lepton-only measurement – interpreted as F_A on carbon, insensitive to lepton-hadron correlation.
- MINER*ν*A QE results: disagree with *M_A* = 1.35 GeV/*c*²
 - Tension points to nuclear effects.

Measurements with heavier targets



- MiniBooNE inclusive, lepton-only measurement – interpreted as F_A on carbon, insensitive to lepton-hadron correlation.
- MINER νA QE results: disagree with M_A = 1.35 GeV/c²
 - Tension points to nuclear effects.
- Better able to identify nuclear effects with lepton-hadron correlations.

Muon p_{\parallel} vs muon p_t vs total proton kinetic energy MINER ν A Ruterbories et al., 2022¹⁰



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Rationale for reweighting GENIE relativistic Fermi gas (RFG) model to NuWro's spectral function (SF) model.

Low energy era measurement: $MINER\nu A$ developed a set of *transverse kinematics imbalance* techniques for the neutrino sample:

- Events with only a primary muon, at least one proton, and no mesons.
- Compare momentum imbalance between µ and p.
- $\delta p_{\rm Ty}$ measures whether $\vec{p}^{\mu} + \vec{p}^{\rm p}$ balance out along the \vec{p}^{μ}_T axis.



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- $\delta p_{\rm Ty}$ measures whether $\vec{p}^{\mu} + \vec{p}^{\rm p}$ balance out along the \vec{p}_T^{μ} axis.
- NuWro spectral function (solid blue) better describes data.



Cai et al., 201912



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Blobbing algorithm



Probability for interacting neutron to have main candidate

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Constraining MINER νA 's GEANT4 neutron model with external data.

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Neutron model from Modular Neutron Array (MoNA)

- The MoNA collaboration collected (Del Guerra, 1976¹⁴) and modeled (Kohley et al., 2012¹³) neutron cross section on CH.
- ¹²C(n,np)¹¹B the dominant interaction channel
- We tune each channel to the MoNA cross sections based on secondary daughter particles.



Fig. 3. Inelastic neutron–carbon reaction cross–sections are shown as a function of the incident neutron energy. MENATE_R uses the six different discrete reaction channel cross-sections while the G4-Physics uses the total inelastic reaction crosssections taken from the JENDL-HE library [37].

GEANT4 reweight – nuisance variables

- We looked at "nuisance" variables to see if the MONA data improved the data/MC agreement
- These variables are not used in the fit and not tuned
- Example includes energy of neutron candidates, and distance of the candidate to vertex
- will show neutron candidate energy deposits in each Q_{OE}^2 bin.

Neutron candidate energy deposits vs $Q_{\rm QE}^2$ Without MoNA.



E (MeV)

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E (MeV)

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Background constraint method

MC cateogries (**C**) with significant contributions in each Q_{QE}^2 bin and angular region is used as template. QELike: CCQE, 2p2h, resonant. Non-QELike: π^0 , π^{\pm} .

$$\chi^{2} = \sum_{S,i} \frac{\left(\left[\sum_{C} w_{C,i} N_{C,S,i}^{\text{mc}}\right] - N_{S,i}^{\text{data}}\right)^{2}}{N_{S,i}^{\text{data}}} + \chi^{2} \text{ term}$$
$$\lambda_{S} \sum_{C} \sum_{j=1}^{N-2} (w_{C,j} + w_{C,j+2} - 2w_{C,j+1})^{2}. \text{ regularizaton term}$$

S: sideband – angular regions participating in fit. **i**: $Q_{\rm QE}^2$ bin. **w**: Weight for each category in each bin.

 $\lambda_{\rm S}$ controls the strength of the regularization, affects smoothness of the fit, and obtained through an L-curve style scan.

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Validating background constraint method with neutrino sample



We select events with trackable protons in a **neutrino sample**. Different final states and available kinematics. Apply same fitting mechanism. Data and MC agree within uncertainty except $Q^2 \in (0.2, 0.4) (\text{GeV}/c)^2$.

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Validating background constraint method with neutrino sample



Neutrino mode:

 100% 2p2h uncertainty covers data/simulation discrepancy.

Antineutrino mode:

- Small 2p2h contribution.
- Sufficiently covered by total uncertainty.

Explanation of the Weights

- The CCQE weight is affected by
 - Base GENIE model
 - Various model tunes:
 - ► RPA, NuWro SF
 - Final state tunes:
 - FSI tunes



Non-QE validation region 1 – fitted



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Non-QE validation region 2 - fitted



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Non-QE fit region - fitted



Non-QE meson fit region - fitted



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Effects of background constraint on angular regions



CCE signal region

Ratio of data to (left) unfitted model, (right) post-fitted model.

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Effects of background constraint on angular regions



QE fit region

Ratio of data to (left) unfitted model, (right) post-fitted model.

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Effects of background constraint on angular regions



QE validation region

Ratio of data to (left) unfitted model, (right) post-fitted model.

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Effects of background constraint on angular regions



Non-QE validation 1

Ratio of data to (left) unfitted model, (right) post-fitted model.

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Effects of background constraint on angular regions



Non-QE validation 2

Ratio of data to (left) unfitted model, (right) post-fitted model.

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Effects of background constraint on angular regions



Non-QE fit region

Ratio of data to (left) unfitted model, (right) post-fitted model.

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Effects of background constraint on angular regions



Non-QE and mesons fit region

Ratio of data to (left) unfitted model, (right) post-fitted model.

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MINERvA $\bar{\nu}_{\mu}$ -CH result – ratio of data to base model

Bashyal et al., 2022¹⁵



FIG. 4. Comparisons of the cross section predicted by various tunes applied on GENIE with respect to the baseline GENIE 2.12.6 (black) as a function of Q_{QE}^2 (left). MINERvA Tune v1 (blue) is the standard simulation tuned to the MINERvA low energy data. MINERvA Tune v2 (red) is MINERvA Tune v1 with the non-resonant pions suppressed in the low Q_{OF}^2 region [30]. The remaining curves show the effect of enabling different corrections to the base model. The plots on the right show comparisons of cross sections predictions for GENIE v3.0.6 (dotted lines) with the MINERvA tuned GENIE predictions.Inner ticks in the data are statistical and the outer ticks are the systematic uncertainties.

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Migration matrix and warping study

- Row normalized migration matrix: Unfolding matrix obtained by
 - D'Agostini iterative unfolding stops at 4th iteration.



Migration matrix and warping study

Ratio of "warped prediction" to base prediction.

- Performed warping study to check stability of regularization.
- "Warped" prediction with ratio, and
- and generated ensemble of 1000 statistical variations.
- Obtain average and median \(\chi^2\) between unfolded and truth of the ensemble for each iteration.



Systematics Uncertainties



(left) all systematics, (right) others

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Systematics Uncertainties



(left) FSI model, (right) cross section model

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Systematics Uncertainties



(left) flux, (right) muon reconstruction

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Systematics Uncertainties



low recoil fit

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Neutron secondary scattering

- Ejected protons responsible for majority of secondary neutron interactions.
- Inelastic scattering from carbon dominates (see previous slide)
- MINER vA doesn't have access to the most updated GEANT4 because we need to access MINOS.

Fragment Particles Type vs Energy Loss


Effect of MONA Reweight - Blob Energy

Without MoNA



E (MeV)

Effect of MONA Reweight - Blob Energy

With MoNA



E (MeV)

Effect of MONA Reweight – Blob Distance to Vertex

Without MoNA



R (mm)

Effect of MONA Reweight – Blob Distance to Vertex

With MoNA



R (mm)

z-expansion parameters

z-expansion formalism and constraints on a_k

$$F_{A}(Q^{2}) = \sum_{k=0}^{k_{\max}} a_{k} z^{k}$$

$$z = \frac{\sqrt{t_{\text{cut}} + Q^{2}} - \sqrt{t_{\text{cut}} - t_{0}}}{\sqrt{t_{\text{cut}} + Q^{2}} + \sqrt{t_{\text{cut}} - t_{0}}}$$

$$\sum_{k=n}^{\infty} k(k-1) \dots (k-n+1)a_{k} = 0, n \in (0, 1, 2, 3)$$

$$\chi^{2} = \Delta X \cdot \text{cov}^{-1} \cdot \Delta X + \lambda \left[\sum_{k=1}^{5} \left(\frac{a_{k}}{5a_{0}} \right)^{2} + \sum_{k=5}^{k_{\max}} \left(\frac{ka_{k}}{25a_{0}} \right)^{2} \right]$$

 $\Delta X = \text{data} - \text{prediction}$

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z-expansion parameters II

Central value fit: $k_{\rm max} = 8, \lambda = 0.13$

- \blacksquare Scan through large range of λ
- Data χ^2 for $k_{\max} = 8$ can be less than $k_{\max} = 6$
- λ chosen at point of maximum curvature.



Dipole Fit

- $M_A = 1.15(10) \text{ GeV}$
- Fit $\chi^2 = 10.2$
- Comparable with z-expansion fit
 - $k_{\text{max}} = 6$
 - $\blacktriangleright \ \lambda = 0$
 - ▶ $\chi^2 = 9.64$



Changing GENIE's initial state model



Transform GENIE RFG to NuWro SF in $|p_i| - Q^2$ space

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Antineutrino-proton scattering at MINERvA

Relativistic Fermi Gas (RFG), Local Fermi Gas (LFG) and Spectral Function (SF)

- RFG: non-interacting fermions in a potential well with fixed Fermi momentum K_F.
- GENIE RFG includes an additional tail.
- LFG: Fermi gas with location-dependent K_F.
- SF: a nuclear-shell model.



Initial state momentum distribution.

Deuterium Fit³, BBBA2007⁴, LQCD⁵

$\ensuremath{\mathit{F}_{\mathrm{A}}}\xspace$ fit and axial radius of the nucleon

Favors larger F_A at higher Q^2 . Calculate proton radius from F_A for $Q^2 \rightarrow 0$. $F_A(Q^2) = F_A(0) \left(1 - \frac{\langle r_A^2 \rangle}{3!}Q^2 + \frac{\langle r_A^4 \rangle}{5!}Q^4 + \dots\right),$ $\frac{1}{F_A(0)} \frac{dF_A}{dQ^2}\Big|_{Q^2=0} = -\frac{1}{6} \langle r_A^2 \rangle$ **a** $\langle r_A^2 \rangle = 0.53(25) \text{fm}^2$ **b** $\sqrt{\langle r_A^2 \rangle} = 0.73(17) \text{fm}$



Deuterium Fit³, BBBA2007⁴, LQCD⁵

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Filled circle: full error budget. Open square: incomplete. Red band: this result. Courtesy of Aaron Meyer.

Reference

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