HADRON SCATTERING FROM LATTICE QCD

Andrei Alexandru

APS 2023 Meeting
GHP workshop

THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

OVERVIEW

- Motivation and scattering from lattice QCD
- Two pion scattering from lattice QCD
- Three pion and kaon scattering
  - Non-resonant channels and three-body force
  - A first look at a resonant channel
- Summary and outlook

GWQCD COLLABORATORS

Lattice QCD

- Frank Lee
- Craig Pelissier
- Dehua Guo
- Chris Culver
- Ruairi Brett

EFT/Phenomenology

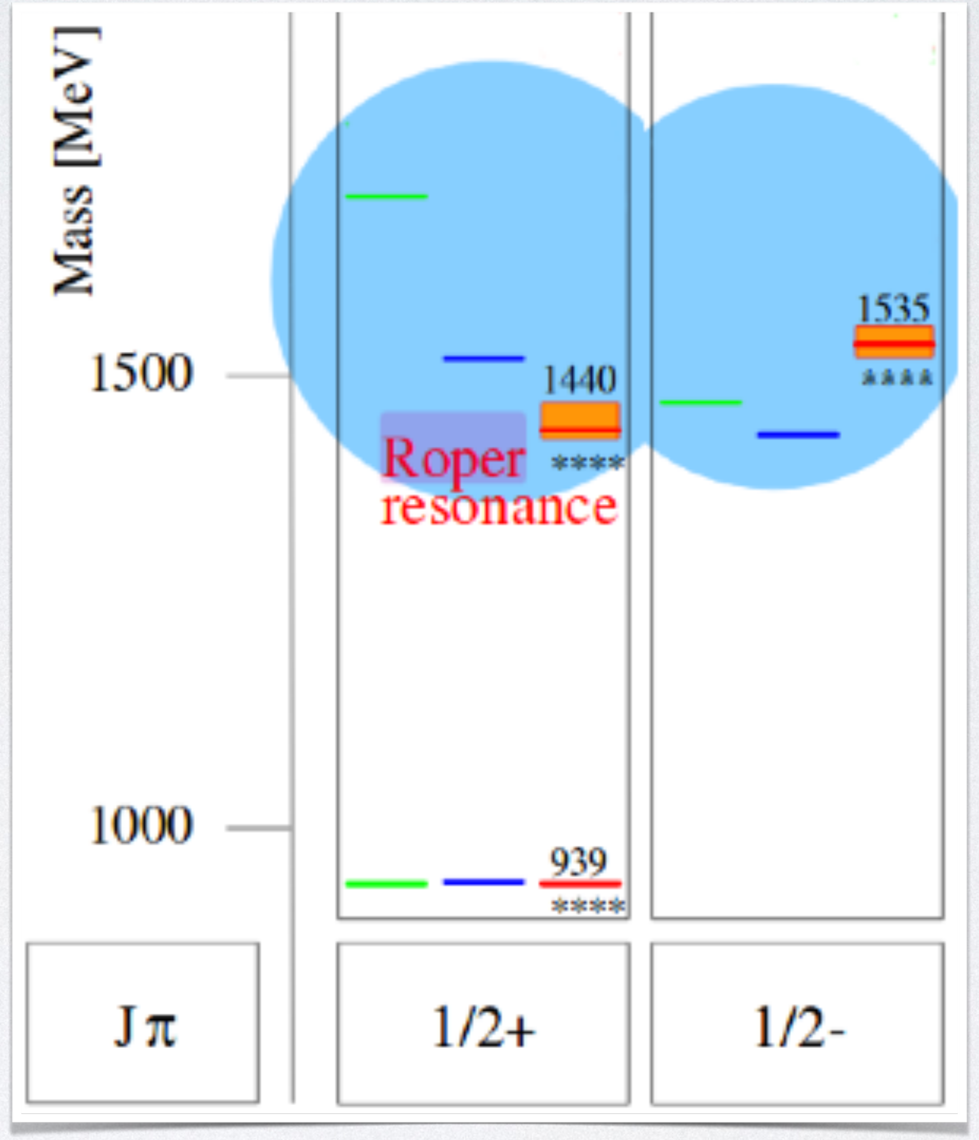
- Michael Doering
- Maxim Mai
- Raquel Molina
- Bin Hu
- Daniel Sadasivan

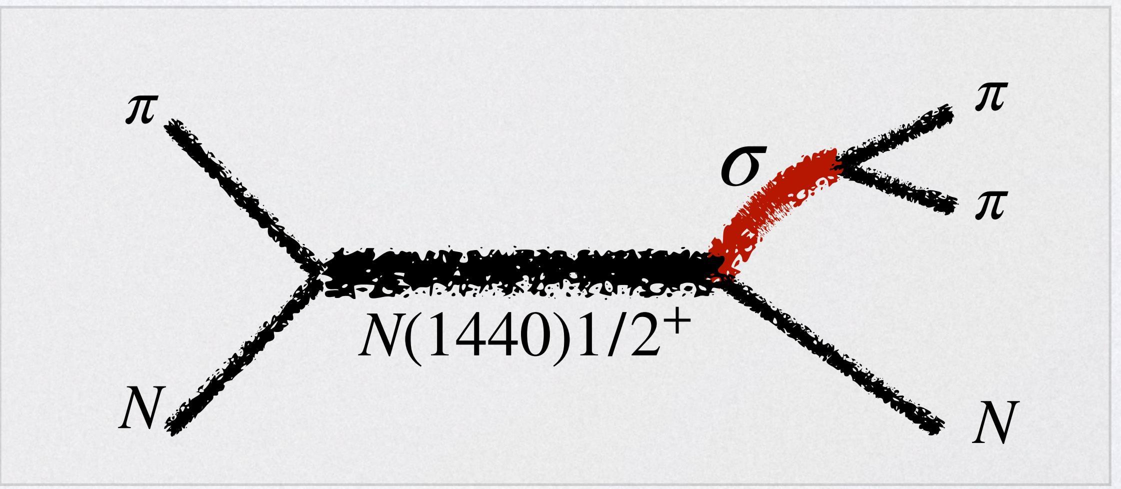
MOTIVATION

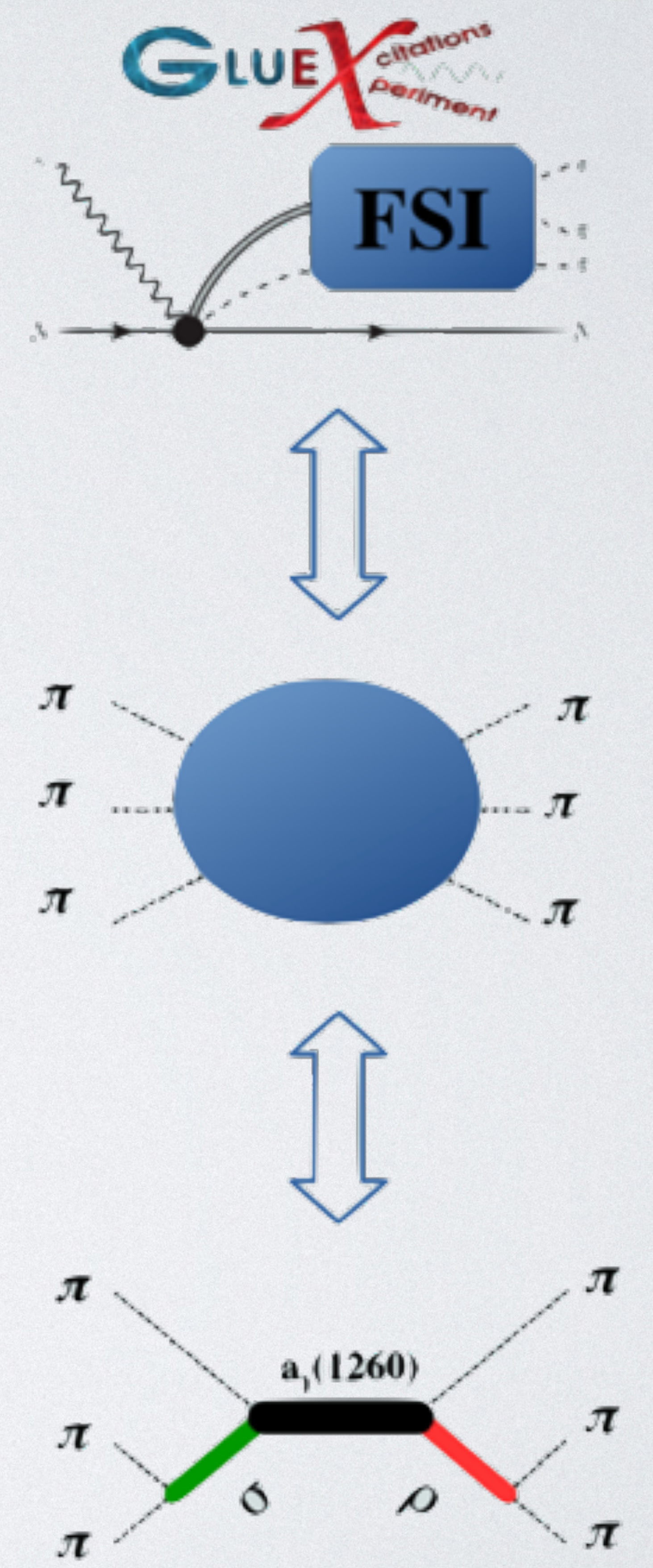
The spectrum of hadronic resonances is an open problem: expected baryonic resonances are missing, exotic resonances are expected in light and charmed mesons, etc.

Many unsolved puzzles in the hadron spectrum where three body states play a relevant role

- Roper N(1440) 1/2$^+$: mass spectrum is inverted presumably due to large branching ratio to N$\pi\pi$
- a_1(1260) decays to 3π but not to 2π. The decay is proceeding through $\rho\pi$ and $\sigma\pi$ intermediate states. This is expected to be a test case for spin-exotics probing the gluon degrees of freedom.
- X(3872), XYZ states, etc.

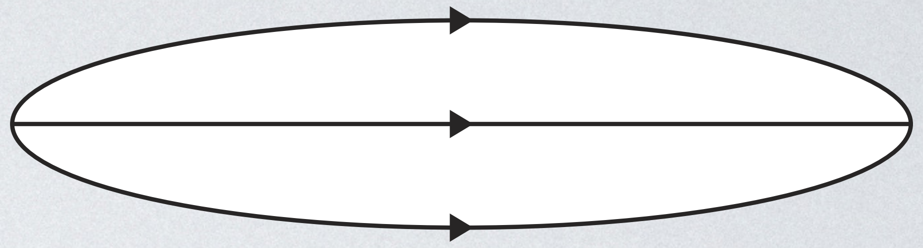


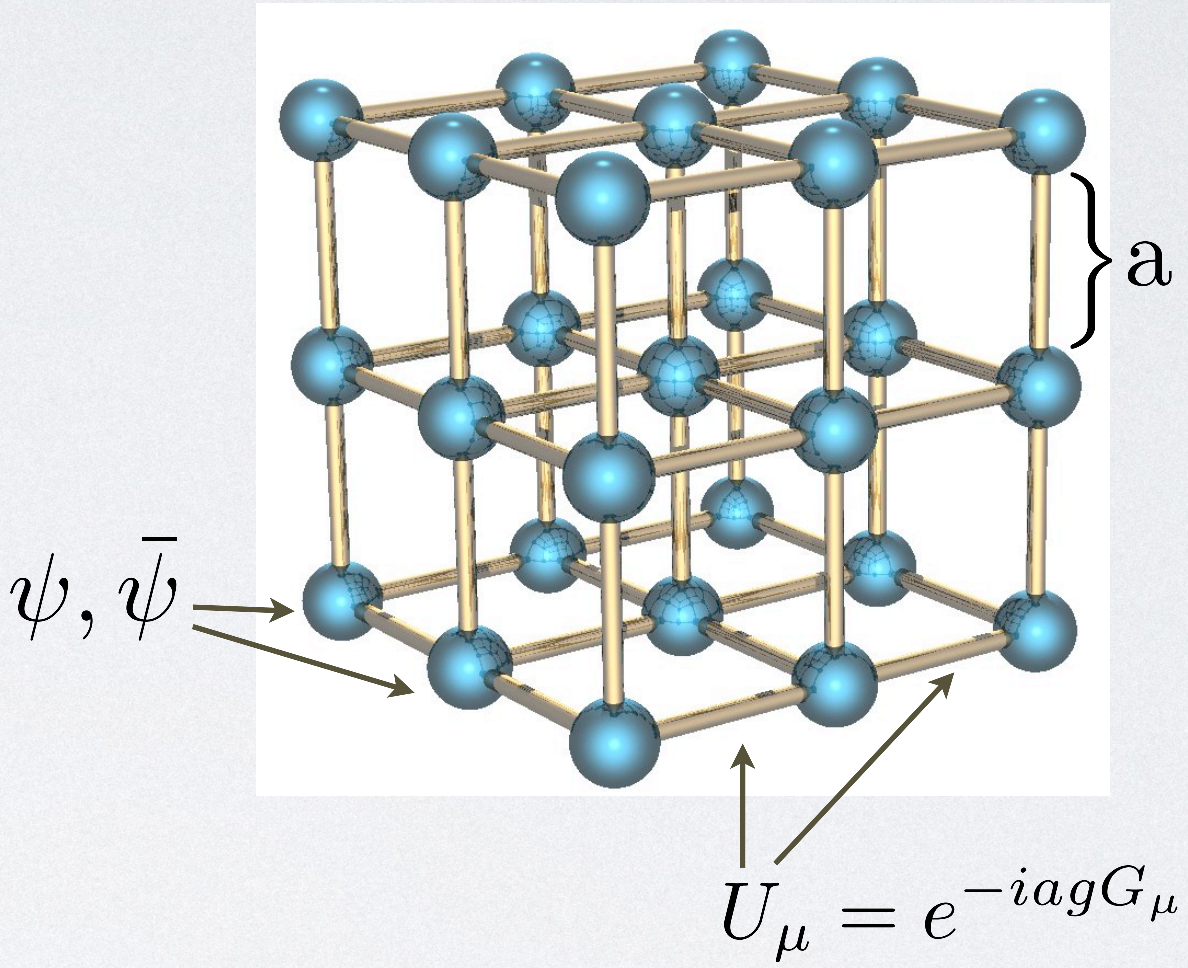


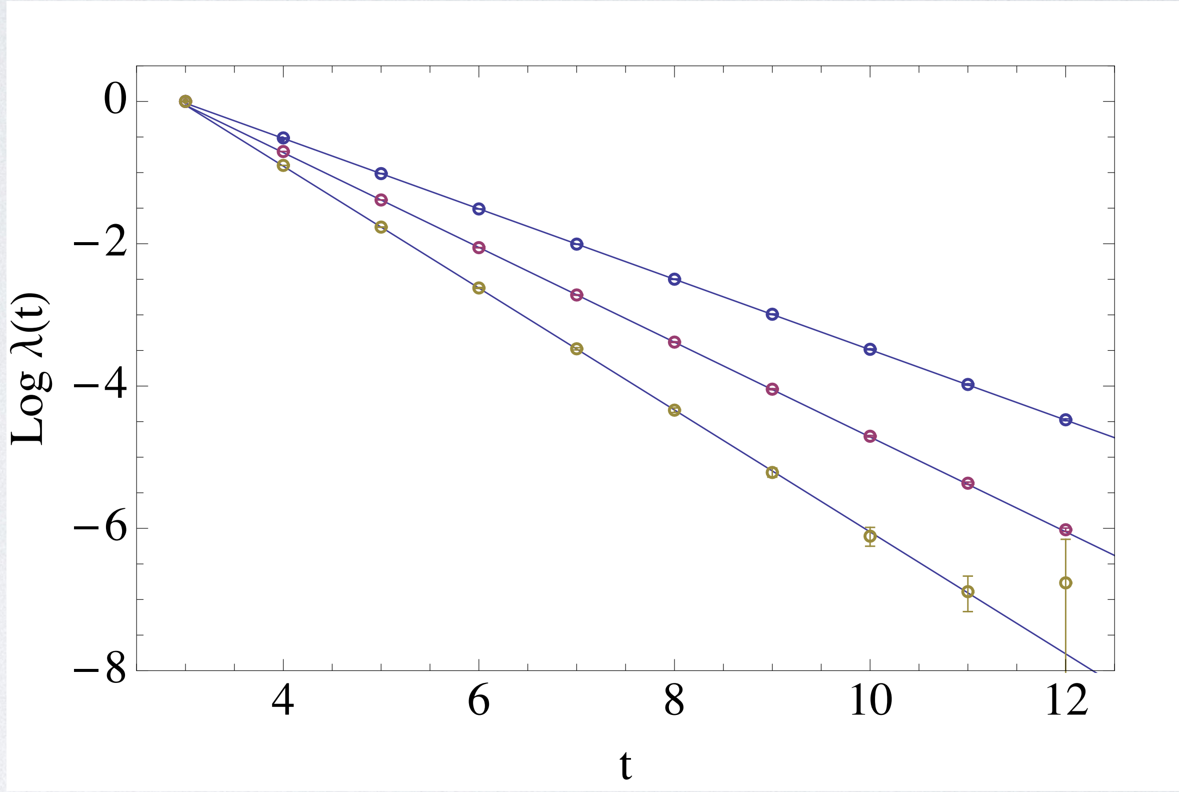


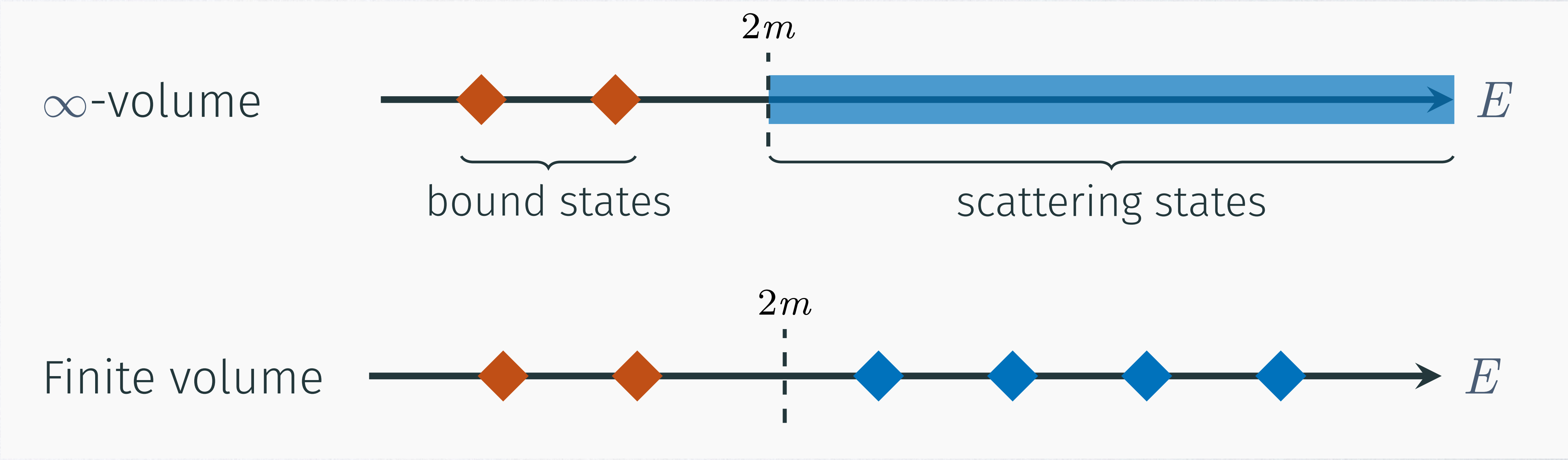
SCATTERING FROM LATTICE QCD

- The basic degrees of freedom are quarks and gluons with QCD interactions. Hadrons operators are composite functions of quark&glue fields.
- The action in **Euclidean** time. Hadron state energies computed from two-point correlation functions.
- For numerical simulations the spatial volume and "temporal" extent is finite.
- Scattering information is accessed indirectly by computing the energy of multi-hadron states.
- The spatial extent of the box controls the set of momenta available.









QUANTIZATION CONDITIONS

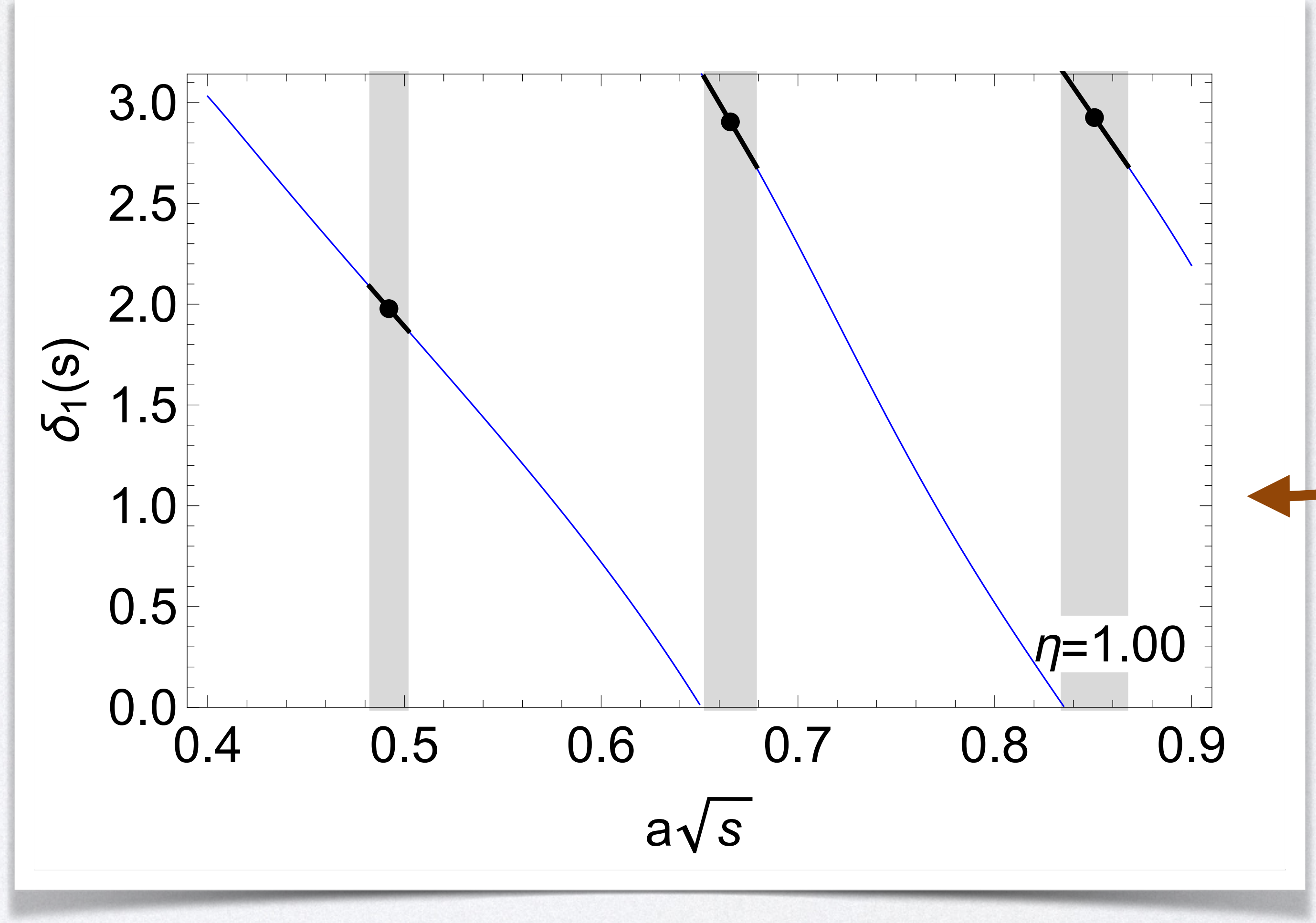
- The quantization conditions (QC) connect infinite volume amplitude to the finite volume energies
- For two-body states these conditions were worked out in the 80s by Lüscher
- In general QC connect an infinite tower of partial waves to the finite volume energies
- For elastic scattering in channels dominated by smallest partial-waves, phase-shifts can be extracted directly from finite volume energies

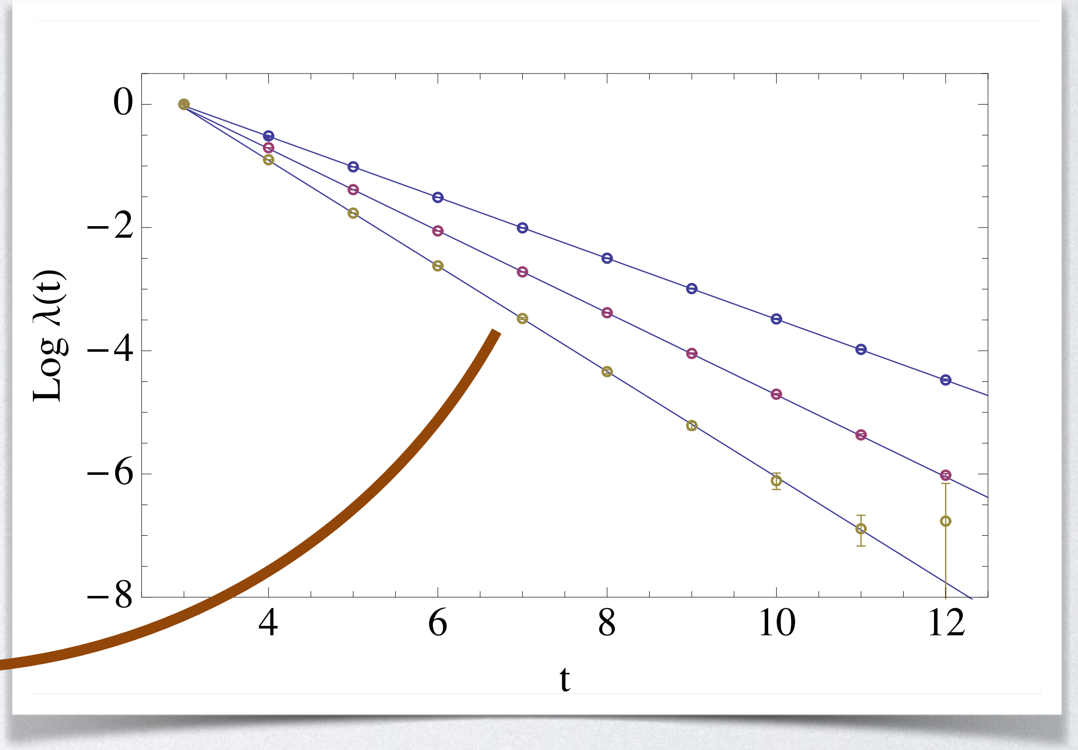
2-body QC

$$\det[\,\widetilde{K}^{-1}(E_{\rm cm}) - B(E_{\rm cm}, L)\,] = 0$$

$\widetilde{K}_\ell^{-1} \sim \cot\delta_l$

known, mixes partial waves

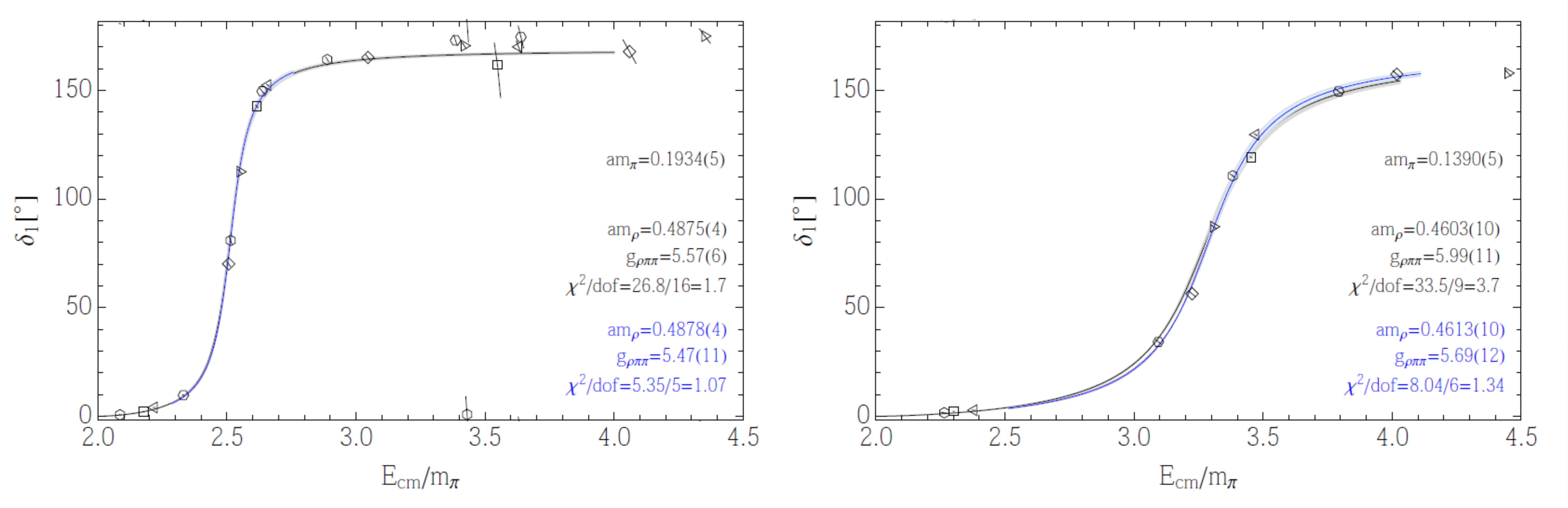




Lattice QCD correlators

TWO PION SCATTERING
I=1
(ρ RESONANCE CHANNEL)

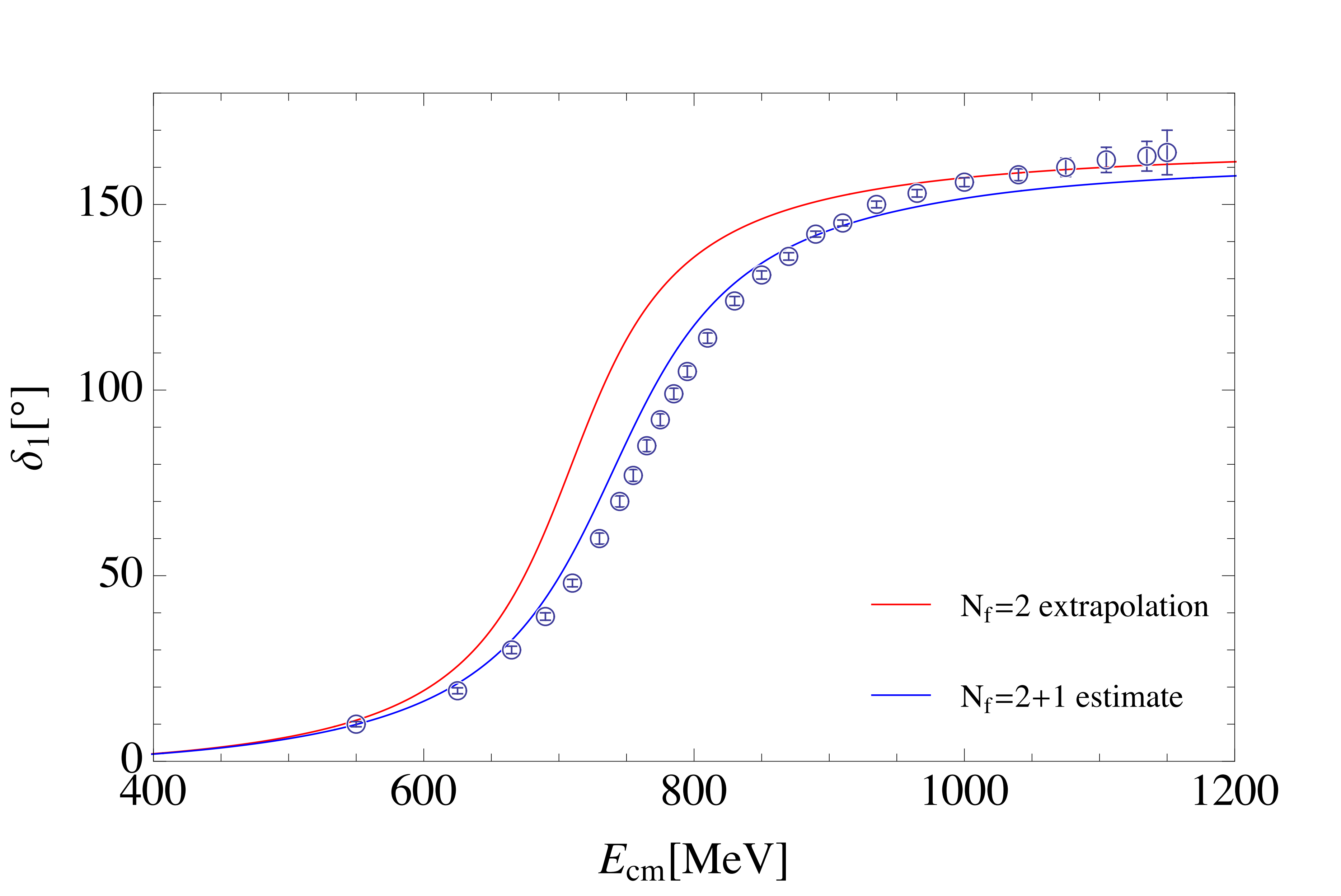
I=1 PHASE SHIFTS (ρ REGION)

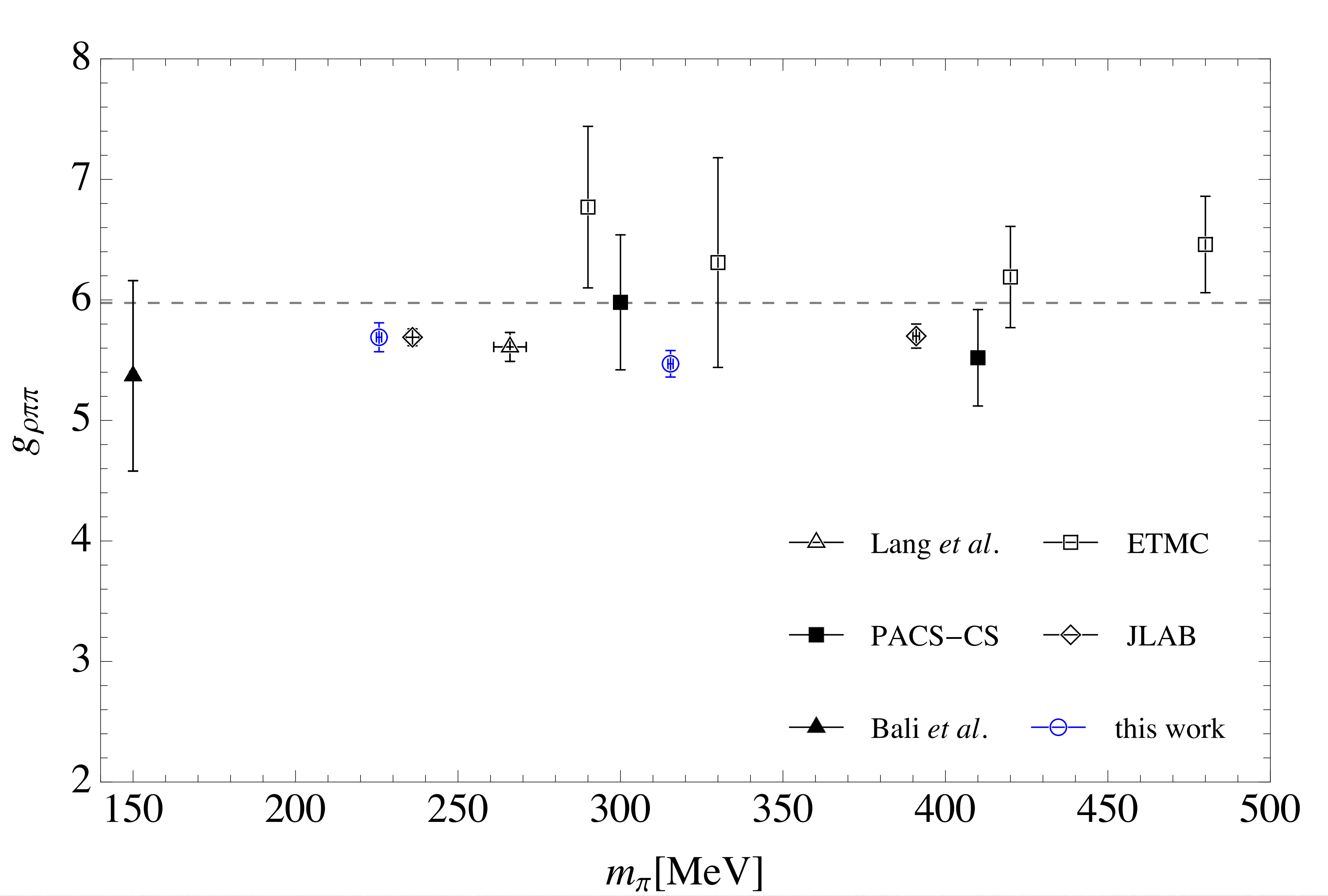


C. Pelissier and AA, *Phys.Rev. D87 (2013) 014503*, [arXiv:1211.0092]

D. Guo, AA, R. Molina, and M. Doering, *Phys. Rev. D94 (2016), no. 3 034501*, [arXiv:1605.03993]

CHIRAL EXTRAPOLATION





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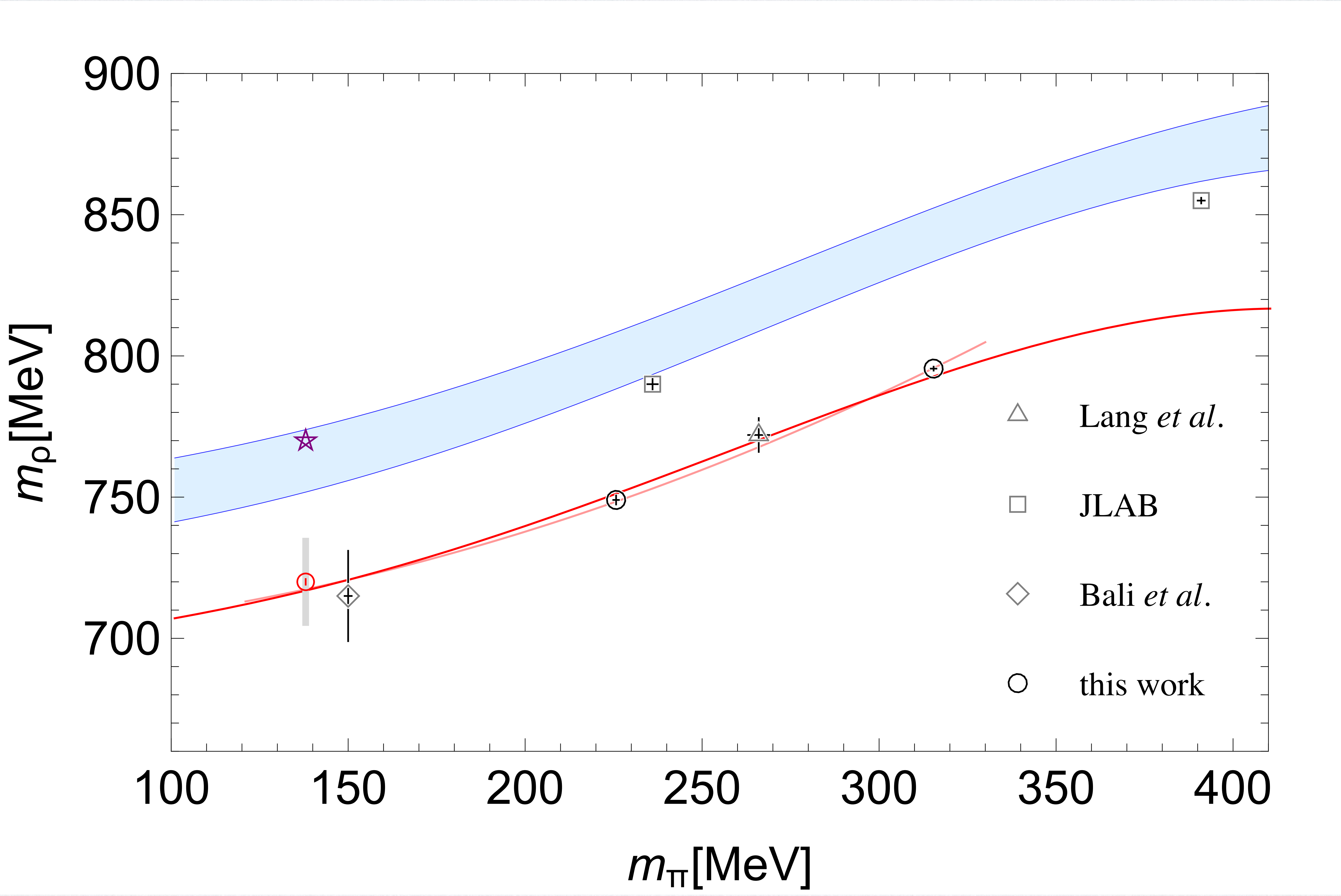
D. Guo, AA, R. Molina, and M. Doering, *Phys. Rev. D94 (2016), no. 3 034501*, [arXiv:1605.03993]

ρ-MASS CHIRAL EXTRAPOLATION

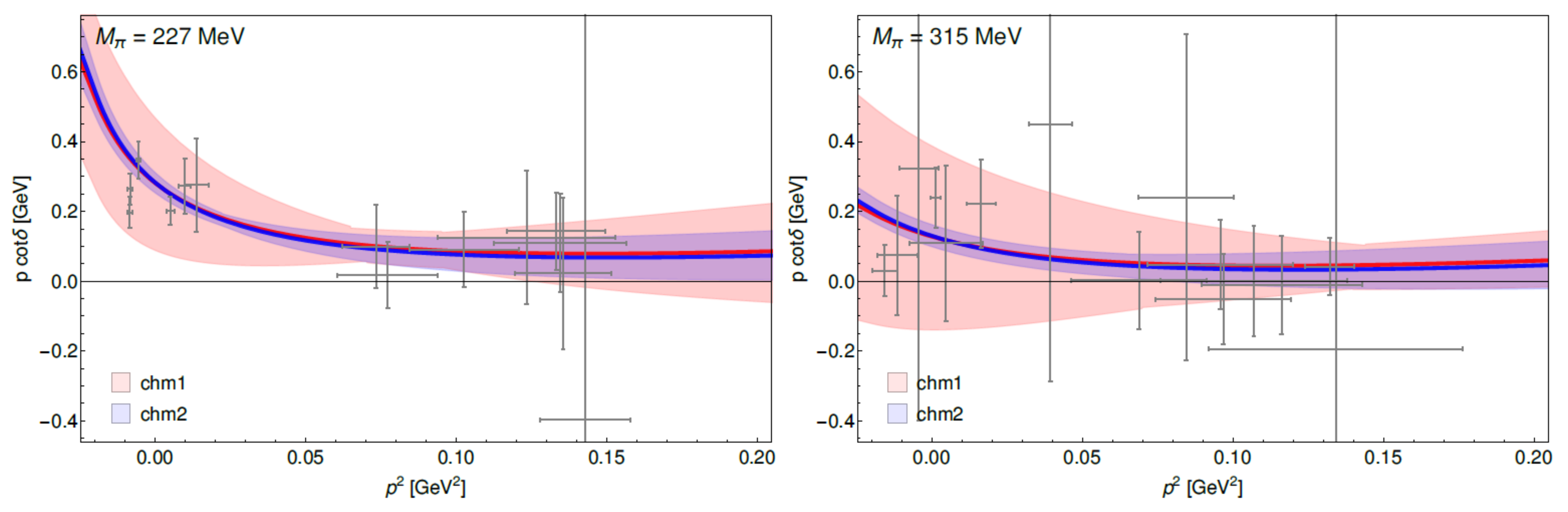


B. Hu, R. Molina, M. Doering, and AA, *Phys. Rev. Lett. 117 (2016), no. 12 122001*, [arXiv:1605.04823]

D. Guo, AA, R. Molina, and M. Doering, *Phys. Rev. D94 (2016), no. 3 034501*, [arXiv:1605.03993]

TWO PION SCATTERING
I=0
(σ RESONANCE CHANNEL)

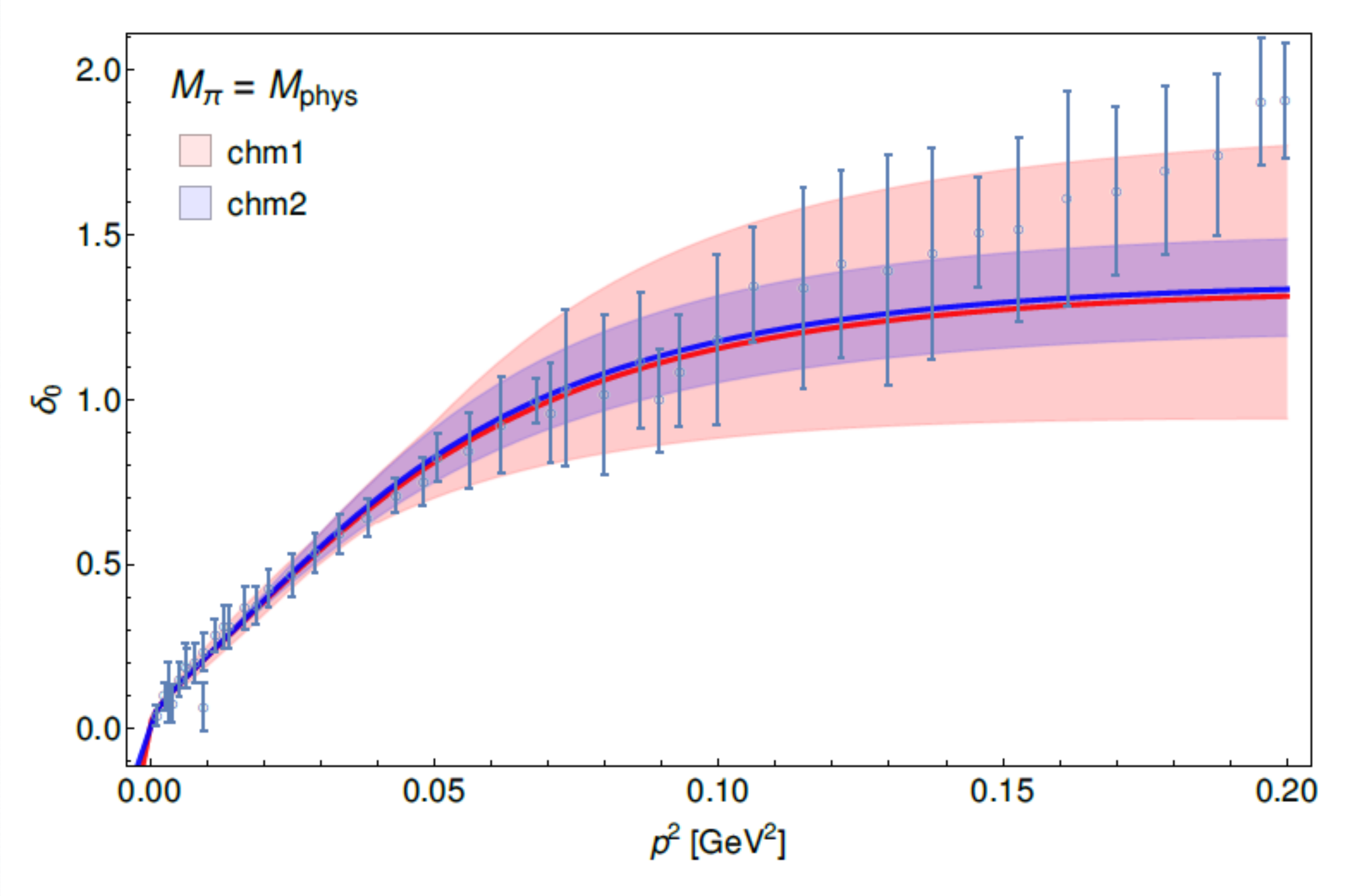
I=0 PHASE SHIFTS—UχPT FITS



$$K_{00}(s) = \frac{3(M_\pi^2 - 2s)^2}{6f_\pi^2(M_\pi^2 - 2s) + 8(L_a M_\pi^4 + s(L_b M_\pi^2 + L_c s))}, \qquad K_{11}(s) = \frac{4M_\pi^2 - s}{3(f_\pi^2 - 8\,\hat{l}_1 M_\pi^2 + 4\,\hat{l}_2 s)},$$

J. A. Oller, E. Oset, and J. R. Pelaez, *Phys. Rev. D59, 074001 (1999)*

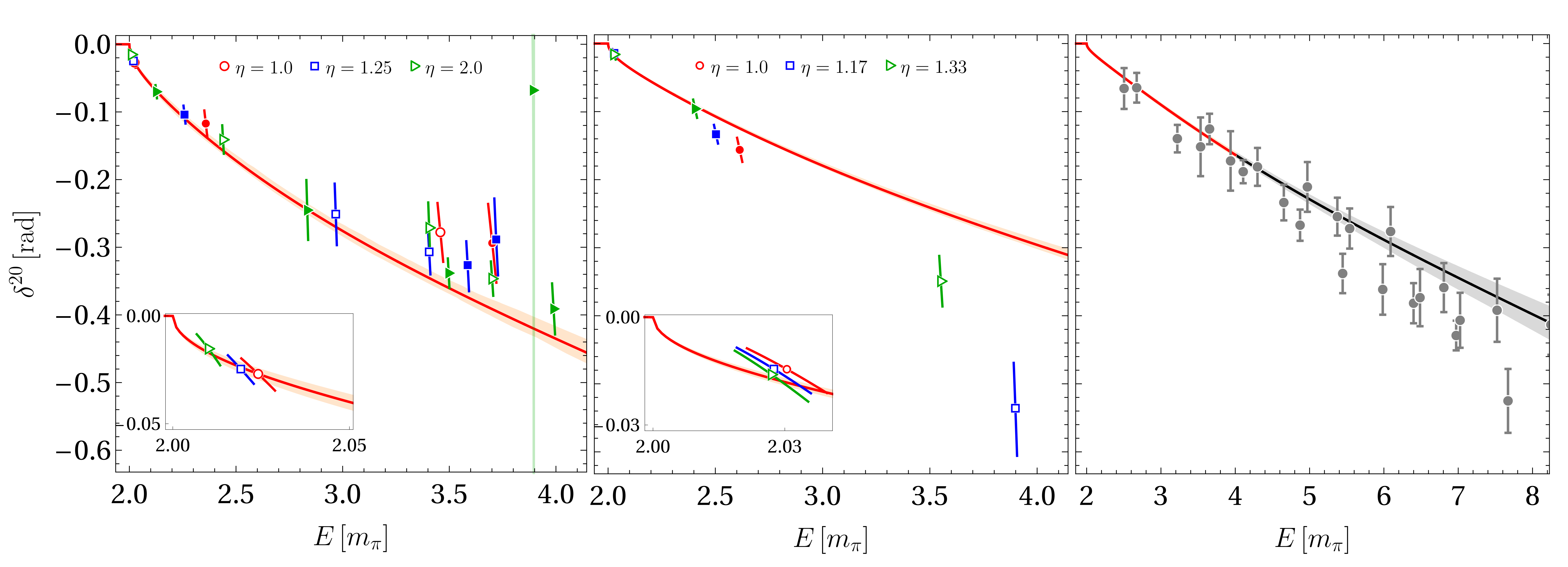
EXTRAPOLATED PHASE-SHIFTS



D. Guo, AA, R. Molina, M. Mai, and M. Döring, *Phys. Rev. D98 (2018), no. 1 014507*, [arXiv:1803.02897]

TWO PION SCATTERING
I=2
(NON-RESONANT CHANNEL)

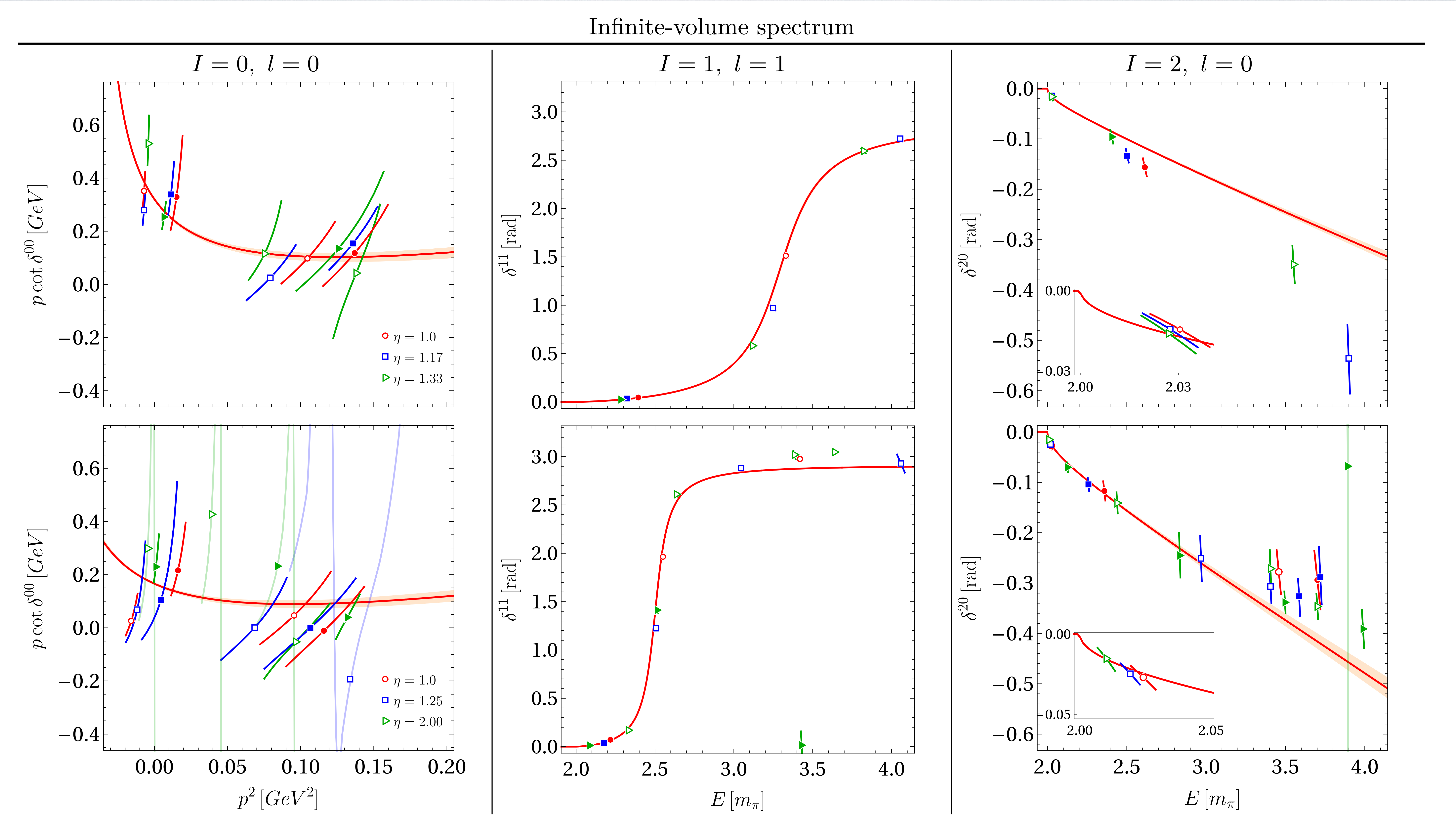
PHASE SHIFTS



C. Culver, M. Mai, AA, M. Dring, and F. X. Lee, *Phys. Rev. D100 (2019), no. 3 034509*, [arXiv:1905.10202]

TWO PION SCATTERING CROSS-CHANNELS FIT

GLOBAL FIT



M. Mai, C. Culver, AA, M. Doering, and F. X. Lee, *Phys. Rev. D100 (2019), no. 11 114514*, [arXiv:1908.01847]

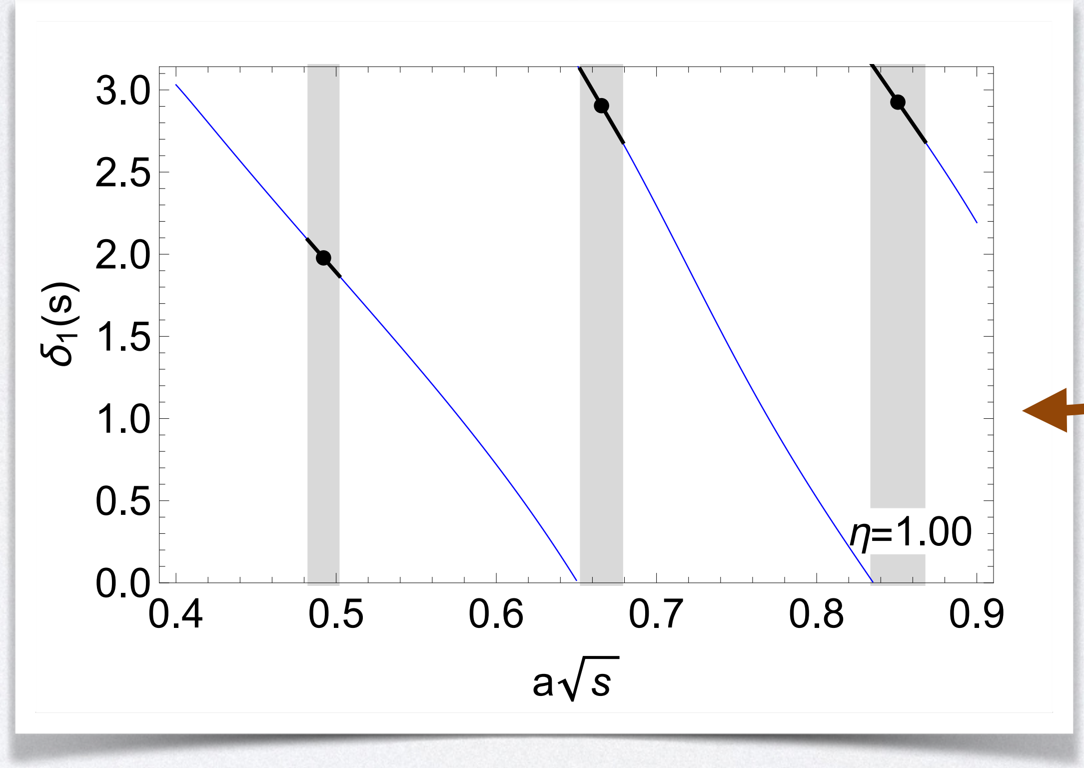
3-BODY QUANTIZATION CONDITIONS

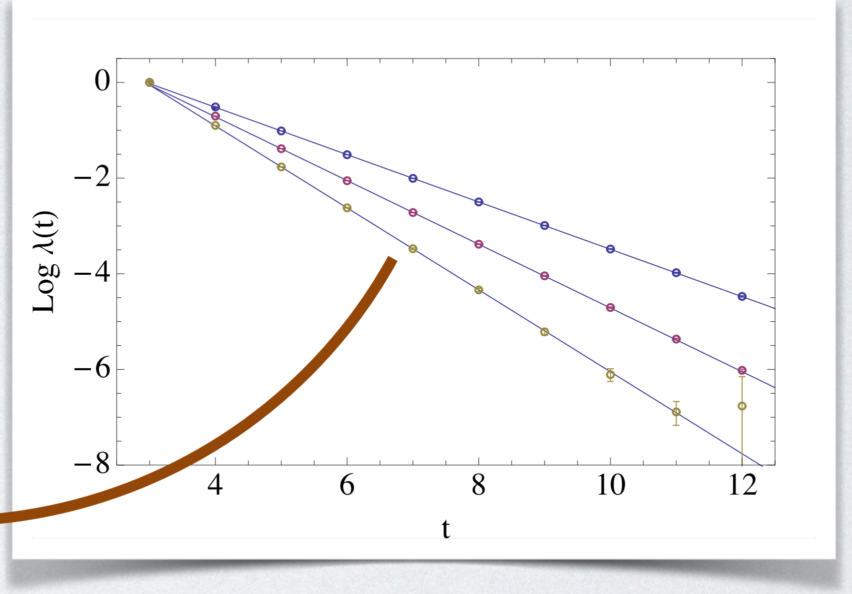
- The quantization conditions (QC) connect infinite volume amplitude to the finite volume energies
- Compared to two-body, the three-body problem is significantly more complex: 8 kinematic variables in infinite volume, etc.
- Recently QC were developed for 3body states
  - RFT (Hansen, Sharpe, …) diagrammatic approach
  - FVU (Döring, Mai) built on unitarity
  - NREFT (Rusetsky, Peng, …) (non-)relativistic EFT
- FVU and RFT were found to be equivalent

2-body QC

$$\det[\,\widetilde{K}^{-1}(E_{\rm cm}) - B(E_{\rm cm}, L)\,] = 0$$

$\widetilde{K}_\ell^{-1} \sim \cot\delta_l$ → $\widetilde{K}^{-1}$; B ↑ known, mixes partial waves

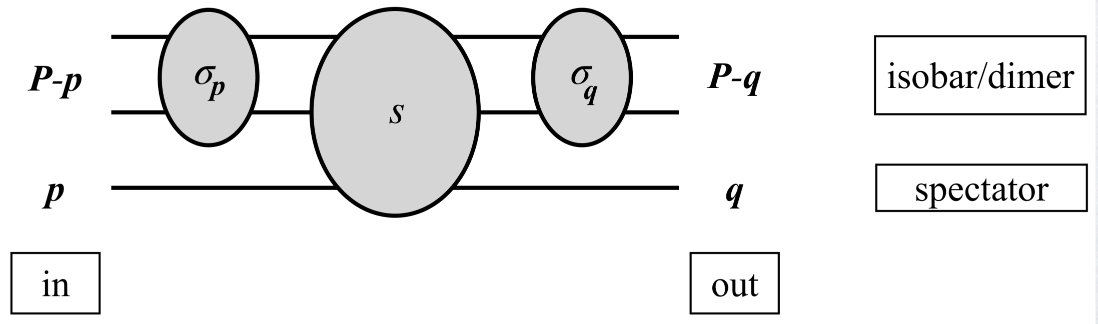




Lattice QCD correlators

3-body QC

$$\det \mathcal{Q} \equiv \det\left[B_0(\sqrt{s}) + C_0(\sqrt{s}) + E_{L\eta}\tau^{-1}_{L\eta P}(\sqrt{s})\right] = 0.$$

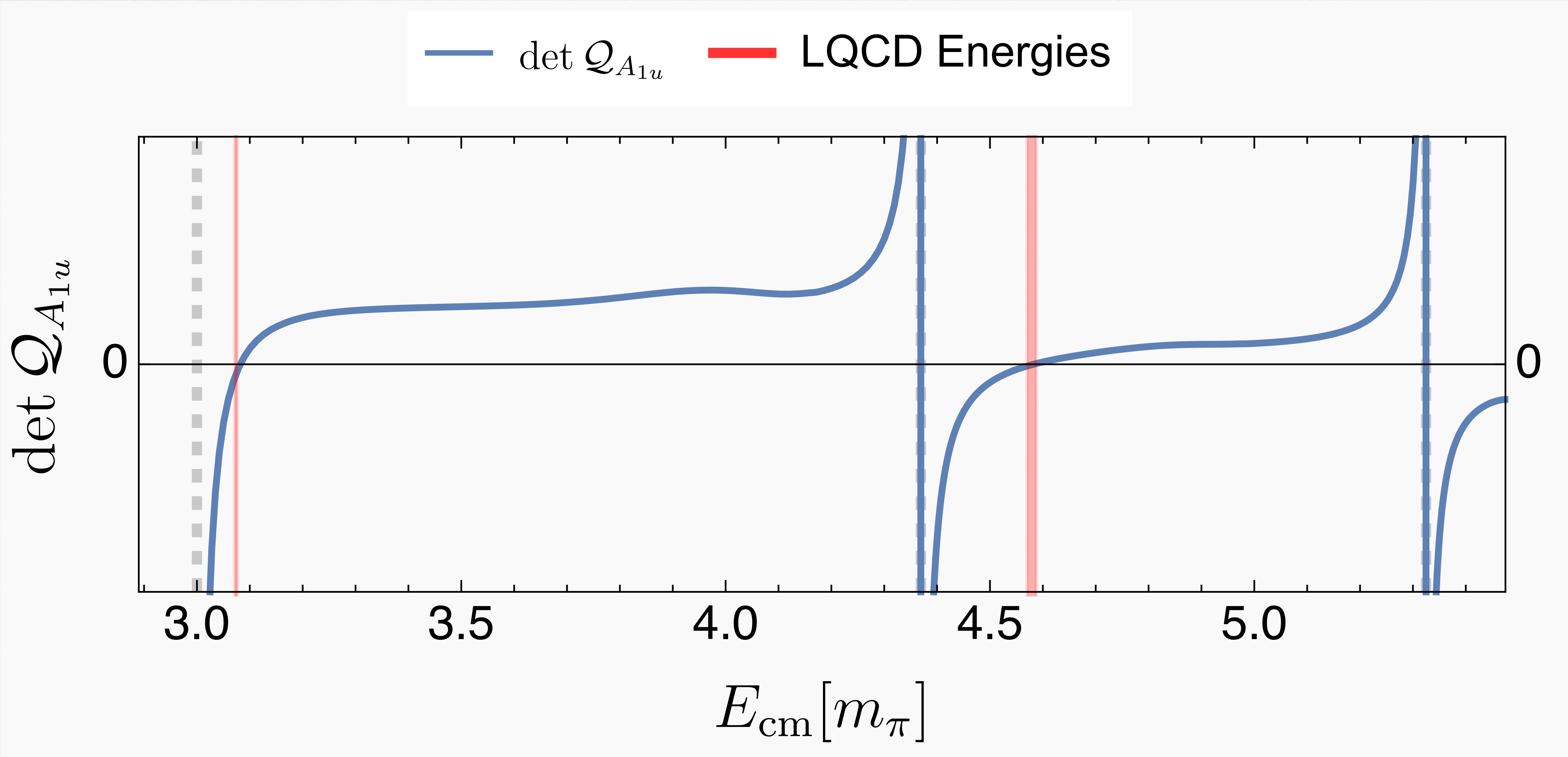


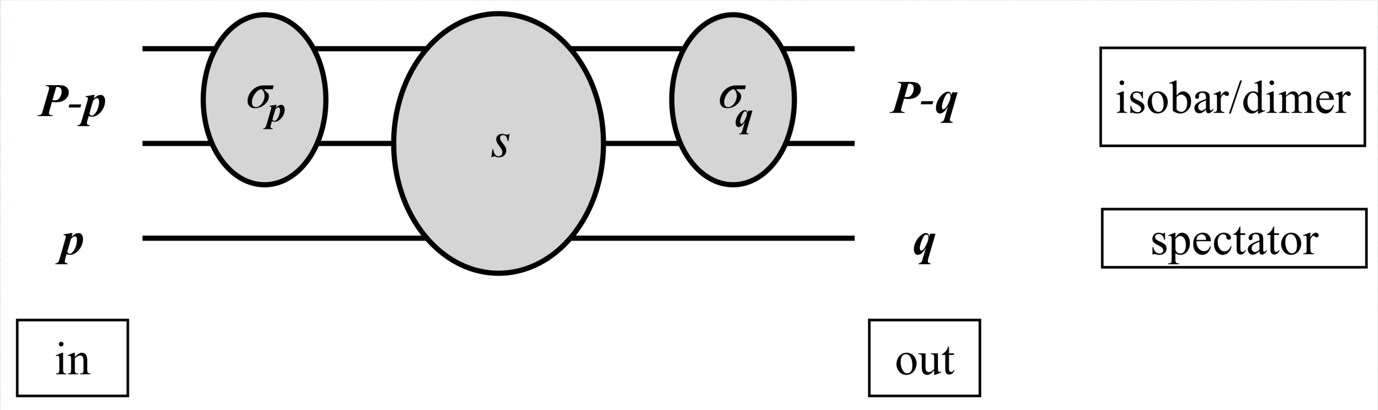
Reviews: M. Hansen, S. Sharpe, arXiv:1901.00483 M. Mai, M. Döring, A. Rusetsky , arXiv:2103.00577
RTF & FVU comparison: A. Jackura *et al*, arXiv:1905.12007

FVU QUANTIZATION CONDITION

Three body state finite volume energies $\sqrt{s}$ satisfy

$$\det \mathcal{Q} \equiv \det \left[B_0(\sqrt{s}) + C_0(\sqrt{s}) + E_{L\eta}\tau^{-1}_{L\eta P}(\sqrt{s}) \right] = 0.$$





M. Mai and M. Döring, *Eur. Phys. J. A53 (2017), no. 12 240*, [arXiv:1709.08222]

R. Brett, C. Culver, M. Mai, AA, M. Doering, and F. X. Lee, *Phys. Rev. D 104 (2021), no. 1 014501*, [arXiv:2101.06144]

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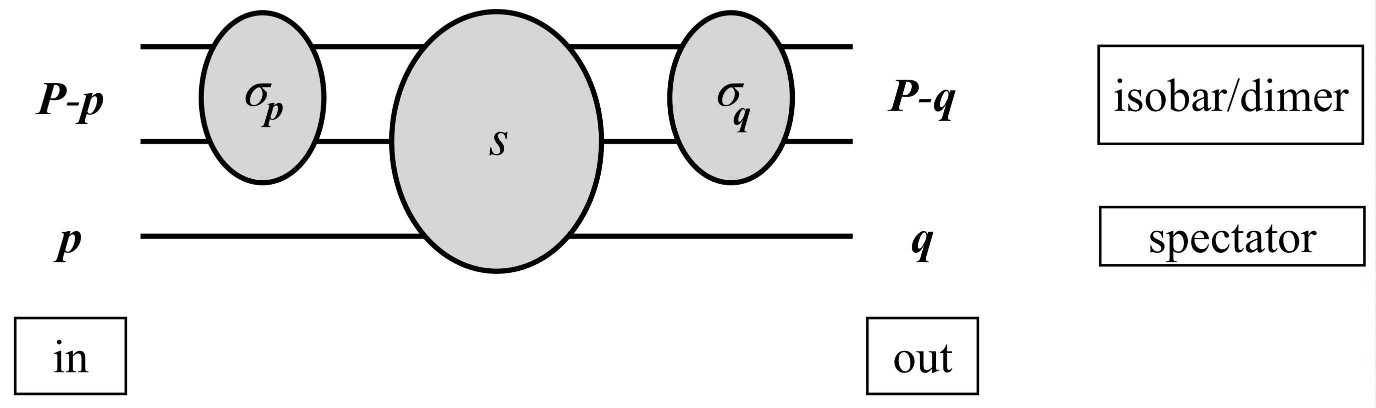

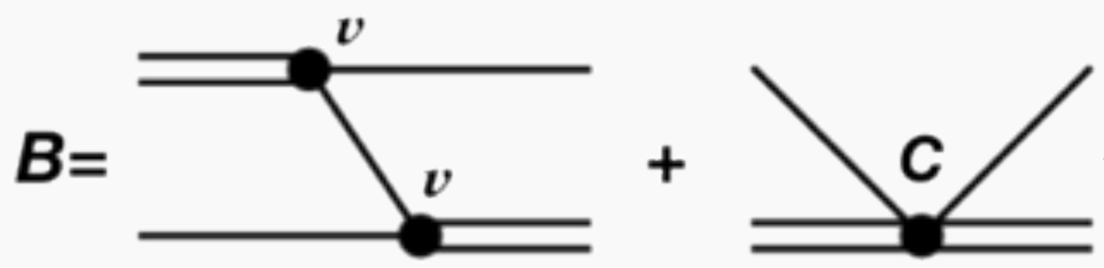

B: one particle exchange
C: isobar-spectator interaction

M. Mai and M. Döring, *Eur. Phys. J. A53 (2017), no. 12 240*, [arXiv:1709.08222]
R. Brett, C. Culver, M. Mai, AA, M. Doering, and F. X. Lee, *Phys. Rev. D 104 (2021), no. 1 014501*, [arXiv:2101.06144]

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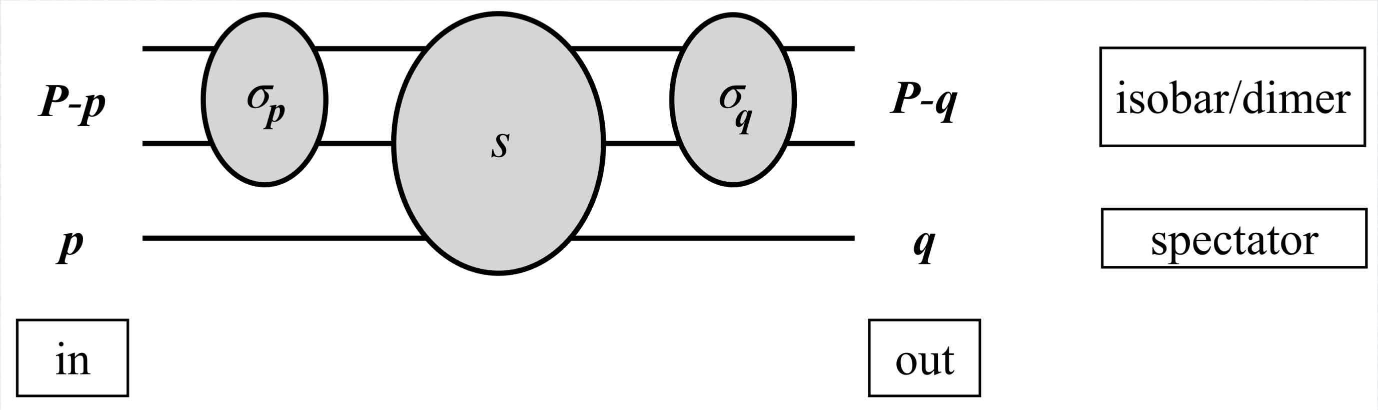

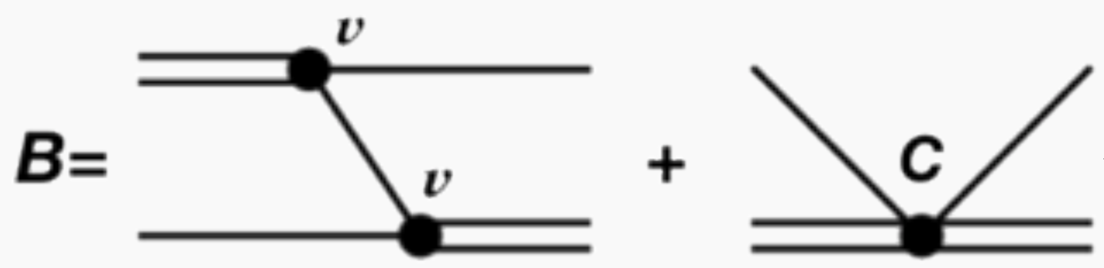

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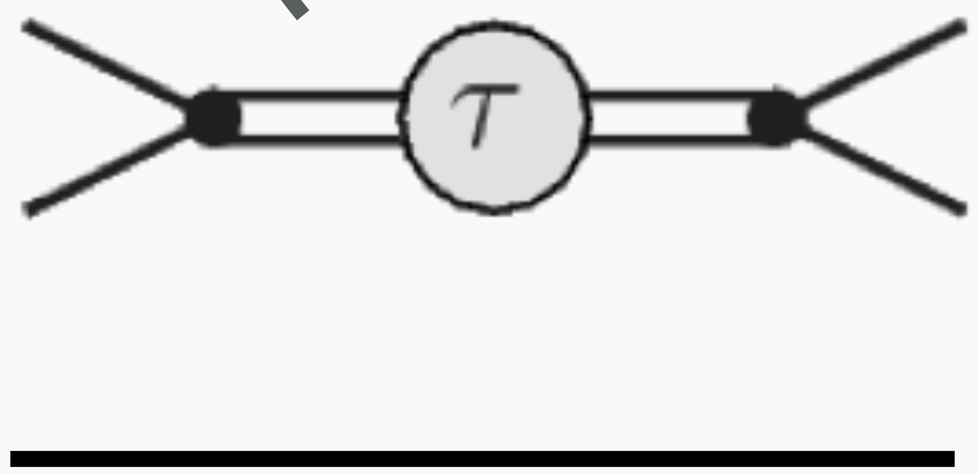
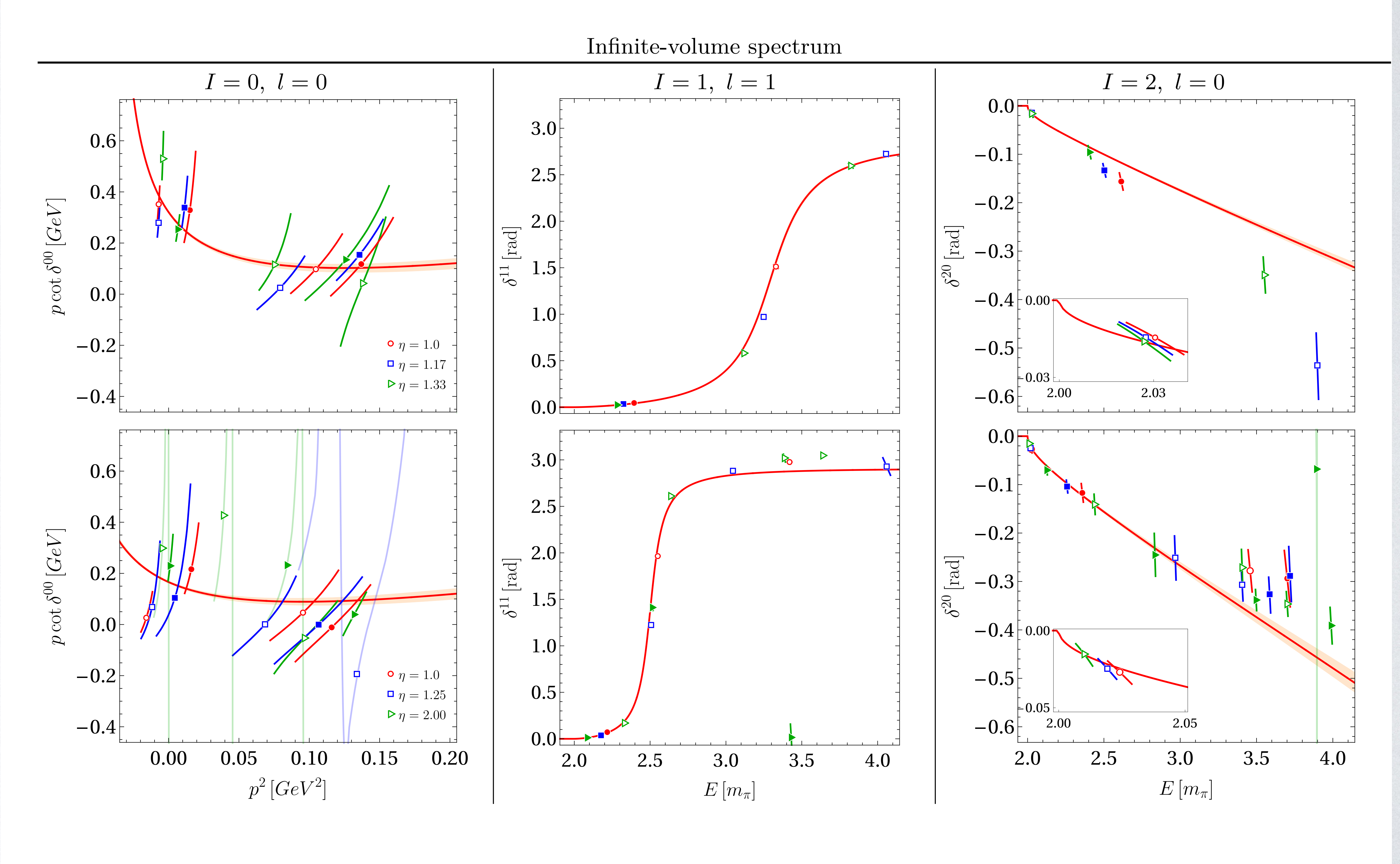


τ: 2-body input
K-matrix, effective range,
(m)IAM, etc

M. Mai and M. Döring, *Eur. Phys. J. A53 (2017), no. 12 240*, [arXiv:1709.08222]
R. Brett, C. Culver, M. Mai, AA, M. Doering, and F. X. Lee, *Phys. Rev. D 104 (2021), no. 1 014501*, [arXiv:2101.06144]

THREE PIONS SCATTERING

- Maximal isospin: sub channels are not resonant (I=2)
- (m)IAM for 2-body interactions, only s-wave set to match ChPT at NLO with LECs set by two strategies:
  - GL: J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984).
  - GW: cross-channel fit to lattice QCD data (M. Mai, C. Culver, A. Alexandru, M. Döring, and F. X. Lee, (2019), arXiv:1908.01847 [hep-lat].)
- Six different ensembles: 2 pion masses (220&315MeV) and 3 different geometries.
- We set the three body contact term C to zero.

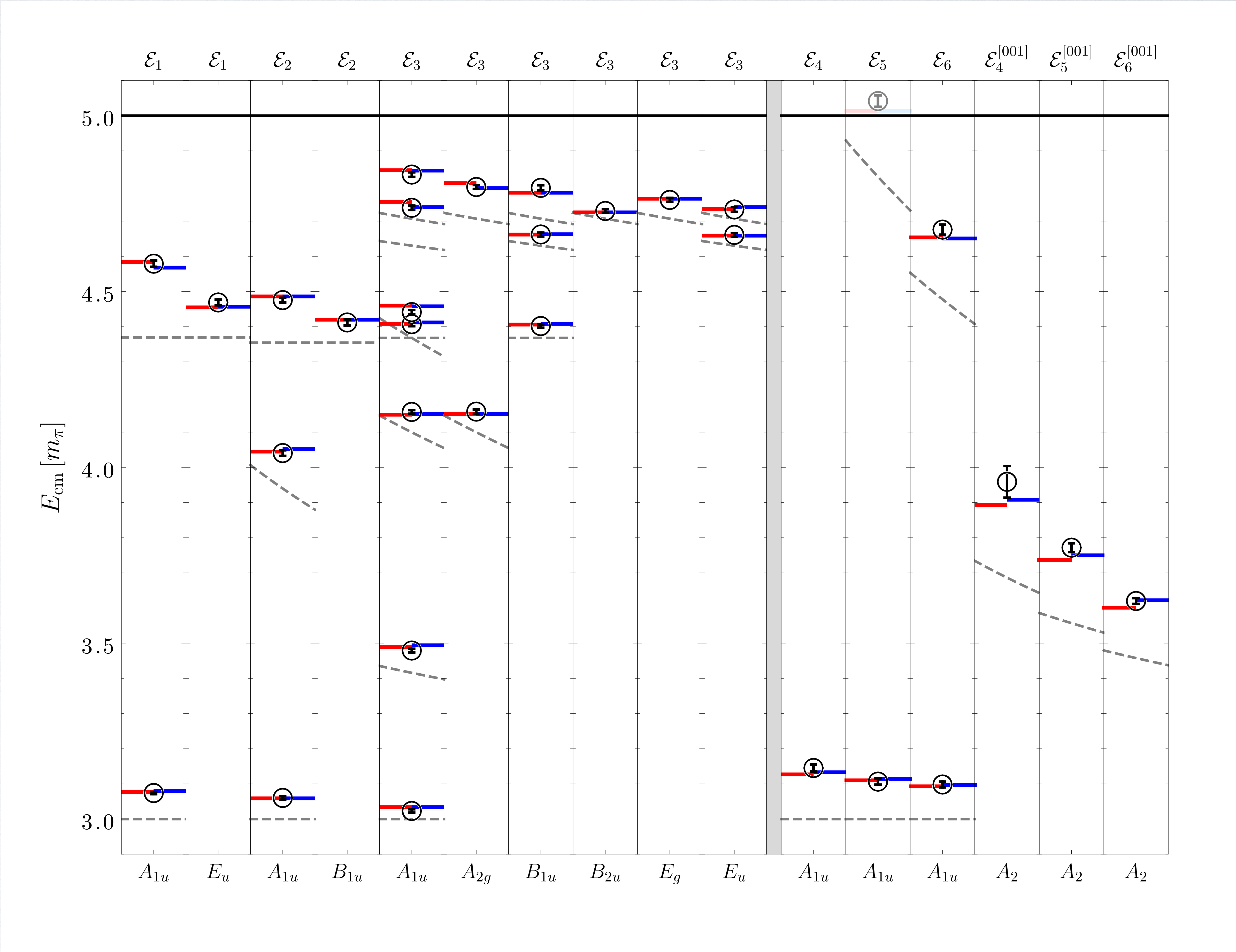


M. Mai, C. Culver, AA, M. Döring, and F. X. Lee, *Phys. Rev. D100 (2019), no. 11 114514*, [arXiv:1908.01847]

C. Culver, M. Mai, R. Brett, AA, and M. Döring, *Phys. Rev. D 101 (2020), no. 11 114507*, [arXiv:1911.09047]

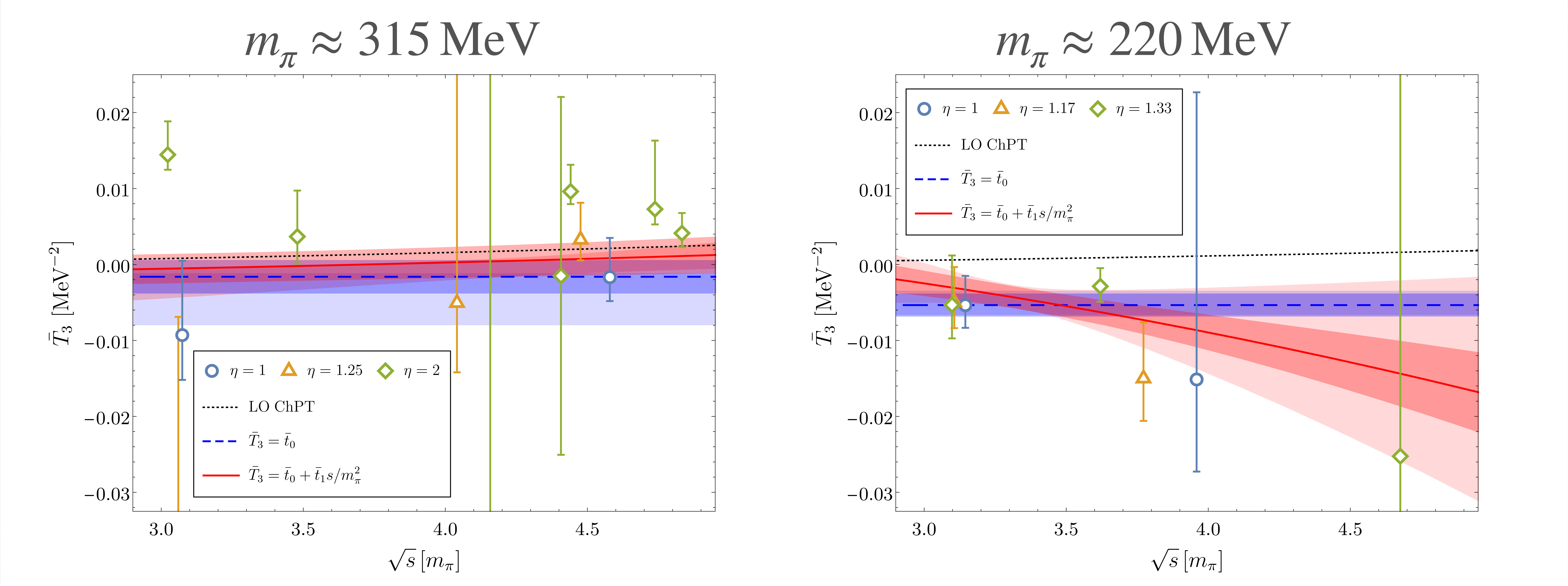
THREE PIONS SCATTERING

- We measured 30 different energy levels for 3π states.
- The predictions from the quantization conditions agree well with the lattice QCD levels.
- The two different set of LECs produce slightly different predictions: $\chi^2/\mathrm{dof} \approx 2.68$ (GW) and $\chi^2/\mathrm{dof} \approx 4.86$ (GL). GW LECs produce better agreement, as expected.
- The disagreement is small, but statistically significant.
- One possible source for this tension is the 3-body force term, C, which was set to zero. This gives hope that we can constrain its value from our data.



C. Culver, M. Mai, R. Brett, AA, and M. Döring, *Phys. Rev. D 101 (2020), no. 11 114507*, [arXiv:1911.09047]

THREE PION "CONTACT" TERM

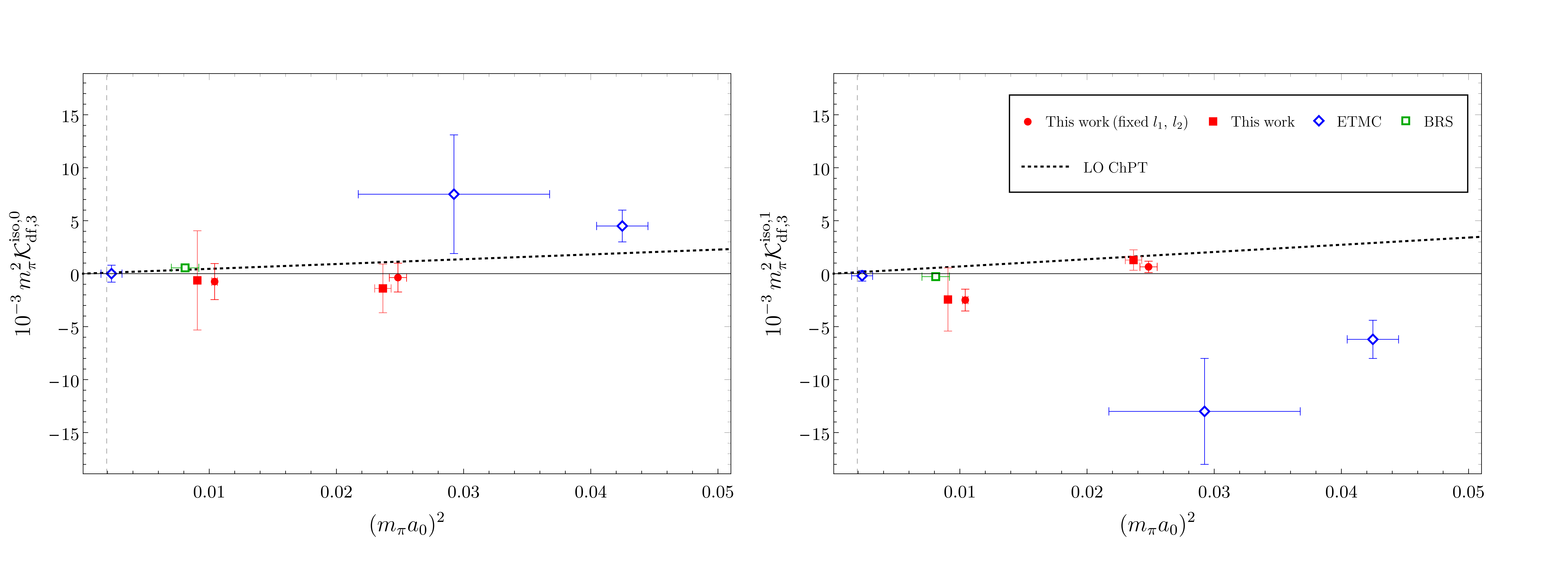


$$\bar{T}_3 = \frac{3}{2}\left(\frac{K^{-1}}{32\pi}\right)^{-1} \frac{C_0}{\mathbb{1} - C_0 E_{L\eta}^{-1}\left(\frac{K^{-1}}{32\pi}\right)^{-1}} \left(\frac{K^{-1}}{32\pi}\right)^{-1}$$

Leading order ChPT: $\bar{T}_3 = \dfrac{1}{27 f_\pi^4}\left(4s - 9m_\pi^2\right)$.

R. Brett, C. Culver, M. Mai, AA, M. Doering, and F. X. Lee, *Phys. Rev. D 104 (2021), no. 1 014501*, [arXiv:2101.06144]

THREE PION "CONTACT" TERM



$$\mathcal{K}_{\mathrm{df},3}^{\mathrm{iso},0} \simeq 6(\bar{t}_0 + 9\bar{t}_1) \qquad \mathcal{K}_{\mathrm{df},3}^{\mathrm{iso},1} \simeq 54\bar{t}_1$$

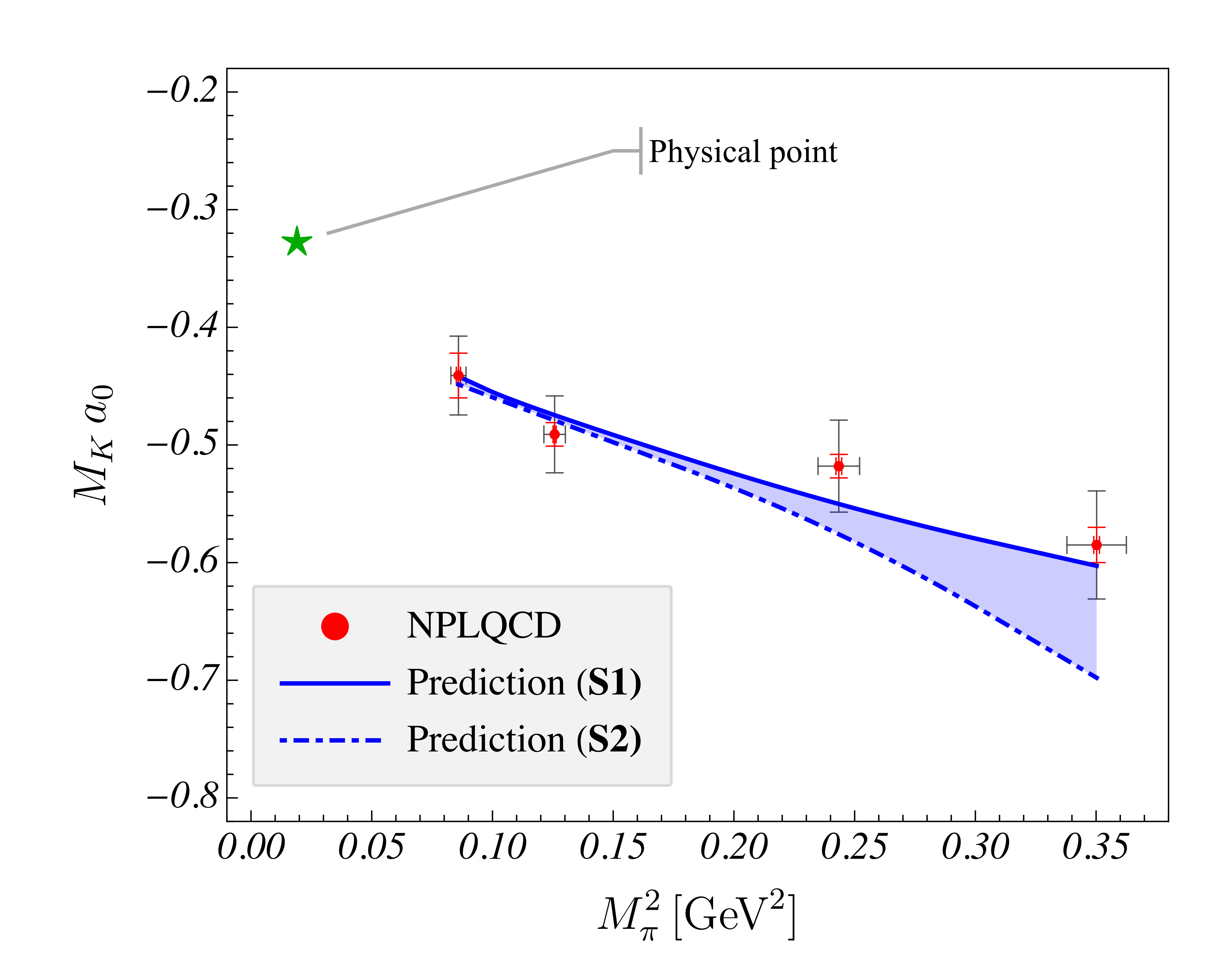
BRS: T. D. Blanton, F. Romero-Lopez, and S. R. Sharpe, *Phys. Rev. Lett. 124 (2020), no. 3 032001*, [arXiv:1909.02973]

ETMC: M. Fischer, B. Kostrzewa, L. Liu, F. Romero-Lopez, M. Ueding, and C. Urbach, *Eur. Phys. J. C 81 (2021), no. 5 436*, [arXiv:2008.03035]

R. Brett, C. Culver, M. Mai, AA, M. Döring, and F. X. Lee, *Phys. Rev. D 104 (2021), no. 1 014501*, [arXiv:2101.06144]

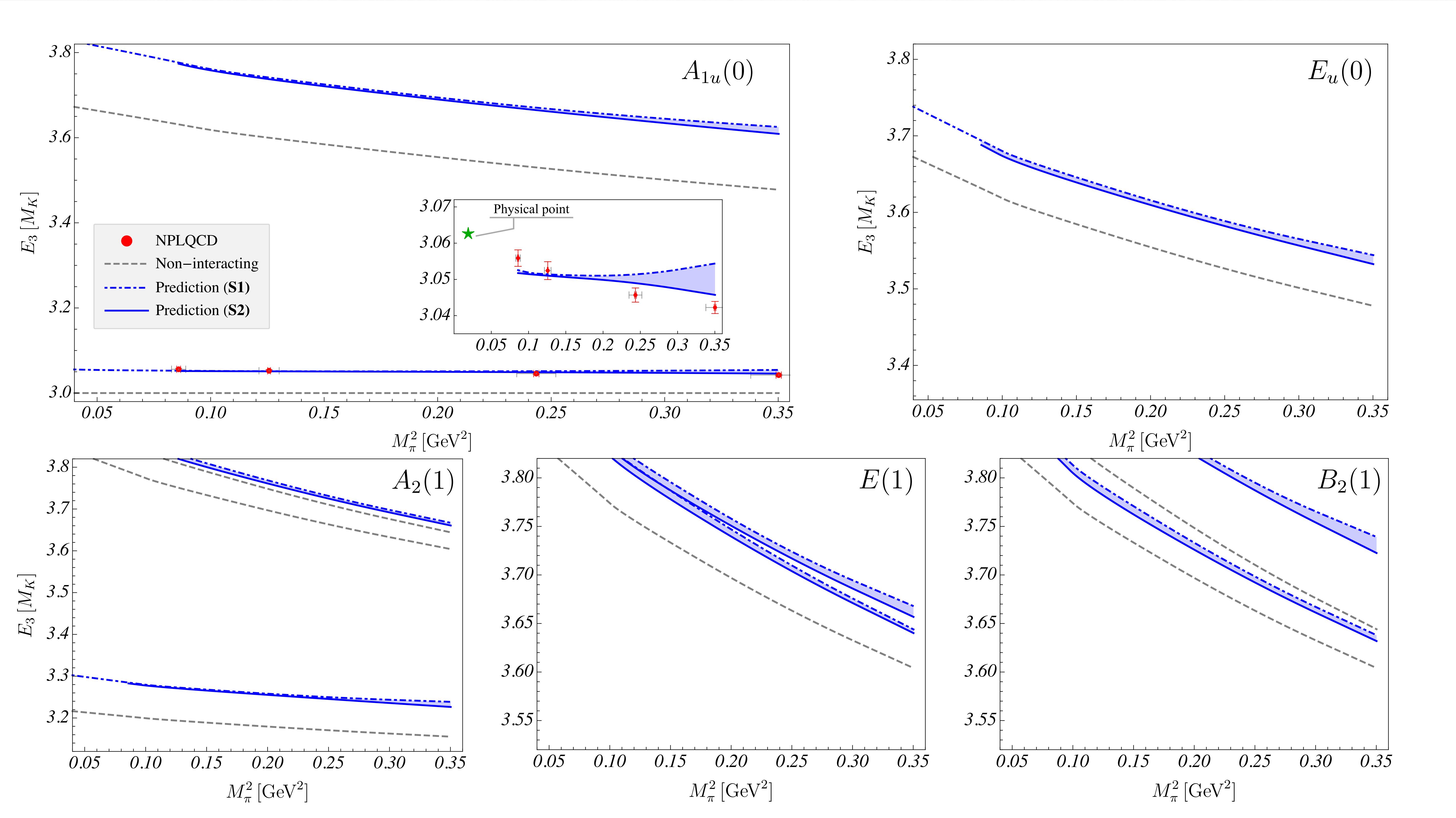
THREE KAONS SCATTERING

- Maximal isospin: sub channels are not resonant
- IAM for 2-body interactions, only s-wave set to match ChPT at NLO with LECs set by two strategies:
  - S1: f_π extrapolated using physical point dar and NLO chiral expressions
  - S2: meson decay constant measured from lattice QCD
- Scattering length matches lattice QCD measurements with small systematics for $m_\pi \lessapprox 500\,\mathrm{MeV}$.
- We set the three body contact term C to zero.



AA, R. Brett, C. Culver, M. Döring, D. Guo, F. X. Lee, and M. Mai, *Phys. Rev. D 102 (2020), no. 11 114523*, [arXiv:2009.12358]

THREE KAONS SCATTERING

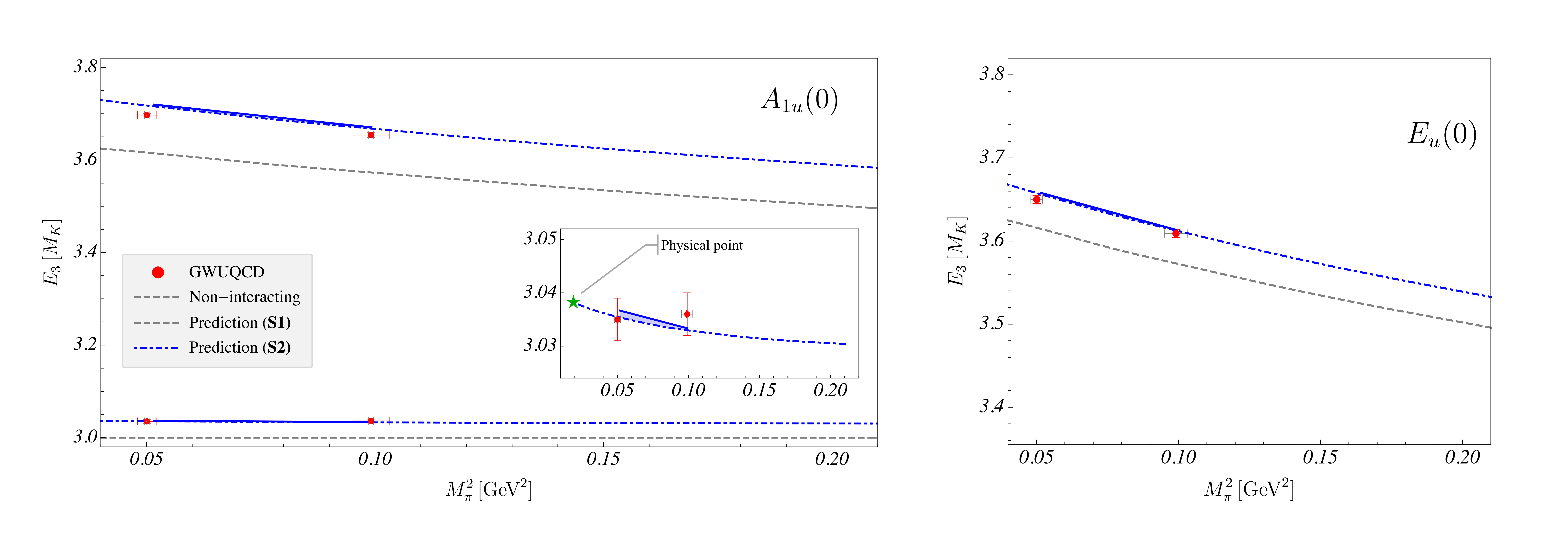


- NPLQCD computed ground states for three-kaons for pion masses in the range 300-600 MeV.
- Ground state predictions match well calculations from NPLQCD.

W. Detmold, K. Orginos, M. J. Savage, and A. Walker-Loud, *Phys. Rev. D 78 (2008) 054514*, [arXiv:0807.1856]

AA, R. Brett, C. Culver, M. Döring, D. Guo, F. X. Lee, and M. Mai, *Phys. Rev. D 102 (2020), no. 11 114523*, [arXiv:2009.12358]

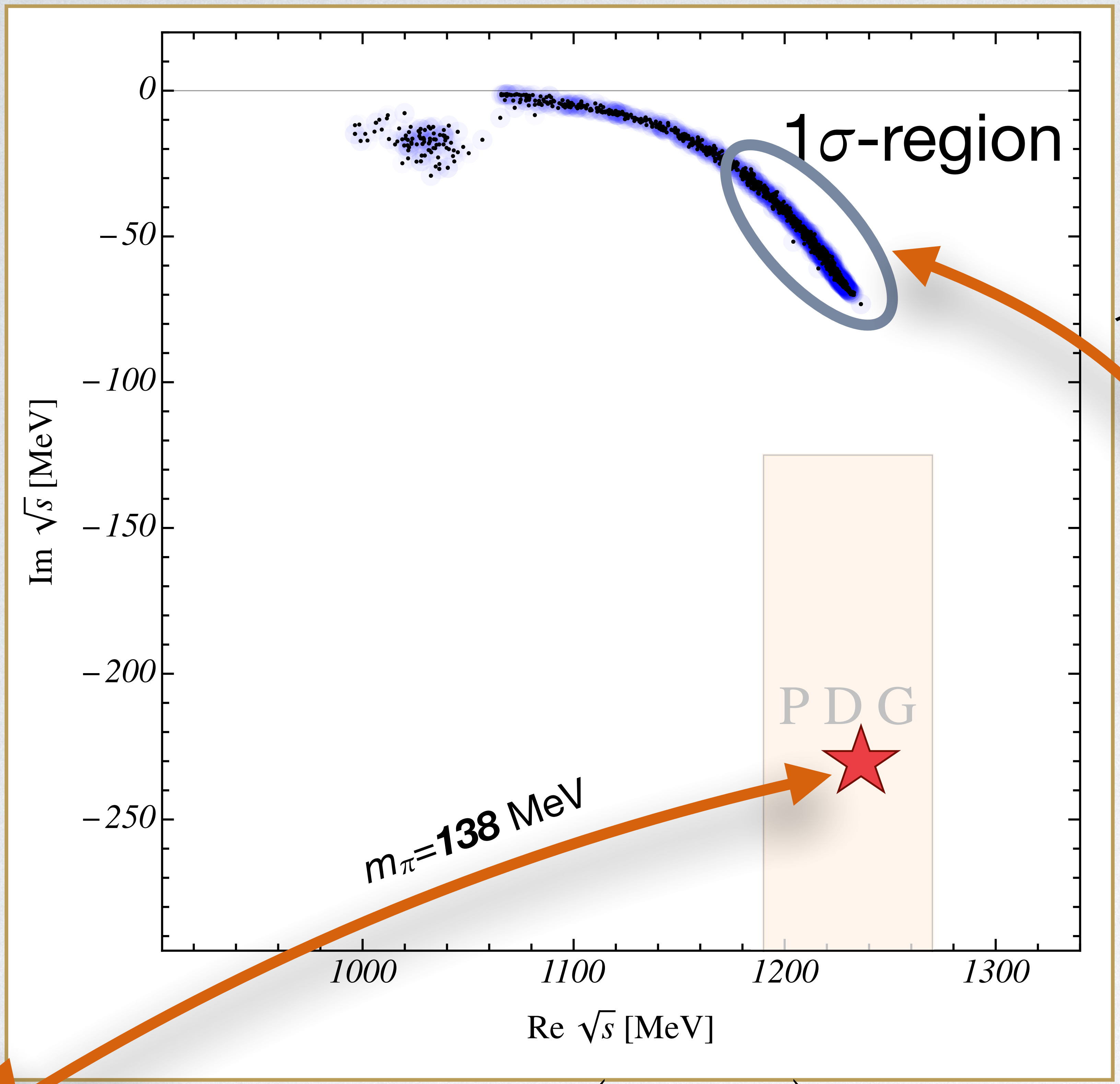
THREE KAONS SCATTERING



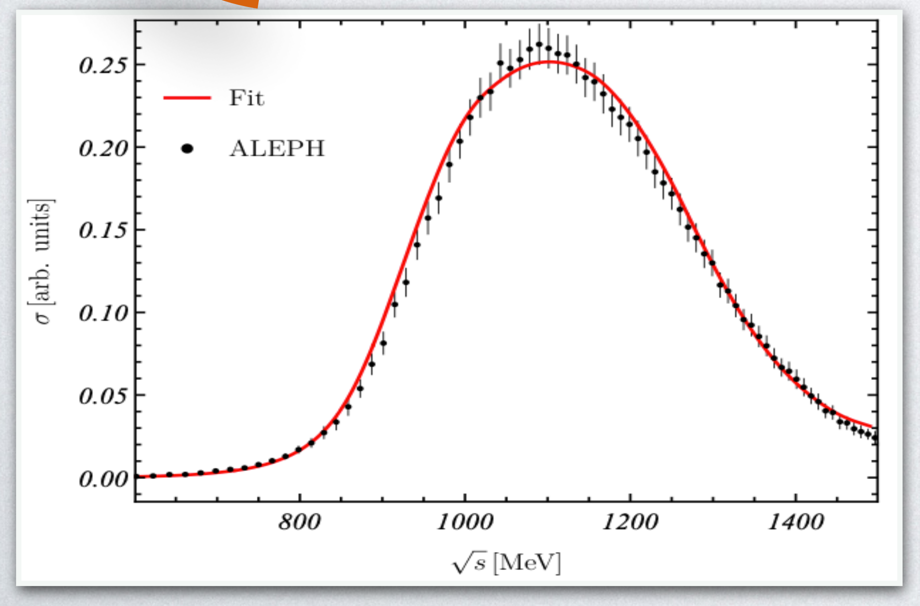
- On a set of N_f=2 ensembles we calculated ground and excited 3K states for m_π=220MeV and 315 MeV.
- We found good agreement with predictions in both irreducible representations we studied.
- Small tension might be due to quenching of strange quarks, or missing contact term.

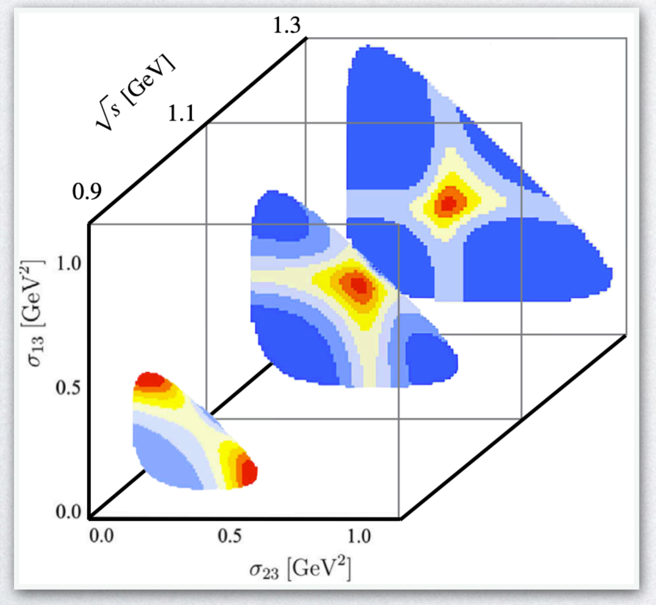
AA, R. Brett, C. Culver, M. Döring, D. Guo, F. X. Lee, and M. Mai, *Phys. Rev. D 102 (2020), no. 11 114523*, [arXiv:2009.12358]

$a_1(1260)$ FROM LATTICE QCD

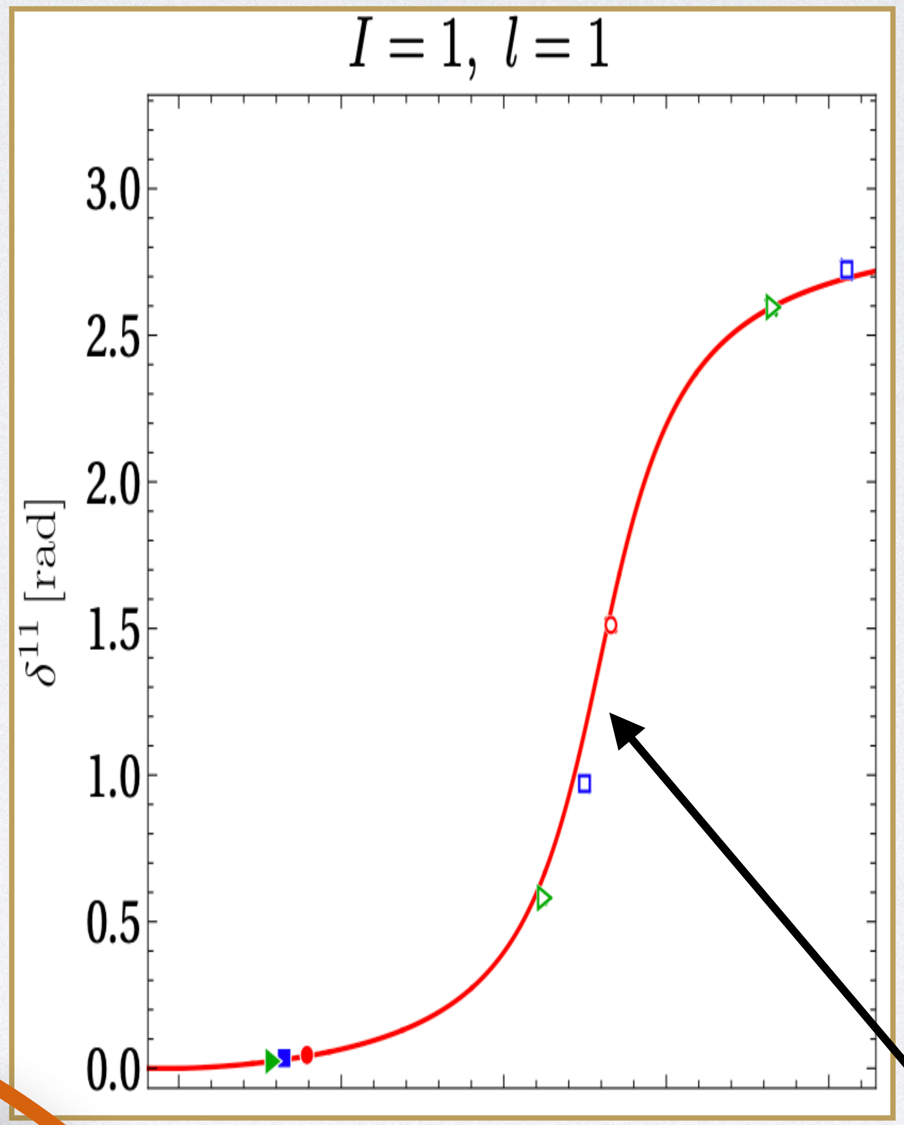


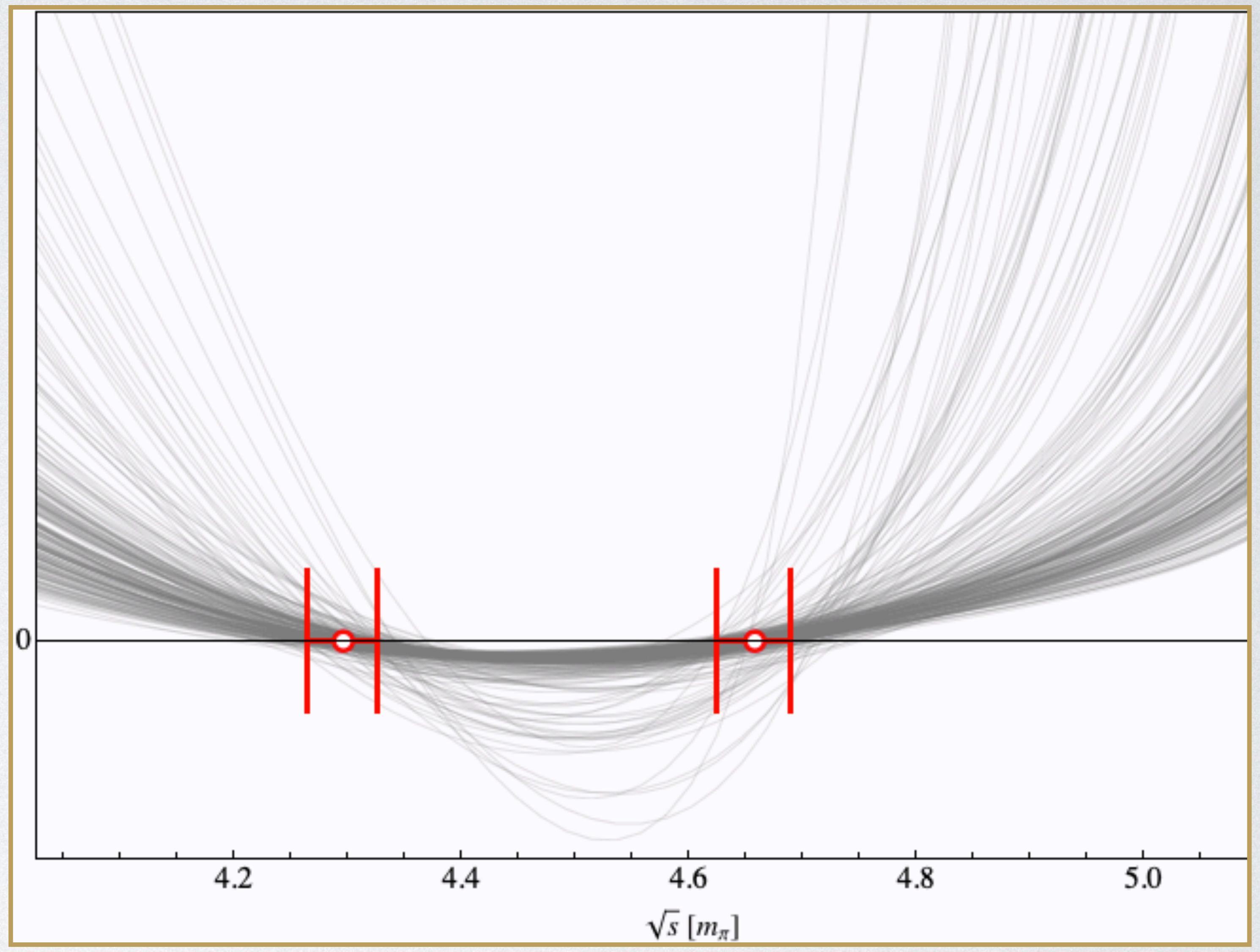
$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$





Our Universe





$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

Heavier Universe

M. Mai, AA, R. Brett, C. Culver, M. Doering, F. Lee, and D. Sadasivan, *Phys. Rev. Lett. 127 (2021), no. 22 222001*, [arXiv:2107.03973]

D. Sadasivan, AA, et al, *Phys. Rev. D 105 (2022), no. 5 054020*, [arXiv:2112.03355]

TAKE HOME

- Two-body and three-body (meson) spectra can be computed with high-precision from lattice QCD
- In the two-meson sector the phase-shifts and resonance parameters can be extracted reliably
- For the three meson sector we found that 3 kaons and 3 pions at maximal isospin the quantization conditions match lattice QCD results
- For 3-hadrons case more energy levels are needed to constrain the amplitudes than in two-body scattering
- For the three-pion case the contact term can be constrained using lattice QCD data (albeit poorly)
- Next challenge for lattice QCD is including resonant sub-channels (add both $\pi\sigma$ & $\pi\rho$ channels for a_1)
- The door is open towards studying three-body resonances