Nuclear short-range correlations with the Generalized Contact Formalism

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LA-UR-23-23577

Outline

- The Generalized Contact Formalism (GCF)

- Towards a systematic short-range expansion
  - Three-nucleon correlations
  - Next-order corrections

- Application: $0\nu\beta\beta$ matrix elements

The Generalized Contact Formalism (GCF)

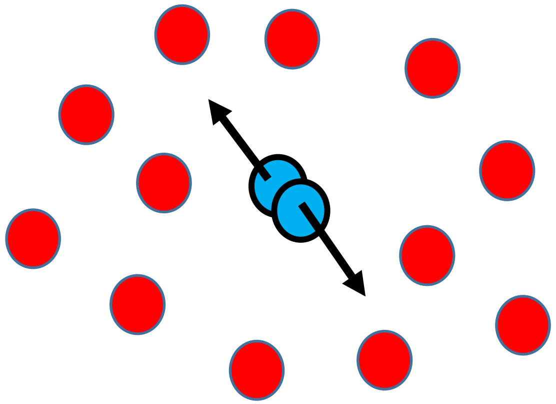
RW, B. Bazak, N. Barnea

Generalized Contact Formalism

- Generalizing Tan's work for atomic systems

S. Tan, Ann. Phys. (N.Y.) 323, 2952 (2008); Ann. Phys. (N.Y.) 323, 2971 (2008); Ann. Phys. (N.Y.) 323, 2987 (2008)

- Starting point – Short-range factorization



$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$

Universal function
(but depends on the potential)

Nucleus-dependent function

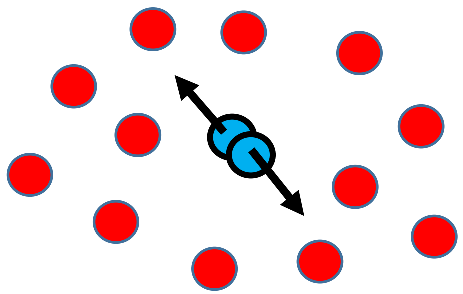
$\varphi(r) \equiv$ Zero-energy solution of the **two-body** Schrodinger Eq.

RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

Generalized Contact Formalism

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2})$$

universal function



For any **short-range** two-body operator $\hat{O}$

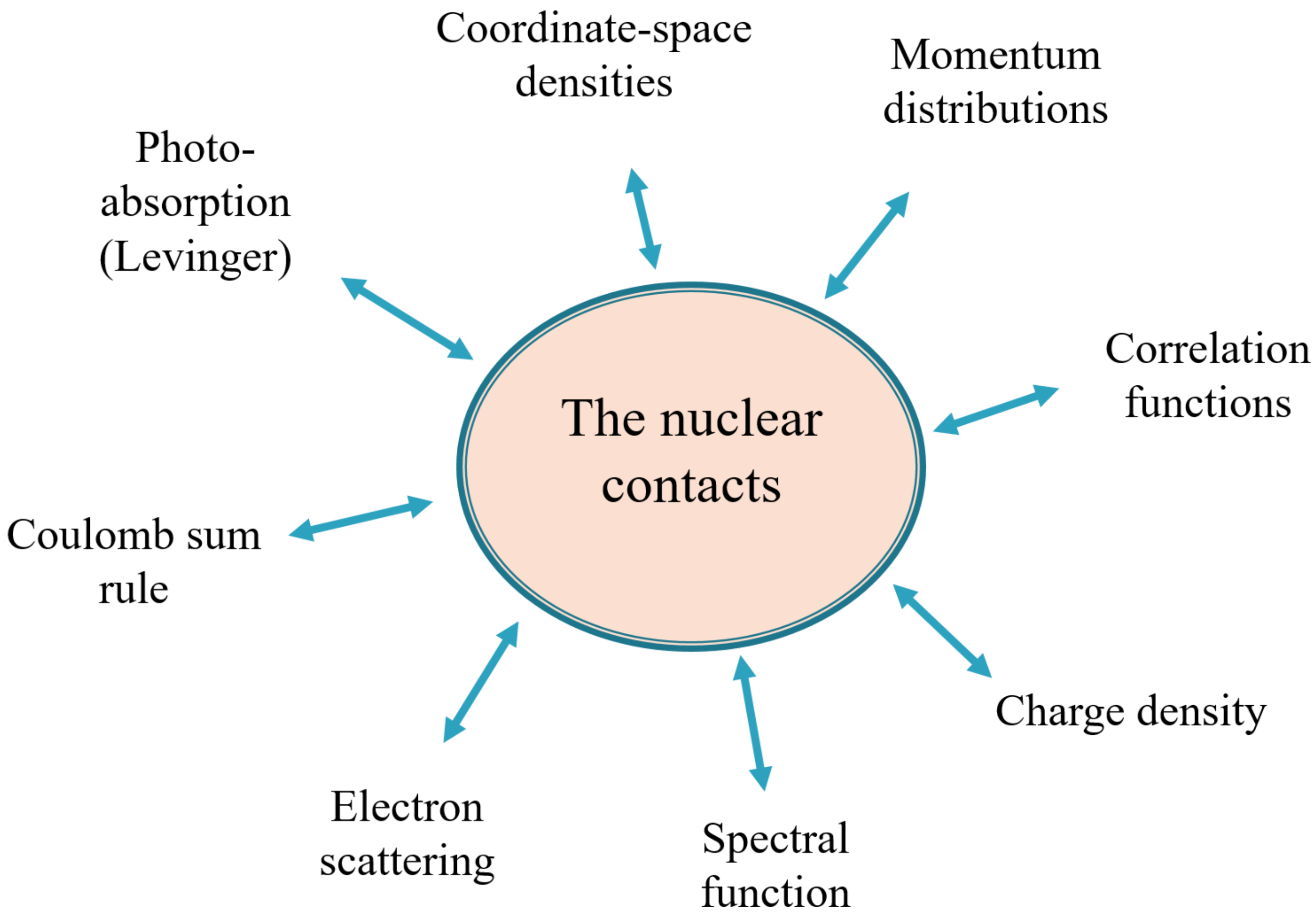
$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \, C \qquad C \propto \langle A | A \rangle$$

- Two-body dynamics
- Universal for all nuclei
- Simply calculated

- The "contact"
- Number of correlated pairs
- Depends on the nucleus
- Independent of the operator

RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

The nuclear contact relations



RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

RW, B. Bazak, N. Barnea, PRL 114, 012501 (2015)

RW, R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB 780, 211 (2018)

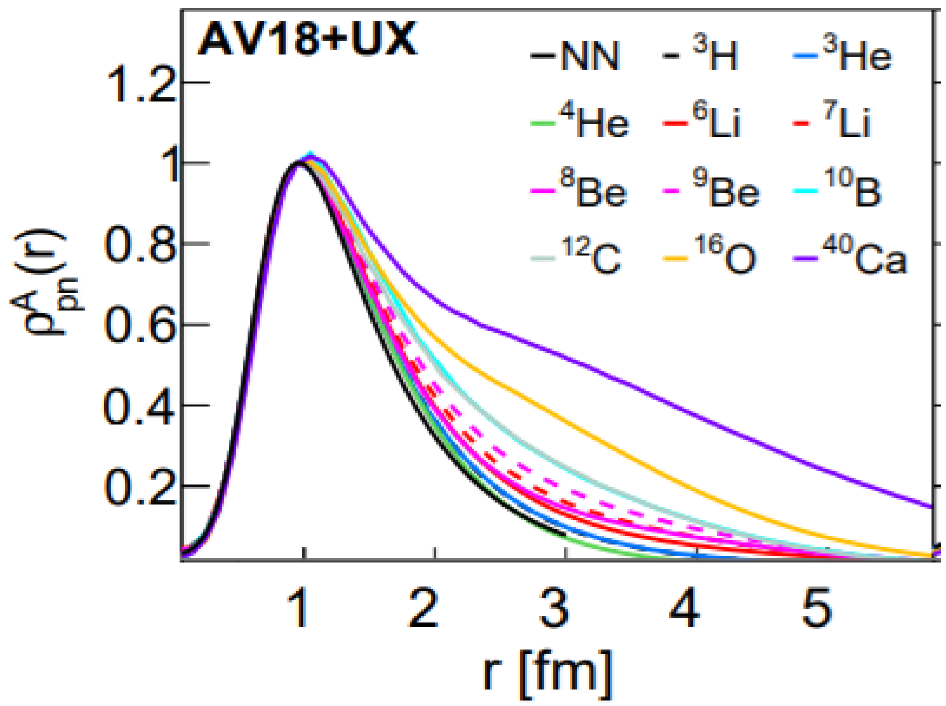
RW, I. Korover, E. Piasetzky, O. Hen and N. Barnea, PLB 791, 242 (2019)

R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)

A. Schmidt, J.R. Pybus, RW, et al., Nature 578, 540 (2020)

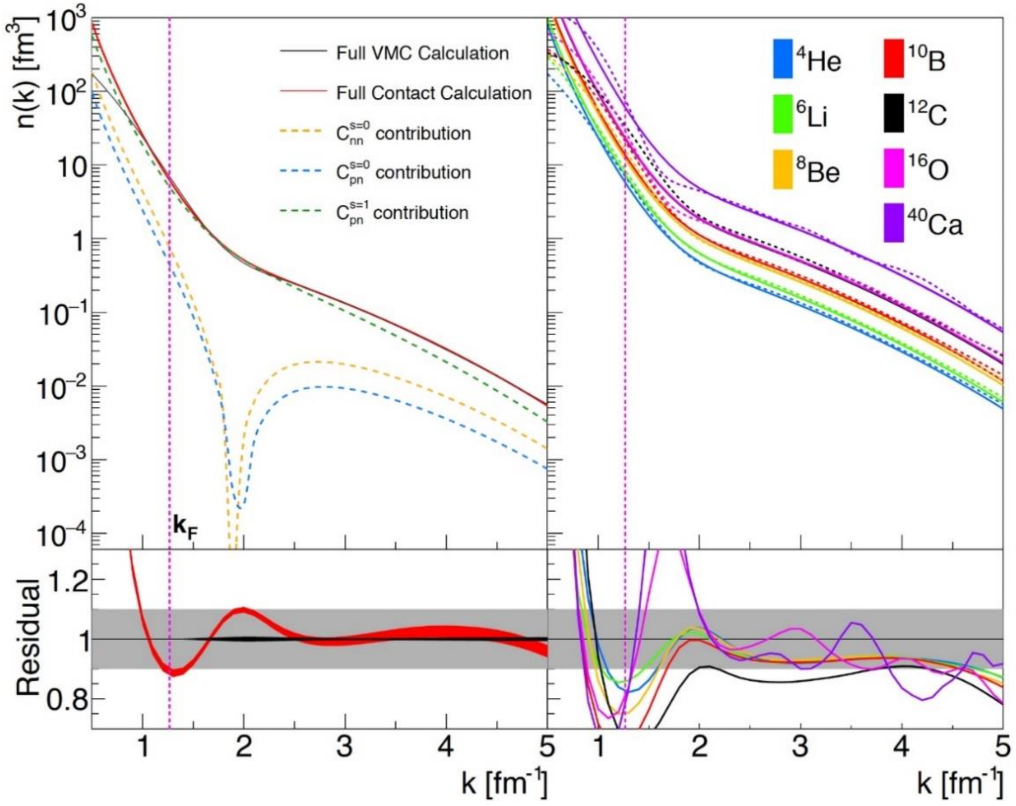
The nuclear contact relations

Two-body density



R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)

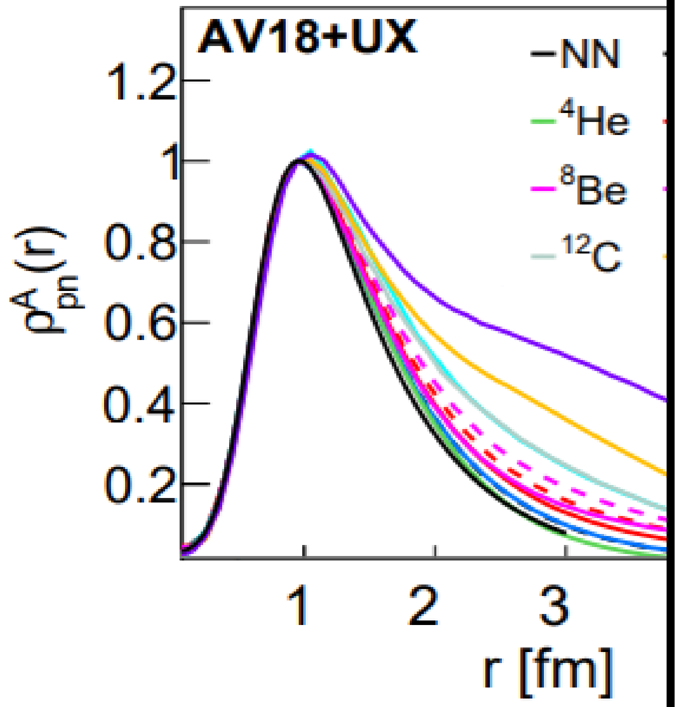
Momentum distribution



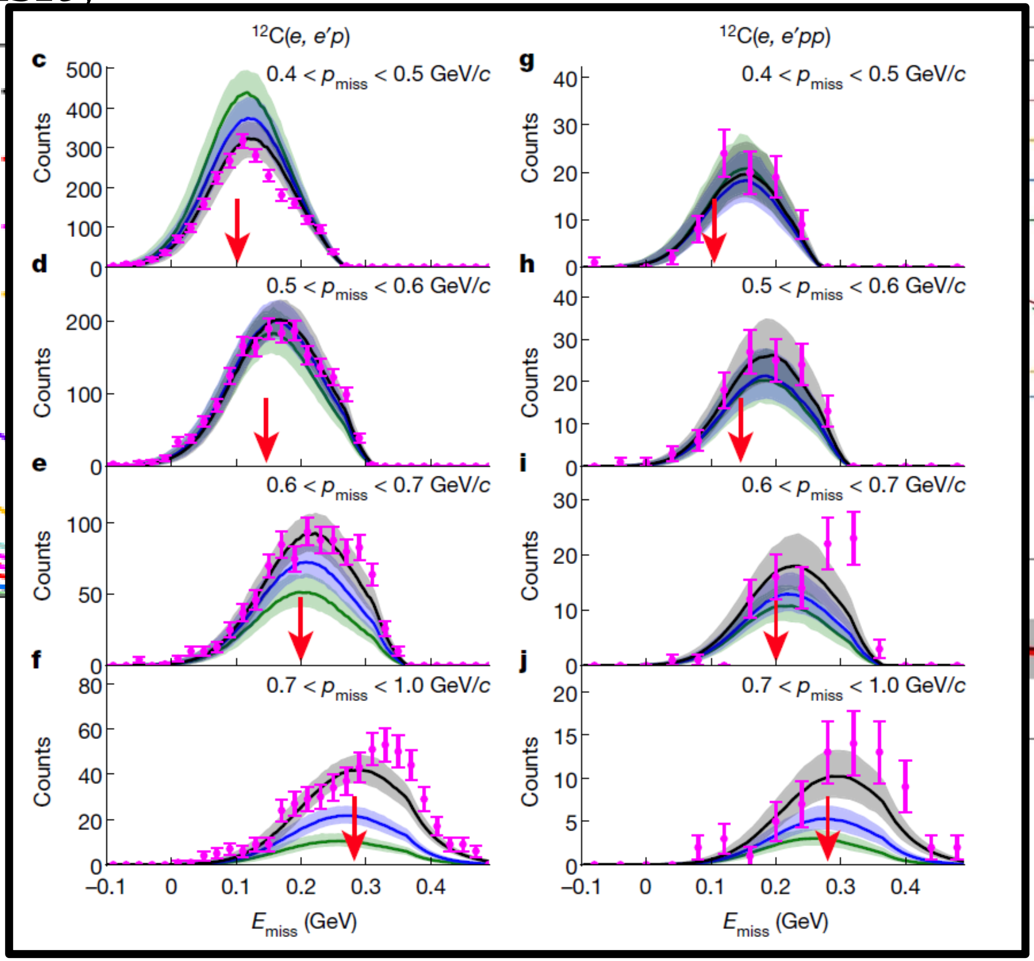
RW, R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB 780, 211 (2018)

The nuclear contact relations

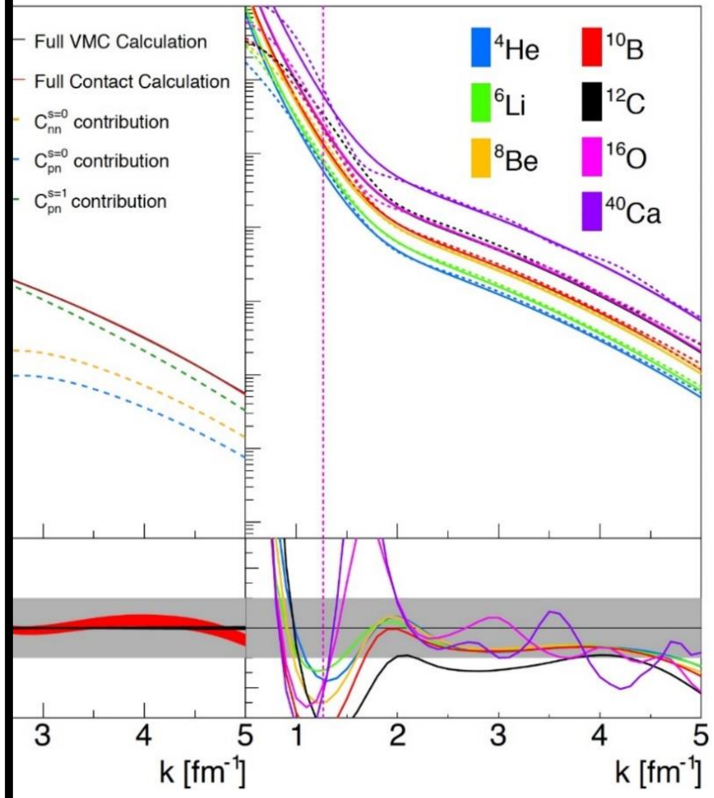
Two-body density



Electron scattering



Momentum distribution



A. Schmidt, J.R. Pybus, RW, E. P. Segarra, A. Hrnjic, A. Denniston, O. Hen, *et al. (CLAS collaboration), Nature 578, 540 (2020)*

Towards a systematic short-range expansion:

Corrections to the GCF

Corrections to the GCF

- GCF is based on the short-range factorization

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}\rightarrow 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$

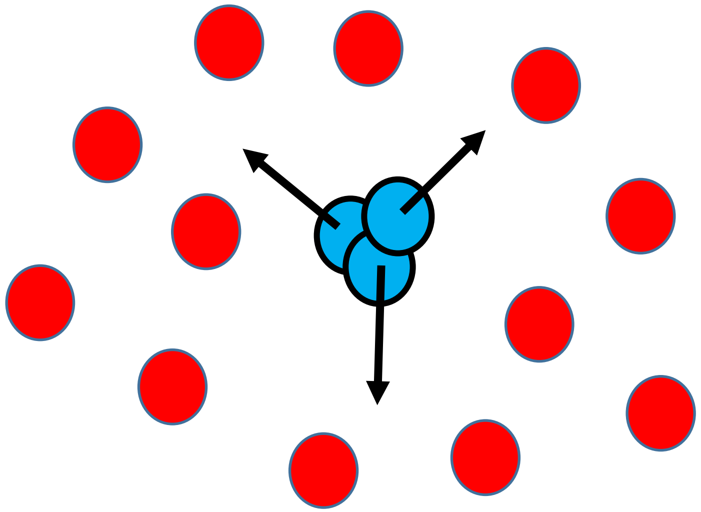
- Possible corrections:
  - Three-body correlations
  - Next-order terms in the description of the pair

Three-body correlations

RW and S. Gandolfi, arXiv:2301.09605 [nucl-th] (2023)

Three-body correlations

- There is no clear experimental signal of 3N SRCs
- No ab-initio calculations sensitive to 3N SRC features



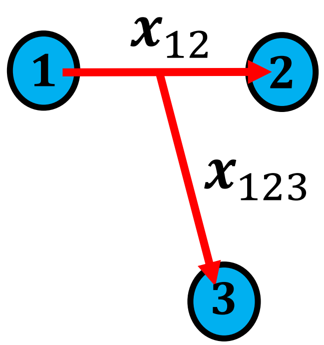
- Factorization?

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}, r_{13}, r_{23} \to 0} \varphi(\boldsymbol{r}_{12}, \boldsymbol{r}_{13}) \times B\left(\boldsymbol{R}_{123}, \{\boldsymbol{r}_k\}_{k \neq 1,2,3}\right)$$

?

universal function

Three-body correlations



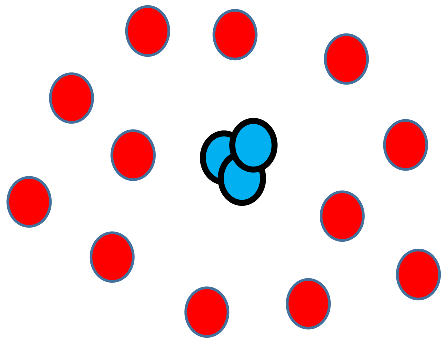
$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}, r_{13}, r_{23} \to 0} \varphi(\boldsymbol{x}_{12}, \boldsymbol{x}_{123}) \times B\left(\boldsymbol{R}_{123}, \{\boldsymbol{r}_k\}_{k \neq 1,2,3}\right)$$

- A **single** leading channel:

$$j^{\pi} = \frac{1}{2}^{+}, t = \frac{1}{2}$$

- The same quantum numbers as ^{3}He

- Therefore, **at short-distances** we expect:
  - $\boldsymbol{T = 1/2}$ **dominance** (over $T = 3/2$)
  - **Universality** - All nuclei should behave like ^{3}He

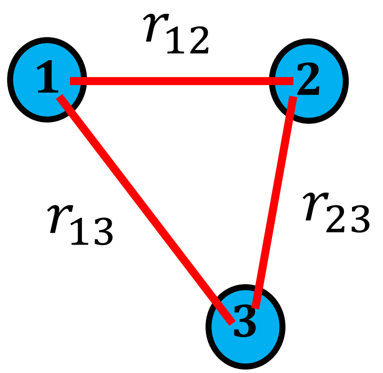


Three-body density

Ab-initio calculations – AFDMC (with Stefano Gandolfi):

$$\rho_3(r_{12}, r_{13}, r_{23}) \equiv \binom{A}{3} \langle \Psi | \delta(|\boldsymbol{r}_1 - \boldsymbol{r}_2| - r_{12}) \delta(|\boldsymbol{r}_1 - \boldsymbol{r}_3| - r_{13}) \delta(|\boldsymbol{r}_2 - \boldsymbol{r}_3| - r_{23}) | \Psi \rangle$$

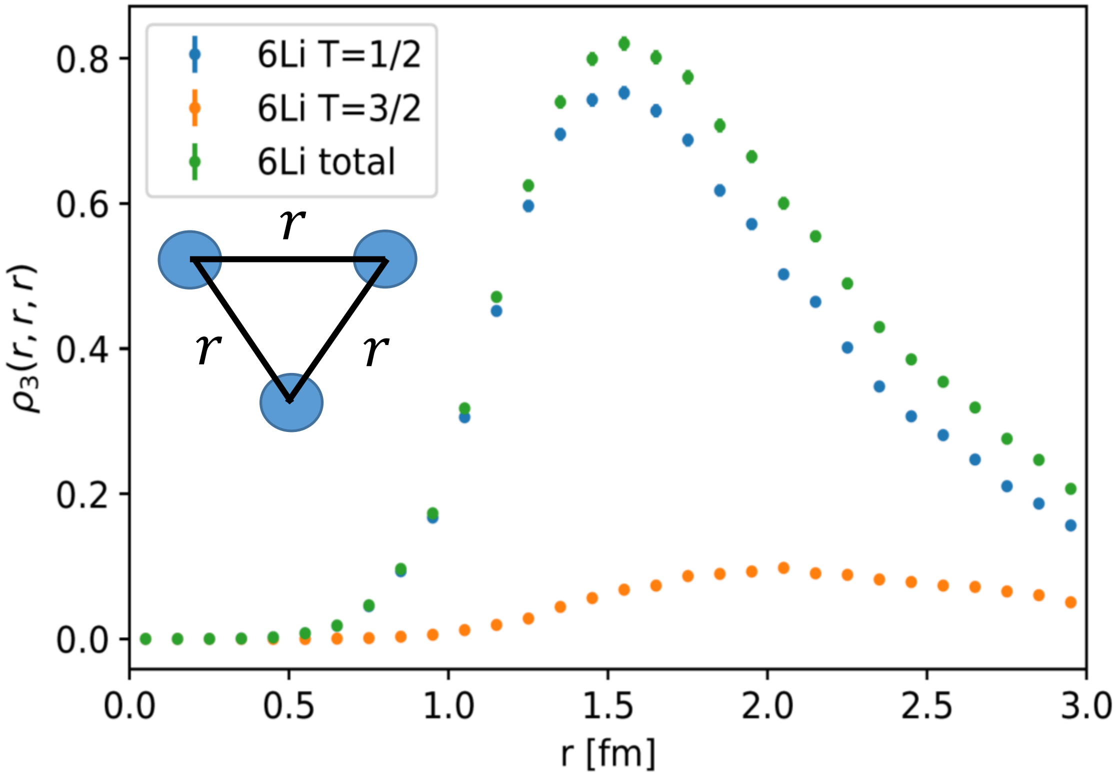
- Projections to $T = \frac{1}{2}$ and $T = \frac{3}{2}$
- N2LO($R = 1.0$ fm)E1 local chiral interaction
- Nuclei: ^{3}He, ^{4}He, ^{6}Li, , ^{16}O



Three-body density

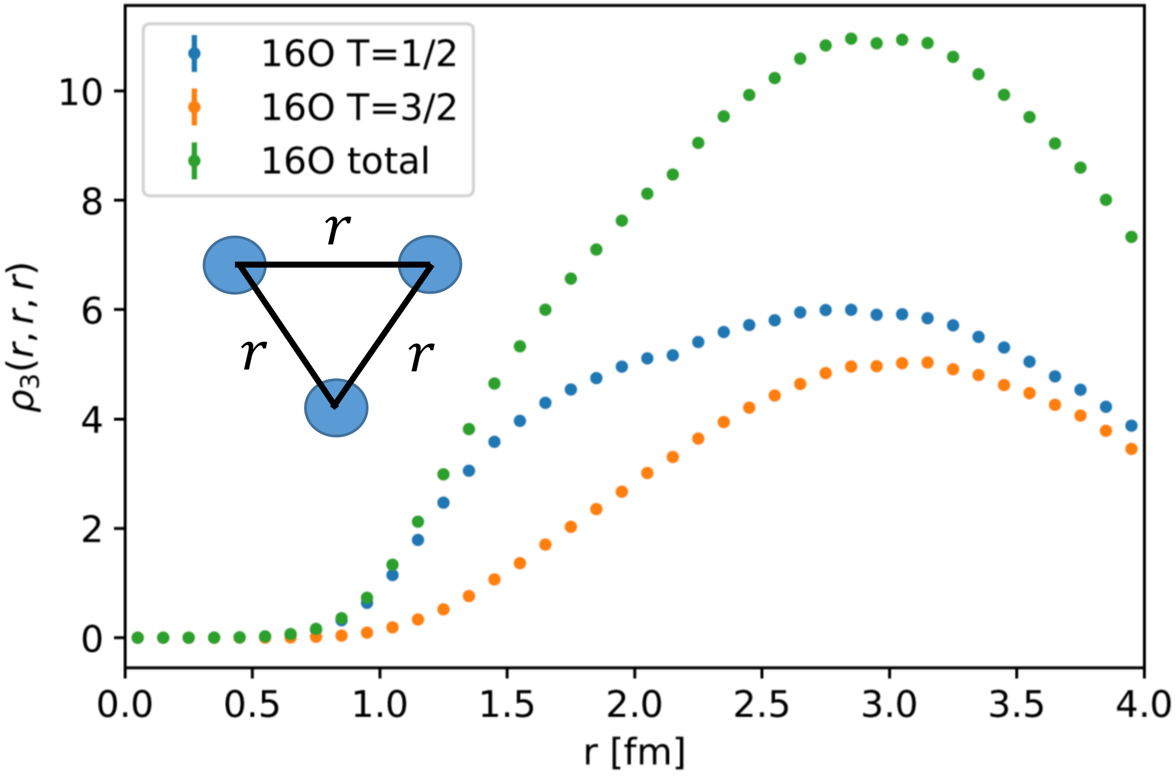
$$T = 1/2 \quad \text{vs} \quad T = 3/2$$

^{6}Li



Total number of triplets: $T = \frac{1}{2}: 16 \quad ; \quad T = \frac{3}{2}: 4$

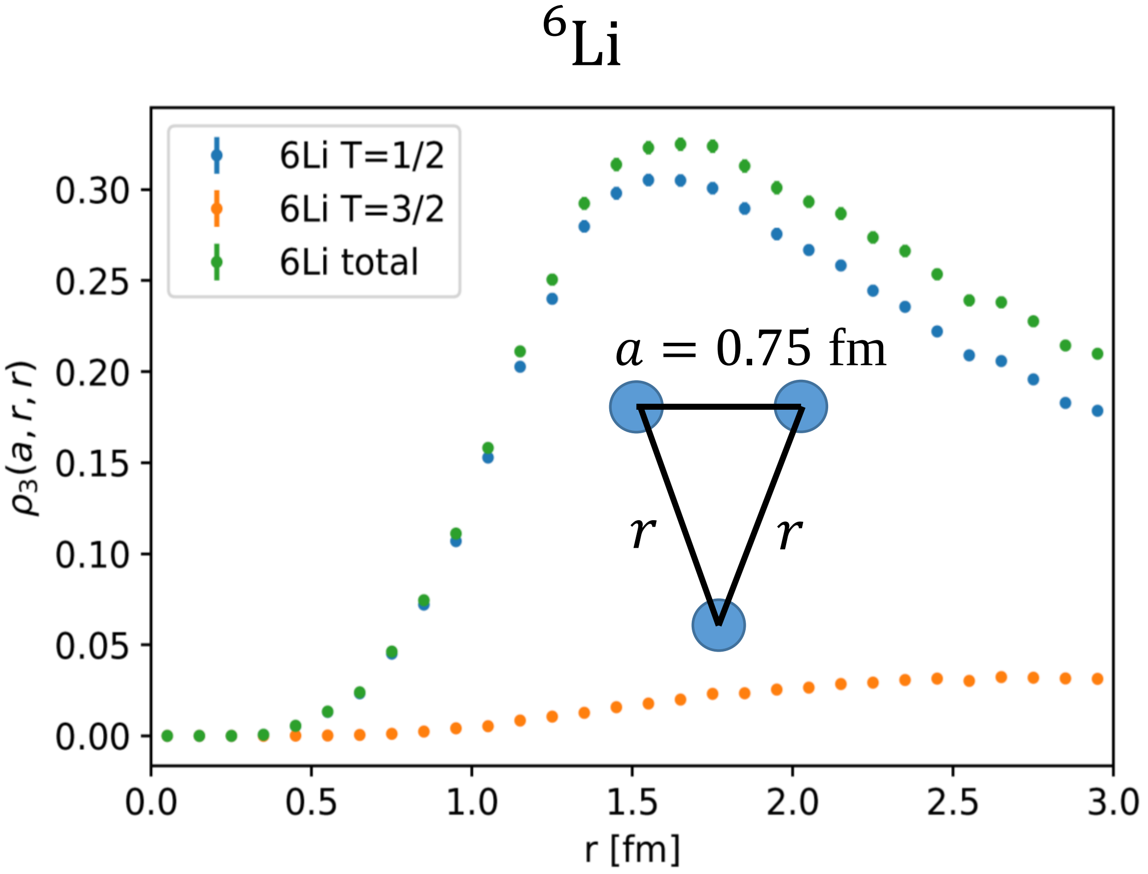
^{16}O



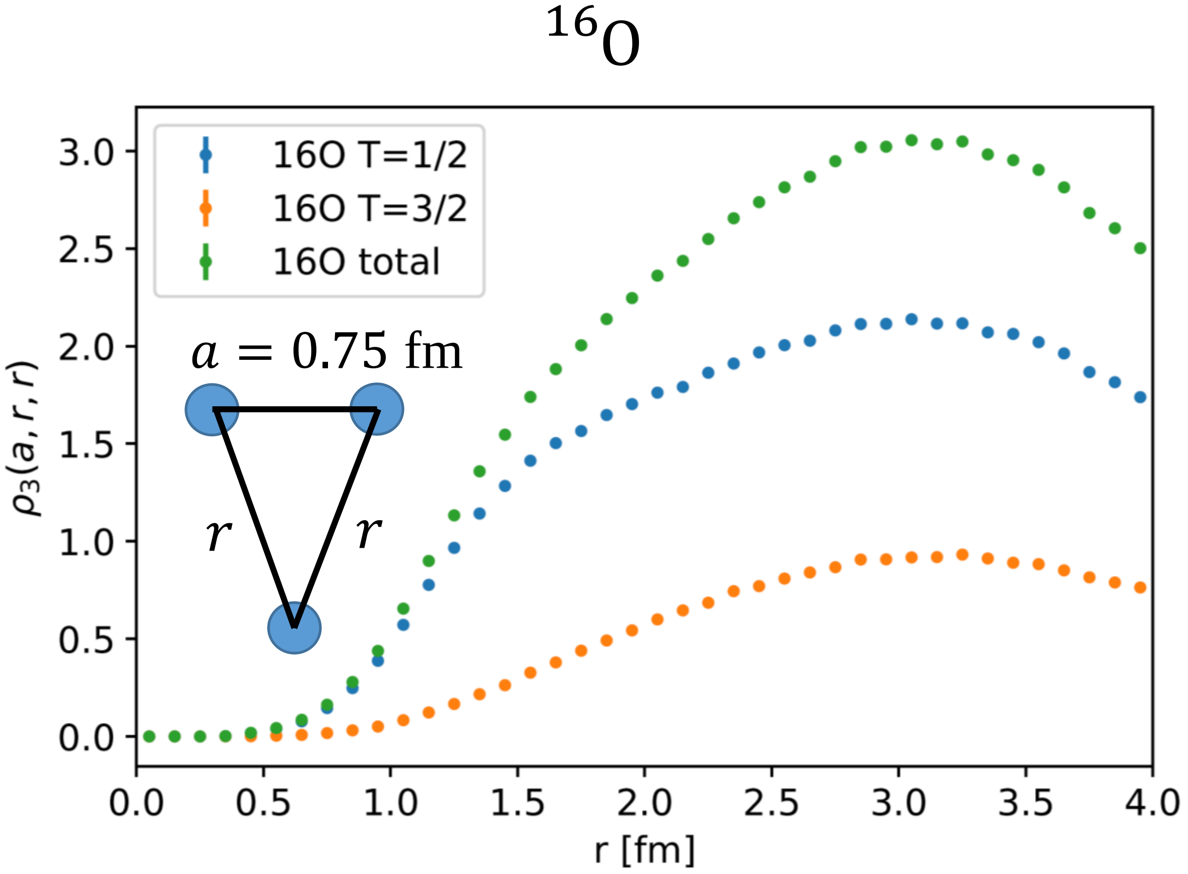
Total number of triplets: $T = \frac{1}{2}: 336 \quad ; \quad T = \frac{3}{2}: 224$

Three-body density

$T = 1/2 \quad \text{vs} \quad T = 3/2$



Total number of triplets: $T = \frac{1}{2}: 16 \quad ; \quad T = \frac{3}{2}: 4$



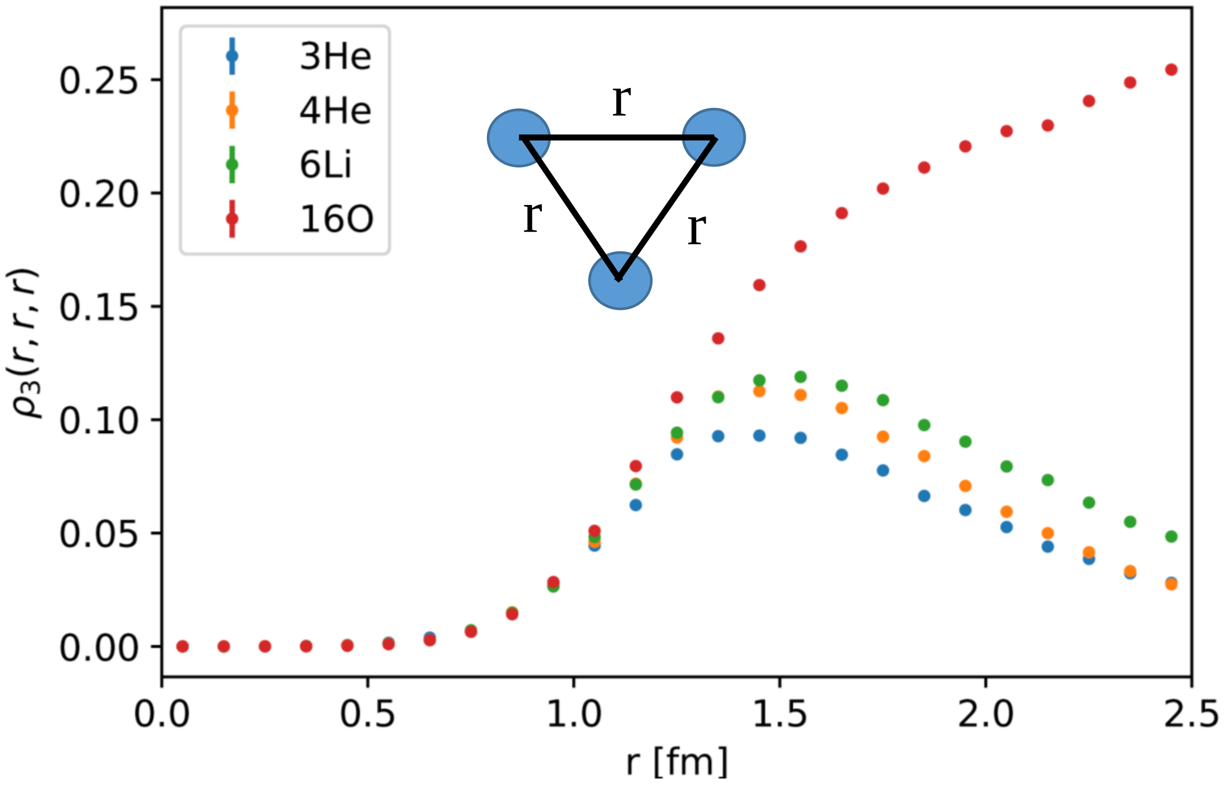
Total number of triplets: $T = \frac{1}{2}: 336 \quad ; \quad T = \frac{3}{2}: 224$

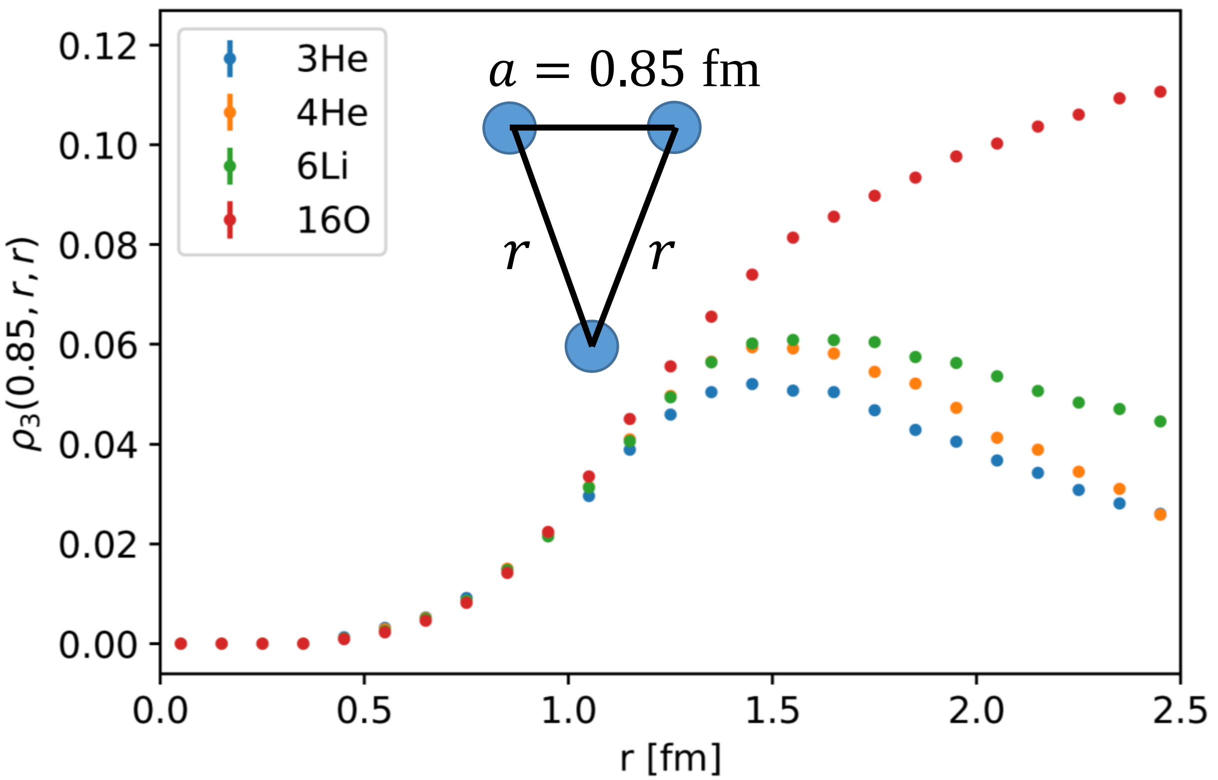
Three-body density

Same scaling factor for all geometries!

$$\boldsymbol{T = \frac{1}{2}}\ \textbf{universality:}$$

rescaled densities





Three-body contact values ($T = 1/2$)

$$\frac{C(^4\mathrm{He})}{C(^3\mathrm{He})} \times \frac{3}{4} = 3.8 \pm 0.3$$

$$\frac{C(^6\mathrm{Li})}{C(^3\mathrm{He})} \times \frac{3}{6} = 3.1 \pm 0.3$$

$$\frac{C(^{16}\mathrm{O})}{C(^3\mathrm{He})} \times \frac{3}{16} = 4.2 \pm 0.5$$

Three-body contact values ($T = 1/2$)

$$\frac{C(^4\mathrm{He})}{C(^3\mathrm{He})} \times \frac{3}{4} = 3.8 \pm 0.3 \qquad \frac{C(^6\mathrm{Li})}{C(^3\mathrm{He})} \times \frac{3}{6} = 3.1 \pm 0.3 \qquad \frac{C(^{16}\mathrm{O})}{C(^3\mathrm{He})} \times \frac{3}{16} = 4.2 \pm 0.5$$

Can be compared to inclusive cross section ratios (in the appropriate kinematics)

$$a_3(A) = \frac{3}{A} \frac{\sigma_{eA}}{(\sigma_{e\,^3\mathrm{He}} + \sigma_{e\,^3\mathrm{H}})/2}$$

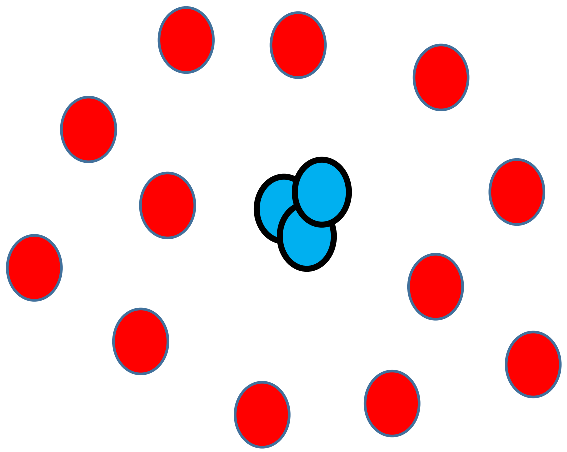
For a symmetric nucleus A

$$a_3(A) = \frac{3}{A} \frac{C(A)}{C(^3\mathrm{He})}$$

Three-body correlations

Future work:

- Dominant configurations
- Model dependence – Additional interactions
- Sensitivity to three-body force, tensor force
- Impact on momentum distributions
- Spectral function, electron scattering…



Subleading terms for SRC pairs:

Beyond factorization

RW et. al., in preparation

Short-range expansion

- Exact expansion:

GCF factorization — Next-order terms

$$\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\dots,\boldsymbol{r}_A)=\sum_\alpha \varphi_\alpha^{E=0}(\boldsymbol{r}_{12})A_\alpha^{(0)}+\sum_\alpha\left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)A_\alpha^{(1)}+\sum_\alpha\left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)A_\alpha^{(2)}+\cdots$$

- Two-body density:

$$\rho_2(r)=\sum_\alpha\left(\left|\varphi_\alpha^{E=0}(r)\right|^2 C_\alpha^{00}+\varphi_\alpha^{E=0}(r)\frac{d}{dE}\varphi_\alpha^{E=0}(r)C_\alpha^{01}+\cdots\right)$$

- Subleading contacts:

$$C_\alpha^{mn}\propto\langle A_\alpha^{(m)}|A_\alpha^{(n)}\rangle$$

Short-range expansion

$$\rho_2(r) = \sum_\alpha \left(\left|\varphi_\alpha^{E=0}(r)\right|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r) \frac{d}{dE} \varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$

- Power counting is needed

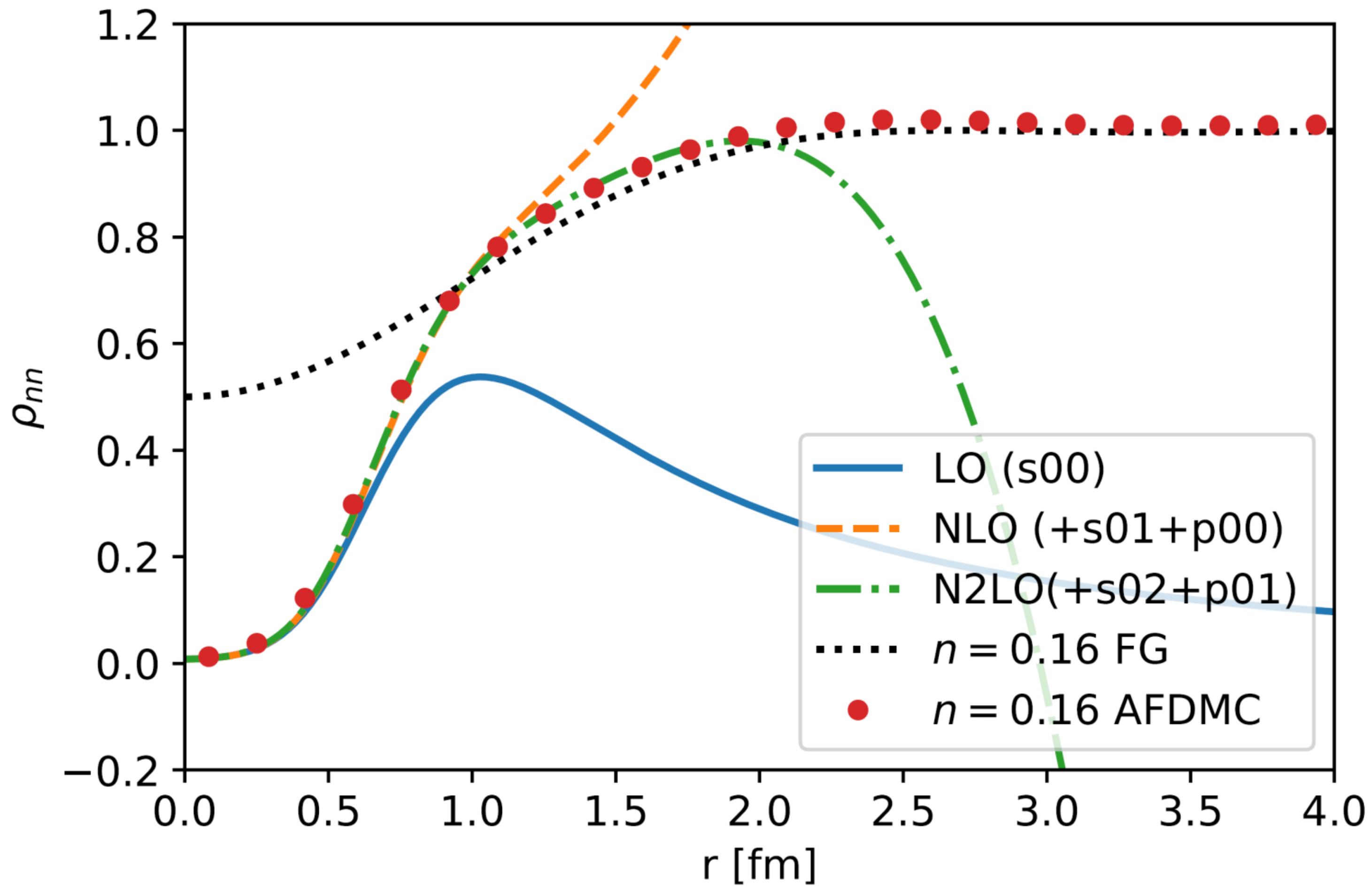
- Two relevant parameters:
  - Number of energy derivatives
  - Orbital angular momentum ($s, p, d, \ldots$)

Short-range expansion

$$\rho_2(r) = \sum_\alpha \left(\left|\varphi_\alpha^{E=0}(r)\right|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r) \frac{d}{dE} \varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$

- Neutron matter:

AFDMC by Diego Lonardoni & Stefano Gandolfi:
AV4' n = 0.16 fm^{-3}



$(S + \ell = \text{Even})$

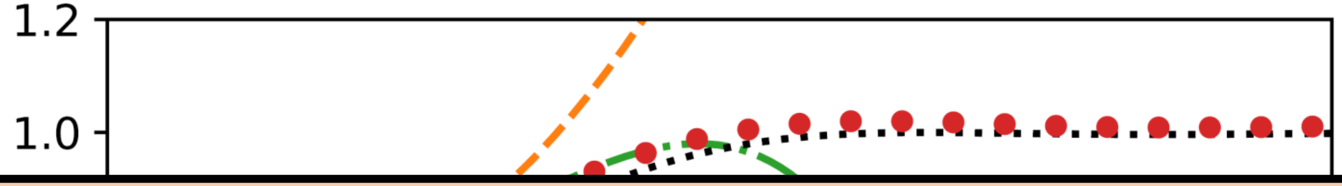
s-wave: $\ell = 0, S = 0, j = 0$

p-wave: $\ell = 1, S = 1, j = 0/1/2$

Short-range expansion

$$\rho_2(r) = \sum_\alpha \left(\left|\varphi_\alpha^{E=0}(r)\right|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r) \frac{d}{dE} \varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$

- Neutron matter:



AFDMC by D
& Stefan
AV4' n =

$(S + \ell = \text{Even})$

s-wave: $\ell = 0, S =$

p-wave: $\ell = 1, S = 1, J = 0/1/2$

- Good description over larger and larger distances
- Using only two-body wave-functions
- More quantities can be calculated!
- Subleading corrections to previous results
- Connecting SRC experiments to more general framework
- Motivate new experimental data (e.g., spin measurement)

Application: Neutrinoless double beta decay

RW, P. Soriano, A. Lovato, J. Menendez, R. B. Wiringa, PRC 106, 065501 (2022)

Neutrinoless double beta decay

$$nn \rightarrow pp + 2e$$

Measurement of the decay will provide information about:

- Majorana nature of neutrinos
- Matter dominance of the universe
- Neutrino mass
- …

Nuclear matrix elements (NMEs) are needed

$$^{76}\mathrm{Ge},\ ^{100}\mathrm{Mo},\ ^{130}\mathrm{Te},\ ^{136}\mathrm{Xe}$$

Our approach: GCF-SM method

Quantum Monte Carlo	+	Shell Model	+	Generalized Contact Formalism
Accurate solution for light nuclei		Long-range physics		Short-range physics

NMEs and transition densities

Light Majorana neutrino exchange mechanism

$$M^{0\nu} = \langle \Psi_f | O^{0\nu} | \Psi_i \rangle$$

$$O^{0\nu} = O_F^{0\nu} + O_{GT}^{0\nu} + O_T^{0\nu} + O_S^{0\nu}$$

V. Cirigliano, et. al., PRL 120, 202001 (2018)

$$M_\alpha^{0\nu} = \int_0^\infty dr\, \rho_\alpha^{0\nu}(r)$$

$r < 1$ fm

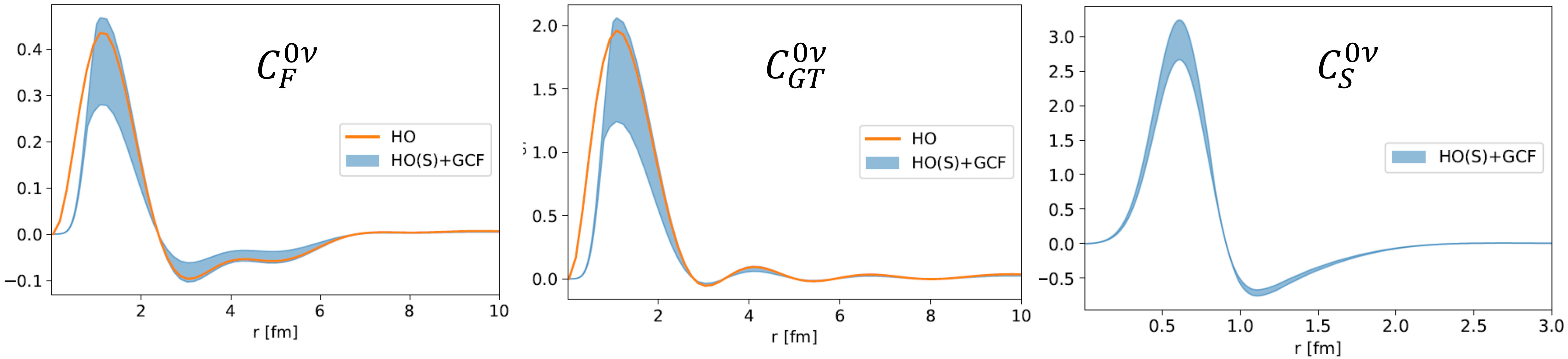
GCF

$r > 1$ fm

Shell model

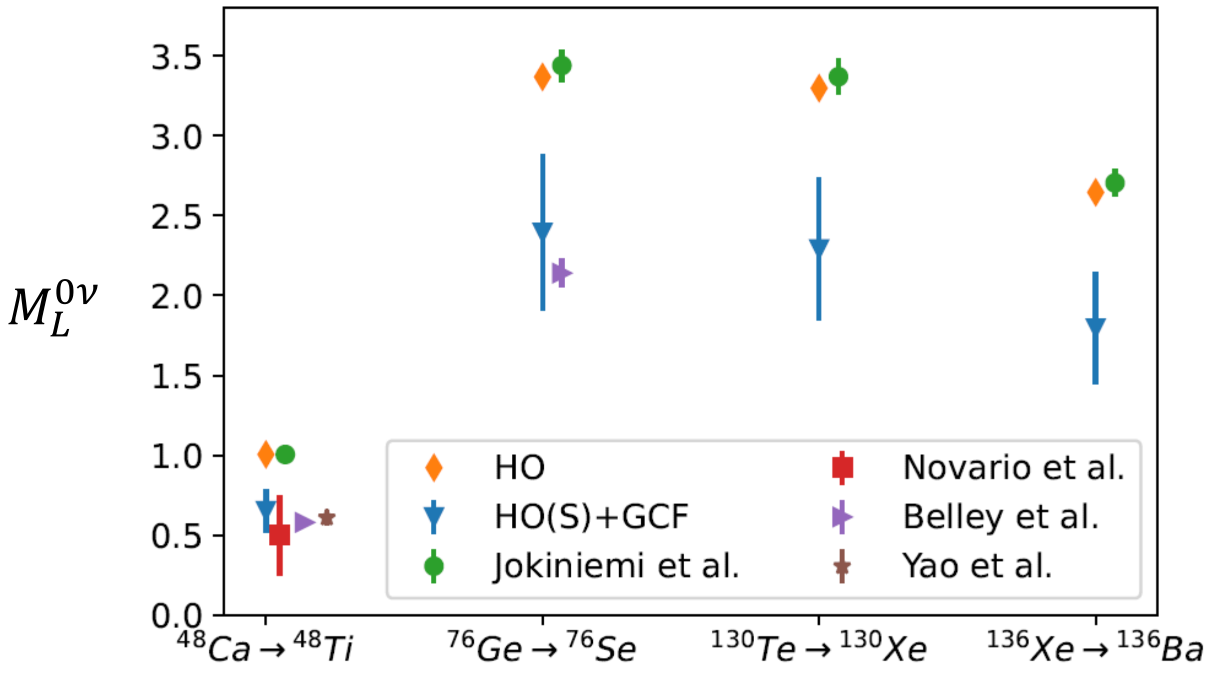
Results – heavy nuclei (AV18)

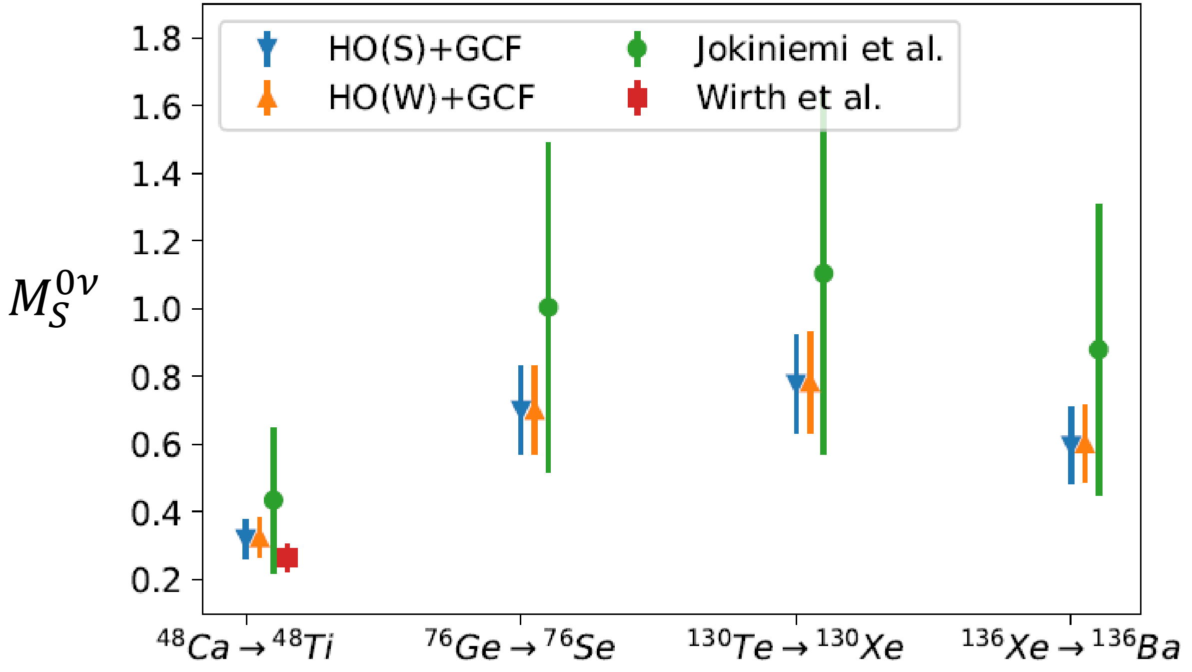
- Transition densities (using $A = 6, 10, 12$ to predict heavy nuclei):



$^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}$

Results – heavy nuclei (AV18)





$$M_F + M_{GT} + M_T$$

Significant reduction due to SRCs

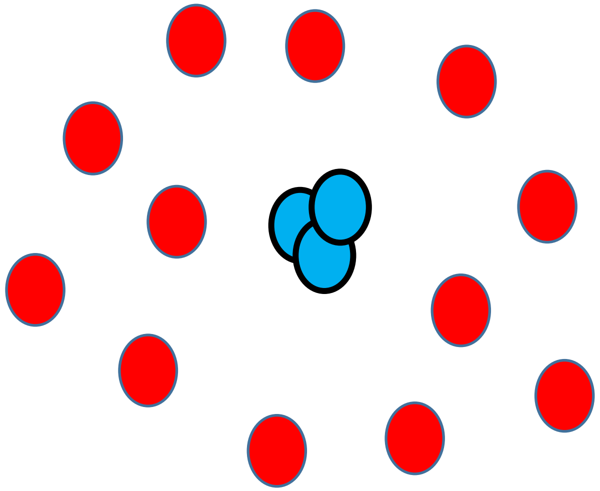
Summary

Summary

- Leading-order GCF provides **consistent and comprehensive description of short-range correlated pairs** $\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$

- **3N SRCs** – clear signal of correlated triplets
  - Wave function factorization
  - Single leading channel - $j^{\pi} = \frac{1}{2}^{+}, t = \frac{1}{2}$
  - Universal behavior of SRC triplets
  - Extracted scaling factors – 3N contact ratios

    Relevant for inclusive scattering (a_3)

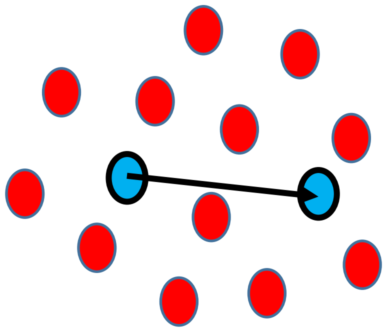


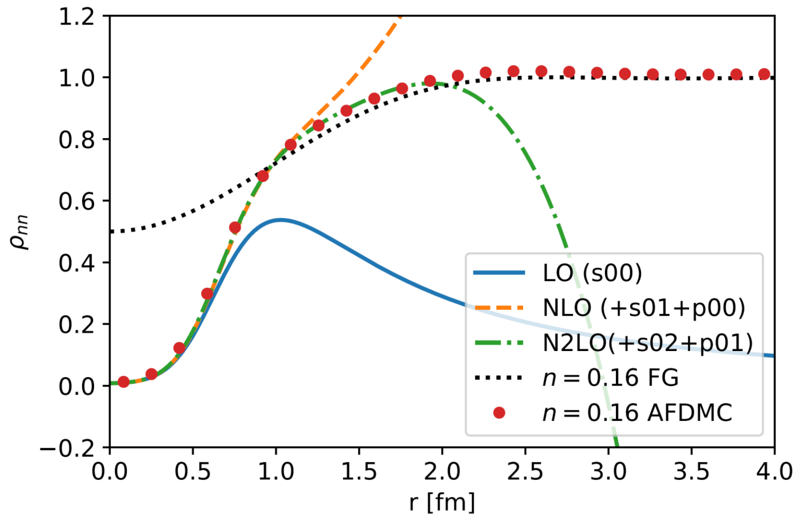
Summary

- **Short-range expansion** – subleading terms

$$\rho_2(r) = \sum_\alpha \left(\left|\varphi_\alpha^{E=0}(r)\right|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r) \frac{d}{dE} \varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$

  - Systematic expansion
  - Valid for larger distances
  - More observables can be described
  - Improved data analysis
  - Motivates new experiments





- Application: $0\nu\beta\beta$ NME

BACKUP

Generalized Contact Formalism

$$\Psi \xrightarrow{r_{ij}\to 0} \sum_\alpha \varphi_{ij}^\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij},\{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C_{ij}^{\alpha\beta}} \propto \langle A_{ij}^\alpha | A_{ij}^\beta \rangle$$

Channels $\alpha = (\ell_2 S_2) j_2 m_2$

Universal functions

The pair kind $ij \in \{pp, nn, pn\}$

3 matrices of Nuclear Contacts

Main channels:

The **deuteron** channel: $\ell_2 = 0,2\,;\, s_2 = 1\,;\, j_2 = 1\,;\, t_2 = 0$

The **spin-zero** channel: $\ell_2 = 0\,;\, s_2 = 0\,;\, j_2 = 0\,;\, t_2 = 1$

RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

Generalized Contact Formalism

$$\Psi \xrightarrow{r_{ij}\to 0} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C}_{\boldsymbol{ij}}^{\boldsymbol{\alpha\beta}} \propto \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

Channels α
$= (\ell_2 S_2) j_2 m_2$

Universal
functions

The pair kind
$ij \in \{pp, nn, pn\}$

3 matrices of
Nuclear Contacts

This factorized form can be derived using:

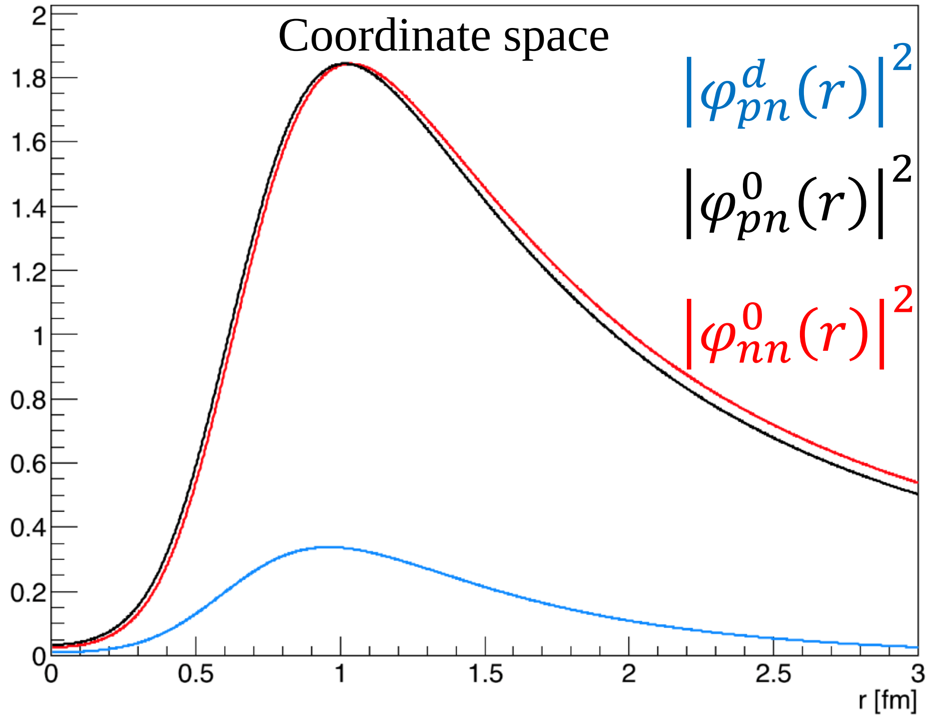
- RG arguments — S. K. Bogner and D. Roscher, Phys. Rev. C 86, 064304 (2012).
  A. J. Tropiano, S. K. Bogner, and R. J. Furnstahl, Phys. Rev. C 104, 034311 (2021)
- Coupled Cluster expansion — S. Beck, RW, N. Barnea, arXiv:2212.13412 [nucl-th] (2022)

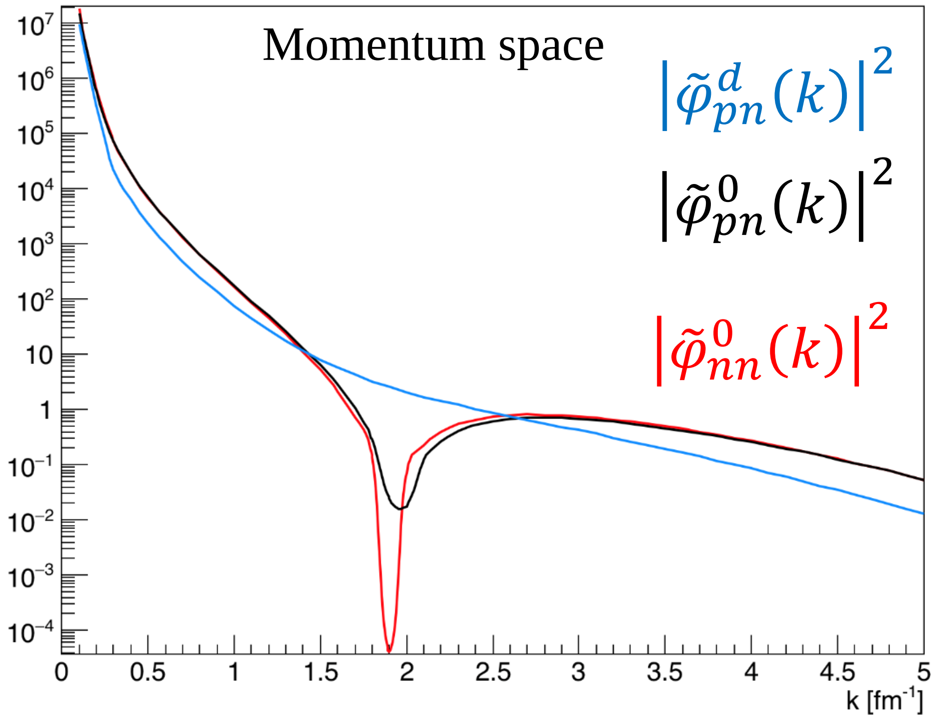
RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

Generalized Contact Formalism

$$\Psi \xrightarrow{r_{ij\to 0}} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C}_{\boldsymbol{ij}}^{\boldsymbol{\alpha\beta}} \propto \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

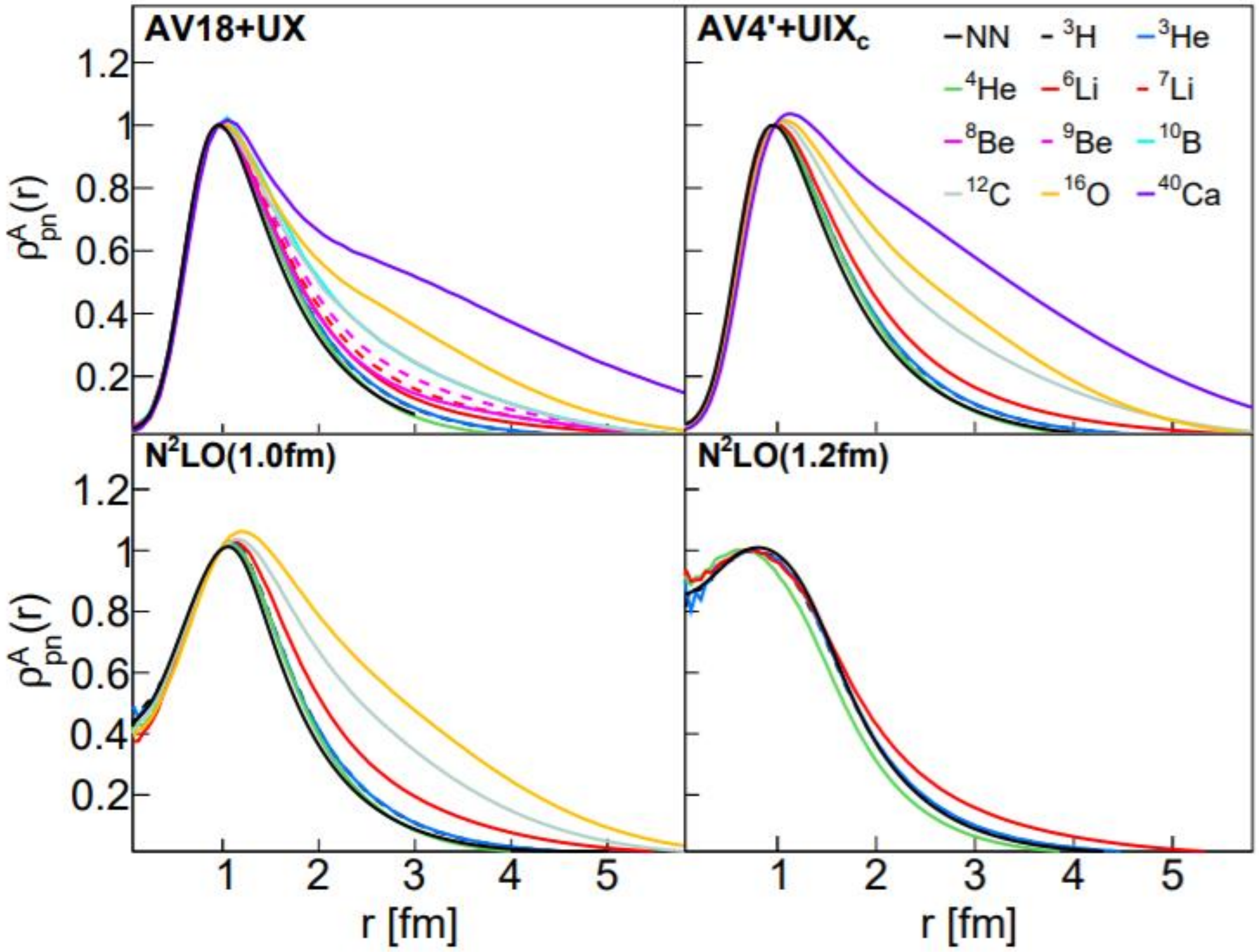
The universal functions using the AV18 potential





Two-body density

$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \, C$$



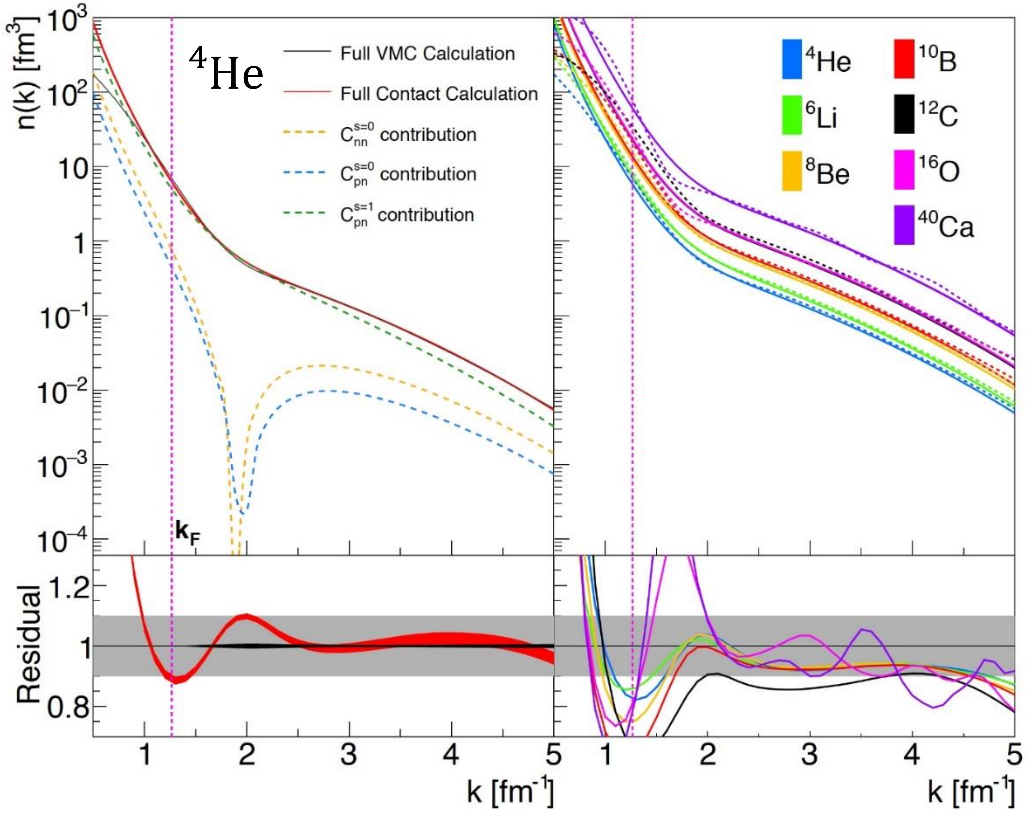
Shows the validity of the factorization

R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)

One-body momentum distribution

$$n_p(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^2 + 2C_{pp}^{0}\left|\varphi_{pp}^{0}(k)\right|^2$$

$n_p(k)$



No fitting parameters!

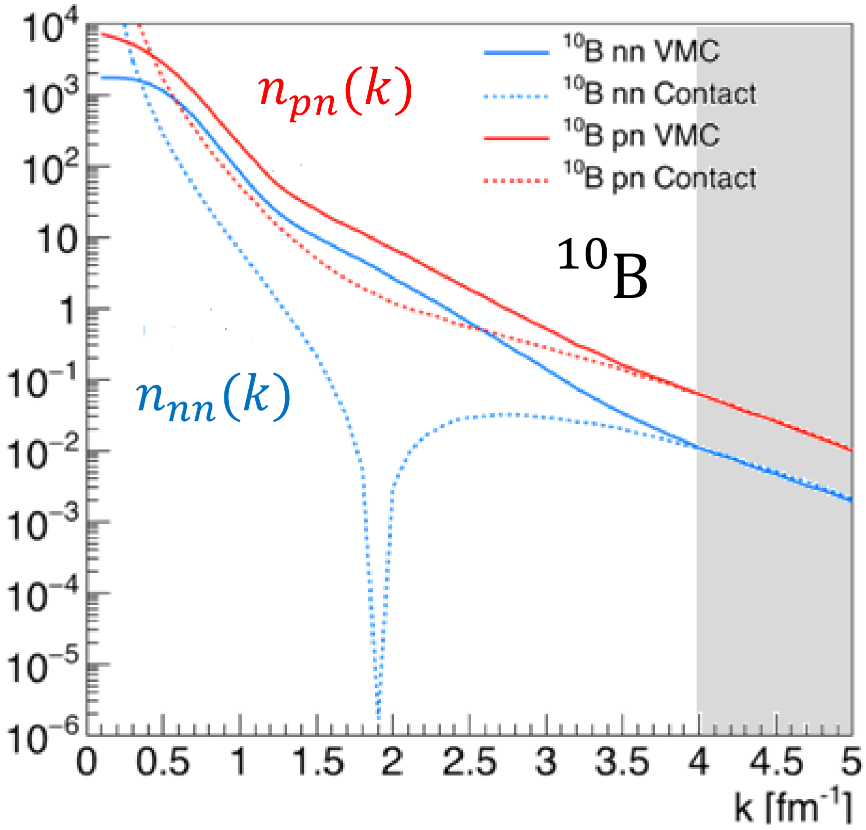
RW, R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB 780, 211 (2018)

Two-body momentum distribution

Relative momentum distribution

$$n_{pn}(k_{rel}) \xrightarrow[k\to\infty]{} C^d_{pn}\left|\varphi^d_{pn}(k_{rel})\right|^2 + C^0_{pn}\left|\varphi^0_{pn}(k_{rel})\right|^2$$

$$n_{nn}(k_{rel}) \xrightarrow[k\to\infty]{} C^0_{nn}\left|\varphi^0_{nn}(k_{rel})\right|^2$$

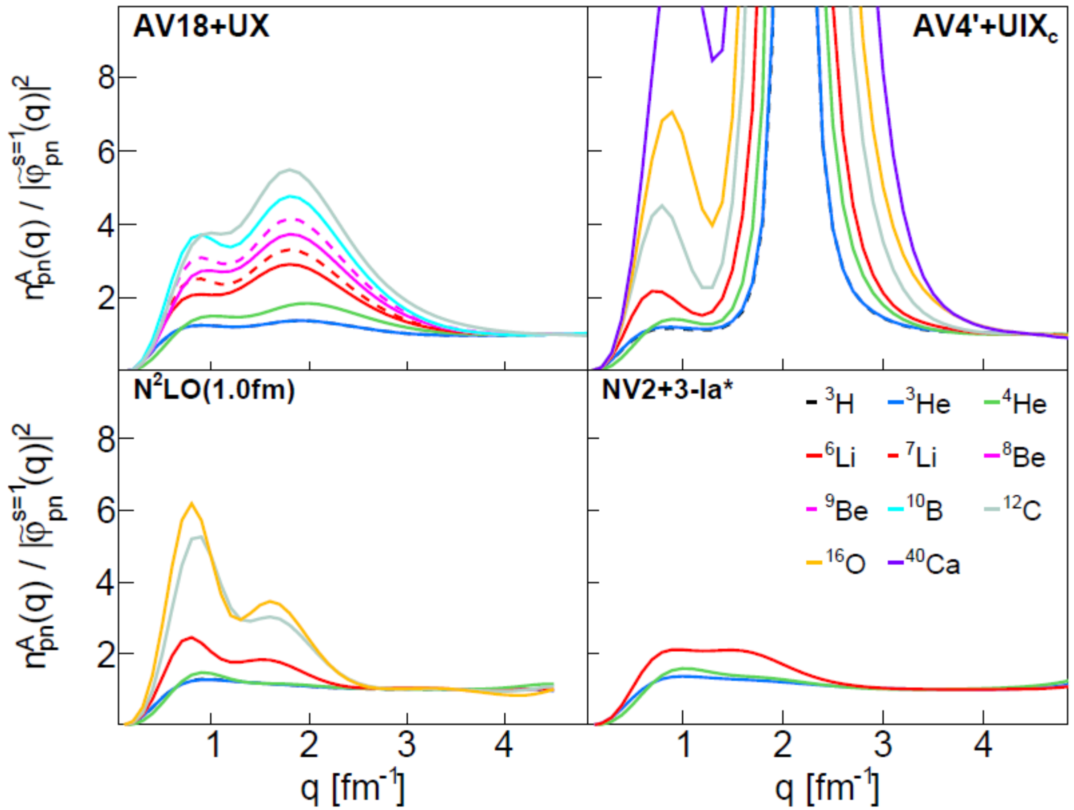


Two-body momentum distribution

Relative momentum distribution

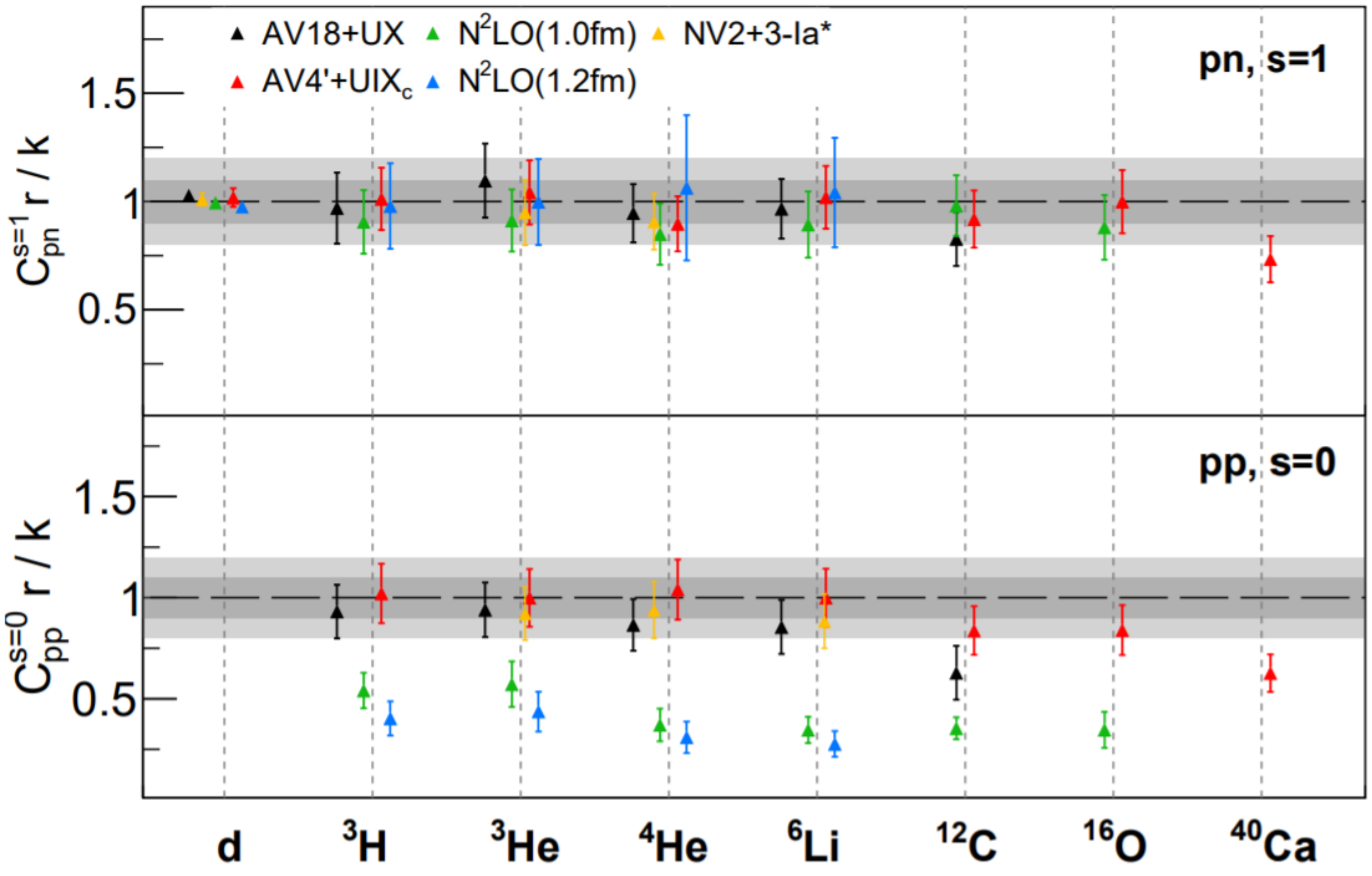
$$n_{pn}(k_{rel}) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k_{rel})\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k_{rel})\right|^2$$

$$n_{nn}(k_{rel}) \xrightarrow[k\to\infty]{} C_{nn}^{0}\left|\varphi_{nn}^{0}(k_{rel})\right|^2$$



R. Cruz-Torres, D. Lonardoni, RW, et al., arXiv: 1907.03658 [nucl-th], Nature physics (2020)

Consistency: k-space vs r-space



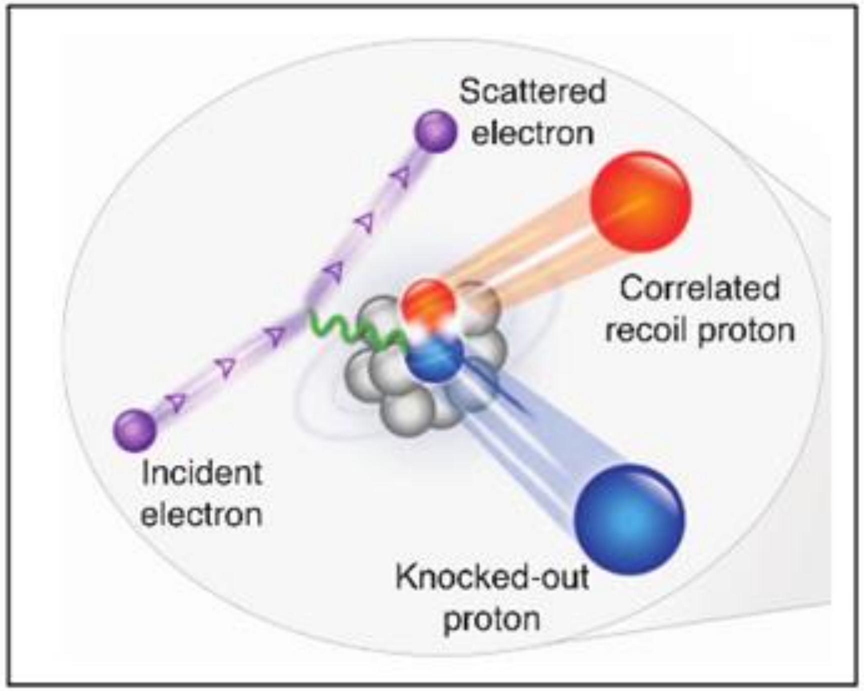
R. Cruz-Torres, D. Lonardoni, RW, et al., arXiv: 1907.03658 [nucl-th], Nature physics (2020)

Electron-scattering experiments

- $A(e,e'N)$ and $A(e,e'NN)$ cross sections

- In PWIA - described using the spectral function (the probability to find nucleon with momentum $\boldsymbol{p}_1$ and energy ϵ_1 in the nucleus)

- Using the GCF:



$$S^p(\boldsymbol{p_1},\epsilon_1) = C_{pn}^1 S_{pn}^1(\boldsymbol{p_1},\epsilon_1) + C_{pn}^0 S_{pn}^0(\boldsymbol{p_1},\epsilon_1) + 2C_{pp}^0 S_{pp}^0(\boldsymbol{p_1},\epsilon_1)$$

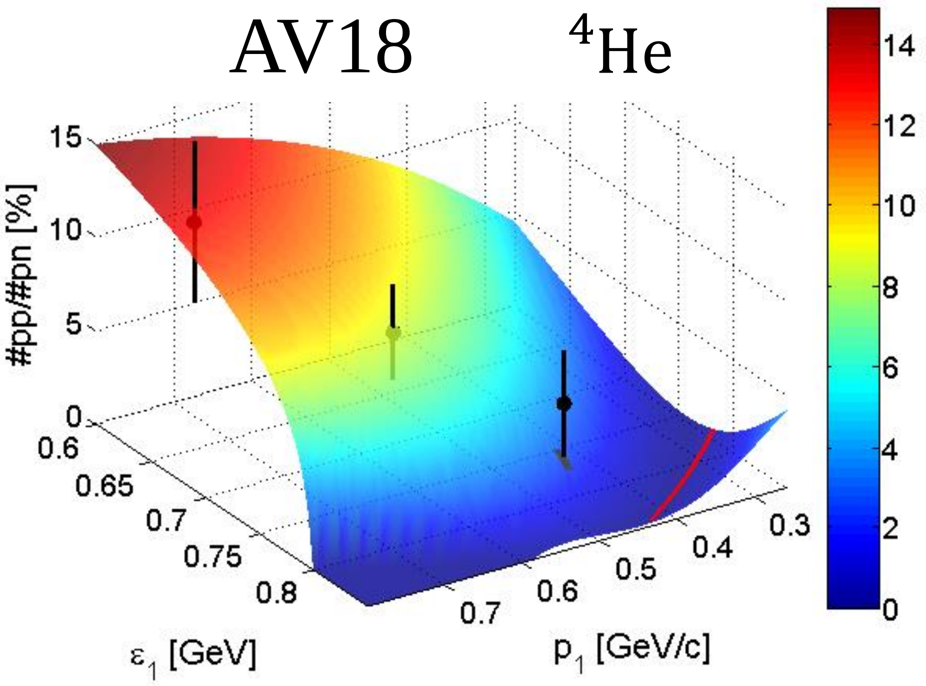
$(p_1 > k_F)$

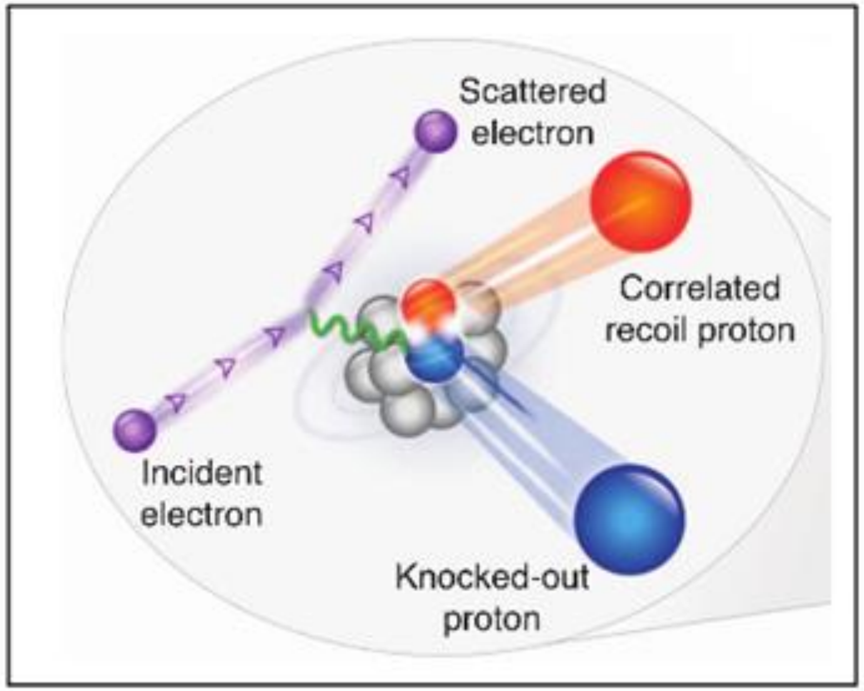
RW, I. Korover, E. Piasetzky, O. Hen and N. Barnea, PLB 791, 242 (2019)

Electron-scattering experiments

$$\frac{\#pp}{\#pn} = \frac{S^0_{pp}(p_1,\epsilon_1)}{\frac{\boldsymbol{C^1_{pn}}}{\boldsymbol{C^0_{pp}}} S^1_{pn}(p_1,\epsilon_1) + S^0_{pn}(p_1,\epsilon_1)}$$

AV18 ^{4}He





$$\frac{\boldsymbol{C^d_{pn}}}{\boldsymbol{C^0_{pp}}}(\,^4He) = 20 \pm 5$$

Previous results

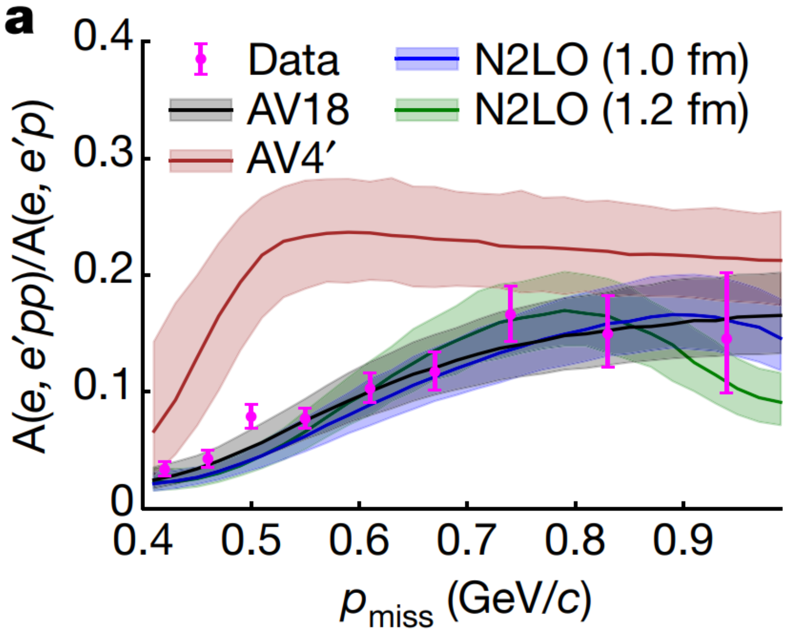
$$\frac{C^d_{pn}}{C^0_{pp}}(\,^4He) = 17 - 21$$

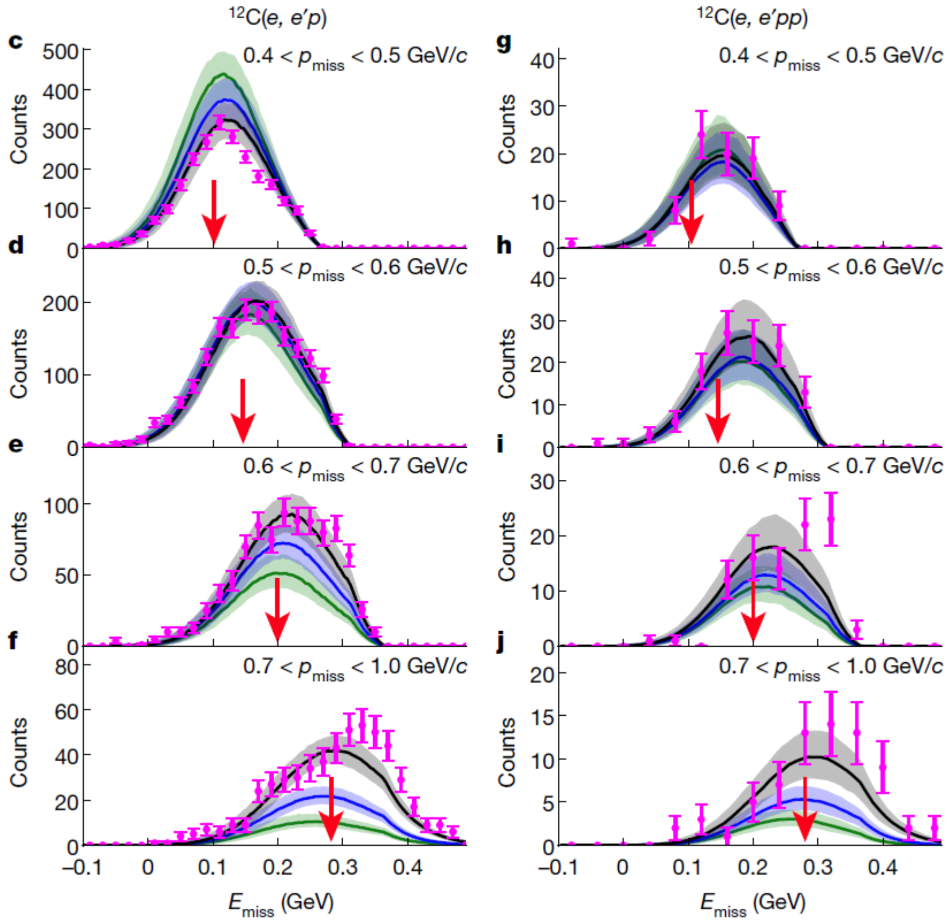
RW, I. Korover, E. Piasetzky, O. Hen and N. Barnea, PLB 791, 242 (2019)

Data from: *PRL 113, 022501 (2014)*

Electron-scattering experiments

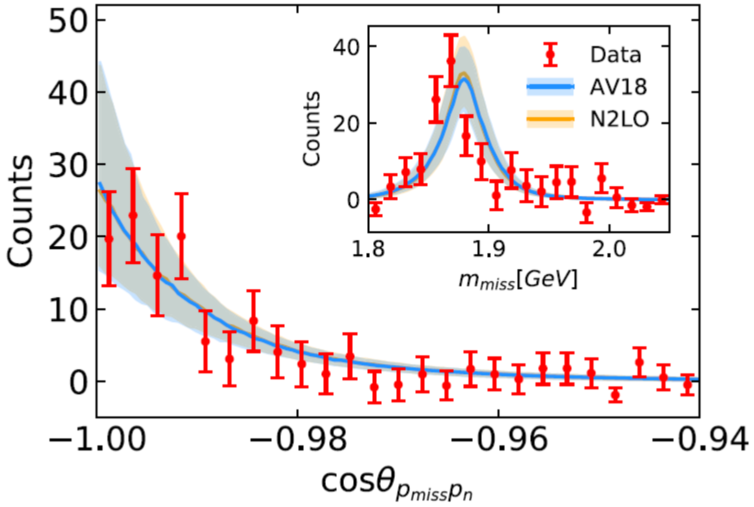
- Good description of experimental data:

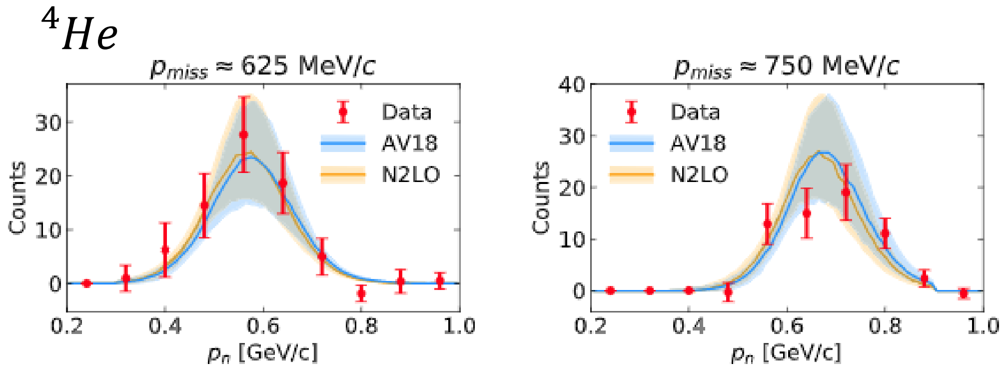




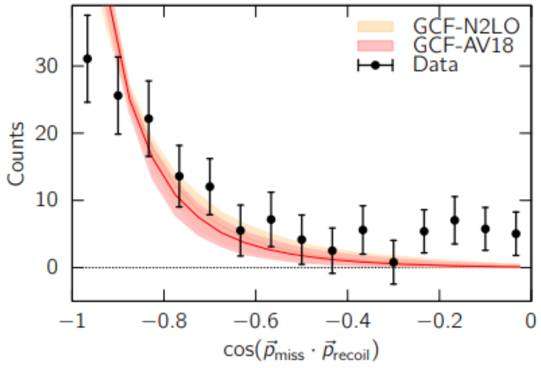
A. Schmidt, J.R. Pybus, RW, E. P. Segarra, A. Hrnjic, A. Denniston, O. Hen, *et al. (CLAS collaboration), Nature 578, 540 (2020)*

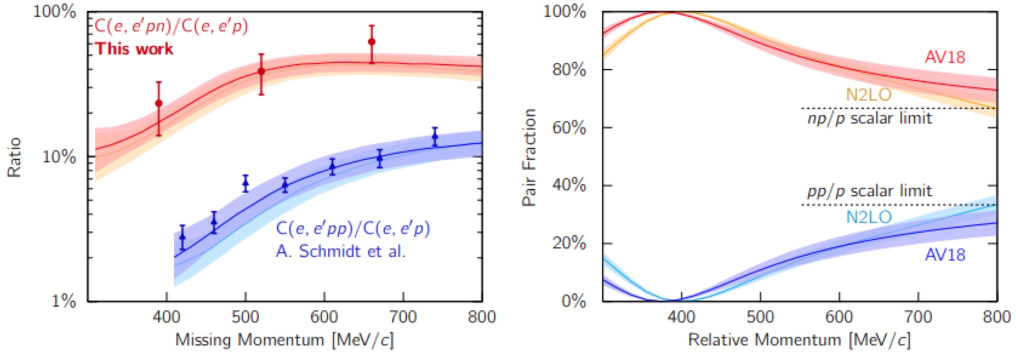
Electron-scattering experiments





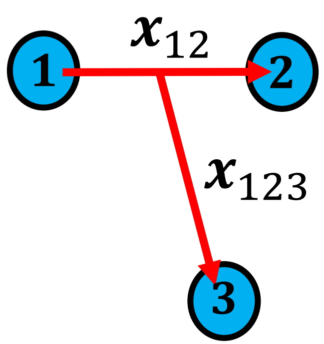
J.R. Pybus et al., PLB 805, 135429 (2020)





I. Korover et al., arXiv:2004.07304 (2020)

Three-body correlations



$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}, r_{13}, r_{23} \to 0} \varphi(\boldsymbol{x}_{12}, \boldsymbol{x}_{123}) \times B\left(\boldsymbol{R}_{123}, \{\boldsymbol{r}_k\}_{k \neq 1,2,3}\right)$$

Three-body wave functions – Quantum numbers: π, j, m, t, t_z

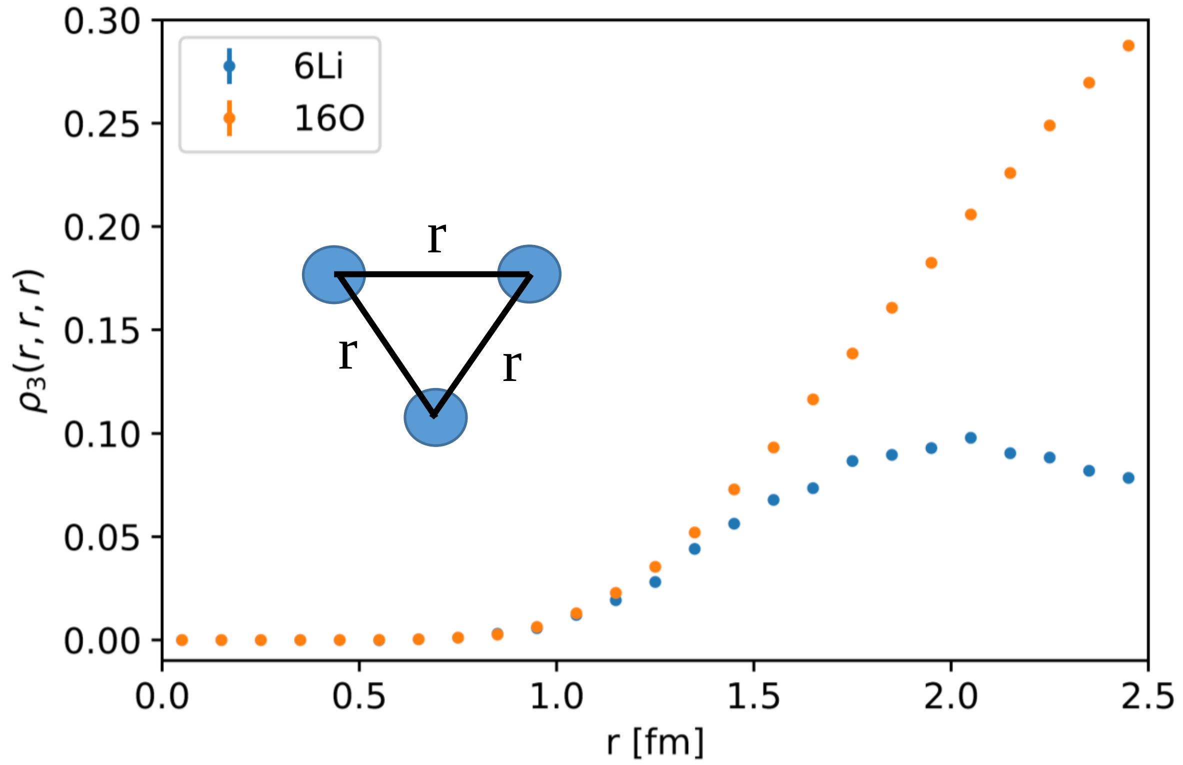
- S-wave dominance at short distances $\ell = 0$ ⟹ $\boxed{\pi = +}$
- Spin $S = \frac{1}{2}, \frac{3}{2}$ + $\ell = 0$ ⟹ $j = \frac{1}{2}, \frac{3}{2}$
- Isospin $t = \frac{3}{2}$ (symmetric function) – suppressed due to Pauli blocking ⟹ $\boxed{t = 1/2}$
- Spin $S = \frac{3}{2}$ (symmetric function) – suppressed due to Pauli blocking ⟹ $\boxed{j = 1/2}$

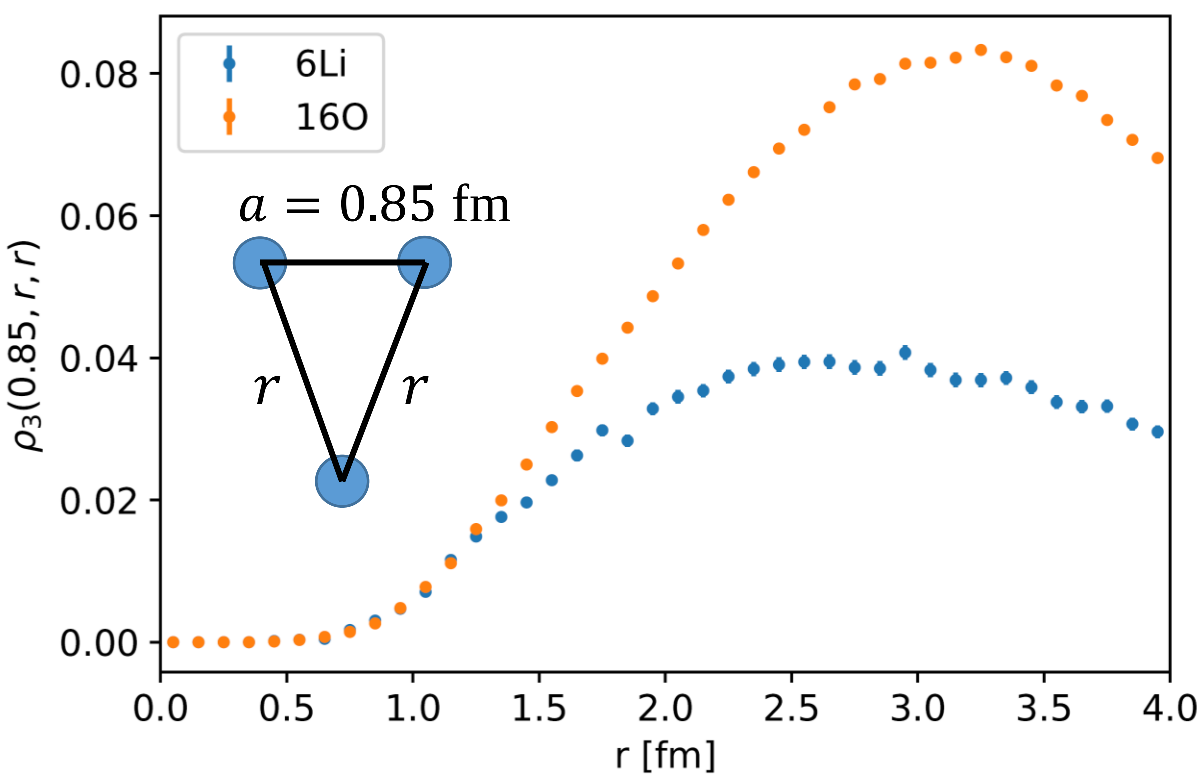
Three-body density

Same scaling factor for all geometries!

$$T = \frac{3}{2} \textbf{ universality:}$$

rescaled densities



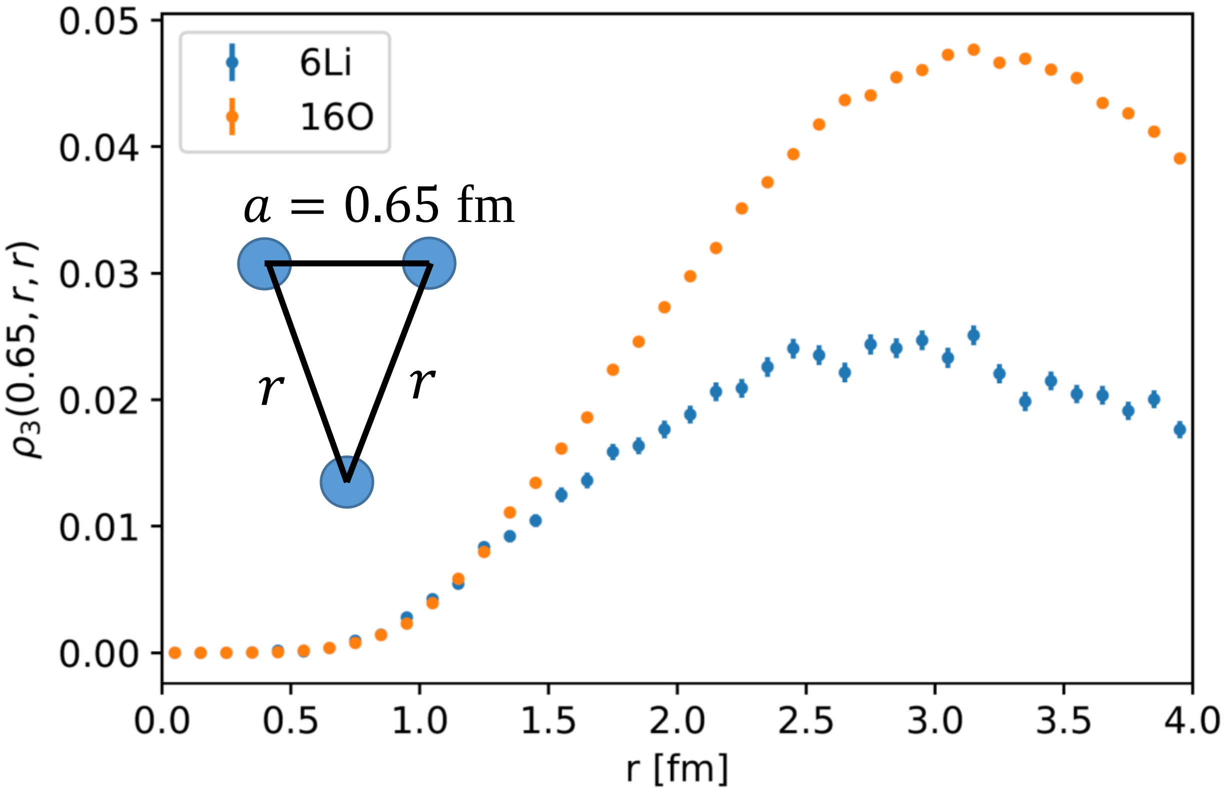


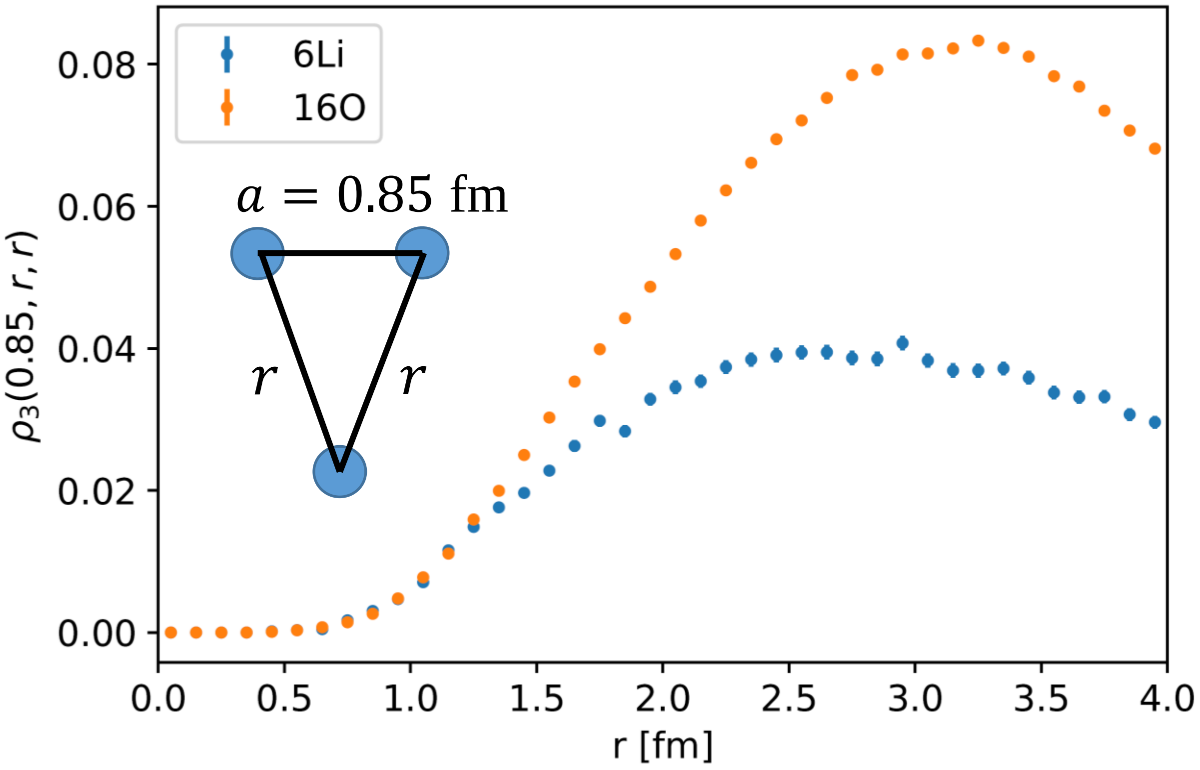
Three-body density

Same scaling factor for all geometries!

$\boldsymbol{T = \frac{3}{2}}$ **universality:**

rescaled densities





Three-body contact values ($T = 1/2$)

$$\frac{C(^4\mathrm{He})}{C(^3\mathrm{He})} \times \frac{3}{4} = 3.8 \pm 0.3 \qquad \frac{C(^6\mathrm{Li})}{C(^3\mathrm{He})} \times \frac{3}{6} = 3.1 \pm 0.3 \qquad \frac{C(^{16}\mathrm{O})}{C(^3\mathrm{He})} \times \frac{3}{16} = 4.2 \pm 0.5$$

Sargsian et al predicted [PRC 100, 044320 (2019)]:

$$a_3(A) = 1.12\,\frac{a_2(A)^2}{a_2(^3\mathrm{He})^2} \quad \Rightarrow \quad a_3(^4\mathrm{He}) \approx 3.15$$

Three-body contact values ($T = 1/2$)

$$\frac{C(^4\mathrm{He})}{C(^3\mathrm{He})} \times \frac{3}{4} = 3.8 \pm 0.3 \qquad \frac{C(^6\mathrm{Li})}{C(^3\mathrm{He})} \times \frac{3}{6} = 3.1 \pm 0.3 \qquad \frac{C(^{16}\mathrm{O})}{C(^3\mathrm{He})} \times \frac{3}{16} = 4.2 \pm 0.5$$

Additional effects might be important:

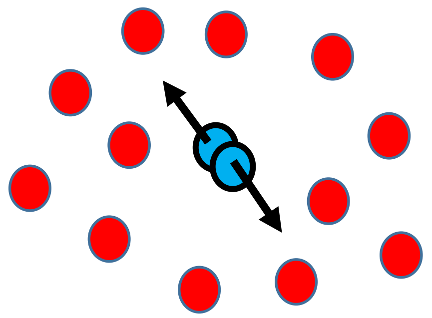
- CM motion of the triplet in nucleus A
- Energy of the $A - 3$ system
- Contribution of $t = 3/2$ triplets (*e.g.*: ppp, nnn)

Detailed reaction calculations are needed

Short-range expansion

- Factorization for short distances

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$



- $\varphi(r) \equiv$ Zero-energy solution of the two-body Schrodinger Eq.

- The two-body system:

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi^E(r) = E\varphi^E(r)$$

- For $r \to 0$: The energy becomes negligible

$$E \ll \frac{\hbar^2}{mr^2}$$

Short-range expansion

- The two-body system:

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi^E(r) = E\varphi^E(r)$$

- Taylor expansion around $E = 0$:

$$\varphi^E(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$

GCF
leading term

Next-order
terms

Short-range expansion

- The two-body system:

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi^E(r) = E\varphi^E(r)$$

- Taylor expansion around $E = 0$:

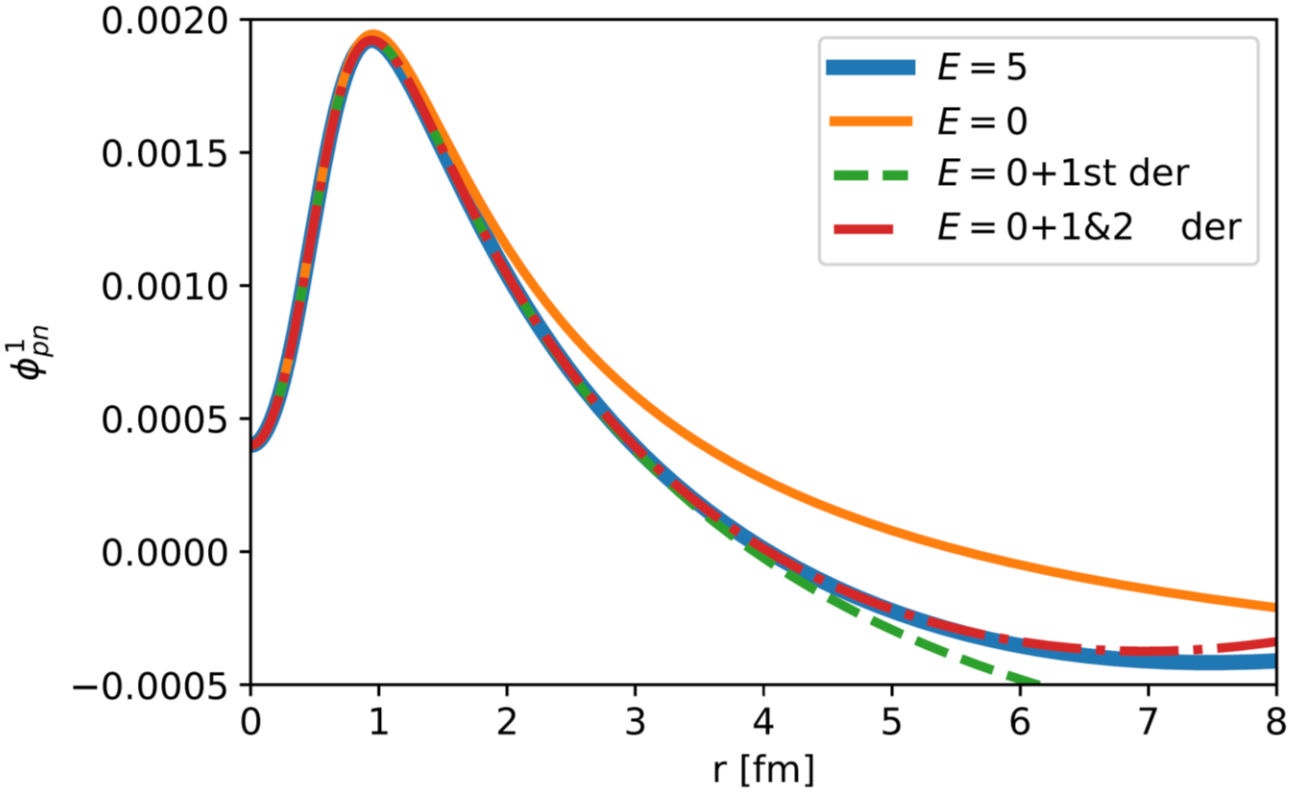
$$\varphi^E(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$

GCF leading term

At short distances: energy derivative is small → Short-range expansion

Short-range expansion

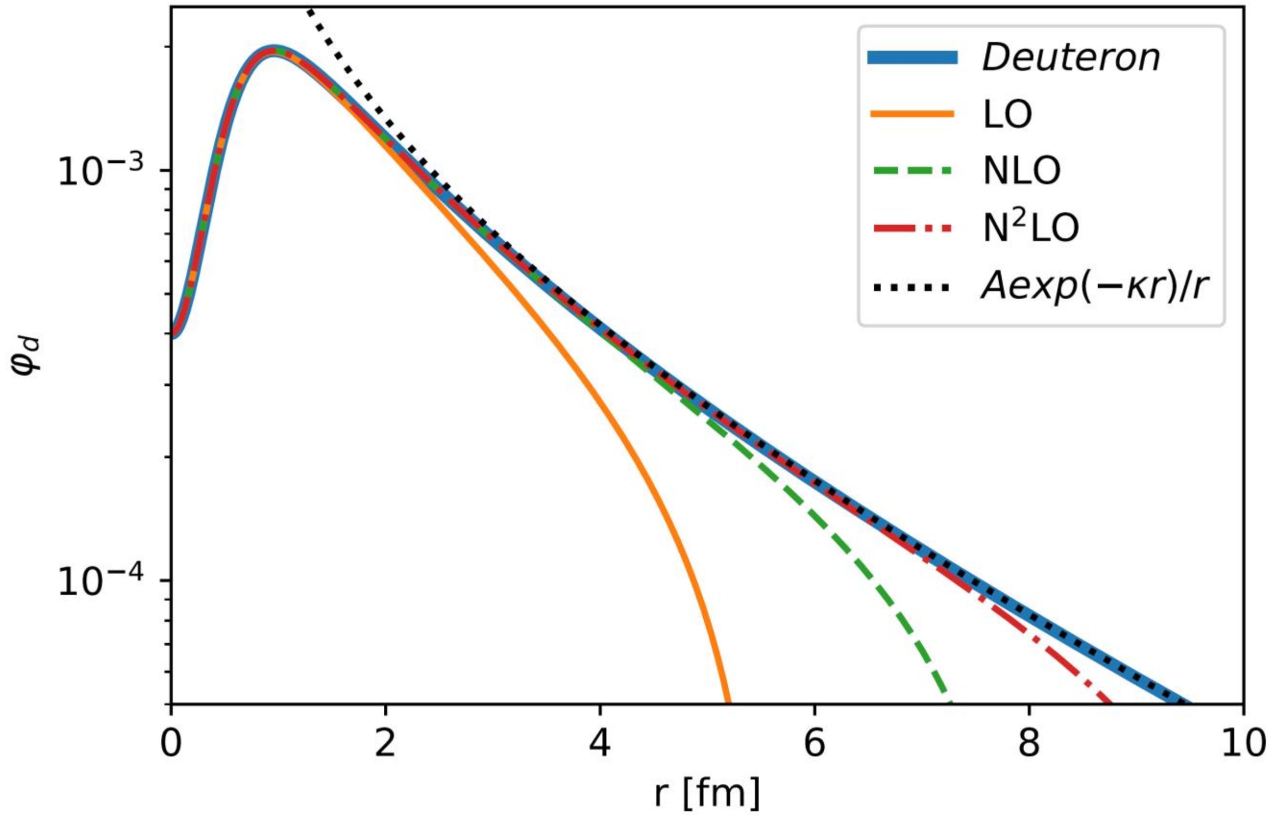
$$\varphi^{E}(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$



AV4'
Deuteron channel
Scattering state

Short-range expansion

$$\varphi^{E}(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$



AV4'
Deuteron channel
Bound state

Short-range expansion

- The many-body case: Exact expansion

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) = \sum_{E,\alpha} \varphi_\alpha^E(\boldsymbol{r}_{12}) A_\alpha^E(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A)$$ (α – quantum numbers)

- Taylor expansion around $E = 0$:

$$\varphi_\alpha^E(\boldsymbol{r}) = \varphi_\alpha^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) E^2 + \cdots$$

GCF factorization | Next-order terms

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) = \sum_\alpha \varphi_\alpha^{E=0}(\boldsymbol{r}_{12}) A_\alpha^{(0)} + \sum_\alpha \left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) A_\alpha^{(1)} + \sum_\alpha \left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) A_\alpha^{(2)} + \cdots$$

Short-range expansion: Next order terms

The many-body case:

$$\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\dots,\boldsymbol{r}_A)=\sum_\alpha \varphi_\alpha^{E=0}(\boldsymbol{r}_{12})A_\alpha^{(0)}+\sum_\alpha\left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)A_\alpha^{(1)}+\sum_\alpha\left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)A_\alpha^{(2)}+\cdots$$

$$A_\alpha^{(0)}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)=\sum_E A_\alpha^E(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)$$

$$A_\alpha^{(1)}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)=\sum_E E A_\alpha^E(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)$$

$$A_\alpha^{(2)}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)=\frac{1}{2!}\sum_E E^2 A_\alpha^E(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)$$

GCF-SM: Short distances ($r < 1$ fm)

- New contacts

$$C(f,i) = \frac{A(A-1)}{2}\langle A(f)|A(i)\rangle$$

$$\rho_{\alpha}^{0\nu}(r) \propto |\phi(r)|^2 C(f,i)$$

Contact values are extracted based on model independence of ratios

$$\frac{C^{V_1}(X)}{C^{V_1}(Y)} = \frac{C^{V_2}(X)}{C^{V_2}(Y)}$$

Model independence of contact ratios

- For $0\nu2\beta$:

$$\frac{C^{AV18}(f_1, i_1)}{C^{AV18}(f_2, i_2)} = \frac{C^{SM}(f_1, i_1)}{C^{SM}(f_2, i_2)}$$

$$C^{AV18}(f_1, i_1) = \frac{C^{SM}(f_1, i_1)}{C^{SM}(f_2, i_2)} C^{AV18}(f_2, i_2)$$

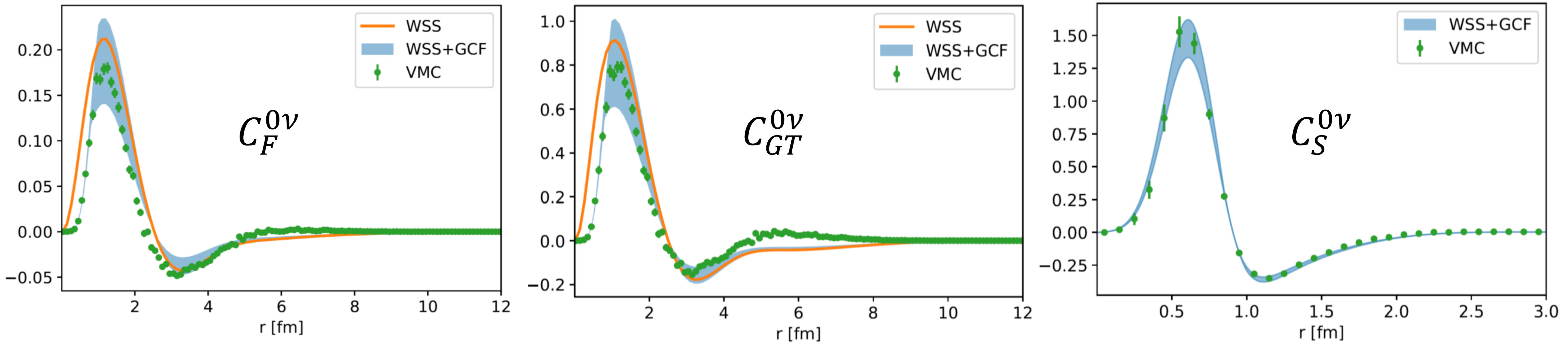
- For example

Exact QMC calculations

$$C^{AV18}\left({}^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}\right) = \frac{C^{SM}\left({}^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}\right)}{C^{SM}\left({}^{12}\mathrm{Be} \rightarrow {}^{12}\mathrm{C}\right)} C^{AV18}\left({}^{12}\mathrm{Be} \rightarrow {}^{12}\mathrm{C}\right)$$

Validation using light nuclei (AV18)

Using $^{6}He \rightarrow$ ^{6}Be and $^{10}Be \rightarrow$ ^{10}C to "predict" $^{12}Be \rightarrow$ ^{12}C



Short distances - GCF

Long distances – Shell model

NMEs and transition densities

Light Majorana neutrino exchange mechanism

$$M^{0\nu} = \langle \Psi_f | O^{0\nu} | \Psi_i \rangle$$

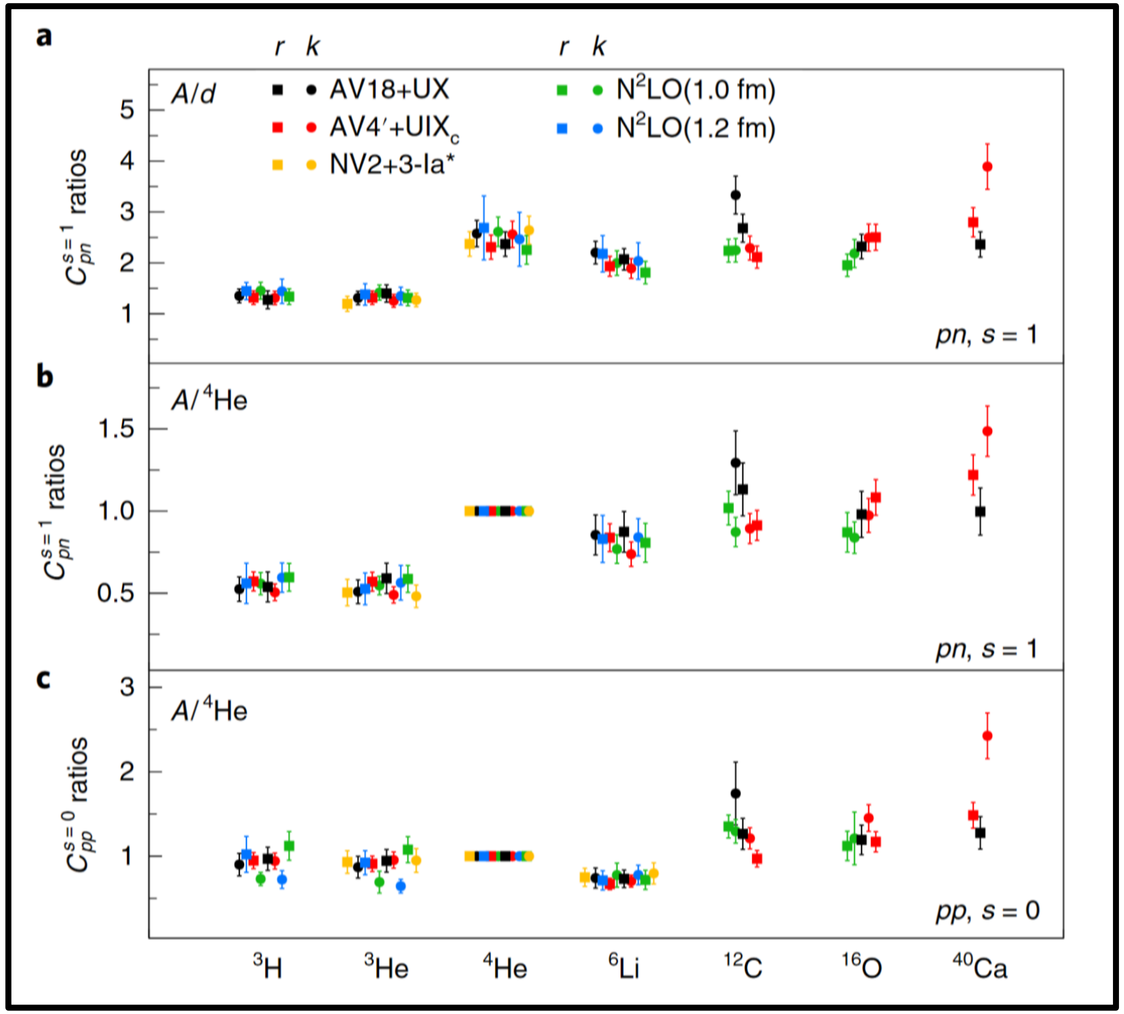
$$O^{0\nu} = O_F^{0\nu} + O_{GT}^{0\nu} + O_T^{0\nu} + O_S^{0\nu}$$

$$4\pi r^2 \rho_F(r) = \langle \Psi_f | \sum_{a<b} \delta(r - r_{ab}) \tau_a^+ \tau_b^+ | \Psi_i \rangle$$

$$C_\alpha^{0\nu}(r) \equiv (8\pi R_A) 4\pi r^2 \rho_\alpha(r) V_\alpha^{0\nu}(r)$$

$$M_\alpha^{0\nu} = \int_0^\infty dr\, C_\alpha^{0\nu}(r)$$

Model independence of contact ratios



$$\frac{C^{V_1}(X)}{C^{V_1}(Y)} = \frac{C^{V_2}(X)}{C^{V_2}(Y)}$$