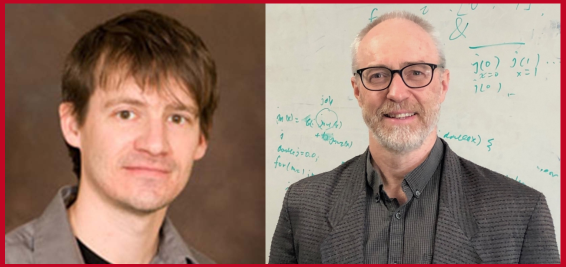
A coupled channel analysis of e^+e^- annihilation in the bottomonium region

Nils Hüsken, Ryan Mitchell, Eric Swanson

10th Workshop of the APS Topical Group on Hadron Physics
Minneapolis, April 14th 2023



Phys. Rev. D106 (2022) 9, 094013

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MOTIVATION

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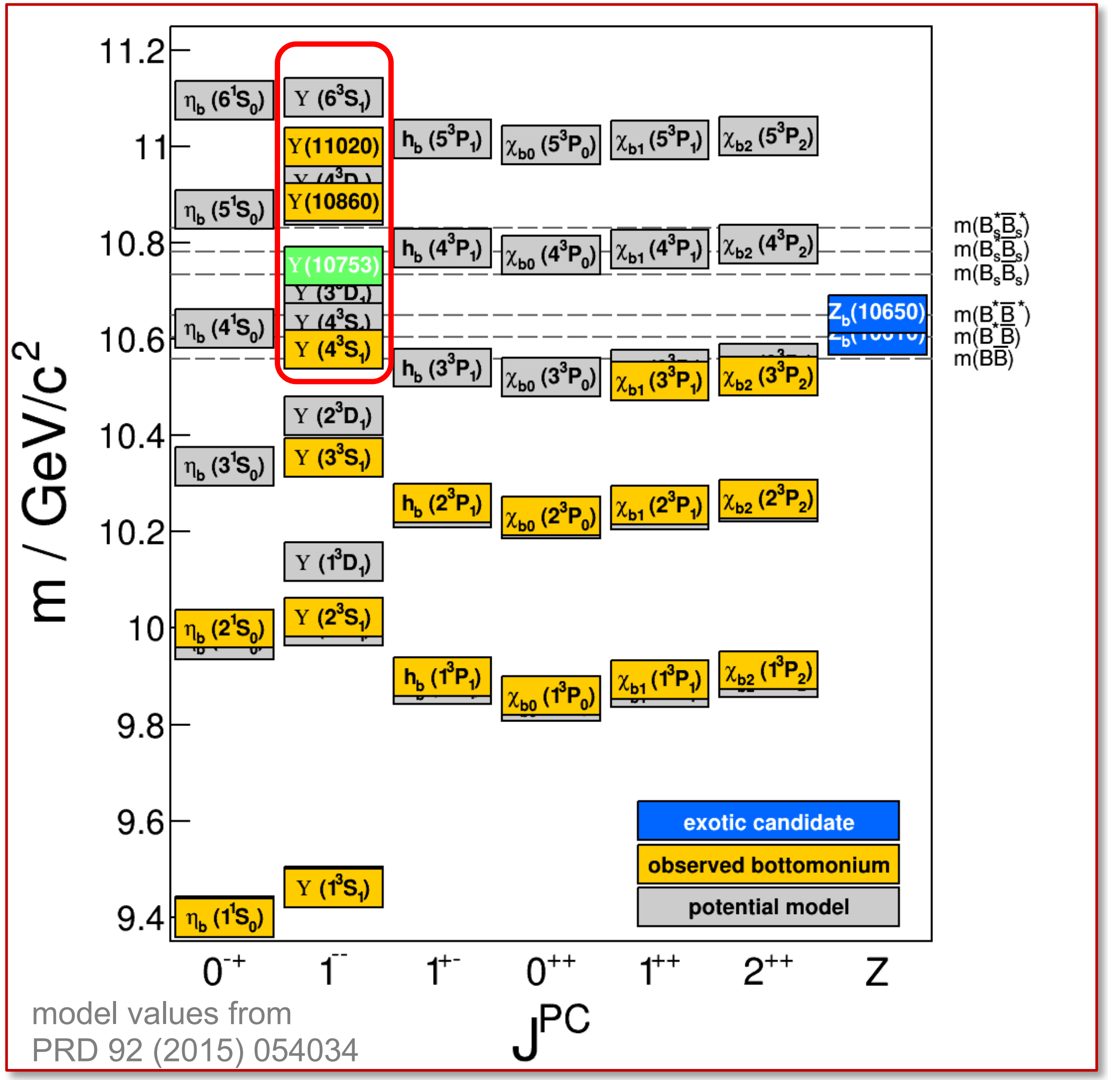

model values from
PRD 92 (2015) 054034

- we seek a comprehensive, unitary analysis of all channels contributing to $e^+e^- \to b\bar{b}$
- extract the properties of Υ-states above $B\bar{B}$ threshold
  - thus far, masses and widths limited to Breit-Wigner, Gaussian
  - branching ratios limited to $BR_i = \frac{\sigma_i}{\sigma_{tot}}(\sqrt{s} = m_{peak})$ (only valid if peak is isolated)
- new data highlights importance of thresholds, so fits should move beyond sums of Breit-Wigners
- investigate $\Upsilon(10753)$, potentially exotic state recently seen by Belle JHEP 10 (2019) 220

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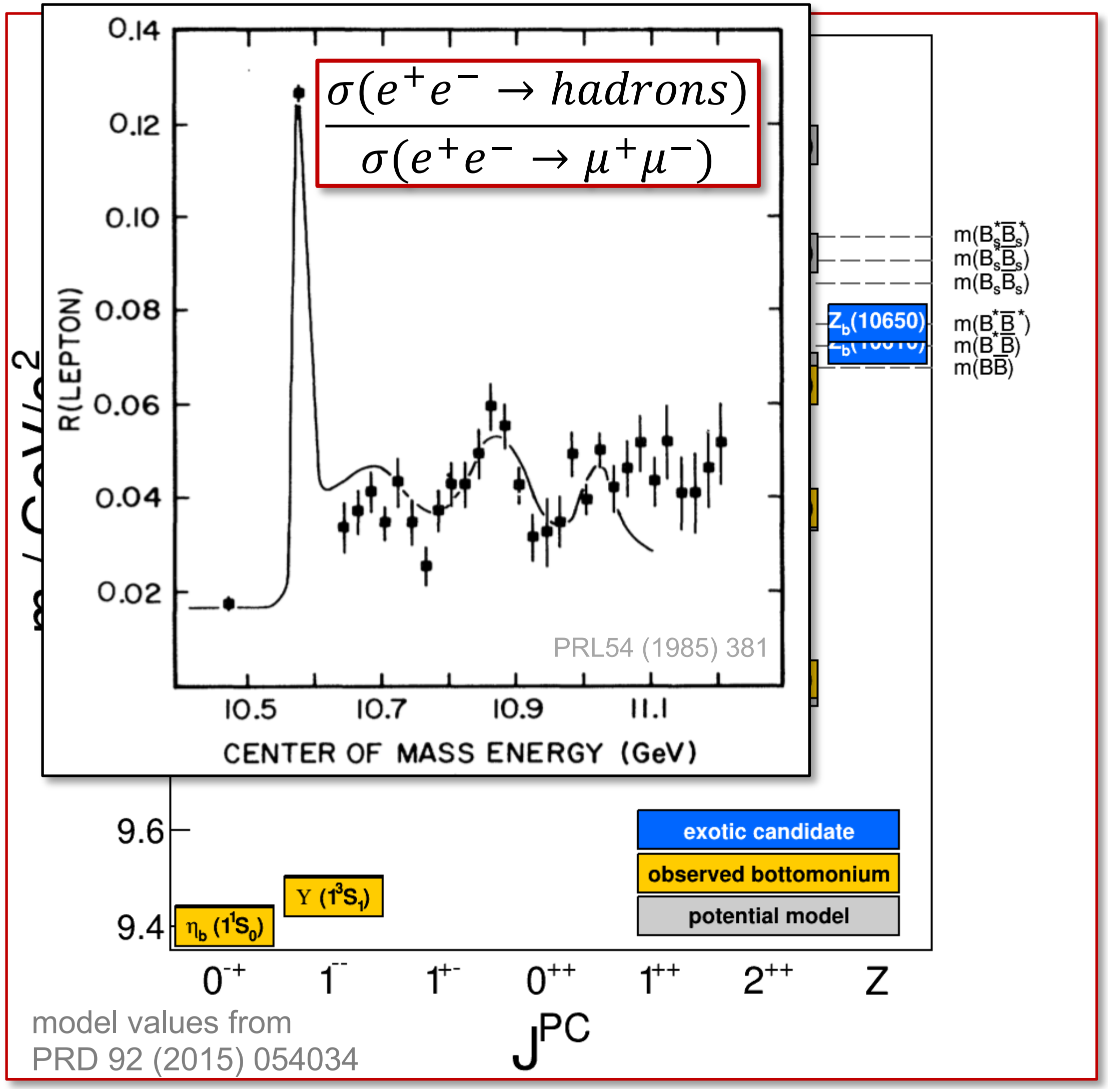

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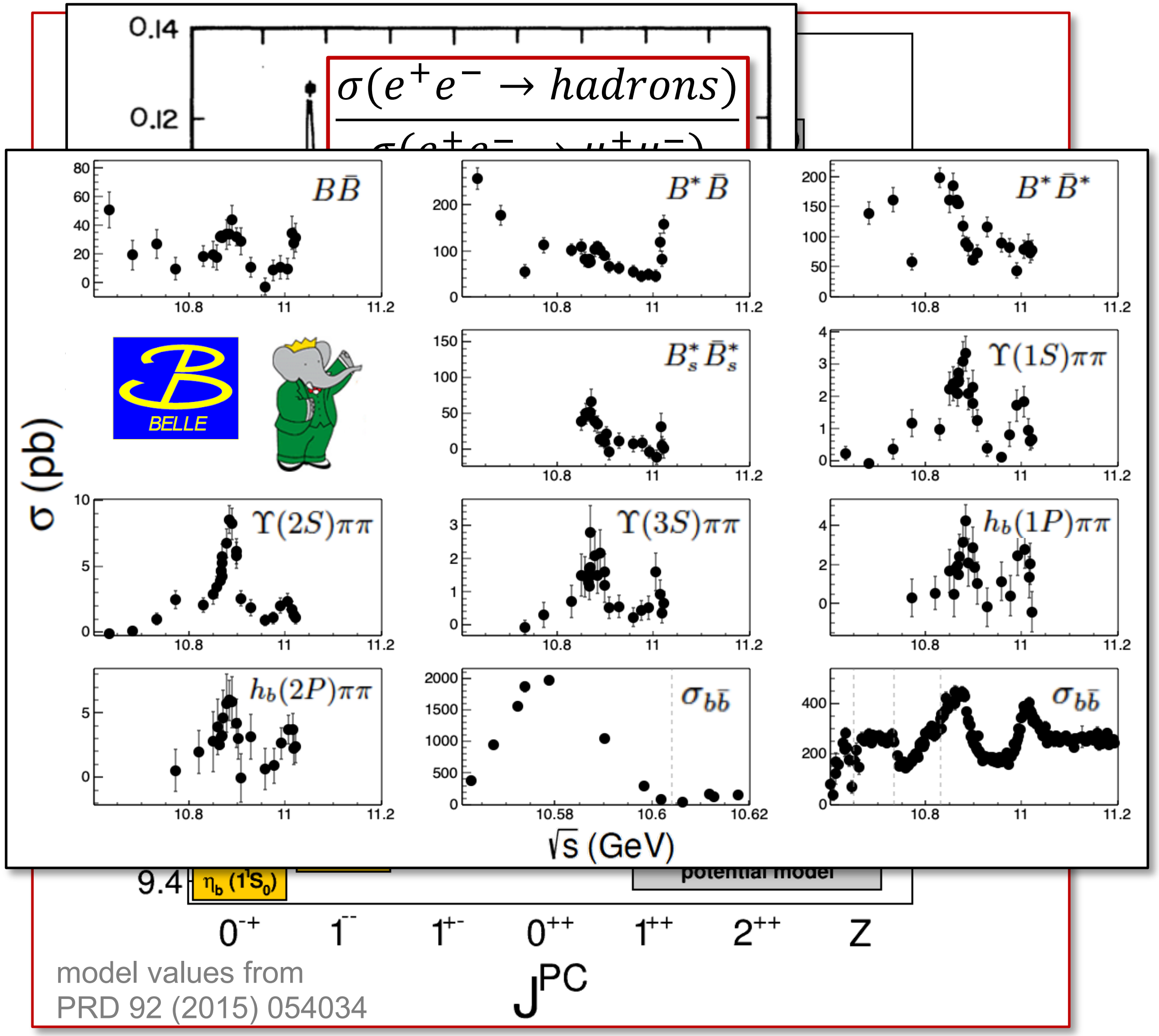

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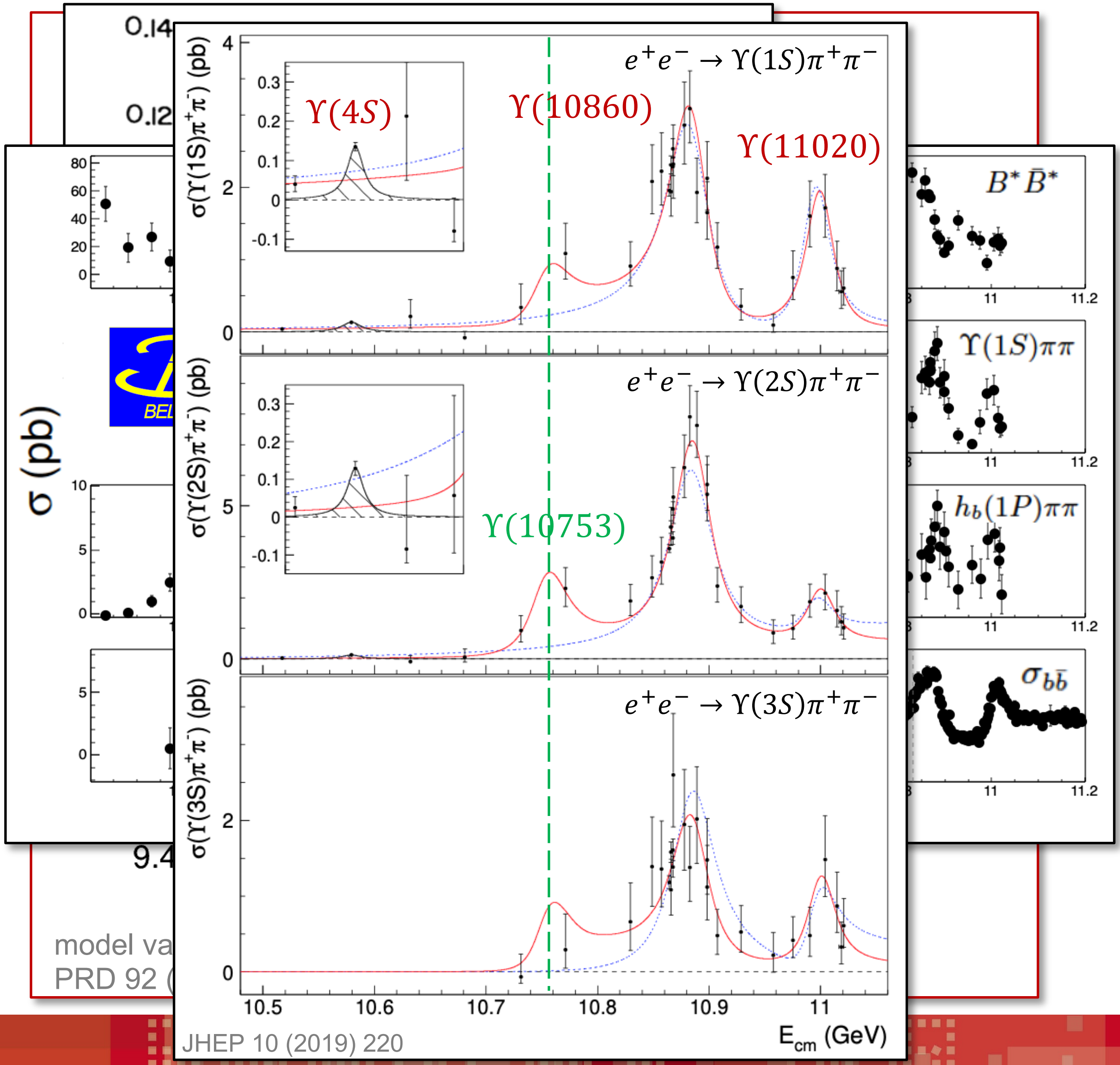

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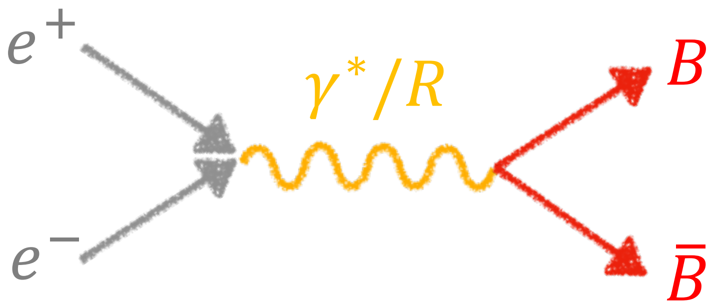
FORMALISM

FORMALISM

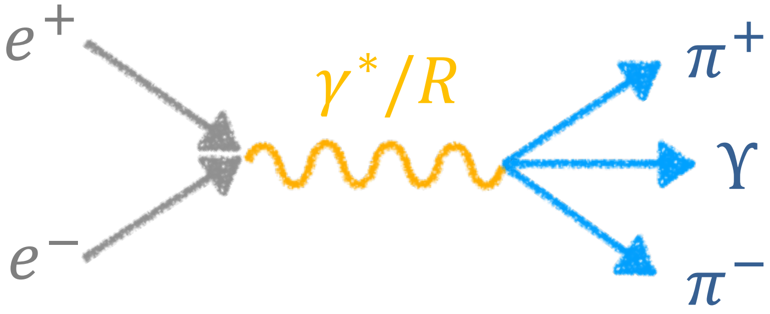
- cross section for $e^+e^- \rightarrow$ two-body final state:

$$\sigma(e^+e^- \rightarrow \mu) = \frac{1}{16\pi s}\frac{k_\mu}{k_{ee}}\overline{|\mathcal{M}_{\mu,ee}|^2}$$



- cross section for $e^+e^- \rightarrow$ three-body final state:

$$d\sigma = \frac{1}{(2\pi)^3}\frac{1}{64 s\sqrt{s}k_i}\overline{|\mathcal{M}_{345:12}|^2}\,dm_{12}^2\,dm_{23}^2$$



FORMALISM

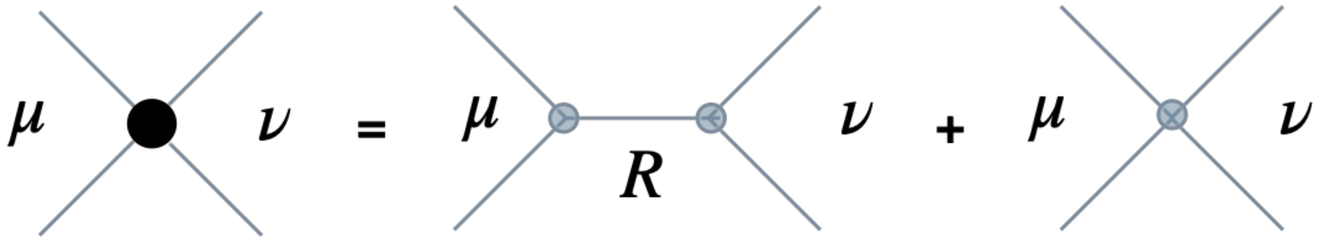
K-matrix formalism:

EPJ C80 (2020) 5, 453
PRD 91, no. 5 (2015) 054008
PRD 16 (1977) 657

$$\mathcal{M} = (1 + KC)^{-1}K = K(1 + CK)^{-1}$$
$$= K(K + KCK)^{-1}K.$$

Chew-Mandelstam

$$K_{\mu,\nu} = \sum_R \frac{g_{R:\mu}g_{R:\nu}}{m_R^2 - s} + f_{\mu,\nu}$$

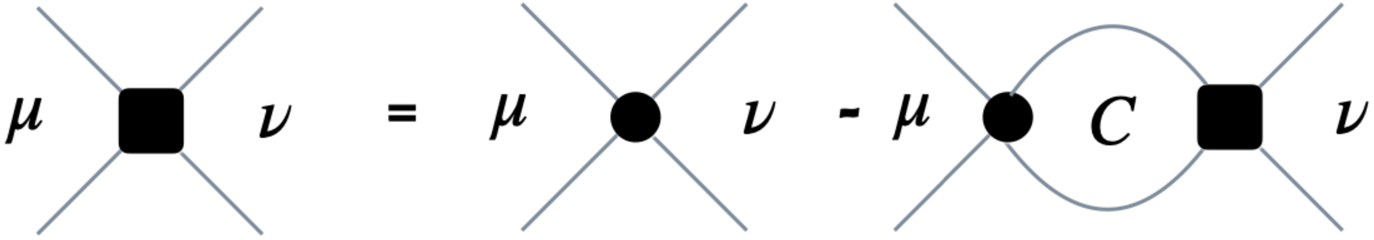


Aitchison's P-vector approach:

$$\mathcal{M}_{\mu,ee} = \sum_\nu (1 + \hat{K}\hat{C})^{-1}_{\mu,\nu} P_\nu$$

in restricted channel space
$(g_{R:ee} \ll g_{R:had})$

$$P_\nu = K_{\nu,ee}$$



sketches from E. Swanson, CIPANP 2022

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Chew-Mandelstam

$$K_{\mu,\nu} = \sum_R \frac{g_{R:\mu} g_{R:\nu}}{m_R^2 - s} + f_{\mu,\nu}$$

couplings:

$$g_{R:\mu}(s) = \hat{g}_{R:\mu} \left(\frac{k_\mu(s)}{\beta} \right)^{\ell_\mu} \cdot \exp[-k_\mu^2(s)/\beta^2]$$

$$f_{\mu,\nu} = \hat{f}_{\mu,\nu} \cdot \left(\frac{k_\mu(s)}{\beta} \right)^{\ell_\mu} \cdot \left(\frac{k_\nu(s)}{\beta} \right)^{\ell_\nu} \cdot \exp\left[- \left(\frac{k_\nu^2(s) + k_\mu^2(s)}{\beta^2} \right) \right]$$

we test $\beta = 0.8, 1.0, 1.2$ GeV

Aitchison's P-vector approach:

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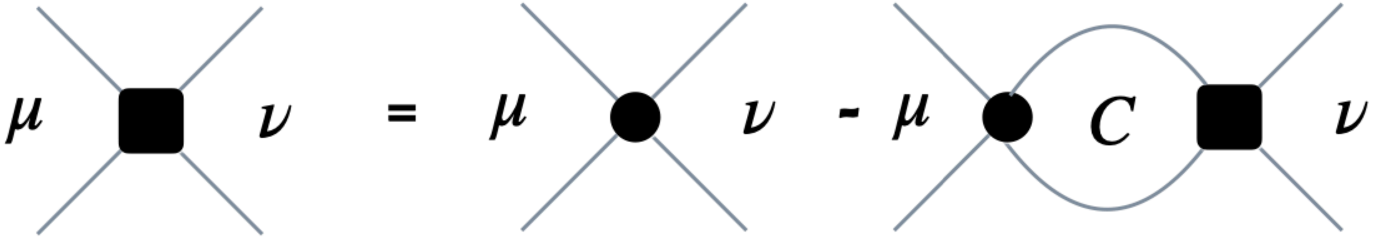

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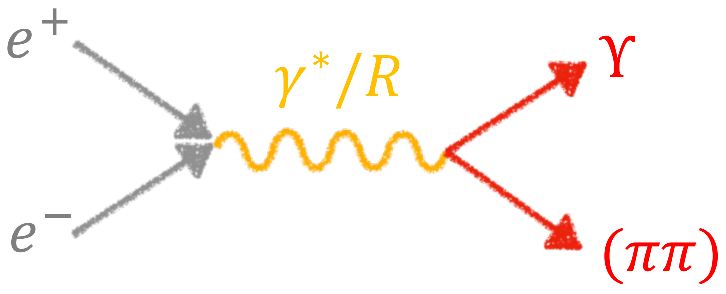
FORMALISM

Three-body channels:

(1) pretend they are two-body channels
(as is commonly done)



(2) perturbative treatment
(similar to Aitchison's P-vector, but for final state)

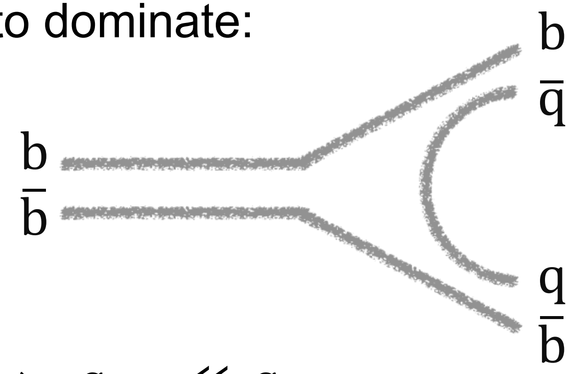
$$\mathcal{M}_{\Delta,ee} = \sum_{\mu} F_{\mu}^{(\Delta)} (1 + \hat{C}\hat{K})^{-1}_{\mu,ee}$$

three-body channels

in two-body channel space

„final state matrix" coupling
two- to three-body channels

expect these processes
to dominate:



$\rightarrow \quad g_{R:\Delta} \ll g_{R:\mu}$

FORMALISM

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three-body channels

„final state matrix“ coupling
two- to three-body channels

in two-body channel space

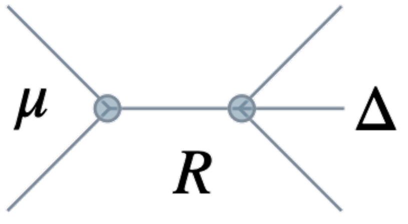
Three-body models:

(2a) non-resonant production

$$F_\mu^{(\Delta)} = f_{\Delta:\mu}$$

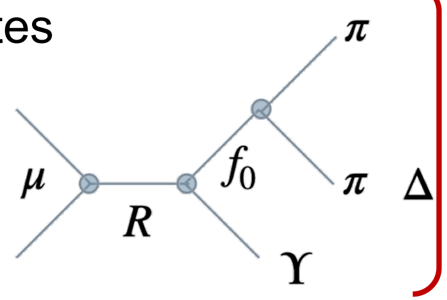
(2b) resonant production

$$F_\mu^{(\Delta)} = \sum_R \frac{g_{R:\Delta}\, g_{R:\mu}}{m_R^2 - s}$$



(2c) including intermediate states

$$F_\mu^{(\Delta=\pi\pi\Upsilon)} = \frac{g_{R:\mu} \cdot g_{R:f_0\Upsilon} \cdot g_{f_0:\pi\pi}}{(m_R^2 - s)\,(m_{f_0}^2 - s_{\pi\pi})}$$



sketches from E. Swanson, CIPANP 2022

RESULTS

FIT RESULTS

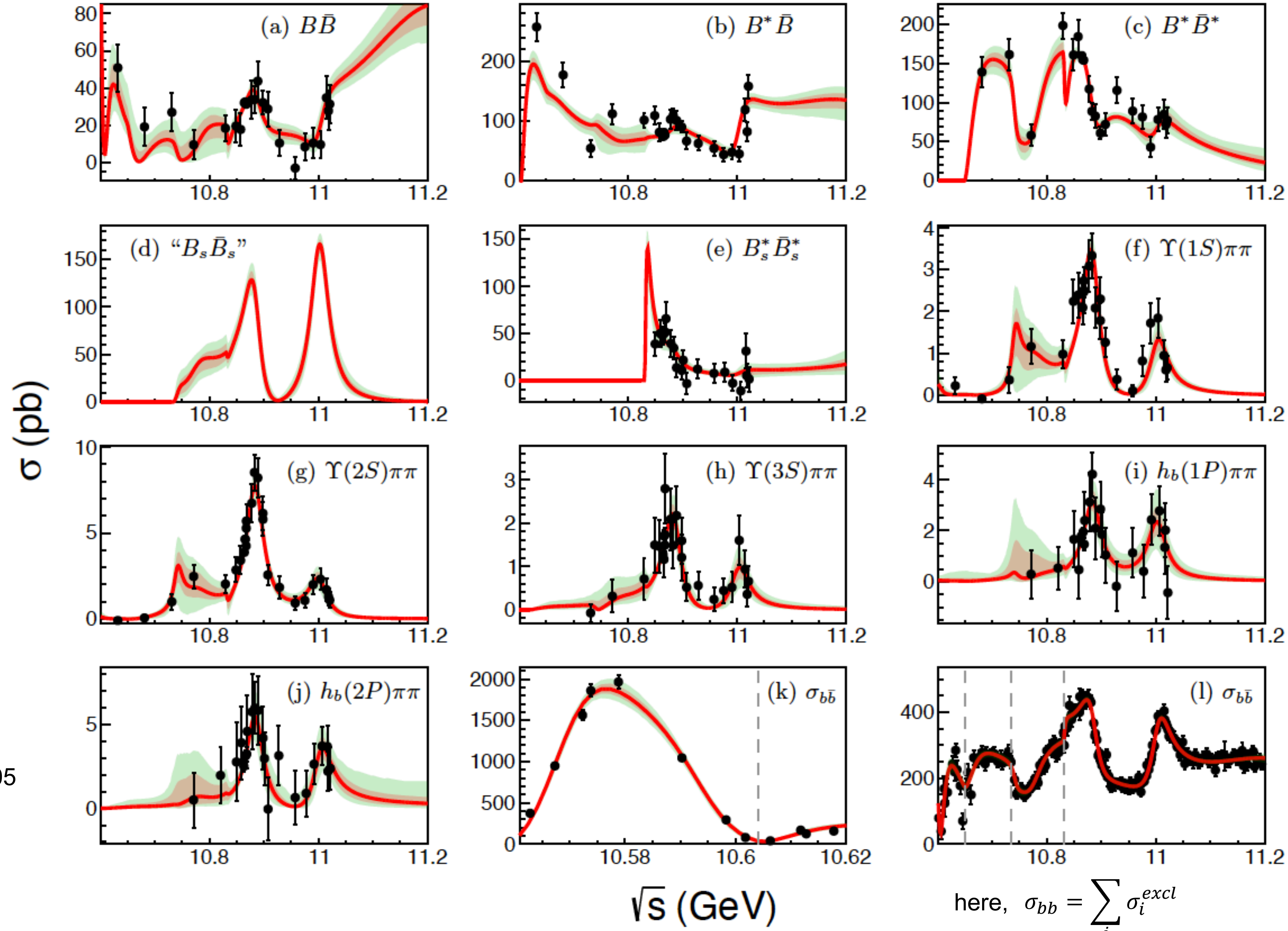
- data

quasi two-body model, $\beta = 1.0$ GeV

68% CL

90% CL

(a-c): JHEP 06 (2021) 137
(e): arXiv:1609.08749
(f-h): JHEP 10 (2019) 220, PRD96 (2017) 052005
(i-j): PRL117 (2016) 14, 142001
(k, l): CPC 44 (2020) no.8 083001



here, $\sigma_{bb} = \sum_i \sigma_i^{excl}$

FIT RESULTS

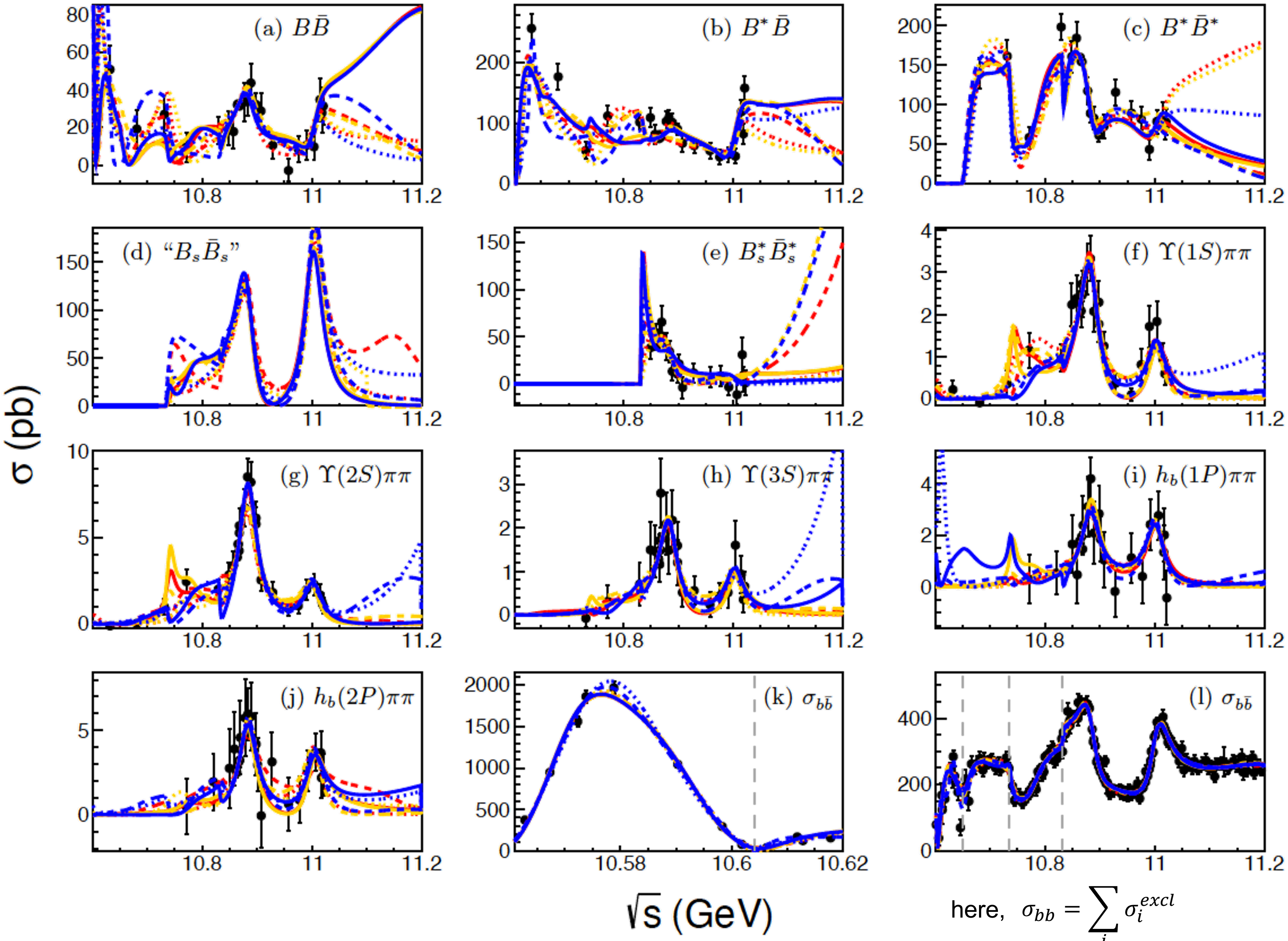
- data

quasi two-body models

three-body resonant

three-body non-resonant

→ model variation recapitulates bootstrap variation



here, $\sigma_{bb} = \sum_i \sigma_i^{excl}$

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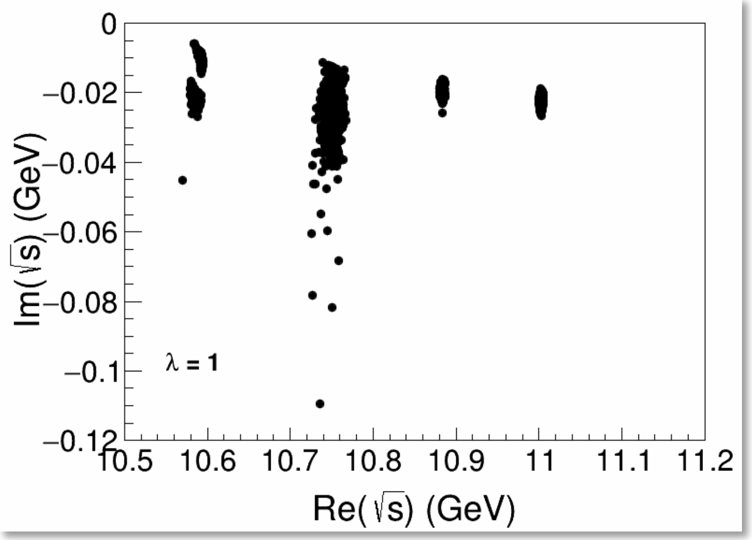
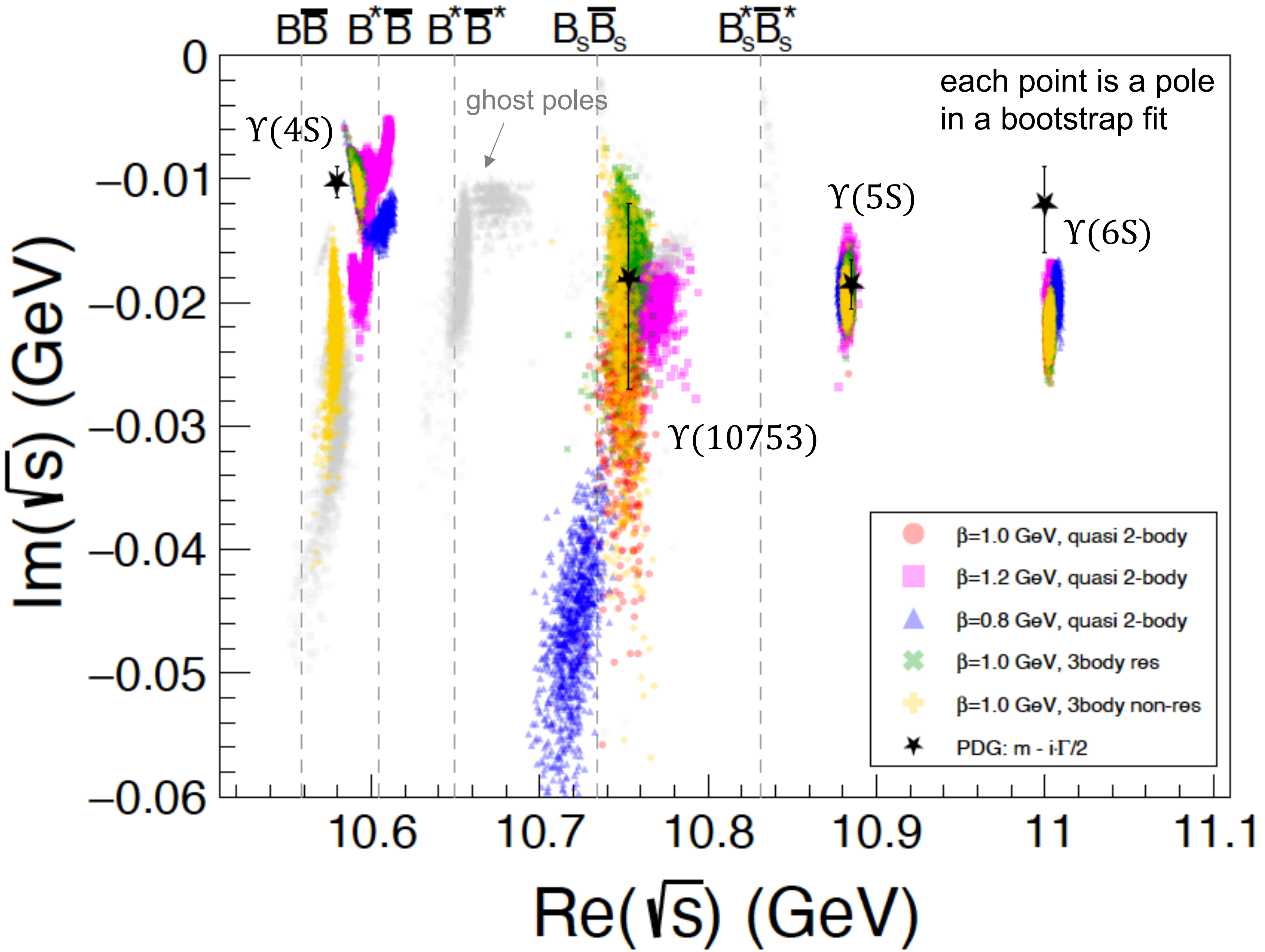
FIT RESULTS

- identify poles in the complex plane
- partial widths from pole residues
- stat. uncertainties from bootstrapping
- identify ghost poles from pole trajectories (with $g' = \lambda g$, $\lambda \to 0$)





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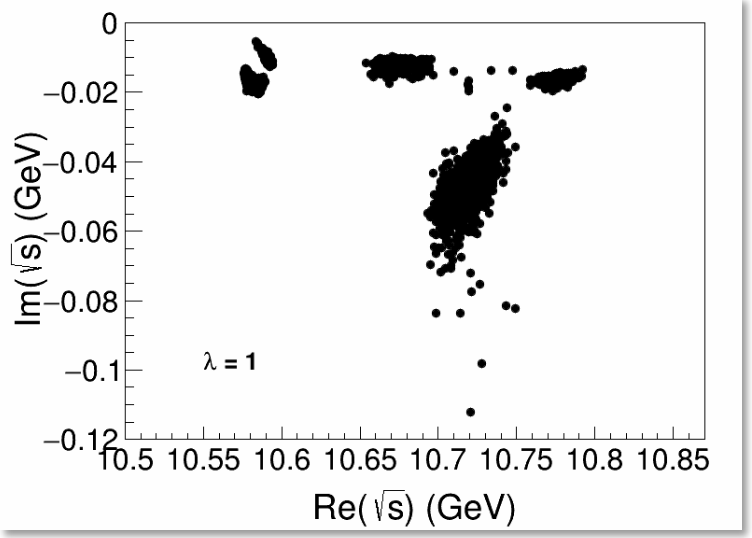
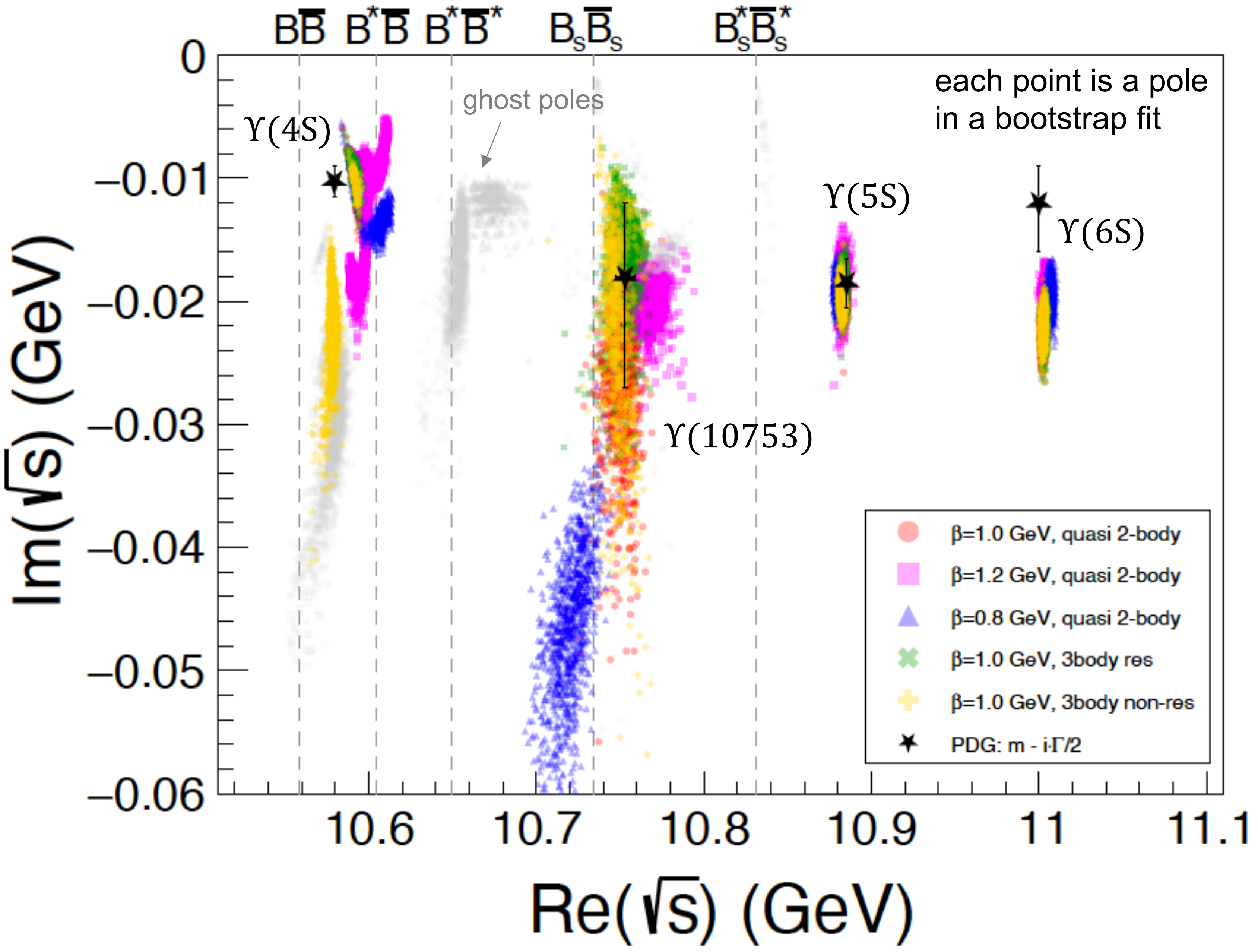
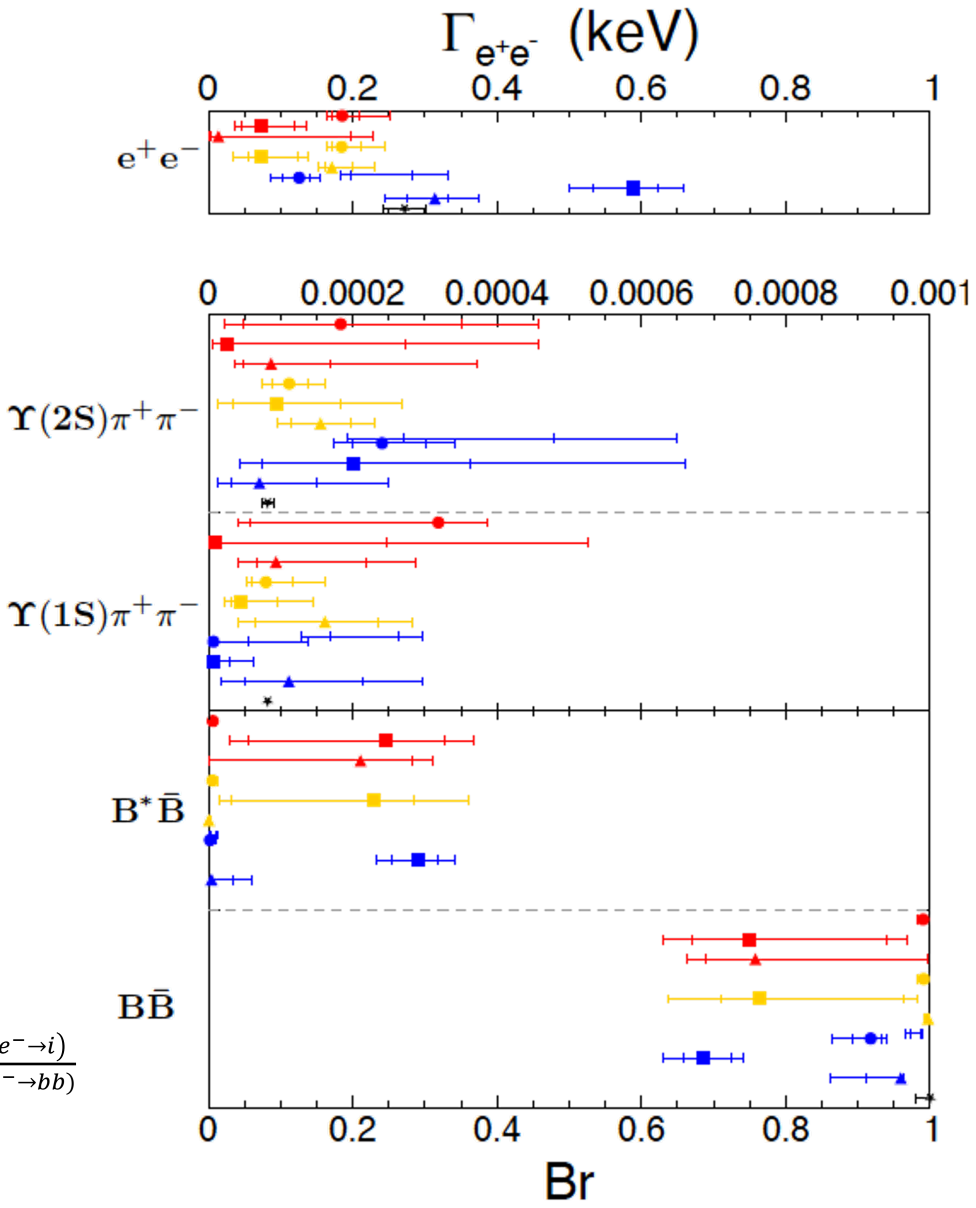
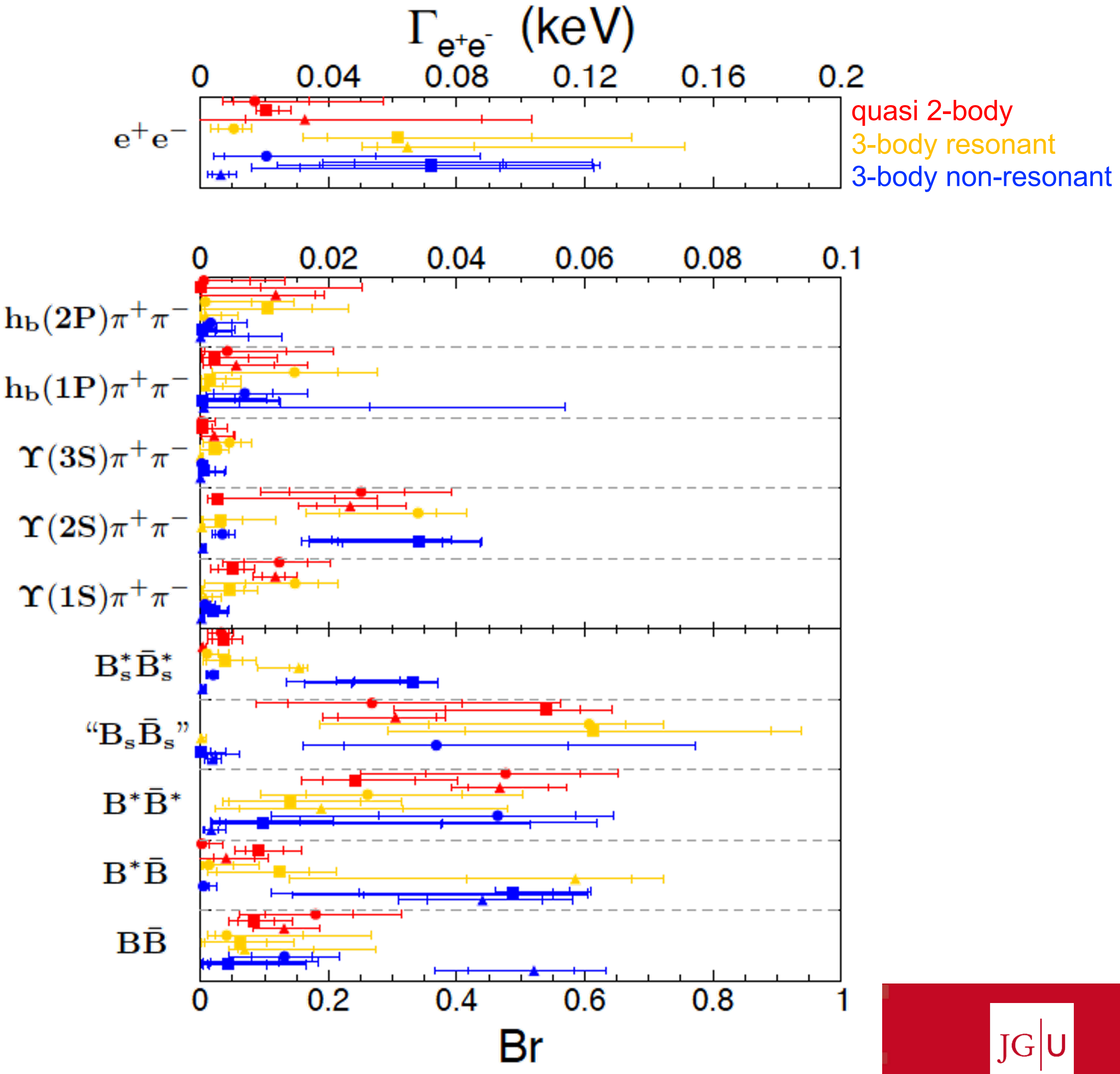




FIT RESULTS

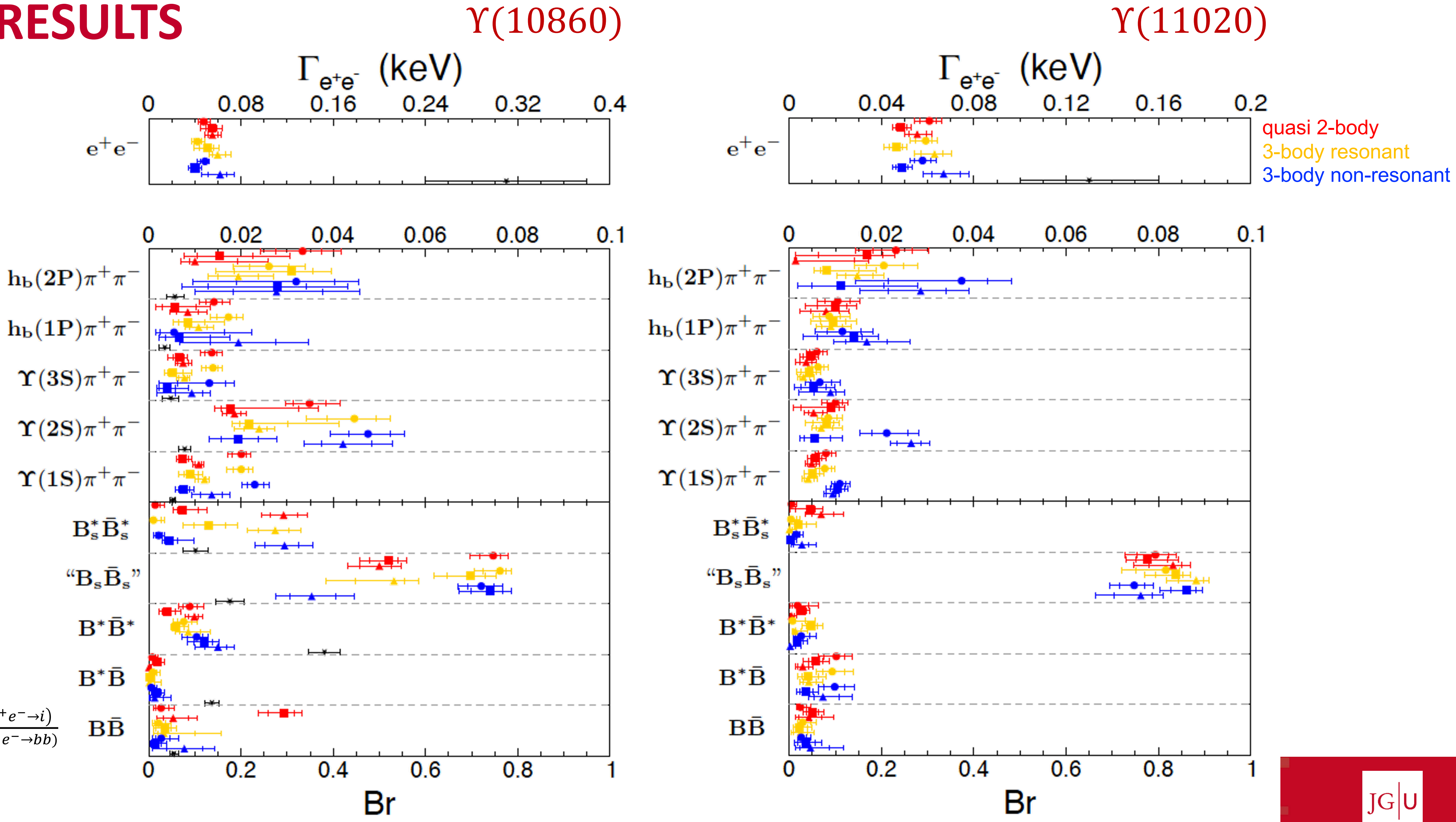
$\Upsilon(4S)$ $\Upsilon(10753)$





PDG: $\mathrm{BR_i} = \frac{\sigma(e^+e^-\to i)}{\sigma(e^+e^-\to bb)}$

FIT RESULTS



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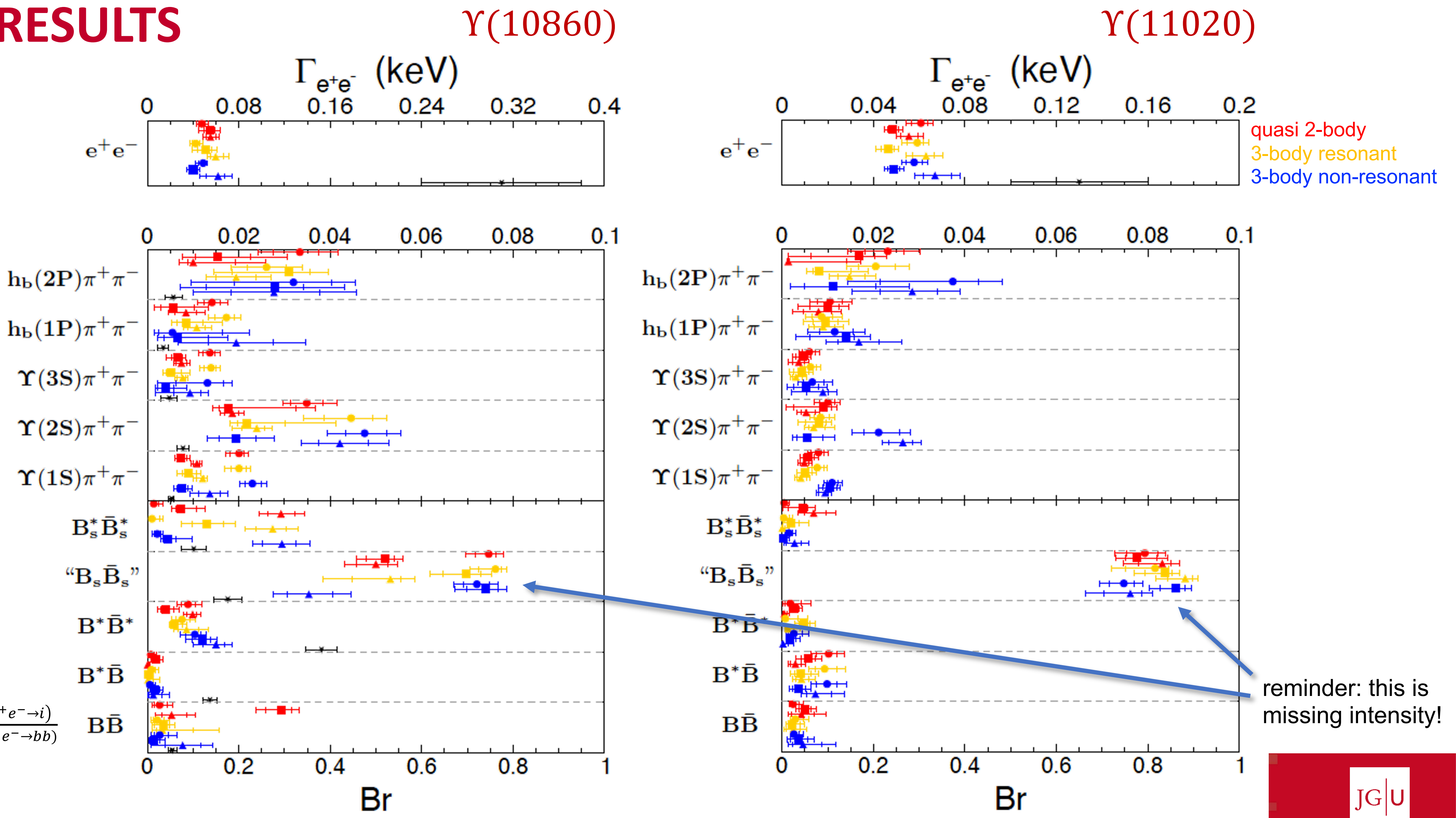

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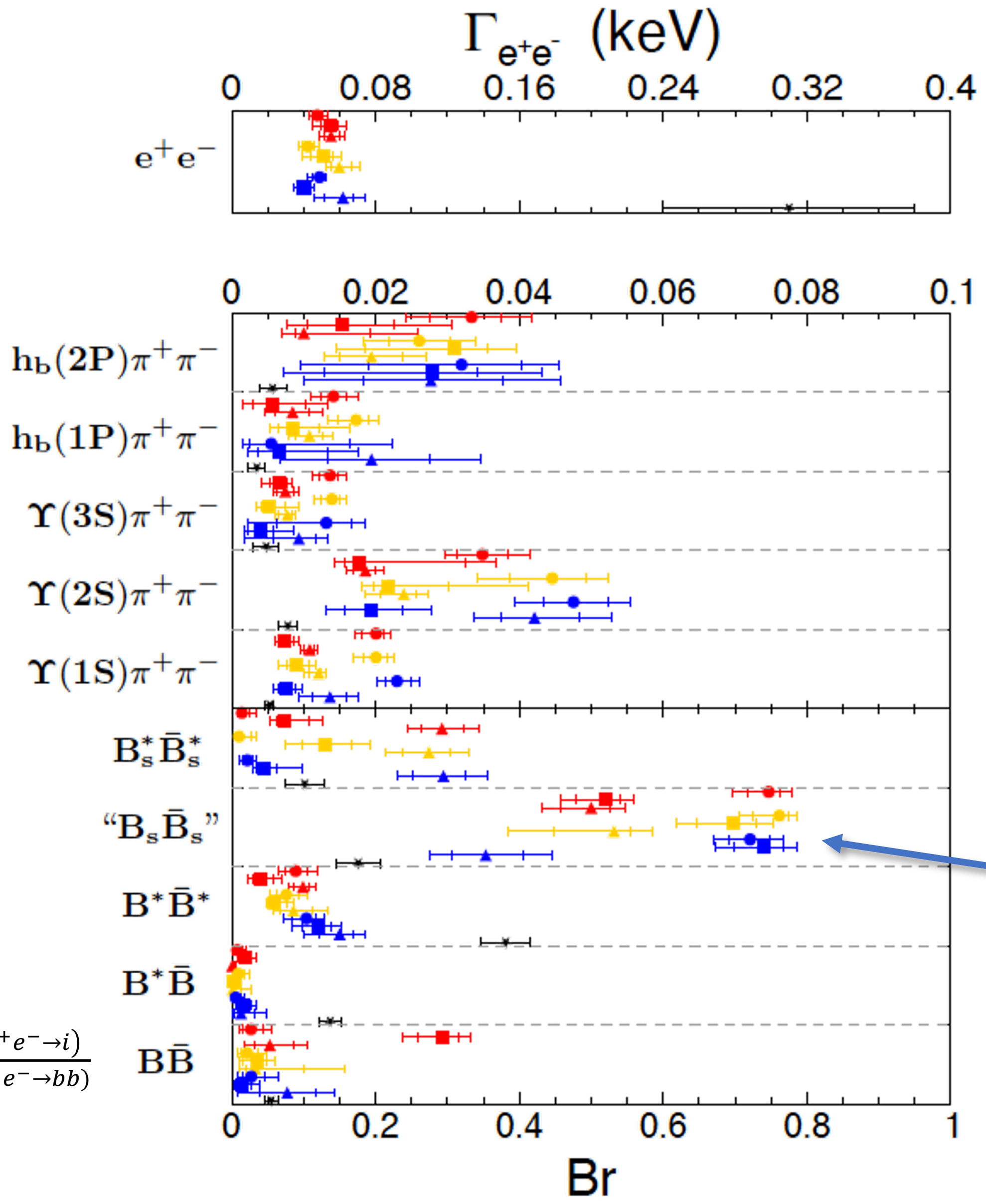
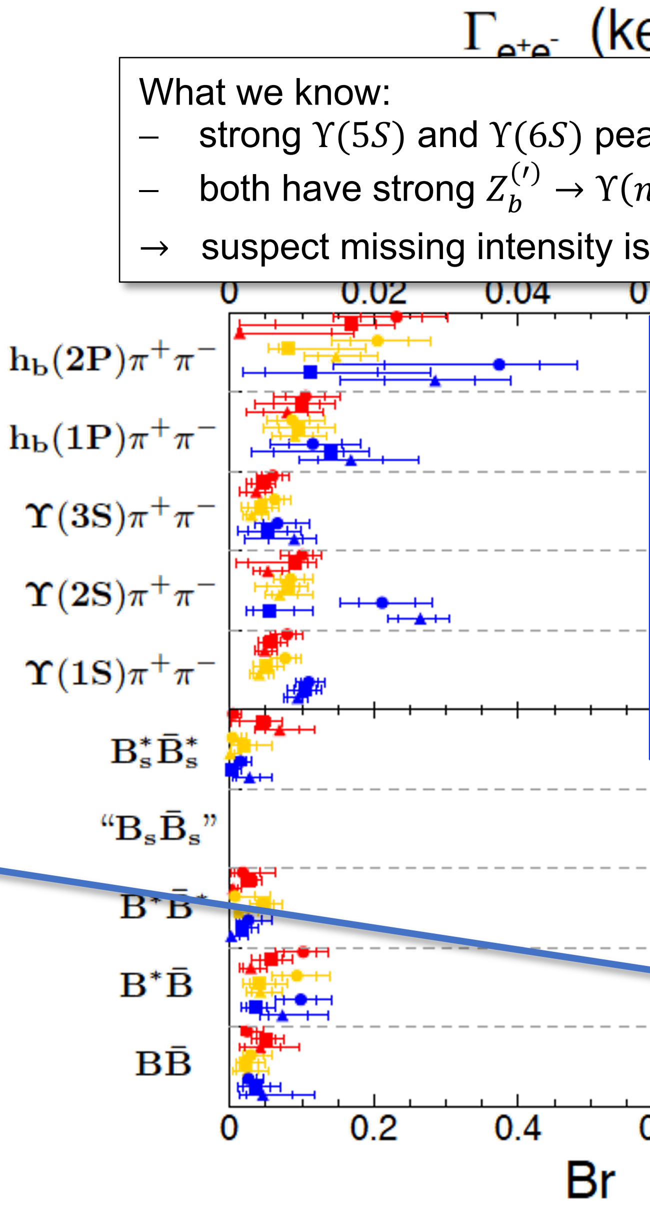
FIT RESULTS

$\Upsilon(10860)$



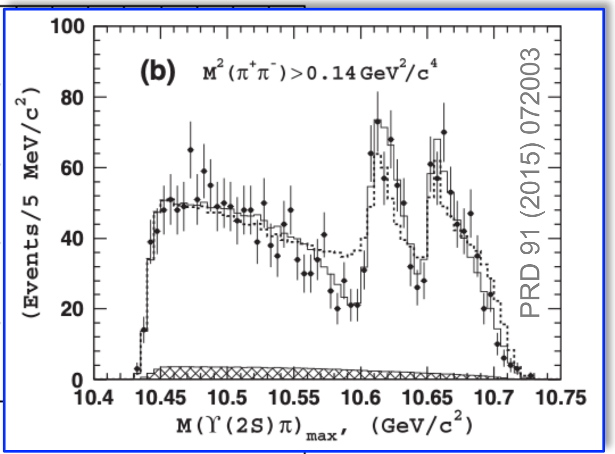
$\Upsilon(11020)$



What we know:

- strong $\Upsilon(5S)$ and $\Upsilon(6S)$ peaks in $\Upsilon(nS)\pi^+\pi^-$ and $h_b(nP)\pi^+\pi^-$
- both have strong $Z_b^{(\prime)} \to \Upsilon(nS)\pi,\ h_b(nP)\pi$ contributions

→ suspect missing intensity is in $Z_b^{(\prime)}\pi$, with $Z_b^{(\prime)} \to B^*\bar{B}^{(*)}$



reminder: this is missing intensity!

PDG: $BR_i = \frac{\sigma(e^+e^-\to i)}{\sigma(e^+e^-\to bb)}$

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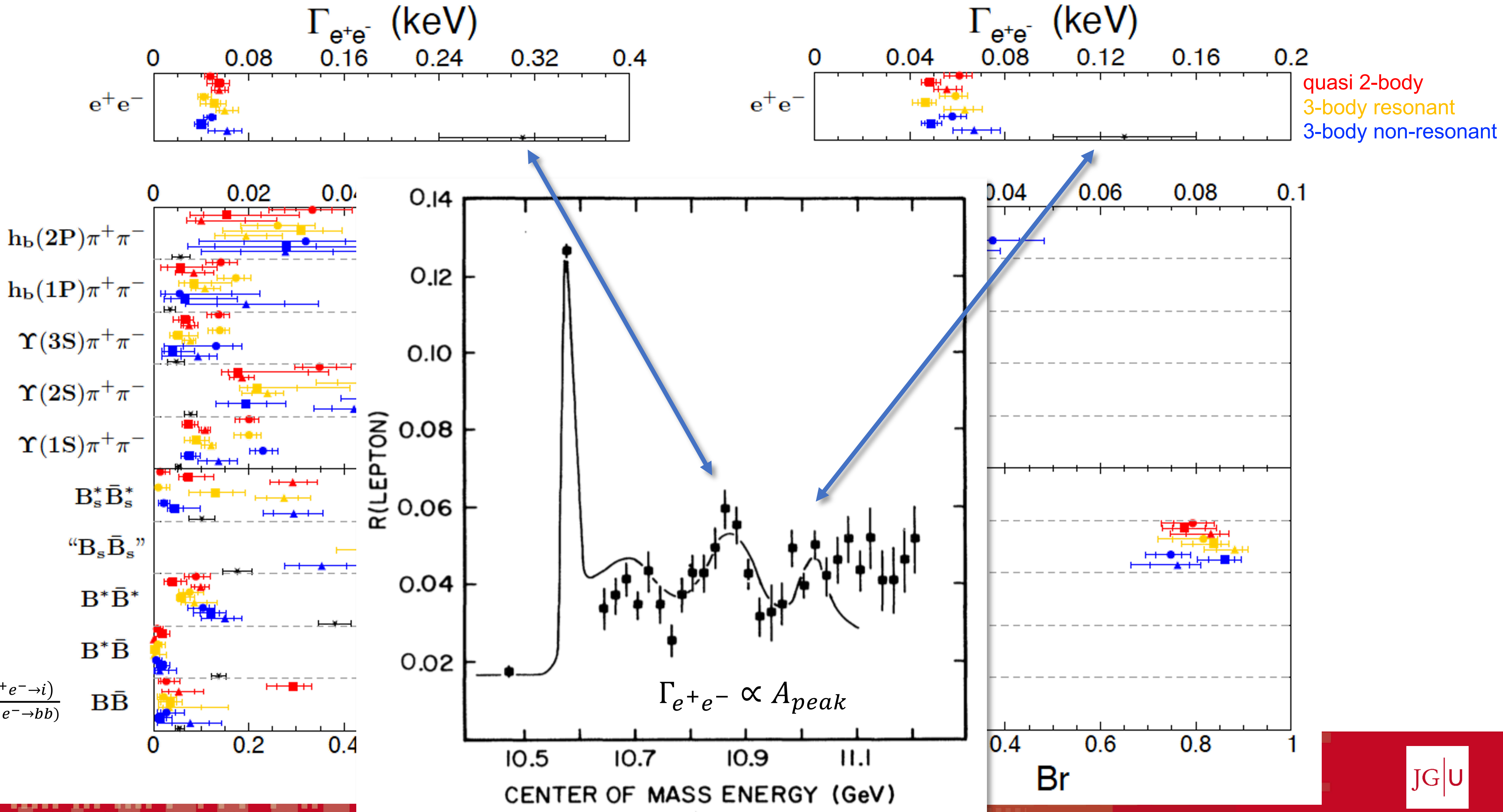
FIT RESULTS

$\Upsilon(10860)$ $\Upsilon(11020)$



$\Gamma_{e^+e^-} \propto A_{peak}$

PDG: $\mathrm{BR_i} = \frac{\sigma(e^+e^- \to i)}{\sigma(e^+e^- \to bb)}$

FIT RESULTS

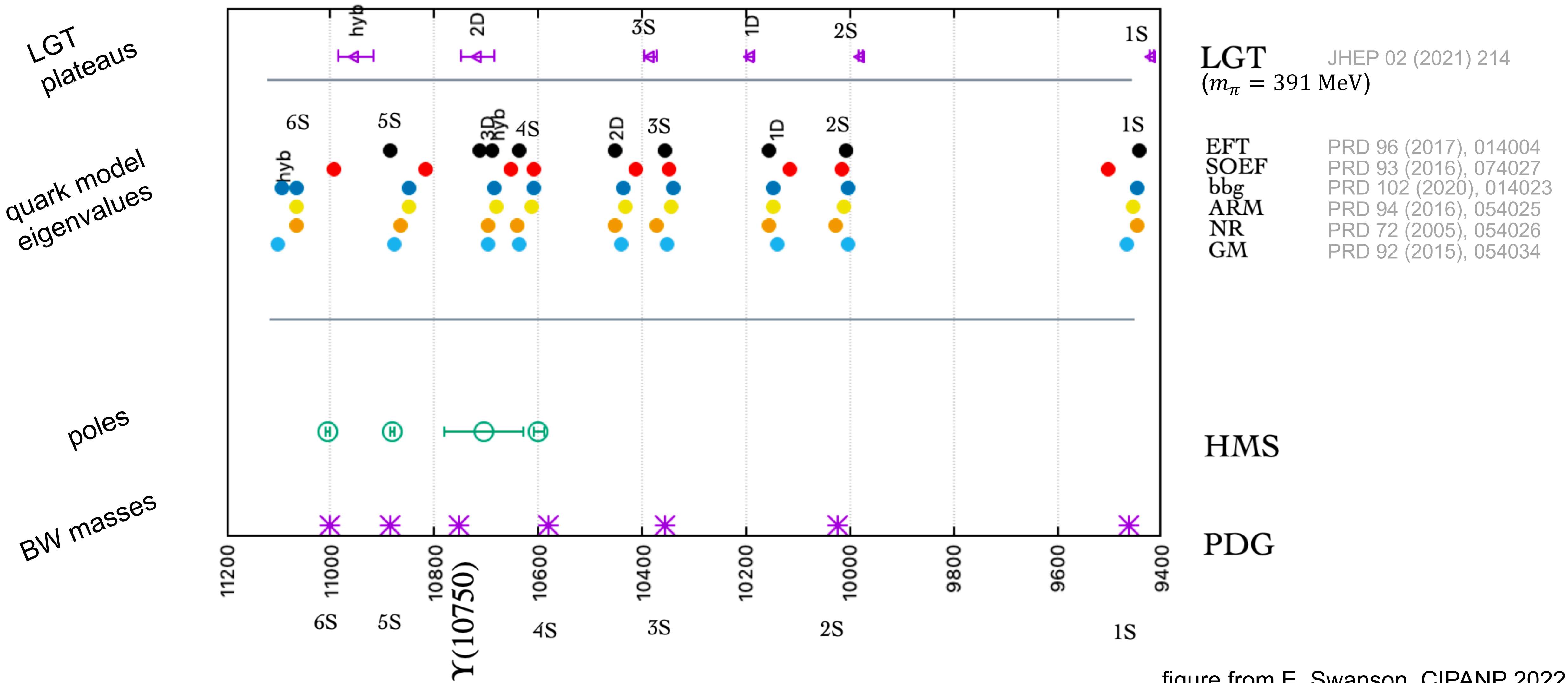


figure from E. Swanson, CIPANP 2022

WHAT DO WE CONCLUDE?

- first comprehensive analysis of $e^+e^- \to b\bar{b}$
- first determination of absolute BR
  - before $BR_i \equiv \frac{\sigma(e^+e^-\to i)}{\sigma(e^+e^-\to b\bar{b})}$, assuming Υ-states to be isolated
- we need a $\Upsilon(10753)$, but its parameters are currently not well determined
  - conventional $\Upsilon(3D)$ within large range of possibilities
  - additional around 10.75 GeV would be beneficial → BelleII
- we find $\Upsilon(4S)$ mass about 10-20 MeV higher than PDG, $\Upsilon(11020)$ about twice as broad
- electronic widths $\Gamma_{e^+e^-}$ significantly smaller than previously thought
- missing channels: measurements of $e^+e^- \to B^*\bar{B}^{(*)}\pi$ would be helpful (promising?)

Thank you for your attention!

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