A continuum Schwinger method for pion GPDs

Observable implications at future electron ion colliders

Jorge Segovia

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Email: jsegovia@upo.es

Generalized parton distributions (GPDs)

(GPD) – Generalized parton distributions:

Non-local quark and gluon operators, evaluated between hadron states in non-forward kinematics and projected onto the light front.

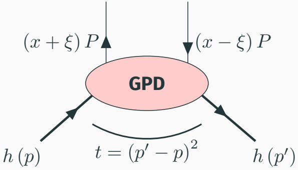
[Fortsch.Phys.:42(1994)101]
[Phys.Lett.B:380(1996)417]
[Phys.Rev.D:55(1197)7114]

Example: Leading twist chiral-even GPDs of a spinless hadron.

$$H^q_\pi(x,\xi,t) = \frac{1}{2}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\left\langle\pi(p')\right|\bar{\psi}^q(-\lambda n/2)\,\gamma^\mu\psi^q(\lambda n/2)\left|\pi(p)\right\rangle n_\mu\,,$$

$$H^g_\pi(x,\xi,t) = \int\frac{d\lambda}{2\pi}e^{i\lambda x}\left\langle\pi(p')\right|G^{\mu\alpha}(-\lambda n/2)\,G^\nu_\alpha(\lambda n/2)\left|\pi(p)\right\rangle n_\mu n_\nu$$

GPDs: Definition and properties



x: Momentum fraction of P.

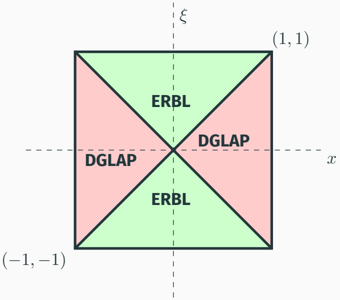
ξ: Fraction of momentum longitudinally transferred.

t: Momentum transfer.

Kinematics:

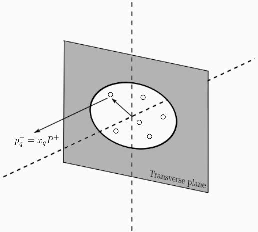
[Phys.Rept:388(2003)41]

- **DGLAP** ($|x| > |\xi|$): Emits/takes a quark ($x > 0$) or antiquark ($x < 0$).
- **ERBL:** ($|x| < |\xi|$): Emits pair quark-antiquark.



GPDs: Definition and properties

Probabilistic interpretation:
Probability amplitude of finding a parton at a given position in transverse plane carrying a momentum fraction "x" of the hadron's averaged light-cone momentum.
[Phys.Rev.D:66(2002)119903]



Striking properties:

"3D picture" of hadrons

- PDFs as forward limit.
- Electromagnetic and gravitational FFs as Mellin moments.
- Parametrize DVCS amplitudes through CFFs.
  [e.g. Phys.Rev.D:59(1999)07009, Phys.Rev.D:56(1997)2982]

GPDs: Definition and properties

- **Support:** **Analyticity/Causality**
  [Phys.Lett.B:428(1998)359]

$$(x,\xi) \in [-1,1] \otimes [-1,1]$$

- **Polynomiality:** Order-m Mellin moments are degree-$(m+1)$ polynomials in ξ. **Lorentz invariance**
  [X.Ji-JPG:1181(24)1998, A.Radyushkin-PLB:81(449)1999]

$$\int_{-1}^{1} dx x^m H(x,\xi,t) = \sum_{\substack{k=0 \\ k \text{ even}}}^{m+1} c_k^{(m)}(t)\,\xi^k$$

- **Positivity:** **Positivity of Hilbert space norm**
  [P.V.Pobylitsa-PRD:114015(65)2002, B.Pire et al.-EPJC:103(8)1999]

$$|H^q(x,\xi,t=0)| \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)} \quad , \qquad |x| \geq \xi$$

- **Low energy soft-pion theorem** **PCAC/Axial-Vector WTI**
  [M.V.Polyakov-NPB:231(555)1999, C.Mezrag et al.-PLB:190(741)2015]

GPD modeling: State of the art

1. **Overlap representation**
   [Nucl.Phys.B596(2001)33]

   Based on LFWFs, $\Psi^q\left(x, k_\perp^2\right)$

   } Polynomiality ? Positivity ✓

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2. **Double Distribution representation**
   [Fortsch.Phys.:42(1994)101, hep-ph/0101225]

   Relying on Radon transform, $\mathcal{R}$

   } Polynomiality ✓ Positivity ?

Problem: Different modeling strategies and different issues

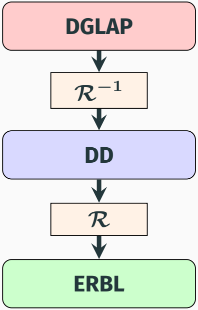
Solution

Covariant extension: Given a DGLAP GPD, the covariant extension allows for computing the corresponding ERBL GPD such that polynomiality is satisfied. [Eur.Phys.J.C:77(2017)12,906]

Covariant extension

Covariant extension: Given a DGLAP GPD, the covariant extension allows for computing the corresponding ERBL GPD such that polynomiality is satisfied.[Eur.Phys.J.C:77(2017)12,906]

$$H\left(x,\xi\right)=\mathcal{R}\left[h\left(\beta,\alpha\right)\right]+\boldsymbol{\frac{1}{|\xi|}D^{+}\left(\frac{x}{\xi}\right)+sign\left(\xi\right)D^{-}\left(\frac{x}{\xi}\right)}$$



1. Build positive DGLAP GPD

2. Covariant extension: ERBL GPD

GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Poby.-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓

3. Soft pion theorem: fix $D^{\pm}\left(\alpha,0\right)$

GPD modeling: DGLAP region (I)

1. Overlap representation [Nucl.Phys.B596(2001)33]

$$H^q(x,\xi,t)\big|_{|x|\geq\xi} = \int \frac{d^2k_\perp}{16\pi^3}\Psi^{q*}\left(x_-,k^2_{\perp,-}\right)\Psi^q\left(x_+,k^2_{\perp,+}\right)$$

2. Assume factorization of the LFWF

$$\Psi^q\left(x,k_\perp^2\right) \propto \varphi(x)\,\phi\left(k_\perp^2\right)$$

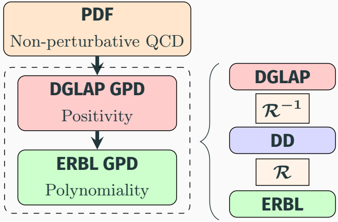
$\Big\downarrow$ (Overalp rep.)

$$H^q(x,\xi,t)\big|_{|x|\geq\xi} = \sqrt{q\left(\frac{x-\xi}{1-\xi}\right)q\left(\frac{x+\xi}{1+\xi}\right)}\,\Phi(x,\xi,t)$$

$\Big\downarrow$ $(t=0)$

$$H^q(x,\xi,0)\big|_{|x|\geq\xi} = \sqrt{q\left(\frac{x-\xi}{1-\xi}\right)q\left(\frac{x+\xi}{1+\xi}\right)}$$

[Phys. Lett. B 815 (2021) 136158]
[Chin. Phys. C 46 (2022) 1, 013105]
[Phys. Rev. D 105 (2022) 9, 094012]



PDFs are the sole ingredient needed to build positive DGLAP GPD

GPD modeling: DGLAP region (II)

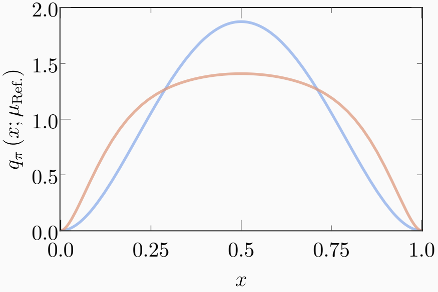
1. Under certain PTIR, chiral symmetry allows to factorize LFWF:

$$\Psi_\pi^{\lambda_1\lambda_2}\left(x,k_\perp^2\right)=\sqrt{q_\pi\left(x\right)}\frac{i^{\lambda_1\lambda_2}M^2}{\left(k_\perp^2+M^2\right)^2}$$

2. **Positive** pion GPD **[Phys. Rev. D 105 (2022) 9, 094012]**

$$H_\pi^q\left(x,\xi,t\right)|_{\text{DGLAP}}=\frac{\sqrt{q_\pi\left(x_-\right)q_\pi\left(x_+\right)}}{\left(1+z^2\right)^2}\left[3+\frac{1-2z}{1+z}\frac{\text{arctanh}\left(\sqrt{\frac{z}{1+z}}\right)}{\sqrt{\frac{z}{1+z}}}\right]$$

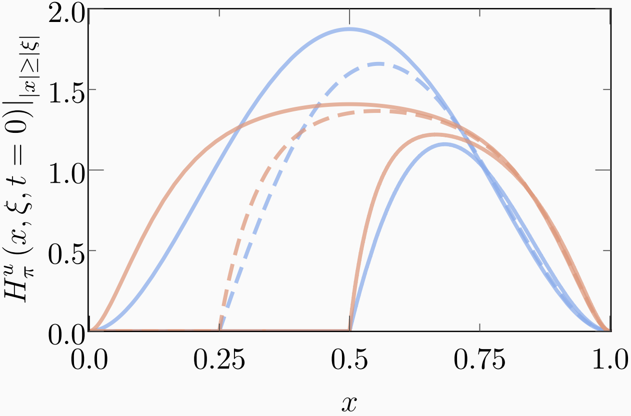
$$z=-t\left(1-x\right)^2/4M^2\left(1-\xi^2\right)$$



Two models:

- Algebraic model
  $q_\pi\left(x\right)=30x^2\left(1-x\right)^2$
- Realistic model (DSE)
  [Phys.Rev.D:101(2020)5,054014]
  $q_\pi\left(x\right)=\mathcal{N}_q x^2\left(1-x\right)^2$
  $\times\left[1+\gamma x\left(1-x\right)+\rho\sqrt{x\left(1-x\right)}\right]$

GPD modeling: DGLAP region (III)



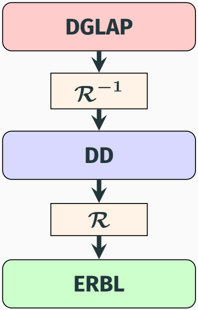
GPD modeling: ERBL region (I)

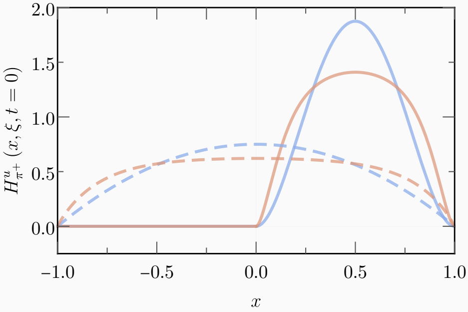
Covariant extension:

$$H^q(x,\xi,t) = \mathcal{R}\left[h(\beta,\alpha,t)\right] + \frac{1}{|\xi|} D^+\left(\frac{x}{\xi},t\right) + sgn(\xi)\, D^-\left(\frac{x}{\xi},t\right)$$

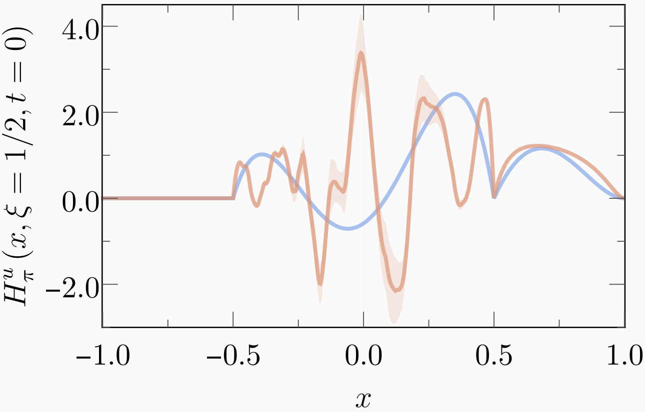
Fix D-terms with soft pion theorem:
[Nucl.Phys.B:555(1999)231, Phys.Lett.B:741(2015)190]

$$H_\pi^{I=0}(x,\xi,t)\Big|_{\xi=1,t=0} = H_\pi(x,\xi,t) - H_\pi(-x,\xi,t)\Big|_{\xi=1,t=0} = 0$$
$$H_\pi^{I=1}(x,\xi,t)\Big|_{\xi=1,t=0} = H_\pi(x,\xi,t) + H_\pi(-x,\xi,t)\Big|_{\xi=1,t=0} = \varphi_\pi\left(\frac{1+x}{2}\right)$$





GPD modeling: ERBL region (II)



Phenomenology of pion GPDs: Sullivan process (I)

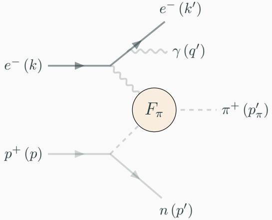
Sullivan process [Phys.Rev.D:5(1972)1732]

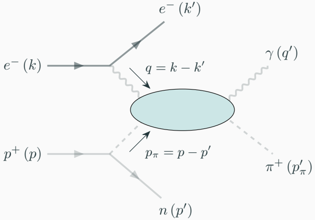
Deep inelastic electron-proton scattering with πn fixed final states.

One-pion-exchange approximation: [Eur.Phys.J.C:58(2008)179]

- $|t|^{\text{Max.}} = 0.6\,\text{GeV}^2$
- Factorization: $\sigma_L^{\gamma^*} >> \sigma_\perp^{\gamma^*}$

} Met at EIC [arXiv[phys.ins-det]2103.05419]

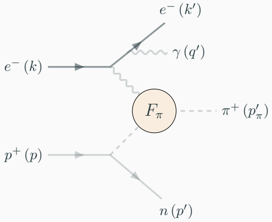


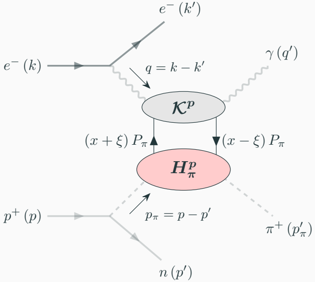


Employed for EFFs
[Phys.Rev.C:78(2008)045203]

Provide access to pion GPDs

Phenomenology of pion GPDs: Sullivan process (II)

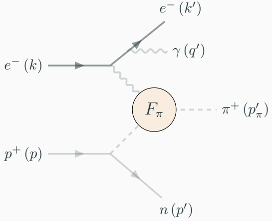


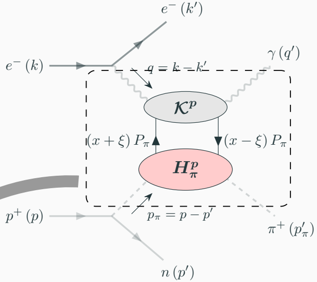


$$\mathcal{M}_{\text{Sullivan}} = i\sqrt{2}g_{\pi NN}\bar{u}_\sigma(p')\gamma_5 u_\sigma(p)\frac{i}{t-m_\pi^2}F(t;\Lambda)\mathcal{M}_{e\pi\to e\gamma\pi}$$

$$\mathcal{M}_{e\pi\to e\gamma\pi} = \mathcal{M}_{\text{BH}}+\mathcal{M}_{\text{DVCS}} \Rightarrow \left|\mathcal{M}_{e\pi\to e\gamma\pi}\right|^2 = \left|\mathcal{M}_{\text{DVCS}}\right|^2+\left|\mathcal{M}_{\text{BH}}\right|^2\pm\mathcal{I}(\lambda)$$

Phenomenology of pion GPDs: Sullivan process (II)





$$\mathcal{M}_{\text{Sullivan}} = i\sqrt{2} g_{\pi NN} \bar{u}_\sigma(p') \gamma_5 u_\sigma(p) \frac{i}{t - m_\pi^2} F(t;\Lambda) \mathcal{M}_{e\pi\to e\gamma\pi}$$

$$\mathcal{M}_{e\pi\to e\gamma\pi} = \mathcal{M}_{\text{BH}} + \mathcal{M}_{\text{DVCS}} \Rightarrow |\mathcal{M}_{e\pi\to e\gamma\pi}|^2 = |\mathcal{M}_{\text{DVCS}}|^2 + |\mathcal{M}_{\text{BH}}|^2 \pm \mathcal{I}(\lambda)$$

DVCS amplitudes are parametrized by hadron GPDs through CFFs.
[Phys.Rev.D:55(1997)7114]

$$\mathcal{M}_{\text{DVCS}} \propto \mathcal{H}_\pi(\xi, t; Q^2) = \sum_{p=q,g} \int_{-1}^{1} \frac{dx}{\xi} \mathcal{K}^p\left(\frac{x}{\xi}, \frac{Q^2}{\mu_F^2}, \alpha_S(\mu_F^2)\right) H_\pi^p(x, \xi, t; \mu_F^2)$$

Phenomenology of pion GPDs: The path towards DVCS

1. Start from state-of-the-art **models for pion GPDs**.
2. Leading order **scale evolution** for GPDs.
3. Convolution with coefficient functions: **CFFs**.

GPD Modeling

[Phys.Lett.B 815 (2021) 136158]

[Chin. Phys. C 46 (2022) 1, 013105]

[Phys. Rev. D 105 (2022) 9, 094012]

+

QCD evolution

[Eur. Phys. J. A 58 (2022) 1, 10]

[Phys. Rev. D 105 (2022)9, L091502]

[Eur. Phys. J. C 82 (2022) 10, 888]

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LO/NLO CFFs

[Phys. Rev. D 105 (2022) 9, 094012]

[Phys. Rev. Lett. 128 (2022) 20, 202501]

Phenomenological study of pion's structure

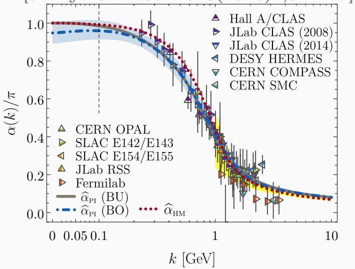
Phys. Rev. Lett. 128 (2022) 20, 202501

Scale evolution: Hadron scale

Reference scale: $\mu_{\text{Ref.}} = 331$ MeV

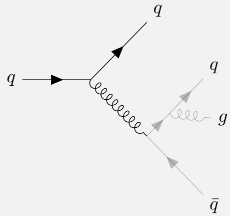
<u>*No gluons nor sea quarks*</u>

[J.Phys.Conf.Ser.1643(2020)1,012177]



Parton splitting (DGLAP)

[Nucl.Phys.B:126(1977)298]

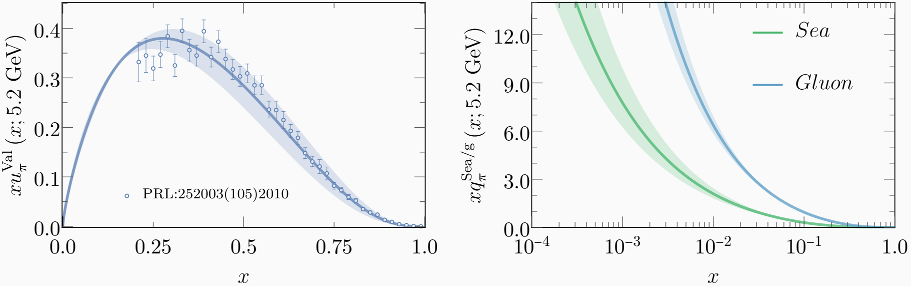


Renormalization group equations for GPD operators yield:

[Phys.Rev.D:55(1997)7114, Eur.Phys.J.C:82(2022)10,888]

$$\frac{dH^{(\pm)}\left(x,\xi,t;\mu^2\right)}{d\log\mu^2} = \frac{\alpha_s\left(\mu^2\right)}{4\pi}\int_x^\infty \frac{dy}{y}\mathcal{P}^{(\pm)}\left(y,\kappa,\alpha_S\left(\mu^2\right)\right)H^{(\pm)}\left(\frac{x}{y},\xi,t;\mu^2\right)$$

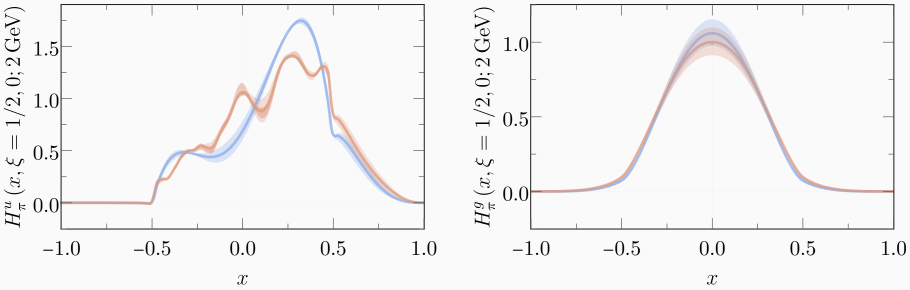
Scale evolution: PDFs and GPDs



- Non-zero gluon distribution generated by scale evolution.

Scale evolution: PDFs and GPDs

[Phys. Rev. D 105 (2022) 9, 094012]

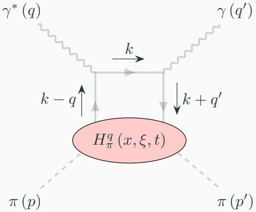


- Non-zero gluon distribution generated by scale evolution.
- Uncertainty band narrowed.
- Continuity along $x = \xi$ lines.

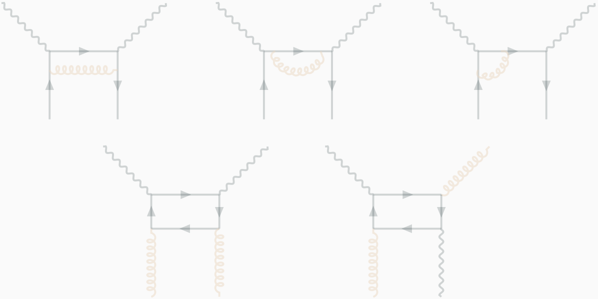
Compton form factors (I)

$$\mathcal{H}_\pi^p\left(\xi, t; Q^2\right) = \int_{-1}^{1} \frac{dx}{\xi} \mathcal{K}^p\left(\frac{x}{\xi}, \frac{Q^2}{\mu_F^2}, \alpha_s\left(\mu_F^2\right)\right) H_\pi^p\left(x, \xi, t; \mu_F^2\right)$$

Leading order

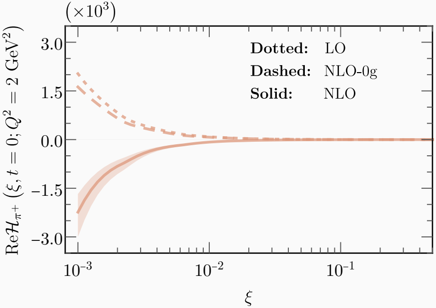


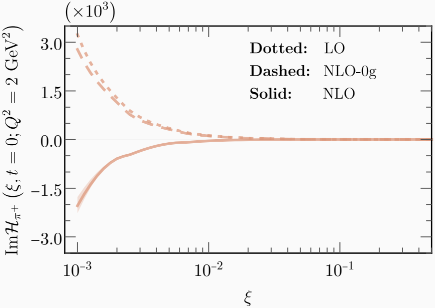
Next to leading order



Compton Form Factors (II)

[Phys. Rev. D 105 (2022) 9, 094012]



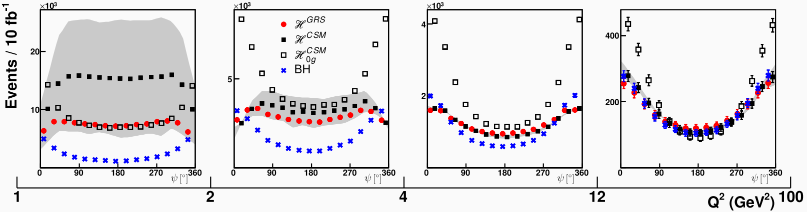


- Small effect of NLO corrections to quark amplitudes.
- Dominant effect of gluons.

Gluon dominance makes essential at least NLO accuracy in any phenomenlogical analysis of DVCS at an EIC.

Phenomenology of pion GPDs: EIC event-rates

[Phys. Rev. Lett. 128 (2022) 20, 202501]



The cross-section can be written as:
[Phys.Rev.D:79(2009)014017s]

$$\left|\mathcal{M}_{e\pi\to e\gamma\pi}\right|^2 = C_{\mathrm{BH}} F_\pi^2\left(t_\pi\right) \pm \frac{F_\pi}{Q}\left(C_{\mathrm{int}}\left(\psi\right)\mathrm{Re}\mathcal{H}_\pi + \lambda S_{\mathrm{Int}}\left(\psi\right)\mathrm{Im}\mathcal{H}_\pi\right) + \frac{1}{Q^2} C_{\mathrm{DVCS}}\left|\mathcal{H}_\pi\right|^2$$

$$\left|\mathcal{H}_\pi\right|^2 = \mathrm{Re}^2\left(\mathcal{H}_\pi\right) + \mathrm{Im}^2\left(\mathcal{H}_\pi\right) \sim \mathrm{Re}^2\left(\left|\mathcal{H}_\pi^{q,\mathrm{LO}}\right| - \left|\mathcal{H}_\pi^g\right|\right) + \mathrm{Im}^2\left(\left|\mathcal{H}_\pi^{q,\mathrm{LO}}\right| - \left|\mathcal{H}_\pi^g\right|\right)$$

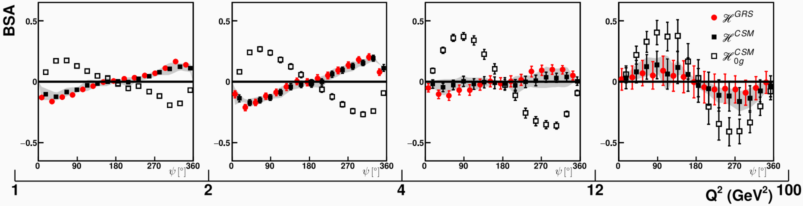
Low energy, $Q^2 < 2\ \mathrm{GeV}^2$ Intermediate energies

$$\left|\mathcal{H}_\pi\right|^2_{Q^2<2\ \mathrm{GeV}^2} \sim \left|\mathcal{H}_\pi^g\right|^2_{Q^2<2\ \mathrm{GeV}^2} > \left|\mathcal{H}_\pi^{q,\mathrm{LO}} - \mathcal{H}_\pi^g\right|^2_{2<Q^2<12\ \mathrm{GeV}^2} \sim \left|\mathcal{H}_\pi\right|^2_{2<Q^2<12\ \mathrm{GeV}^2}$$

Quark-gluon “interference”: Modulates expected number of events

Phenomenology of pion GPDs: EIC beam-spin asymmetries

[Phys. Rev. Lett. 128 (2022) 20, 202501]



$$\mathcal{A}(\psi) = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}} \sim s^{1}_{\text{Int}} \text{Im}\mathcal{H}_{\pi} \sin\psi$$

$$\text{Im}\left(\mathcal{H}_{\pi}\right) = \text{Im}\left(\mathcal{H}_{\pi}^{q,\text{LO}}\right) + \text{Im}\left(\mathcal{H}_{\pi}^{q,\text{NLO}}\right) + \text{Im}\left(\mathcal{H}_{\pi}^{g}\right) \sim \text{Im}\left|\mathcal{H}_{\pi}^{q,\text{LO}}\right| - \text{Im}\left|\mathcal{H}_{\pi}^{g}\right|$$

Sign inversion: **Smoking gun** for gluon dominance

Summary and perspectives

1. *Can we obtain "theoretically complete" pion GPDs?* **YES!**

- Positivity: "Factorization hypothesis"
- Polynomiality: Covariant extension
- Support and soft-pion theorem
- Continuity along $x = \xi$

2. *What can we learn from them about the real world?*

- Hadron scale hypothesis
- GPDs might be accessible in future experiments
- Crucial role of gluons within pions
  - Modulate expected statistics
  - Manifest themselves through beam-spin asymmetries

Future work

- "Incomplete" covariant extension: Trigger Lattice efforts
- Refine evolution
- Neutral pions: Direct window to GPDs?