Gravitational form factors and mechanical properties of the proton

Hadron Physics
Beyond Tomography

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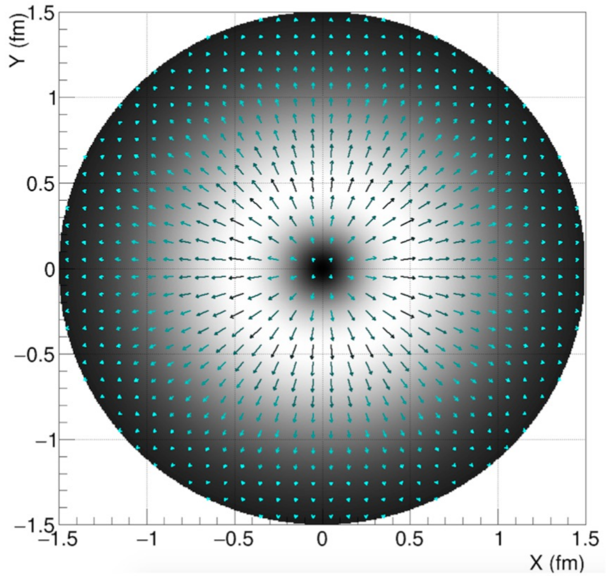
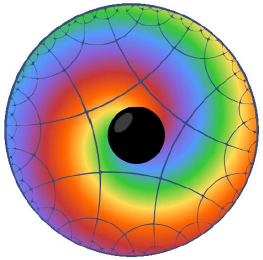
10th workshop of the APS Topical Group on Hadronic Physics - April 14, 2023



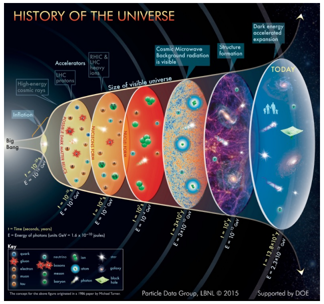
V. Burkert., L. Elouadrhiri, F.X. Girod
Nature 557 (2018) no.7705, 396-399
Colloquium: Submitted to RMP

Basic questions about the proton

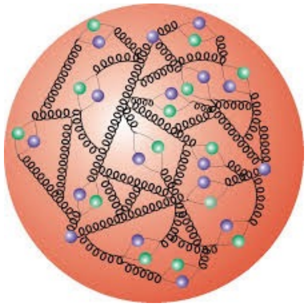
- Proton's make up nearly 90% of the (normal) matter in the universe. Elementary valence quarks contribute only few percent to the proton mass. What is the origin of its mass?



- How did quarks hadronize and form protons as the universe cooled below the Hagedorn temperature? What is the origin of confinement ?



- How are the strong forces distributed in space to keep quarks confined and make protons stable particles.



Fundamental global properties of the proton

The structure of strongly interacting particles can be probed by means of the other fundamental forces: ***electromagnetic*****,** ***weak*****, and (in principle)** ***gravity*****.**

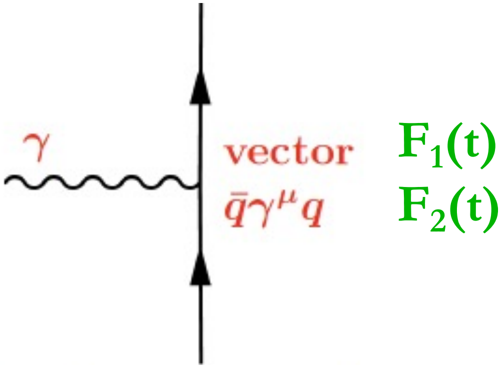
em:	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N'\|J^\mu_{\text{em}}\|N\rangle$	$\longrightarrow$	Q_{prot}	$=$	$1.602176487(40) \times 10^{-19}\text{C}$
	vector			μ_{prot}	$=$	$2.792847356(23)\mu_N$
weak:	PCAC	$\langle N'\|J^\mu_{\text{weak}}\|N\rangle$	$\longrightarrow$	g_A	$=$	$1.2694(28)$
	axial			g_p	$=$	$8.06(0.55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\text{grav}}\|N\rangle$	$\longrightarrow$	M_{prot}	$=$	$938.272013(23)\,\text{MeV}/c^2$
				J	$=$	$\frac{1}{2}$
	tensor			D	$=$	**?**

P. Schweitzer et al., arXiv:1612.0672, 2016.

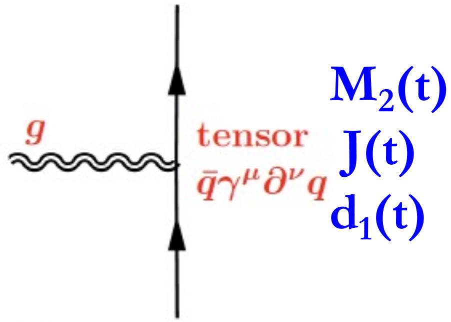
The D-term is the “last unknown global property” of the nucleon

Probing basic properties of the proton

- **Electromagnetic properties:** probed with photons
  - **Charge** - electromagnetic form factors, inelastic structure functions, proton charge radius, charge densities and current densities for N & N*
  - **Magnetic moment** - helicity densities



- **Gravitational properties:** probed with gravitons
  - **Mass:** energy and mass densities
  - **Spin:** angular momentum distribution
  - **D-term:** dynamical stability, normal and shear forces, pressure distribution



2018 Review of Particle Physics.
M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)

GAUGE AND HIGGS BOSONS

graviton $J = 2$

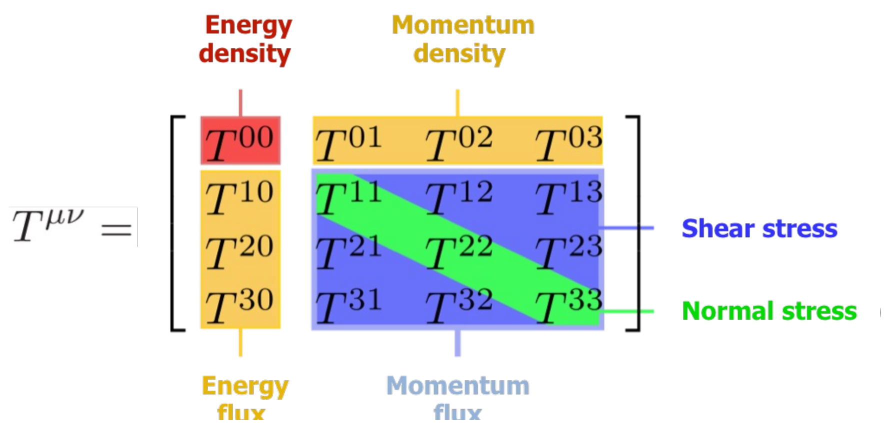
graviton MASS	$< 6 \times 10^{-32}$ eV

Probing mechanical properties of the proton?

Gravitational Interaction of Fermions

Yu. Kobzarev and L.B. Okun, JETP 16, 5 (1963)

Energy-Momentum Structure Form Factors of Particles *Heinz Pagels, Phys. Rev. 144 (1966) 1250-1260*

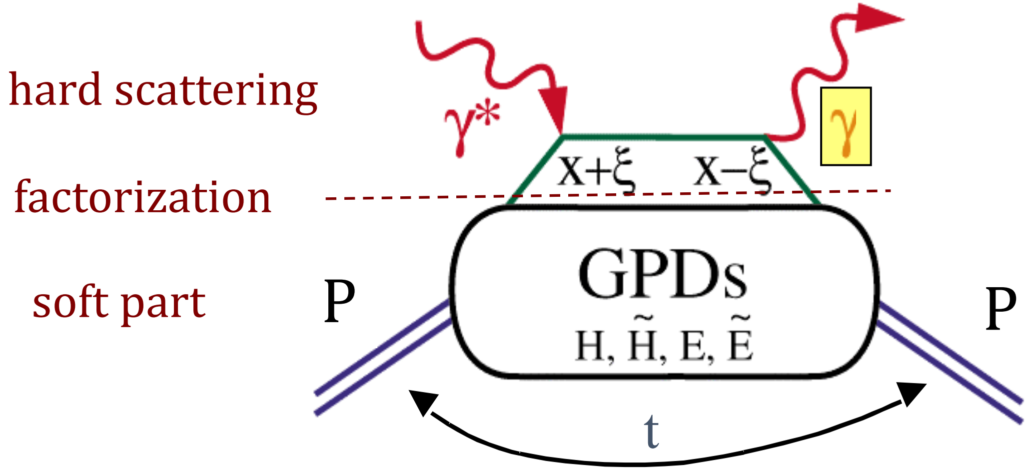


$$T_{ij}(\vec{r}\,) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(r)\delta_{ij}$$

"….. , there is very little hope of learning anything about the detailed mechanical structure of a particle, because of the extreme weakness of the gravitational interaction" (*H. Pagels)*

Generalized Parton Distributions (GPDs)

Deeply virtual Compton scattering (DVCS)



$$\xi = \frac{x_B}{2 - x_B}$$

(in the Bjorken regime)

GPD: *H(x,ξ, t), …*

Proton stays intact

D. Müller (1994) *X. Ji (1996)* *A.Radyushkin (1996)*

D. Müller et al., F.Phys. 42,1994
X. Ji, PRL 78, 610, 1997
A. Radyushkin, PLB 380, 1996

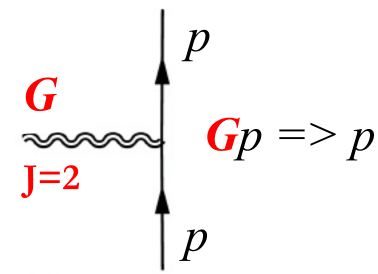
GPDs – GFFs Relations

Nucleon matrix element of the Energy-Momentum Tensor contains three scalar form factors and can be written as:

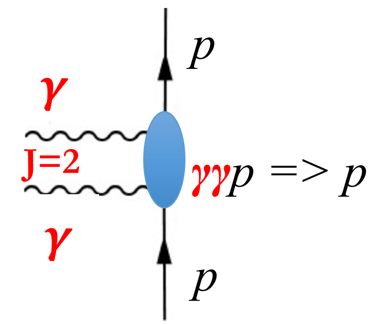
$$\langle p_2|\hat{T}^q_{\mu\nu}|p_1\rangle = \bar{U}(p_2)\left[M_2^q(t)\frac{P_\mu P_\nu}{M} + J^q(t)\frac{\imath(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + d_1^q(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M}\right]U(p_1)$$

$M_2(t)$: Mass/energy distribution inside the nucleon
$J(t)$: Angular momentum distribution
$d_1(t)$: Forces and pressure distribution

Graviton – proton scattering



DVCS



GPDs ⟺ GFFs

$$\int dx\, x\,[H(x,\xi,t) + E(x,\xi,t)] = 2J(t)$$

$$\int dx\, x H(x,\xi,t) = M_2(t) + \frac{4}{5}\xi^2 d_1(t),$$

X. Ji, Phys. Rev. D55, 7114 (1997)

GPD – GFF correspondence

Wikipedia:

"It can be shown that any massless spin-2 field would give rise to a force indistinguishable from gravitation, because a massless spin-2 field would couple to the stress–energy tensor in the same way that gravitational interactions do."

GPDs & Compton Form Factors

- GPDs cannot directly be determined from current DVCS measurements alone.
- We can determine the Compton Form Factor $\mathcal{H}(\xi, t)$
- $\mathcal{H}(\xi, t)$ is related to the corresponding GPD $H(x,\xi,t)$ through an integral over the quark longitudinal momentum fraction x.

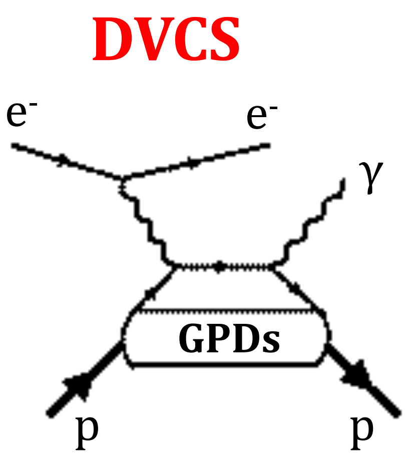
$$\mathcal{H}(\xi,t) = \int_{-1}^{+1} dx H(x,\xi,t) \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right)$$

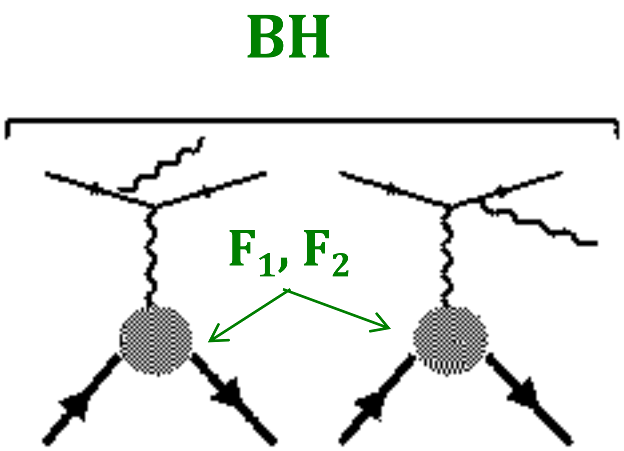
M. Polyakov (2003)

To determine the complex CFF $\mathcal{H}(\xi, t)$ we exploit the interference of the DVCS amplitude with the Bethe-Heitler amplitude that results in a polarized beam spin asymmetry.

M. Polyakov, Phys. Lett. B555 (2003) 57

From GPD to GFF $d_1(t)$ to s(r) and p(r)





F_1: Dirac FF; **F_2**: Pauli FF

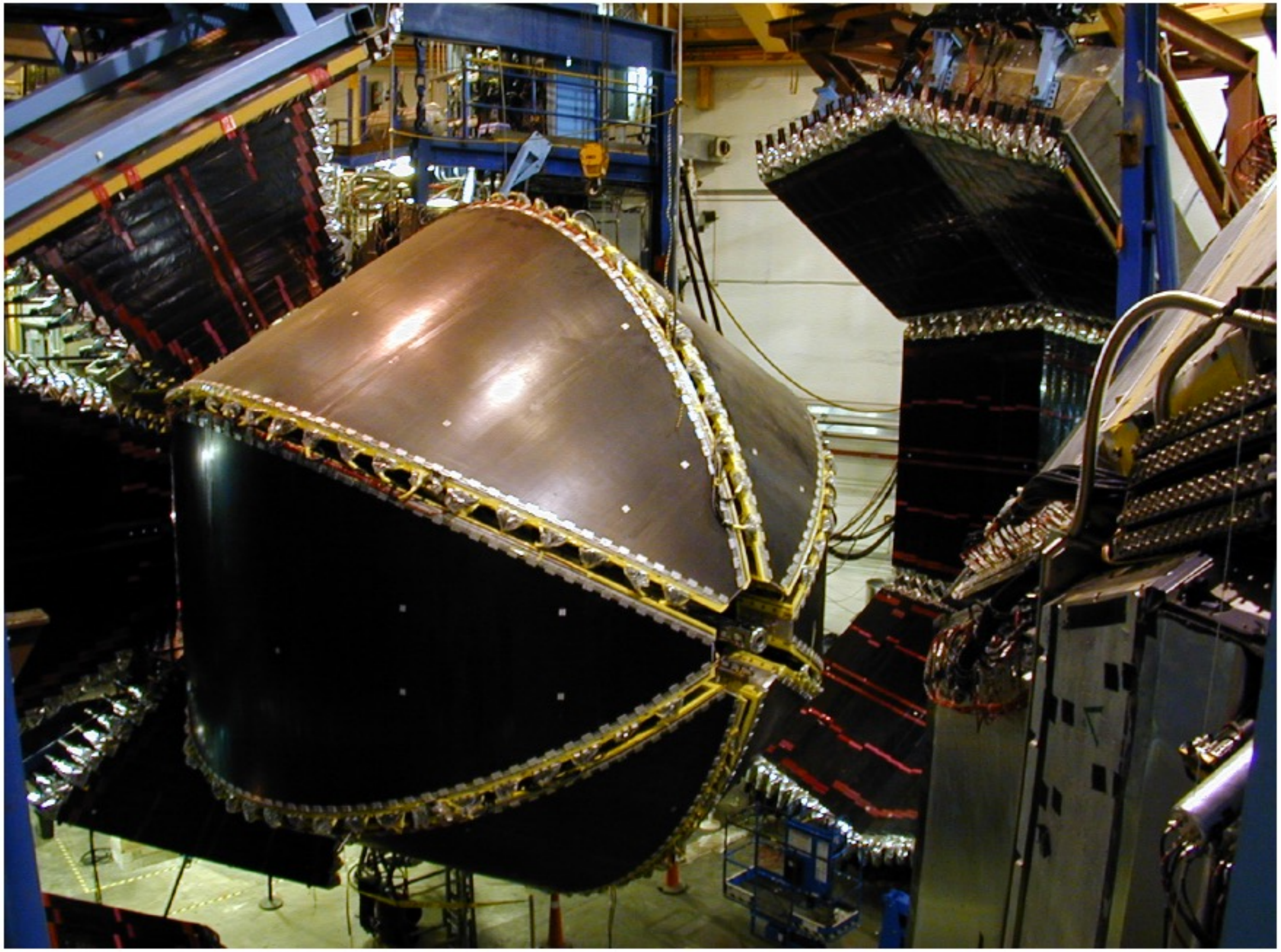
Polarized beam, unpolarized target:

$$\Delta\sigma_{LU} \sim \sin\phi \,\mathrm{Im}\{F_1 \mathcal{H} + ..\} \Rightarrow \mathcal{H}(\xi,t) \Rightarrow d_1(t)$$

Bessel Integral relates $d_1(t)$ to the pressure distribution

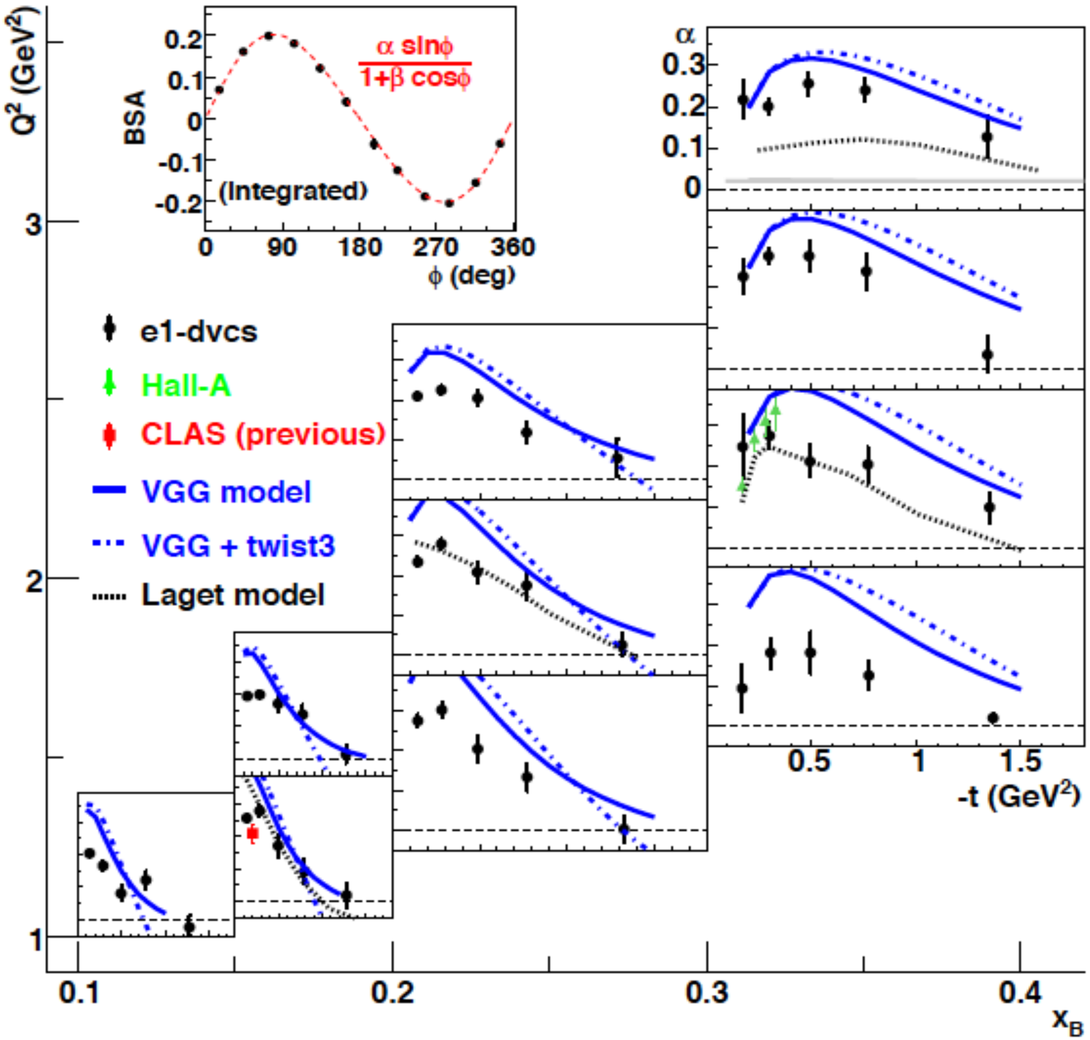
$$d_1(t) \propto \int d^3\mathbf{r}\, \frac{j_0(r\sqrt{-t})}{2t}\, p(r)$$

CLAS Experiment



In operation 1997 – 2012

DVCS Beam Spin Asymmetry

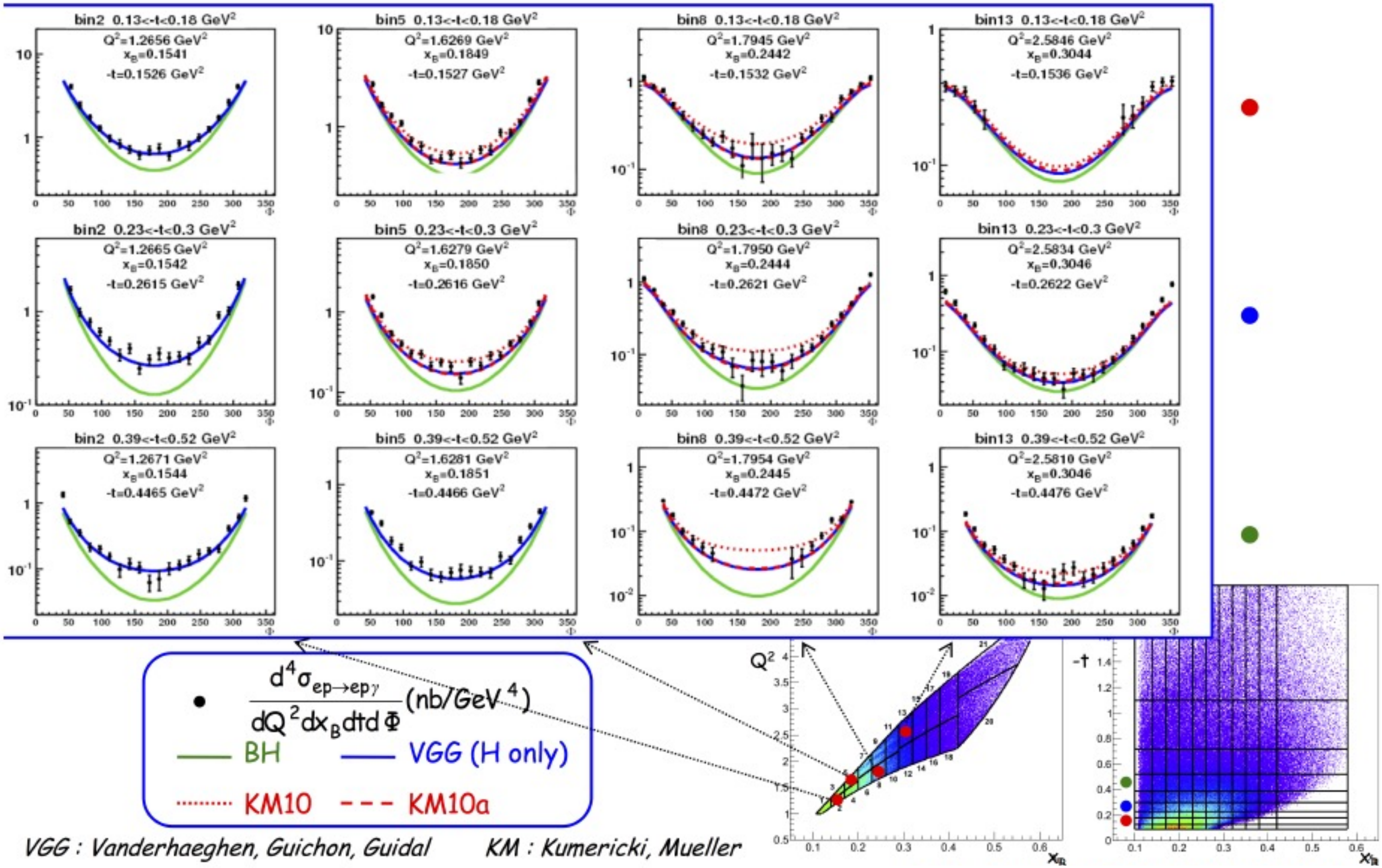


$$F_1\mathcal{H} + \xi G_M\tilde{\mathcal{H}} - \frac{t}{4M^2}F_2\mathcal{E}$$

Precision in a large phase space Q^2, x_B, t

F.X. Girod et al. Phys.Rev.Lett. 100 162002 (2008)

DVCS Unpolarized Cross-Sections



H.S. Jo et al., Phys.Rev.Lett. 115 (2015)

Dispersion Relation Analysis and Global Fits

Compton Form Factor $\mathcal{H}$

$$\mathrm{Re}\mathcal{H}(\xi,t) + i\mathrm{Im}\mathcal{H}(\xi,t) = \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H(x,\xi,t)$$

Beam Spin Asymmetries

$$\mathrm{Im}\mathcal{H}(\xi,t) = \frac{r}{1+\xi}\left(\frac{2\xi}{1+\xi}\right)^{-\alpha(t)}\left(\frac{1-\xi}{1+\xi}\right)^{b}\left(\frac{1-\xi}{1+\xi}\frac{t}{M^2}\right)^{-1}$$

*K. Kumericki, D. Müller, Nucl. Phys. B **841,** 1-58, 2010*
D. Müller, T. Lautenschlager, K, Passek-Kumericki, G. Schaefer, Nucl.B. 884, 438, 2014

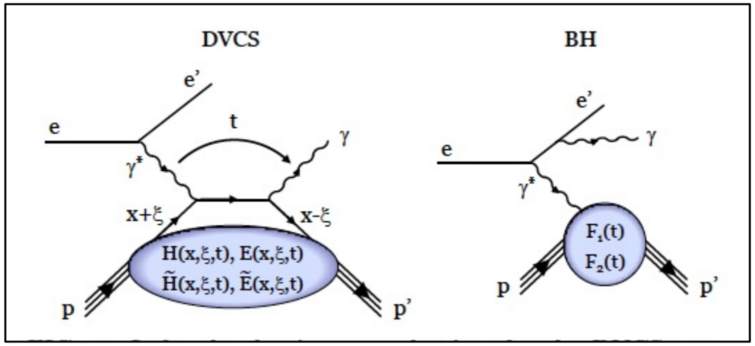
Unpolarized cross sections

Use Dispersion Relations:

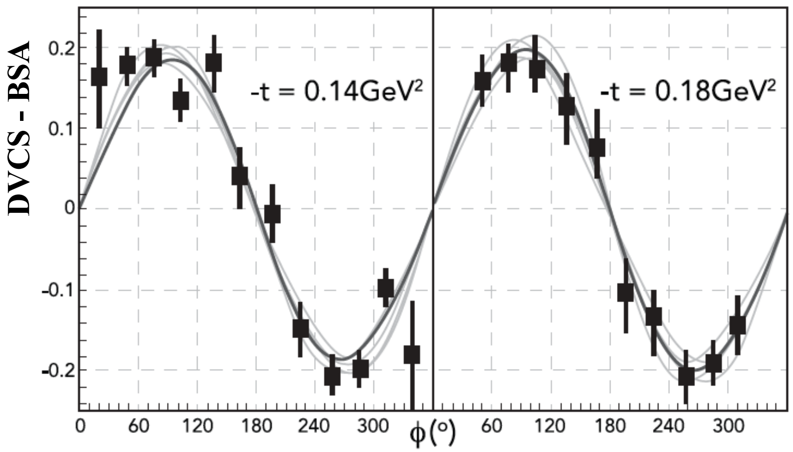
$$\mathrm{Re}\mathcal{H}(\xi,t) \stackrel{\mathrm{LO}}{=} D(t) + \mathcal{P}\int_{-1}^{1} dx \left(\frac{1}{\xi - x} - \frac{1}{\xi + x}\right) \mathrm{Im}\mathcal{H}(x,t)$$

M. Polyakov , C. Weiss, Phys.Rev. D60 (1999) 114017

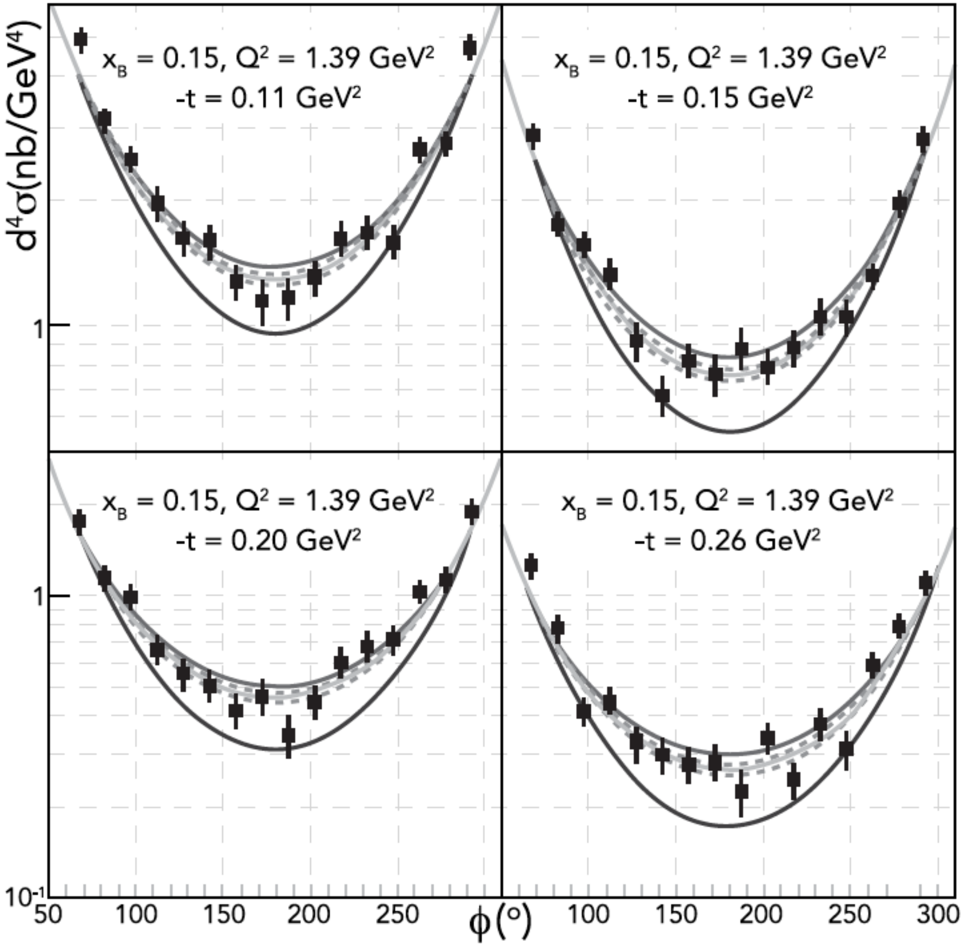
Fit to DVCS data to determine *D*-Term



Samples of Beam Spin Asymmetry

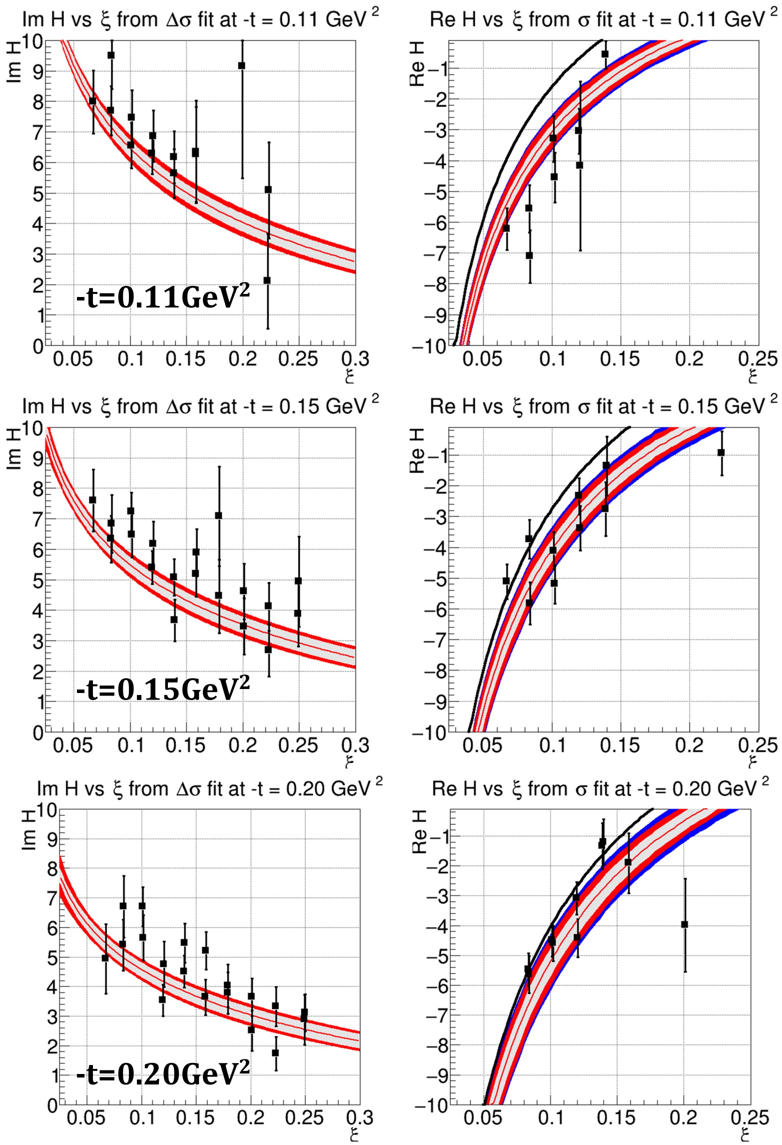


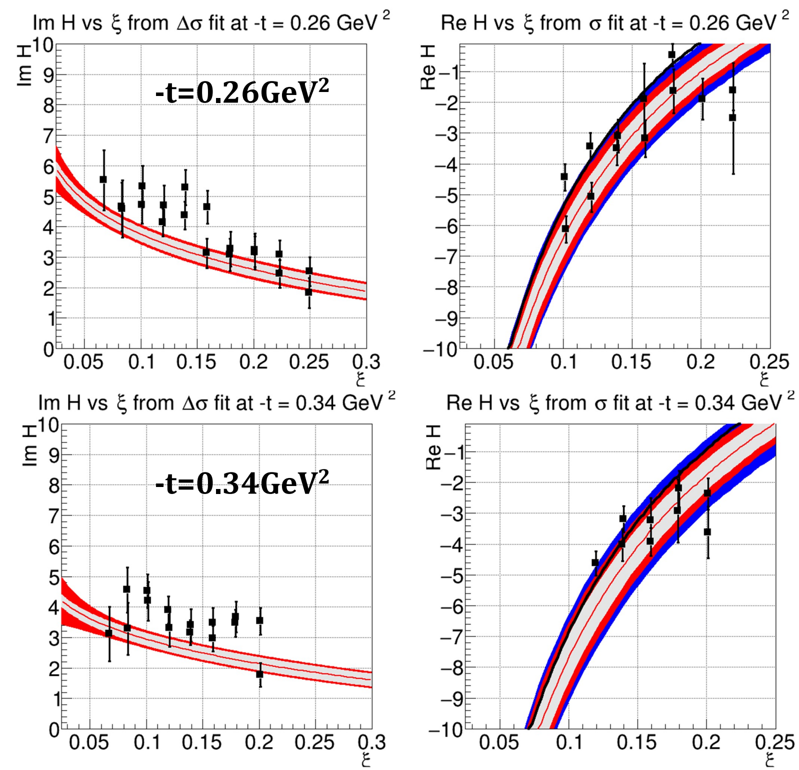
Samples of differential cross sections with fits



F.X. Girod et al., Phys.Rev.Lett. 100 (2008) 162002 ; H.S. Jo et al., Phys.Rev.Lett. 115 (2015) 212003,

Extraction of Compton Form Factor $\mathcal{H}(\xi,t)$

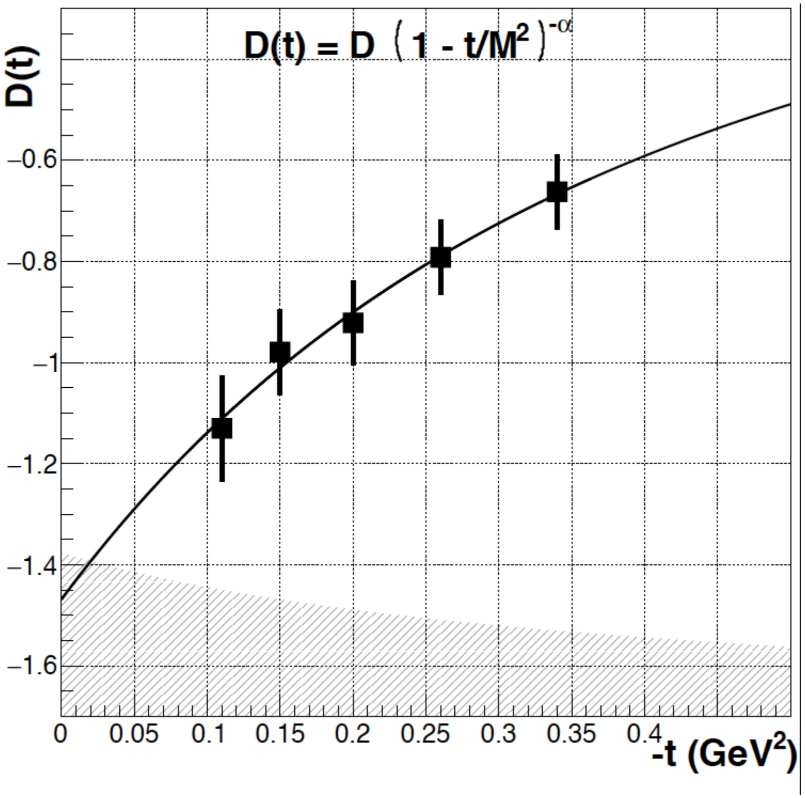




The Real and Imaginary parts of Compton FF $\mathcal{H}(\xi,t)$ for different ξ and t values, resulting from the fit to the BSA and cross section data.

Extraction of D^q (t) for quark distribution

D(t) from CLAS 6 GeV data

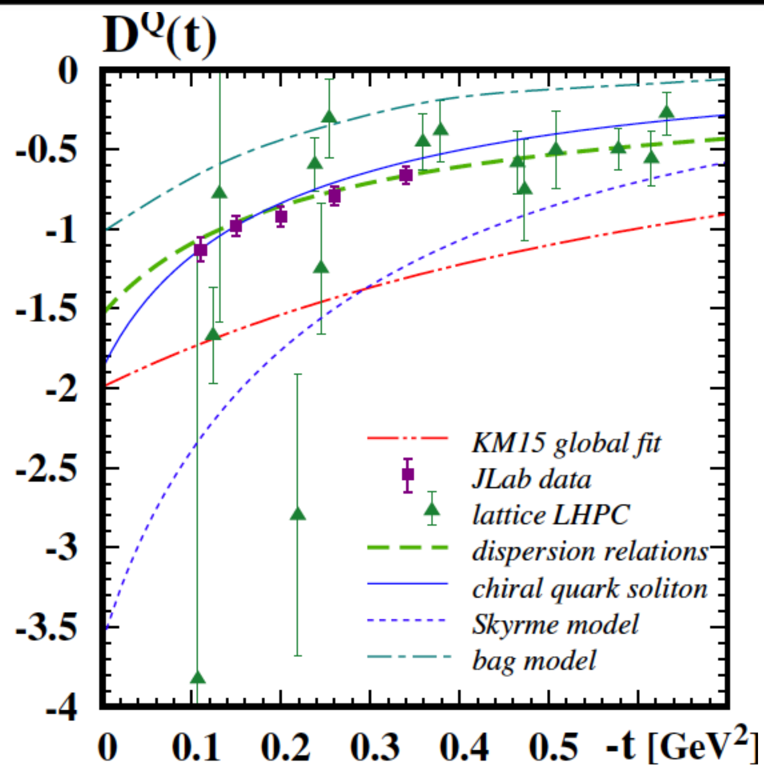


$D^Q(0) = -1.47 \pm 0.10 \pm 0.24$

$M^2 = 1.06 \pm 0.10 \pm 0.15$

$\alpha = 2.76 \pm 0.25 \pm 0.50$

Comparison of $D^Q(t)$ with models



- Chiral Quark Soliton Model
- Dispersion Relations, normalized at t=0.
- Lattice QCD LHPC, no disconnected diagrams
- Global fit – *K.L.M. EPJ A52 (2016) 6 , 157*

M.V. Polyakov, P. Schweitzer
Int. J. Mod. Phys. A 33 (26), 1830025,
arXiv:1805.06596 [hep-ph].

em: $\partial_\mu J^\mu_{\rm em} = 0$	$\langle N'\vert J^\mu_{\rm em}\vert N\rangle$	$\longrightarrow$	$Q = 1.602176487(40)\times 10^{-19}{\rm C}$	
			$\mu = 2.792847356(23)\mu_N$	
weak: PCAC	$\langle N'\vert J^\mu_{\rm weak}\vert N\rangle$	$\longrightarrow$	$g_A = 1.2694(28)$	
			$g_p = 8.06(55)$	
gravity: $\partial_\mu T^{\mu\nu}_{\rm grav} = 0$	$\langle N'\vert T^{\mu\nu}_{\rm grav}\vert N\rangle$	$\longrightarrow$	$m = 938.272013(23)\,{\rm MeV}/c^2$	
			$J = \frac{1}{2}$	
			$\boldsymbol{D} = \boldsymbol{?}$	

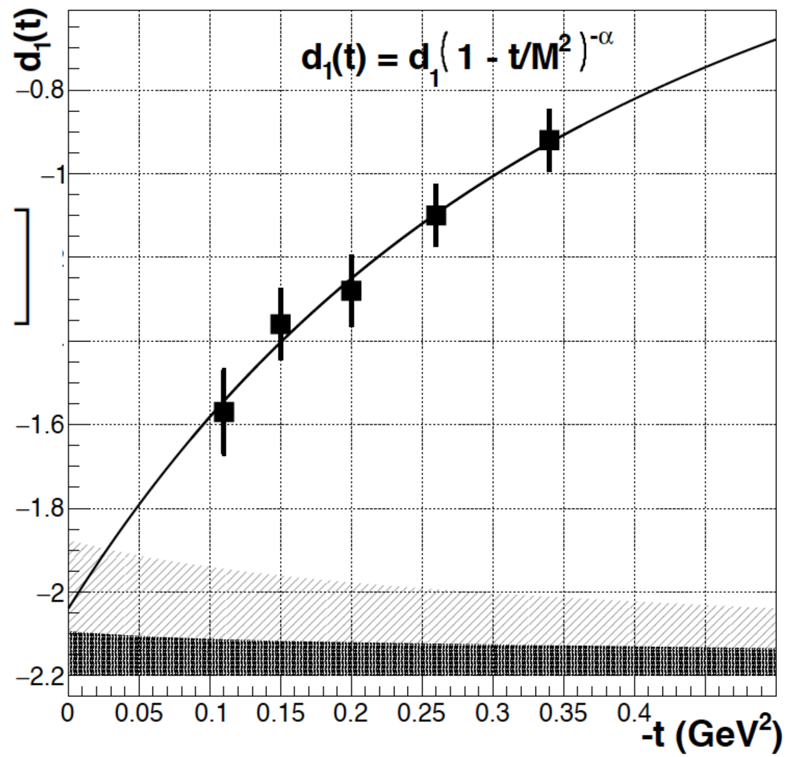
$d^Q_1(t)$ - Gravitational Form Factor

Expansion in Gegenbauer polynomials

$$D(t) = \frac{1}{2}\int_{-1}^{1} \mathrm{d}z \frac{D(z,t)}{1-z}$$ with

$$D(z,t) = (1-z^2)\left[d_1(t) C_1^{3/2}(z) + \cdots\right]$$

$$-1 < z = \frac{x}{\xi} < 1$$



$d_1(0) < 0$ dynamical stability of bound state

$d^Q_1(0) = -2.04 \pm 0.14 \pm 0.33$

First determination of new fundamental quantity.

The pressure distribution inside the proton

$$d_1(t) \propto \int \mathrm{d}^3\mathbf{r}\, \frac{j_0(r\sqrt{-t})}{2t}\, p(r)$$

Repulsive pressure near center

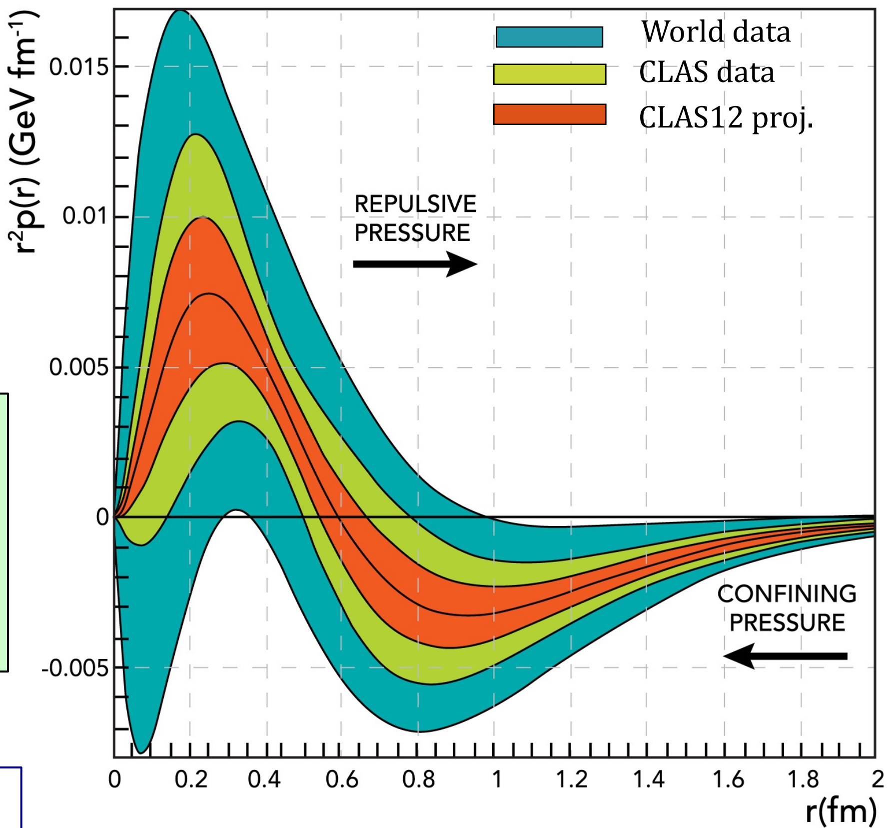
$p(r=0) = 10^{35}$ Pa

Confining pressure at r > 0.6 fm

(in χQSM due to the pion field)

Atmospheric pressure: 10^5 Pa

Pressure in the center of neutron stars ≤ 10^{34} Pa



V.B., L. Elouadrhiri, F.X. Girod

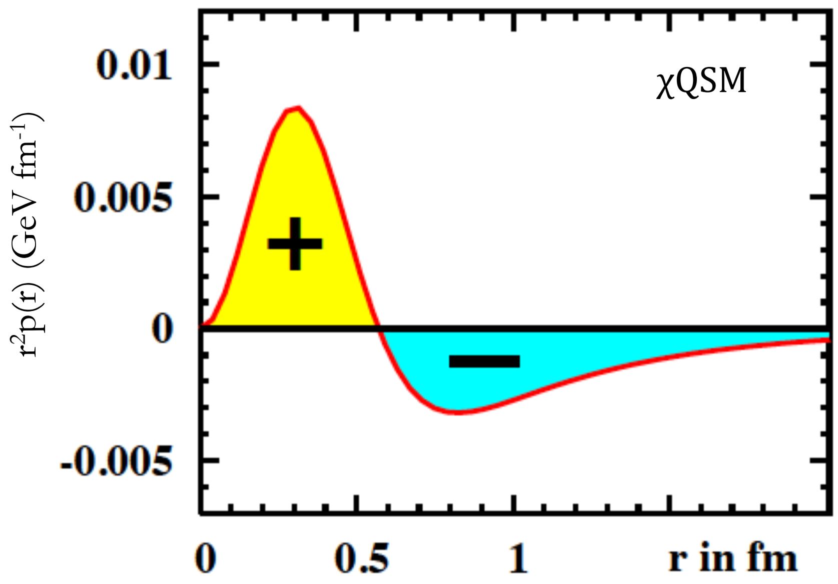
Nature 557 (2018) no.7705, 396-399

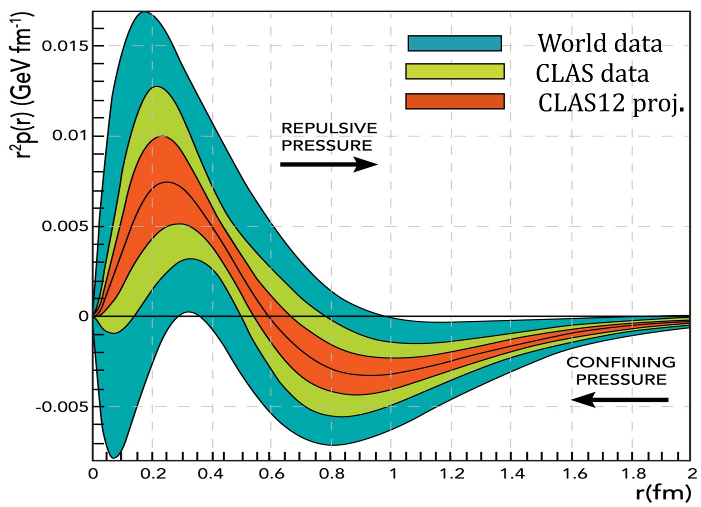
A new direction in experimental nuclear/hadronic physics.

Comparison with χQSM

- Gravitational form factors may be computed in Lattice QCD. No results exist for *p(r)*.
- In the chiral quark-soliton model (χQSM) the proton is modeled as a chiral soliton with the constituent quarks bound by a self-consistent pion field.

K. Goeke et al, Phys.Rev. D75 (2007) 094021





$\mathrm{Int}(r^2 p(r)dr = 0$

The $d_1(t=0) < 0$ is rooted in the spontaneous chiral symmetry breaking (χSB).
In the χQSM the pion field provides the confining pressure at the proton's periphery.

Distribution of forces in the proton

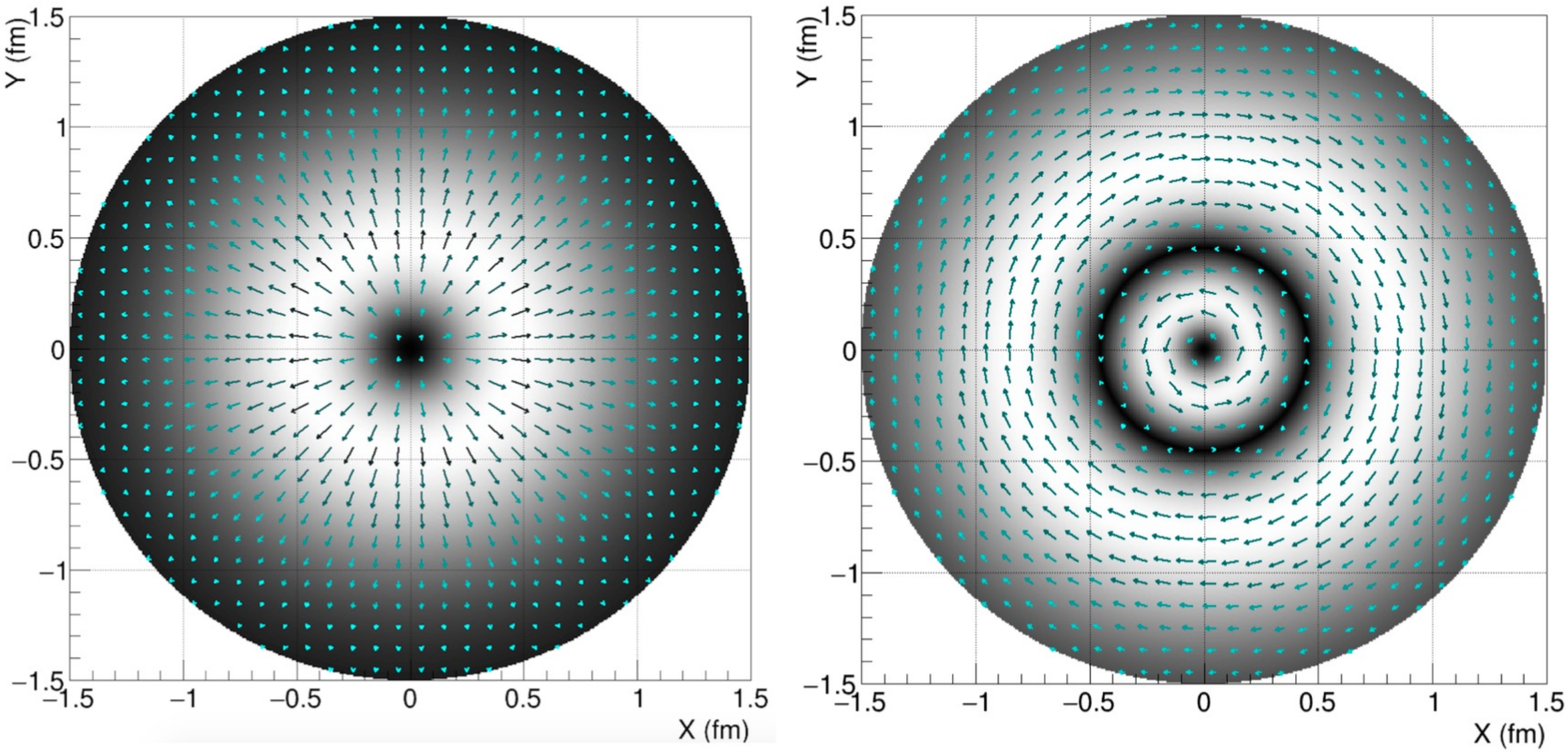
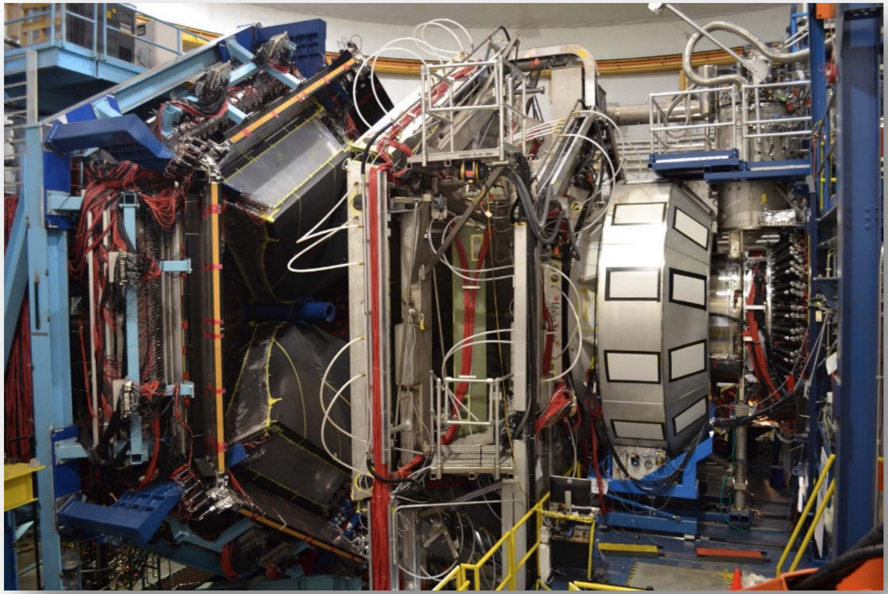


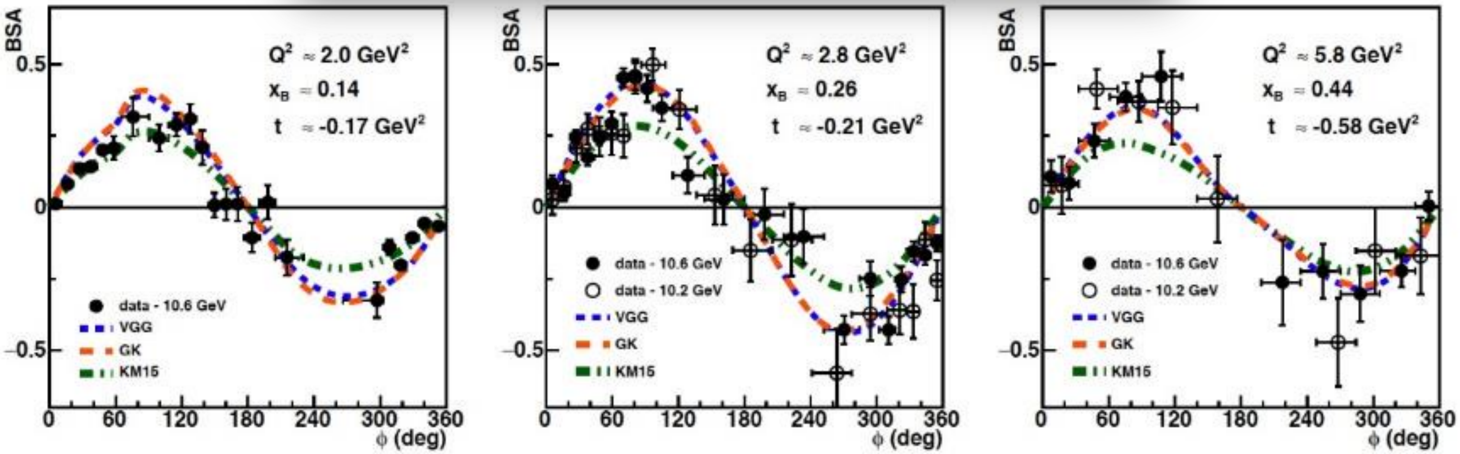
FIG. 14 2D display of the quark contribution to the distribution of forces in the proton as a function of the distance from the proton's center (Burkert et al., 2021b). The light gray shading and longer arrows indicate areas of stronger forces, the dark shading and shorter arrows indicate areas of weaker forces. Left panel: Normal forces as a function of distance from the center. The arrows change magnitude and point always radially outwards. Right panel: Tangential forces as a function of distance from the center. The forces change direction and magnitude as indicated by the direction and lengths of the arrows. They change sign near 0.4 fm from the proton center.

Colloquium: Gravitational Form Factors of the Proton (submitted to RMP)
V. D. Burkert, L. Elouadrhiri, F. X. Girod,1 C. Lorće, P. Schweitzer, and P. E. Shanahan

DVCS Beam-Spin Asymmetries in Extended Valence Region with CLAS12

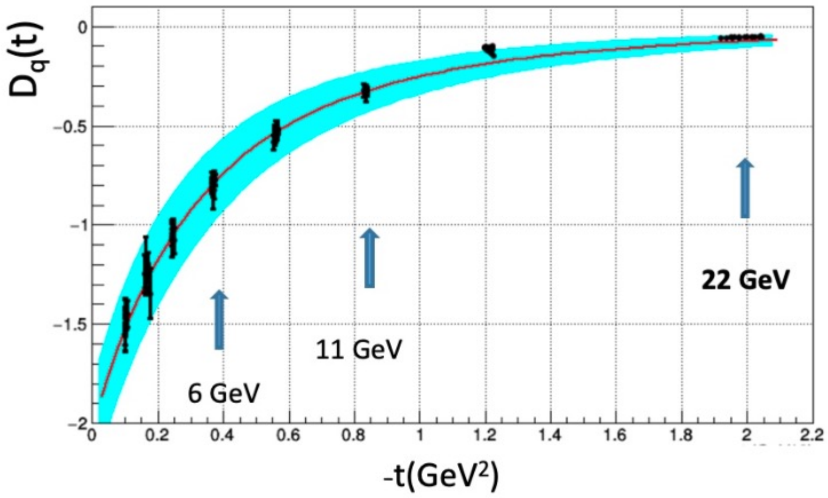
[V.D. Burkert, L. Elouadrhiri, et al., Nuclear Inst. and Methods in Physics Research, A 959 (2020) 163419]

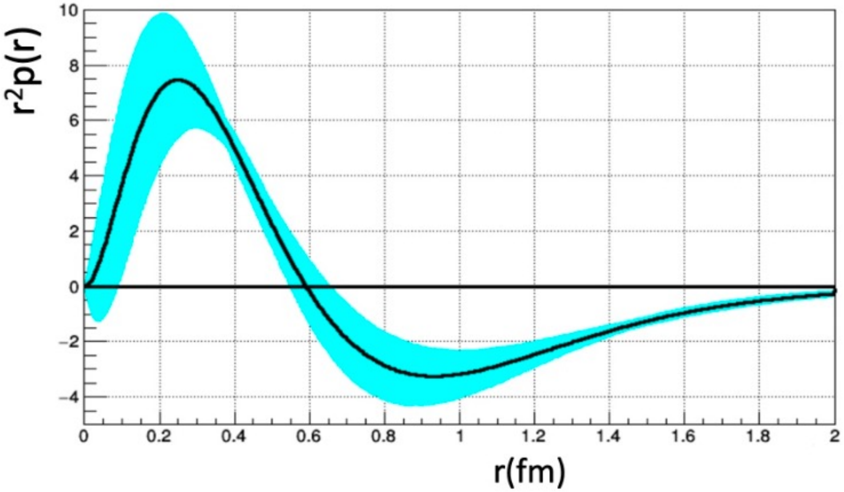




G. Christiaens to be submitted to PRL

Gravitational Structure of the proton with JLab 22GeV





Summary and Outlook

- A new perspective on experimental exclusive reaction physics
- **First determination of the proton Gravitational Form Factor $D^Q(t)$**
- Determination of the last unknown global property of the proton *D*.
- Opens a new avenue to test Confinement Mechanism
- Access the Partonic Energy Momentum Tensor
- New CLAS12 DVCS data double the t-range
- Exciting times at with the 12 GeV high precision era
- Program essential part of the Jefferson Lab 22 GeV, Jefferson Lab science program with positron beam and EIC program as well